

Advection-Diffusion Equation Neural Network Parameter Modification Results

Data Denoising 500 Neurons within Hidden Layer

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	6.53e-04	4.01e-01	1.06e-02	7.25e-01
	$\sigma = 01$	ANN	3.83e-04	4.67e-01	2.61e-02	4.64e-01
	$\sigma = 05$	ANN	1.33e-03	2.44e+00	7.08e-02	1.01e+00
	$\sigma = 10$	ANN	1.05e-03	2.66e+00	2.93e-02	9.44e-01
	$\sigma = 25$	ANN	6.29e-03	3.89e+00	9.17e-02	6.50e-01
	$\sigma = 50$	ANN	9.14e-02	1.01e+01	4.00e-01	3.23e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.812203u_x + 0.010296u_{xx}$
0.01	ANN	$u_t = -0.807979u_x + 0.010927u_{xx}$
0.05	ANN	$u_t = -0.806153u_x + 0.008983u_{xx}$
0.10	ANN	$u_t = -0.804392u_x + 0.009624u_{xx}$
0.25	ANN	$u_t = -0.803864u_x + 0.008451u_{xx}$
0.50	ANN	$u_t = -0.767790u_x + 0.007857u_{xx}$

Data Denoising 1000 Neurons within Hidden Layer

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	4.09e-04	5.19e-01	3.79e-02	4.08e-01
	$\sigma = 01$	ANN	5.06e-04	7.01e-01	1.60e-02	6.90e-01
	$\sigma = 05$	ANN	5.90e-04	9.44e-01	4.71e-02	5.39e-01
	$\sigma = 10$	ANN	1.13e-03	3.30e-01	1.00e-02	6.04e-01
	$\sigma = 25$	ANN	7.94e-03	1.19e+00	4.13e-02	6.92e-01
	$\sigma = 50$	ANN	5.86e-02	9.15e+00	3.22e-01	1.14e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.805233u_x + 0.010173u_{xx}$
0.01	ANN	$u_t = -0.808772u_x + 0.010293u_{xx}$
0.05	ANN	$u_t = -0.809023u_x + 0.011093u_{xx}$
0.10	ANN	$u_t = -0.811688u_x + 0.010791u_{xx}$
0.25	ANN	$u_t = -0.790162u_x + 0.007800u_{xx} + 0.000793u_x^2$
0.50	ANN	$u_t = -0.779393u_x + 0.007553u_{xx}$

Data Denoising 2000 Neurons within Hidden Layer

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	3.57e-04	2.54e-01	1.22e-02	4.17e-01
	$\sigma = 01$	ANN	4.13e-04	9.58e-02	7.04e-03	6.34e-01
	$\sigma = 05$	ANN	7.94e-04	1.88e-01	3.69e-02	6.65e-01
	$\sigma = 10$	ANN	6.46e-04	6.79e-01	4.40e-02	9.14e-01
	$\sigma = 25$	ANN	4.24e-03	1.26e+00	4.28e-02	7.76e-01
	$\sigma = 50$	ANN	7.26e-02	2.88e+00	1.46e-01	1.44e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.805402u_x + 0.010823u_{xx}$
0.01	ANN	$u_t = -0.806860u_x + 0.010454u_{xx}$
0.05	ANN	$u_t = -0.811765u_x + 0.010091u_{xx}$
0.10	ANN	$u_t = -0.810993u_x + 0.010362u_{xx}$
0.25	ANN	$u_t = -0.799108u_x + 0.010361u_{xx} + -0.000046u_x^2 + 0.000579u_x^2$
0.50	ANN	$u_t = -0.788453u_x + 0.007749u_{xx}$

Data Denoising 3000 Neurons within Hidden Layer

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	4.45e-04	3.40e-01	2.09e-02	4.41e-01
	$\sigma = 01$	ANN	3.76e-04	5.79e-01	3.40e-02	5.12e-01
	$\sigma = 05$	ANN	3.75e-04	1.43e-01	1.08e-02	5.80e-01
	$\sigma = 10$	ANN	9.79e-04	7.03e-01	2.35e-02	8.76e-01
	$\sigma = 25$	ANN	6.93e-03	1.48e+00	4.89e-02	5.87e-01
	$\sigma = 50$	ANN	8.48e-02	3.13e+00	1.28e-01	1.63e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.805673u_x + 0.010307u_{xx}$
0.01	ANN	$u_t = -0.808025u_x + 0.010820u_{xx}$
0.05	ANN	$u_t = -0.813860u_x + 0.010578u_{xx}$
0.10	ANN	$u_t = -0.812516u_x + 0.010463u_{xx}$
0.25	ANN	$u_t = -0.802582u_x + 0.009429u_{xx}$
0.50	ANN	$u_t = -0.773982u_x + 0.008161u_{xx}$

Training Validation Split: 70/30 (1000 Neurons per Hidden Layer)

