Prob 5

a)

$Y=y$ \ $X=x$	-1	0	1
-1	0.06	0.18	0.06
0	0.08	0.24	0.08
1	0.06	0.18	0.06

$P(X=x \wedge Y=y)$

b1)

$$E(V|W=0) = \frac{\sum_{x,y} (x^2+y^2) P(X=x \wedge Y=y)\, I\{x+y=0\}}{P(X+Y=0)} = \frac{2\cdot 0.06 + 2\cdot 0.06}{0.06+0.24+0.06} = \frac{4\cdot 0.06}{0.36} = \frac{2}{3}$$

b2)

$$cov(V,W) = cov(X^2+Y^2, X+Y) = \cancel{cov((X+Y)^2, X+Y) - cov\, 2XY, X+Y}$$

$$= E(X^2+Y^2)(X+Y) - EX^2+Y^2 E(X+Y) = -1$$

$E(X^2+Y^2)(X+Y) = 2(-2)\cdot 0.06 + 1(-1)(0.18+0.08) + 1\cdot 1\,(0.08+0.06) + 2\cdot 2\; 0.06 = 0$

$EX^2+EY^2 = 0.2+0.2+0.3+0.3 = 1$

$EX+EY = 0$

c) as $cov(V,W) \neq 0 \Rightarrow$ V and W are not uncorrelated

as $cov(V,W) \neq 0 \Rightarrow$ V and W are not independent

Problem 6.

a) ξ - random driver η - ~~salary~~ income of ξ

$P(\xi \text{ pays}) = P(\eta \geq 60000) + \frac{1}{10} P(\eta < 60000) = 1 - F_\eta(60000) + \frac{1}{10} F_\eta(60000) = 1 - \frac{9}{10} F(60000) \boxed{=}$

$F_\eta(x) = \int_0^x f_\eta(y)\,dy = 0$ in $x \leq x_m = 15000$

$\int_{x_m}^{x} \frac{\alpha x_m^\alpha}{y^{\alpha+1}}\,dy = -y^{-\alpha} x_m^\alpha \Big|_{x_m}^{x} = 1 - \left(\frac{x_m}{x}\right)^\alpha$

$\boxed{=}\ 1 - \frac{9}{10}\left(1 - \left(\frac{1}{4}\right)^\alpha\right) = \frac{1}{10} + \frac{9}{10}\left(\frac{1}{4}\right)^\alpha =: \beta = 0.28276$

b) Expected value of drivers from A to B during the week is $5 \cdot 10000 + 2 \cdot 3000 = 56000$

=> expected number of payers is $\beta\, 56000$

=> expected weekly revenue is $500 \cdot 56000 \cdot \beta \approx 7\,917\,190$ rub

Prob 7.

A - event that this client delayed payment

B - event that this client delayed 2nd payment

C - event that this client is from 1st group

$P(C) = \frac{100}{600} = \frac{1}{6}$

$$P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{P(B\cap A|C)P(C) + P(B\cap A|\bar{C})P(\bar{C})}{P(A|C)P(C) + P(A|\bar{C})P(\bar{C})} =$$

$$= \frac{0.6^2 \cdot 1/6 + 0.06^2 \; 5/6}{0.6 \cdot 1/6 + 0.06 \cdot 5/6} = \frac{0.6^2 + 5 \cdot 0.06^2}{0.6 + 5 \, 0.06} = \frac{0.36 + 0.018}{0.9} = \frac{0.378}{0.9} = 0.42$$

Expected result for all the work 24/100

Prob 8.

a) Let A be event that both ball are black, and B - both are white

$$P(A \text{ or } B) = P(A) + P(B) = \frac{C_5^2}{C_{10}^2} + \frac{C_5^2}{C_{10}^2} = 2\frac{\frac{5\cdot4}{2}}{\frac{10\cdot9}{2}} = \frac{4}{9}$$

prob of they are the same color

C - event that they are the same color

$$\text{expected value gain} = 1\cdot P(C) + (-1)P(\overline{C}) = \frac{4}{9} + (-1)\left(1-\frac{4}{9}\right) = \frac{8}{9} - 1 = -\frac{1}{9}$$

b) $X_1 \dots X_n \dots$ - events that ith shot reached target (at the 1st gun)

