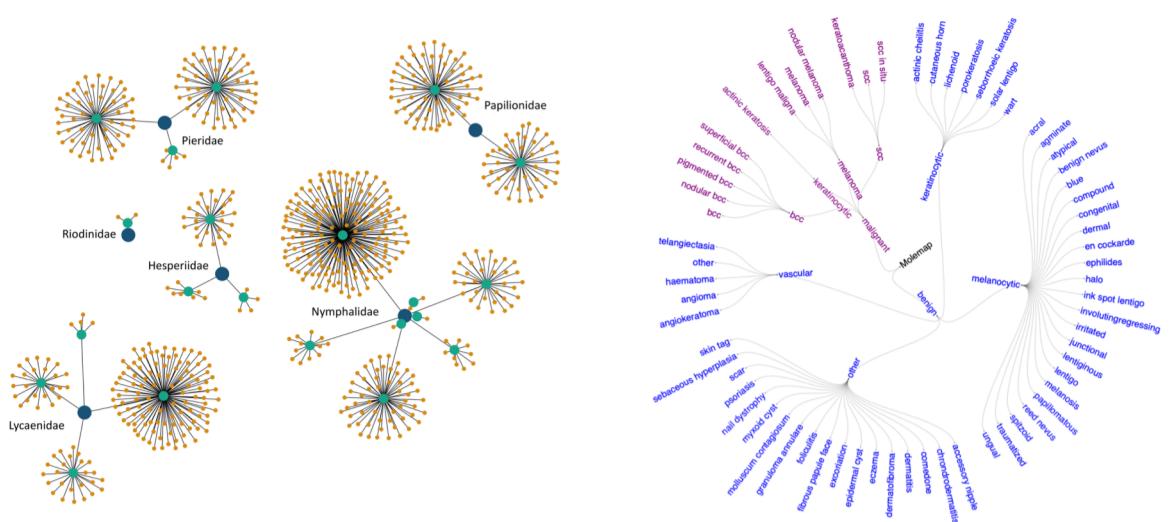


# Designing Hierarchies for Optimal Hyperbolic Embedding

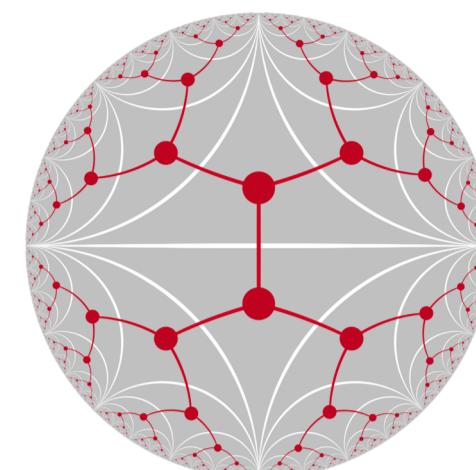
Melika Ayoughi, Max van Spengler, Pascal Mettes, Paul Groth



