Fundamentals of Programming
Spring Term 2019
LAB-14 GPS

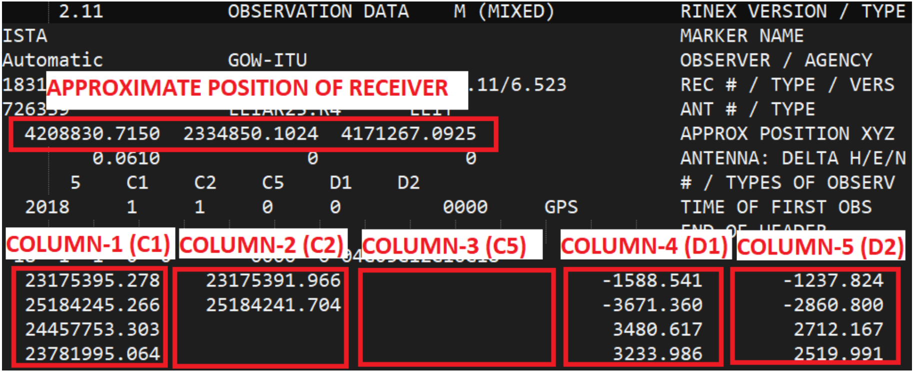
May 14, 2019

LabWork14.1 - Global Positioning System

Find the position of ISTA station using pseudo-range equations?

Observation file (RINEX): "ista0010.18o"
Precise orbit file: "igs19821.sp3"

LabWork14.1 - RINEX Observation File


```
     2.11           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
ISTA                                                        MARKER NAME
Automatic           GOW-ITU                                 OBSERVER / AGENCY
1831                                     .11/6.523          REC # / TYPE / VERS
7263                                                        ANT # / TYPE
  4208830.7150  2334850.1024  4171267.0925                  APPROX POSITION XYZ
        0.0610             0             0                  ANTENNA: DELTA H/E/N
     5    C1    C2    C5    D1    D2                        # / TYPES OF OBSERV
  2018     1     1     0     0         0000     GPS         TIME OF FIRST OBS
  23175395.278    23175391.966                    -1588.541        -1237.824
  25184245.266    25184241.704                    -3671.360        -2860.800
  24457753.303                                     3480.617         2712.167
  23781995.064                                     3233.986         2519.991
```


Right now, we are only interested in Column-1 which is code pseudo-range measurement. It is in meter unit and represents the distance between satellite and receiver (approximately 20000-25000km)

```
#cP2018  1  1  0  0  0.00000000      96 ORBIT IGS14 HLM  IGS
## 1982  86400.00000000   900.00000000 58119 0.0000000000000
/* FINAL ORBIT COMBINATION FROM WEIGHTED AVERAGE OF:
```

SV	X Coordinate	Y Coordinate	Z Coordinate	Clock bias
G01	-7746.389210	-13820.659479	-21494.943319	-20.753728
G02	-16146.095026	14527.214173	15627.540391	240.422316
G03	-12362.480870	-22753.026810	-6038.321404	-17.744063
G05	-4900.810713	16693.999959	19909.026570	-18.204317
G06	-24712.780585	7688.503452	6021.512570	414.280208
G07	-22163.680755	-6866.636968	13521.019272	258.005521
G08	4333.533776	-25046.338006	-7426.494909	-86.110705
G09	-12036.085259	-10539.771655	21163.356509	488.854175

NOTE: Coordinates are in kilometer and clock bias are in microsecond unit.

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Pseudo-range Equation (neglecting some terms for ease of computation):

$$R_i(t) = \rho_i(t) + c \times (\delta t_{rec} - \delta t_{SV})$$

where t is time of measurement, $R_i(t)$ is pseudo-range measurement, $\rho_i(t)$ is geometric distance between satellite-receiver, c is speed of light (299792458.0 m/s), δt_{SV} is satellite clock bias, δt_{rec} is receiver clock bias (which is unknown here).
First, we need to compute the geometric distances between satellite and receiver,

$$\rho_i(t) = \sqrt{(X_{SV} - X_{rec})^2 + (Y_{SV} - Y_{rec})^2 + (Z_{SV} - Z_{rec})^2}$$

where X_{SV}, Y_{SV} and Z_{SV} are cartesian coordinates of satellite vehicle, X_{rec}, Y_{rec} and Z_{rec} are approximate cartesian coordinates of receiver position.

