Advanced Econometrics Project

Mattia Elezi

05/05/2022

Contents

Goldsmiths College, University of London.

Advanced Econometrics.

Introduction

This project relies on various statistical methods to provide empirical content on the economic relationship between the Capacity Utilisation Rate and the Inflation Rate of the USA. Indeed, Inflation is highly correlated with the nominal interest rate as when inflation rises, the central bank raises the nominal interest rate to counter it, thus reducing the growth rate of GDP to maintain inflation at acceptable levels for economic functionality. However, the Federal Reserve ends up reducing both demand and supply in the process and, thus, capacity utilisation decreases. Initially, the project analyses the trend and correlation of the variables and collects evidence on the possible presence of unit roots in the variables in levels and in first differences through the ADF, PP, and KPSS tests. Afterwards, cointegration between the two series is analysed through the Engle-Granger and Johansen Tests. The subsequent statistical method of the project is the ARDL model, testing how Capacity Utilisation is adapting in response to Inflation, but the model is not correctly specified. Therefore, as cointegration test results appear to be ambiguous, and as there is evidence of the absence of unit roots in the variables in first differences, the Bivariate VAR Model in first differences is computed, followed by the Granger-Causality test, Cholesky decompositions for orthogonal errors, Impulse Response Functions (IRFs), and VAR forecast. Finally, a forecast of inflation in the next 10 months is constructed through the ARIMA model, and a within forecast through the SARIMA model is compared to it.

```
library(forecast)
library(tseries)
library(nlme)
library(pdfetch)
library(zoo)
library(urca)
library(vars)
library(car)
library(dynlm)
library(tsDyn)
library(gets)
library(readxl)
library(aod)
library(aTSA)
library(rmarkdown)
library(tinytex)


rm(list=ls())
```

Dataset Properties

The dataset for this project is comprised of TCU and Inflation with 663 values in levels from 01/1967 to 04/2022, obtained from the Federal Reserve Economic Data (FRED).

The Consumer Price Index for All Urban Consumers (CPIAUCSL) is a price index of a basket of goods and services paid by urban consumers. Percent changes in the price index measure the inflation rate between any two time periods. The CPI is used to derive the monthly inflation rate correctly calculated year-on-year and aligned to the properties of the TCU data.

```
CPI = pdfetch_FRED("CPIAUCSL")
names(CPI) = "CPI"

Inflation = diff(log(CPI), lag = 12) * 100
names(Inflation) = "Inflation"

Inflation = ts(Inflation, start=c(1947, 1), frequency=12)
Inflation = na.omit(Inflation)
```

Capacity Utilisation: Total Industry (TCU) is the amount of capacity being used from the total available capacity to produce demanded finished goods and services. It is the percentage of resources used by corporations and factories to produce goods in manufacturing, mining, and electric and gas utilities for all facilities located in the USA.

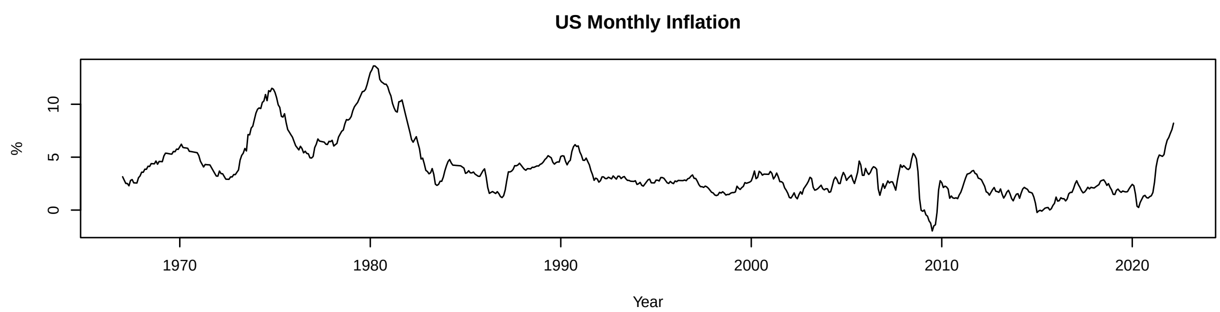
```
TCU = pdfetch_FRED("TCU")
names(TCU) = "TCU"
TCU = ts(TCU, start=c(1967, 1), frequency=12)


data.set = na.omit(
           ts.intersect(

       Inflation,
       TCU,

           dframe=TRUE))


Inflation   = ts( data.set$Inflation,    start=c(1967, 1),  frequency=12)
TCU         = ts( data.set$TCU,          start=c(1967, 1),  frequency=12)


data.set = ts(data.set, start=c(1967, 1), frequency=12)
```

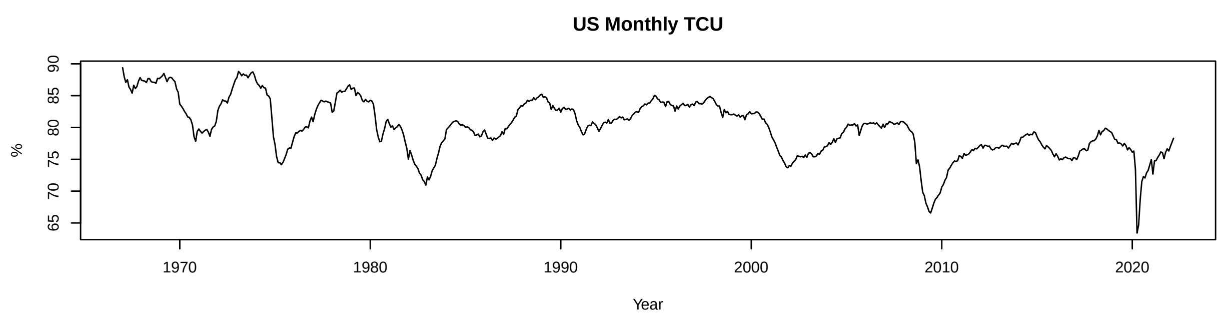
Variables in Levels

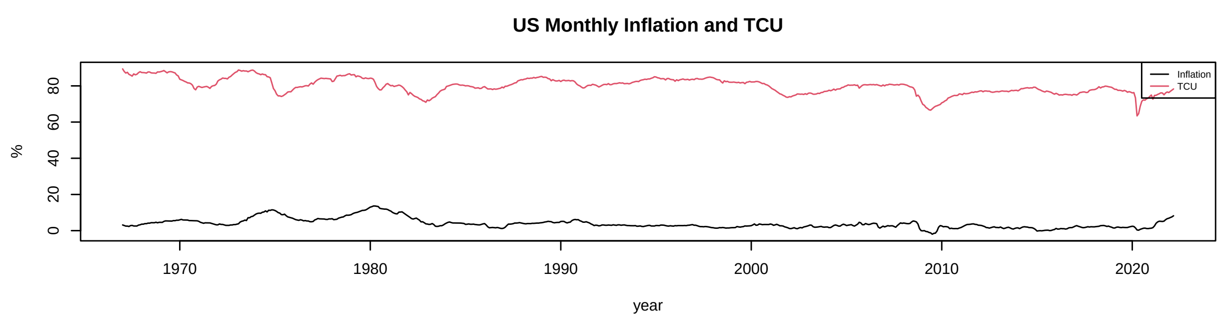
Trend and Correlation Plots in Levels

```
par(mfrow=c(3,1))

plot(Inflation,
main = "US Monthly Inflation",
xlab = "Year",
ylab = "%")
plot(TCU,
```

```
main = "US Monthly TCU",
xlab = "Year",
ylab = "%")
plot(data.set,
main = "US Monthly Inflation and TCU",
xlab = "year",
ylab = "%",
plot.type="single", col = 1:ncol(data.set))
legend("topright", colnames(data.set), col=1:ncol(data.set), lty=1, cex=.65)
```





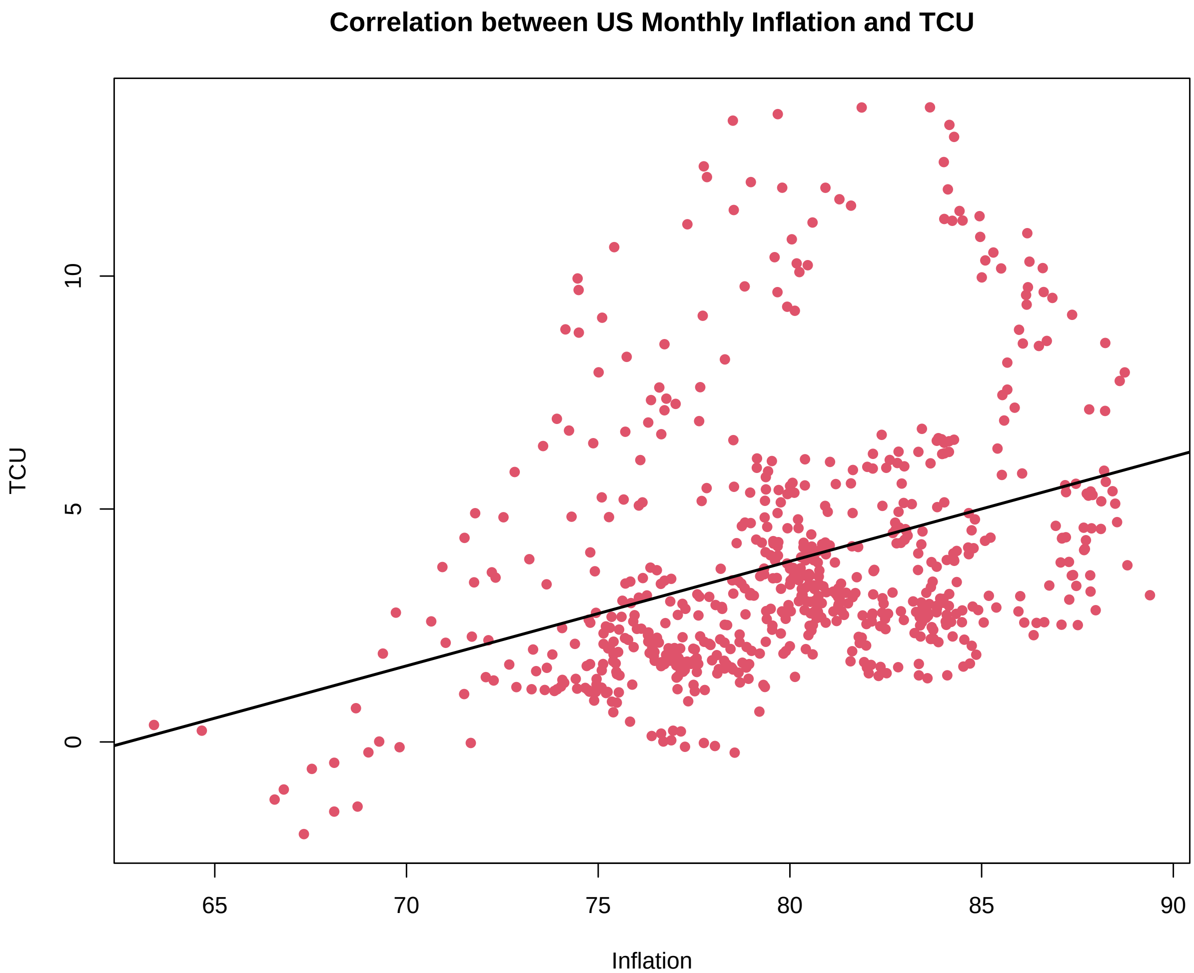


As it can be inferred from the plots, there is a graphical indication of the presence of a unit root in both series in levels as the respective means of the stochastic processes appear to be decreasing and not reverting over time. Furthermore, the plots suggest cointegration as the series seem to follow a similar trend.

```
cor(Inflation, TCU)
```

```
## [1] 0.3605283
```

```
plot(Inflation ~ TCU,
pch = 16, col = 2,
main = "Correlation between US Monthly Inflation and TCU",
xlab = "Inflation",
ylab = "TCU")
lm_Inflation <- lm(Inflation ~ TCU)
abline(coef(lm_Inflation), lwd = 2)
```



Inflation and TCU are positively correlated.

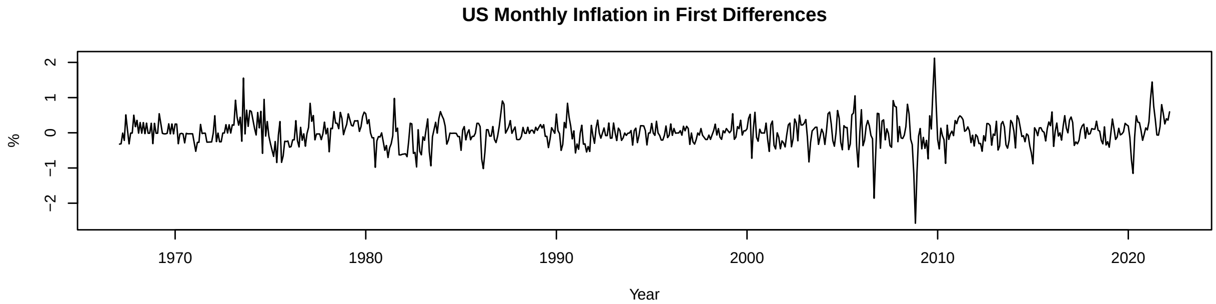
Variables in First Differences

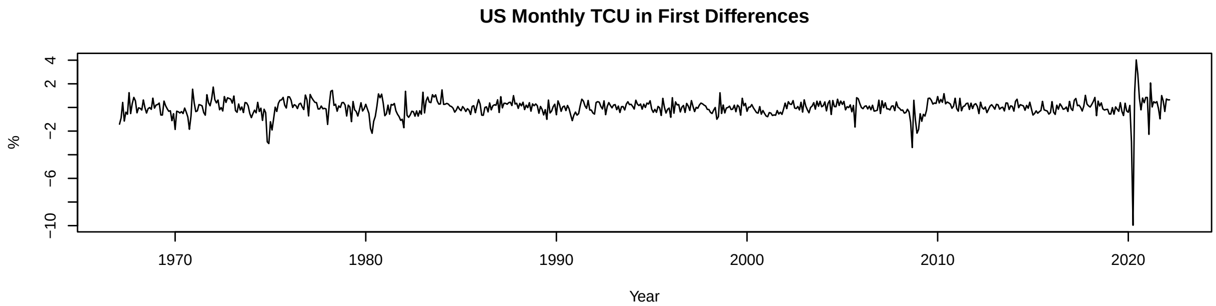
```
Inflation     =  diff(Inflation)
TCU           =  diff(TCU)
```

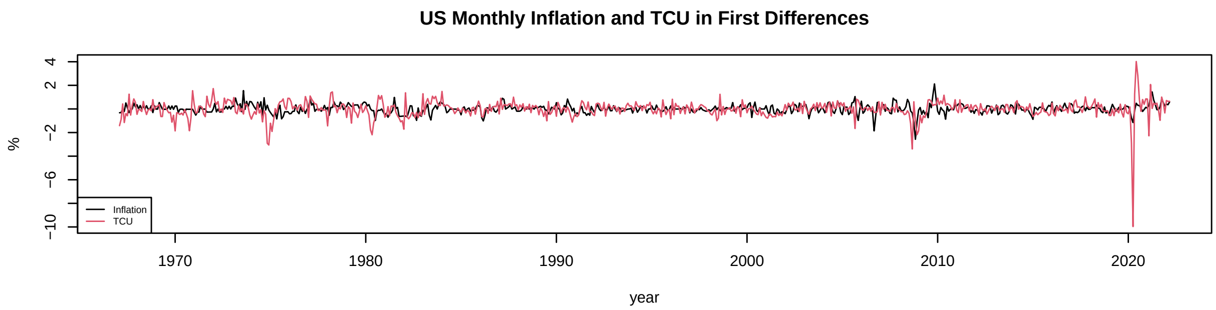
```
data.set = na.omit(
           ts.intersect(

       Inflation,
       TCU,

           dframe=TRUE))


Inflation   = ts( data.set$Inflation,    start=c(1967, 2),  frequency=12)
TCU         = ts( data.set$TCU,          start=c(1967, 2),  frequency=12)

  data.set = ts(data.set, start=c(1967, 2), frequency=12)
```

Trend and Correlation Plots in First Differences

```
par(mfrow=c(3,1))

plot(Inflation,
main = "US Monthly Inflation in First Differences",
xlab = "Year",
ylab = "%")
plot(TCU,
main = "US Monthly TCU in First Differences",
xlab = "Year",
ylab = "%")
plot(data.set,
main = "US Monthly Inflation and TCU in First Differences",
xlab = "year",
ylab = "%",
plot.type="single", col = 1:ncol(data.set))
legend("bottomleft", colnames(data.set), col=1:ncol(data.set), lty=1, cex=.65)
```





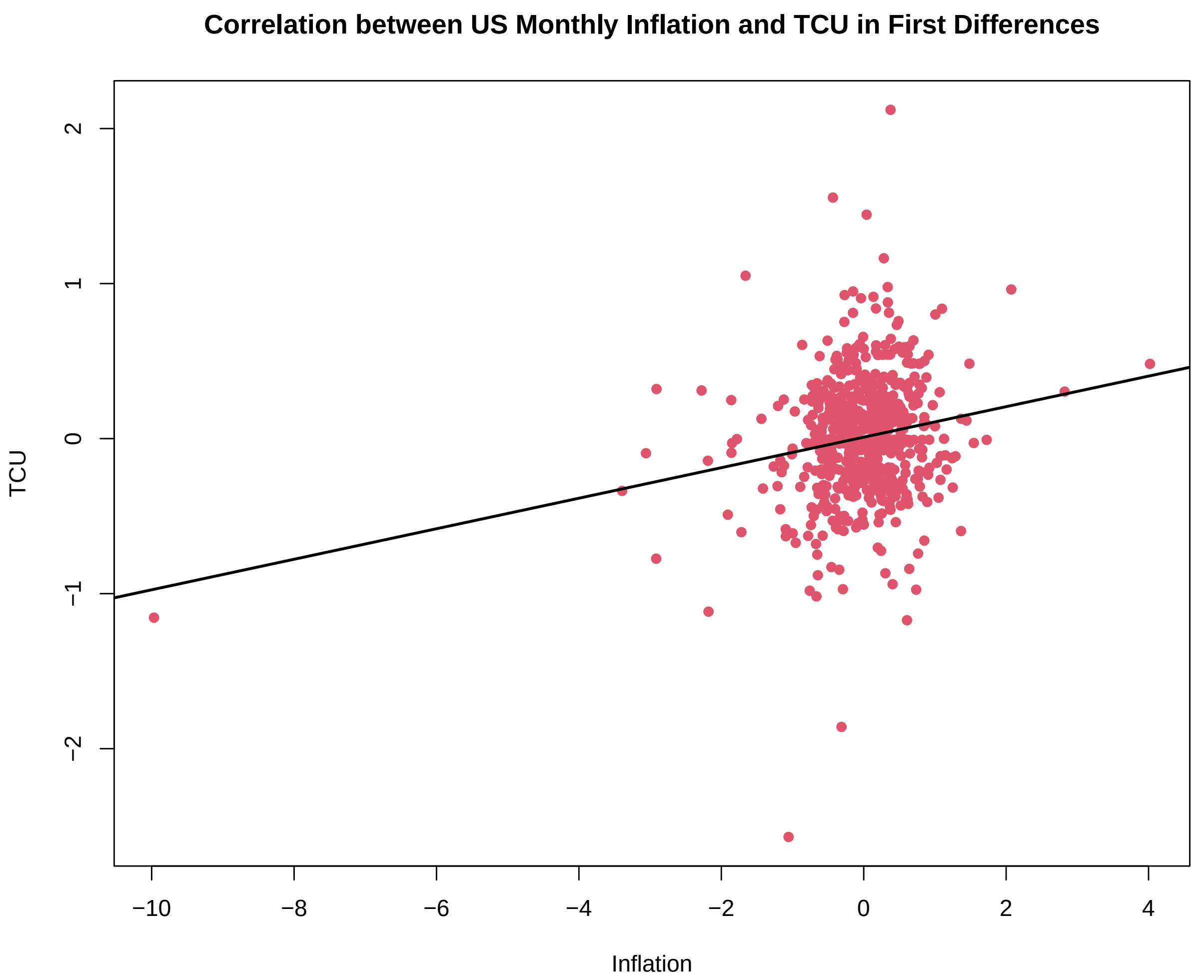


As it can be inferred from the plots, there is a graphical indication of the absence of a unit root in both series in first differences as the respective means of the possibly stationary processes appear to be reverting over time.

```
cor(Inflation, TCU)
```

```
## [1] 0.1909527
```

```
plot(Inflation ~ TCU,
pch = 16, col = 2,
main = "Correlation between US Monthly Inflation and TCU in First Differences",
xlab = "Inflation",
ylab = "TCU")
lm_Inflation <- lm(Inflation ~ TCU)
abline(coef(lm_Inflation), lwd = 2)
```



Correlation is weaker in first differences.

