Audio Source Separation

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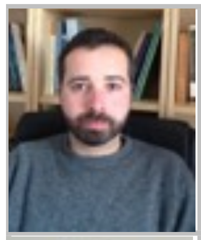	Alexandros Tsilfidis, PhD Founder & CEO	·Electrical and Computer Engineer ·MPhil and PhD in digital audio
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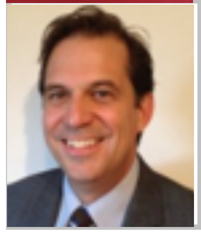	Michael Tzannes, PhD Board member and advisor	·Founder and ex-CEO of Aware Inc (Nasdaq: AWRE) ·Founder and Executive Manager at Tzannes Patent management LLC

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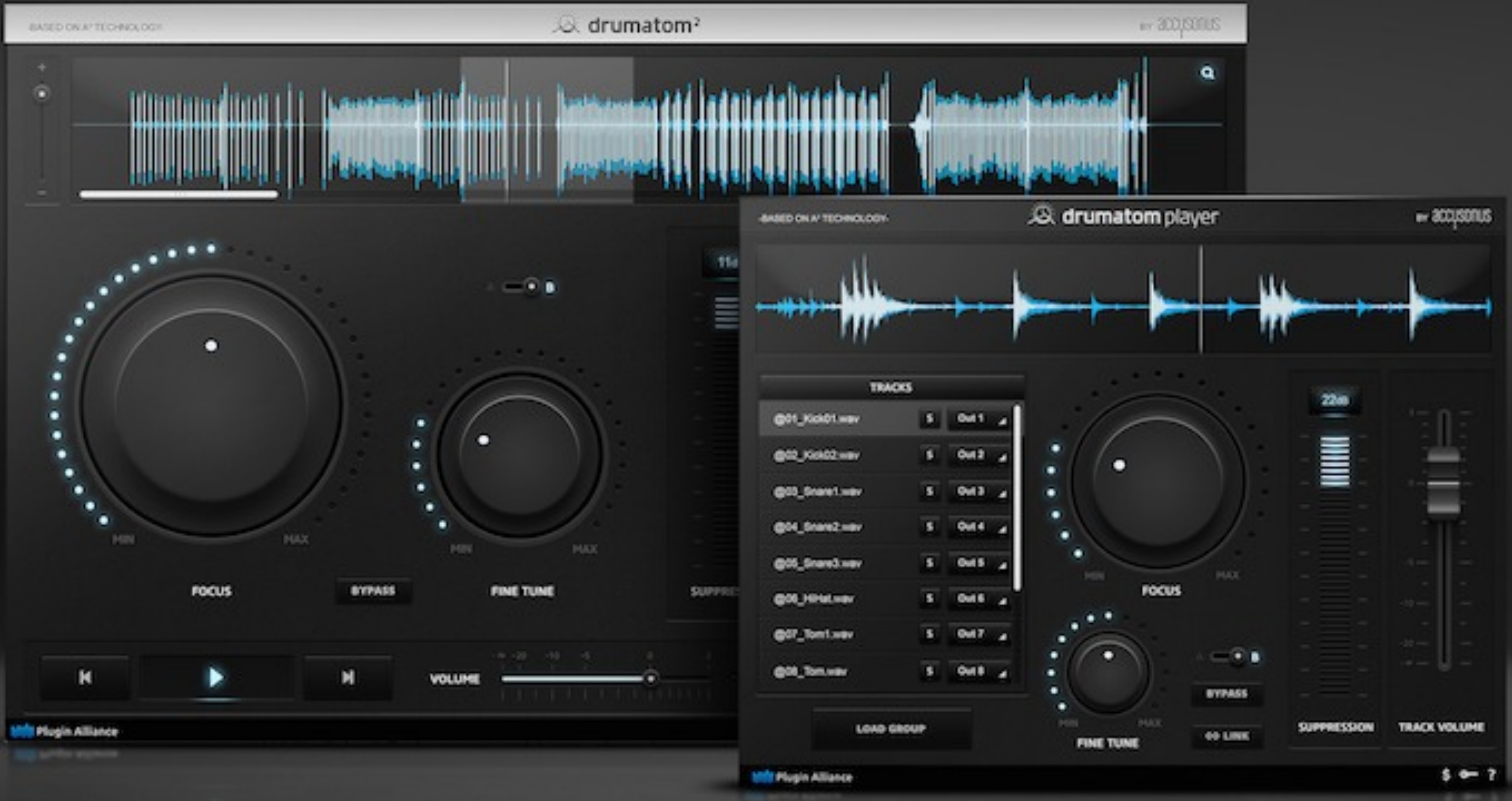


drumatom²
drumatom player
TRACKS
@01_Kick01.wav
@02_Kick02.wav
@03_Snare1.wav
@04_Snare2.wav
@05_Snare3.wav
@06_HiHat.wav
@07_Tom1.wav
@08_Tom.wav
LOAD GROUP
FOCUS
FINE TUNE
BYPASS
LINK
SUPPRESSION
TRACK VOLUME
VOLUME
Plugin Alliance

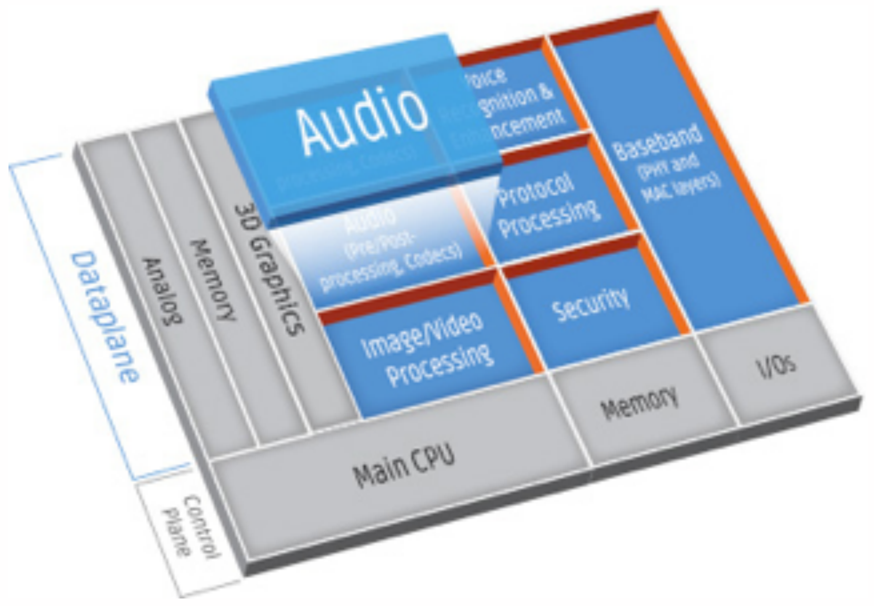
Focus Voice Engine

Focus-BDR/Focus-MDR

- Patent-pending
- The first dereverberation product in the market
- The first dual channel dereverberation in the market
- Combines state-of-the-art dereverberation algorithm with room acoustics

Focus-DNR

- State-of-the art denoise algorithm reengineered for tight integration with dereverberation



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https://github.com/EliasKokkinis/audio-source-separation

Audio Source Separation

Part I - Why do we need it?

