1、$\alpha_{2\psi}(\theta) = 1 - \beta_\psi(X \in W)$

$= 1 - P_{\theta=\theta_1}(X_1 = X_2 = X_3 = 1)$

$= 1 - (\theta_1^2)^3$

$= 1 - \theta_1^6$

2.(1) $L(p) = \prod_{i=1}^{130} p(1-p)^{y_i - 1}$

$= \prod_{i=1}^{12} \left(p(1-p)^{i-1}\right)^{x_i}$

$= \prod_{i=1}^{12} p^{x_i} (1-p)^{(i-1)x_i}$

$= p^{130} (1-p)^{\sum_{i=2}^{12}(i-1)x_i}$

$l(p) = 130 \ln p + \sum_{i=2}^{12}(i-1)x_i \ln(1-p)$

令 $\dfrac{dl(p)}{dp} = \dfrac{130}{p} - \dfrac{\sum_{i=1}^{12}(i-1)x_i}{1-p} = 0$

得 $\dfrac{1}{p} - 1 = \dfrac{\sum_{i=2}^{12}(i-1)x_i}{130} = \dfrac{233}{130} = 1.7923$

$p = 0.358$

(2). H_0: "X服从几何分布"成立 $\Longleftrightarrow$ H_1: "X服从几何分布"不成立

k	1	2	3	4	5	6	≥ 7	n
频数	48	31	20	9	6	5	11	130.
np	np_1	np_2	np_3	np_4	np_5	np_6	$n(1-\sum_{j=1}^{6} p_j)$	

$Z_0 = \sum_{i=1}^{6} \dfrac{n_i^2}{n\,p(1-p)^{i-1}} + \dfrac{n_7^2}{n(1-\sum_{j=1}^{6} p_j)} - n.$

$= \dfrac{1}{np} \sum_{i=1}^{6} \dfrac{n_i^2}{(1-p)^{i-1}} + \dfrac{n_7^2}{n(1-\sum_{j=1}^{6} p(1-p)^{j-1})} - n.$

$= 131.868 - 130 = 1.868$

而 $\chi^2_{7-1-1}(0.05) = \chi^2_5(0.05) = 11.071$

当 $Z_0 > \chi^2_5(0.05)$ 时可以拒绝 H_0

但 $z_0 = 1.868 < \chi^2(0.05) = 11.071$

故不能拒绝 H_0，即可认为“X服从几何分布”是正确的

3. 列列联表 H_0: 两班水平大致相当 $\longleftrightarrow$ H_1 两班水平不同

							$n_{i\cdot}$
	8	27	10	6	8	6	65
	15	25	8	7	6	4	65
$n_{\cdot j}$	23	52	18	13	14	10	130

$$z = \sum_{i=1}^{2}\sum_{j=1}^{6} \frac{(n n_{ij} - n_{i\cdot} n_{\cdot j})^2}{n n_{i\cdot} n_{\cdot j}}$$

$$= \sum_{j=1}^{6} \frac{(130 n_{1j} - 65 n_{\cdot j})^2}{65\times130\, n_{\cdot j}} + \sum_{j=1}^{6} \frac{(130 n_{2j} - 65 n_{\cdot j})^2}{65\times130\, n_{\cdot j}}$$

$$= \sum_{j=1}^{6} \frac{(2n_{1j} - n_{\cdot j})^2}{2n_{\cdot j}} + \sum_{j=1}^{6} \frac{(2n_{2j} - n_{\cdot j})^2}{2n_{\cdot j}}$$

$$\overset{n_{1j}+n_{2j} = n_{\cdot j}}{=\!=} \sum_{j=1}^{6} \frac{(n_{1j} - n_{2j})^2}{2n_{\cdot j}} \cdot 2 = \sum_{j=1}^{6} \frac{(n_{1j} - n_{2j})^2}{n_{\cdot j}}$$

$$= \frac{7^2}{23} + \frac{2^2}{52} + \frac{2^2}{18} + \frac{1^2}{13} + \frac{2^2}{14} + \frac{2^2}{10}$$

$$= 3.1922$$

而 $\chi^2_{1\times5}(0.05) = \chi^2_5(0.05) = 11.071$

由 $z < \chi^2_5(0.05)$ 故不能拒绝 H_0

可以认为两班英语水平相当.