Market Regime Detector

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1 Introduction

Market regimes are long-term, persistent states that can describe what state a market is in. Markets can be the bond, real estate, currencies, or commodities markets. This report focuses on the stock market with the two classic regimes, Bull and Bear. Our goal is to classify different time periods of the stock market using a Hidden Markov Model. Doing this will give a user the ability to predict changes in regimes and quickly adjust trading, investing and risk management strategies.

2 Data Description

To capture the overall stock market we look at 4 popular indices:

a. S&P 500 from January 1$^{\text{st}}$, 1928 to December 30$^{\text{th}}$, 2020

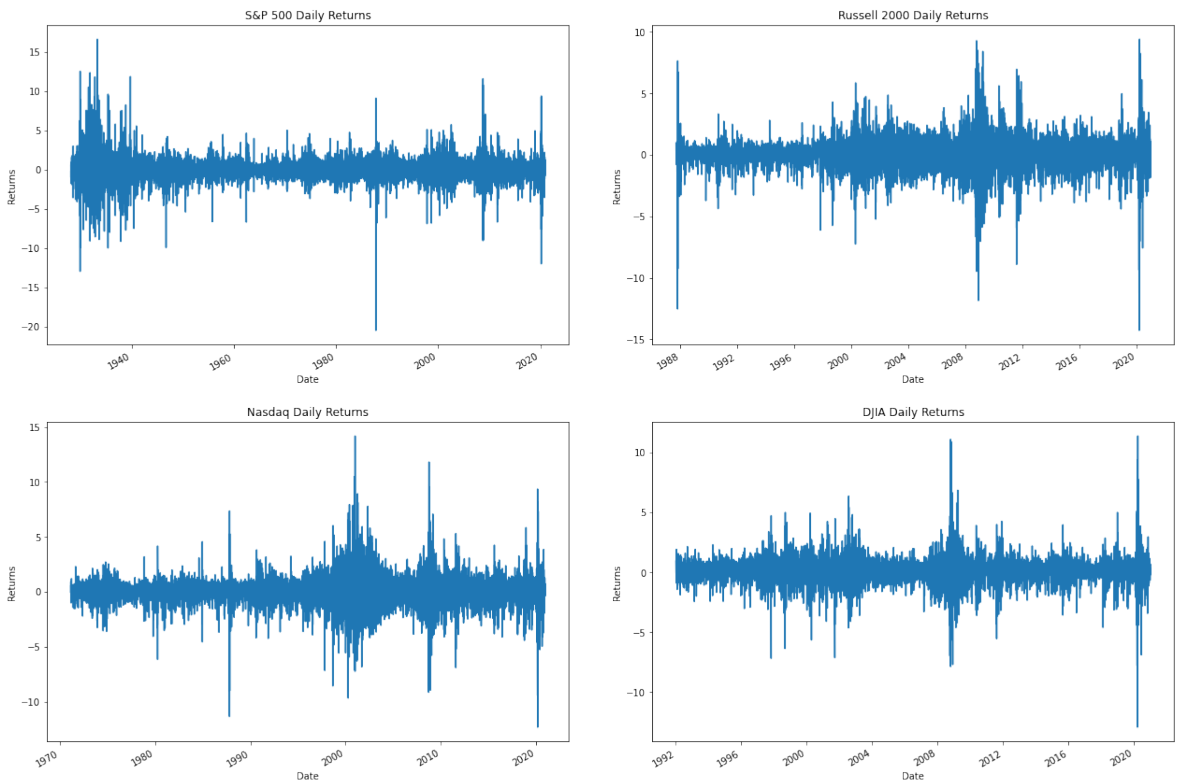
b. Russell 2000 from September 10$^{\text{th}}$, 1987 to December 30$^{\text{th}}$, 2020

c. Nasdaq Composite from February 5$^{\text{th}}$, 1971 to December 30$^{\text{th}}$, 2020

d. Dow Jones Industrial Average (DJIA) from January 2$^{\text{nd}}$, 1992 to December 30$^{\text{th}}$, 2020