Speech

- Can you understand what is being said?

Music

- Can you focus on the piano?
- Can you focus on the guitar?

Man vs. Machine

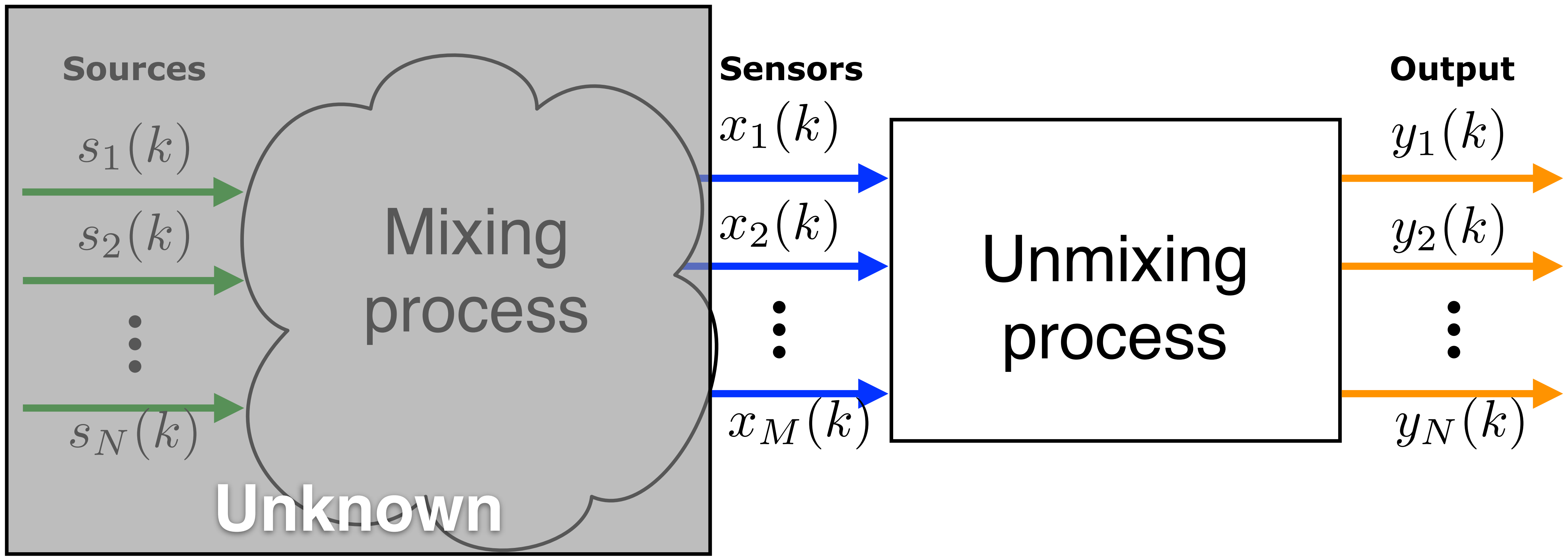
- Would a computer be able to do it?
- Probably **no!**
- We need audio source separation to *help computers understand* auditory events
- Applications:
  - speech enhancement and ASR performance improvement
  - music information retrieval applications (MIR)
  - better audio in studio and live (**that's what we do!**)
  - and many more...

Audio Source Separation

Part II - Blind source separation

Problem formulation

- Consider **N** sources and **M** sensors (microphones)



Problem formulation

- Consider **N** sources and **M** sensors (microphones)
- For each time instant

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$

- **A** is the MxN **mixing matrix**
  - instantaneous mixing
  - three distinct cases:
    - overdetermined (M > N)
    - determined (M = N)
    - underdetermined (M < N). when M = 1 things get really tough...

Problem formulation

- Consider **N** sources and **M** sensors (microphones)
- For each time instant

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$

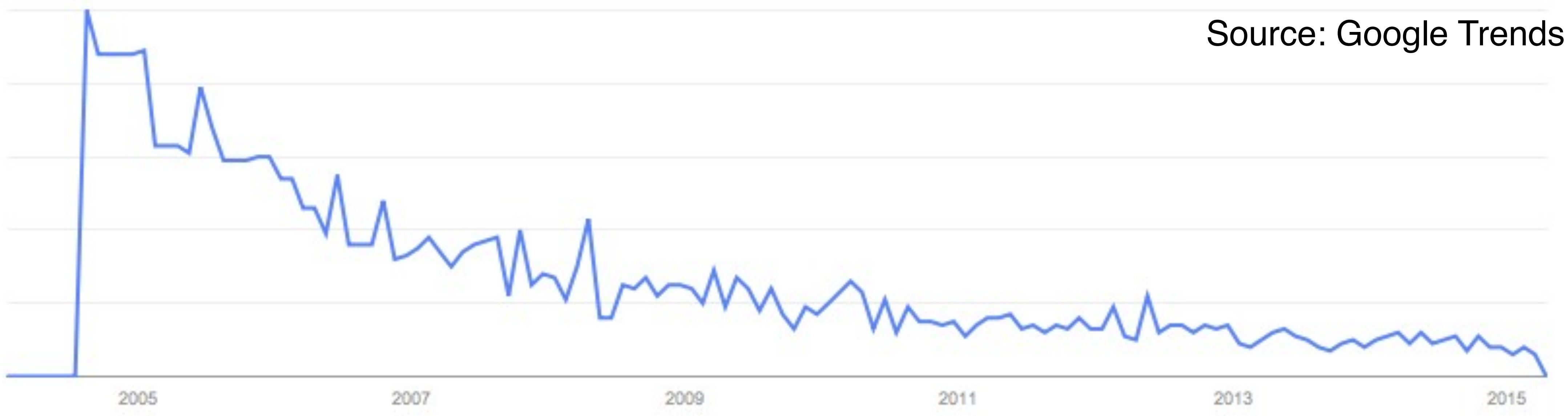
- Blind source separation
  - estimate an **unmixing matrix** **W** such that

$$\hat{\mathbf{s}}(t) = \mathbf{W}\mathbf{x}(t)$$

  - with **only** **x**(t)
  - we **don't** know anything about **A** or **s**(t)

Problem formulation

- The problem **cannot** be solved completely blind
- We have to make some **assumptions** about the source signals
- BSS was first introduced in the 1980s
- A very active research field for more than 20 years



Source: Google Trends

ICA

- Main assumption: **statistical independence (SI)**
  - the most widely used assumption
  - leads to Independent Component Analysis (ICA)
- Intuition:
  - knowing something about one signal does not give you any information about another signal
  - this is violated when signals are mixed

ICA

- ICA methods estimate an unmixing matrix so that the estimated signals are as SI as possible
- *How can we measure statistical independence?*
  - non-Gaussianity
  - mixed SI signals tend towards Gaussian (via CLT)
- *How can we measure non-Gaussianity?*
  - kurtosis: how “spiky” is a pdf
    - 4th order statistic, sensitive to outliers
  - negentropy: difference between the entropy of Gaussian and a given distribution
    - difficult to calculate

FastICA

- We want to maximize a measure of non-Gaussianity

$$\mathcal{J}_G = \sum_{i=1}^{N} \mathrm{E}\{G(\mathbf{w}_i^T \mathbf{x})\}$$

- G is a non-linear function
  - this function defines the measure that is maximized
- FastICA: an iterative method to maximize this cost function