Unit Root Tests

Dataset in Levels for Unit Root Tests

```
CPI = pdfetch_FRED("CPIAUCSL")
names(CPI) = "CPI"

Inflation = diff(log(CPI), lag = 12) * 100
names(Inflation) = "Inflation"

Inflation = ts(Inflation, start=c(1947, 1), frequency=12)
Inflation = na.omit(Inflation)

TCU = pdfetch_FRED("TCU")
names(TCU) = "TCU"
TCU = ts(TCU, start=c(1967, 1), frequency=12)
```

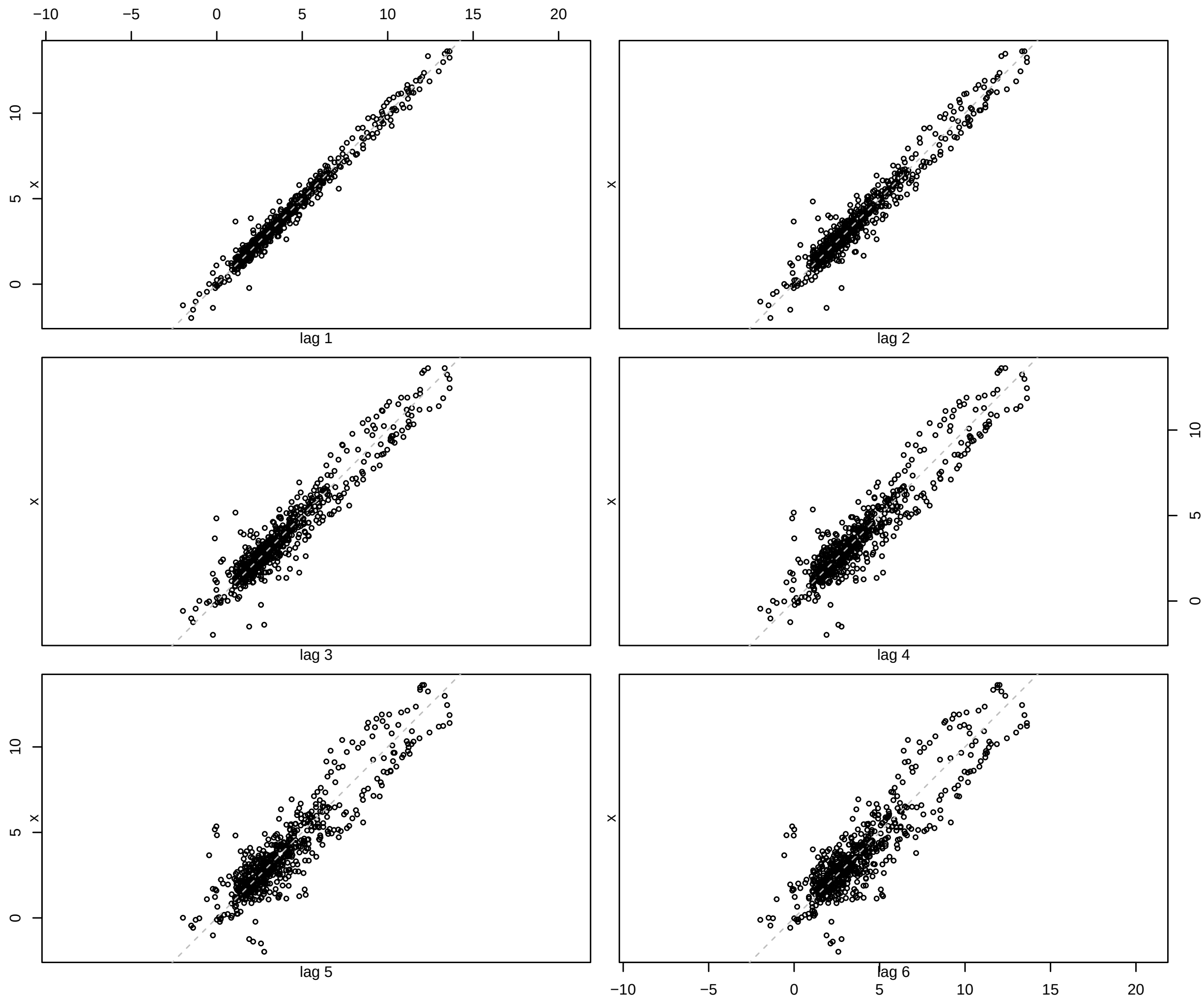
```r
data.set = na.omit(
           ts.intersect(

       Inflation,
       TCU,

           dframe=TRUE))


Inflation   = ts( data.set$Inflation,    start=c(1967, 1),  frequency=12)
TCU         = ts( data.set$TCU,          start=c(1967, 1),  frequency=12)



data.set = ts(data.set, start=c(1967, 1), frequency=12)
```

Plots of the Lag Correlation

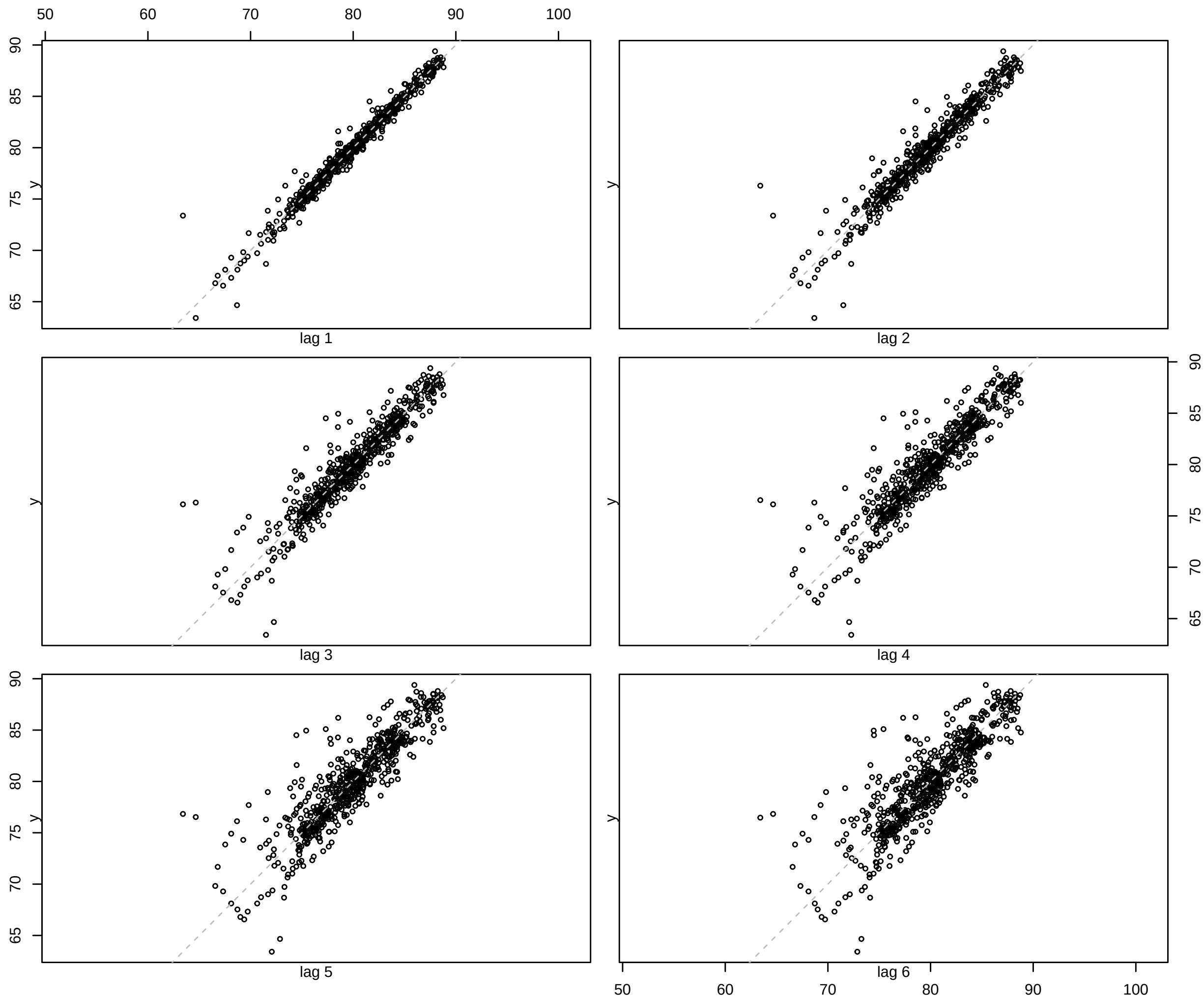
The plot shows the correlation of the variable with itself throughout 6 months. The closer the dots are to the line, the higher the correlation. Correlation over time is autocorrelation which indicates the presence of a unit root.

The correlation of Inflation and TCU in levels is persistent over 6 months. There is a strong suggestion for unit root presence.

```r
x = Inflation

lag.plot(x, 6, do.lines=FALSE)
```

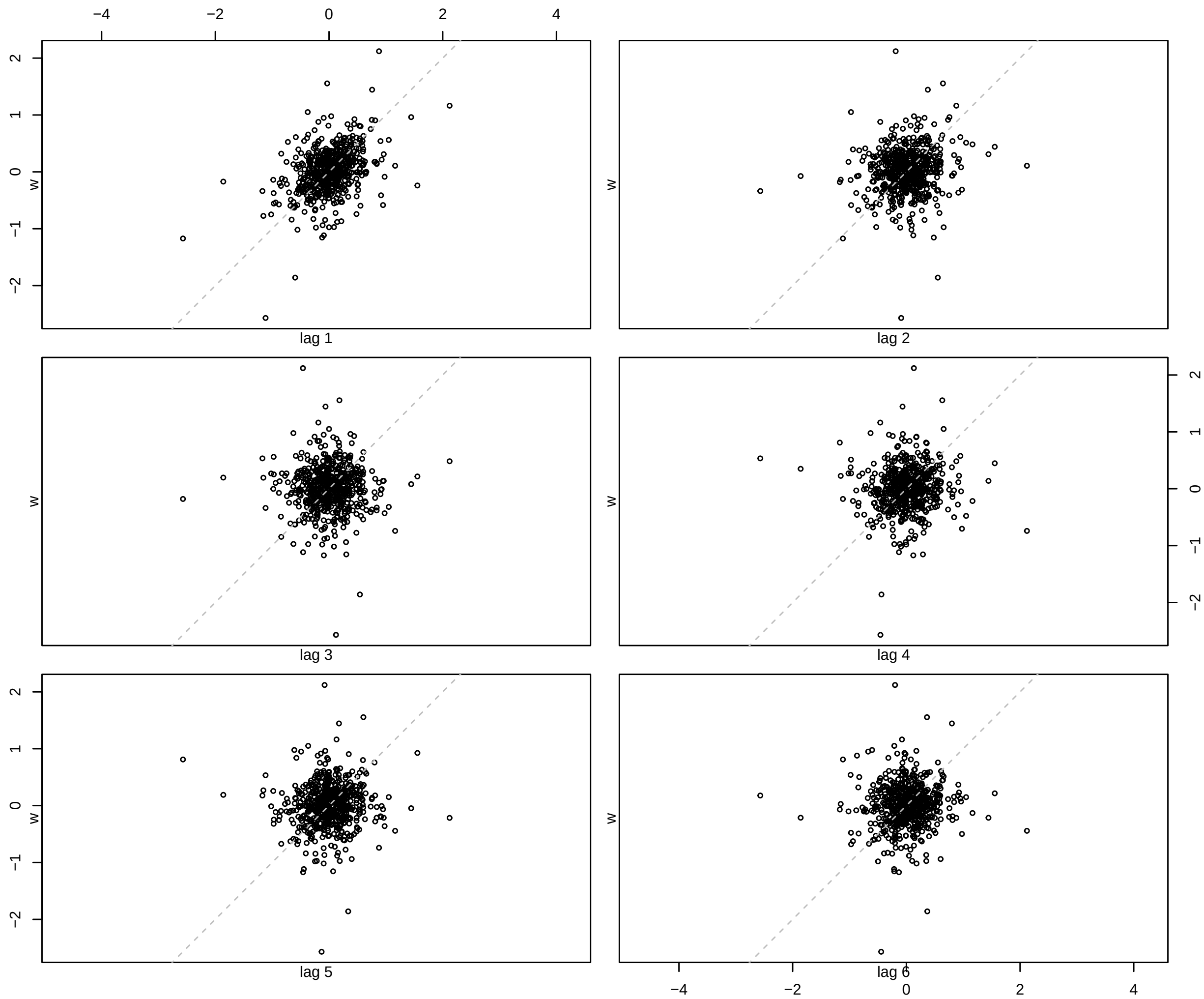



```r
y = TCU

lag.plot(y, 6, do.lines=FALSE)
```

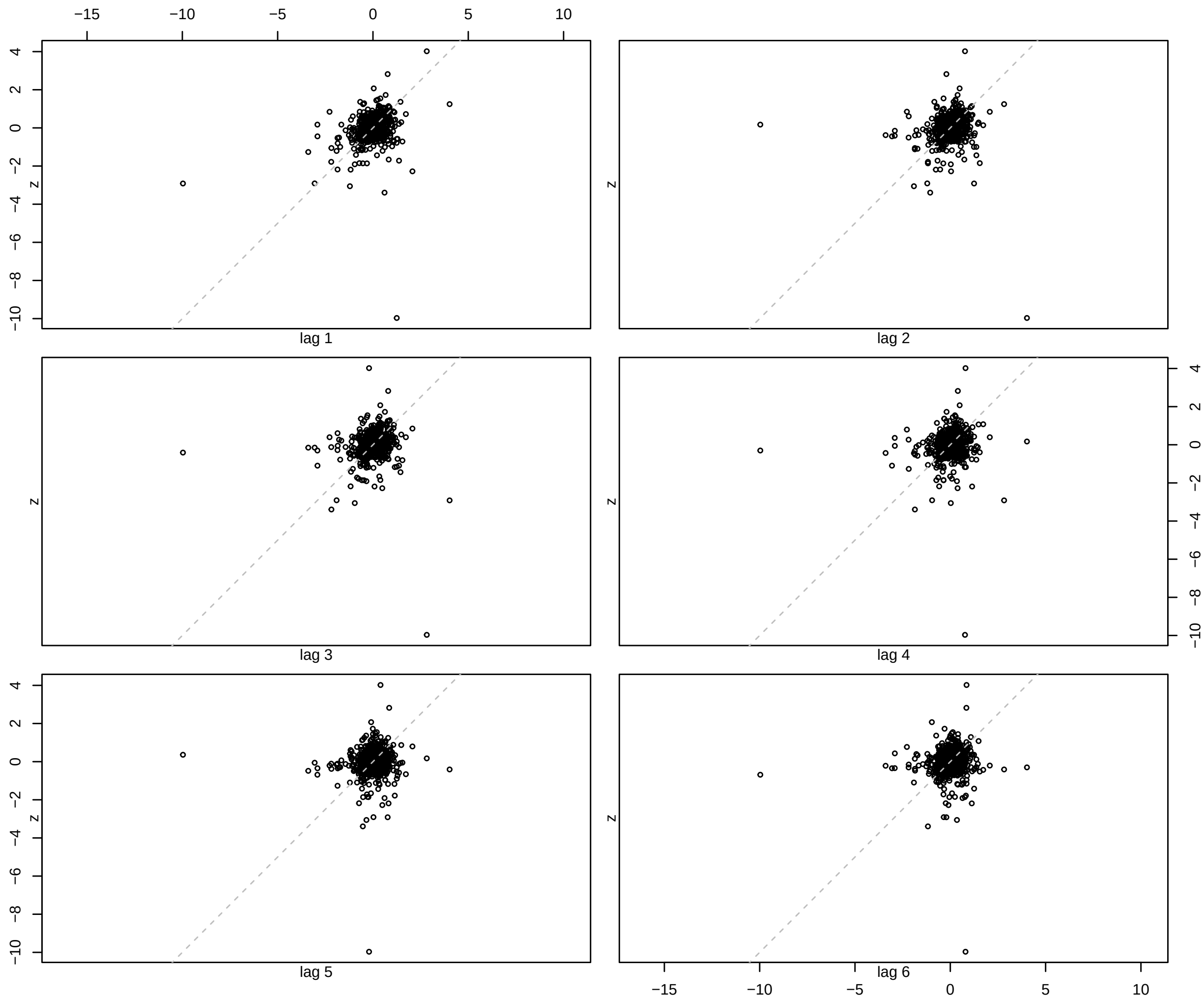


For Inflation and TCU in first differences, correlation is weakened as it can be suggested that the differential of the variable eliminates the unit root.

```
w = diff(Inflation)

lag.plot(w, 6, do.lines=FALSE)
```



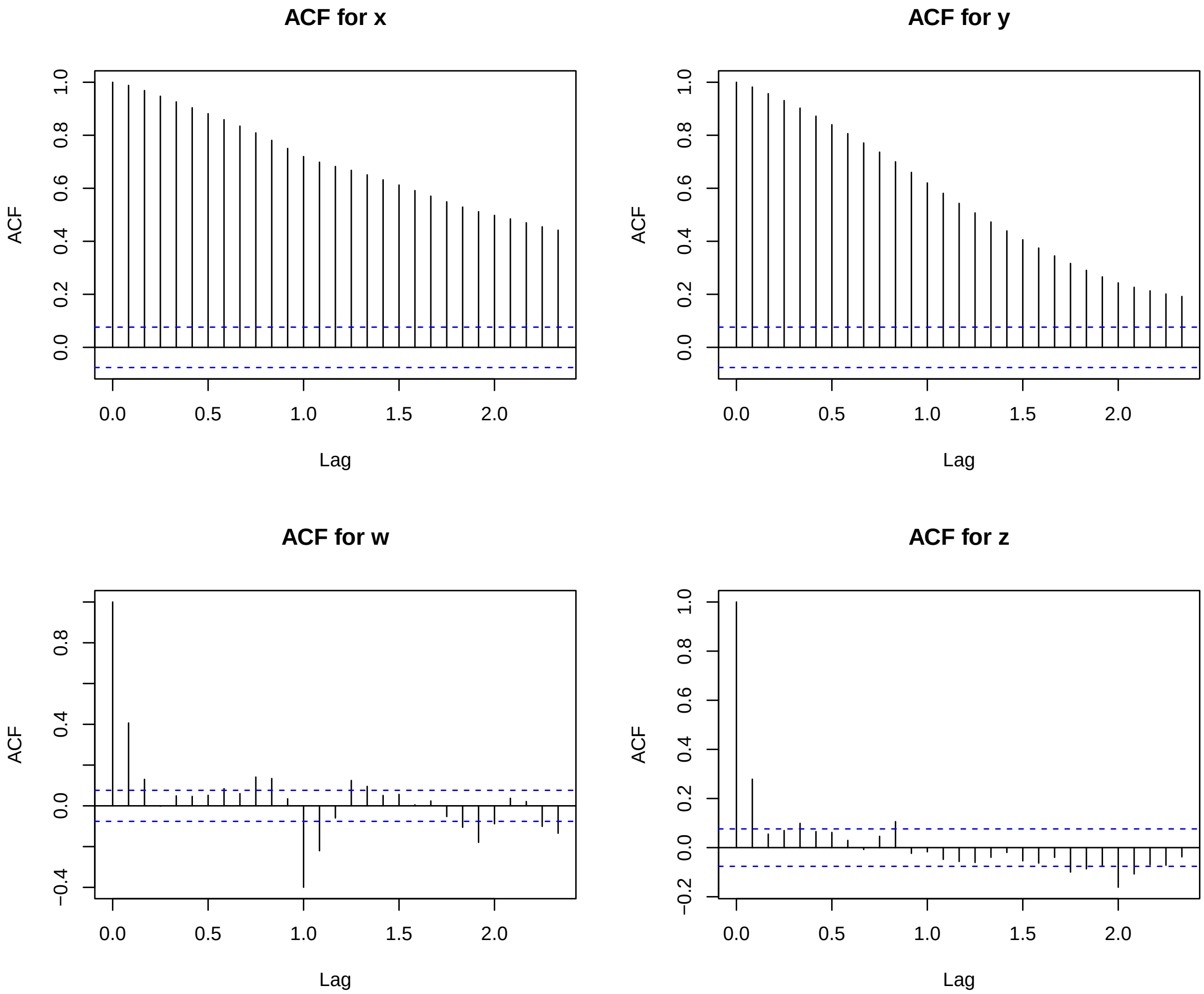

```r
z = diff(TCU)

lag.plot(z, 6, do.lines=FALSE)
```



ACF and PACF Plots

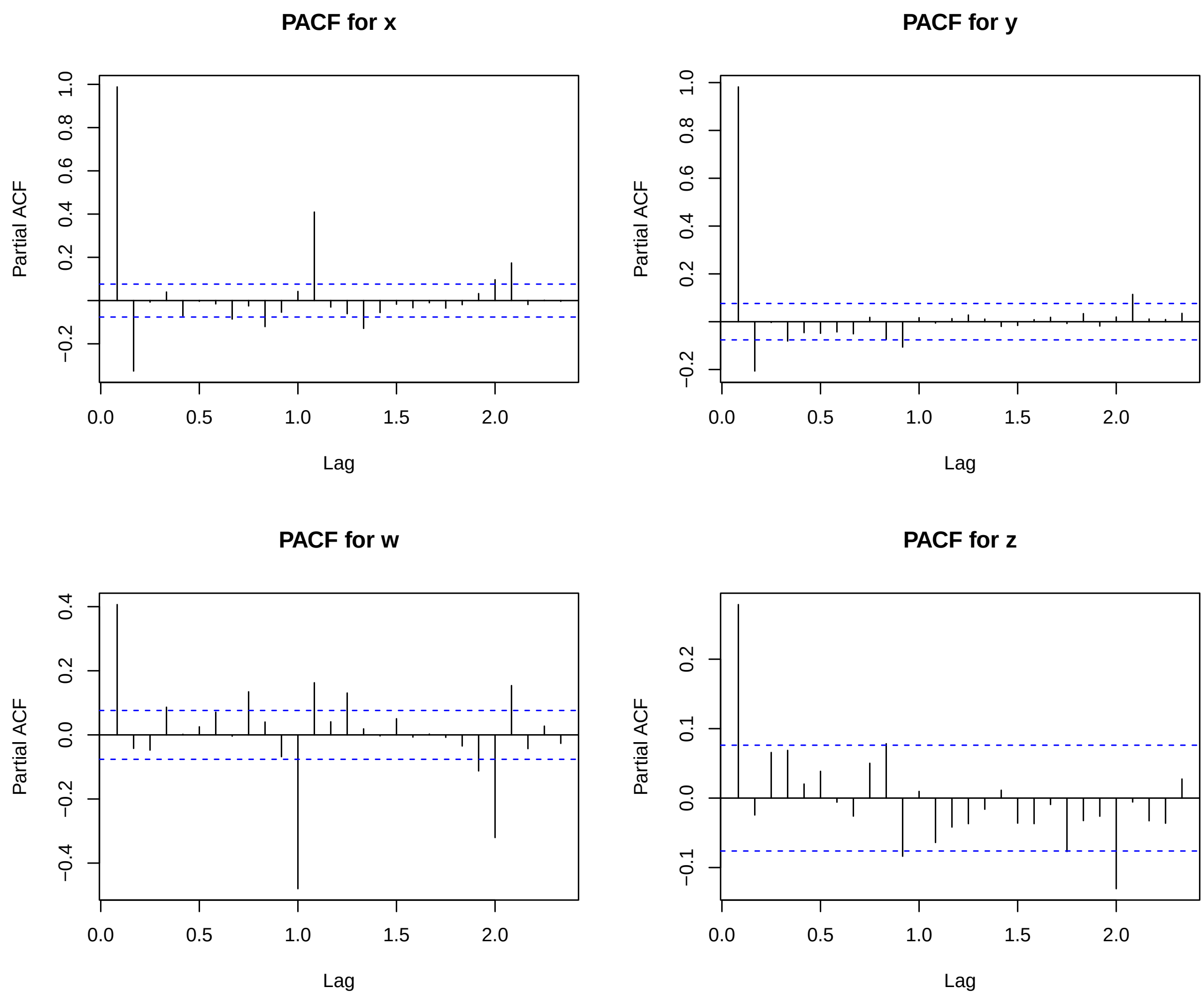
Inflation and TCU in levels show a strong persistence, whereas in first differences the persistence weakens.

```
par(mfrow=c(2,2))
acf      (x,          main="ACF for x")
acf      (y,          main="ACF for y")
acf      (w,          main="ACF for w")
acf      (z,          main="ACF for z")
```



Inflation and TCU are auto-regressive processes of order 1.

```
par(mfrow=c(2,2))
pacf    (x,          main="PACF for x")
pacf    (y,          main="PACF for y")
pacf    (w,          main="PACF for w")
pacf    (z,          main="PACF for z")
```



TCU and Inflation seem non-stationary I(1) in levels and stationary I(0) in first differences. Nevertheless, unit root tests are required.

ADF Test

Ho: residuals have a unit root and therefore the series is not stationary.

```
max.lags = trunc(  (12 * (  (length(x)/100)^(1/4)  ) ) )
max.lags
```

```
## [1] 19
```

```
x.adf.drift <- ur.df(x, selectlags="BIC", type="drift", lags=max.lags )
summary(x.adf.drift)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
```

```
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.6647 -0.1577  0.0023  0.1633  1.6169
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.053224   0.021344   2.494 0.012900 *
## z.lag.1      -0.012932   0.004633  -2.791 0.005411 **
## z.diff.lag1   0.461735   0.039411  11.716  < 2e-16 ***
## z.diff.lag2  -0.003637   0.043448  -0.084 0.933313
## z.diff.lag3   0.039029   0.043134   0.905 0.365905
## z.diff.lag4   0.035328   0.037133   0.951 0.341778
## z.diff.lag5   0.017493   0.036812   0.475 0.634817
## z.diff.lag6  -0.013282   0.036805  -0.361 0.718321
## z.diff.lag7   0.079936   0.036744   2.175 0.029968 *
## z.diff.lag8  -0.022462   0.036842  -0.610 0.542292
## z.diff.lag9   0.070908   0.036772   1.928 0.054265 .
## z.diff.lag10  0.069312   0.036817   1.883 0.060219 .
## z.diff.lag11  0.135759   0.036913   3.678 0.000255 ***
## z.diff.lag12 -0.537218   0.037414 -14.359  < 2e-16 ***
## z.diff.lag13  0.144639   0.043157   3.351 0.000852 ***
## z.diff.lag14 -0.012447   0.043530  -0.286 0.775026
## z.diff.lag15  0.154626   0.039972   3.868 0.000121 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.293 on 626 degrees of freedom
## Multiple R-squared:  0.431,  Adjusted R-squared:  0.4164
## F-statistic: 29.63 on 16 and 626 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -2.7912 3.9323
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

The test statistic of Inflation in levels is -2.79 which is less negative than the critical value at the 5% significance level (-2.86). Therefore, Ho cannot be rejected.