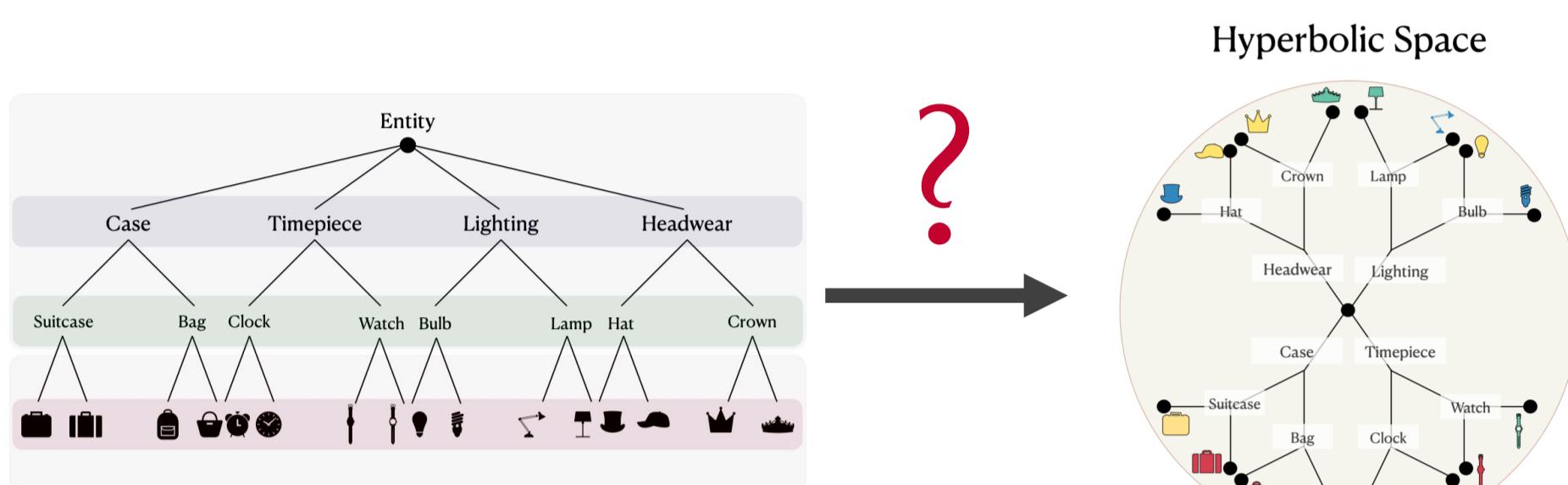
## Problem Statement



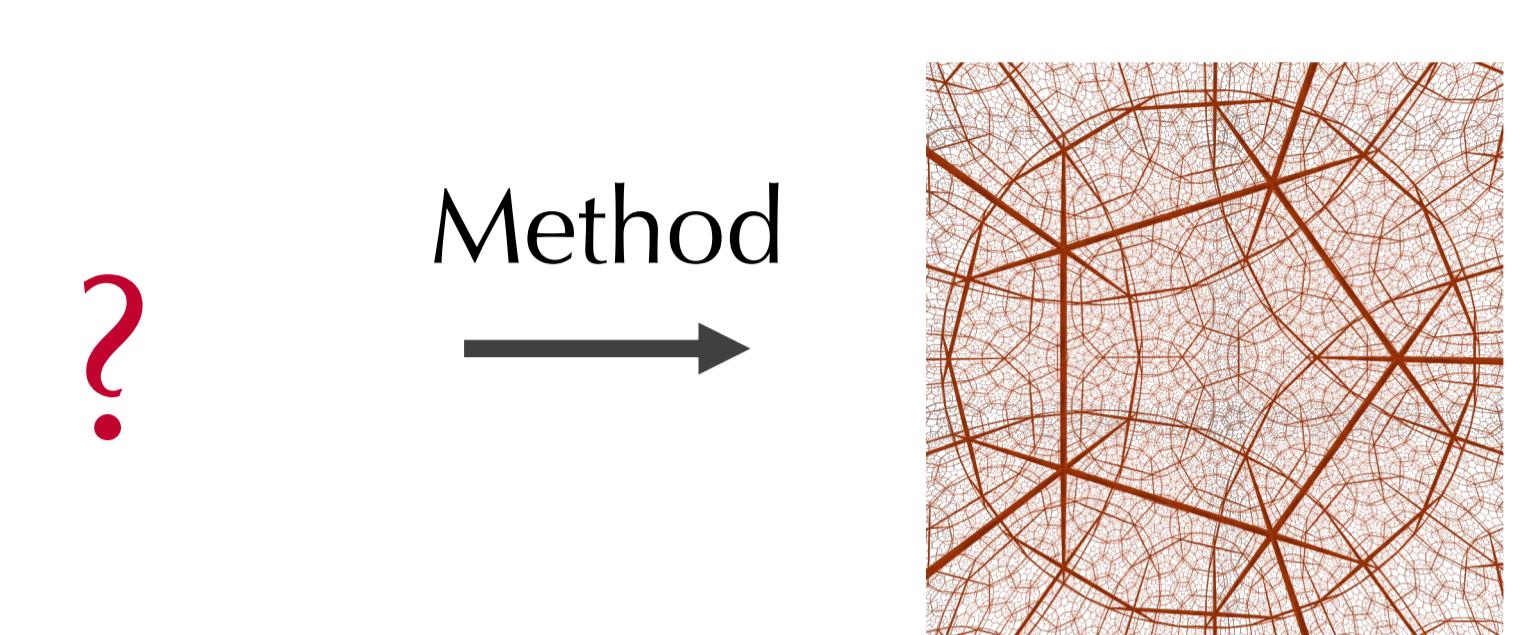
Knowledge graphs and ontologies