Hyvärinen, A.; Oja, E. (2000). "Independent component analysis: Algorithms and applications". *Neural Networks* **13** (4–5): 411–430

FastICA

- The update for FastICA is

$$\mathbf{w}_i = \mathrm{E}\{\mathbf{x}g(\mathbf{w}_i^T\mathbf{x})\} - \mathrm{E}\{g'(\mathbf{w}_i^T\mathbf{x})\}\mathbf{w}_i$$

  - g() is the derivative of G()
  - g'() is the derivative of g()

- For kurtosis maximization

$$g(y) = y^3 \qquad g'(y) = 3y^2$$

- For negentropy maximization

$$g(y) = \tanh(y) \qquad g'(y) = 1 - (\tanh(y))^2$$

FastICA

- Implementation issues
  - Orthonormalization of W after each iteration

$$\bar{\mathbf{W}} = \mathbf{W}\left(\mathbf{W}^T\mathbf{W}\right)^{-\frac{1}{2}}$$

  - Preprocessing
    - Center the data (remove the mean)
    - Whiten the data: uncorrelated data with unity variance (use eigenvalues)

Ambiguities

- Ideally

$$\mathbf{W}\mathbf{A} = \mathbf{I}$$

- BSS suffers from a set of indeterminancies
- Formally this is stated as

$$\mathbf{W}\mathbf{A} = \mathbf{P}\mathbf{\Lambda}\mathbf{I}$$

  - permutation matrix
  - scaling matrix (diagonal)
  - identity matrix

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- Permuted and scaled signals are still SI!

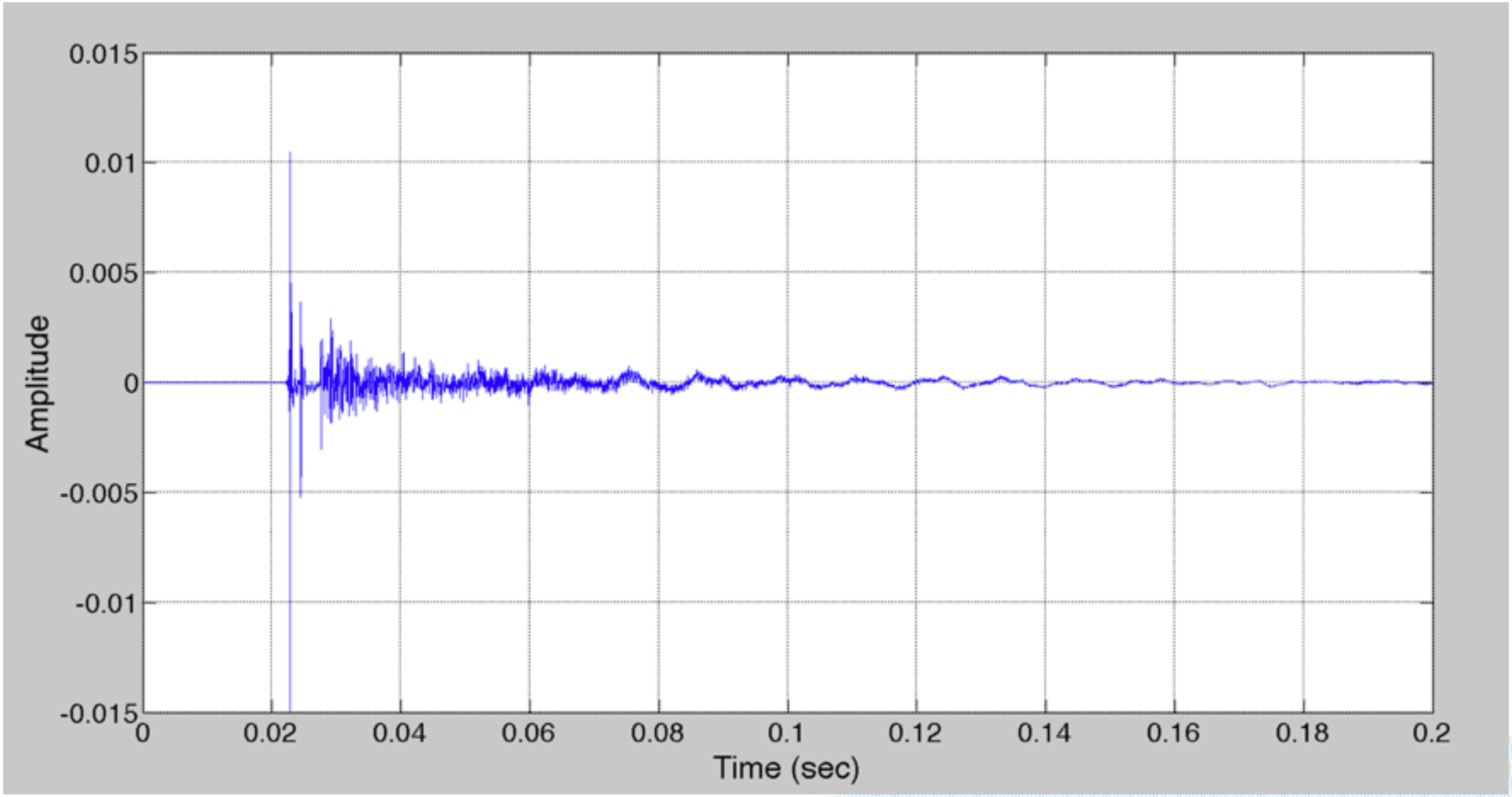
Implementation

Convolutive mixing

- Instantaneous mixing does not actually occur!
  - but it is very useful of biomedical applications
- Audio sources are typically inside rooms!
  - The mixing matrix becomes a set of filters
  - The number of parameters to estimate increases drastically!
- Convolutive BSS: a much harder problem!