```
max.lags = trunc(  (12 * (  (length(y)/100)^(1/4)  ) ) )
max.lags
```

```
## [1] 19
```

```r
y.adf.drift <- ur.df(y, selectlags="BIC", type="drift", lags=max.lags )
summary(y.adf.drift)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.2341 -0.3011  0.0271  0.3207  3.8488
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.693835   0.525794   3.221  0.00134 **
## z.lag.1     -0.021350   0.006581  -3.244  0.00124 **
## z.diff.lag   0.295130   0.037700   7.828 2.06e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7044 on 640 degrees of freedom
## Multiple R-squared:  0.09655,    Adjusted R-squared:  0.09372
## F-statistic:  34.2 on 2 and 640 DF,  p-value: 7.759e-15
##
##
## Value of test-statistic is: -3.244 5.3194
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

For TCU in levels, Ho can be rejected at the 5% significance level but not at the 1% significance level.

```r
max.lags = trunc(  (12 * (  (length(w)/100)^(1/4)  ) ) )
max.lags
```

```
## [1] 19
```

```r
w.adf.drift <- ur.df(w, selectlags="BIC", type="drift", lags=max.lags )
summary(w.adf.drift)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
```

```
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.65492 -0.15612  0.00057  0.16625  1.60708
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.003252   0.011638   0.279 0.780025
## z.lag.1      -0.478635   0.084014  -5.697 1.88e-08 ***
## z.diff.lag1  -0.062491   0.082403  -0.758 0.448519
## z.diff.lag2  -0.073195   0.078405  -0.934 0.350894
## z.diff.lag3  -0.043453   0.072052  -0.603 0.546677
## z.diff.lag4  -0.010724   0.071751  -0.149 0.881235
## z.diff.lag5   0.001653   0.070501   0.023 0.981305
## z.diff.lag6  -0.016958   0.068452  -0.248 0.804414
## z.diff.lag7   0.056639   0.065761   0.861 0.389409
## z.diff.lag8   0.027800   0.063396   0.439 0.661161
## z.diff.lag9   0.091425   0.060079   1.522 0.128581
## z.diff.lag10  0.153569   0.056928   2.698 0.007172 **
## z.diff.lag11  0.280657   0.053739   5.223 2.40e-07 ***
## z.diff.lag12 -0.266698   0.050319  -5.300 1.61e-07 ***
## z.diff.lag13 -0.126680   0.045966  -2.756 0.006023 **
## z.diff.lag14 -0.144420   0.040078  -3.603 0.000339 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2948 on 626 degrees of freedom
## Multiple R-squared:  0.5097, Adjusted R-squared:  0.4979
## F-statistic: 43.38 on 15 and 626 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -5.6971 16.2508
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

For Inflation in first difference, Ho can be rejected at the 5% significance level.

```r
max.lags = trunc(  (12 * (  (length(z)/100)^(1/4)  ) ) )
max.lags
```

```
## [1] 19
```

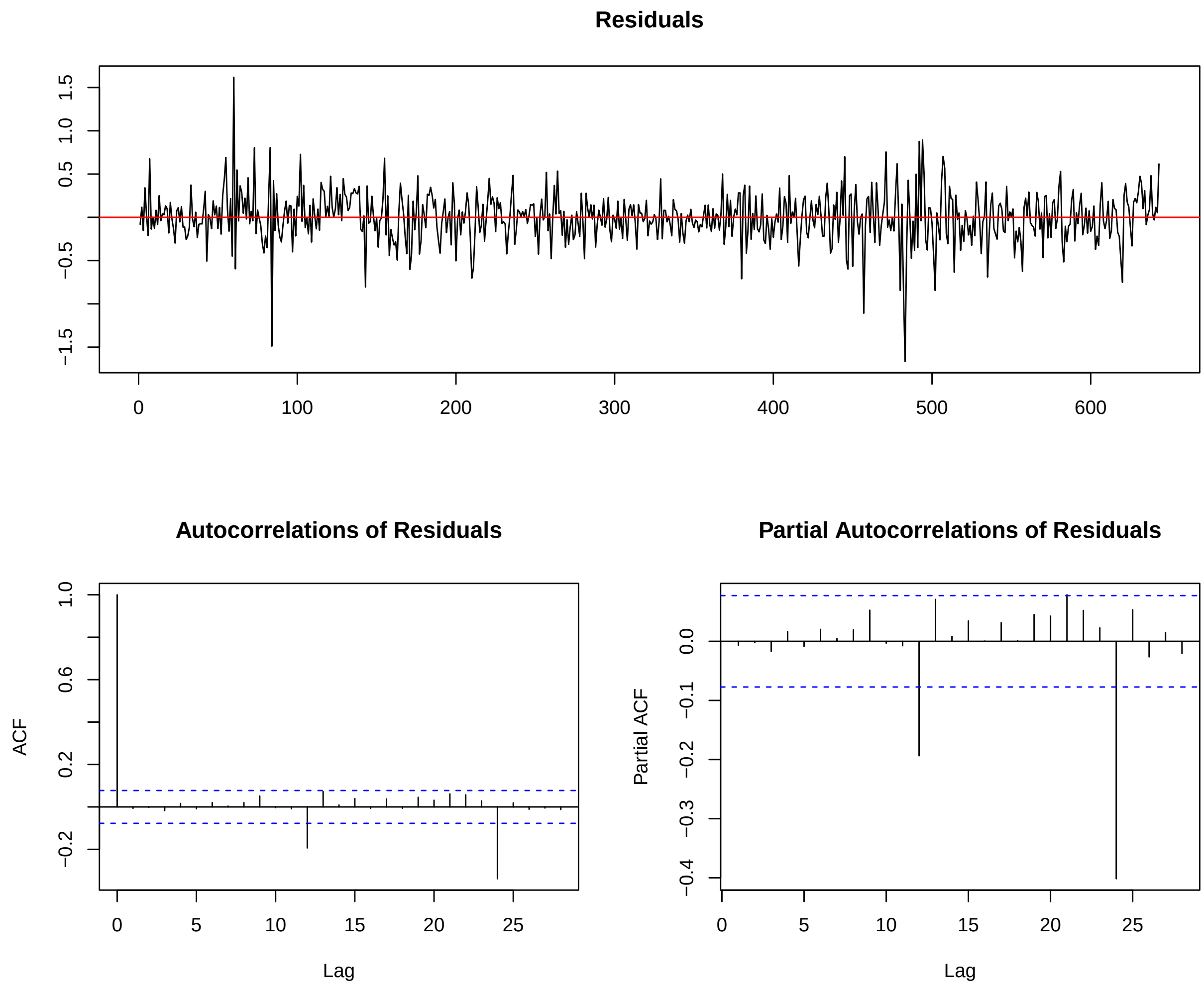
```r
z.adf.drift <- ur.df(z, selectlags="BIC", type="drift", lags=max.lags )
summary(z.adf.drift)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.0963 -0.3009  0.0078  0.2997  4.1054
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.009808   0.028050  -0.350    0.727
## z.lag.1     -0.733657   0.047321 -15.504   <2e-16 ***
## z.diff.lag   0.027480   0.039582   0.694    0.488
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7105 on 639 degrees of freedom
## Multiple R-squared:  0.3572, Adjusted R-squared:  0.3552
## F-statistic: 177.5 on 2 and 639 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -15.5039 120.1864
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

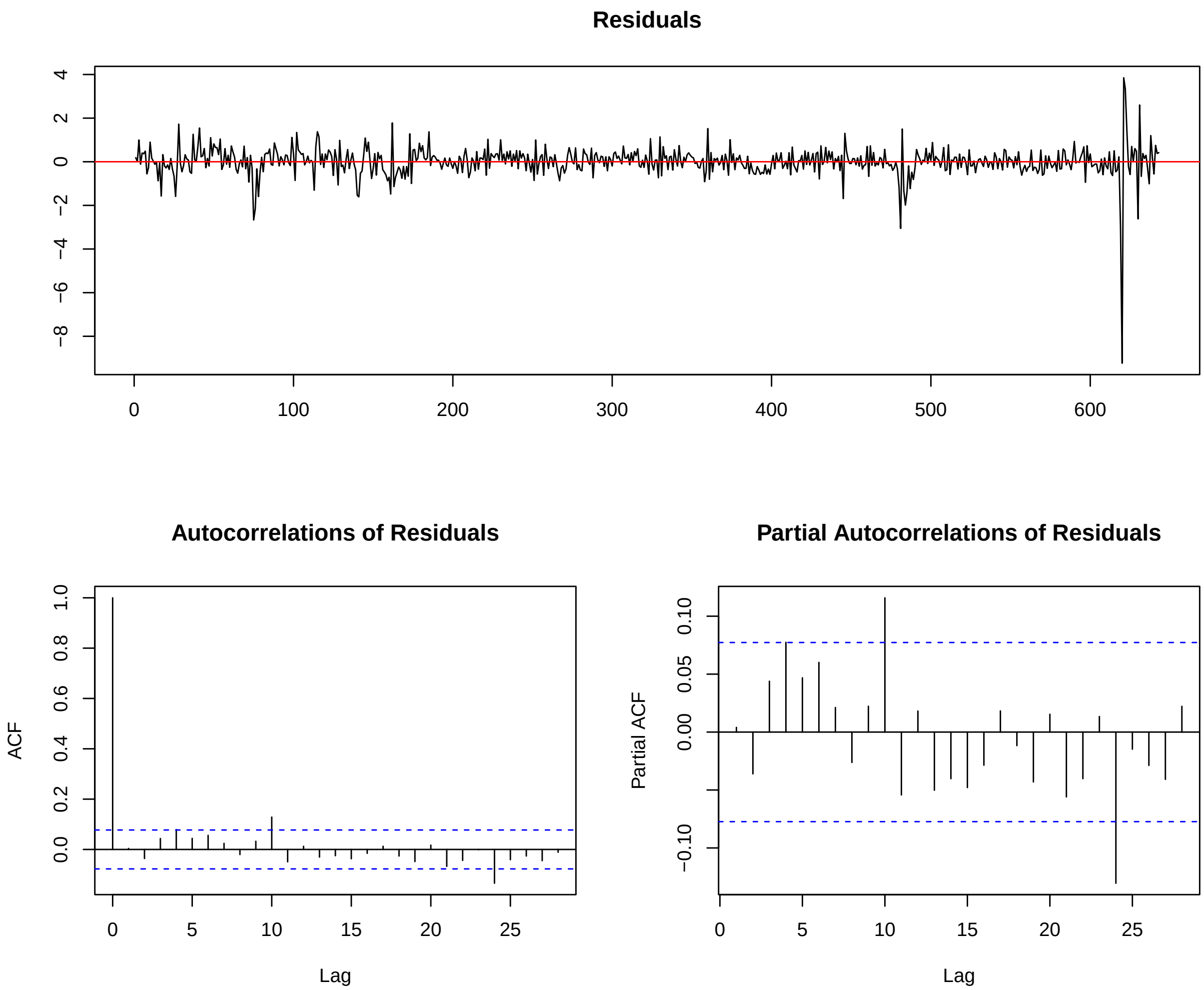
For TCU in first difference, Ho can be rejected at the 5% significance level.

According to the ADF tests at the 5% significance level, Inflation in levels has a unit root I(1) and does not in first differences I(0). TCU does not have a unit root in levels I(0) and first differences I(0).

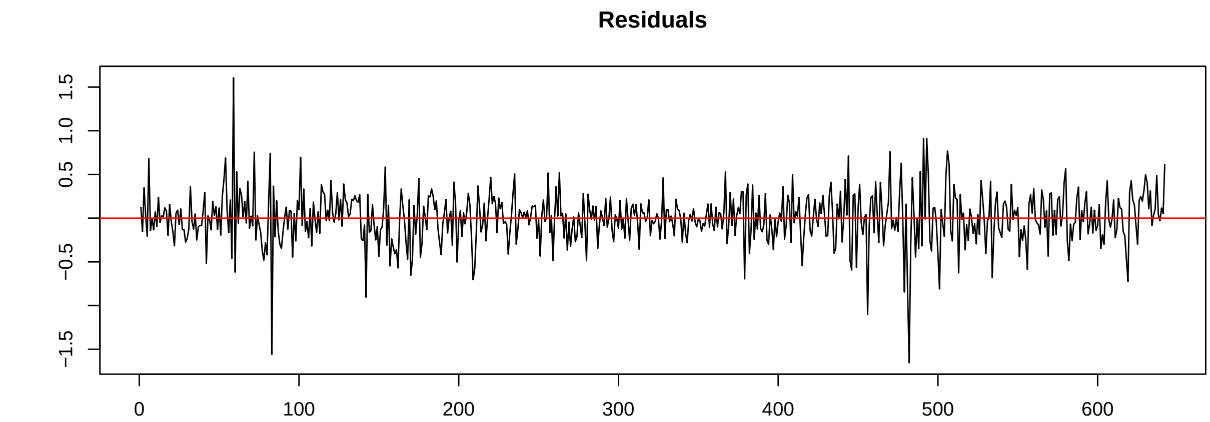
```r
plot(x.adf.drift)
```

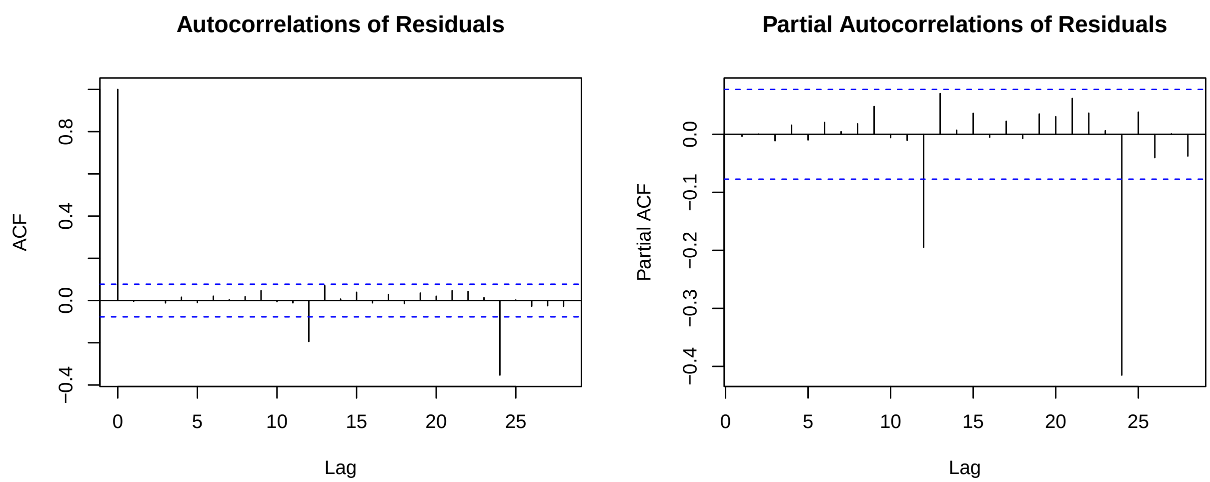



```
plot(y.adf.drift)
```

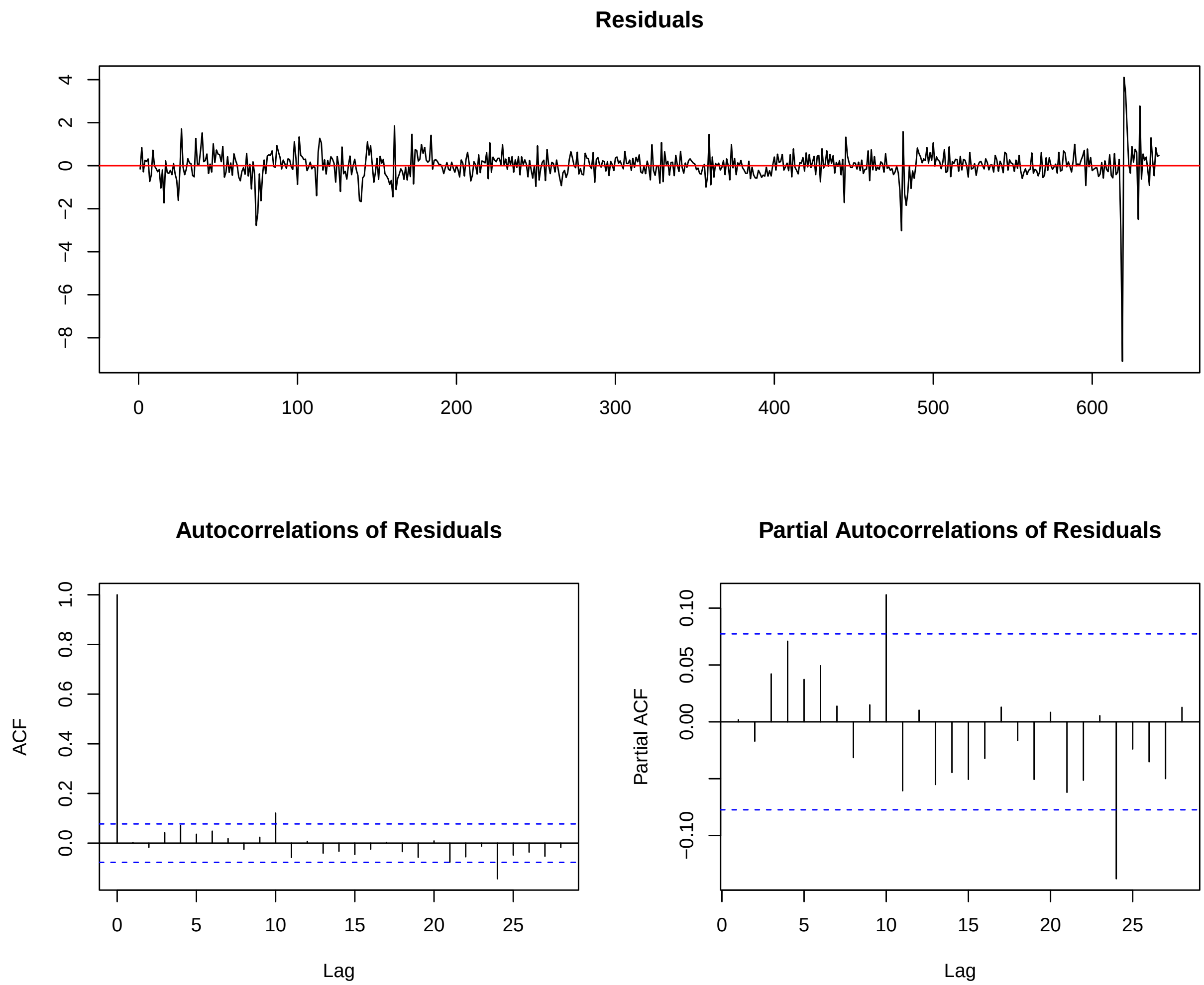



```
plot(w.adf.drift)
```






```
plot(z.adf.drift)
```



PP Test

Ho: residuals have a unit root.

```
x.pp <- ur.pp(x, type="Z-tau", model="constant", lags="long")
summary(x.pp)
```

```
##
## ##################################
## # Phillips-Perron Unit Root Test #
## ##################################
##
## Test regression with intercept
##
##
## Call:
## lm(formula = y ~ y.l1)
##
## Residuals:
```

```
##      Min       1Q   Median       3Q      Max
## -2.57851 -0.21179 -0.01974  0.20530  2.08039
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.038532   0.025779   1.495    0.135
## y.l1        0.992015   0.005461 181.653   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3802 on 660 degrees of freedom
## Multiple R-squared:  0.9804, Adjusted R-squared:  0.9804
## F-statistic: 3.3e+04 on 1 and 660 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic, type: Z-tau  is: -2.5704
##
##          aux. Z statistics
## Z-tau-mu            2.2978
##
## Critical values for Z statistics:
##                      1pct      5pct     10pct
## critical values -3.442629 -2.866255 -2.569282
```

The test statistic which is -2.57 is less negative than the critical values at the 5% significance level. The series is non-stationary because Ho cannot be rejected. the PP test just as the ADF test indicates the presence of a unit root in the inflation rate in levels.

```r
y.pp <- ur.pp(y, type="Z-tau", model="constant", lags="long")
summary(y.pp)
```

```
##
## ##################################
## # Phillips-Perron Unit Root Test #
## ##################################
##
## Test regression with intercept
##
##
## Call:
## lm(formula = y ~ y.l1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.0680  -0.3112   0.0572   0.3570   3.7640
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.406049   0.525563   2.675  0.00765 **
## y.l1        0.982211   0.006561 149.701  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.7346 on 660 degrees of freedom
## Multiple R-squared:  0.9714, Adjusted R-squared:  0.9713
## F-statistic: 2.241e+04 on 1 and 660 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic, type: Z-tau  is: -3.5943
##
##          aux. Z statistics
## Z-tau-mu            3.5676
##
## Critical values for Z statistics:
##                      1pct      5pct     10pct
## critical values -3.442629 -2.866255 -2.569282
```

For TCU in levels. Ho can be rejected at the 5% significance level.

```
w.pp <- ur.pp(w, type="Z-tau", model="constant", lags="long")
summary(w.pp)
```

```
##
## ##################################
## # Phillips-Perron Unit Root Test #
## ##################################
##
## Test regression with intercept
##
##
## Call:
## lm(formula = y ~ y.l1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.0967 -0.1717  0.0004  0.1844  1.6472
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.005388   0.013530   0.398    0.691
## y.l1        0.407999   0.035618  11.455   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3478 on 659 degrees of freedom
## Multiple R-squared:  0.166,  Adjusted R-squared:  0.1648
## F-statistic: 131.2 on 1 and 659 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic, type: Z-tau  is: -16.7923
##
##          aux. Z statistics
## Z-tau-mu            0.4009
##
## Critical values for Z statistics:
##                      1pct      5pct     10pct
## critical values -3.442643 -2.866261 -2.569286
```

For Inflation in first differences, Ho is rejected at the 5% significance level.

```r
z.pp <- ur.pp(z, type="Z-tau", model="constant", lags="long")
summary(z.pp)
```

```
##
## ###################################
## # Phillips-Perron Unit Root Test #
## ###################################
##
## Test regression with intercept
##
##
## Call:
## lm(formula = y ~ y.l1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.1445 -0.3005  0.0098  0.3025  4.0372
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.00968    0.02754  -0.352    0.725
## y.l1         0.27896    0.03732   7.474 2.48e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7078 on 659 degrees of freedom
## Multiple R-squared:  0.07815,    Adjusted R-squared:  0.07675
## F-statistic: 55.87 on 1 and 659 DF,  p-value: 2.479e-13
##
##
## Value of test-statistic, type: Z-tau  is: -19.9491
##
##          aux. Z statistics
## Z-tau-mu           -0.3671
##
## Critical values for Z statistics:
##                      1pct      5pct     10pct
## critical values -3.442643 -2.866261 -2.569286
```

For TCU in first differences, the Ho is rejected at the 5% significance level.

According to the PP tests, TCU is I(0) in levels and first differences, whereas Inflation is I(1) and I(0) respectively.

