Question 3 ID:18962022

3i

$$\lim_{n \to \infty} \frac{\ln(n)+4}{5n^4+7n^2+6}$$

L'hopital's rule

$$\lim_{n \to \infty} \left(\frac{\frac{d}{dn}(\ln(n)+4)}{d/dn(5n^4+7n^2+6)} \right)$$

$$\lim_{n \to \infty} \left(\frac{\frac{1}{n}}{20n^3+14n} \right)$$

$$\lim_{n \to \infty} \left(\frac{1}{20n^3+14n} \right)$$

$$= \lim_{n \to \infty} \left(\frac{1}{n(20n^2+14)} \right)$$

$$\lim_{n \to \infty} (n(20n^2+14))$$

$$= 0$$

2. $\lim_{n \to \infty} \frac{2^n}{\log_2(n)}$ ID: 18962022

$$\lim_{n \to \infty} \left(\frac{2^n(\ln 2)}{\frac{1}{n \ln(2)}} \right)$$

$$\lim_{n \to \infty} \ln^2(2) \times 2^n n$$

$$\ln^2(2) \times \lim_{n \to \infty} (2^n \cdot n)$$

$$\ln^2(2) \times \lim_{n \to \infty} 2^n \times \lim_{n \to \infty} n$$

Taking them one by one

$\cancel{\ln^2(2)}$

$\lim_{n \to \infty} 2^n = \infty$

$\lim_{n \to \infty} (n) = \infty$

$\therefore \ln^2(2) \times \infty + \infty = \infty$

$$\therefore \lim_{n \to \infty} \frac{2^n}{\log_2(n)} = \infty$$

Approximation.

i)
$$\sum_{k=0}^{30} k^2$$

k = 0 to 30

$0 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 +$
$12^2 + 13^2 + 14^2 + 15^2 + 16^2 + 17^2 + 18^2 + 19^2 + 20^2 + 21^2 +$
$22^2 + 23^2 + 27^2 + 25^2 + 26^2 + 27^2 + 28^2 + 29^2 + 30^2$

$\Rightarrow 9455$

ii) $\sum_{k=0}^{100} k^3$: start at 0 end at 100.

$0^3 + 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 + 11^3 + 12^3$
$13^3 + 14^3 + 15^3 + 16^3 + 17^3 + 18^3 + 19^3 + 20^3 + 21^3 + 22^3 +$
$23^3 + 24^3 + 25^3 + \ldots\ 99^3 + 100^3$

$= 25502500$

Induction

) $2\sum_{k=0}^{n} 3^k = 3^{n+1} - 1$

Assume $n = i$

$2\sum_{k=0}^{i} 3^k = 3^{i+1} - 1$; Show $i+1$

$2\sum_{k=0}^{i+1} 3^k = 3^{(i+1)+1} - 1$

induction step

$2\sum_{k=0}^{i+1} 3^k = 2(3^0 + 3^1 + 3^2 \ldots 3^i + 3^{(i+1)}) = 3^{(i+1)+1} - 1$

But

$2(3^0 + 3^1 + 3^2 \ldots 3^i) = 2\sum_{k=0}^{i} 3^k$

$2\sum_{k=0}^{i+1} = 2\sum_{k=0}^{i} 3^k + (3^{i+1}) = 3^{(i+1)+1} - 1$

From induction:

$3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$

$2\sum_{k=0}^{i+1} 3^k = 3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$

$= 3^{i+1} + 3^{i+1} - 1 = 3^{(i+1)+1} - 1$

$3^{(i+1)+1} - 1 = 3^{(i+1)+1} - 1$

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Master Theorem

i) $T(n) = 7T(n/2) + n^2$

Theorem =

$aT(n/b) + fn$

$\therefore a = 7, \; b = 2 \; ; \; fn = k = 2$

if $a = 7 \; ; \; b^k = 4$

$a > b$; case 3 applies

$T(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_2 7})$

$\Rightarrow \log_2 7 = 2.807$

$\therefore \Theta(n^{\log_2 7}) = \Theta(n^{2.807})$

Order $= \Theta(n^{\log_a b}) = \Theta(n^{2.807})$

ii) $T(n) = 5T(n/3) + O(n)$

$aT(n/b) + fn$

$5T(n/3) + O(n) = 5T(n/3) + n$

$= 5T(n/3) + n^1$

$a = 5; \; b = 3, \; fn = n^1 \therefore D = 1$ and $k = 0$ because

$n^1 = n^1 \times 1 = n^1 \times \log^0 n \; , \; \log^0 n = 0$

$\therefore$ Check if $D >, <$ or $= \log_3 5$

But $\log_3 5 = 1.465 > D$

$\therefore$ Case 3

$= \Theta(n^{\log_b a}) = \Theta(n^{\log_3 5}) = \Theta(n^{1.465})$

Order $= \Theta(n^{1.465})$

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iii) Master theorem

$T(n) = 3T(n/2) + 3/4\, n + 1$

$T(n) = aT(n/b) + fn$

$a = 3 \quad b = 2 \quad fn = 3/4$

$n^{\log_b a} = n^{\log_2 3} = n^{1.584}$

Since $\log_b a \geq fn \approx 0.75$

Case 3 holds.

$\theta(n^{\log_b a}) = \theta(n^{\log_2 3})$

$= \theta(n^{1.584})$