

Integer Programming

Melissa Paniagua

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Assignment 6 | Module 11

The purpose of this assignment is to formulate and solve an integer programming problem.

AP

AP is a shipping service that guarantees overnight delivery of packages in the continental US. The company has various hubs at major cities and airports across the country. Packages are received at hubs, and then shipped to intermediate hubs or to their final destination. The manager of the AP hub in Cleveland is concerned about labor costs, and is interested in determining the most effective way to schedule workers. The hub operates seven days a week, and the number of packages it handles varies from one day to another. The table below provides an estimate of the number of workers needed each day of the week.

Day of the Week	Workers Required
Sunday	18
Monday	27
Tuesday	22
Wednesday	26
Thursday	25
Friday	21
Saturday	19

Figure 1: Number of Workers

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is \$750 per week. Workers working on Saturday or Sunday receive an additional \$25 per day. The possible shifts and salaries for package handlers are:

Shift	Days Off	Wage
1	Sunday and Monday	\$775

2	Monday and Tuesday	\$800
3	Tuesday and Wednesday	\$800
4	Wednesday and Thursday	\$800
5	Thursday and Friday	\$800
6	Friday and Saturday	\$775
7	Saturday and Sunday	\$750

Figure 2: Salaries for Handlers

Questions

The manager wants to keep the total wage expenses as low as possible while ensuring that there are sufficient number of workers available each day. Formulate and solve the problem. What was the total cost? How many workers are available each day?

Hint: The number of available workers each day can exceed, but can not be below the required amount.

Formulate and solve the problem

Let x_i = Number of workers under shift i schedules with specific salaries, where $i = 1, 2, 3, 4, 5, 6, 7$

Objective function:

$$\text{Min } Z = 775x_1 + 800x_2 + 800x_3 + 800x_4 + 800x_5 + 775x_6 + 750x_7$$

ST

$$\text{Sunday} = x_2 + x_3 + x_4 + x_5 + x_6 \geq 18$$

$$\text{Monday} = x_3 + x_4 + x_5 + x_6 + x_7 \geq 27$$

$$\text{Tuesday} = x_1 + x_4 + x_5 + x_6 + x_7 \geq 22$$

$$\text{Wednesday} = x_1 + x_2 + x_5 + x_6 + x_7 \geq 26$$

$$\text{Thursday} = x_1 + x_2 + x_3 + x_6 + x_7 \geq 25$$

$$\text{Friday} = x_1 + x_2 + x_3 + x_4 + x_7 \geq 21$$

$$\text{Saturday} = x_1 + x_2 + x_3 + x_4 + x_5 \geq 19$$

$$x_i \text{ is integer, for } i = 1, 2, 3, 4, 5, 6, 7.$$

```
# Load libraries needed
library(lpSolveAPI)

# Read the lp file
ap <- read.lp("AP.lp")

# See the file
ap
```

Model name:

	x1	x2	x3	x4	x5	x6	x7		
Minimize	775	800	800	800	800	775	750		
Sunday	0	1	1	1	1	1	0	>=	18
Monday	0	0	1	1	1	1	1	>=	27
Tuesday	1	0	0	1	1	1	1	>=	22
Wednesday	1	1	0	0	1	1	1	>=	26
Thursday	1	1	1	0	0	1	1	>=	25
Friday	1	1	1	1	0	0	1	>=	21
Saturday	1	1	1	1	1	0	0	>=	19
Kind	Std	Std	Std	Std	Std	Std	Std		
Type	Int	Int	Int	Int	Int	Int	Int		
Upper	Inf	Inf	Inf	Inf	Inf	Inf	Inf		
Lower	0	0	0	0	0	0	0		

```
# See if the model converges
solve(ap)
```

```
[1] 0
```

Here we can see that the model is solving the problem.

What was the total cost?

```
# See the solution of Objective Function
get.objective(ap)
```

```
[1] 25675
```

In order to satisfy the minimum number of employees working per day, the minimum total salaries payed are \$25,765 per week. In other words, the total salary's cost per week is \$25,675.

```
# See the optimal number of the variables
get.variables(ap)
```

```
[1] 2 4 5 0 8 1 13
```

This is the number of workers with the corresponding shift schedule and salary, which are explained as follow:

- There are 2 employees with the shift 1 schedule (having Sundays and Mondays off), which receive a salary of 775.
- There are 4 employees with the shift 2 schedule (having Mondays and Tuesdays off), which receive a salary of 800.
- There are 5 employees with the shift 3 schedule (having Tuesdays and Wednesdays off), which receive a salary of 800.
- There are 0 employeeed under this schedule
- There are 8 employees with the shift 5 schedule (having Thursdays and Fridays off), which receive a salary of 800.
- There are 1 employee with the shift 6 schedule (having Fridays and Saturdays off), which receives a salary of 775.
- There are 13 employees under shift 7 schedule (having Saturdays and Sundays off), which receive a salary of 750.

\$25,675 in total.

How many workers are available each day?

```
# See the constraints
get.constraints(ap)
```

```
[1] 18 27 24 28 25 24 19
```

Here we can see the number of workers available per day. It also allow us to determine that the constraints are satisfied. Almost all days have the minimum required except Tuesdays and Wednesdays, which there are two extra people, and Fridays three more people.