We can look at the returns for each one,

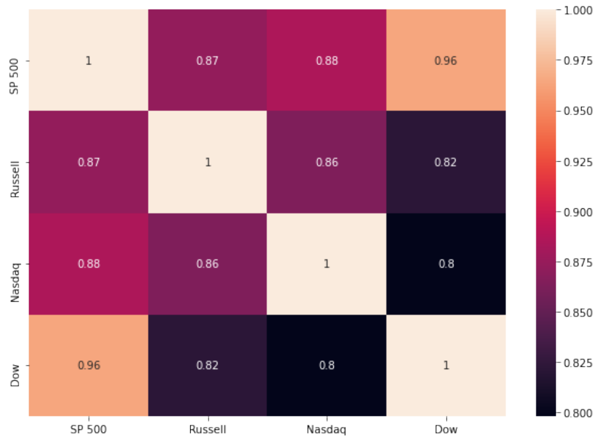
Figure 1



and see the 4 indices indeed share some similarities but they themselves have their own distinct features.

Looking at their correlations,

Figure 2



we see there is high positive correlations between the indices as expected. Consequently, we can use one of the indices to build the model. Since the S&P 500 has the most historical data we use it to build the model. We then use the model to predict the regimes on all four indices.

3 Markov Chains

A Markov Chain is a model that tells us something about the probabilities of a sequence of random variables, each of which can take on values from some set of states. An important assumption that these models make is the future is dependent solely on the present state and not the past.

More formerly, we call a stochastic process $\{X_n, n \in \mathbb{N}\}$ a Markov Chain if the following holds

a. The state space S of $\{X_n, n \in \mathbb{N}\}$ is at most countable (finite or a countably infinite set)

b. Markov property. For all $n \in \mathbb{N}$ and all states $x_0, x_1, \ldots, x_n, x_{n+1} \in S$

$$\mathbb{P}(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \ldots, X_0 = x_0) = \mathbb{P}(X_{n+1} = x_{n+1} | X_n = x_n)$$

For this report we will use N finite states. The probability of transiting to another state is govern by the transition probability matrix,

$$P = [P_{i,j}]_{i,j \in S} = \begin{bmatrix} P_{0,0} & P_{0,1} & \ldots & P_{0,j} & \ldots & P_{0,N} \\ P_{1,0} & P_{1,1} & \ldots & P_{1,j} & \ldots & P_{1,N} \\ & & \vdots & & & \\ P_{i,0} & P_{i,1} & \ldots & P_{i,j} & \ldots & P_{i,N} \\ & & \vdots & & & \\ P_{N,0} & P_{N,1} & \ldots & P_{N,j} & \ldots & P_{N,N} \end{bmatrix}$$

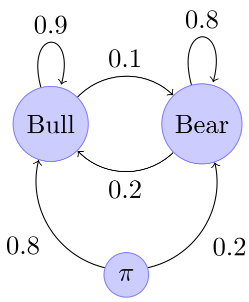
$$P_{i,j} = \mathbb{P}(X_{n+1} = j | X_n = i) = \mathbb{P}(X_1 = j | X_0 = i) \geq 0 \quad \forall i, j \in S$$

A Markov Chain is characterized by its transition probability matrix and the initial probability vector. The initial probability vector is,

$$\pi = \pi_0 = (\mathbb{P}(X_0 = 0), \mathbb{P}(X_0 = 1), \ldots, \mathbb{P}(X_0 = N))$$

The figure below is a visualization of a Markov chain with made up transition probabilities. We start with the initial probability vector and then can transition to either Bull or Bear state given the probabilities on the edges.

Figure 3



4 Hidden Markov Models

When using Markov Chains, we have observable events but that's not always the case. In this application, the returns are observable but the underlying regime state is hidden. Thus, the returns are indirect observations that are influenced by these regime states.

Hidden Markov models (HMM) are similar to Markov Chains but there's 2 Markov Chains X and Y, where X has hidden states and Y has observable states that are dependent on X. Thus, we can learn X through Y. In addition to the transition probability matrix and the initial probability vector the HMM model also has,

$$Y = \{y_1, y_2, \dots, y_T\}$$

a sequence of T observations and,

$$B = b_i(y_t)$$

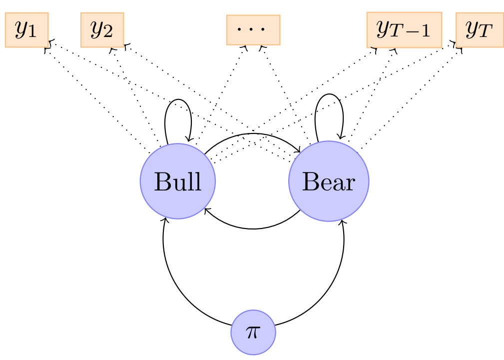
emission probabilities, each expressing the probability of an observation y_t being generated from a state i.