Hyperbolic: ideal geometry for hierarchies



Existing works: What's the best method?

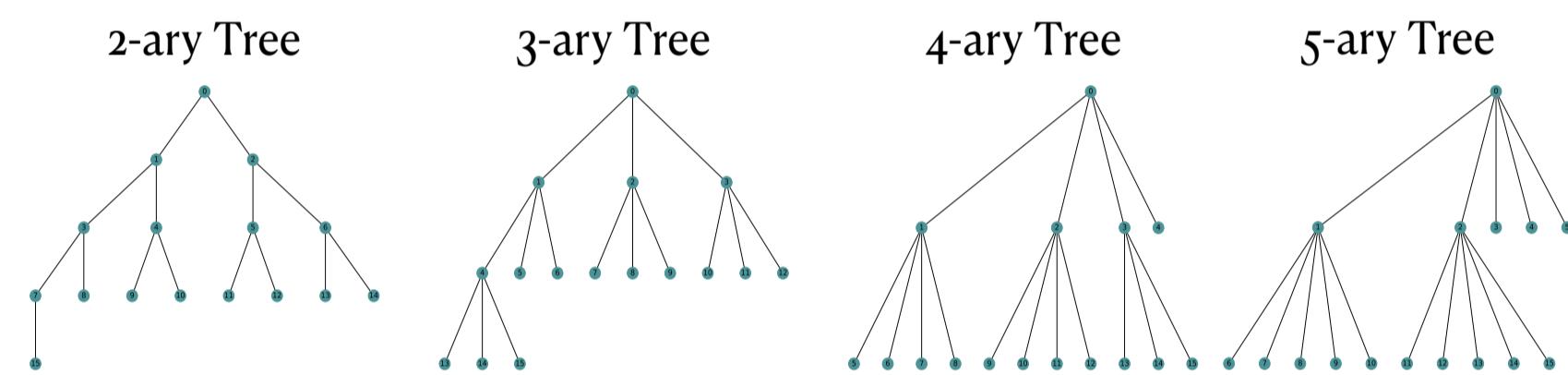


Our work: What's the best hierarchy?

