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A FREQUENCY ASSIGNMENT PROBLEM

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Abstract A frequency assignment problem is introduced and a binary programming model for solving it is formulated. The model is implemented using AIMMS modelling language. As there are situations where AIMMS fails to solve the integer linear programming problem, a new dynamic model for finding an optimal frequency assignment is developed and also implemented in C++ programming language.

1. PROBLEM FORMULATION

As a result of the large number of mobile communication systems, there is an increasing need to allocate and re-allocate frequencies for point to point communications. Frequency allocations remain operational for seconds/minites in cellular communications, days/weeks in military communication systems or months/years in television systems. During these operational periods, usually the volume of traffic changes significantly, which causes point-to-point capacity and interference problems. Hence, in practice frequency assignment is a recurring process. In this paper a specific frequency allocation problem is modelled as described next.

We consider a satellite communication system in which a station A sends a signal through a satellite to a station B. A *link* in such a communication system is any pair A, B of communicating ground stations. The *frequency domain* is a specific range of frequencies available for allocation. Usually the frequency span is continuous, but for modelling reasons it can be divided up into fixed-width sufficiently small portions, referred to as *channels.* Any specific link requires a pre-specified number of adjacent channels, which is referred to as *frequency interval* or *link width.* The concepts of *channel* and *frequency interval* are represented in fig. 1. *Link interference* represents a combined measure of the transmitter and receiver interference as caused by other existing communication systems.

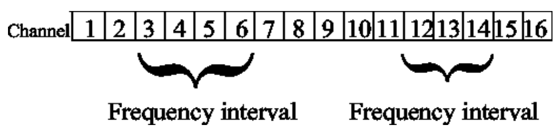

Fig. 1. Channels and intervals in a frequency span.

A *frequency allocation* is the assignment of a frequency interval to at most one link in the communication system. An *optimal frequency allocation* is one in which some measure of total interference is minimized.

For a given link, the overall level of interference is depending on the interference over its entire frequency interval. In our model formulation, we assume that interference data are available on a per channel basis, for each link between two stations. Furthermore, for each interval-link combination, it is assumed that the overall interference of the link is equal to the value of the mean channel interference (one can also consider the value of maximum) that is found in the interval.

To illustrate this, we consider a small example data set consisting of three links with seven adjacent channels available for transmission. The first link requires one channel for transmission, while the other two links must be allocated a frequency interval containing three channels. Table 1 presents the interference level for each link on a per channel basis. Using this data, the overall interference of each interval-link is found by averaging the interferences in the corresponding frequency interval. The values are also presented in Table 1.

	Channel	link	interference	Interval	link	interference
	Link 1	Link 2	Link 3	Link 1	Link 2	Link 3
Channel 1	4	4	3	4	5	3
Channel 2	3	5	4	3	6	4
Channel 3	6	6	2	6	7	3
Channel 4	8	7	6	8	8	3
Channel 5	3	8	1	3	9	2
Channel 6	2	9	2	2	-	-
Channel 7	1	10	3	1	-	-

Table 1: Channel and interval interference data.

2. THE INTEGER PROGRAMMING MODEL AND ITS EXTENSION

In order to formulate our model, the following notations are introduced:

Sets:

Number of links L with index $l \in 1, 2, ..., L$;

Number of channels F with index $f \in 1, 2, ..., F$;

Parameters:

The width of each link $w_l, \forall l \in 1, 2, ..., L$;
The interference function $Loss_l(f), \forall l \in 1, 2, ..., L, \forall f \in 1, 2, ..., F$;
Variable:
The decision variable x_{lf} takes value 1 if the link l starts at channel f and 0 otherwise.

Having the above notations, the frequency assignment problem can be formulated with the aid of the following integer programming model:

$$\text{Minimize:} \quad \sum_l \sum_f x_{lf} \cdot Loss_l(f) \tag{1}$$

$$\text{Subject to:} \quad \sum_l c_{lf} \leq 1 \qquad \forall f \in 1, 2, ..., F \tag{2}$$

$$\sum_f x_{lf} = 1 \qquad \forall l \in 1, 2, ..., L \tag{3}$$

$$c_{lf} = \sum_{i=f-w_l+1}^{f} x_{li} \qquad \forall l \in 1, 2, ..., L, \forall f \in 1, 2, ..., F \tag{4}$$

$$x_{lf} \in \{0, 1\} \qquad \forall l \in 1, 2, ..., L, \forall f \in 1, 2, ..., F \tag{5}$$

$$c_{lf} \in \{0, 1\} \qquad \forall l \in 1, 2, ..., L, \forall f \in 1, 2, ..., F \tag{6}$$

The objective of the model is to minimize the total loss (1), as defined by the loss-function. The first constraint (2) assures that every frequency can be used maximal one time and, thus, there is no overlapping of links. Each link has to start exactly one time and this is stated in the second constraint (3). The third constraint (4) takes care of the fact that every link l has to be placed in a continuous interval of width w_l. In this way all the constraints of frequency allocation are fulfilled.

