Comparison with [5]

July 21, 2018

This report is supplementary material of RTSS 2018 submission [3]. The aim of this report is to experimentally compare the method in [5] with our methods proposed in [3]. Our conclusion is that the methods proposed in [3] yields much smaller WCRT than the method in [5].

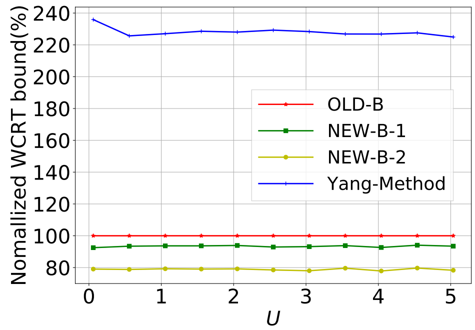
1 Experiments with Randomly Generated Workload

We compare the WCRT yielded by the following four methods:

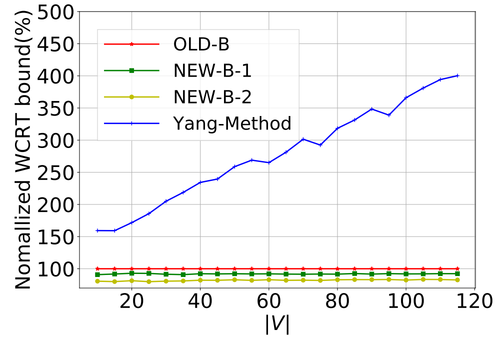
- **Yang-Method**: the method in [5] transforms a DAG task into an independent task set. The **Yang-Method** used two techniques to assign the relative deadline to each vertex: 1) the relative deadline of each vertex equals to the period of the task it belongs to (implicit deadline) 2) the relative deadline of each vertex is assigned by a linear programming (LP-based deadline) to minimize the end-to-end WCRT of each task. The second method gives smaller WCRT than the first method. The results shown in the following for **Yang-Method** are using the second deadline assignment technique.
- **OLD-B**: the baseline method in [4].
- **NEW-B-1**: our first new method in [3].
- **NEW-B-2**: our second new method in [3].

The standard setting of these experiments are as following:

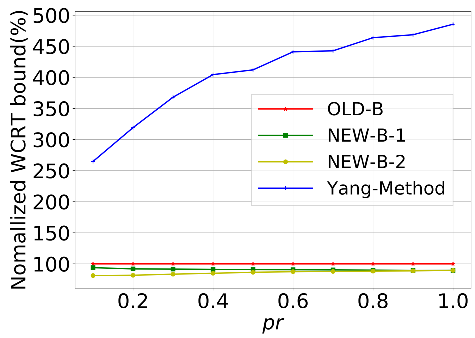
- The number of types $|S|$ is randomly chosen in the range $[5, 10]$, and the number of cores M_s of each type s is randomly chosen in $[2, 11]$.
- The DAG structure of the task is generated by the method proposed in [2], where the number of vertices $|V|$ is randomly chosen in the range $[70, 100]$ and the parallelism factor p_r is randomly chosen in $[0.08, 0.1]$ (the larger p_r, the more sequential is the graph).
- The total utilization U of the typed DAG task is randomly chosen in $[1, 3]$, and thus the total WCET of the task $vol(G) = U \times 100$.