## Controlled Experiments

### Methods

- I. Gradient-based
- II. Construction-based



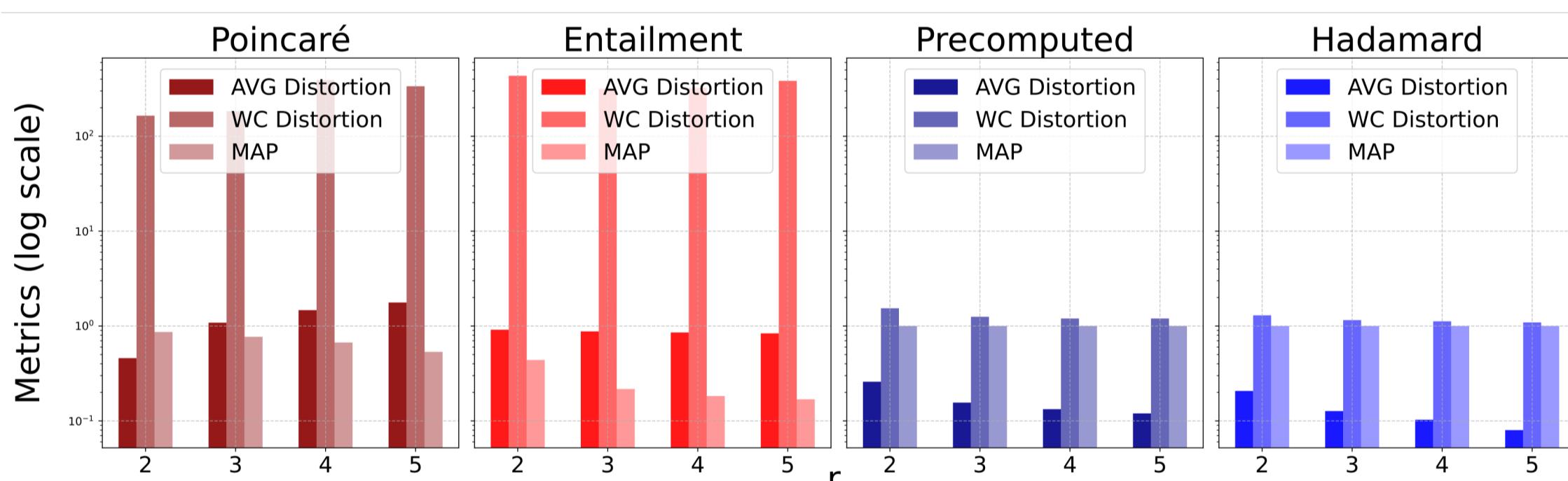
### Diverse Hierarchies

### Evaluation Metrics

- $\downarrow D_{avg}$ : Average distortion
- $\downarrow D_{wc}$ : Worst-case distortion
- $\uparrow MAP$ : Reconstruction

## Results

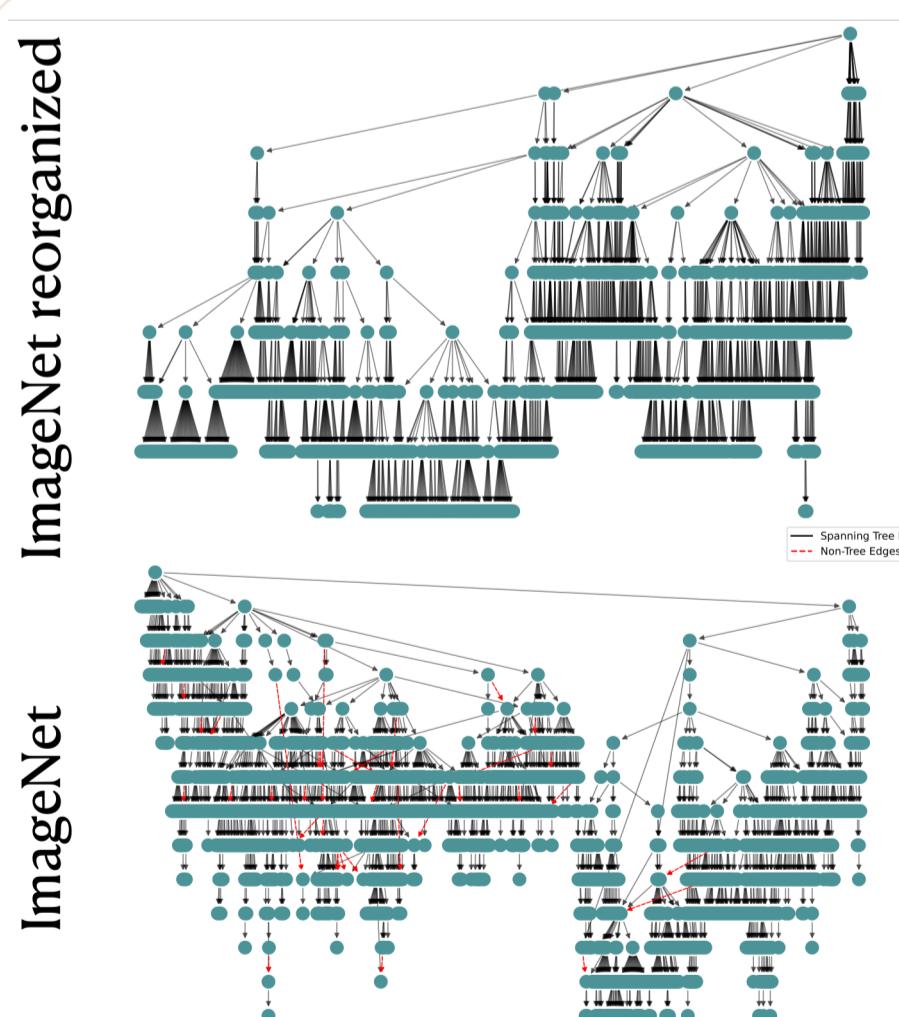
All methods except Poincaré: wide and shallow, lower distortion!