3. IMPLEMENTATION IN AIMMS

The model is implemented in AIMMS as a mixed integer problem (MIP). AIMMS is an optimization modelling language developed by Paragon Decision Technology B.V., the Netherlands. Reasonable instances of the frequency assignment problem for this model are instances with 20 links, a frequency span of 100 to 200 and a link-width of 5 to 10 channels. Though, in practice, about 1.5 times this amount of channels are needed. Therefore, we are interested in the maximum instance size that can still be solved by this MIP, using AIMMS. In Table 2 the results of some run instances are shown.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	-	-
[illegible]	[illegible]	1	[illegible]	[illegible]	-	[illegible]	-
[illegible]	[illegible]	1	[illegible]	[illegible]	-	[illegible]	-
[illegible]	[illegible]	1	[illegible]	[illegible]	-	[illegible]	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1	[illegible]	-	[illegible]	-	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	-	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	-	-
[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	-	-
[illegible]	[illegible]	1	[illegible]	[illegible]	-	[illegible]	-

[illegible]

From these results we conclude that in general, apart from large instances (frequency-span > 500 and > 20 links), the program has problems in solving instances of the MIP where the links cover almost the whole frequency span.

For instances that cannot be solved by MIP, we use the linear program (LP) to find a feasible solution. Though in many cases even the LP, where the integrality conditions (5) are relaxed (i.e. $0 \leq x_{lf} \leq 1$), cannot be solved. This is possibly due to the fact that the constraint for width-cover still has to be fulfilled.

In case that a solution for the LP problem is found, this solution might be noninteger and, thus, not an optimal solution to the initial problem. The solution is a lower bound and we can use this lower bound to find a feasible integer solution.

From the noninteger solution, we can generate sequences of links. As the decision variables in the LP problem are between 0 and 1 and sum up to 1, they can be interpreted as probabilities.

Example. Suppose we found the following noninteger solution for the LP problem:

	channel →																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
link ↓ 1		0.5									0.5							
2					1													
3								0.25								0.75		
4													1					

Table 3: A noninteger solution of the LP problem

From Table 3 we see that, with probability 0.5, link 1 is sequenced first and, with probability 0.5, it is sequenced after link 2. Link 3 is sequenced with probability 0.25 before link 4 and, with probability 0.75, after link 4. We find four possible sequences out of this LP solution:

Sequence 1: Link 1, Link 2, Link 3, Link 4;

Sequence 2: Link 1, Link 2, Link 4, Link 3;
Sequence 3: Link 2, Link 3, Link 1, Link 4;
Sequence 4: Link 2, Link 1, Link 4, Link 3.
Given such a sequence of links, we develop a dynamic procedure such that, using the frequency interval width for each link, no link overlapping occurs and the total interference to be minimum. This model is presented in the next section.

4. A DYNAMIC MODEL FOR FINDING AN OPTIMAL FREQUENCY ASSIGNMENT

In this section we give a method for big size instances of frequency assignment problem that can not be solved by AIMMS. The method is implemented and tested in C++ programming language.

For such instances, AIMMS collapses to solve the MIP problem. However, it will be able to solve the LP problem (in most of the cases), so the solution that AIMMS will provide is fractional. From this solution, it can be derived a so called "reasonable" sequence of links; it means that we know that the links should be assigned on the frequency spectrum in a certain order.

Let $\mathcal{F} = \{1, 2, \ldots, F\}$ be the set of frequencies , $\mathcal{L} = \{1, 2, \ldots, N\}$ the set of links, $loss(l, f)$ the loss when link l starts at frequency f and w_l the width of a link.

As input to our method, we consider the sequence of links $\sigma = (L_N, L_{N-1}, \ldots, L_1)$ after a reindexation (for example if we know that the links should be assigned in the order $(\text{link9}, \text{link5}, \ldots, \text{link7})$, then $L_N = 9, L_{N-1} = 5, \ldots, L_1 = 7$).

Our problem is to find the optimal frequency assignment for σ, given that $f_{L_N} < f_{L_{N-1}} < \ldots < f_{L_1}$, with f_{L_i} the starting frequency of link L_i , $i = 1, \ldots, N$, so that to have no overlapping and the total loss be minimum.

We develop an inductive procedure that finds an optimal frequency assignment for the sub-sequence $(L_i, \ldots, L_1)$, given an optimal frequency assignment for the sub-sequence $(L_{i-1}, \ldots, L_1)$ for $i = 2, \ldots, N$.

We denote by $C(L_i, f)$ the minimum cost of a frequency assignment for $(L_i, \ldots, L_1)$, knowing that the first link L_i starts at frequency f.

It follows that $C(L_1, f)$ is the minimum cost of assigning the sequence (L_1) to start from frequency f, and is given by the formula

$$C(L_1, f) = \begin{cases} loss(L_1, f), & 1 \leq f \leq F - w_{L_1} + 1 \\ \infty \qquad\qquad\;\, , & \text{otherwise} \end{cases}$$

In case that $i = 2, \ldots, N$, because link L_i starts from frequency f, it means that we just have to assign sub-sequence $(L_{i-1}, \ldots, L_1)$ in an optimal way. But this sub-sequence can start from any of the frequency $g \in \{f + w_{L_i}, \ldots, F\}$.

It follows that the minimum cost is defined by the recursive formula

$$C(L_i, f) = loss(L_i, f) + \min_{g \in \{f + w_{L_i}, \ldots, F\}} C(L_{i-1}, g)$$

for $i = 2, \ldots, N$ and $1 \leq f \leq F$.

For the initial sequence $\sigma = (L_N, L_{N-1}, \ldots, L_1)$ the minimum cost is given by

$$min\{C(L_N, 1), C(L_N, 2), \ldots, C(L_N, F)\}.$$

It means that we calculate the cost when the first link L_N starts from all possible frequencies $1, 2, \ldots, F$, and then we take the minimum.

The above presented method gives the optimal frequency assignment for an initial sequence of links σ.

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