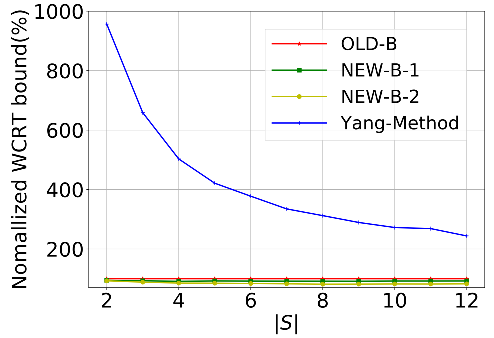
(a) Norm. bound with changing U



(b) Norm. bound with changing $|V|$



(c) Norm. bound with changing pr



(d) Norm. bound with changing $|S|$

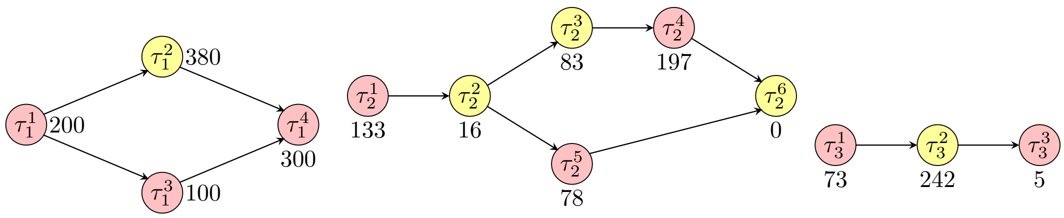
Figure 1: Comparison of analysis precision of different bounds.

- We use the Unnifast method [1] to distribute the total WCET to each individual vertex.
- Each vertex is randomly assigned a type in S.

Each subfigure in Figure 1 has a sample space of size 3000, shows the results with one particular task generation parameter varying (the other task generation parameters follow the above standard setting). The results are displayed in the form of normalized WCRT with respect to **OLD-B**. From the experiment results we can see that the transformation-based approach **Yang-Method** yields significantly larger WCRT than the other approaches. The gap is even larger when the DAG has larger scale or is more sequential, or the system has smaller number of core types.

2 Case Study

We use the case study provided in [5] to illustrate the computation procedure of **Yang-Method** in detail. Figure 2 shows the three typed DAG tasks. Each task uses two types of cores (marked by red and yellow respectively). We assume $M_1 = M_2 = 2$.



(a) G_1 (b) G_2 (c) G_3

Figure 2: DAG tasks in this case study each task with 2 types vertices.

2.1 Case study 1

The task G_1 in this case study is shown in Figure Fig.2(a). The WCET of each vertex is shown in the Fig.2(a) next to the vertex and $T_1 = 500$. When the relative deadline of each vertex equals its period, the end-to-end WCRT of G_1 is computed as following:

$$\begin{aligned}R_1^1 &= \frac{1}{2}\left(500 \times \frac{200+100+300}{500} + \frac{200}{500}(500-500) + \frac{100}{500}(500-500) + \frac{300}{500}(500-500)\right)\\ &+ \max\{200, 100, 300\} + \frac{2-1}{2} \times 200\\ &= 700\end{aligned}$$

$$\begin{aligned}R_1^2 &= \frac{1}{2}\left(500 \times \frac{380}{500} + \frac{380}{500}(500-500)\right) + \max\{380\} + \frac{2-1}{2} \times 380\\ &= 760\end{aligned}$$

$$\begin{aligned}R_1^3 &= \frac{1}{2}\left(500 \times \frac{200+100+300}{500} + \frac{200}{500}(500-500) + \frac{100}{500}(500-500) + \frac{300}{500}(500-500)\right)\\ &+ \max\{200, 100, 300\} + \frac{2-1}{2} \times 100\\ &= 650\end{aligned}$$

$$\begin{aligned}R_1^4 &= \frac{1}{2}\left(500 \times \frac{200+100+300}{500} + \frac{200}{500}(500-500) + \frac{100}{500}(500-500) + \frac{300}{500}(500-500)\right)\\ &+ \max\{200, 100, 300\} + \frac{2-1}{2} \times 300\\ &= 750\end{aligned}$$

$$\varphi_1^2 = \varphi_1^3 = \varphi_1^1 + R_1^1 = 700$$

$$\varphi_1^4 = \max\{\varphi_1^3 + R_1^3, \varphi_1^2 + R_1^2\} = 1460$$

$$R_1 = 1460 + 750 = 2210$$

When using the LP-based deadline assignment to minimize the maximum end-to-end WCRT of task G_1. The relative deadline of each vertex is in a range, such as $0 \leq D_1^1 \leq 500$, $0 \leq D_1^2 \leq 500$, $0 \leq D_1^3 \leq 500$, $0 \leq D_1^4 \leq 500$. We can compute the end-to-end WCRT of G_1 as following:

$$R_1 = \max\{R_1^1 + R_1^2 + R_1^4, R_1^1 + R_1^3 + R_1^4\} \tag{1}$$

$$\begin{aligned}
R_1^1 + R_1^2 + R_1^4 = &\frac{1}{2}\left(D_1^1 \times \frac{200+100+300}{500} + \frac{200}{500}(500 - D_1^1) + \frac{100}{500}(500 - D_1^3) + \frac{300}{500}(500 - D_1^4)\right) \\
&+ 300 + \frac{200}{2} \\
&+ \frac{1}{2}\left(D_1^2 \times \frac{380}{500} + \frac{380}{500}(500 - D_2^2)\right) \\
&+ 380 + \frac{380}{2} \\
&+ \frac{1}{2}\left(D_1^4 \times \frac{200+100+300}{500} + \frac{200}{500}(500 - D_1^1) + \frac{100}{500}(500 - D_1^3) + \frac{300}{500}(500 - D_1^4)\right) \\
&+ 300 + \frac{300}{2} \\
&= 700 + \frac{1}{2}\left(\frac{4}{5}D_1^1 - \frac{1}{5}D_1^3 - \frac{3}{5}D_1^4\right) + 760 \\
&+ 750 + \frac{1}{2}\left(\frac{3}{5}D_1^4 - \frac{1}{5}D_1^3 - \frac{2}{5}D_1^1\right) \\
&= 2210 + \frac{2}{10}\left(D_1^1 - D_1^3\right)
\end{aligned}$$

Similar we can get:

$$
\begin{aligned}
R_1^1 + R_1^3 + R_1^4 = & \frac{1}{2}\left(D_1^1 \times \frac{200+100+300}{500} + \frac{200}{500}(500 - D_1^1) + \frac{100}{500}(500 - D_1^3) + \frac{300}{500}(500 - D_1^4)\right) \\
& + 300 + 200/2 \\
& + \frac{1}{2}\left(D_1^3 times \frac{200+100+300}{500} + \frac{200}{500}(500 - D_1^1) + \frac{100}{500}(500 - D_1^3) + \frac{300}{500}(500 - D_1^4)\right) \\
& + 300 + \frac{2-1}{2} \times 100 \\
& + \frac{1}{2}\left(D_1^4 \times \frac{200+100+300}{500} + \frac{200}{500}(500 - D_1^1) + \frac{100}{500}(500 - D_1^3) + \frac{300}{500}(500 - D_1^4)\right) \\
& + 300 + 300/2 \\
& = 700 + \frac{1}{2}\left(\frac{4}{5}D_1^1 - \frac{1}{5}D_1^3 - \frac{3}{5}D_1^4\right) \\
& + 650 + \frac{1}{2}\left(D_1^3 - \frac{2}{5}D_1^2 - \frac{3}{5}D_1^4\right) \\
& + 750 + \frac{1}{2}\left(\frac{3}{5}D_1^4 - \frac{1}{5}D_1^3 - \frac{2}{5}D_1^1\right) \\
& = 2160 + \frac{3}{10}\left(D_1^3 - D_1^4\right)
\end{aligned}
$$

Finally, when $D_1^1 = 0, D_1^2 = 500, D_1^3 = 500, D_1^4 = 500$, one get the minimum end-to-end WCRT of G_1 as $R_1 = 2110$.

