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DSC 540: Machine Learning

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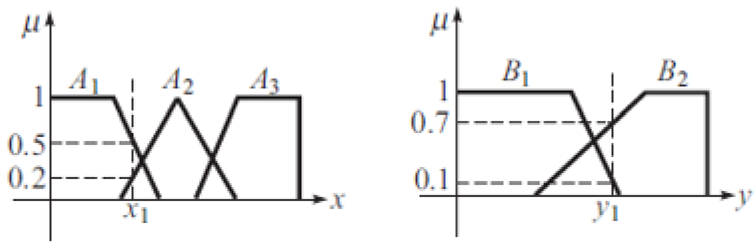
Part 1 – Theory

Let's take the input member functions below. With a crisp input x1 and crisp input y1



We find the following from the graphs:

$$\mu(x = A_1) = 0.5$$

$$\mu(x = A_2) = 0.2$$

$$\mu(y = B_1) = 0.1$$

$$\mu(y = B_2) = 0.7$$

Take the fuzzified inputs above and apply them to antecedents of the fuzzy rules. If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of antecedent evaluation. This number is then applied to the consequent membership function.

To evaluate the disjunction of rule antecedents we use the OR fuzzy operation.

$$\mu_A \cup \mu_B(x) = \max\,[\mu_A(x), \mu_B(x)]$$

In order to evaluate the conjunction of rule antecedents, we apply the AND fuzzy operation intersection. (Sharif University, n.d.)

$$\mu_A \cap \mu_B(x) = \min\,[\mu_A(x), \mu_B(x)]$$

Rule (1): If x is A_3 (0) $OR\ y\ is\ B_1$ (0.1) $THEN\ z\ is\ C_1$ (0.1)

Rule (2): If x is A_2 (0.2) $AND\ y\ is\ B_2$ (0.7) $THEN\ z\ is\ C_2$ (0.2)

Rule (3): If x is A_1 (0.5) $THEN\ z\ is\ C_3$ (0.5)

With the OR statement, we take the max of 0 and .1 which is how I got 0.1. For rule 2, there is a AND statement which is the min of the two values which is how I got .2.

Now we find the intersections of C_1 and C_2 at 0.1:

$$\frac{z-25}{10} = 0.1$$

$$z = 26$$

The intersection of C_2 and C_3 at 0.2:

$$\frac{z-60}{10} = 0.2$$

$$z = 62$$

Therefore, the function would be:

$$0\ if\ z < 0$$

$$0.1\ if\ 0 \leq z \leq 26$$

$$\frac{z-25}{10}\ if\ 26 \leq z < 27$$

$$0.2\ if\ 27 \leq z < 62$$

$$\frac{z-60}{10}\ if\ 62 \leq z < 65$$

$$0.5\ if\ 65 \leq z \leq 100$$

$$0\ if\ z > 100$$

I will use the Center-Average Method of defuzzification with this equation to approximate z with the more precise boundaries I found for the function (Gopal, M., 2019):

$$y' = \frac{\sum_{r=1}^{R} b^{(r)}\,\alpha^{(r)}}{\sum_{r=1}^{R} \alpha^{(r)}}$$

$$Weighted\ Average = (26 * 0.1) + (65 * 0.2) + (100 * .5) = 2.6 + 13 + 50 = 65.6$$

Z is approximately 65.6.

Task 2: Takagi-Sugeno Fuzzy Model and ANFIS

The problem of determining this relation is called the ocean color direct problem. There goal is to optimize Takagi-Sugeno type fuzzy rule-based systems. Accuracy and complexity are the two competitive objectives to be simultaneously optimized. TS-type FRBSs are implemented as an artificial neural network. By training the neural network, the parameters of the fuzzy model are adjusted. “The evolutionary optimization coarsely identifies the structure of the TS-type FRBSs, while the corresponding neural networks finely tune their parameters. As a result, a set of TS-type FRBSs with different trade-offs between accuracy and complexity is provided at the end of the optimization process.” (Cococcioni, M. et al., 2008) Takagi-Sugeno type fuzzy rule-based systems were generated to estimate concentrations and compare characteristics of the Takagi-Sugeno type fuzzy rule-based systems to the solutions with the most similar MSEs for help in validating the solution by using their complexities. They conclusion is that they “approached the ocean color inverse problem by using a Pareto-based multi-objective evolutionary algorithm, namely, the (2+2)M-PAES, so as to generate a set of TS-type FRBSs with different trade-offs between accuracy and complexity.” (Cococcioni, M. et al., 2008) The TStype FRBSs are embedded in an ANFIS and they used rules form the Takagi-Sugeno type fuzzy rule-based systems doe the ANFIS and they are expressed in Figure 1.