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	5.94e-04	2.82e-01	1.51e-02	9.51e-01
	$\sigma = 01$	ANN	6.70e-04	3.26e-01	1.13e-02	8.70e-01
	$\sigma = 05$	ANN	5.35e-04	7.50e-01	6.51e-02	5.61e-01
	$\sigma = 10$	ANN	9.61e-04	2.06e-01	1.09e-02	1.07e+00
	$\sigma = 25$	ANN	5.64e-03	3.62e+00	1.91e-02	1.11e+00
	$\sigma = 50$	ANN	5.60e-02	3.53e+00	8.93e-02	1.11e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.811403u_x + 0.010230u_{xx}$
0.01	ANN	$u_t = -0.812346u_x + 0.010192u_{xx}$
0.05	ANN	$u_t = -0.809346u_x + 0.011278u_{xx}$
0.10	ANN	$u_t = -0.809029u_x + 0.009698u_{xx}$
0.25	ANN	$u_t = -0.806192u_x + 0.010489u_{xx} + -0.000022u^2u_{xx} + 0.000347u_x^2$
0.50	ANN	$u_t = -0.786944u_x + 0.010370u_{xx} + -0.000070u^2u_{xx} + 0.000769u_x^2$

Training Validation Split: 80/20 (1000 Neurons per Hidden Layer)

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	6.13e-04	4.82e-01	1.40e-02	7.45e-01
	$\sigma = 01$	ANN	4.86e-04	1.56e+00	1.03e-02	4.89e-01
	$\sigma = 05$	ANN	4.66e-04	1.63e+00	6.39e-02	4.89e-01
	$\sigma = 10$	ANN	7.07e-04	4.02e-01	1.73e-02	8.81e-01
	$\sigma = 25$	ANN	5.29e-03	4.18e-01	2.40e-02	5.78e-01
	$\sigma = 50$	ANN	6.94e-02	5.89e+00	9.94e-02	1.24e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.809315u_x + 0.010519u_{xx}$
0.01	ANN	$u_t = -0.804583u_x + 0.011532u_{xx}$
0.05	ANN	$u_t = -0.806814u_x + 0.010571u_{xx}$
0.10	ANN	$u_t = -0.809572u_x + 0.009974u_{xx}$
0.25	ANN	$u_t = -0.804438u_x + 0.008788u_{xx}$
0.50	ANN	$u_t = -0.781217u_x + 0.007786u_{xx}$

Training Validation Split: 90/10 (1000 Neurons per Hidden Layer)

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	4.09e-04	5.19e-01	3.79e-02	4.08e-01
	$\sigma = 01$	ANN	5.06e-04	7.01e-01	1.60e-02	6.90e-01
	$\sigma = 05$	ANN	5.90e-04	9.44e-01	4.71e-02	5.39e-01
	$\sigma = 10$	ANN	1.13e-03	3.30e-01	1.00e-02	6.04e-01
	$\sigma = 25$	ANN	7.94e-03	1.19e+00	4.13e-02	6.92e-01
	$\sigma = 50$	ANN	5.86e-02	9.15e+00	3.22e-01	1.14e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.805233u_x + 0.010173u_{xx}$
0.01	ANN	$u_t = -0.808772u_x + 0.010293u_{xx}$
0.05	ANN	$u_t = -0.809023u_x + 0.011093u_{xx}$
0.10	ANN	$u_t = -0.811688u_x + 0.010791u_{xx}$
0.25	ANN	$u_t = -0.790162u_x + 0.007800u_{xx} + 0.000793u_x^2$
0.50	ANN	$u_t = -0.779393u_x + 0.007553u_{xx}$

Training Validation Split: 95/5 (1000 Neurons per Hidden Layer)

	Error	Method	u RMSE	u_t RMSE	u_x RMSE	u_{xx} RMSE
advection_diffusion	$\sigma = 00$	ANN	3.29e-04	4.22e-01	4.59e-02	4.95e-01
	$\sigma = 01$	ANN	4.05e-04	2.15e-01	2.15e-02	5.19e-01
	$\sigma = 05$	ANN	5.09e-04	4.00e-01	3.91e-02	5.76e-01
	$\sigma = 10$	ANN	1.38e-03	1.24e+00	1.99e-02	6.36e-01
	$\sigma = 25$	ANN	4.82e-03	1.38e+00	3.40e-02	7.32e-01
	$\sigma = 50$	ANN	7.73e-02	3.47e+00	1.06e-01	1.79e+00

		True Equation
		$u_t = 0.010000u_{xx} + -0.800000u_x$
σ	Method	Learned Equation
0.0	ANN	$u_t = -0.807162u_x + 0.010820u_{xx}$
0.01	ANN	$u_t = -0.809184u_x + 0.010687u_{xx}$
0.05	ANN	$u_t = -0.811571u_x + 0.011481u_{xx}$
0.10	ANN	$u_t = -0.802807u_x + 0.009779u_{xx}$
0.25	ANN	$u_t = -0.802345u_x + 0.008861u_{xx} + 0.000514u_x^2$
0.50	ANN	$u_t = -0.790457u_x + 0.009953u_{xx}$