Convolutive mixing

- The room as a filter

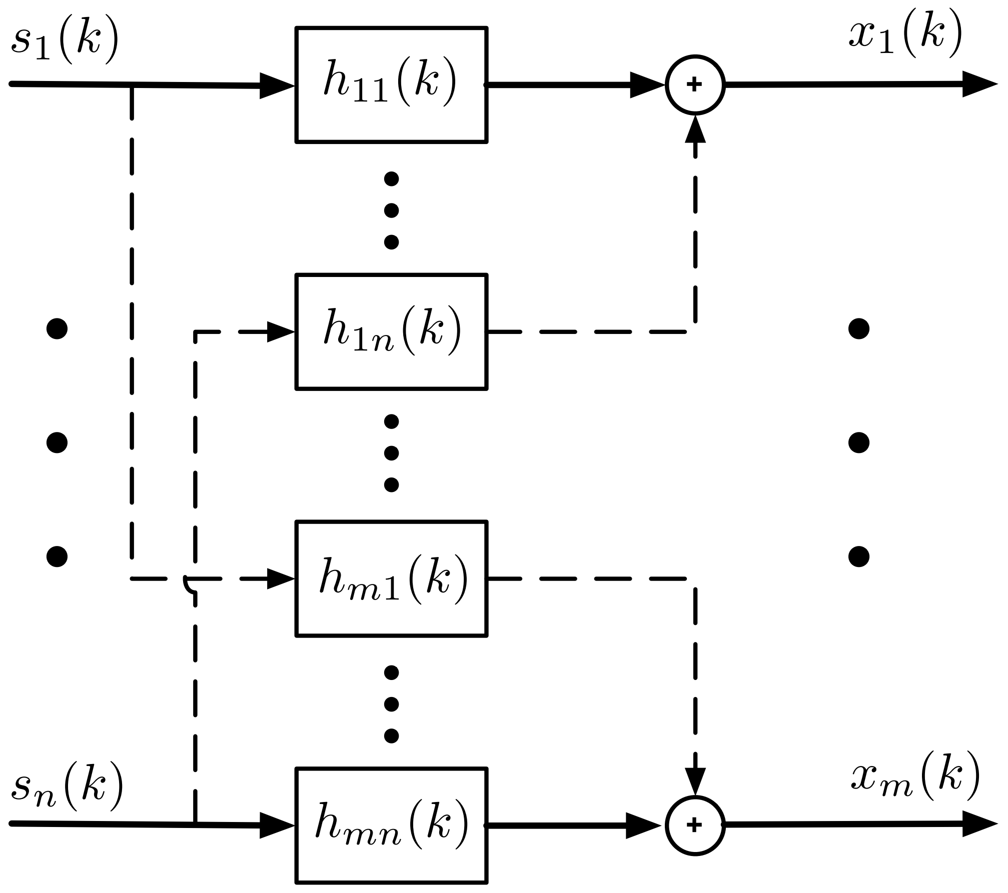


Convolutive mixing

- The room as a filter
  - Modeled as an FIR filter
  - The effect on a signal: convolution
  - The number of coefficients is related to fs and RT60
  - The room impulse response (RIR) characterizes a room
    - for the specific source-microphone position!
  - RIRs are non-minimum phase
    - their inversion is not straightforward!

Convolutive mixing

- Convolutive mixing
  - A block diagram



  - or in equation form

$$x_i(t) = \sum_j \sum_k a_{ij}(t) s_j(t-k)$$

Freq. domain ICA

- An elegant solution: transform to frequency domain
  - convolution becomes a multiplication

$$\mathbf{X}(\omega) = \mathbf{A}(\omega)\mathbf{S}(\omega)$$

- Solve an instantaneous ICA problem for each bin

Freq. domain ICA

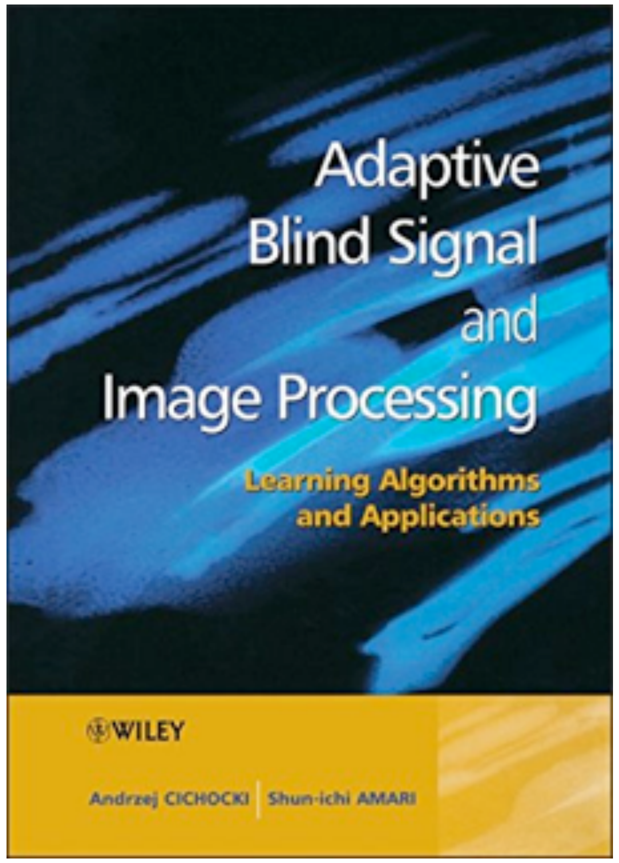
- But: ambiguities need to be resolved!
  - the **permutation** problem is quite complex but can be solved
  - **scaling** ambiguities introduce spectral distortions
- Further issues:
  - the length of the FFT has to be greater than the RIR length for the multiplication assumption to hold (due to **circular convolution**)
  - however a long FFT increases the number of parameters to adjust and **slows convergence**
  - also for longer windows the SI assumption **collapses**

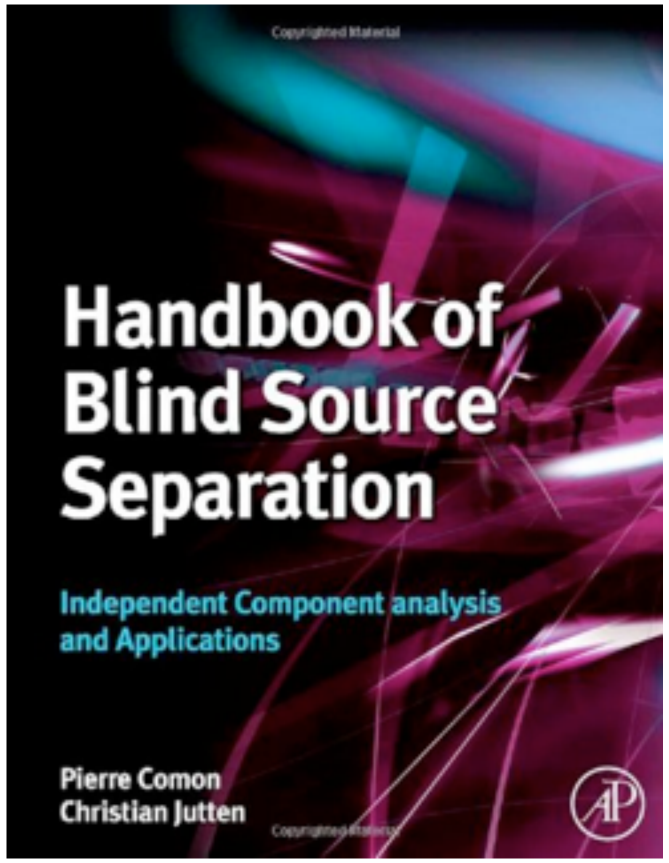
Convolutive BSS

- Convolutive BSS methods are quite involved
  - beyond the scope of this workshop
- They suffer from significant limitations in realistic audio applications

References

- Two very good books


Adaptive Blind Signal and Image Processing
Learning Algorithms and Applications
WILEY
Andrzej CICHOCKI | Shun-ichi AMARI


Handbook of Blind Source Separation
Independent Component analysis and Applications
Pierre Comon
Christian Jutten

- Review chapter *"Convolutive Blind Source Separation Methods"* by M. Pedersen et al. in "Handbook of Speech Processing" (Springer)

