Linear Programming Problem in R

Melissa Paniagua

9/28/2020

Assignment 2 | Module 4

The purpose if this assignment is to solve linear programming problems utilizing R

Weigelt Corporation

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

Linear programming model for this problem.

Let

X_iL = number of large items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

X_iM = number of medium items produced on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

X_iS = number of small items produced on on plant i, where i= 1 (Plant 1), 2 (Plant 2), and 3 (Plant 3).

Maximize profit

$Z = 420 \cdot (X_1L + X_2L + X_3L) + 360 \cdot (X_1M + X_2M + X_3M) + 300 \cdot (X_1S + X_2S + X_3S)$

Constraints:

Total number of size's units produced regardless the plant:

$L = X_1L + X_2L + X_3L$

$M = X_1M + X_2M + X_3M$

$S = X_1S + X_2S + X_3S$

Production capacity per unit by plant each day:

Plant 1 $= X_1L + X_1M + X_1S \leq 750$

Plant 2 $= X_2L + X_2M + X_2S \leq 900$

Plant 3 $= X_3L + X_3M + X_3S \leq 450$

Storage capacity per unit by plant each day:

Plant 1 $= 20X_1L + 15X_1M + 12X_1S \leq 13000$

Plant 2 $= 20X_2L + 15X_2M + 12X_2S \leq 12000$

Plant 3 $= 20X_3L + 15X_3M + 12X_3S \leq 5000$

Sales forecast per day:

$L = X_1L + X_2L + X_3L \leq 900$

$M = X_1M + X_2M + X_3M \leq 1200$

$S = X_1S + X_2S + X_3S \leq 750$

The plants should use the same percentage of their excess capacity to produce the new product.

$$\frac{X_1L + X_1M + X_1S}{750} = \frac{X_2L + X_2M + X_2S}{900} = \frac{X_3L + X_3M + X_3S}{450}$$

It can be simplified as:

a) $900(X_1L + X_1M + X_1S) - 750(X_2L + X_2M + X_2S) = 0$

b) $450(X_2L + X_2M + X_2S) - 900(X_3L + X_3M + X_3S) = 0$

c) $450(X_1L + X_1M + X_1S) - 750(X_3L + X_3M + X_3S) = 0$

All values must be greater or equal to zero

L, M, and $S \geq 0$

X_iL, X_iM, and $X_iS \geq 0$

Solving this problem utilizing R.

Add the LP objective function and constrains

To accomplish it, we will use the "lpSolveAPI" package in R.

```
# Import the lpSolve package.
library(lpSolveAPI)

# Create an lp object named lppoint with 0 constraints and 9 decision variables.
lppoint <- make.lp(0, 9)

# Set the objective function. Minimization problem is the default, but in the
#following steps we will set the problem to maximize.
set.objfn(lppoint, c(420, 420, 420,
                     360, 360, 360,
                     300, 300, 300))

# Here we change the direction to set our problem to maximize.
lp.control(lppoint,sense='max')
```

```r
# Add the 12 constraints based on the plant's number and products made on those plants.
add.constraint(lppoint, c(1, 0, 0, 1, 0, 0, 1, 0, 0), "<=", 750)
add.constraint(lppoint, c(0, 1, 0, 0, 1, 0, 0, 1, 0), "<=", 900)
add.constraint(lppoint, c(0, 0, 1, 0, 0, 1, 0, 0, 1), "<=", 450)

add.constraint(lppoint, c(20, 0, 0, 15, 0, 0, 12, 0, 0), "<=", 13000)
add.constraint(lppoint, c(0, 20, 0, 0, 15, 0, 0, 12, 0), "<=", 12000)
add.constraint(lppoint, c(0, 0, 20, 0, 0, 15, 0, 0, 12), "<=", 5000)

add.constraint(lppoint, c(1, 1, 1, 0, 0, 0, 0, 0, 0), "<=", 900)
add.constraint(lppoint, c(0, 0, 0, 1, 1, 1, 0, 0, 0), "<=", 1200)
add.constraint(lppoint, c(0, 0, 0, 0, 0, 0, 1, 1, 1), "<=", 750)

add.constraint(lppoint, c(900, -750, 0, 900, -750, 0, 900, -750, 0), "=", 0)
add.constraint(lppoint, c(0, 450, -900, 0, 450, -900, 0, 450, -900), "=", 0)
add.constraint(lppoint, c(450, 0, -750, 450, 0, -750, 450, 0, -750), "=", 0)
```

Set bounds for variables.

Remember that all variables had to be non-negative. We don't need to to do it here, as that is the default,we can set bounds explicitly.

```r
# Set bounds for variables.
set.bounds(lppoint, lower = c(0, 0, 0, 0, 0, 0, 0, 0, 0),
           columns = c(1:9)) #Not really needed

# To identify the variables and constraints, we can set variable names and name the constraints
RowNames <- c("PrdCapPlant1", "PrdCapPlant2", "PrdCapPlant3",
              "SrgCapPlant1", "SrgCapPlant2", "SrgCapPlant3",
              "SlsForctLarge", "SlsForctMedium", "SlsForctSmall",
              "PctCapPlant1_2", "PctCapPlant2_3", "PctCapPlant1_3")
ColNames <- c("Plant1Large", "Plant2Large", "Plant3Large",
              "Plant1Medium", "Plant2Medium", "Plant3Medium",
              "Plant1Small", "Plant2Small", "Plant3Small")
dimnames(lppoint) <- list(RowNames, ColNames)

# Now, print out the model
lppoint
```

```
Model name:
  a linear program with 9 decision variables and 12 constraints
```

Solve the LP problem presented structued above

```r
solve(lppoint)
```

```
[1] 0
```

As we can see, the result is "0," which means it was succefully solved.

We now output the value of the objective function, and the variables

```r
# It will give us the maximun profitabily for the optimal solution.
get.objective(lppoint)
```

```
[1] 696000
```

```r
# It will give us the optimal number of size products made on each production plant.
get.variables(lppoint)
```

```
[1] 516.6667   0.0000   0.0000 177.7778 666.6667   0.0000   0.0000 166.6667
[9] 416.6667
```

In order to get the same percentage of plant's excess capacity to produce the new product in three sizes (large, medium, small) on each plant; plant 1, plant 2, and plant 3 should use 92.59% of its capacity.

As we can see above, each plant should produce the following:

Plant 1, 516.67 of large products and 177.78 of medium products.

Plant 2, 666.67 of medium products and 166.67 of small products.

Plant 3, 416.67 of small products.