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General theory of electroadhesion

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Abstract
We present a general theory of electroadhesion assuming layered materials with finite electric conductivity and an air gap resulting from interfacial surface roughness. The theory reduces to the results derived in Persson (2018 *J. Chem. Phys.* **148** 144701) in the appropriate limits. We present numerical results to illustrate the theory.

Keywords: electroadhesion, contact mechanics, contact resistance

(Some figures may appear in colour only in the online journal)

1. Introduction

The Danish engineers Alfred Johnsen and Knud Rahbek discovered a century ago that an attractive force occurs between two contacting materials when there is an electrical potential difference between them [1]. The term 'electroadhesion' was coined to denote this electrostatic attraction [1, 2]. The electrical attraction between a charged surface and a human finger was discovered by Johnsen and Rahbek. In 1953 Mallinckrodt *et al* [3] applied an alternating voltage to insulated metal electrodes and observed an alternating electrostatic force that periodically attract and release the finger from the surface; this is now denoted electrovibration [4–7], and forms the basis for electroadhesion based haptic devices such as touchscreens and tactile displays. For these applications, tactile sensations are produced by the application of a voltage to the conductive layer of an insulated haptic device such as a touchscreen, inducing electroadhesive forces between the device and the approached user finger. If the applied electric voltage is modulated in time the friction force acting on the finger will generate sensorial experiences [4–13].

The Johnsen–Rahbek effect is due to the electrostatic attraction between the polarization charges on two solids resulting from an applied electric potential. In several publications we have addressed different aspects of electrtoadhesion [14, 19–21]. In reference [14] I considered two limiting cases, namely the electroadhesive contact between (a) two metallic objects and (b) between two perfect conducing solids with insulating (dielectric) surface layers. Here we consider the more general case of the contact between two perfect conductors with surface layers with arbitrary conductivity's.

2. Theory

Consider two elastic solids with nominal flat surfaces squeezed together with the pressure p_0. Because of the surface roughness the solids in general only make real contact in a small fraction of the nominal contact area A_0. The electroadhesion force is determined by the electric field at the interface between the two solids.

2.1. The electric field

Consider the contact between two perfect conducting solids with dielectric films with the dielectric functions ϵ_1 and ϵ_2 (see figure 1). We will assume that the upper surface of the bottom solid is perfectly smooth and the bottom surface of the upper solid has roughness described by the $z = h(x, y)$ topography profile. We assume the thickness d_2 of the upper dielectric film is much larger than the amplitude of the surface roughness. With touchscreen applications in mind we assume the substrate is rigid and the upper solid an elastic (layered) material. At least for friction less stationary contact it is easy to include both surface roughness and elasticity of the substrate [22].

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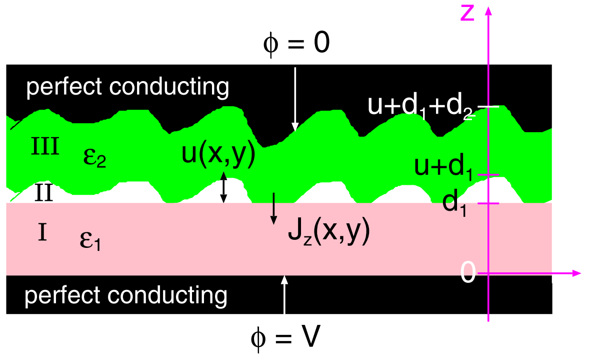

Figure 1. An elastic solid with surface roughness squeezed against a flat substrate.

We assume the electric field oscillate in time as $\exp(-\mathrm{i}\omega t)$ and write the dielectric functions as

$$\epsilon_1 = \epsilon_1^0 - \frac{\mathrm{i}\kappa_1}{\omega\epsilon_0} \tag{1}$$

$$\epsilon_2 = \epsilon_2^0 - \frac{\mathrm{i}\kappa_2}{\omega\epsilon_0}, \tag{2}$$

where $\epsilon_0 \approx 8.85 \times 10^{-12}$ F m^{-1} is the electric constant. In (1) and (2) ϵ_1^0 and ϵ_2^0 are real numbers due to the polarization (induced dipoles) of the materials and κ_1 and κ_2 the electric conductivity of the solids 1 and 2.