The plots of the PP results are shown below.

```r
plot(x.pp)
```

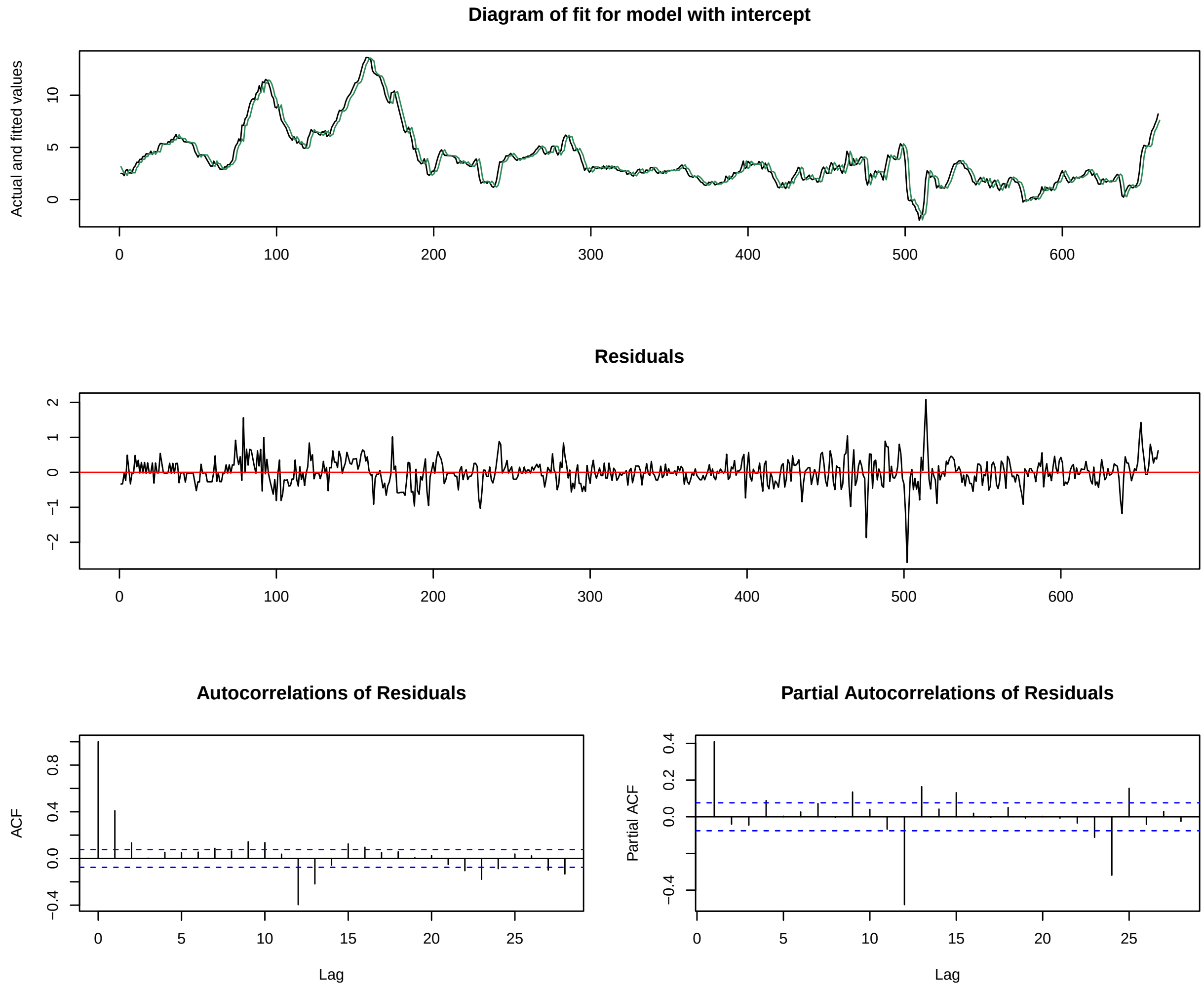


Diagram of fit for model with intercept
Actual and fitted values
Residuals
Autocorrelations of Residuals
ACF
Lag
Partial Autocorrelations of Residuals
Partial ACF
Lag

```
plot(y.pp)
```

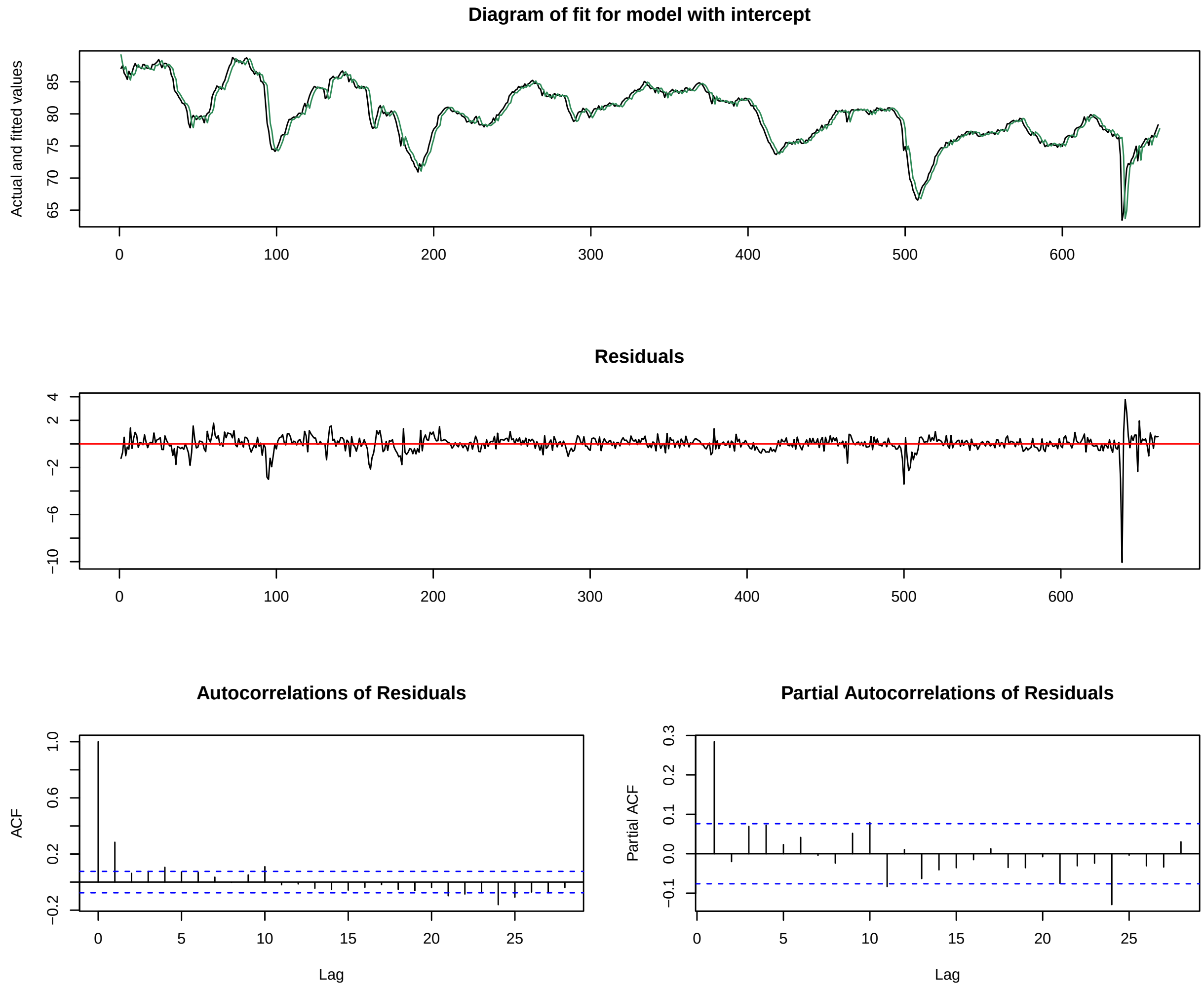


Diagram of fit for model with intercept
Actual and fitted values
65
70
75
80
85
0
100
200
300
400
500
600
Residuals
4
2
−2
−6
−10
0
100
200
300
400
500
600
Autocorrelations of Residuals
ACF
1.0
0.6
0.2
−0.2
0
5
10
15
20
25
Lag
Partial Autocorrelations of Residuals
Partial ACF
0.3
0.2
0.1
0.0
−0.1
0
5
10
15
20
25
Lag

```
plot(w.pp)
```

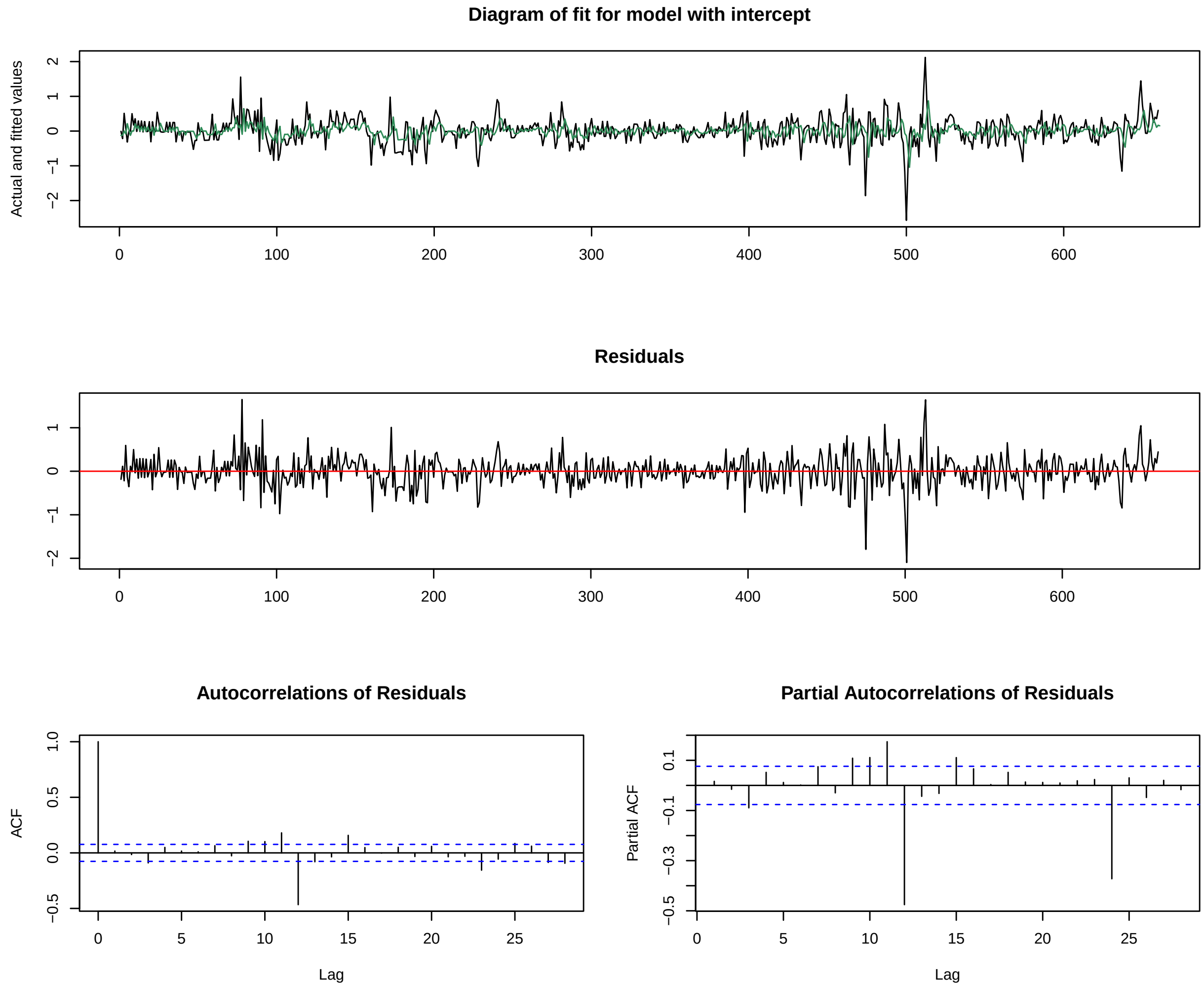
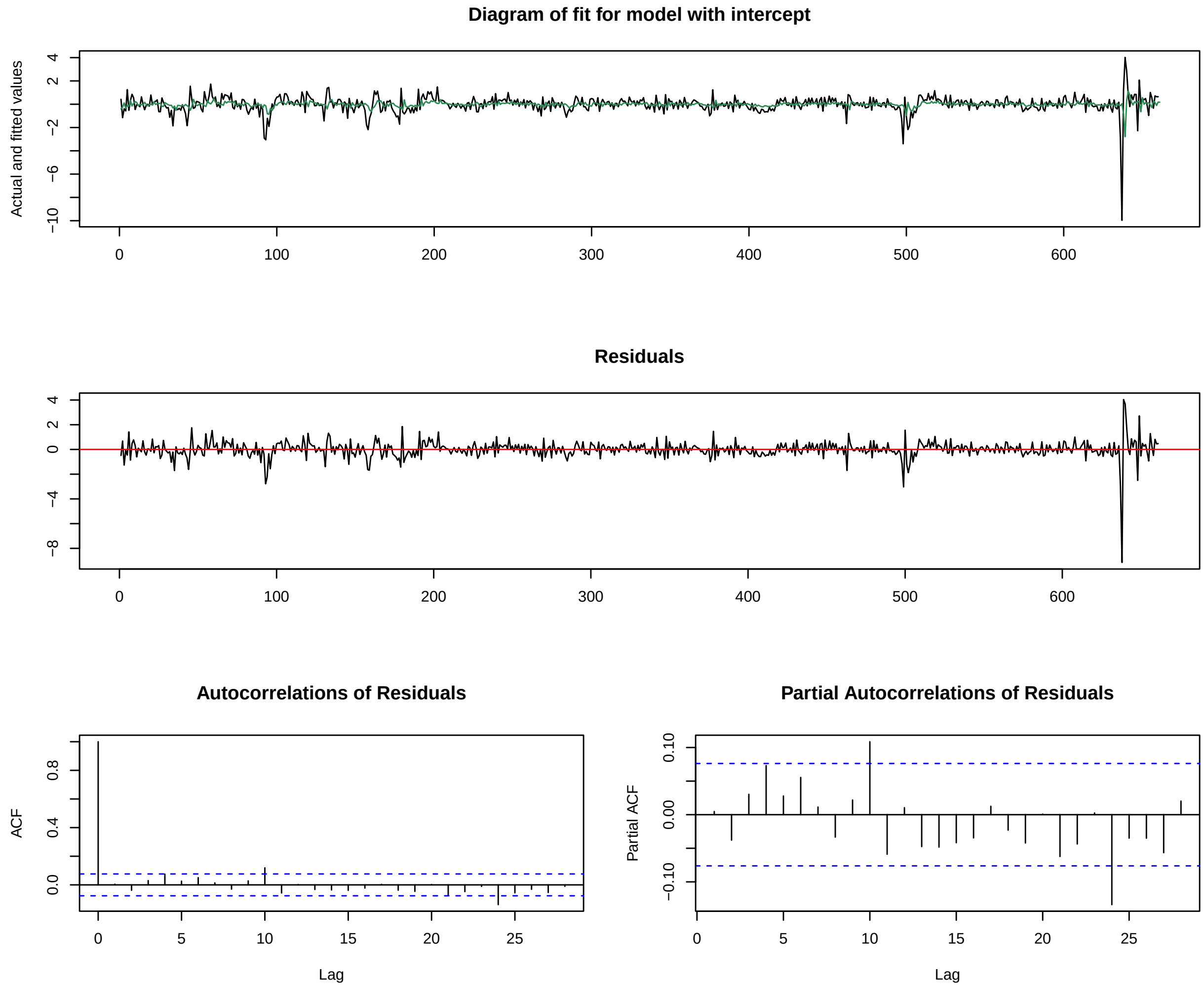


Diagram of fit for model with intercept
Actual and fitted values
Residuals
Autocorrelations of Residuals
ACF
Lag
Partial Autocorrelations of Residuals
Partial ACF
Lag


```
plot(z.pp)
```



KPSS Test

Ho: residuals do not have a unit root and the series is stationary.

```
x.kpss <- ur.kpss(x, type="mu",  lags="long" )
summary(x.kpss)
```

```
##
## #######################
## # KPSS Unit Root Test #
## #######################
##
## Test is of type: mu with 19 lags.
##
## Value of test-statistic is: 1.5785
##
## Critical value for a significance level of:
##                 10pct  5pct 2.5pct  1pct
## critical values 0.347 0.463  0.574 0.739
```

For Inflation in levels, the test statistic is 1.576 which is greater than the critical value at the 5% significance level. Ho is rejected, and the series cannot be stationary because there is a unit root.

```
y.kpss <- ur.kpss(y, type="mu",  lags="long" )
summary(y.kpss)
```

```
##
## #######################
## # KPSS Unit Root Test #
## #######################
##
## Test is of type: mu with 19 lags.
##
## Value of test-statistic is: 1.4505
##
## Critical value for a significance level of:
##                 10pct  5pct 2.5pct  1pct
## critical values 0.347 0.463  0.574 0.739
```

For TCU in levels, Ho is rejected at the 5% significance level.

```
w.kpss <- ur.kpss(w, type="mu",  lags="long" )
summary(w.kpss)
```

```
##
## #######################
## # KPSS Unit Root Test #
## #######################
##
## Test is of type: mu with 19 lags.
##
## Value of test-statistic is: 0.0775
##
## Critical value for a significance level of:
##                 10pct  5pct 2.5pct  1pct
## critical values 0.347 0.463  0.574 0.739
```

For Inflation in first differences, Ho is not rejected at the 5% significance level.

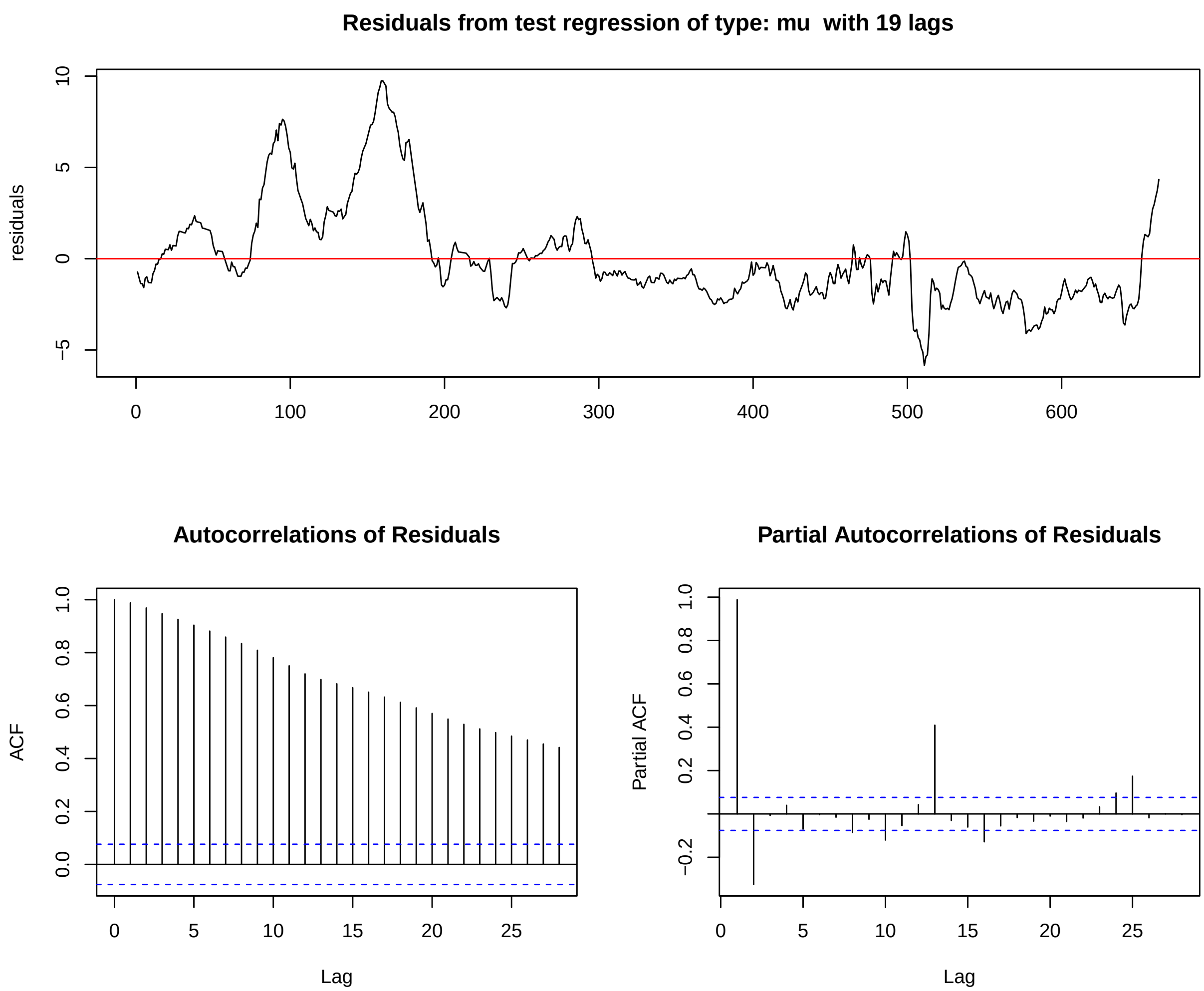
```
z.kpss <- ur.kpss(z, type="mu",  lags="long" )
summary(z.kpss)
```

```
##
## #######################
## # KPSS Unit Root Test #
## #######################
##
## Test is of type: mu with 19 lags.
##
## Value of test-statistic is: 0.0364
##
## Critical value for a significance level of:
##                 10pct  5pct 2.5pct  1pct
## critical values 0.347 0.463  0.574 0.739
```

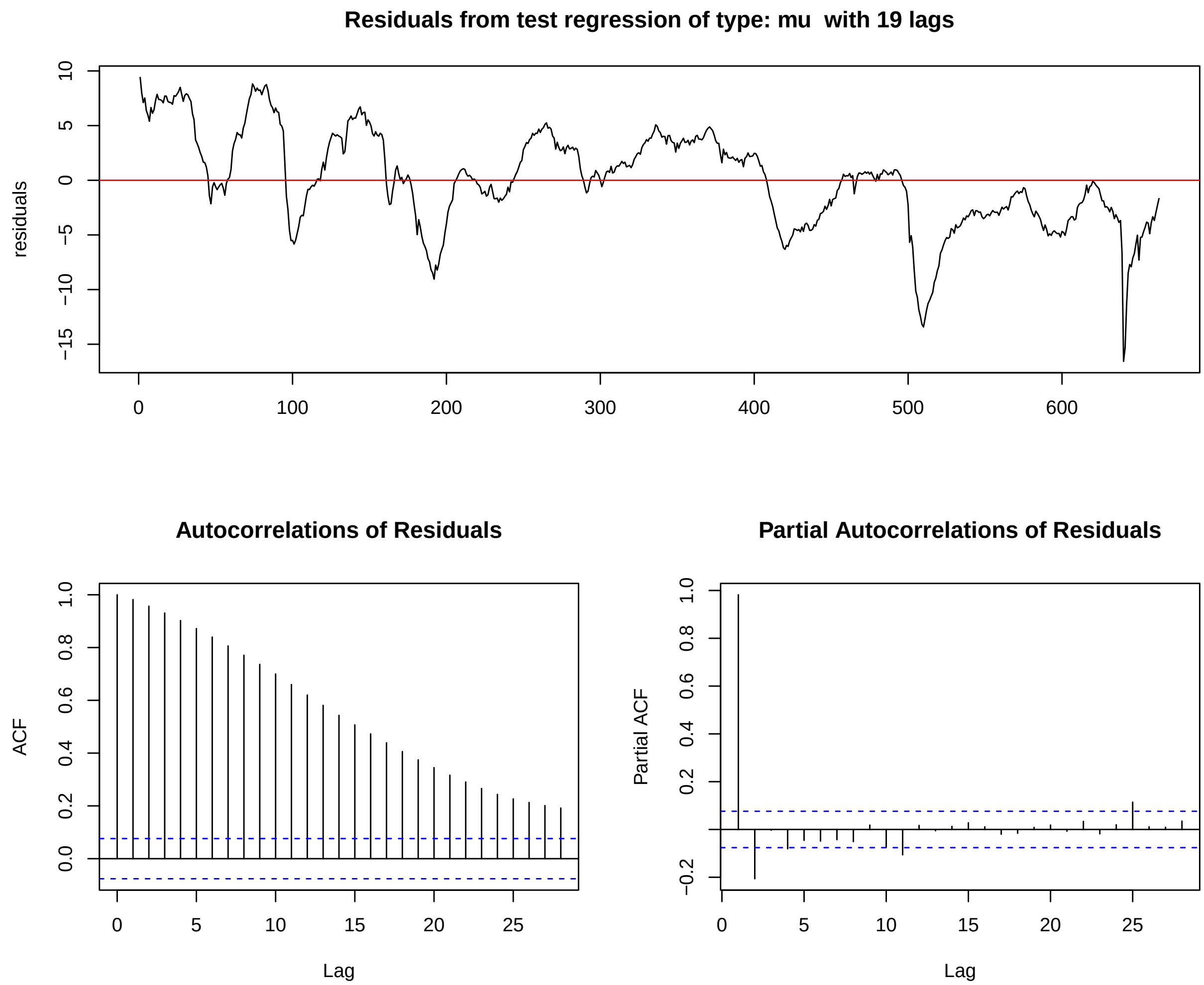
For TCU in first differences, Ho is not rejected at the 5% level of significance.

