第五次作业答案

章海🄵

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目录

第一题

对于 $n=2, l=0$ 或 1 。对于 $l=0, m_l=0$; 对于 $l=1$, $m_l=-1$ 0 或 1 。对于任意 $m_l, m_s=+\frac{1}{2}$ 或 $-\frac{1}{2}$ 。有八个电子态: $(n,l,m_t,m_s)=\left(2,0,0,+\frac{1}{2}\right)$, $\left(2,0,0,-\frac{1}{2}\right)$, $\left(2,1,-1,+\frac{1}{2}\right)$, $\left(2,1,-1,-\frac{1}{2}\right)$, $\left(2,1,0,+\frac{1}{2}\right)$, $\left(2,1,0,-\frac{1}{2}\right)$, $\left(2,1,1,+\frac{1}{2}\right)$, $\left(2,1,1,-\frac{1}{2}\right)$ 。

第二题

解:
$$\begin{aligned}
\overline{V(r)} &= \int_0^\infty r^2\left|R_{10}(r)\right|^2 V(r)\mathrm{d}r = -\frac{1}{4\pi\varepsilon_0} * \int_0^\infty r^2\left|R_{10}(r)\right|^2 \frac{e^2}{r}\ \mathrm{d}r \\
&= -\frac{1}{4\pi\varepsilon_0} * e^2\int_0^\infty \frac{4r}{a_{\mathrm{B}}^3}\mathrm{e}^{-2r/a_{\mathrm{B}}}\mathrm{d}r = -\frac{1}{4\pi\varepsilon_0} * \frac{e^2}{a_{\mathrm{B}}}\int_0^\infty x\mathrm{e}^{-x}\ \mathrm{d}x \quad \left(x=\frac{2r}{a_{\mathrm{B}}}\right) \\
&= -\frac{1}{4\pi\varepsilon_0} * \frac{e^2}{a_{\mathrm{B}}}\left[-(x+1)\mathrm{e}^{-x}\right]_0^\infty = -\frac{1}{4\pi\varepsilon_0} * \frac{e^2}{a_{\mathrm{B}}}.
\end{aligned}$$

第四题

解:
$$\psi = \frac{1}{\sqrt{\pi}a_0^{\frac{3}{2}}}e^{-\frac{r}{a_0}}$$
$$-\frac{\hbar^2}{2m}\Delta\psi < 0.$$

因而有:
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial e^{-\frac{r}{a_0}}}{\partial r}\right) = e^{-\frac{r}{a_0}}\left(\frac{r^2}{a_0^2}-\frac{2r}{a_0}\right) < 0.$$

得到: $r > 2a_0$

$$p(T<0)=\int_{r>2a_0} d^3\vec{r}\frac{1}{\pi a_0^3}e^{-\frac{2r}{a_0}}=\frac{13}{e^4}.$$

算毕。

第六题

解:

$$\psi(t)=\frac{1}{\sqrt{2}}\left(\psi_{200}e^{-i\frac{E_2}{\hbar}t}-\psi_{100}e^{-i\frac{E_1}{\hbar}t}\right)=$$

$$\frac{1}{\sqrt{2}}\left(R_{20}e^{-i\frac{E_2}{\hbar}t}-R_{10}e^{-i\frac{E_1}{\hbar}t}\right)Y_{00}(\theta,\phi)$$

$$\langle r\rangle_t=\frac{1}{2}\int r^2\cdot dr\left(R_{20}e^{i\frac{E_2}{\hbar}t}-R_{10}e^{i\frac{E_1}{\hbar}t}\right)r\left(R_{20}e^{-i\frac{E_2}{\hbar}t}-R_{10}e^{-i\frac{E_1}{\hbar}t}\right)=$$

$$\frac{15a_0}{4}+\frac{32}{81}\sqrt{2}a_0\cos\frac{E_2-E_1}{\hbar}t=\frac{15a_0}{4}+\frac{32}{81}\sqrt{2}a_0\cos\frac{3E_0}{4\hbar}t$$

这里 $E_0=-\frac{\hbar^2}{2\mu a_0^2}$ 。

第七题

解:

$$\psi(x,y,z)=\frac{\alpha^{\frac{5}{2}}}{\sqrt{\pi}}z\exp\left(-\alpha\left(x^2+y^2+z^2\right)\right)=\frac{\alpha^{\frac{5}{2}}}{\sqrt{\pi}}r\cos\theta\exp\left(-\alpha r^2\right);$$

$$L_z\psi=-i\hbar\frac{\partial}{\partial\phi}\psi(r,\theta,\phi)=0;$$

$$L^2\psi=-\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)\psi=2\hbar^2\psi$$

得证。本征值为 $\left(2\hbar^2,0\right)$

第九题

解:

$$L_{\theta,\phi}=\cos\theta L_z+\sin\theta\cos\phi L_x+\sin\theta\sin\phi L_y;$$

$$\langle L_{\theta,\phi}\rangle=\int d^3\vec{r}\psi^*\left(\cos\theta L_z+\sin\theta\cos\phi L_x+\sin\theta\sin\phi L_y\right)\psi=$$

$$\int d^3\vec{r}\psi^*\cos\theta L_z\psi=m\hbar\cos\theta.$$

第十一题

解:

$$[L_j, T] = \frac{1}{2m}\left[L_j, P_i^2\right] = \frac{1}{2m}\left(P_i\left[L_j, P_i\right] + \left[L_j, P_i^2\right] P_i\right) =$$

$$\frac{i\hbar}{2m}\left(\epsilon_{jik} P_i P_k + \epsilon_{jik} P_k P_i\right) = \frac{i\hbar}{2m}\left(\epsilon_{jki} + \epsilon_{jik}\right) P_k P_i = 0.$$

算毕。