ANFIS is an artificial neural network which embeds a TS-type FRBS : the learning capability of ANFIS allows adapting the FRBS. They modeled the input-output relation using ANFIS. ANFIS example is shown in Figure 1.

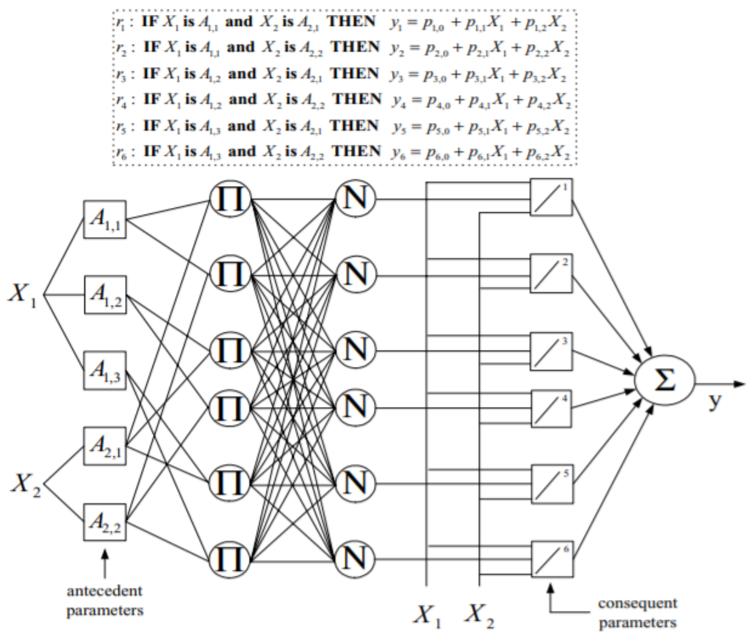


Figure 1

The neurons of the first layer of an ANFIS compute the membership degrees of each component of an input vector to each fuzzy set defined over the universe of each variable. The first layer has therefore a specific amount of neurons. The neurons of the second layer model the antecedents of the TS-type rules and compute the activation degrees. The neurons of the third layer normalize the activation degrees by computing the values. The neurons of the fourth layer compute the weighted output associated with the output of each rule. The single neuron of the fifth layer computes the output of ANFIS by summing up the weighted outputs provided by each rule. "The learning phase of ANFIS simultaneously tunes the antecedent and consequent parameters using the following hybrid learning procedure: once the antecedent parameters have been computed by adopting the gradient descent method, the consequent parameters are determined by means of the recursive least squares method. The procedure is repeated for a number of epochs decided by the user and typically chosen to achieve the convergence." (Cococcioni, M. et al., 2008) They also used Multi-objective evolutionary optimization of ANFIS. They compares the total number of parameters and the mean squared error between the output of ANFIS and the expected output. Therefore, the ANFIS helped them verify their results. The learning algorithm of ANFIS was used to estimate the consequent parameters and to tune the antecedent parameters.

Part 2 – Fuzzy Models

Refer to the readings assigned for this topic and provide solutions to the following problems, using Jupyter notebooks. Include formal and detailed explanations to accompany the code.

Problem 1. Consider a two-dimensional *sinc* equation defined by:

$$y = \mathrm{sinc}(x_1, x_2) = \frac{\sin(x_1)\sin(x_2)}{x_1 x_2}$$

Training data are sampled uniformly from the input range [–10, 10] × [–10, 10]. With two symmetric triangular membership functions assigned to each input variable, construct a Takagi-Sugeno fuzzy model (linear static mappings) for the *sinc* function. Provide defining equations for determination of the premise and consequent parameters of the model.