$Y_1 \dots Y_n \dots$ - events that ith shots at 2nd gun reached target

$X_i \perp Y_j \quad i \neq j$

$\perp Y_k \perp Y_l \quad k \neq l.$

$X_i \sim Bern(p_1)$

$Y_i \sim Bern(p_2)$

$\xi_1 = \min i \mid X_i = 1 \qquad E\xi_1 = 10$

$\xi_2 = \min j \mid Y_j = 1 \qquad E\xi_2 = 15$

$$E\xi_1 = \sum_{i=1}^{\infty} i\, P(X_1, \dots X_{i-1} = 0 \wedge X_i = 1) = \sum_{i=\phi}^{\infty} i(1-p_1)^{i-1} p_1 = p_1 \sum_{i=\phi}^{\infty} i(1-p_1)^{i-1} = p_1 \frac{1}{p_1^2} = \frac{1}{p_1}$$

$$P(X_1=0)\dots P(X_{i-1}=0)\, P(X_i=1)$$

$$\frac{1}{(1-(1-p_1))^2}$$

$x<0 \qquad \frac{1}{1-x} = \sum_{i=0}^{\infty} x^i$

$$\left(\frac{1}{1-x}\right)' = \left(\sum_{i=0}^{\infty} x^i\right)' = \sum_{i=\phi}^{\infty} i x^{i-1}$$

$$= \frac{+1}{(1-x)^2}$$

$$E\xi_2 = \frac{1}{p_2} \Rightarrow \quad p_1 = 1/10, \quad p_2 = 1/15$$

$Z_i := \max(X_i, Y_i) \qquad Z_i \perp Z_j \quad i \neq j$

$$P(Z_i = 1) = P(X_1 = 1 \wedge Y_1 = 1) = 1 - P(X_1 = 0 \vee Y_1 = 0) = 1 - (1-p_1)(1-p_2) =$$

$$= p_1 + p_2 - p_1p_2 = \frac{1}{10} + \frac{1}{15} - \frac{1}{150} = \frac{25-1}{150} = \frac{24}{150}$$

$P(Z_1 = 0) = 1 - P(Z_i = 1)$

$\Rightarrow Z_i \sim Bern(p_1 + p_2 - p_1p_2) =: p_3$

$$\Rightarrow \text{Exp number of gulps} = \frac{1}{p_3} = \frac{1}{p_1+p_2-p_1p_2} = \frac{150}{24} = 6.25$$

Prob 8

c) $X, Y \sim U[0,1] \quad X \perp Y \qquad p_{(X,Y)}(x,y) = \underbrace{I\{x \in [0,1]\}}_{= I\{x\}} \underbrace{I\{y \in [0,1]\}}_{= I\{y\}}$

$$\mathrm{Cov}(XY, X) = EX^2Y - EXY\,\underbrace{EX}_{= 1/2}$$

$$\cancel{EXY = \int_{\mathbb{R}^2} xy\, p_{(X,Y)}(x,y)\,dx\,dy = \int_{[0,1]\times[0,1]} xy\,dx\,dy = \left(\int_{[0,1]} x\,dx\right)^2 =}$$

as $X \perp Y$ $\quad EXY = EX\,EY = 1/4$

$$EX^2Y = EX^2EY = \frac{1}{2}EX^2 = \frac{1}{6}$$

as $X \perp Y \Rightarrow X^2 \perp Y$

$$EX^2 = \int_0^1 x^2\,dx = 1/3$$

$$\mathrm{cov}(XY, X) = 1/3 \cdot \frac{1}{2} - \frac{1}{8} = \frac{2}{24} = \frac{1}{12}$$