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Remember from "Fundamentals of Surveying" that every measurements contain errors. In this case, the true (actually most probable) range is

$$R_{true} = R_i + v_i$$

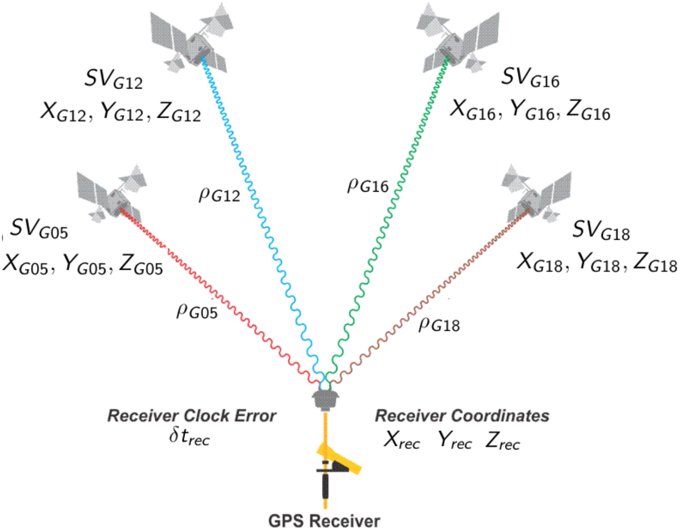
Adding a correction term to the pseudo-range equation yields,

$$R_i(t) + v_i(t) = \rho_i(t) + c \times (\delta t_{rec} - \delta t_{SV})$$

Re-arranging the formula to leave the correction term alone:

$$v_i(t) = \rho_i(t) - R_i(t) + c \times (\delta t_{rec} - \delta t_{SV})$$

LabWork14.1 - Visible Satellite Vehicles



LabWork14.1 - Global Positioning System

In "ista0010.18o" observation file, there are 4 visible satellite vehicles. Pseudo-range equations for each visible satellites,

$$\rho_{G05} = \sqrt{(X_{G05} - X_{rec})^2 + (Y_{G05} - Y_{rec})^2 + (Z_{G05} - Z_{rec})^2}$$

$$\rho_{G12} = \sqrt{(X_{G12} - X_{rec})^2 + (Y_{G12} - Y_{rec})^2 + (Z_{G12} - Z_{rec})^2}$$

$$\rho_{G16} = \sqrt{(X_{G16} - X_{rec})^2 + (Y_{G16} - Y_{rec})^2 + (Z_{G16} - Z_{rec})^2}$$

$$\rho_{G18} = \sqrt{(X_{G18} - X_{rec})^2 + (Y_{G18} - Y_{rec})^2 + (Z_{G18} - Z_{rec})^2}$$

LabWork14.1 - Global Positioning System

Approximate Position of receiver:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{meter} = \begin{bmatrix} 4208830.7150 \\ 2334850.1024 \\ 4171267.0925 \end{bmatrix}$$

Pseudo-range measurements:

$$\begin{bmatrix} R_{G05} \\ R_{G12} \\ R_{G16} \\ R_{G18} \end{bmatrix}_{meter} = \begin{bmatrix} 23175395.278 \\ 25184245.266 \\ 24457753.303 \\ 23781995.064 \end{bmatrix}$$

Precise Position of Satellite Vehicles:

$$\begin{bmatrix} X \\ Y \\ Z \\ \delta t \end{bmatrix}_{G05} = \begin{bmatrix} 4208830.7150 \\ 2334850.1024 \\ 4171267.0925 \\ -0.000018204317 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ \delta t \end{bmatrix}_{G12} = \begin{bmatrix} 9185357.848 \\ 23532040.166 \\ -8696332.711 \\ 0.000357695350 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ \delta t \end{bmatrix}_{G16} = \begin{bmatrix} 5256644.115 \\ -16163001.999 \\ 20153075.239 \\ 0.000034718998 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ \delta t \end{bmatrix}_{G18} = \begin{bmatrix} 20772761.087 \\ 13717705.750 \\ -8894618.323 \\ 0.000633574972 \end{bmatrix}$$

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Correction for each satellite is computed as follow:

$$v_{G05} = \rho_{G05} - R_{G05} + c \times (0 - \delta t_{G05})$$

$$v_{G12} = \rho_{G12} - R_{G12} + c \times (0 - \delta t_{G12})$$

$$v_{G16} = \rho_{G16} - R_{G16} + c \times (0 - \delta t_{G16})$$

$$v_{G18} = \rho_{G18} - R_{G18} + c \times (0 - \delta t_{G18})$$

$$\begin{bmatrix} v_{G05} \\ v_{G12} \\ v_{G16} \\ v_{G18} \end{bmatrix} = \begin{bmatrix} 39.039 \\ 56.768 \\ -87.967 \\ -66.011 \end{bmatrix}_{meter}$$

LabWork14.1 - SOLUTION

From GPS library import the function called"pseudorange" to solve the pseudorange equations and find the corrected position for ISTA station.

```
from GPS import pseudorange
ista = pseudorange("ista0010.18o","igs19821.sp3")
```

Expected output:

$$\begin{bmatrix} X \\ Y \\ Z \\ \delta t_{rec} \end{bmatrix} = \begin{bmatrix} 4208584.916 \\ 2334867.060 \\ 4171133.229 \\ -7.61013413433 5274e-08 \end{bmatrix}$$