Takagi-Sugeno method follows an indirect approach: identification of the nonlinear system in terms of a T-S fuzzy model is first carried out; the T-S fuzzy model is then used for designing decision-support system. (Gopal, M., 2019) Looking at the two-dimensional sinc function with a input range of [–10, 10] × [–10, 10] we can visualize the graph in Figure 1.

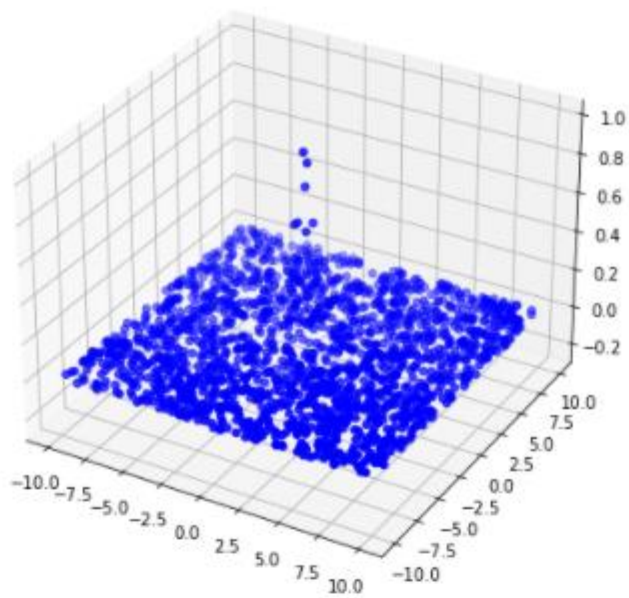


Figure 1

The fuzzy sets define for the variable will require different parameters depending on their shape. In the case of triangular sets, for example, three parameters are needed, two for the lower and upper extremes having a degree of membership of 0 and one for the apex which has a degree-of-membership of 1. I created two input variables with 100 values, x1 and x2. I created the system to have the inputs as high or low and came up with the following system In figure 2 using python.

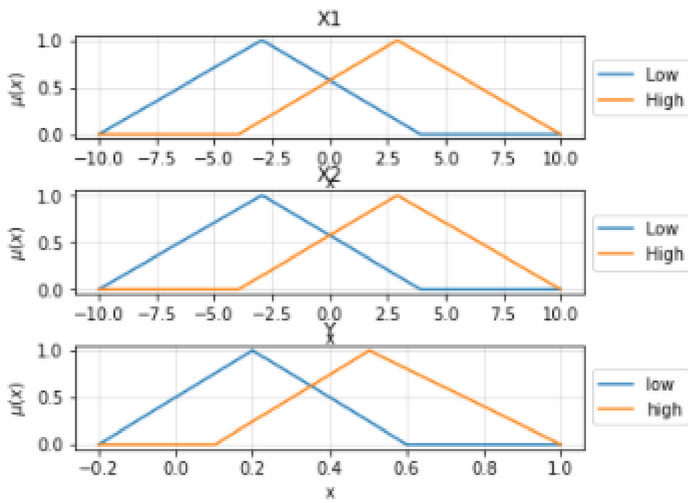


Figure 2

Problem 2. To identify the non-linear system

$$y = (1 + (x_1)^{0.5} + (x_2)^{-1} + (x_3)^{-1.5})^2$$

Assign two membership functions to each input variable. Training and testing data are sampled uniformly from the input ranges:

Training data: [1, 6] × [1, 6] × [1, 6]

Testing data: [1.5, 5.5] × [1.5, 5.5] × [1.5, 5.5]

Extract *Takagi-Sugeno fuzzy rules* from the numerical input-output training data that could be employed in an *ANFIS* model.

List and explain all the rules.