According to the KPSS Inflation and TCU are I(1) in levels and I(0) in first differences at the 5% significance level.

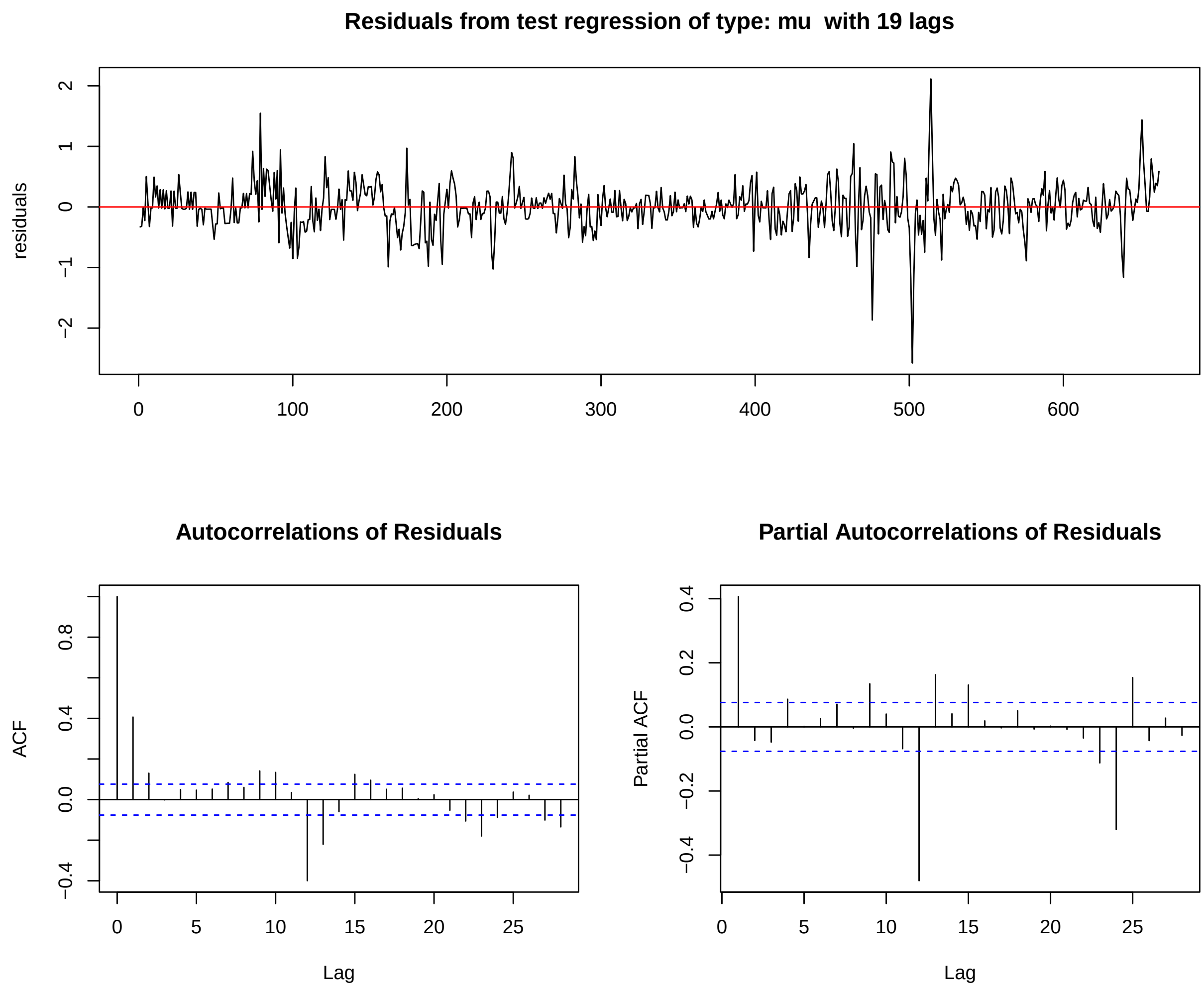
```
plot(x.kpss)
```



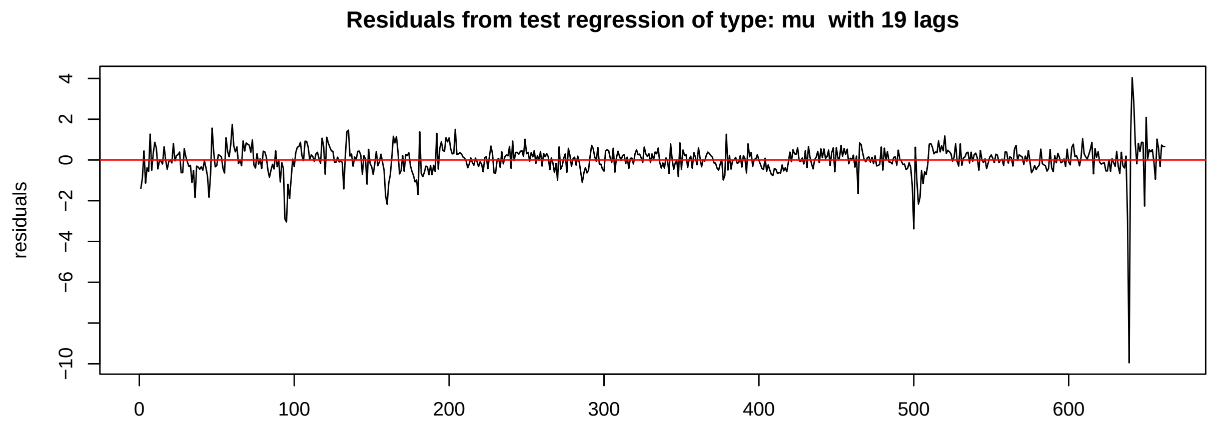

```
plot(y.kpss)
```

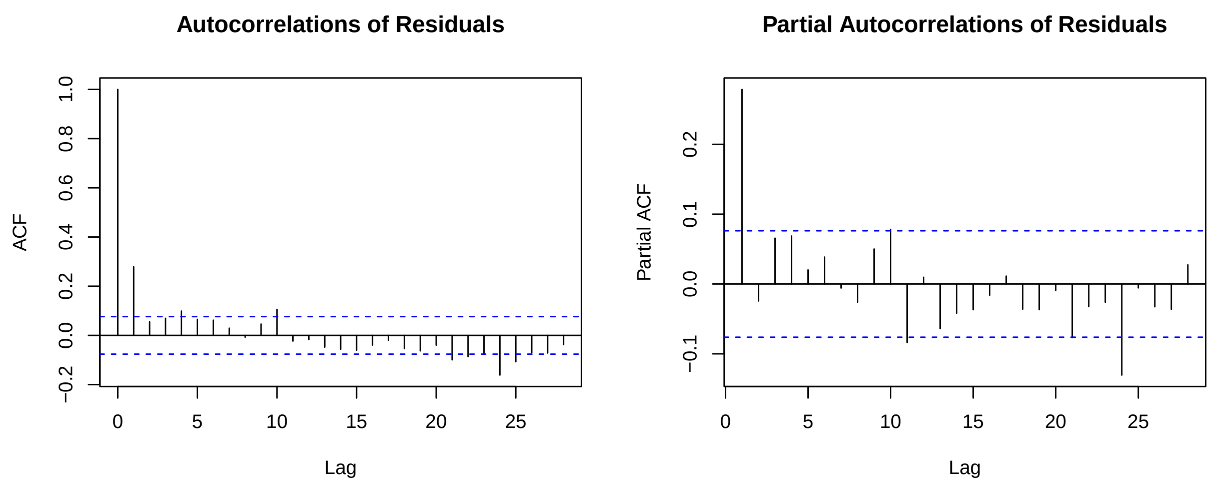

Residuals from test regression of type: mu with 19 lags
residuals
10
5
0
−5
−10
−15
0
100
200
300
400
500
600
Autocorrelations of Residuals
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
Lag
Partial Autocorrelations of Residuals
Partial ACF
1.0
0.8
0.6
0.4
0.2
−0.2
0
5
10
15
20
25
Lag

```
plot(w.kpss)
```


Residuals from test regression of type: mu with 19 lags
residuals
Autocorrelations of Residuals
ACF
Lag
Partial Autocorrelations of Residuals
Partial ACF
Lag

```
plot(z.kpss)
```





The plots show that the trend of Inflation and TCU is a random walk with drift, accounted for by running the tests with a constant.

Cointegration Analysis

As the unit root tests results are inconclusive in levels at the 5% significance level, but both series are I(0) in first differences they might be cointegrated.

Engle-Granger Methodology

Ho: no cointegration

```
coint.test(Inflation, TCU,           d = 0, nlag = NULL, output = TRUE)
```

```
## Response: Inflation
## Input: TCU
## Number of inputs: 1
## Model: y ~ X + 1
```

```
## --------------------------------
## Engle-Granger Cointegration Test
## alternative: cointegrated
##
## Type 1: no trend
##     lag      EG p.value
##  6.0000 -2.6428  0.0876
## -----
##  Type 2: linear trend
##     lag      EG p.value
##  6.0000  0.0228  0.1000
## -----
##  Type 3: quadratic trend
##     lag      EG p.value
##    6.00   -0.48    0.10
## -----------
## Note: p.value = 0.01 means p.value <= 0.01
##     : p.value = 0.10 means p.value >= 0.10
```

Ho cannot be rejected at the 5% significance level (p value=0.0876).

Inflation and TCU seem to not be cointegrated in levels at the 5% significance level.

Johansen Test

Ho: no cointegration vector.

```
VARselect(data.set, lag.max=10, type="const", season = NULL,
          exogen = NULL)$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      2      2      2      2
```

```
optimal.lags = 2


johansen.const = ca.jo(data.set, type="eigen", ecdet="const",  K = optimal.lags,
                       spec="longrun")
    summary(johansen.const)
```

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: maximal eigenvalue statistic (lambda max) , without linear trend and constant in cointegra
##
## Eigenvalues (lambda):
## [1] 2.943962e-02 2.049640e-02 3.469447e-18
##
## Values of teststatistic and critical values of test:
##
```

```
##           test 10pct  5pct  1pct
## r <= 1 | 13.69  7.52  9.24 12.97
## r = 0  | 19.75 13.75 15.67 20.20
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##              Inflation.l2     TCU.l2   constant
## Inflation.l2      1.00000  1.0000000   1.000000
## TCU.l2          -10.31221 -0.1915511  -1.891966
## constant        814.48319 11.3603475 197.589867
##
## Weights W:
## (This is the loading matrix)
##
##              Inflation.l2      TCU.l2      constant
## Inflation.d -0.0006959788 -0.01709435  1.116010e-19
## TCU.d        0.0021677903 -0.02522300 -1.054726e-17
```

The test statistic 19.75 is greater than the critical value 15.67 at the 5% significance level. Ho is rejected at the 5% significance level as there is evidence of a cointegrating vector. However, Ho is also rejected when the rank of the pi matrix is equal to 1, which is logically impossible as the variables are 2.

```
johansen.const = ca.jo(data.set, type="trace", ecdet="const",  K = optimal.lags,
                       spec="longrun")
    summary(johansen.const)
```

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: trace statistic , without linear trend and constant in cointegration
##
## Eigenvalues (lambda):
## [1] 2.943962e-02 2.049640e-02 3.469447e-18
##
## Values of teststatistic and critical values of test:
##
##           test 10pct  5pct  1pct
## r <= 1 | 13.69  7.52  9.24 12.97
## r = 0  | 33.44 17.85 19.96 24.60
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##              Inflation.l2     TCU.l2   constant
## Inflation.l2      1.00000  1.0000000   1.000000
## TCU.l2          -10.31221 -0.1915511  -1.891966
## constant        814.48319 11.3603475 197.589867
##
## Weights W:
## (This is the loading matrix)
```

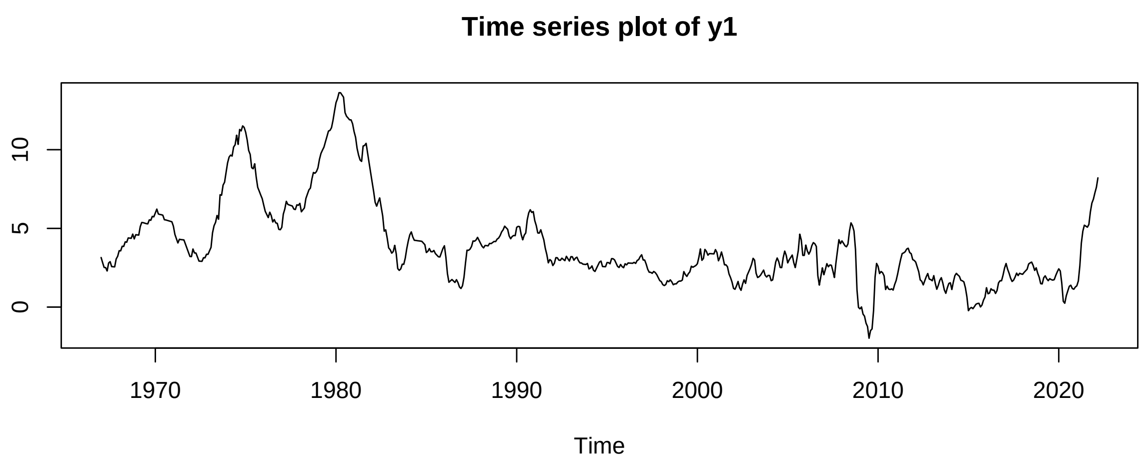
```
##
##               Inflation.l2      TCU.l2      constant
## Inflation.d -0.0006959788 -0.01709435  1.116010e-19
## TCU.d        0.0021677903 -0.02522300 -1.054726e-17
```

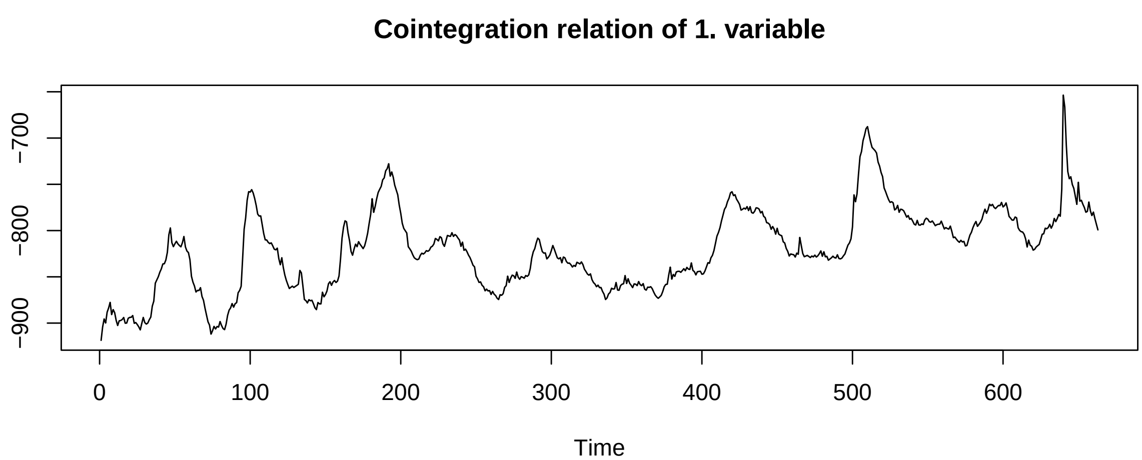
Ho is rejected at the 5% significance level for both ranks again.

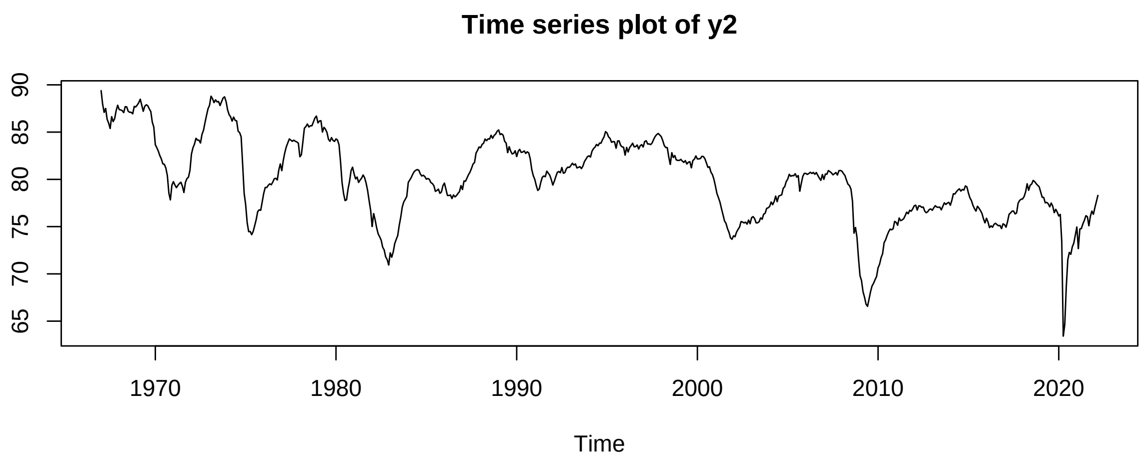
The tests cannot establish a meaningful correlation.

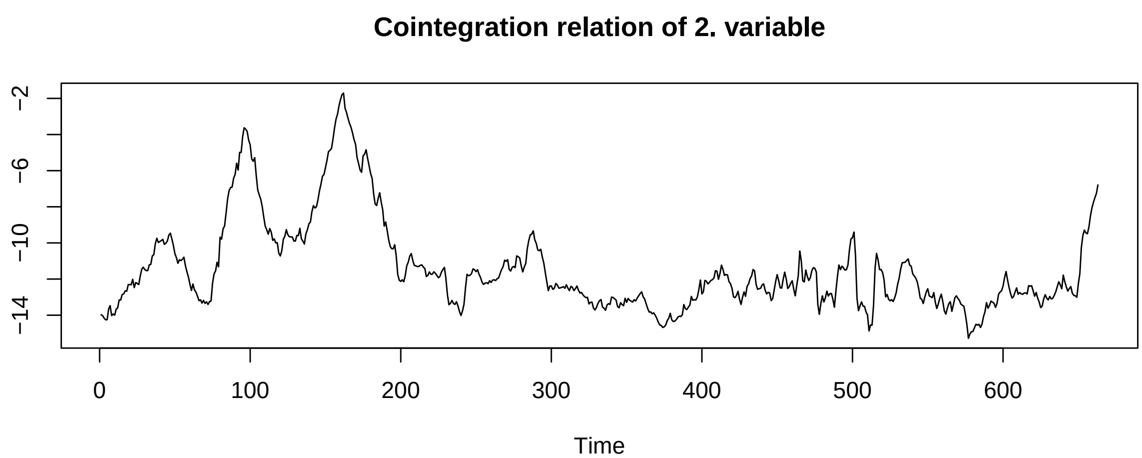
The Cointegration relation seems to be stationary for TCU but not for Inflation.

```
plot(johansen.const)
```

