Name: Melissa Boateng

1. $\lim_{n \to \infty} \frac{\ln(n)+4}{5n^4+7n^2+6}$

L'hopital's rule

$\lim_{n \to \infty} \left(\frac{d/dn(\ln(n)+4)}{d/dn(5n^4+7n^2+6)} \right) = \left(\frac{\frac{1}{n}}{20n^3+14n} \right)$

$\lim_{n \to \infty} \left(\frac{1}{20n^3+14n} \right)$

$\lim_{n \to \infty} \left(\frac{1}{n(20n^2+14)} \right)$

$\lim_{n \to \infty} n(20n^2+14)$

$= 0$

Limit 2

Q. $\lim_{n \to \infty} \frac{2^n}{\log_2 n}$

$$\lim_{n \to \infty} \left(\frac{2^n(\ln(2))}{1/n\ln(2)} \right)$$

$$\lim_{n \to \infty} \ln^2(2) \times 2^n n$$

$$\ln^2(2) \times \lim_{n \to \infty} 2^n n$$

$$\ln^2(2) \times \lim_{n \to \infty} 2^n \times \lim_{n \to \infty} n$$

$$\lim_{n \to \infty} 2^n = \infty$$

$$\lim_{n \to \infty} n = \infty$$

$\therefore \ln^2(2) \times \infty \times \infty = \infty$

$\therefore \lim_{n \to \infty} \frac{2^n}{\log_2 n} = \infty$

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Induction.

ii) $2\sum_{k=0}^{n} 3^k = 3^{n+1} - 1$

Assume $n = i$

$$2\sum_{k=0}^{i} 3^k = 3^{i+1} - 1$$

Now showing $i+1$

$$2\sum_{k=0}^{i+1} 3^k = 3^{(i+1)+1} - 1$$

Induction step

$$2\sum_{k=0}^{i+1} 3^k = 2(3^0 + 3^1 \dots 3^i + 3^{i+1}) = 3^{(i+1)+1} - 1$$

b.c

$$2(3^0 + 3^1 \dots 3^i) = 2\sum_{k=0}^{i} 3^k$$

$$2\sum_{k=0}^{i+1} = 2\sum_{k=0}^{i} 3^k + 3^{(i+1)} = 3^{(i+1)+1} - 1$$

From induction

$$3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$$

$$\Rightarrow 2\sum_{k=0}^{i+1} 3^k = 3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$$

$$= 3^{(i+1)+1} - 1 = 3^{(i+1)+1} - 1$$

Melissa Boateng

Approximation

$$\sum_{k=0}^{30} k^2 = \sum_{k=0}^{n} k^2 = \frac{n^3}{3}$$

$$\therefore \sum_{k=0}^{30} k^2 \Rightarrow \frac{30^3}{3}$$

$$= 9000$$

i) $$\sum_{k=0}^{100} k^3 = \sum_{k=0}^{n} k^3 \approx \frac{n^4}{4}$$

$$\Rightarrow \frac{100^4}{4}$$

$$= 25\,000\,000$$

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Master Theorem

i) $T(n) = 7T(n/2) + n^2$

Theorem = $aT(n/b) + fn$

$a = 7$, $b = 2$, $fn \to k = 2$

if $a = 7$, $b^k = 4$

$a > b^k$ $\therefore$ case 3

$T(n) \in \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 7})$

$= \Theta(n^{2.807})$

ii) $T(n) = 5T(n/3) + O(n)$

$a = 5$, $b = 3$ $fn = n^1$ $\therefore$ $D = 1$ and $k = 0$

since $a > b$ case 3

$\Rightarrow \Theta(n^{\log_b a})$

$= \Theta(n^{\log_3 5})$

$= \Theta(n^{1.465})$

Master theorem

iii) $T(n) = 3T(n/2) + 3/4 n + 1$

$a = 3$ $b = 2$ $fn = 3/4$

$\log_b a > fn$

$\therefore$ case 3 holds:

$\Theta(n^{\log_b a}) = \Theta(n^{\log_2 3})$

$= \Theta(n^{1.584})$