Audio Source Separation

Part III - Non-negative matrix factorization

Definition

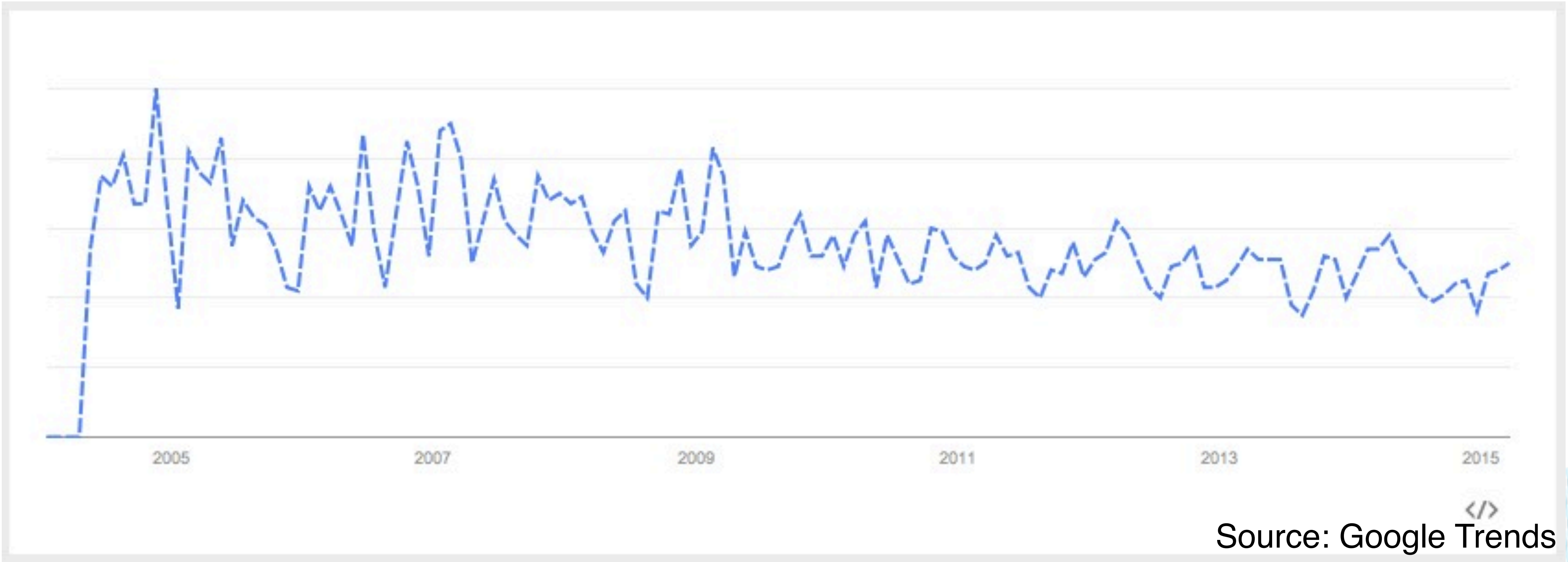
- Non-negative matrix factorization (NMF)
  - Given a non-negative matrix **V**,
  - find two matrices **W** and **H** such that

$$\mathbf{V} \approx \mathbf{W}\mathbf{H}$$

$$\mathbb{R}_+ : \{x \in \mathbb{R}, x \geq 0\}$$
$$\mathbf{V} \in \mathbb{R}_+^{F \times N}, \mathbf{W} \in \mathbb{R}_+^{F \times K}, \mathbf{H} \in \mathbb{R}_+^{K \times N}$$

A bit of history

- First introduced by Paatero and Tapper (1994)
- Became widely known with Lee and Seung (1999)
- Google Scholar reports more than 132.000 results
- A very strong research field even after 15 years.



Source: Google Trends

The matrices (1)

- **V** of size FxN is a collection of column vectors $\mathbf{v}_n$
  - each vector is a **data point**
  - each vector is F-dimensional (or it has F features)
  - there are N data points in total

$$\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \cdots \mathbf{v}_N]$$

$$\mathbf{v}_n = \begin{bmatrix} v_{1n} \\ v_{2n} \\ \vdots \\ v_{Fn} \end{bmatrix}$$

The matrices (2)

- **W** is the basis matrix of size FxK
- **W** has K columns of size F
- The columns of **W** span a linear space

The matrices (3)

- **H** is the weight matrix of KxN
- **H** has N columns of size K
  - The n-th column of **H** represents the coordinates of the n-th data point in the space defined by **W**

$$\mathbf{v}_n \approx \mathbf{W}\mathbf{h}_n$$

Dimensions

- K is the **rank** of the factorization
  - Typically K is chosen such that K ≪ FN
  - This leads to a dimensionality reduction in the data
  - For K > F, matrix **W** becomes overcomplete.
  - Overcomplete basis matrices are related to sparse signal processing.
  - K is the number of “building blocks”

Terminology & Notation

- A note on terminology:
  - **W** is called basis matrix
    - The columns of W are called basis or *atoms*
    - **W** is also called a dictionary (in spase signal processing)
  - **H** is called the weight or *gain* matrix
  - K is the *rank* of the factorization (also called number of components)

NMF discovers parts!

- **Non-negativity** leads to **parts-based** decomposition
  - The “building blocks” of **W** must be combined **constructively**
  - The algorithm is forced to discover the parts that make up **V**

Applications

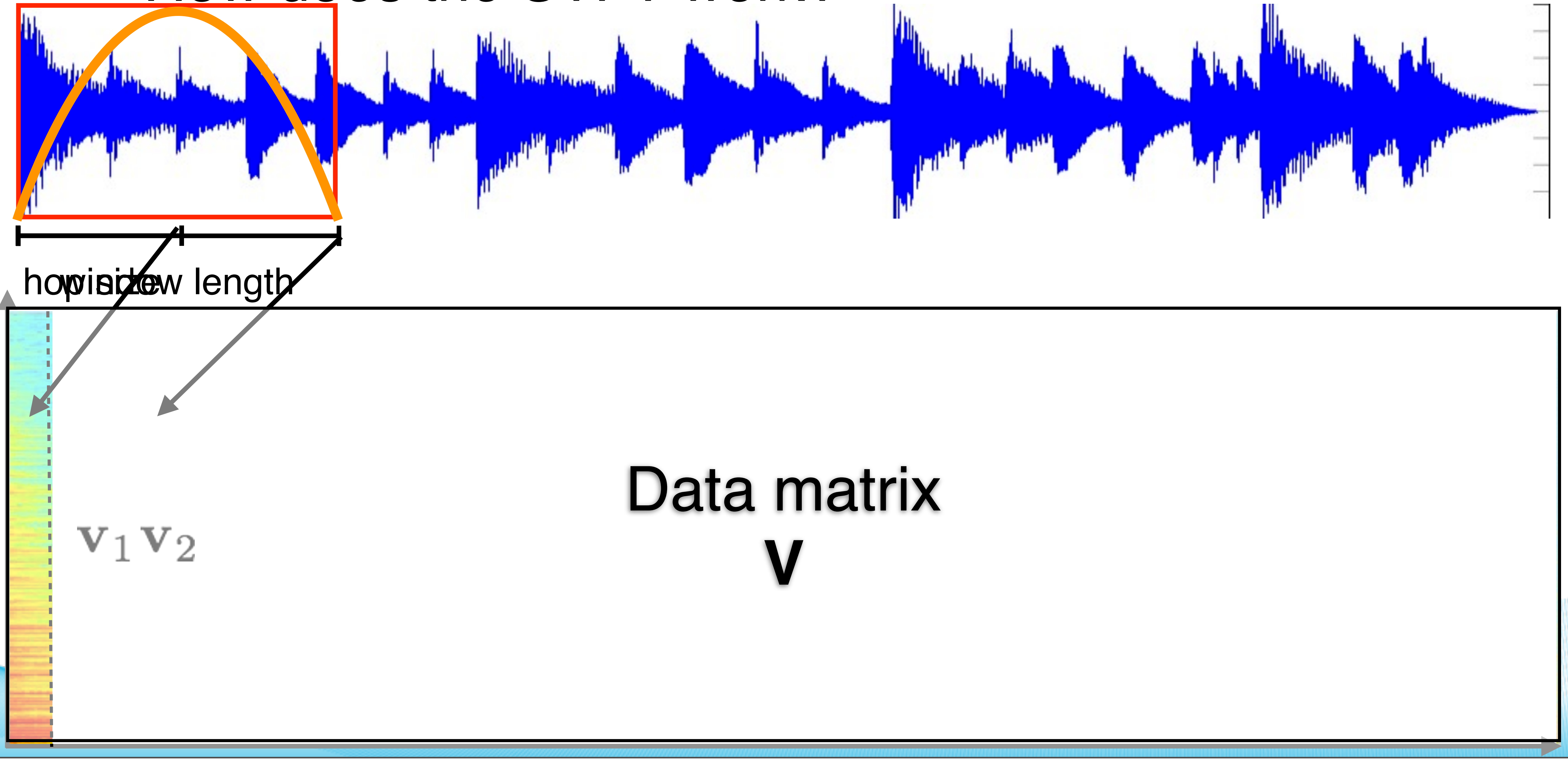
- Some applications of NMF
  - image processing
  - text mining
- We are concerned with audio applications.
- *How can we interpret **V, W** and **H** in audio?*