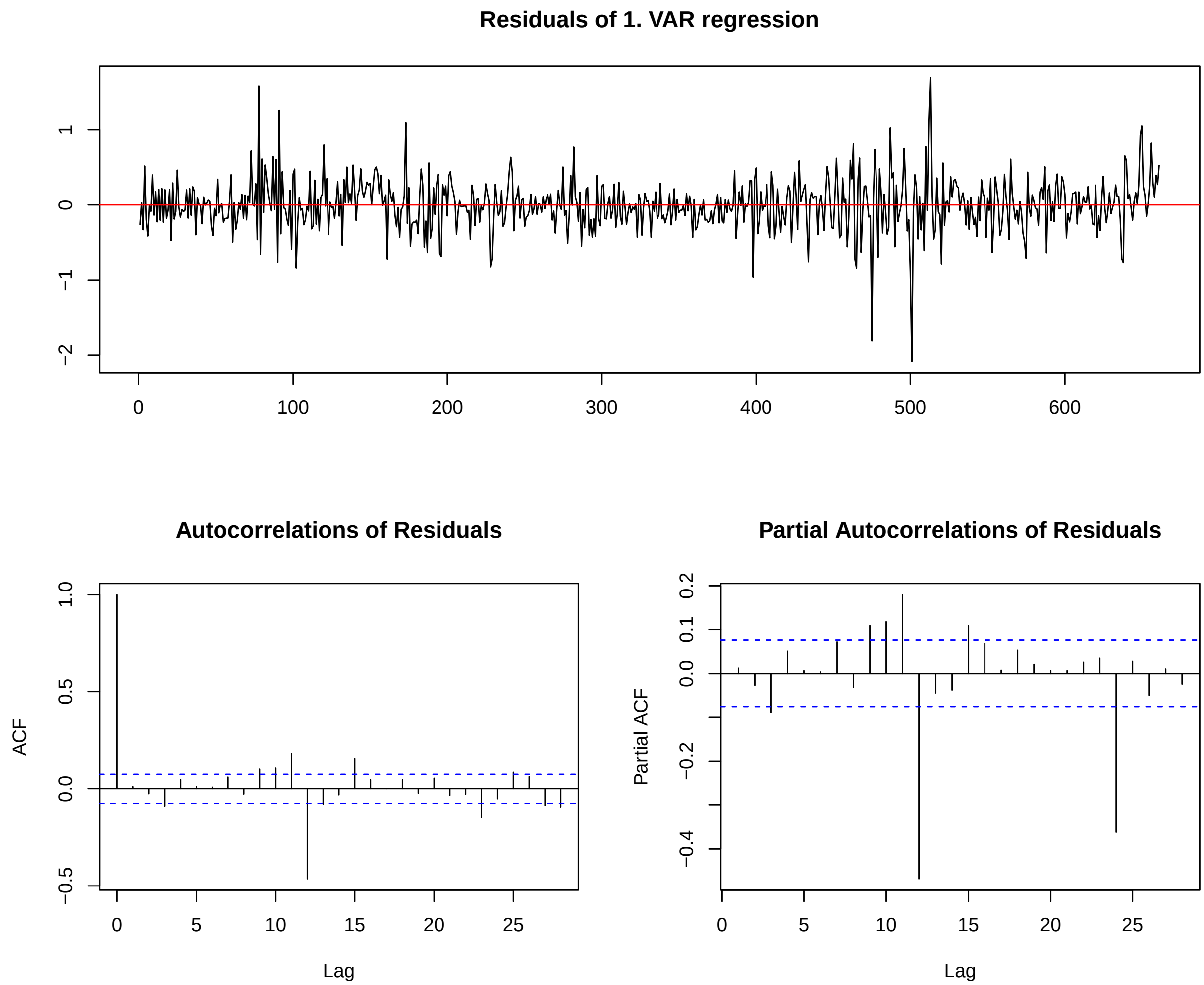


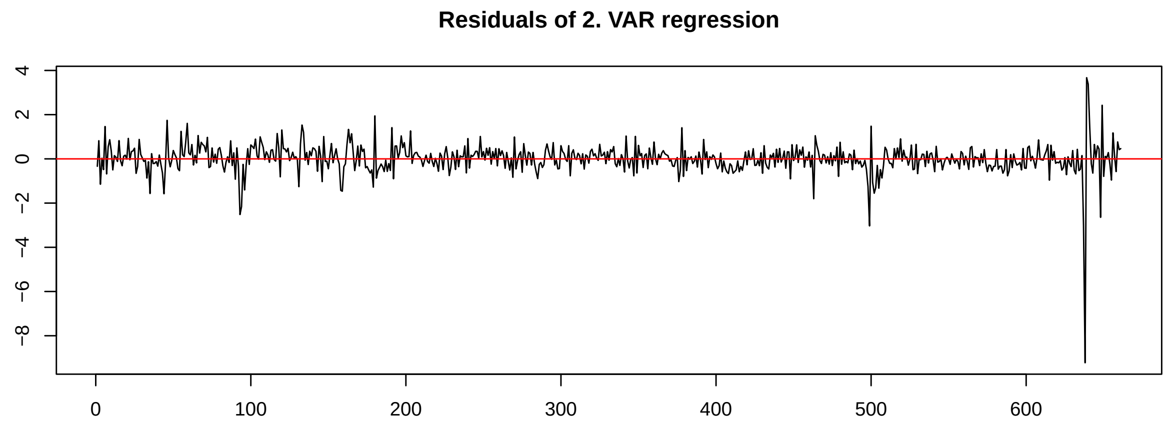


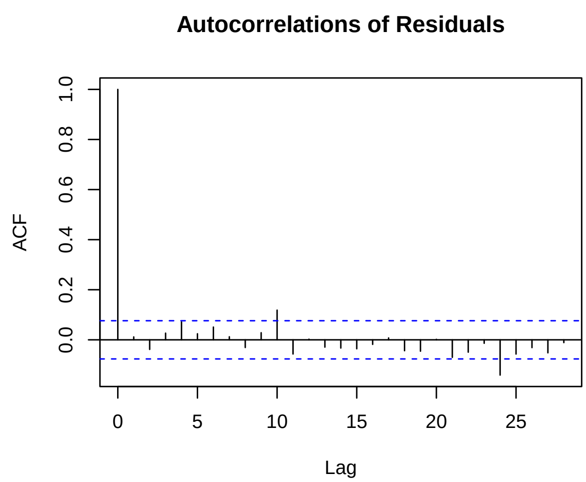


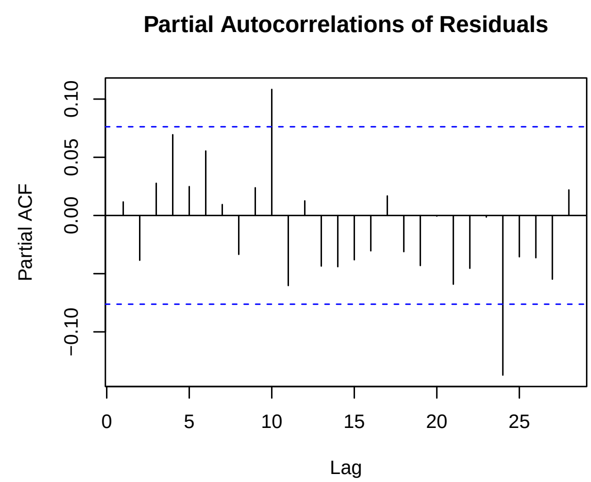



```
plotres(johansen.const)
```


Residuals of 1. VAR regression
1
0
−1
−2
0
100
200
300
400
500
600
Autocorrelations of Residuals
ACF
1.0
0.5
0.0
−0.5
0
5
10
15
20
25
Lag
Partial Autocorrelations of Residuals
Partial ACF
0.2
0.1
0.0
−0.2
−0.4
0
5
10
15
20
25
Lag







Inflation and TCU seem to not be cointegrated in levels.

ARDL

Dataset in Levels for the ARDL

```
library(dynamac)

CPI = pdfetch_FRED("CPIAUCSL")
names(CPI) = "CPI"

Inflation = diff(log(CPI), lag = 12) * 100
names(Inflation) = "Inflation"

Inflation = ts(Inflation, start=c(1947, 1), frequency=12)
Inflation = na.omit(Inflation)

TCU = pdfetch_FRED("TCU")
```

```r
names(TCU) = "TCU"
TCU = ts(TCU, start=c(1967, 1), frequency=12)

data.set = na.omit(
           ts.intersect(

       Inflation,
       TCU,

           dframe=TRUE))


Inflation   = ts( data.set$Inflation,    start=c(1967, 1),  frequency=12)
TCU         = ts( data.set$TCU,          start=c(1967, 1),  frequency=12)
```

The model in error correction is computing the impact of Inflation on TCU.

```r
set.seed(123)

lags = 2

ARDL = dynardl(
                      TCU ~ Inflation,

                lags = list("Inflation" = 1,          "TCU" = 1              ),
               diffs =    c("Inflation"                                      ),
            lagdiffs = list("Inflation" = c(1:lags),  "TCU" = c(1:lags)      ),

                  ec = TRUE,
            constant = TRUE,
               trend = FALSE,

            simulate = FALSE,
            shockvar = "Inflation",
               range = 50,
                sims = 1000,
            fullsims = TRUE,

                data = data.set)
```

```
## [1] "Error correction (EC) specified; dependent variable to be run in differences."
```

```r
summary(ARDL)
```

```
##
## Call:
## lm(formula = as.formula(paste(paste(dvnamelist), "~", paste(colnames(IVs),
##     collapse = "+"), collapse = " ")))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.9456 -0.3277  0.0147  0.3346  3.4830
```

```
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      1.565174   0.539942   2.899 0.003872 **
## l.1.TCU         -0.018845   0.006928  -2.720 0.006696 **
## ld.1.TCU         0.258712   0.038931   6.645 6.39e-11 ***
## ld.2.TCU        -0.031717   0.039031  -0.813 0.416733
## d.1.Inflation    0.287377   0.078837   3.645 0.000288 ***
## l.1.Inflation   -0.018357   0.011036  -1.663 0.096713 .
## ld.1.Inflation   0.124342   0.085131   1.461 0.144608
## ld.2.Inflation  -0.084717   0.079575  -1.065 0.287444
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6919 on 652 degrees of freedom
##   (3 observations deleted due to missingness)
## Multiple R-squared:  0.1266, Adjusted R-squared:  0.1172
## F-statistic:  13.5 on 7 and 652 DF,  p-value: 2.395e-16
```

The model with the current variables is not a good fit because the r-squared is 0.1266.

PSS Test

```
pssbounds(ARDL)
```

```
##
##  PESARAN, SHIN AND SMITH (2001) COINTEGRATION TEST
##
##  Observations: 660
##  Number of Lagged Regressors (not including LDV) (k): 1
##  Case: 3 (Unrestricted intercept; no trend)
##
##  ------------------------------------------------------
##  -                       F-test                       -
##  ------------------------------------------------------
##                 <------- I(0) ------------ I(1) ----->
##  10% critical value       4.04            4.78
##  5% critical value        4.94            5.73
##  1% critical value        6.84            7.84
##
##
##  F-statistic = 7.9
##  ------------------------------------------------------
##  -                       t-test                       -
##  ------------------------------------------------------
##                 <------- I(0) ------------ I(1) ----->
##  10% critical value       -2.57           -2.91
##  5% critical value        -2.86           -3.22
##  1% critical value        -3.43           -3.82
##
##
```

```
##  t statistic = -2.72
##  ------------------------------------------------------
##  F-statistic note: Asymptotic critical values used.
##  t-statistic note: Asymptotic critical values used.
```

Ho of no long-run relationship is rejected because the F-statistic 7.9 is greater than the critical value at the 1% significance level. The lagged levels are statistically significant.

```
pssbounds(ARDL, restriction=TRUE)
```

```
##
##  PESARAN, SHIN AND SMITH (2001) COINTEGRATION TEST
##
##  Observations: 660
##  Number of Lagged Regressors (not including LDV) (k): 1
##  Case: 2 (Intercept included in F-stat restriction; no trend)
##
##  ------------------------------------------------------
##  -                       F-test                       -
##  ------------------------------------------------------
##                  <------- I(0) ------------ I(1) ----->
##  10% critical value       3.02            3.51
##  5% critical value        3.62            4.16
##  1% critical value        4.94            5.58
##
##
##  F-statistic = 5.333
##
##  ------------------------------------------------------
##  F-statistic note: Asymptotic critical values used.
##  t-statistic note: Critical values do not currently exist for Case II.
```

Ho is not rejected at the 1% significance level but it is rejected at the 5% significance level.

Diagnostic Checks

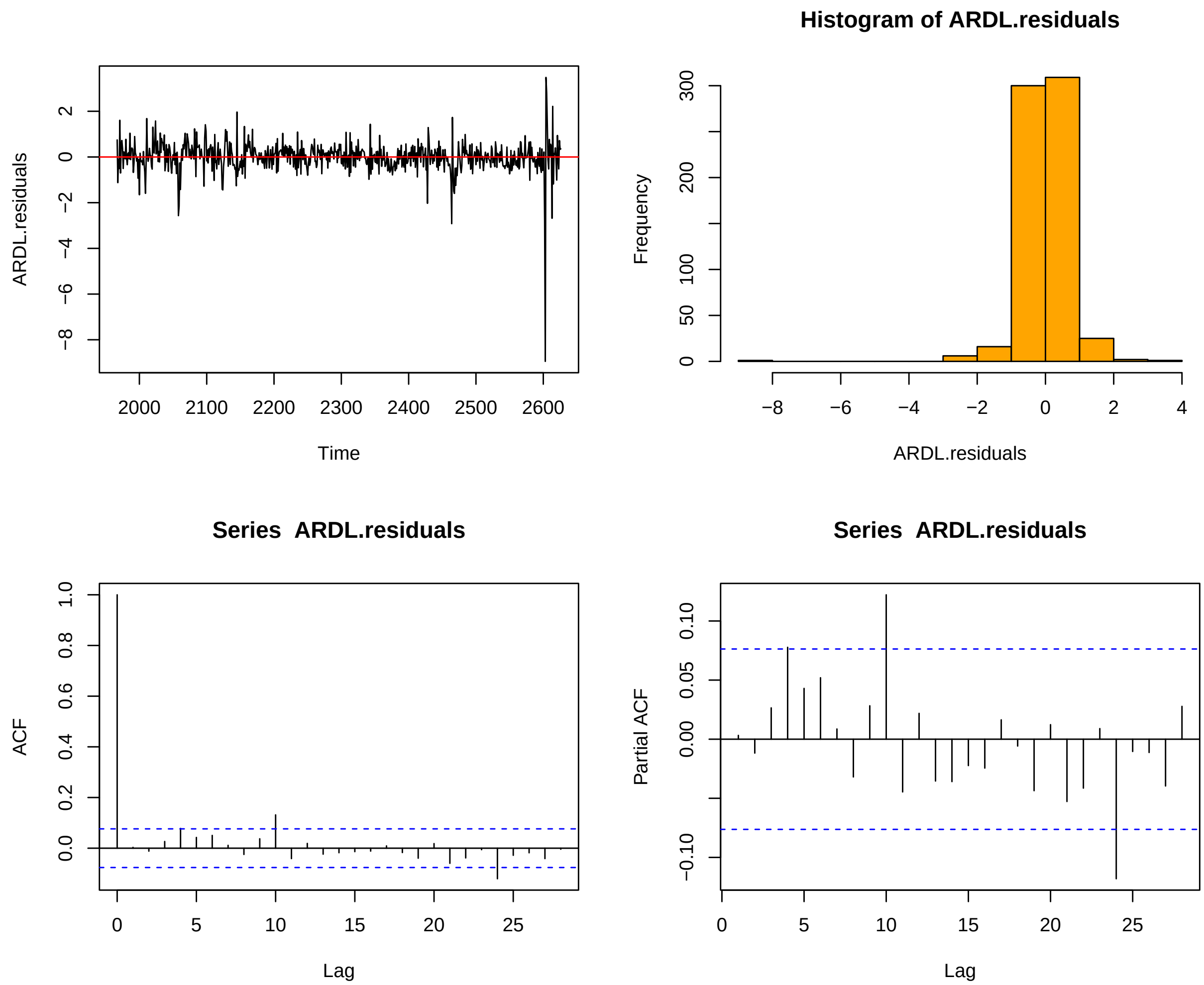
```
dynardl.auto.correlated(ARDL)
```

```
##
##  ------------------------------------------------------
##  Breusch-Godfrey LM Test
##  Test statistic: 2.043
##  p-value: 0.153
##  H_0: no autocorrelation up to AR 1
##
##  ------------------------------------------------------
##  Shapiro-Wilk Test for Normality
##  Test statistic: 0.791
##  p-value: 0
##  H_0: residuals are distributed normal
```

```
##
##  ------------------------------------------------------
##  Log-likelihood: -689.392
##  AIC: 1396.784
##  BIC: 1437.215
##  Note: AIC and BIC calculated with k = 8 on T = 660 observations.
##
##  ------------------------------------------------------
## Shapiro-Wilk test indicates we reject the null hypothesis of normality at p < 0.01.
```

Breusch-Godfrey test's Ho of no autocorrelation cannot be rejected at the 5% significance level. Shapiro-Wilk test's Ho of normal distribution is rejected at the 1% significance level.

```
ARDL.residuals =    ARDL$model$residuals
ARDL.residuals = ts(ARDL$model$residuals,  start=c(1967, 1) ,  frequency=1)




  par(mfrow = c(2, 2))
      plot( ARDL.residuals )
    abline( h=0, col="red" )
      hist( ARDL.residuals, col="orange")
       acf( ARDL.residuals )
      pacf( ARDL.residuals )
```



The distribution is left-skewed with a very long negative spike on the right side of the residuals plot. The ACF and PACF indicate no serial correlation in the residuals.

```
jarque.bera.test(ARDL.residuals)
```

```
##
##  Jarque Bera Test
##
## data:  ARDL.residuals
## X-squared = 56909, df = 2, p-value < 2.2e-16
```

Ho of normality is rejected at the 1% significance level.

```
bptest(ARDL$model)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  ARDL$model
## BP = 53.505, df = 7, p-value = 2.946e-09
```

Ho of no heteroskedasticity is rejected at the 1% significance level.

Heteroskedasticity needs to be corrected.

```
coeftest(ARDL$model, vcov.=vcovHC )
```

```
##
## t test of coefficients:
##
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       1.5651740  0.6907645  2.2659  0.02379 *
## l.1.TCU          -0.0188453  0.0083571 -2.2550  0.02446 *
## ld.1.TCU          0.2587120  0.1755596  1.4736  0.14106
## ld.2.TCU         -0.0317172  0.1430188 -0.2218  0.82456
## d.1.Inflation     0.2873770  0.1393642  2.0621  0.03960 *
## l.1.Inflation    -0.0183574  0.0141885 -1.2938  0.19619
## ld.1.Inflation    0.1243421  0.0855743  1.4530  0.14670
## ld.2.Inflation   -0.0847166  0.0909339 -0.9316  0.35187
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Standard errors are greater and t values are lower when heteroskedasticity is corrected. The regressors are less significant.

IRFs

The model is run with robust errors to plot the IRFs for the bootstrapped confidence interval over 50 months, with the exogenous shock coming from Inflation.

```
ARDL = dynardl(
                      TCU ~ Inflation,

                lags = list("Inflation" = 1,          "TCU" = 1             ),
               diffs =    c("Inflation"                                     ),
            lagdiffs = list("Inflation" = c(1:lags),  "TCU" = c(1:lags)     ),

                  ec = TRUE,
            constant = TRUE,
               trend = FALSE,

            simulate = TRUE,
            shockvar = "Inflation",
               range = 50,
                sims = 1000,
            fullsims = TRUE,

                data = data.set)
```

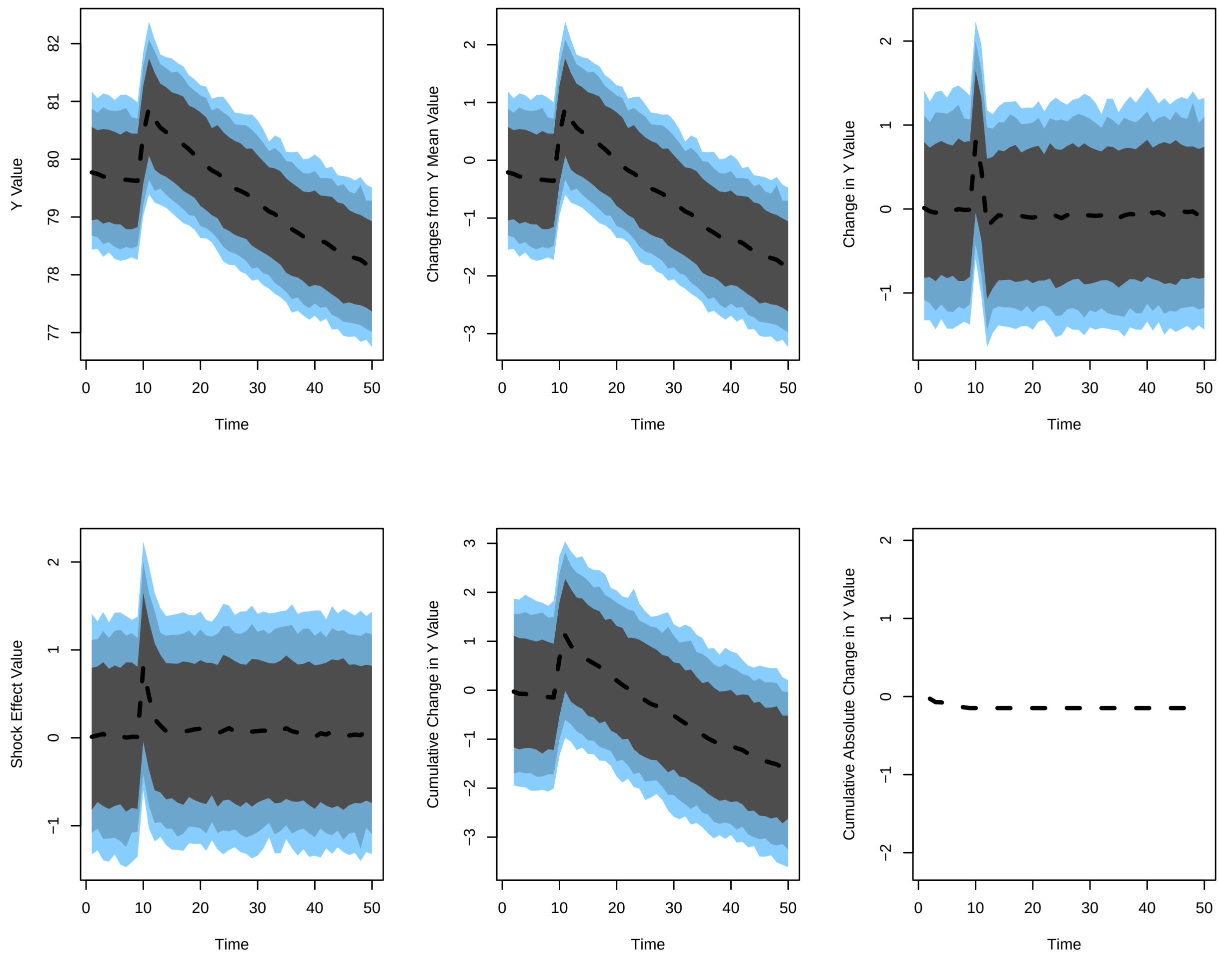
```
## [1] "Error correction (EC) specified; dependent variable to be run in differences."
## [1] "Inflation shocked by one standard deviation of Inflation by default."
## [1] "dynardl estimating ..."
##   |                                                                              |
```

```r
summary(ARDL)
```

```
##
## Call:
## lm(formula = as.formula(paste(paste(dvnamelist), "~", paste(colnames(IVs),
##     collapse = "+"), collapse = " ")))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.9456 -0.3277  0.0147  0.3346  3.4830
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)     1.565174   0.539942   2.899 0.003872 **
## l.1.TCU        -0.018845   0.006928  -2.720 0.006696 **
## ld.1.TCU        0.258712   0.038931   6.645 6.39e-11 ***
## ld.2.TCU       -0.031717   0.039031  -0.813 0.416733
## d.1.Inflation   0.287377   0.078837   3.645 0.000288 ***
## l.1.Inflation  -0.018357   0.011036  -1.663 0.096713 .
## ld.1.Inflation  0.124342   0.085131   1.461 0.144608
## ld.2.Inflation -0.084717   0.079575  -1.065 0.287444
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6919 on 652 degrees of freedom
##   (3 observations deleted due to missingness)
## Multiple R-squared:  0.1266, Adjusted R-squared:  0.1172
## F-statistic:  13.5 on 7 and 652 DF,  p-value: 2.395e-16
```

According to the coefficients of the model, in explaining the current value TCU, the differenced first lag of Inflation is significant at the 0.1 % level and has a positive impact on the TCU.