Because the number of types are the same, so the results of OLD-B equals with NEW-B-1, which is computed as following:

$$
R = \frac{200+380+300}{2} + \frac{200+380+300+100}{2} = 930
$$

The result of NEW-B-2 is : $R = 880$

2.2 Case study 2

The task G_2 in this case study is shown in Figure Fig.2(b). The WCET of each vertex is shown in the Fig.2(b) next to the vertex and $T_2 = 1000$. When the relative deadline of each vertex equals its period, the end-to-end WCRT of G_2 is computed as following:

$$
\begin{aligned}
R_2^1 = & \frac{1}{2}\left(1000 \times \frac{133+197+78}{1000} + \frac{133}{1000}(1000-1000) + \frac{197}{1000}(1000-1000) + \frac{78}{1000}(1000-1000)\right) \\
& + \max\{133, 197, 78\} + \frac{2-1}{2} \times 133 \\
& = 467.5
\end{aligned}
$$

$$
\begin{aligned}
R_2^2 = & \frac{1}{2}\left(1000 \times \frac{16+83}{1000} + \frac{16}{1000}(1000-1000) + \frac{83}{1000}(1000-1000)\right) + \max\{16, 83\} + \frac{2-1}{2} \times 16 \\
& = 140.5
\end{aligned}
$$

$$R_2^3 = \frac{1}{2}\left(1000 \times \frac{16+83}{1000} + \frac{16}{1000}(1000-1000) + \frac{83}{1000}(1000-1000)\right) + \max\{16, 83\} + \frac{2-1}{2} \times 83$$
$$= 174$$

$$R_2^4 = \frac{1}{2}\left(1000 \times \frac{133+197+78}{1000} + \frac{133}{1000}(1000-1000) + \frac{197}{1000}(1000-1000) + \frac{78}{1000}(1000-1000)\right)$$
$$+ \max\{133, 197, 78\} + \frac{2-1}{2} \times 197$$
$$= 499.5$$

$$R_2^5 = \frac{1}{2}\left(1000 \times \frac{133+197+78}{1000} + \frac{133}{1000}(1000-1000) + \frac{197}{1000}(1000-1000) + \frac{78}{1000}(1000-1000)\right)$$
$$+ \max\{133, 197, 78\} + \frac{2-1}{2} \times 78$$
$$= 440$$

According to the assumption of [5] $R_2^6 = 0$, we have:

$$\varphi_2^2 = 0 + 467.5 = 467.5,\ \varphi_2^3 = \varphi_2^2 + R_2^2 = 608,\ \varphi_2^4 = \varphi_2^3 + R_2^3 = 782, \varphi_2^5 = \varphi_2^2 + R_2^2 = 608$$

$$\varphi_2^6 = \max\{\varphi_2^4 + R_2^4, \varphi_2^5 + R_2^5\} = 1281.5$$

$$R_2 = 1281.5 + 0 = 1281.5$$

When using the LP-based deadline assignment to minimize the maximum end-to-end WCRT of task G_2. The relative deadline of each vertex is in a range, such as $0 \leq D_2^1 \leq 1000$, $0 \leq D_2^2 \leq 1000$, $0 \leq D_2^3 \leq 1000$, $0 \leq D_2^4 \leq 1000$, $0 \leq D_2^5 \leq 1000$. We can compute the minimum end-to-end WCRT of G_2 as following.

There are only two paths in G_2, $\pi_1 = \{\tau_2^1, \tau_2^2, \tau_2^3, \tau_2^4, \tau_2^6\}$ and $\pi_2 = \{\tau_2^1, \tau_2^2, \tau_2^5, \tau_2^6\}$. We have:

$$R_2 = \max\{R_2^1 + R_2^2 + R_2^3 + R_2^4 + R_2^6, R_2^1 + R_2^2 + R_2^5 + R_2^6\} \tag{2}$$

$$R_2^1 + R_2^2 + R_2^3 + R_2^4 + R_2^6 = \frac{1}{2}\left(\frac{142}{1000}D_2^1 + \frac{14}{1000}D_2^4 - \frac{156}{1000}D_2^5 + \frac{67}{1000}D_2^2 - \frac{67}{1000}D_2^3\right) + 1281.5$$

$$R_2^1 + R_2^2 + R_2^5 + R_2^6 = \frac{1}{2}\left(\frac{142}{1000}D_2^1 + \frac{83}{1000}D_2^2 - \frac{83}{1000}D_2^3 - \frac{394}{1000}D_2^4 - \frac{252}{1000}D_2^5\right) + 1048$$

Finally, when $D_2^1 = 0, D_2^2 = 0, D_2^3 = 1000, D_2^4 = 0, D_2^5 = 1000$, one get the minimum end-to-end WCRT of G_2 as $R_1 = 1170$.