NMF in audio context

- **V** represents the audio spectrogram
  - F is the number of frequency bins
  - N is the number of audio frames
- **W** consists of *spectral profiles*
  - i.e. the magnitude spectra of elementary sources in **V**
- **H** represents the *gains or activation functions* of the spectral profiles
  - i.e. in a given frame which elementary sources are active

The spectrogram

- The spectrogram is the magnitude of the STFT
- How does the STFT work?



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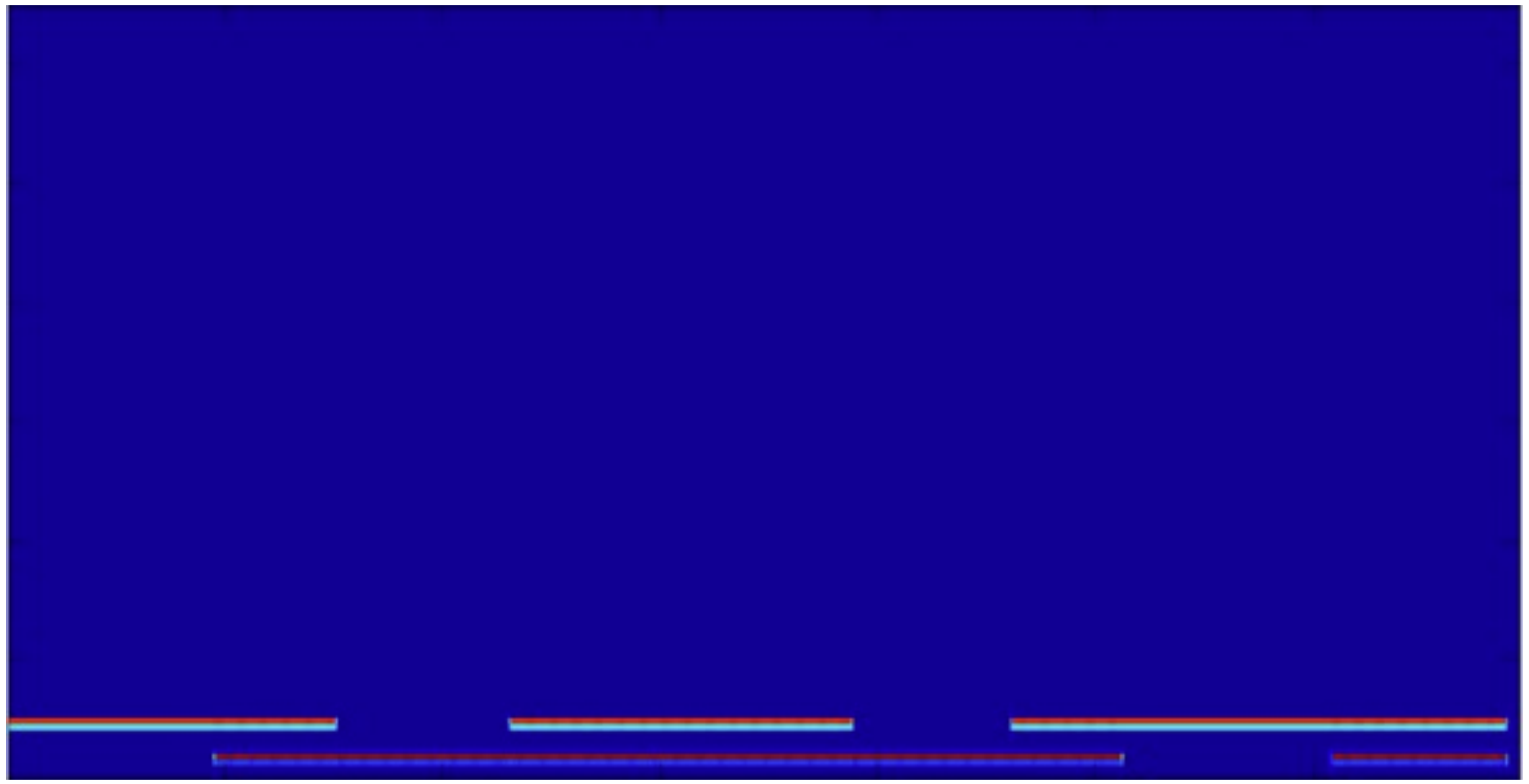
Toy example

- Two sine waves that come and go...



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NMF algorithms

- NMF can be expressed as an optimization problem

$$\min_{\mathbf{W}\geq 0, \mathbf{H}\geq 0} \mathcal{D}\left(\mathbf{V}, \mathbf{WH}\right)$$

- The distance between the matrices is calculated element-wise

$$\mathcal{D}\left(\mathbf{V}, \mathbf{WH}\right) = \sum_f \sum_n \underbrace{\boxed{d(v_{fn}, w_{fk}h_{kn})}}_{\text{scalar distance}}$$