We will assume the small-slope approximation where $|\nabla h(x,y)| \ll 1$. In this case the electric potential will satisfy $\partial^2\phi/\partial z^2 \approx 0$. The electric potential in the three regions $0 < z < d_1$ (region I), $d_1 < z < d_1 + u$ (region II) and $d_1 + u < z < d_1 + d_2 + u$ (region III) can be written as

$$\phi = V + b_1 z \qquad \text{region I} \tag{3}$$

$$\phi = a + b(z - d_1) \qquad \text{region II} \tag{4}$$

$$\phi = b_2(z - u - d_1 - d_2) \qquad \text{region III} \tag{5}$$

Note that $\phi = V$ for $z = 0$ and $\phi = 0$ for $z = u + d_1 + d_2$.

The electric potential must be continuous at $z = d_1$ and for $z = d_1 + u$ which gives

$$V + b_1 d_1 = a \tag{6}$$

$$a + bu = -b_2 d_2 \tag{7}$$

Combining these equations gives

$$V + b_1 d_1 + b_2 d_2 = -bu \tag{8}$$

An electric current will flow from one solid to the other through the area of real contact. If ϕ_1' and ϕ_2' are the electric potential very close (but not too close) to the interface then the electric contact conductivity α is defined so that [15]

$$J_z = \alpha(\phi_2' - \phi_1'), \tag{9}$$

where J_z is the electric current close to the interface. Here 'close' mean at such a distance δ from the interface that ϕ_1' and ϕ_2' and hence J_z are nearly independent of the x and y coordinates. The distance δ is of order the distance between the macroasperity contact regions, which in turn is determined by the wavenumber of the roll-off region in the surface roughness power spectrum (see reference [15]). It is assumed that $\delta << d_1$ and $\delta << d_2$. We can also take ϕ_1' and ϕ_2' as the lateral averaged potential at the interface of solid 1 and 2. In the treatment below we will not perform this lateral averaging as it result in a more complex theory but with similar numerical result. Thus, the final expression for the electroadhesive force derived below reduces to the results derived in reference [14] in the appropriate limits.

For the contact between elastic solids we know that [15–17]

$$\alpha = \frac{2\kappa}{E^*} K_\perp, \tag{10}$$

where the mechanical contact stiffness

$$K_\perp = -\frac{\mathrm{d}p}{\mathrm{d}\bar{u}}. \tag{11}$$

Where $\bar{u}$ is the average surface separation and p the applied (uniform) pressure. The effective Young's modulus E^* is defined by

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_1^2}{E_2} \tag{12}$$

and the effective conductivity κ is defined by

$$\frac{1}{\kappa} = \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \tag{13}$$

Note that (10) and (11) are not valid for viscoelastic solids, or when plasticity is important.

Using (3) and (5) in (9) we get

$$J_z = -\alpha V + b_1 d_1 + b_2 d_2.$$

Using (8) this gives

$$J_z = \alpha b u \tag{14}$$

If q denote the surface charge density per unit surface area, ensemble averaged (or averaged over the xy-plane), moving from one solid to the other, then from the charge continuity equation we deduce that

$$J_z = -\dot{q}$$

or if the time dependency is $\exp(-\mathrm{i}\omega t)$:

$$J_z = \mathrm{i}\omega q$$

Using (14) this gives

$$\alpha b u = \mathrm{i}\omega q. \tag{15}$$

From the Maxwell equation

$$\frac{\mathrm{d}D_z}{\mathrm{d}z} \approx \frac{\rho}{\epsilon_0},$$

where $D_z = \epsilon E_z$ we get at the solid–air interface $z = d_1$:

$$E_z(d_1 + 0^+) - \epsilon_1 E_z(d_1 - \delta) = q/\epsilon_0, \tag{16}$$

and at the air–solid interface $z = d_1 + u$:

$$\epsilon_2 E_z(d_1 + u + \delta) - E_z(d_1 + u - 0^+) = -q/\epsilon_0. \qquad (17)$$