Because the number of types are the same, so the results of OLD-B equals with NEW-B-1, which is computed as following:

$$R = \frac{133+16+83+197}{2} + \frac{133+16+83+197+78}{2} = 468$$

And the result of NEW-B-2 is : $R = 468$

2.3 Case study 3

The task G_3 in this case study is shown in Figure Fig.2(c). The WCET of each vertex is shown in the Fig.2(c) next to the vertex and $T_3 = 1000$. When the relative deadline of each vertex equals with period, the end-to-end WCRT of G_3 is computed as following:

$$\begin{aligned} R_3^1 &= \frac{1}{2}\left(1000 \times \frac{73+5}{1000} + \frac{73}{1000}(1000-1000) + \frac{5}{1000}(1000-1000)\right) \\ &+ \max\{73, 5\} + \frac{2-1}{2} \times 73 \\ &= 148.5 \end{aligned}$$

$$\begin{aligned} R_3^2 &= \frac{1}{2}\left(1000 \times \frac{242}{1000} + \frac{242}{1000}(1000-1000)\right) + 242 + \frac{2-1}{2} \times 242 \\ &= 484 \end{aligned}$$

$$\begin{aligned} R_3^3 &= \frac{1}{2}\left(1000 \times \frac{73+5}{1000} + \frac{73}{1000}(1000-1000) + \frac{5}{1000}(1000-1000)\right) \\ &+ \max\{73, 5\} + \frac{2-1}{2} \times 5 \\ &= 114.5 \end{aligned}$$

$$\varphi_3^2 = 0 + 148.5 = 148.5,\ \ \varphi_3^3 = \varphi_3^2 + R_3^2 = 632.5$$

$$R_3 = 632.5 + 114.5 = 747$$

When using the LP-based deadline to minimize the maximum end-to-end WCRT of task G_3. The relative deadline of each vertex is in a range, such as $0 \le D_3^1 \le 1000$, $0 \le D_3^2 \le 1000$, $0 \le D_3^3 \le 1000$. We can compute the minimum end-to-end WCRT of G_2 as following:

$$\begin{aligned} R_3 &= R_3^1 + R_3^2 + R_3^3 \\ &= \frac{1}{2}\left(\frac{68}{1000}D_3^3 - \frac{68}{1000}D_3^1\right) + 747 \end{aligned} \tag{3}$$

Finally, when $D_3^1 = 1000, D_3^2 = 1000, D_3^3 = 0, D_2^4 = 0$, one get the minimum end-to-end WCRT of G_3 as $R_3 = 713$.

Because the number of types are the same, so the results of OLD-B equals with NEW-B-1, which is computed as following:

$$R = \frac{73 + 242 + 5}{2} + \frac{73 + 5 + 242}{2} = 320$$

And the result of NEW-B-2 is : $R = 320$

2.4 Summary

The following table summarizes the results of these three examples. We can see that the WCRT yielded by **Yang-Method** is about 2-3 times of the results by our methods. As we discussed in last section, the gap between **Yang-Method** when is even larger with large scale tasks. However, even with the small DAGs the gap between **Yang-Method** and our methods is still significant.

Table 1: Results of 3 examples

task	Yang-Method, implicit deadline	Yang-Method, LP-based deadline	NEW-B-1(OLD-B)	NEW-B-2
G_1	2210	2110	930	880
G_2	1281.5	1170	468	468
G_1	747	713	320	320

References

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