- Only **V** is known.
  - NMF performs **unsupervised** learning

NMF algorithms

- Which distance to choose?
  - The most straightforward: Euclidean **distance**

$$d_{EUC}(x|y) = \frac{1}{2}(x-y)^2$$

  - (Generalized) Kullback-Leibler **divergence**

$$d_{KL}(x|y) = x \log \frac{x}{y} - x + y$$

  - Itakura-Saito **divergence**

$$\boxed{d_{IS}(x|y) = \frac{x}{y} - \log \frac{x}{y} - 1}$$

Itakura-Saito distance

- Proposed by Itakura and Saito in 1968
- Measures the difference between two **spectra**
- Reflects **perceptual similarity** between two spectra
  - used as a speech enhancement performance metric
- It is **scale invariant**
  - low and high energy data are treated the same

$$d_{IS}(x|y) = d_{IS}(\alpha x|\alpha y)$$

NMF algorithms

- A statistical insight behind the choice of distance
  - Euclidean distance: ML estimation of W, H in AGN

$$\mathbf{V} = \mathbf{W}\mathbf{H} + \mathbf{E}$$

  - KL divergence: ML estimation of W, H in Poisson noise
  - IS divergence: ML estimation of W, H in Gamma multiplicative noise

$$\mathbf{V} = \mathbf{W}\mathbf{H} \odot \mathbf{E}$$

NMF algorithms

- All the previous measures belong to the family of **beta divergences**:

$$d_\beta(x|y) = \begin{cases} \frac{x}{y} - \log\frac{x}{y} - 1 & \beta = 0 \\ x\log\frac{x}{y} + (y - x) & \beta = 1 \\ \frac{1}{\beta(\beta-1)}\left(x^\beta + (\beta-1)y^\beta - \beta x y^{\beta-1}\right) & \beta \in \mathbb{R}\backslash\{0,1\} \end{cases}$$

NMF algorithms

- How can we **solve** the optimization problem?

$$\min_{\mathbf{W}\geq 0, \mathbf{H}\geq 0} \mathcal{D}\left(\mathbf{V}, \mathbf{WH}\right)$$

- It cannot be solved for **both** **W** and **H.**
    - Keep **W** **fixed** and optimize **H** and vice versa
- Common minimization approach: **gradient descent**

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta \nabla \mathcal{J}(\boldsymbol{\theta})$$

- batch approach - all data are considered
- calculate the gradient of the distance w.r.t. W and H
- rearrange terms in order to remove negative terms
- choose the appropriate step size

NMF algorithms

- Let's do this for **W**
  - The derivative of the beta divergence

$$d_\beta(x|y) = y^{\beta-2}(y-x)$$

  - The gradient with respect to **W**

$$\nabla_W D_\beta(\mathbf{V}|\mathbf{WH}) = \left(\mathbf{WH}^{\beta-2} \odot (\mathbf{WH}-\mathbf{V})\right)\mathbf{H}^T$$

  - Rearrange

$$\nabla_W D_\beta(\mathbf{V}|\mathbf{WH}) = \mathbf{WH}^{\beta-1}\mathbf{H}^T - \left(\mathbf{WH}^{\beta-2} \odot \mathbf{V}\right)\mathbf{H}^T$$

$$\nabla^+_W D_\beta(\mathbf{V}|\mathbf{WH}) \qquad \nabla^-_W D_\beta(\mathbf{V}|\mathbf{WH})$$

NMF algorithms

- Let’s do this for **W**
  - The gradient descent method

$$\mathbf{W}^{i+1} \leftarrow \mathbf{W}^{i} - \eta \left(\nabla_W D_\beta(\mathbf{V}|\mathbf{WH}) \right)$$

$$\mathbf{W}^{i+1} \leftarrow \mathbf{W}^{i} - \eta \left(\nabla^+_W D_\beta(\mathbf{V}|\mathbf{WH}) - \nabla^-_W D_\beta(\mathbf{V}|\mathbf{WH}) \right)$$

  - Choose the step size as

$$\eta = \frac{\mathbf{W}^i}{\nabla^+_W D_\beta(\mathbf{V}|\mathbf{WH})}$$

  - Substitute

$$\mathbf{W}^{i+1} \leftarrow \mathbf{W}^{i} \frac{\nabla^-_W D_\beta(\mathbf{V}|\mathbf{WH})}{\nabla^+_W D_\beta(\mathbf{V}|\mathbf{WH})}$$

NMF algorithms

- NMF multiplicative update rules for beta divergence

$$\mathbf{W} \leftarrow \mathbf{W}\frac{\left((\mathbf{WH})^{\beta-2}\odot\mathbf{V}\right)\mathbf{H}^T}{(\mathbf{WH})^{\beta-1}\mathbf{H}^T}$$

$$\mathbf{H} \leftarrow \mathbf{H}\frac{\mathbf{W}^T\left((\mathbf{WH})^{\beta-2}\odot\mathbf{V}\right)}{\mathbf{W}^T(\mathbf{WH})^{\beta-1}}$$

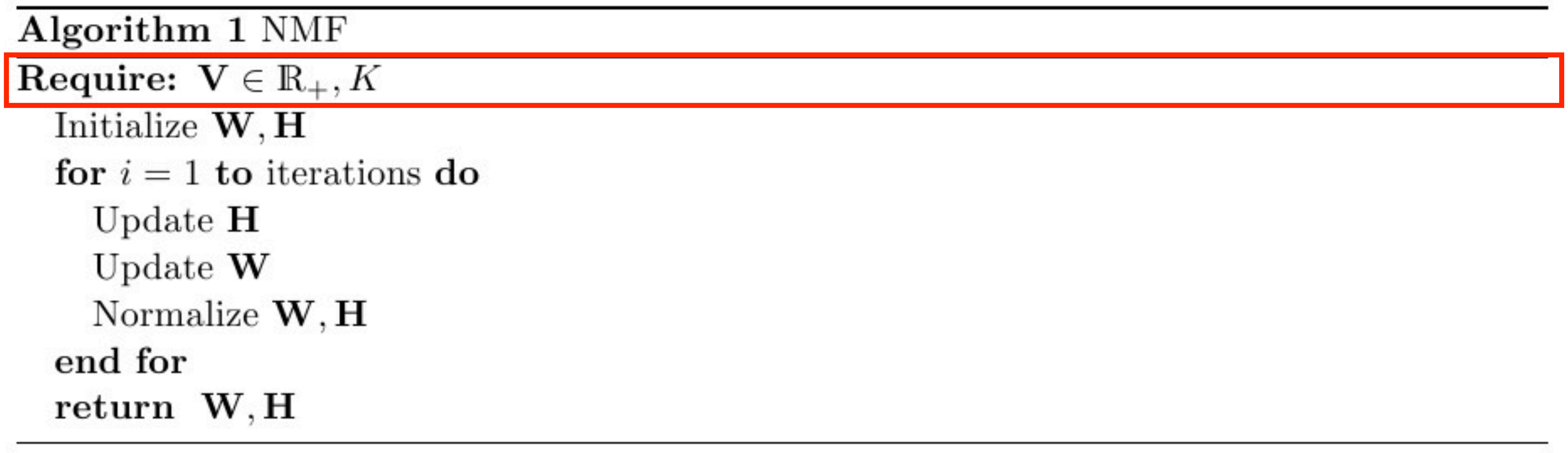
- Why multiplicative?
  - non-negativity is preserved
  - easy to implement

NMF algorithms

- The MU updates are proved to **converge**
    - Lee & Seung’s classic paper
- The NMF problem is **not convex for both** **W** and **H**
    - it is convex for W or H
    - It is **not guaranteed** that a global minimum will be found

NMF algorithms

- The basic NMF algorithm



```
Algorithm 1 NMF
Require: $\mathbf{V} \in \mathbb{R}_+, K$
  Initialize $\mathbf{W}, \mathbf{H}$
  for $i = 1$ to iterations do
    Update $\mathbf{H}$
    Update $\mathbf{W}$
    Normalize $\mathbf{W}, \mathbf{H}$
  end for
  return $\mathbf{W}, \mathbf{H}$
```

How to start?