Using (3)–(5) we can write (16) and (17) as

$$-b + \epsilon_1 b_1 = q/\epsilon_0 \qquad (18)$$

$$-\epsilon_2 b_2 + b = -q/\epsilon_0 \qquad (19)$$

giving

$$b_1 = \frac{1}{\epsilon_1}\left(b + \frac{q}{\epsilon_0}\right) \qquad (20)$$

$$b_2 = \frac{1}{\epsilon_2}\left(b + \frac{q}{\epsilon_0}\right) \qquad (21)$$

Substituting (20) and (21) in (8) gives

$$-bu = V + h_0\left(b + \frac{q}{\epsilon_0}\right), \qquad (22)$$

where

$$h_0 = \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}$$

From (22) we get

$$b = -\frac{V + h_0 q/\epsilon_0}{u + h_0}$$

Using (15) we get

$$b = -\frac{V + \eta b u}{u + h_0}, \qquad (23)$$

where

$$\eta = \frac{\alpha h_0}{\mathrm{i}\omega\epsilon_0}$$

From (23) we get

$$b = -\frac{V}{u + h_0 + \eta u} \qquad (24)$$

The electric field in the air gap is $E_z = -b$. Using (24) this gives

$$E_z = \frac{V}{u + h_0 + \eta u} \qquad (25)$$

In the limit of infinite high resistivity for both solids we have $\kappa = 0$ and hence $\alpha = \eta = 0$. In this case there is no electric leak-current and (25) reduces to the standard result derived in reference [14]:

$$E_z \approx \frac{V}{u + h_0}.$$

In the limit of very small resistivity, or small frequency, $\epsilon_1 \to -\mathrm{i}\kappa_1/(\omega\epsilon_0) \to \infty$ and similar for ϵ_2. Hence $h_0 \to 0$ and

$$\eta = \frac{\alpha h_0}{\mathrm{i}\omega\epsilon_0} \to \frac{\alpha}{\mathrm{i}\omega\epsilon_0}\left(\frac{d_1}{(-\mathrm{i}\kappa_1/(\omega\epsilon_0))} + \frac{d_2}{(-\mathrm{i}\kappa_2/(\omega\epsilon_0))}\right)$$
$$= \alpha\left(\frac{d_1}{\kappa_1} + \frac{d_2}{\kappa_2}\right)$$

giving

$$E_z \approx \frac{V}{u\left[1 + \alpha\left(\frac{d_1}{\kappa_1} + \frac{d_2}{\kappa_2}\right)\right]}, \qquad (26)$$

which was derived already in reference [14] as another limiting case.

If the contact pressure is not too high [18]

$$p = \beta E^* \mathrm{e}^{-\bar{u}/u_0}.$$

In this case $K_\perp = p/u_0$ and

$$\alpha = \frac{2\kappa}{u_0}\frac{p}{E^*}.$$

If we assume $d_1/\kappa_1 \gg d_2/\kappa_2$ and $\kappa_2 \gg \kappa_1$ we get

$$\alpha\left(\frac{d_1}{\kappa_1} + \frac{d_2}{\kappa_2}\right) \approx \alpha\frac{d_1}{\kappa_1} = \frac{2\kappa_1}{u_0}\frac{p}{E^*}\frac{d_1}{\kappa_1} = \frac{2d_1}{u_0}\frac{p}{E^*}$$

Note that if one neglect the leak-current, then for very low frequencies from (25) $E_z = V/u$ which is just the electric field obtained if all the potential drop would occur over the air gap. In this case screening charges of opposite sign would be located at the two air–solid interfaces. When the leak-current is included one instead get at very low frequencies, assuming $d_1/\kappa_1 \gg d_2/\kappa_2$ and $\kappa_2 \gg \kappa_1$,

$$E_z \approx \frac{V}{u\left[1 + \frac{2d_1}{u_0}\frac{p}{E^*}\right]}.$$

Thus in this case the electroadhesion will be smaller than when the leak-current is neglected.

2.2. Mean-field theory of electroadhesion

From a knowledge of the electric field in the air gap one can calculate the electroadhesion force as described in references [14, 19–21], and which we briefly summarize here.

From the electric field E_z we obtain the electroadhesion stress from the zz-component of the electrostatic Maxwell stress tensor:

$$\sigma_{zz} = \frac{1}{2}\epsilon_0 E_z^2.$$

Note that σ_{zz} depends on the lateral coordinate $\mathbf{x} = (x, y)$ since the interfacial separation $u = u(\mathbf{x})$ depends on the coordinate $\mathbf{x}$.

Assume that the two solids are squeezed together with the pressure p_0. In the simplest (mean-field) approach one include the electrostatic attraction as a contribution to the external load. Thus we write the effective loading pressure as

$$p = p_0 + p_\mathrm{a}, \qquad (27)$$

where $p_\mathrm{a} = \langle\sigma_{zz}\rangle$ is the electroadhesion stress averaged over the surface, or, equivalently ensemble averaged. Intuitively, one expect this approach to be accurate when the interaction force between the surfaces is long-range, and a similar approach has been used for the attraction resulting from capillary bridges [23, 24] and also in an earlier study of electroadhesion [14, 19, 20]. If $P(p, u)$ denote the probability distribution of interfacial separations from (27) we get

$$p = p_0 + V_0^2\int_0^\infty \mathrm{d}u\ P(p, u)G(p, u), \qquad (28)$$

where $G(p, u) = \sigma_{zz}/V_0^2$.

