张晨阳-第10次作业

10. 由算法 3，得到 $N'=\{S,A_1,A_2,A_3,A_4,A_5\}$，因为 $S\in N'$，则变换为无 ε 生成式的文法：
$G_1=(\{S_1,S,A_1,A_2,A_3,A_4,A_5\},\{a,b,d\},P_1,S_1)$,其中 P_1 如下：
$S_1\to\varepsilon|S,\quad S\to A_1|A_2,\quad A_1\to A_3|A_4,\quad A_2\to A_4|A_5$
$A_3\to S|b,\quad A_4\to S|a,\quad A_5\to S|d$

由算法 4，得到 $N_{S_1}=\{S_1,S,A_1,A_2,A_3,A_4,A_5\}$
$N_S=N_{A_1}=N_{A_2}=N_{A_3}=N_{A_4}=N_{A_5}=\{S,A_1,A_2,A_3,A_4,A_5\}$
则变换为无单生成式的 P_1 如下：
$S_1\to\varepsilon|a|b|d,\quad S\to a|b|d,\quad A_1\to a|b|d,\quad A_2\to a|b|d$
$A_3\to a|b|d,\quad A_4\to a|b|d,\quad A_5\to a|b|d$

由算法 1 和算法 2，得到只有 S_1 为可达符号，则变换为没有无用符号的等价文法：
$G_1=(\{S_1\},\{a,b,d\},P_1,S_1)$，其中 P_1 为：$S_1\to\varepsilon|a|b|d$

11. 由算法 3，得到 $N'=\{S\}$，因为 $S\in N'$，则变换为无 ε 生成式的文法：
$G_1=(\{S_1,S,A,B\},\{a,b,\},P_1,S_1)$,其中 P_1 如下：
$S_1\to\varepsilon|S,\quad S\to ASB|AB$
$A\to aAS|aA|a,\quad B\to SBS|SB|BS|B|A|bb$

由算法 4，得到 $N_{S_1}=\{S_1,S\},\quad N_S=\{S\},\quad N_A=\{A\},\quad N_B=\{A,B\}$
则变换为无单生成式的文法：
$G_1=(\{S_1,S,A,B\},\{a,b,\},P_1,S_1)$,其中 P_1 如下：
$S_1\to\varepsilon|ASB|AB,\quad S\to ASB|AB$
$A\to aAS|aA|a,\quad B\to SBS|SB|BS|aAS|aA|a|bb$

由算法 1 和算法 2，本题没有无用符号，则无单生成式的文法即为没有无用符号的文法
最后转换为 CNF：
将 $S_1\to ASB$ 变换为 $S_1\to AC,\quad C\to SB$
将 $S\to ASB$ 变换为 $S\to AC$
将 $A\to aAS|aA$ 变换为 $A\to ED|EA,\quad D\to AS,\quad E\to a$
将 $B\to SBS|aAS|aA|bb$, 变换为 $B\to CS|ED|EA|FF,\quad F\to b$
则相应的等价文法为：$G_1=(\{S_1,S,A,B,C,D,E,F\},\{a,b,\},P_1,S_1)$,其中 P_1 如下：
$S_1\to\varepsilon|AC|AB,\quad S\to AC|AB$
$A\to ED|EA|a,\quad B\to CS|SB|BS|ED|EA|a|FF$
$C\to SB,\quad D\to AS,\quad E\to a,\quad F\to b$

15. (1) 将 1 式代入 2 式中得：$D \to DDS|aS|b$

消除直接左递归得到：$D \to aSD'|bD'|aS|b, \quad D' \to DS|DSD'$

回代，得到：$S \to aSD'D|bD'D|aSD|bD|a$

$D' \to aSD'S|aSD'SD'|bD'S|bD'SD'|aSS|aSSD'|bS|bSD'$

则等价的 GNF 为：

$G_1 = (\{S, D, D'\}, a, b, P_1, S)$,其中生成式 P_1 如下：

$S \to asD'D|bD'D|aSD|bD|a$

$D \to aSD'|bD'|aS|b$

$D' \to aSD'S|aSD'SD'|bD'S|bD'SD'|aSS|aSSD'|bS|bSD'$

(2) 首先将其转换为等价的 CNF：

$G_1 = (\{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}, \{a, b\}, P_1, A_1)$,其中 P_1 如下：

$A_1 \to A_3A_4|A_2A_5$

$A_2 \to A_1A_4|A_2A_6|b$

$A_3 \to A_1A_5|A_3A_7|a$

$A_4 \to b$

$A_5 \to a$

$A_6 \to A_2A_5$

$A_7 \to A_3A_4$

接下来进行按顺序的代换、消除直接左递归、最后回代：

对于 A_2:

$A_2 \to A_3A_4A_4|A_3A_4A_4A_2'|bA_2'|b$

$A_2' \to A_5A_4A_2'|A_6A_2'|A_5A_4|A_6$

对于 A_3:

$A_3 \to bA_5A_5|bA_2'A_5A_5|a|bA_5A_5A_3'|bA_2'A_5A_5A_3'|aA_3'$

$A_3' \to A_4A_5|A_4A_4A_5A_5|A_4A_4A_2'A_5A_5|A_7|A_4A_5A_3'|A_4A_4A_5A_5A_3'|A_4A_4A_2'A_5A_5A_3'|A_7A_3'$

对于 A_6:

$A_6 \to bA_5A_5A_4A_4A_5|bA_2'A_5A_5A_4A_4A_5|aA_74A_4A_5|bA_5A_5A_3'A_4A_4A_5|bA_2'A_5A_5A_3'A_4A_4A_5|$
$aA_3'A_4A_4A_5|bA_5A_5A_4A_4A_2'A_5|bA_2'A_5A_5A_4A_4A_2'A_5|aA_4A_4A_2'A_5|bA_5A_5A_3'A_4A_4A_2'A_5$
$|bA_2'A_5A_5A_3'A_4A_4A_2'A_5|aA_3'A_4A_4A_2'A_5|bA_2'A_5|bA_5$

将 A_3 生成式代入 A_7 生成式:

$A_7 \to bA_5A_5A_4|bA_2'A_5A_5A_4|aA_4|bA_5A_5A_3'A_4|bA_2'A_5A_5A_3'A_4|aA_3'A_4$

将 A_5, A_6 生成式代入 A_2' 生成式:

$$
\begin{aligned}
A_2' \to\ & aA_4A_2'|bA_5A_5A_4A_4A_5A_2'|bA_2'A_5A_5A_4A_4A_5A_2'|aA_4A_4A_5A_2'| \\
& bA_5A_5A_3'A_4A_4A_5A_2'|bA_2'A_5A_5A_3'A_4A_4A_5A_2'|aA_3'A_4A_4A_5A_2'| \\
& bA_5A_5A_4A_4A_2'A_5A_2'|bA_2'A_5A_5A_4A_4A_2'A_5A_2'|aA_4A_4A_2'A_5A_2'| \\
& bA_5A_5A_3'A_4A_4A_2'A_5A_2'|bA_2'A_5A_5A_3'A_4A_4A_2'A_5A_2'|aA_3'A_4A_4A_2'A_5A_2'| \\
& bA_2'A_5A_2'|bA_5A_2'|aA_4|bA_5A_5A_4A_4A_5|bA_2'A_5A_5A_4A_4A_5|aA_4A_4A_5| \\
& bA_5A_5A_3'A_4A_4A_5|bA_2'A_5A_5A_3'A_4A_4A_5|aA_3'A_4A_4A_5|bA_5A_5A_4A_4A_2'A_5| \\
& bA_2'A_5A_5A_4A_4A_2'A_5|aA_4A_4A_2'A_5|bA_5A_5A_3'A_4A_4A_2'A_5 \\
& |bA_2'A_5A_5A_3'A_4A_4A_2'A_5|aA_3'A_4A_4A_2'A_5|bA_2'A_5|bA_5
\end{aligned}
$$

将 A_4, A_7 生成式代入 A_3' 生成式:

$$
\begin{aligned}
A_3' \to\ & aA_5|aA_4A_5A_5|aA_4A_2'A_5A_5|aA_5A_3'|aA_4A_5A_5A_3'|aA_4A_2'A_5A_5A_3'| \\
& bA_5A_5A_4|bA_2'A_5A_5A_4|aA_4|bA_5A_5A_3'A_4|bA_2'A_5A_5A_3'A_4|aA_3'A_4|bA_5A_5A_4A_3'| \\
& bA_2'A_5A_5A_4A_3'|aA_4A_3'|bA_5A_5A_3'A_4A_3'|bA_2'A_5A_5A_3'A_4A_3'|aA_3'A_4A_3'
\end{aligned}
$$

由此得出等价的 GNF:

$G_1 = (\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_2', A_3'\}, \{a, b\}, P_1, A_1)$,其中 P_1 如下:

$$
\begin{aligned}
A_1 \to\ & bA_5A_5A_4|bA_2'A_5A_5A_4|aA_4|bA_5A_5A_3'A_4|bA_2'A_5A_5A_3'A_4|aA_3'A_4|bA_5A_5A_4A_4A_5| \\
& bA_2'A_5A_5A_4A_4A_5|aA_4A_4A_5|bA_5A_5A_3'A_4A_4A_5|bA_2'A_5A_5A_3'A_4A_4A_5|aA_3'A_4A_4A_5| \\
& bA_5|bA_5A_5A_4A_4A_2'A_5|bA_2'A_5A_5A_4A_4A_2'A_5|aA_4A_4A_2'A_5|bA_5A_5A_3'A_4A_4A_2'A_5| \\
& bA_2'A_5A_5A_3'A_4A_4A_2'A_5|aA_3'A_4A_4A_2'A_5|bA_2'A_5 \\
A_2 \to\ & bA_5A_5A_4A_4|bA_2'A_5A_5A_4A_4|aA_4A_4|bA_5A_5A_3'A_4A_4|bA_2'A_5A_5A_3'A_4A_4|aA_3'A_4A_4| \\
& bA_5A_5A_4A_4A_2'|bA_2'A_5A_5A_4A_4A_2'|aA_4A_4A_2'|bA_5A_5A_3'A_4A_4A_2'| \\
& bA_2'A_5A_5A_3'A_4A_4A_2'|aA_3'A_4A_4A_2'|bA_2'|b \\
A_3 \to\ & bA_5A_5|bA_2'A_5A_5|a|bA_5A_5A_3'|bA_2'A_5A_5A_3'|aA_3' \\
A_4 \to\ & b \\
A_5 \to\ & a \\
A_6 \to\ & bA_5A_5A_4A_4A_5|bA_2'A_5A_5A_4A_4A_5|aA_74A_4A_5|bA_5A_5A_3'A_4A_4A_5|bA_2'A_5A_5A_3'A_4A_4A_5| \\
& aA_3'A_4A_4A_5|bA_5A_5A_4A_4A_2'A_5|bA_2'A_5A_5A_4A_4A_2'A_5|aA_4A_4A_2'A_5|bA_5A_5A_3'A_4A_4A_2'A_5 \\
& |bA_2'A_5A_5A_3'A_4A_4A_2'A_5|aA_3'A_4A_4A_2'A_5|bA_2'A_5|bA_5 \\
A_7 \to\ & bA_5A_5A_4|bA_2'A_5A_5A_4|aA_4|bA_5A_5A_3'A_4|bA_2'A_5A_5A_3'A_4|aA_3'A_4 \\
A_2' \to\ & aA_4A_2'|bA_5A_5A_4A_4A_5A_2'|bA_2'A_5A_5A_4A_4A_5A_2'|aA_4A_4A_5A_2'| \\
& bA_5A_5A_3'A_4A_4A_5A_2'|bA_2'A_5A_5A_3'A_4A_4A_5A_2'|aA_3'A_4A_4A_5A_2'| \\
& bA_5A_5A_4A_4A_2'A_5A_2'|bA_2'A_5A_5A_4A_4A_2'A_5A_2'|aA_4A_4A_2'A_5A_2'| \\
& bA_5A_5A_3'A_4A_4A_2'A_5A_2'|bA_2'A_5A_5A_3'A_4A_4A_2'A_5A_2'|aA_3'A_4A_4A_2'A_5A_2'| \\
& bA_2'A_5A_2'|bA_5A_2'|aA_4|bA_5A_5A_4A_4A_5|bA_2'A_5A_5A_4A_4A_5|aA_4A_4A_5| \\
& bA_5A_5A_3'A_4A_4A_5|bA_2'A_5A_5A_3'A_4A_4A_5|aA_3'A_4A_4A_5|bA_5A_5A_4A_4A_2'A_5| \\
& bA_2'A_5A_5A_4A_4A_2'A_5|aA_4A_4A_2'A_5|bA_5A_5A_3'A_4A_4A_2'A_5 \\
& |bA_2'A_5A_5A_3'A_4A_4A_2'A_5|aA_3'A_4A_4A_2'A_5|bA_2'A_5|bA_5 \\
A_3' \to\ & aA_5|aA_4A_5A_5|aA_4A_2'A_5A_5|aA_5A_3'|aA_4A_5A_5A_3'|aA_4A_2'A_5A_5A_3'|
\end{aligned}
$$

$bA_5A_5A_4|bA_2'A_5A_5A_4|aA_4|bA_5A_5A_3'A_4|bA_2'A_5A_5A_3'A_4|aA_3'A_4|bA_5A_5A_4A_3'|$
$bA_2'A_5A_5A_4A_3'|aA_4A_3'|bA_5A_5A_3'A_4A_3'|bA_2'A_5A_5A_3'A_4A_3'|aA_3'A_4A_3'$