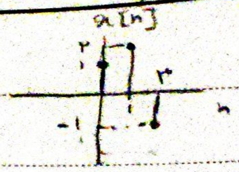
Subject:
Date

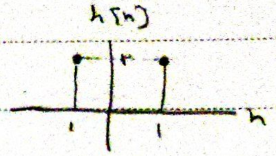
سیگنال سیستم - تمرین فصل ۲ - نرگس نوروزی‌راد - ۴۰۲۳۳۴۰۱۱۹۰۹۵

①

a) $y_1[n] = x[n] * h[n] = \sum h[k]x[n-k]$ $\qquad x[n] =$ 

$h[-1]x[n+1] + h[1]x[n-1] = 2x[n+1] + 2x[n-1]$

$\Rightarrow y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$

b) $y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n+2-k]$ $\qquad h[n] =$ 

$\Rightarrow y_1[n+2]$

c) $y_3[n] = x[n] * h[n+2] = x[n+2] * h[n] = y_1[n+2]$

②

$h[n-k] = \begin{cases} (\frac{1}{2})^{n-k-1} & A \leq k \leq B \\ 0 & o.w \end{cases} \Rightarrow h[k] = (\frac{1}{2})^{k-1}(u[n+3] - u[n-10])$

خلاف فرض

$\Rightarrow h[n-k] = (\frac{1}{2})^{n-k-1}(u[n+3-k] - u[n-10-k])$ (مشخصاً $h[k]$ در بازه $-9 \leq k \leq -3$ قرار دارد)

$\Rightarrow \boxed{n-9 \leq k \leq 3+n}$

③

$x[n] * h[n] = x_1[n-2] * h_1[n+2] = \sum_m x_1[m] \cdot h[n-m]$

$n-2 = m$

$\Rightarrow y[n] = 2\left[1 - (\frac{1}{2})^{n+1}\right]u[n]$

④

$y[n] = x[n] * h[n] = \sum x[k]h[n-k] = x[3]h[n-3] + x[4]h[n-4]$

$+ x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$

$\Rightarrow y[n] = \begin{cases} n-6 & 7 \leq n \leq 11 \\ 6 & 12 \leq n \leq 18 \\ 24-n & 19 \leq n \leq 23 \\ 0 & o.w \end{cases}$

⑤

$y[n] = x[n] * h[n] = \sum_{k=0}^{9} x[k]h[n-k] = \sum_{k=0}^{9} h[n-k]$

$\Rightarrow y[n] \begin{cases} \text{مثبت} & 0 \leq n \leq N+9 \\ 0 & o.w \end{cases} + \; y[14] = 0 \Rightarrow N+9 = 14 \Rightarrow N = 5$

کلی اشتباه $y[14]=0$ برقرار نشد $\Leftarrow N=4$

$\begin{cases} y[4] = 5 \\ y[14] = 0 \end{cases} \Rightarrow \cap \Rightarrow N = 4$

PAPCO

$y[n] = x[n] * h[n] \Rightarrow \sum_{\infty} x[k]h[n-k] = \sum_{\infty} \left(\frac{1}{3}\right)^{-k} u[-k-1]\,u[n-k-1]$ ⑥

$= \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k} u[n-k-1] = \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k} u[n+k-1] \xrightarrow{k=p-1} \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} u[n+p]$

$\xrightarrow{n\geq 0} y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \frac{1}{3}\,\frac{1}{1-\frac{1}{3}} = \frac{1}{2} \Rightarrow y[n] = \begin{cases} 3^n/2 & n<0 \\ 1/2 & n\geq 0 \end{cases}$

⑦

$y[n] = \sum^{\infty} x[k]\,g[n-2k] \qquad g[n] = u[n] - u[n-4]$

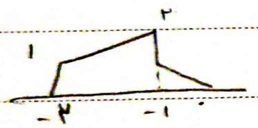
a) $x[n] = \delta[n-1] \longrightarrow y[n] = \sum^{\infty} x[k]g[n-2k] = g[n-2] = u[n-9]$

b) $x[n] = \delta[n-2] \longrightarrow y[n] = \sum^{\infty} x[k]g[n-2k] = g[n-4] = u[n-4] - u[n-8]$

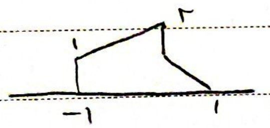
c) LTI نیست

D) $x[n] = u[n] \longrightarrow y[n] = \sum^{\infty} x[k]g[n-2k] = \sum_{0}^{\infty} g[n-2k]$

$x(t) * h(t) = \int_{\infty} h(\tau)\,x(t-\tau)\,d\tau = x(t+2) + 2x(t+1)$ ⑧



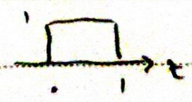
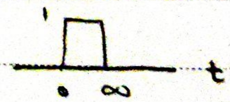
$x(t+2)$

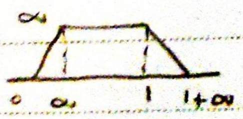


$x(t+1)$

$h(\tau) = e^{2\tau}u(-\tau+4) + e^{-2\tau}u(\tau-5) = \begin{cases} e^{-2\tau} & \tau>5 \\ e^{2\tau} & \tau<4 \\ 0 & 4<\tau<5 \end{cases}$ ⑨

$\Rightarrow h(t-\tau) = \begin{cases} e^{-2(t-\tau)} & \tau<t-5 \\ e^{2(t-\tau)} & \tau>t-4 \\ 0 & t-5<\tau<t-4 \end{cases} \Rightarrow \begin{cases} A = t-5 \\ B = t-4 \end{cases}$

a) $y(t) = x(t) * h(t) \rightarrow x(t)$  $h(t)$  ⑩

$\Rightarrow$ 

b) ?

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a) $y(t) = x(t) * h(t) \rightarrow \int_{-\infty}^{\infty} e^{-2\tau}\left(u(t-\tau-2) - u(t-\tau-5)\right) d\tau$ (11)

$(t-5) < \tau < (t-2)$, $2 < t < 5$ $\Rightarrow \int_{t-5}^{t-2} e^{-2\tau} d\tau = \dfrac{(1-e^{-6})e^{-2(t-5)}}{2}$

b) $\dfrac{dx(t)}{dt} = \delta(t-2) - \delta(t-5) \Rightarrow \dfrac{dx(t)}{dt} * h(t) = e^{-2(t-2)}u(t-2) - e^{-2(t-5)}u(t-5)$

$y(t) = e^{-t}u(t) * \sum_{-\infty}^{\infty} \delta(t-2k) = \cdots + e^{-(t+2)}u(t+2) + \cdots e^{-(t-2)}u(t-2) + \cdots$ (12)

$0 \leq t < 2 \Rightarrow y(t) = e^{-t}\dfrac{1}{1-e^{-2}} \Rightarrow A = \dfrac{1}{1-e^{-2}}$

$h[n] = (\frac{1}{5})^n u[n] \rightarrow$ a) $h[n] - A h[n-1] = \delta[n] \Rightarrow (\frac{1}{5})^n u[n] - (\frac{1}{5})^{n-1} u[n-1]$ (13)

$\Rightarrow n=1 \longrightarrow \frac{1}{5} = A$

b) $h[n] - \frac{1}{5}h[n-1] = \delta[n] \Rightarrow h[n] * (\delta[n] - \frac{1}{5}\delta[n-1]) = \delta[n]$

$\Rightarrow g[n] = \delta[n] - \frac{1}{5}\delta[n-1]$

? (14)

? (15)