```r
dynardl.all.plots(ARDL)
```



The cumulative change in the dependent variable indicates that when Inflation increases, the TCU increases as well. After 1 year, TCU increases by 1 p.p after a shock of one unit to Inflation. Afterwards, TCU exhibits a downward trend.

In conclusion, The ARDL model confirms the argument constructed at the beginning of the project. Nevertheless, the model is not correctly specified because the R-squared is low and the residuals are not well-behaving. Inflation needs to be complemented with more variables to predict TCU.

Bivariate VAR Model

In the ARDL model, the independent variables are exogenous, but it is very difficult to apply such analysis in reality. The VAR model attempts to overcome the ubiquitous endogeneity problem by treating all the variables in the model as endogenous to one another. As there is no evidence of cointegration and the variables are non-stationary in levels, it is necessary to run a VAR model in first differences.

Dataset in First Differences for VAR

```
CPI = pdfetch_FRED("CPIAUCSL")
names(CPI) = "CPI"

Inflation = diff(log(CPI), lag = 12) * 100
names(Inflation) = "Inflation"

Inflation = ts(Inflation, start=c(1947, 1), frequency=12)
Inflation = na.omit(Inflation)

TCU = pdfetch_FRED("TCU")
names(TCU) = "TCU"
TCU = ts(TCU, start=c(1967, 1), frequency=12)


Inflation    =  diff(Inflation)
TCU          =  diff(TCU)



data.set = na.omit(
           ts.intersect(

       Inflation,
       TCU,

           dframe=TRUE))


Inflation   = ts( data.set$Inflation,    start=c(1967, 2),  frequency=12)
TCU         = ts( data.set$TCU,          start=c(1967, 2),  frequency=12)

  data.set = ts(data.set, start=c(1967, 2), frequency=12)
```

Optimal Lag Selection in First Differences

```
VARselect(data.set, lag.max=10, type="const", season = NULL,
          exogen = NULL)$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##     10      1      1     10
```

VAR Model Results

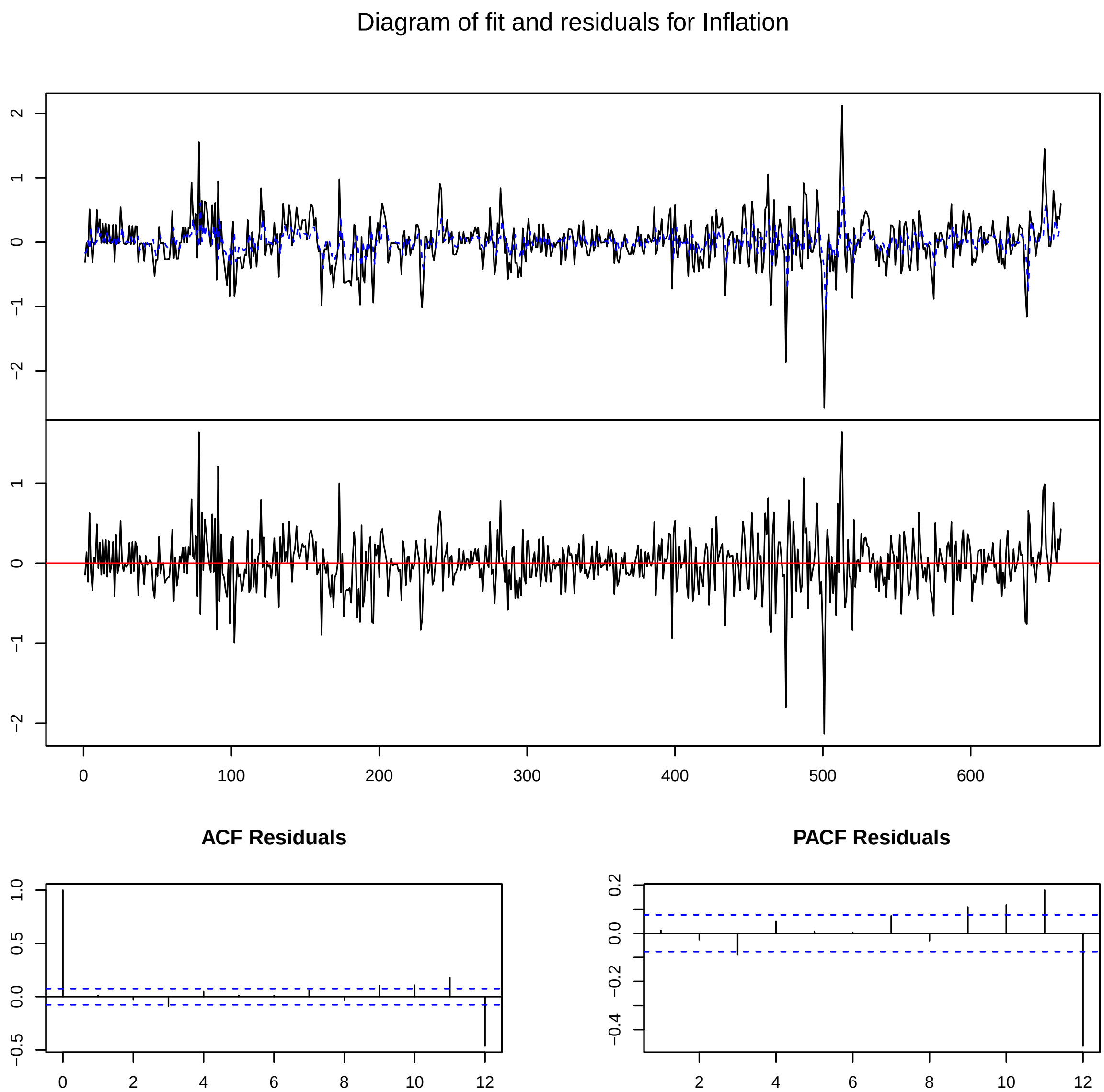
```
optimal.lags = 1


var.model.const  <- VAR(data.set, p=optimal.lags, type="const", exogen=NULL)
summary(var.model.const)
```

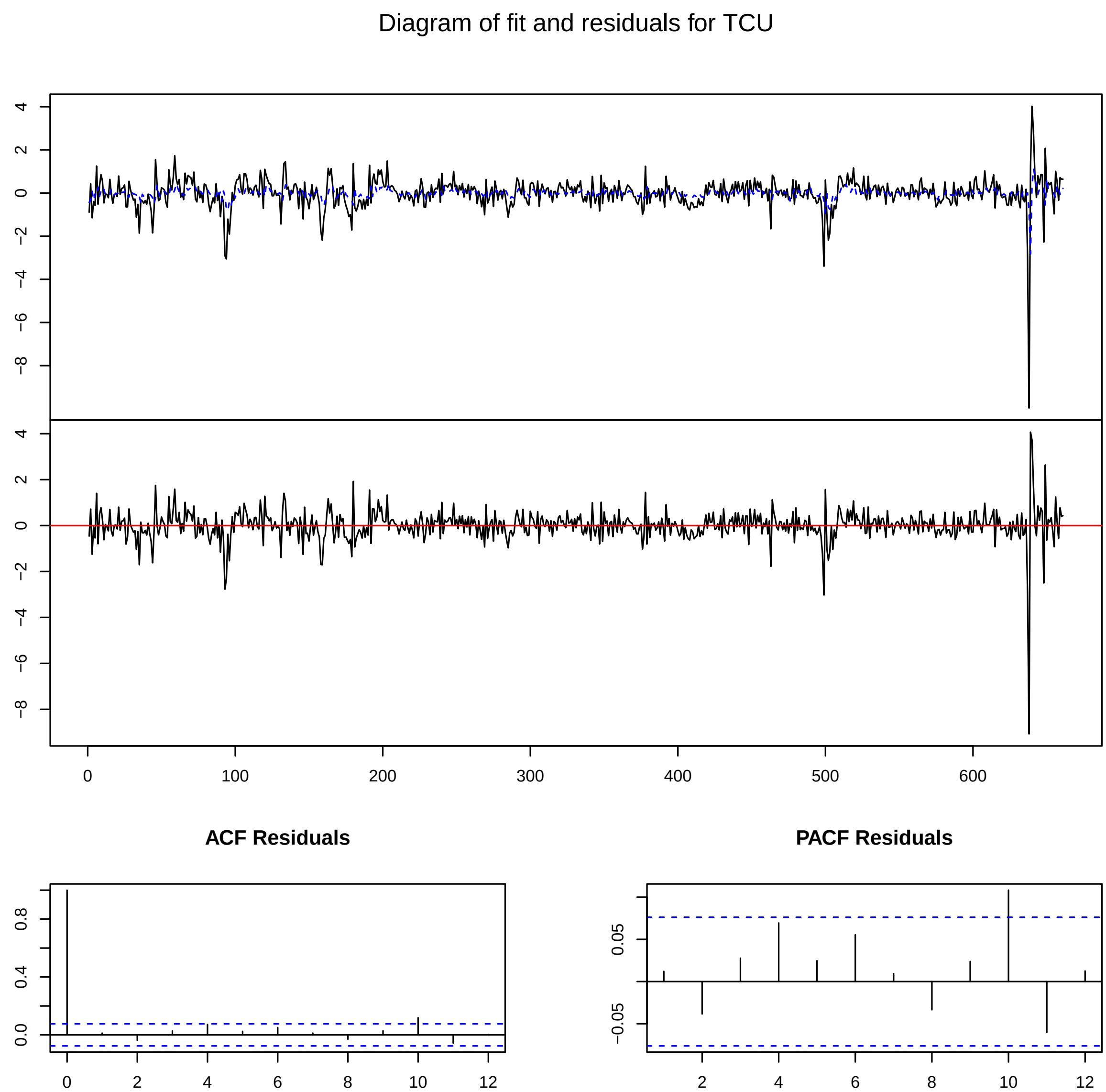
```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: Inflation, TCU
## Deterministic variables: const
## Sample size: 661
## Log Likelihood: -937.279
## Roots of the characteristic polynomial:
## 0.4265 0.2336
## Call:
## VAR(y = data.set, p = optimal.lags, type = "const", exogen = NULL)
##
##
## Estimation results for equation Inflation:
## ==========================================
## Inflation = Inflation.l1 + TCU.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## Inflation.l1 0.395733   0.036214  10.928   <2e-16 ***
## TCU.l1       0.033362   0.018646   1.789    0.074 .
## const        0.006063   0.013513   0.449    0.654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.3472 on 658 degrees of freedom
## Multiple R-Squared: 0.1701,  Adjusted R-squared: 0.1676
## F-statistic: 67.43 on 2 and 658 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation TCU:
## ====================================
## TCU = Inflation.l1 + TCU.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## Inflation.l1  0.14946    0.07365   2.029   0.0428 *
## TCU.l1        0.26439    0.03792   6.972  7.6e-12 ***
## const        -0.01095    0.02748  -0.398   0.6905
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.7062 on 658 degrees of freedom
## Multiple R-Squared: 0.08388, Adjusted R-squared: 0.0811
## F-statistic: 30.12 on 2 and 658 DF,  p-value: 3.034e-13
##
##
##
## Covariance matrix of residuals:
##           Inflation     TCU
## Inflation   0.12057 0.03394
## TCU         0.03394 0.49865
##
## Correlation matrix of residuals:
```

```
##           Inflation    TCU
## Inflation    1.0000 0.1384
## TCU          0.1384 1.0000
```

According to the coefficients of the model, in explaining the current value TCU, the first lag of Inflation is significant at the 5% level. Therefore, the cumulative IRFs are expected to show an increase in TCU after a shock of one unit to Inflation. Nevertheless, the R-squared is very close to 0, thus the model is not correctly specified.

```
plot(var.model.const)
```





There is no persistence in the residuals.

Granger-causality Test with Robust Errors

```
causality(var.model.const, cause="Inflation", boot=FALSE, boot.runs=1000,
          vcov.=vcovHC(var.model.const))
```

```
## $Granger
##
##  Granger causality H0: Inflation do not Granger-cause TCU
##
## data:  VAR object var.model.const
```

```
## F-Test = 2.7224, df1 = 1, df2 = 1316, p-value = 0.09919
##
##
## $Instant
##
##  H0: No instantaneous causality between: Inflation and TCU
##
## data:  VAR object var.model.const
## Chi-squared = 12.426, df = 1, p-value = 0.0004233
```

The p-value is 0.099. There is evidence of Granger-causality from Inflation to TCU at 10% significance level.

```
causality(var.model.const, cause="TCU", boot=FALSE, boot.runs=1000,
          vcov.=vcovHC(var.model.const))
```

```
## $Granger
##
##  Granger causality H0: TCU do not Granger-cause Inflation
##
## data:  VAR object var.model.const
## F-Test = 1.0456, df1 = 1, df2 = 1316, p-value = 0.3067
##
##
## $Instant
##
##  H0: No instantaneous causality between: TCU and Inflation
##
## data:  VAR object var.model.const
## Chi-squared = 12.426, df = 1, p-value = 0.0004233
```

No evidence of Granger-causality from the TCU to Inflation.

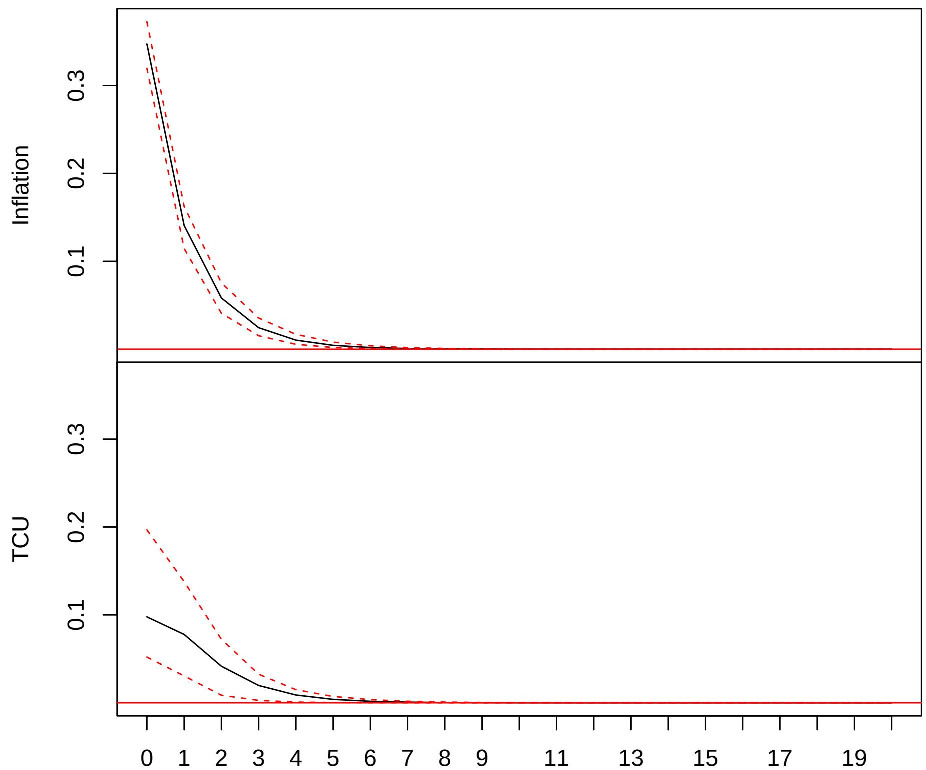
Cholesky Decompositions for Orthogonal Errors

Due to the Granger-causality results, the optimal ordering is from Inflation to TCU. Non-cumulative and cumulative IRFs are computed accordingly.

```
ordered.data.set = data.set[, c("Inflation","TCU")]

var.ordered.const  <- VAR(ordered.data.set,  p=optimal.lags,   type="const",
                          exogen=NULL)
```

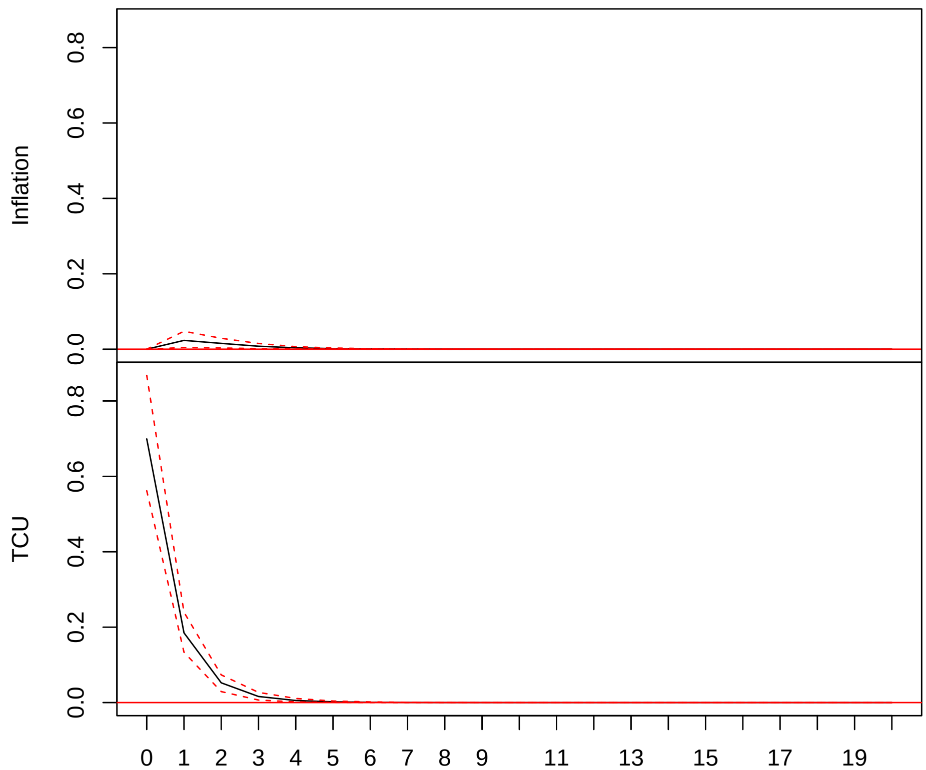
```
plot(irf(var.ordered.const, n.ahead=20, ortho=TRUE, cumulative=FALSE,
         boot=TRUE, ci=0.90, runs=100))
```

Orthogonal Impulse Response from Inflation



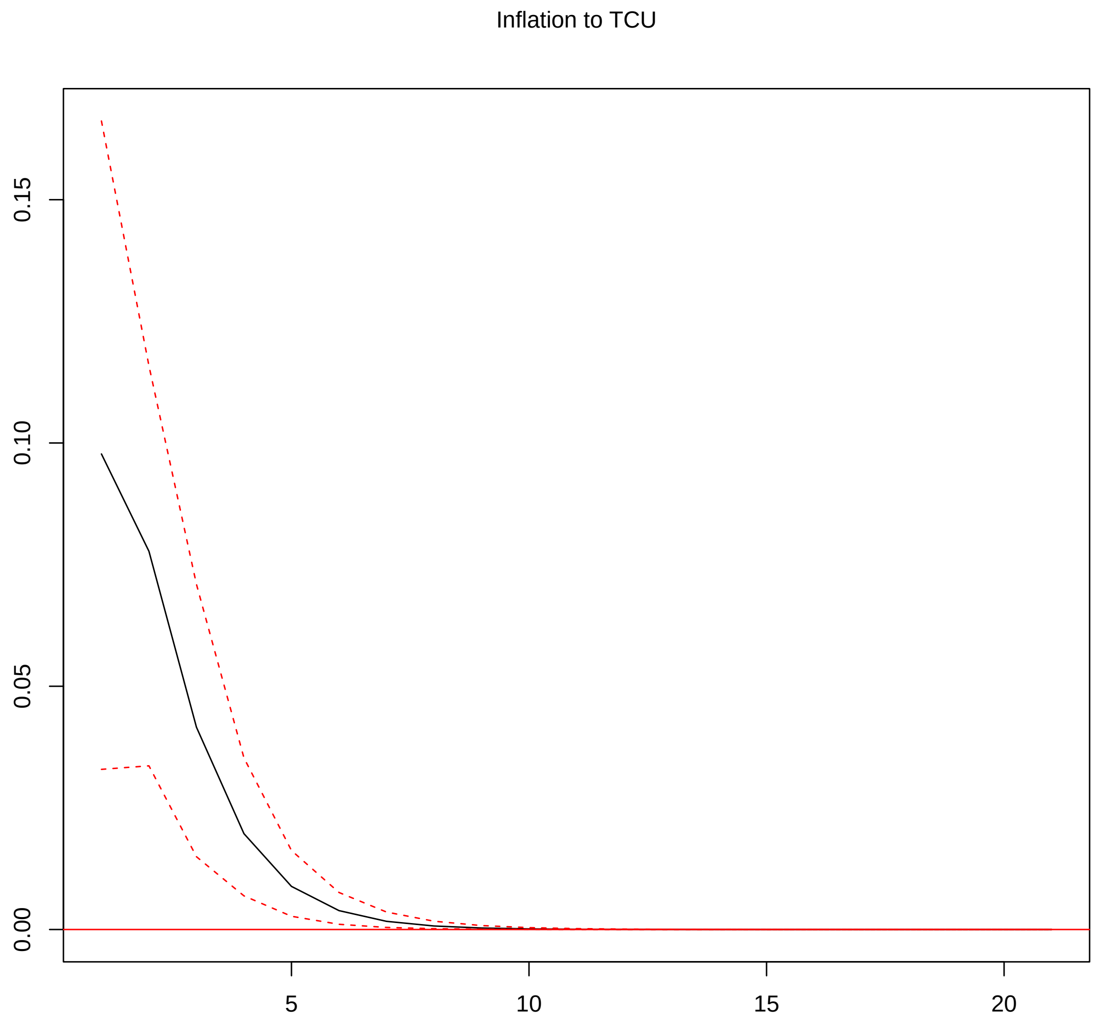
90 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from TCU



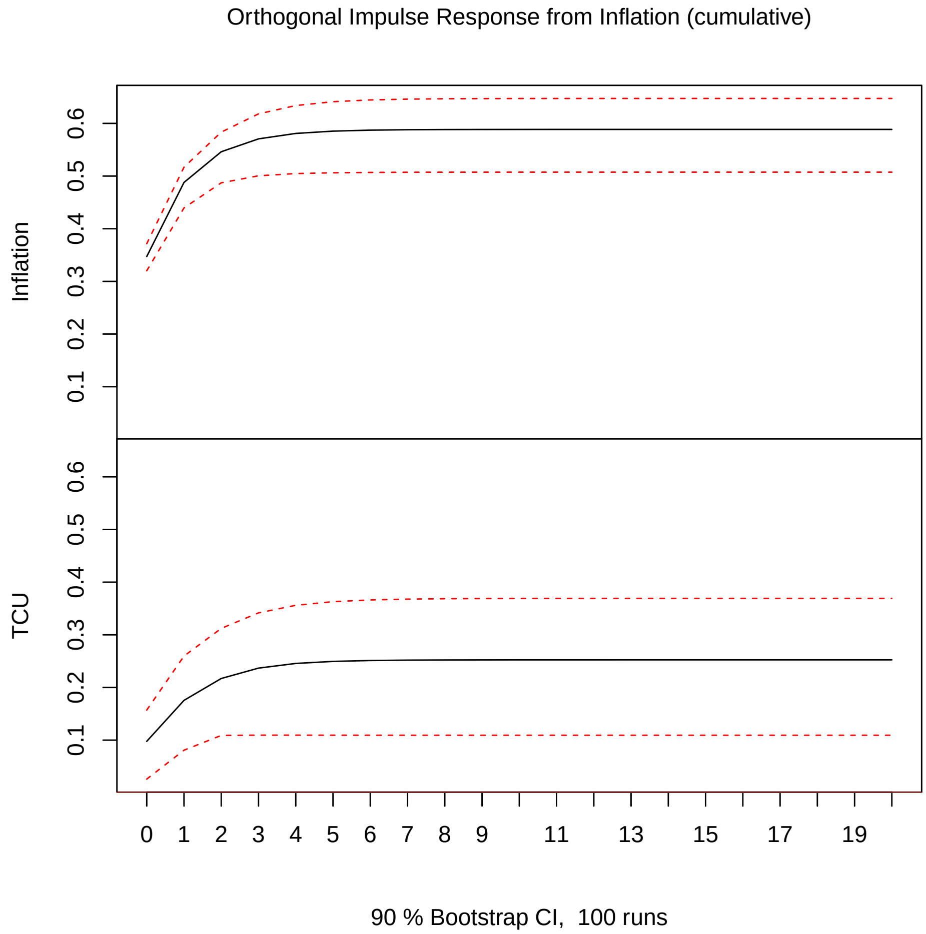
90 % Bootstrap CI, 100 runs

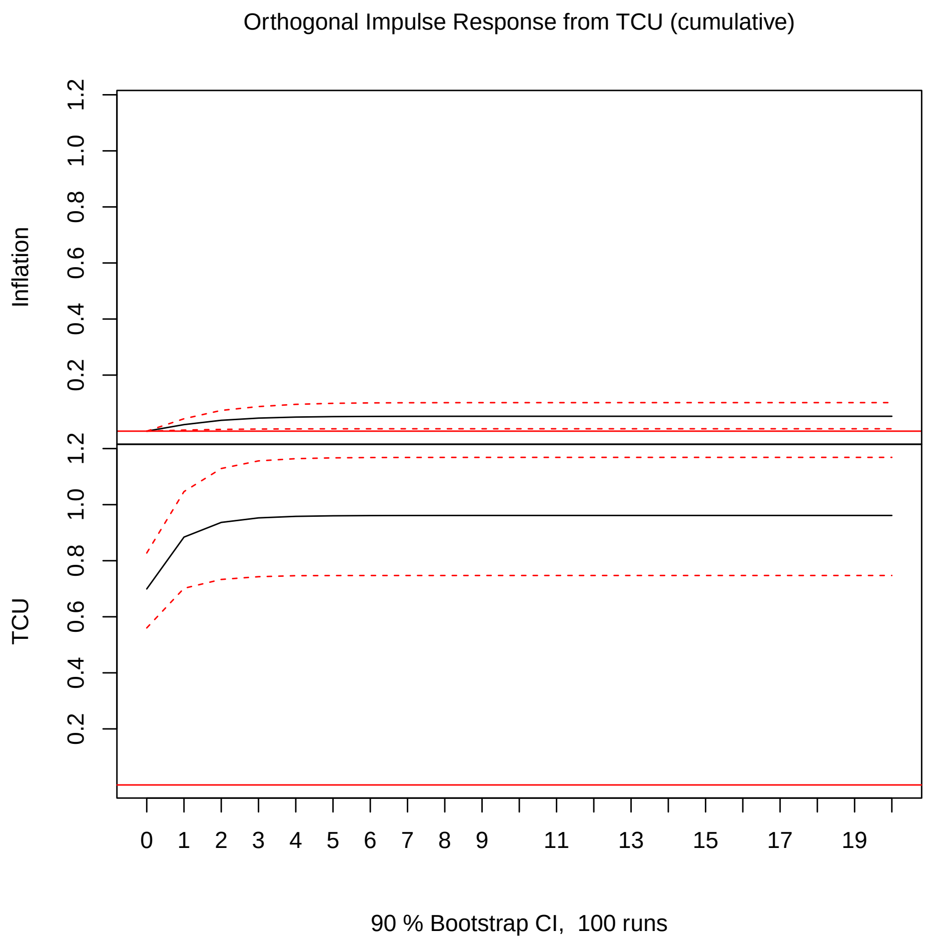
```
plot(irf(var.ordered.const,  impulse="Inflation",
    response="TCU", n.ahead=20, ortho=TRUE, cumulative=FALSE,
    boot=TRUE, ci=0.90, runs=100, seed=NULL),
            main="Inflation to TCU", xlab="Lag", ylab="", sub="", oma=c(3,0,3,0))
```



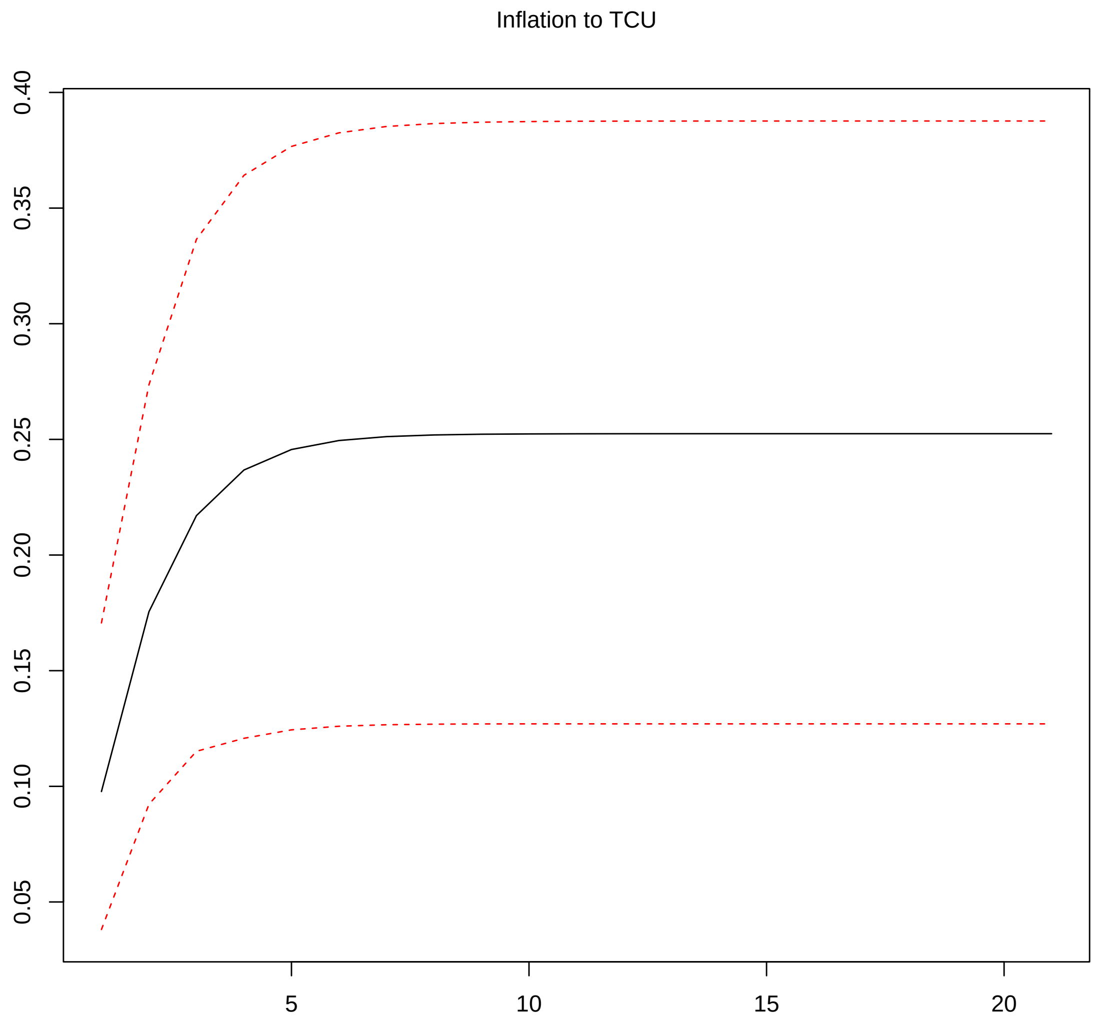
TCU remains inside the red lines and therefore when affected by Inflation behaves well. The IRFs converge to 0 over time, as expected because unit roots are absent.