Thus, we continue to assume the Markov assumption $\mathbb{P}(x_i|x_1, x_2, \dots x_{i-1}) = \mathbb{P}(x_i|x_{i-1})$ and now the probability of an output y_i depends only on the state that produced the observation x_i and not on any other states or any other observations.

$$\mathbb{P}(y_i|x_1, \dots x_i, \dots, x_t, y_1, \dots, y_i, \dots, y_T) = \mathbb{P}(y_i|x_i)$$

In this context, given a sequence of observations, Y, (each term in the sequence being a return for the day) find the hidden sequence, X, of regimes (Bull or Bear) which caused the market to be that price.

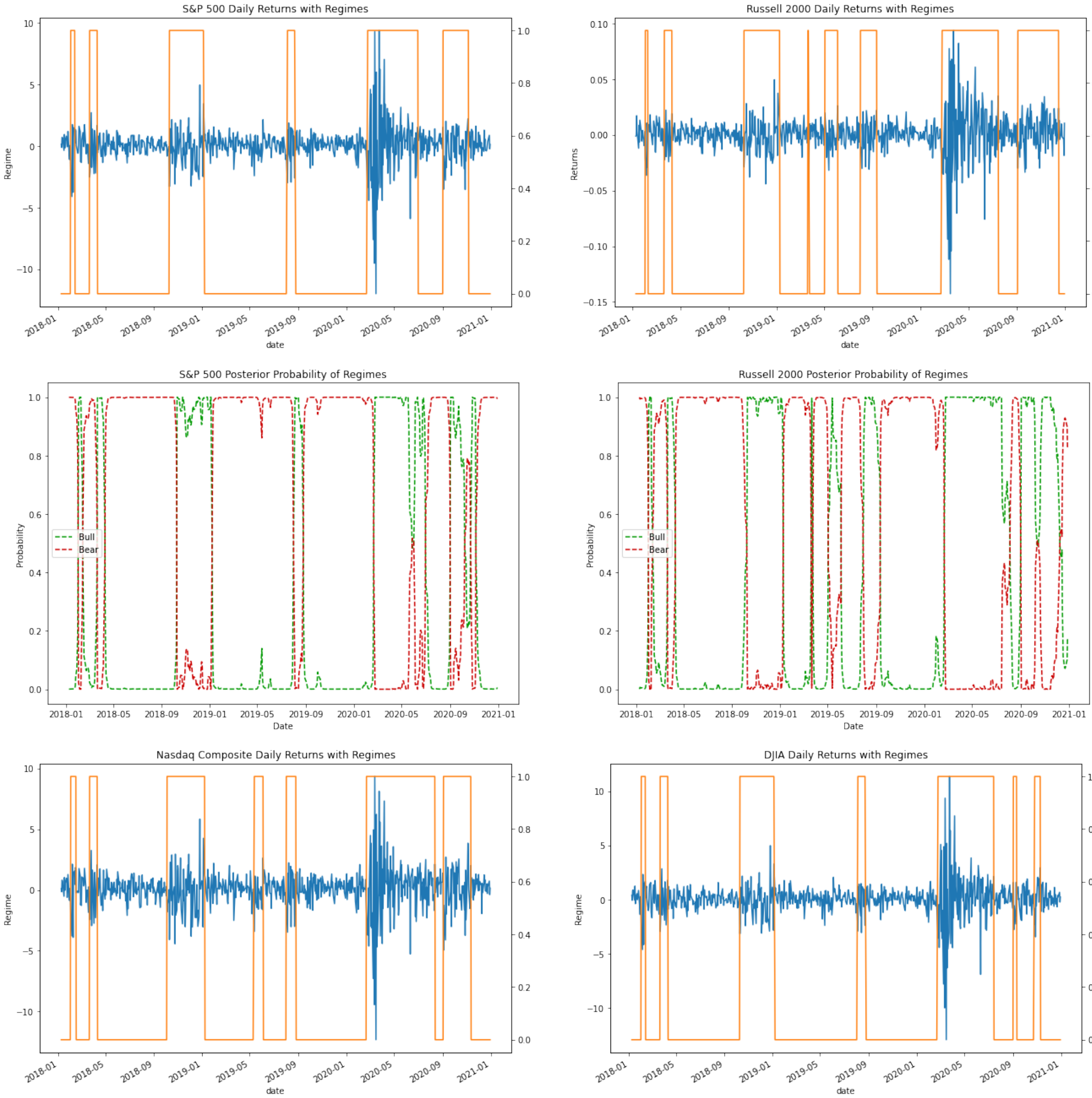
Figure 4

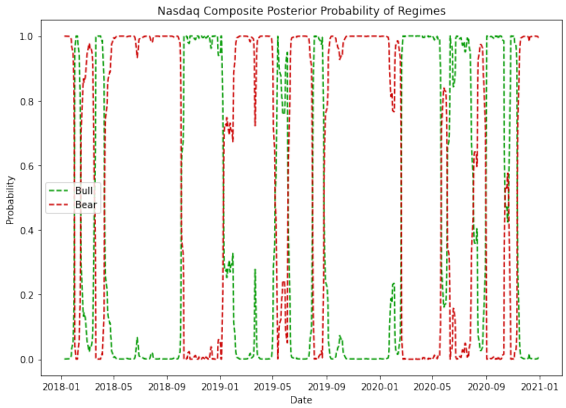


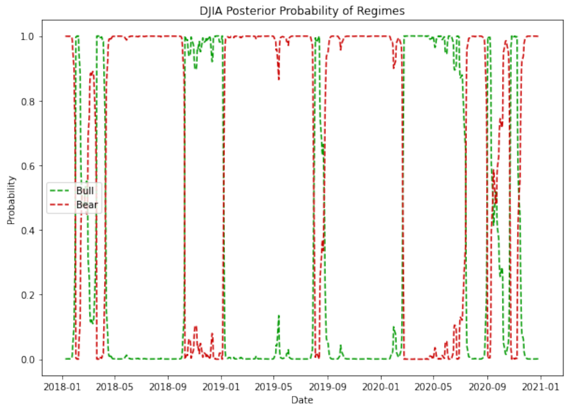
5 Model

Using Python, the HHM is fitted to the historical returns of the S&P 500. The following plots display the model's prediction with the returns for the index. The orange line is the model's prediction with 1 being a Bear market and 0 being a Bull market from the secondary y-axis. The posterior probability plots display the model's probability of being in a Bull or Bear state.

Figure 5







Looking at the mean and standard deviation of returns for each state in the 4 indices,

	S&P 500		Russell 2000		Nasdaq Composite		DJIA	
	Bull	Bear	Bull	Bear	Bull	Bear	Bull	Bear
Mean	0.0617	-0.0952	0.0909	-0.0687	0.0928	-0.0900	0.0648	-0.0738
SD	0.707	2.267	0.730	2.094	0.703	2.168	0.720	2.061

Across all 4 indices the daily mean of the Bear regime is negative while the daily mean of the Bear regime is positive. A major difference between the two regimes is the amount of volatility. Thus, the model makes its classification based on volatility and is able to identify volatility clustering. That is, it is able to identify large changes tend to be followed by large changes, and small changes tend to be followed by small changes.

6 Conclusion

This report showed using a Hidden Markov Model to classify time periods of returns as either in a Bull state or Bear state is reasonable. In addition, since indices that model the stock market are highly positively correlated, we only need historical data on 1 index, the S&P 500. In the analysis we saw the Bull and Bear regimes are influenced mainly by the amount of volatility during the period and the model was able to learn volatility trends. We were also able to calculate the probability of being in these states quantifying our confidence on how the market will behave during the period. It should be noted that the model can easily be extended beyond 2 states to model more specific regimes. Thus, this model can be used to predict the current state of the market and aid in trading, investment and risk management decisions.

Bibliography

[1] Daniel Jurafsky and James H. Martin. Hidden markov models, December 2020.