- Which are the initial values of **W** and **H**?
  - the most common approach: **random non-negative values!**
    - the specific random value distribution may affect results
  - NMF is generally **sensitive** to initial values
  - more involved initialization strategies have been proposed
  - initialization **depends** on application
    - for audio applications, random is enough

When to stop?

- NMF is calculated iteratively.
  - How **many** iterations do we need?
- Typically you check that the cost function decreases more than ε (tolerance) in each iteration
- You can also set a **maximum** number of iterations.
- For audio applications: trial and error
  - a smaller value of the cost function does not mean better perceptual quality

Other issues

- How do we choose K?
  - model order selection is a difficult problem
  - most approaches are trial and error
  - some approaches based on the eigenvalues of **V**
  - the number of components K is not necessarily equal to the number of sources
- Normalization
  - avoid getting stuck by small values due to multiplicative updates
  - normalize columns of **W**
  - the cost function changes!

NMF algorithms

- Adding constraints: the beauty of NMF
  - incorporate **prior knowledge** about the data
  - easy to extend the core method for specific applications
- We now want to solve

$$\min_{\mathbf{W}\geq 0,\mathbf{H}\geq 0} \mathcal{J}\left(\mathbf{W},\mathbf{H}\right)$$

- where the cost function is

$$\mathcal{J}\left(\mathbf{W},\mathbf{H}\right) = \mathcal{D}_\beta\left(\mathbf{V},\mathbf{WH}\right) + c_1\Phi(\mathbf{W}) + c_2\Psi(\mathbf{H})$$

NMF algorithms

- The multiplicative updates have the same form:

$$\mathbf{W} \leftarrow \mathbf{W} \frac{\nabla^-_W \mathcal{J}(\mathbf{W},\mathbf{H})}{\nabla^+_W \mathcal{J}(\mathbf{W},\mathbf{H})}$$

- where each gradient is rearranged to form

$$\boxed{\nabla^+_W \mathcal{J}(\mathbf{W},\mathbf{H}) = \nabla^+_W \mathcal{D}_\beta(\mathbf{V},\mathbf{WH}) + c_1 \nabla^+_W \Phi(\mathbf{W}) + \cancel{c_2 \nabla^+_W \Psi(\mathbf{H})}}$$

$$\boxed{\nabla^-_W \mathcal{J}(\mathbf{W},\mathbf{H}) = \nabla^-_W \mathcal{D}_\beta(\mathbf{V},\mathbf{WH}) + c_1 \nabla^-_W \Phi(\mathbf{W}) + \cancel{c_2 \nabla^-_W \Psi(\mathbf{H})}}$$

- Of course the same holds for **H**

NMF algorithms

- The most common constraint: **sparsity!**
  - each data point is a combination of **some** sources
- How is sparsity measured?
  - There are several ways which can be summarized as

$$\Psi(\mathbf{H}) = \sum_{k,n} f(h_{kn})$$

  - A very simple and efficient way is to choose $f(x) = x$
  - Since all elements are non-negative it is equivalent to the entrywise l_1 norm

$$\Psi(\mathbf{H}) = ||\mathbf{H}||_1$$

NMF algorithms

- So the cost function now is

$$\mathcal{J}(\mathbf{W},\mathbf{H}) = \mathcal{D}_\beta(\mathbf{V},\mathbf{W}\mathbf{H}) + \lambda\Psi(\mathbf{H})$$

- The gradients of the constraint are

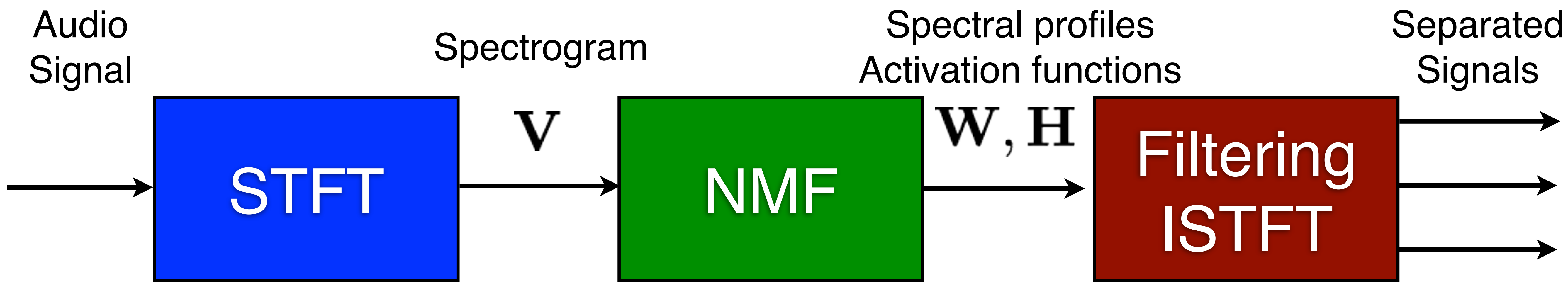
$$\nabla_H^+\Psi(\mathbf{H}) = 1 \qquad \nabla_H^-\Psi(\mathbf{H}) = 0$$

- And the new multiplicative update rules are

$$\mathbf{W} \leftarrow \mathbf{W}\frac{\left((\mathbf{W}\mathbf{H})^{\beta-2}\odot\mathbf{V}\right)\mathbf{H}^T}{(\mathbf{W}\mathbf{H})^{\beta-1}\mathbf{H}^T} \qquad \mathbf{H} \leftarrow \mathbf{H}\frac{\mathbf{W}^T\left(\mathbf{V}\odot(\mathbf{W}\mathbf{H})^{\beta-2}\right)}{\mathbf{W}^T(\mathbf{W}\mathbf{H})^{\beta-1}+\lambda}$$

How to separate using NMF?

- Building blocks of an NMF based separation system



STFT details

- STFT: from time to time-frequency domain
- Why? Audio signals are sparse in this domain
- Time-frequency resolution:
  - trade-off: choose good time OR frequency resolution
  - workaround: decrease hop size
    - this increases the number of data points tremendously
- Other things to consider:
  - analysis window type
  - spectrogram domain (magnitude or power)

Separating with masks

- We know how to get **V**, **W** and **H**.
- How do we get the separated signals?
- Time-frequency masks
  - Recall the Wiener filter

$$H(\omega) = \frac{P_{ss}(\omega)}{P_{ss}(\omega) + P_{nn}(\omega)}$$

  - Pseudo-Wiener masks

$$\mathbf{M}_k = \frac{\mathbf{w}_k \mathbf{h}^k}{\mathbf{W}\mathbf{H}}$$

  - Masks are real. Signal phase is left untouched.

Separating with masks

- Masks are applied on the complex spectrogram **X**
- Each mask produces a new spectrogram
- We perform ISTFT to obtain the separated signal

Implementation

Beyond NMF

- NMF can be trained!
  - Semi-supervised case
  - Supervised case
- Sort components: Make sense of the output data!
- Multichannel extensions: tensor factorizations
- Bayesian formulations
- Probabilistic Latent Component Analysis (PLCA)

References

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Thank you!

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