	Gradient-based						Construction-based						Precomputed			Hadamard			
	Poincaré			Entailment			Precomputed			Hadamard			$D_{avg}$	$D_{wc}$	MAP	$D_{avg}$	$D_{wc}$	MAP	
	$D_{avg}$	$D_{wc}$	MAP	$D_{avg}$	$D_{wc}$	MAP	$D_{avg}$	$D_{wc}$	MAP	$D_{avg}$	$D_{wc}$	MAP	$D_{avg}$	$D_{wc}$	MAP	$D_{avg}$	$D_{wc}$	MAP	
<b>Balanced</b>																			
2-ary	0.459	164.777	0.866	0.914	434.177	0.439	0.259	1.539	1	0.207	1.297	1							
3-ary	1.085	183.974	0.770	0.878	316.338	0.217	0.156	1.252	1	0.127	1.155	1							
4-ary	1.471	390.397	0.671	0.855	323.967	0.183	0.133	1.201	1	0.103	1.121	1							
5-ary	1.770	336.711	0.534	0.837	383.626	0.169	0.120	1.201	1	0.080	1.092	1							
<b>Imbalanced</b>																			
Binomial	1.439	69.530	0.171	0.863	224.731	0.304	0.249	1.542	1	0.186	1.257	1							
BA	2.791	3607.95	0.020	0.802	731.914	0.231	0.140	1.329	1	-	-	-							

Better to have a wide imbalanced tree than a deep balanced one!

Significant increase in the number of nodes only moderately impacts distortion



	Gradient-based			Construction-based			Gradient-based		
	Poincaré	Entailment	Precomputed	Hadamard	Poincaré	Entailment	Precomputed	Hadamard	
Balanced	256	512	1024	256	512	1024	256	512	1024
2-ary	0.880	0.459	0.229	0.816	0.914	0.960	0.220	0.259	0.300
3-ary	1.439	1.085	0.752	0.742	0.878	0.940	0.124	0.156	0.160
4-ary	2.129	1.471	1.092	0.695	0.855	0.928	0.102	0.133	0.137
5-ary	2.472	1.770	1.385	0.657	0.837	0.919	0.115	0.120	0.156
Imbalanced	Binomial	1.736	1.439	0.988	0.717	0.863	0.932	0.207	0.249
	BA	3.444	2.791	2.206	0.595	0.802	0.903	0.108	0.140
							-	-	-

- I. Poincaré: only method to handle multiple inheritance
- II. Hierarchy reorganization leads to better distortion and MAP

## Recommendations

- ✓ Design hierarchies for width
- ✓ Do not worry about balance
- ✓ Hyperbolic embeddings can handle additional node complexity
- ✓ Avoid multiple inheritance