```
plot(irf(var.ordered.const, n.ahead=20, ortho=TRUE, cumulative=TRUE,
         boot=TRUE, ci=0.90, runs=100))
```


Orthogonal Impulse Response from Inflation (cumulative)
Inflation
TCU
0.1
0.2
0.3
0.4
0.5
0.6
0 1 2 3 4 5 6 7 8 9 11 13 15 17 19
90 % Bootstrap CI, 100 runs




```
plot(irf(var.ordered.const,  impulse="Inflation",
         response="TCU", n.ahead=20, ortho=TRUE, cumulative=TRUE,
         boot=TRUE, ci=0.90, runs=100, seed=NULL),
            main="Inflation to TCU", xlab="Lag", ylab="", sub="", oma=c(3,0,3,0))
```



The IRFs do not converge to 0 overtime and the impulse is positive as expected. Residuals seem well-behaving.

Diagnostic Checks

```
serialtest <- serial.test(var.model.const, type = "PT.asymptotic")
serialtest
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object var.model.const
## Chi-squared = 273.31, df = 60, p-value < 2.2e-16
```

There is evidence of autocorrelation (p-value < 0.05).

```
serialtest <- serial.test(var.model.const, type = "PT.adjusted")
serialtest
```

```
##
##  Portmanteau Test (adjusted)
##
## data:  Residuals of VAR object var.model.const
## Chi-squared = 277.87, df = 60, p-value < 2.2e-16
```

There is evidence of autocorrelation.

```r
serialtest <- serial.test(var.model.const, type = "BG")
serialtest
```

```
##
##  Breusch-Godfrey LM test
##
## data:  Residuals of VAR object var.model.const
## Chi-squared = 29.427, df = 20, p-value = 0.07968
```

There is evidence of no autocorrelation.

```r
serialtest <- serial.test(var.model.const, type = "ES")
serialtest
```

```
##
##  Edgerton-Shukur F test
##
## data:  Residuals of VAR object var.model.const
## F statistic = 1.4735, df1 = 20, df2 = 1294, p-value = 0.08126
```

There is evidence of no autocorrelation.

```r
normalitytest <- normality.test(var.ordered.const)
normalitytest
```

```
## $JB
##
##  JB-Test (multivariate)
##
## data:  Residuals of VAR object var.ordered.const
## Chi-squared = 50247, df = 4, p-value < 2.2e-16
##
##
## $Skewness
##
##  Skewness only (multivariate)
##
## data:  Residuals of VAR object var.ordered.const
## Chi-squared = 952.78, df = 2, p-value < 2.2e-16
##
##
## $Kurtosis
##
##  Kurtosis only (multivariate)
```

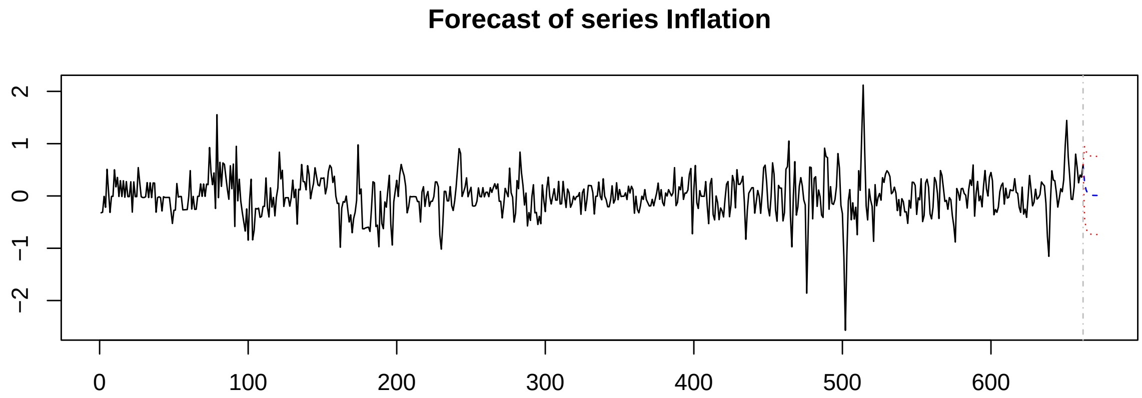
```
##
## data:  Residuals of VAR object var.ordered.const
## Chi-squared = 49294, df = 2, p-value < 2.2e-16
```

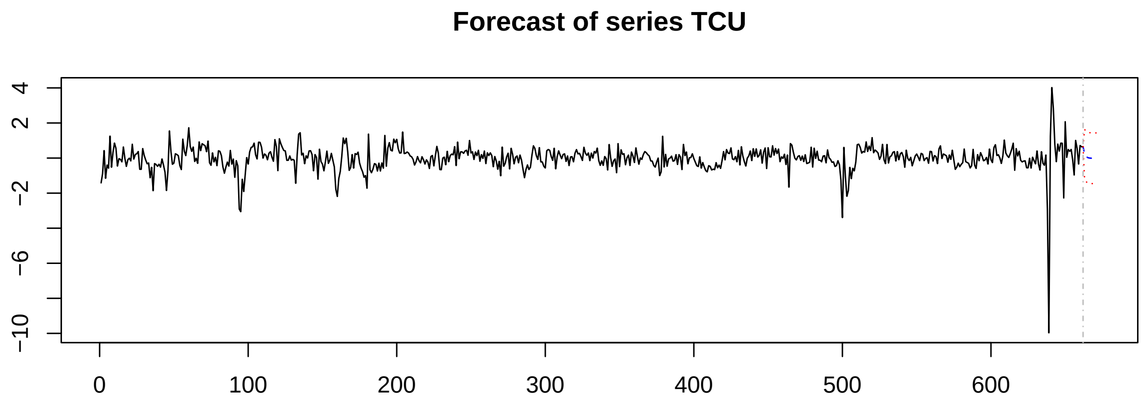
The residuals are not normally distributed.

The residuals might be autocorrelated, and they are not normally distributed. The model is not correctly specified.

VAR Model Forecast

```
var.prd.const <- predict(var.model.const, n.ahead = 10, ci = 0.95)
plot(var.prd.const)
```





Inflation and TCU exhibit a downward forecasted trend in the next 10 months.

ARIMA Model for Forecasting

The ARIMA model uses only one variable, modelled in relation to its past behaviour, to identify how one variable can predict itself in the future.

Dataset in Levels for ARIMA

```
CPI = pdfetch_FRED("CPIAUCSL")
names(CPI) = "CPI"

Inflation = diff(log(CPI), lag = 12) * 100
names(Inflation) = "Inflation"

Inflation = ts(Inflation, start=c(1947, 1), frequency=12)
Inflation = na.omit(Inflation)

TCU = pdfetch_FRED("TCU")
names(TCU) = "TCU"
TCU = ts(TCU, start=c(1967, 1), frequency=12)


data.set = na.omit(
           ts.intersect(

       Inflation,
       TCU,

           dframe=TRUE))


Inflation   = ts( data.set$Inflation,    start=c(1967, 1),  frequency=12)
TCU         = ts( data.set$TCU,          start=c(1967, 1),  frequency=12)



data.set = ts(data.set, start=c(1967, 1), frequency=12)
```

ARIMA Model Selection

The model will predict the behaviour of Inflation.

```
x = Inflation

   arima.model = forecast::auto.arima(x,

                                  D = 1,
                         stationary = FALSE,
                                 ic = c("aicc", "aic", "bic"),
                           stepwise = FALSE,
                      approximation = FALSE,
                           seasonal = TRUE,
```

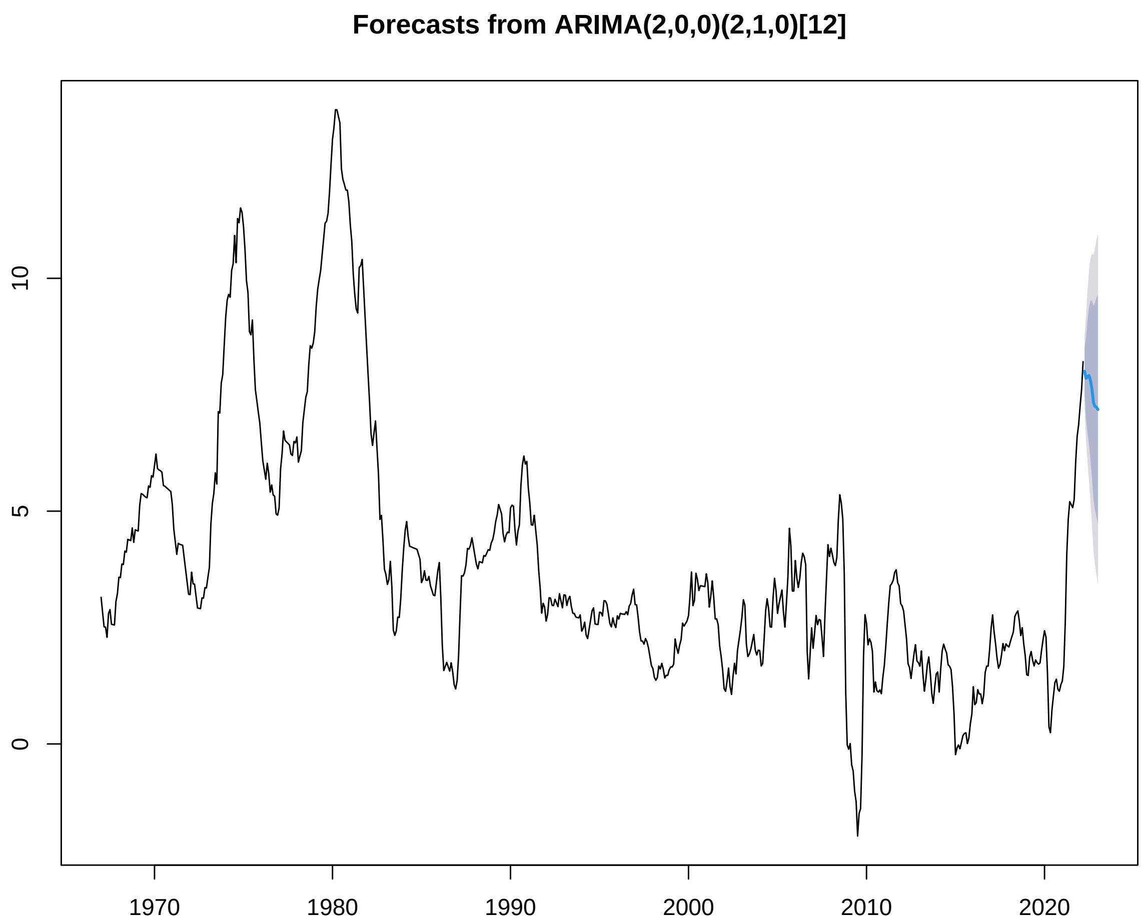
```r
                          allowdrift = TRUE   )


arima.model
```

```
## Series: x
## ARIMA(2,0,0)(2,1,0)[12]
##
## Coefficients:
##          ar1      ar2     sar1     sar2
##       1.4654  -0.4838  -0.9813  -0.5205
## s.e.  0.0348   0.0348   0.0341   0.0336
##
## sigma^2 = 0.15:  log likelihood = -312.71
## AIC=635.41   AICc=635.5   BIC=657.8
```

The best model for predicting the trend of Inflation is ARIMA (2,0,0) (2,1,0) [12]. The model is AR(2), therefore the current value is based on the previous 2 values. Also, the best model is in levels and does not rely on the moving average of past forecasting errors, MA(0).

ARIMA Forecast

```r
ARIMA.forecast = forecast::forecast(arima.model, h = 10)

plot(ARIMA.forecast)
```



In the next 10 months, Inflation decreases, but the confidence intervals are wide.

SARIMA Forecast

To compare the prediction of the ARIMA model, the SARIMA model is computed for a within-sample forecast.

```
library(smooth)

sarima.model = smooth::msarima(x, orders=list(

                                                    ar = c(2,2),
                                                     i = c(0,1),
                                                    ma = c(1,0)),
                                                  lags = c(1,12),
                                                     h = 10,
                                               holdout = TRUE)


sarima.model
```

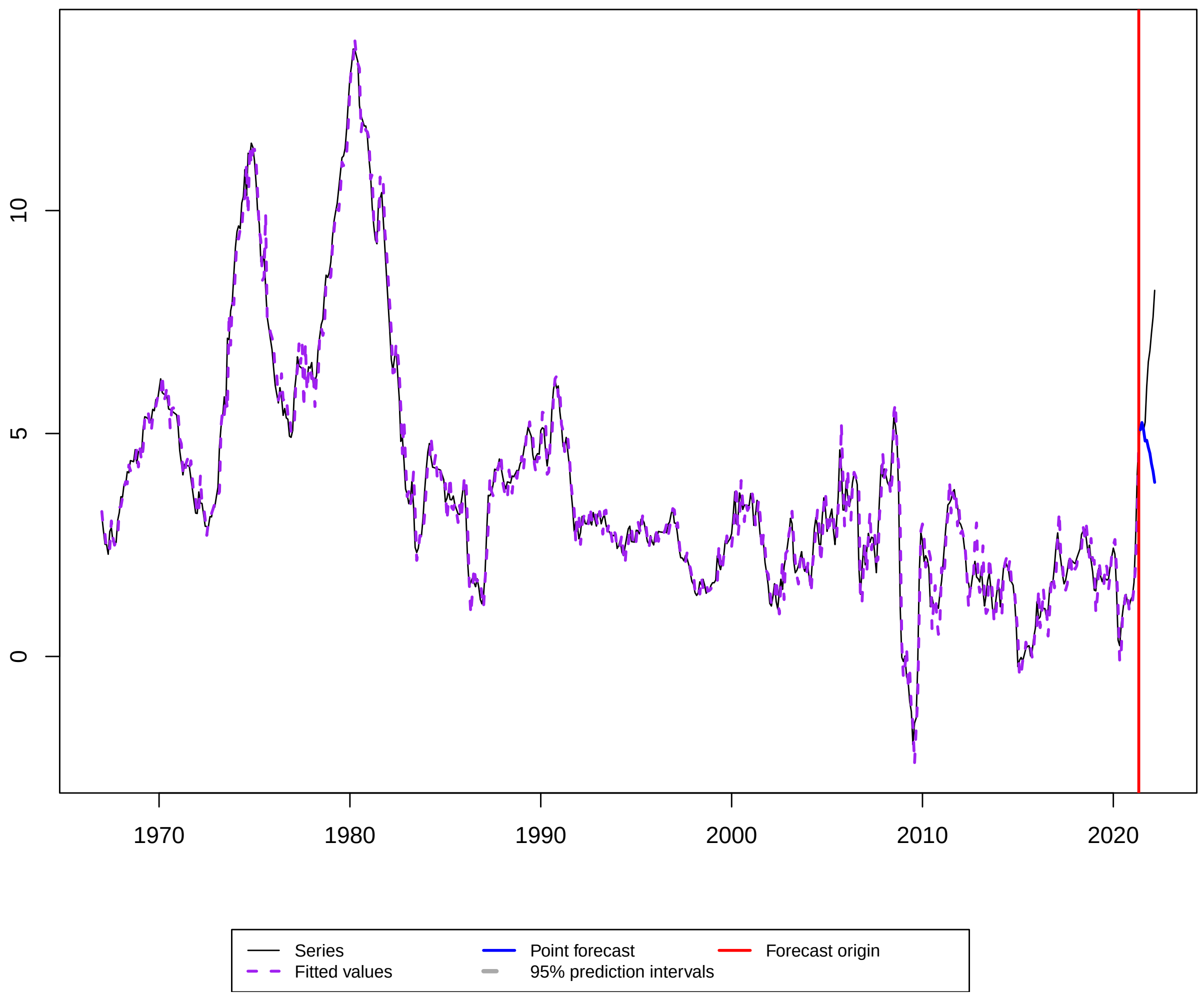
```
## Time elapsed: 1.11 seconds
## Model estimated: SARIMA(2,0,1)[1](2,1,0)[12]
## Matrix of AR terms:
##         Lag 1  Lag 12
## AR(1)  1.4728 -1.0064
## AR(2) -0.4929 -0.5437
## Matrix of MA terms:
##         Lag 1
## MA(1) -0.0121
## Initial values were produced using backcasting.
##
## Loss function type: likelihood; Loss function value: 297.7593
## Error standard deviation: 0.3835
## Sample size: 653
## Number of estimated parameters: 6
## Number of degrees of freedom: 647
## Information criteria:
##      AIC     AICc      BIC     BICc
## 607.5186 607.6487 634.4081 634.8295
##
## Forecast errors:
## MPE: 22.9%; sCE: 428.4%; Asymmetry: 95%; MAPE: 23.3%
## MASE: 6.28; sMAE: 43.4%; sMSE: 33.3%; rMAE: 1.119; rRMSE: 1.203
```

The estimated SARIMA model is (2,0,1)[1] (2,1,0) [12].

```
summary(sarima.model)
```


```
## Time elapsed: 1.11 seconds
## Model estimated: SARIMA(2,0,1)[1](2,1,0)[12]
## Matrix of AR terms:
##         Lag 1  Lag 12
## AR(1)  1.4728 -1.0064
## AR(2) -0.4929 -0.5437
## Matrix of MA terms:
##         Lag 1
## MA(1) -0.0121
## Initial values were produced using backcasting.
##
## Loss function type: likelihood; Loss function value: 297.7593
## Error standard deviation: 0.3835
## Sample size: 653
## Number of estimated parameters: 6
## Number of degrees of freedom: 647
## Information criteria:
##      AIC     AICc      BIC     BICc
## 607.5186 607.6487 634.4081 634.8295
##
## Forecast errors:
## MPE: 22.9%; sCE: 428.4%; Asymmetry: 95%; MAPE: 23.3%
## MASE: 6.28; sMAE: 43.4%; sMSE: 33.3%; rMAE: 1.119; rRMSE: 1.203
```

```r
values = sarima.model

greybox::graphmaker(x,values$forecast,values$fitted,values$lower,values$upper,
                    level=0.95,legend=TRUE)
```



According to the forecast, inflation should have decreased in the last 10 months but according to present values, it has increased. The SARIMA model could not reliably predict the future, thus the ARIMA model might not be meaningful.

Conclusion

The variables appeared to be I(1) in levels and I(0) in first differences. However, the unit root tests results are conflicting as TCU is I(0) in levels and in first differences at the 5% significance level, according to the ADF and PP tests. Nevertheless, both series are I(0) in first differences, and it was assumed that they are I(1) in levels. As cointegration tests results were ambiguous, it was assumed that there is no cointegration between the two variables. The ARDL model was not correctly specified but the PSS tests indicated a long-run relationship at the 5% significance level from Inflation to TCU, and a positive impact could be inferred from the IRFs. Following the results of the project the VAR model was computed and the Granger-causality test indicated a Granger-causality at the 10% significance level from Inflation to TCU. The VAR IRFs were computed and again TCU seemed to adapt to Inflation. A VAR forecast was constructed, showing a decrease in Inflation in the next 10 months. Nevertheless, the residuals were not normally distributed and the R-squared was too low. The ARIMA forecast was constructed and compared to the SARIMA forecast. The SARIMA could not predict the past behaviour of inflation, thus the ARIMA model appears to be inadequate for Inflation forecasting. The results of the project are not adequate to accurately assess the significance of causality from Inflation to TCU and the auto-regressive behaviour of Inflation. Further research is required for meaningful results.