

Name: Melissa Boateng

$$\lim_{n \rightarrow \infty} \frac{\ln(n) + 4}{5n^4 + 7n^2 + 6}$$

L'hospital's rule

$$\lim_{n \rightarrow \infty} \left( \frac{d/dn(\ln(n) + 4)}{d/dn(5n^4 + 7n^2 + 6)} \right) = \left( \frac{\frac{1}{n}}{20n^3 + 14n} \right)$$

$$\lim_{n \rightarrow \infty} \left( \frac{1}{20n^3 + 14n} \right)$$

$$\lim_{n \rightarrow \infty} \left( \frac{1}{n(20n^2 + 14)} \right)$$

$$\lim_{n \rightarrow \infty} n(20n^2 + 14)$$

$$= 0$$

Limit 2

$$Q. \lim_{n \rightarrow \infty} \frac{2^n}{\log_2 n}$$

$$\lim_{n \rightarrow \infty} \left( \frac{2^n (\ln(2))}{1/n \ln(2)} \right)$$

$$\lim_{n \rightarrow \infty} \ln^2(2) \times 2^n n$$

$$\ln^2(2) \times \lim_{n \rightarrow \infty} 2^n n$$

$$\ln^2(2) \times \lim_{n \rightarrow \infty} 2^n \times \lim_{n \rightarrow \infty} n$$

$$\lim_{n \rightarrow \infty} 2^n = \infty$$

$$\lim_{n \rightarrow \infty} n = \infty$$

$$\therefore \ln^2(2) \times \infty \times \infty = \infty$$

$$\therefore \lim_{n \rightarrow \infty} \frac{2^n}{\log_2 n} = \infty$$

18962022

Induction.

$$ii) \sum_{k=0}^n 2 \cdot 3^k = 3^{n+1} - 1$$

Assume  $n=i$

$$\sum_{k=0}^i 2 \cdot 3^k = 3^{i+1} - 1$$

Now showing  $i+1$

$$\sum_{k=0}^{i+1} 2 \cdot 3^k = 3^{(i+1)+1} - 1$$

Induction step

$$\sum_{k=0}^{i+1} 2 \cdot 3^k = 2(3^0 + 3^1 + \dots + 3^i + 3^{i+1}) = 3^{(i+1)+1} - 1$$

Let

$$2(3^0 + 3^1 + \dots + 3^i) = 2 \sum_{k=0}^i 3^k$$

$$\sum_{k=0}^{i+1} 2 \cdot 3^k = 2 \sum_{k=0}^i 3^k + 3^{(i+1)+1} = 3^{(i+1)+1} - 1$$

From induction

$$3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$$

$$\Rightarrow \sum_{k=0}^{i+1} 2 \cdot 3^k = 3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$$

$$= 3^{(i+1)+1} - 1 = 3^{(i+1)+1} - 1$$

Melissa Boateng

Approximation

$$\sum_{k=0}^{30} k^2 = \sum_{k=0}^n k^2 = \frac{n^3}{3}$$

$$\begin{aligned} \therefore \sum_{k=0}^{30} k^2 &\Rightarrow \frac{30^3}{3} \\ &= 9000 \end{aligned}$$

$$c) \sum_{k=0}^{100} k^3 = \sum_{k=0}^n k^3 \approx \frac{n^4}{4}$$

$$\begin{aligned} &\Rightarrow \frac{100^4}{4} \\ &= 25\,000\,000 \end{aligned}$$

18962022

## Master Theorem

$$i) T(n) = 7T(n/2) + n^2$$

$$\text{Theorem} = aT(n/b) + fn$$

$$a=7, b=2, fn=k=2$$

$$\text{if } a=7, b^k=4$$

$$a > b^k \therefore \text{case 3}$$

$$T(n) \in \Theta(n^{\log_b a})$$

$$= \Theta(n^{\log_2 7})$$

$$= \Theta(n^{2.807})$$

$$ii) T(n) = 5T(n/3) + O(n)$$

$$a=5, b=3, fn=n^1 \therefore D=1 \text{ and } k=0$$

$$\text{since } a > b \text{ case 3}$$

$$\Rightarrow \Theta(n^{\log_b a})$$

$$= \Theta(n^{\log_3 5})$$

$$= \Theta(n^{1.465})$$



Master theorem

(ii)  $T(n) = 3T(n/2) + 3/4n + 1$

$$a = 3 \quad b = 2 \quad f(n) = 3/4n + 1$$

$$\log_b a \gg f(n)$$

$\therefore$  case 3 holds;

$$\Theta(n^{\log_b a}) = \Theta(n^{\log_2 3})$$

$$= \Theta(n^{1.584})$$