In the textbook, ANFIS uses two inputs and one output which follows the following four rules which corresponds to Figure 3 (Gopal, M., 2019):

Rule 1: IF x_1 is $\underset{\sim}{A}_{11}$ and x_2 is $\underset{\sim}{A}_{21}$ THEN $y^{(1)} = a_0^{(1)} + a_1^{(1)}x_1 + a_2^{(1)}x_2$

Rule 2: IF x_1 is $\underset{\sim}{A}_{12}$ and x_2 is $\underset{\sim}{A}_{22}$ THEN $y^{(2)} = a_0^{(2)} + a_1^{(2)}x_1 + a_2^{(2)}x_2$

Rule 3: IF x_1 is $\underset{\sim}{A}_{12}$ and x_2 is $\underset{\sim}{A}_{21}$ THEN $y^{(3)} = a_0^{(3)} + a_1^{(3)}x_1 + a_2^{(3)}x_2$

Rule 4: IF x_1 is $\underset{\sim}{A}_{11}$ and x_2 is $\underset{\sim}{A}_{22}$ THEN $y^{(4)} = a_0^{(4)} + a_1^{(4)}x_1 + a_2^{(4)}x_2$

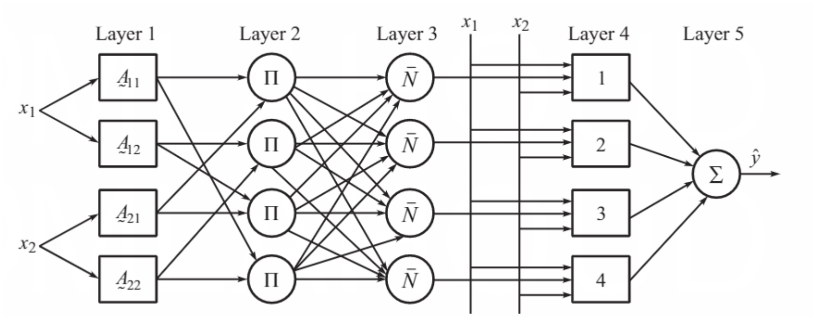


Figure 3

Since we have three input variables, we will have to add more rules which will correspond to Figure 4 below.

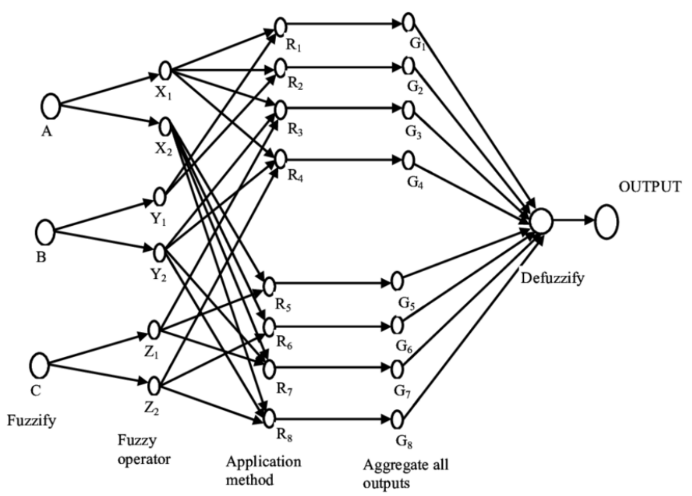


Figure 4

The equation will take values for x1, x2, and x3 between 1 and 6 because that is the bounds for the training set. The output (y) will be between 5.58 and 24.43 (Calculated in python). Although, I cannot finish the problem in python due to lack of knowledge in python, I can see the relationship of extracting Takagi-Sugeno fuzzy rules from the numerical input-output training data that could be employed in an ANFIS model.

Simple rules between input and output values are as $x2$ and/or $x3$ gets larger, y gets smaller. As $x1$ gets larger, y gets larger. We can make these into multiple different rules like the ones above.

References

Cococcioni, M., Corsini, G., Lazzerini, B., & Marcelloni, F. (2008). *Solving the ocean color inverse problem by using evolutionary multi-objective optimization of neuro-fuzzy systems.* International Journal of Knowledge Based Intelligent Engineering Systems, 12(5/6), 339–355.

Gopal, M. (2019). *Applied Machine Learing*. New Delhi, India: McGraw-Hill Education.

Sharif University (n.d.). *Lecture 6&7: Fuzzy Inference Systems*. Retrieved from http://ce.sharif.edu/courses/92-93/1/ce957-1/resources/root/Lectures/Lecture6&7.pdf