Assignment 1 | Module 2

Melissa Paniagua

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The purpose of this assignment is to formulate a linear programming problem and assist you in meeting the following module outcome:

Formulate a linear program model for different scenarios.

Problem 1

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long- term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. Clearly define the decision variables

The decision variable is the quantity of each type of backpack (collegiate backpack and mini backpack) to produce per week.

Let

X_1= Number of collegiate backpacks to produce per week.

X_2= Number of mini backpacks to produce per week.

b. What is the objective function?

The objective function is:

$Z = 32X_1 + 24X_2$

c. What are the constraints?

$X_1 \leq 1000$ max items sold per week

$X_2 \leq 1200$ max items sold per week

$45X_1 + 40X_2 \leq 84000$ minutes per week (35 people * 40 hours * 60minutes)

$3X_1 + 2X_2 \leq 5000$ square-foot of material per week

$X_1 \geq 0$

$X_2 \geq 0$

d. Write down the full mathematical formulation for this LP problem.

Let

X_1= Number of collegiate backpacks to produce per week.

X_2= Number of mini backpacks to produce per week.

Max

$Z = 32X_1 + 24X_2$

Subject To

$X_1 \leq 1000$ max items sold per week

$X_2 \leq 1200$ max items sold per week

$45X_1 + 40X_2 \leq 84000$ minutes per week (35 people * 40 hours * 60minutes)

$3X_1 + 2X_2 \leq 5000$ square-foot of material per week

$X_1 \geq 0$

$X_2 \geq 0$

Now, let's solve this problem utilizing R.

To accomplish this problem, we will utilize a package in R named "lpSolve"

```r
# Import the lpSolve package.
library(lpSolve)

# Set coefficients of the objective function.
my_obfunct <- c(32, 24)

# Set the left hand side of the problem's constraints.
my_lefconst <- matrix(c(1, 0,
                        0, 1,
                        45, 40,
                        3, 2), nrow = 4, byrow = TRUE)

# Set the right hand side of the problem's constraints.
my_rigconst <- c(1000,
                 1200,
                 84000,
                 5000)

# Set the unequality signs
my_signs <- c("<=",
              "<=",
              "<=",
              "<=")

# To get the objective function' solution for the lp problem
lp("max", my_obfunct, my_lefconst, my_signs, my_rigconst)
```

```
## Success: the objective function is 55400
```

```r
# To get the optimal number of backpacks to produce per week
lp("max", my_obfunct, my_lefconst, my_signs, my_rigconst)$solution
```

```
## [1] 1000  975
```

As we can see, the optimal number of backpacks to produce per week are 1000 collegiate backpacks and 975 mini backpacks. It will allow the company to have a maximum profitability of $55,400 per week.

Problem 2

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

The decision variables are the number of units of the new product, regardless its size, that should be produced on each plant to maximize the company's profit.

Note:

$X_i =$ means the number of units produced on each plant, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

L, M, and S = means the product's size, where L = large, M = medium, and S = small.

The decision variables are:

$X_iL =$ number of large items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

$X_iM =$ number of medium items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

$X_iS =$ number of small items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

b. Formulate a linear programming model for this problem.

Let

$X_iL =$ number of large items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

$X_iM =$ number of medium items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

$X_iS =$ number of small items produced on on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

Maximize profit

$Z = 420 \cdot (X_1L + X_2L + X_3L) + 360 \cdot (X_1M + X_2M + X_3M) + 300 \cdot (X_1S + X_2S + X_3S)$

Constraints:

Total number of size's units produced regardless the plant:

$L = X_1L + X_2L + X_3L$

$M = X_1M + X_2M + X_3M$

$S = X_1S + X_2S + X_3S$

Production capacity per unit by plant each day:

Plant $1 = X_1L + X_1M + X_1S \leq 750$

Plant $2 = X_2L + X_2M + X_2S \leq 900$

Plant $3 = X_3L + X_3M + X_3S \leq 450$

Storage capacity per unit by plant each day:

Plant $1 = 20X_1L + 15X_1M + 12X_1S \leq 13000$

Plant $2 = 20X_2L + 15X_2M + 12X_2S \leq 12000$

Plant $3 = 20X_3L + 15X_3M + 12X_3S \leq 5000$

Sales forecast per day:

$L = X_1L + X_2L + X_3L \leq 900$

$M = X_1M + X_2M + X_3M \leq 1200$

$S = X_1S + X_2S + X_3S \leq 750$

The plants should use the same percentage of their excess capacity to produce the new product.

$$\frac{X_1L + X_1M + X_1S}{750} = \frac{X_2L + X_2M + X_2S}{900} = \frac{X_3L + X_3M + X_3S}{450}$$

It can be simplified as:

a) $900(X_1L + X_1M + X_1S) - 750(X_2L + X_2M + X_2S) = 0$

b) $450(X_2L + X_2M + X_2S) - 900(X_3L + X_3M + X_3S) = 0$

c) $450(X_1L + X_1M + X_1S) - 750(X_3L + X_3M + X_3S) = 0$

All values must be greater or equal to zero

$L, M,$ and $S \geq 0$

$X_iL, X_iM,$ and $X_iS \geq 0$

Now, let's solve this problem utilizing R.

To accomplish this problem, we will also use the "lpSolve" package in R.

```r
# Import the lpSolve package.
library(lpSolve)

# Set coefficients of the objective function.
my_obfunct <- c(420, 420, 420,
                360, 360, 360,
                300, 300, 300)

# Set the left hand side of the problem's constraints.
my_lefconst <- matrix(c(1, 0, 0, 1, 0, 0, 1, 0, 0,
                        0, 1, 0, 0, 1, 0, 0 ,1, 0,
```

```r
                         0, 0, 1, 0, 0, 1, 0, 0, 1,
                         20, 0, 0, 15, 0, 0, 12, 0, 0,
                         0, 20, 0, 0, 15, 0, 0, 12, 0,
                         0, 0, 20, 0, 0, 15, 0, 0, 12,
                         1, 1, 1, 0, 0, 0, 0, 0, 0,
                         0, 0, 0, 1, 1, 1, 0, 0, 0,
                         0, 0, 0, 0, 0, 0, 1, 1, 1,
                         900, 900, 900, -750, -750, -750, 0, 0, 0,
                         0, 0, 0, 450, 450, 450, -900, -900, -900,
                         450, 450, 450, 0, 0, 0, -750, -750, -750), nrow = 12, byrow = TRUE)


# Set the right hand side of the problem's constraints.
my_rigconst <- c(750,
                 900,
                 450,
                 13000,
                 12000,
                 5000,
                 900,
                 1200,
                 750,
                 0,
                 0,
                 0)

# Set the unequality signs
my_signs <- c("<=",
              "<=",
              "<=",
              "<=",
              "<=",
              "<=",
              "<=",
              "<=",
              "<=",
              "=",
              "=",
              "=")

# Set up the final lp problem
lp("max", my_obfunct, my_lefconst, my_signs, my_rigconst)
```

```
## Success: the objective function is 684955.8
```

```r
# To get the soluction of the lp problem
lp("max", my_obfunct, my_lefconst, my_signs, my_rigconst)$solution
```

```
## [1] 500.000000   2.654867 161.061947   0.000000 796.460177   0.000000 250.000000
## [8]   0.000000 148.230088
```

In order to get the same percentage of plant's excess capacity to produce the new product in three sizes (large, medium, small); plant 1, plant 2, and plant 3 should use 88.49% of its capacity.

R code source:

Salazar, R. (2019). Linear Programming with R. Retrieved from https://towardsdatascience.com/linear-programming-in-r-444e9c199280