We can also write (17) as:

$$V_0^2 = \frac{p - p_0}{\int_0^\infty \mathrm{d}u\ P(p, u) G(p, u)} \tag{29}$$

from which we can easily calculate V_0 as a function of the nominal contact pressure p. Thus given V_0 and the applied (external) contact pressure p_0 the theory predict the electroadhesion pressure p. Note that when $V_0 = 0$ then $p = p_0$ is equal to the external applied pressure p_0.

To complete the theory we need the probability distribution $P(p, u)$. For randomly rough surfaces we calculate $P(p, u)$ using the theory presented in references [25–27].

In the present study we have neglected the contribution to the adhesion from the van der Waals interaction [28–31], and from capillary bridges. Both effects are easily included in the treatment presented above. Thus the van der Waals interaction can be included by adding to $p_0 + p_\mathrm{a}$ in (27) the term [24]

$$\int_0^\infty \mathrm{d}u\ P(p, u) \frac{H}{12\pi(u + a)^2},$$

where the Hamaker constant H depends on the polarizabilities (or dielectric functions) of the solids, and where a an atomic distance (reference plane for the van der Waals interaction [32]). In a similar way it is easy to include capillary bridges, as described in reference [23].

3. Numerical results and discussion

In a typical smart phone application a human finger is in contact with a thin ($d_2 \approx 1$ μm) insulating glass film below which occur a conducting layer. For this configuration the model above predict negligible influence on the electroadhesion force from charge leakage between the finger and the touch screen. However, if the insulating film is made very thick the theory predict some effect from the leak-current. To illustrate this in figure 2 we show the area of real contact as a function of the applied voltage without the electric leak current (green) and with the leak current (red). In the calculations the two solids have weakly conducting layers of thickness $d_1 = 1$ mm (lower solid) and $d_2 = 100$ μm (upper solid) on top of perfectly conducting semi-infinite solids. The bottom solid is rigid while the upper solid surface layer has the modulus $E = 1$ MPa (as typical for the SC of wet skin) and the solid above it $E = 20$ kPa. We use $\epsilon_1^0 = 5$, $\epsilon_2^0 = 10^4$, $\kappa_1 = 10^{-11}$ $(\Omega\mathrm{m})^{-1}$ and $\kappa_2 = 10^{-5}$ $(\Omega\mathrm{m})^{-1}$. The frequency $f = 10^{-2}$ Hz.

Note that including the leak current reduces the area of real contact and hence the electroadhesion force. This is expected because the leak current removes charges located on the two surfaces at the interface resulting in effective larger charge separation and a weaker electric field in the air gap. (The applied voltage V is a line integral of the electric field, so if the charge separation increases, the electric field must decrease in order for V not to change).

There is another limiting case we have not studied above, and which may have a much more important influence on the electroadhesion. Thus, if there is a conducting surface state

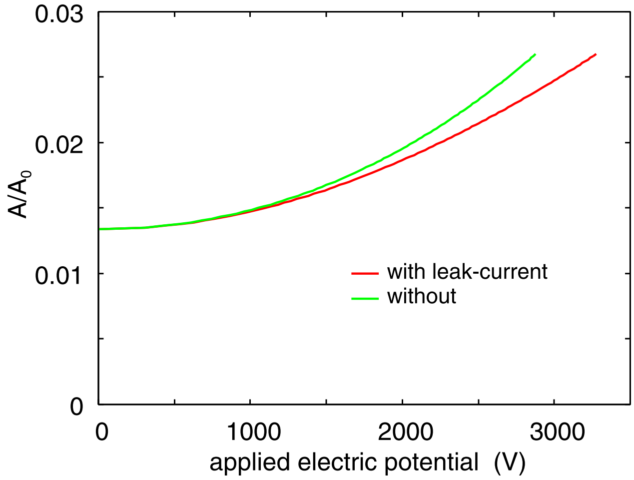

Figure 2. The area of real contact as a function of the applied voltage without the electric leak current (green) and with the leak current (red). In the calculations the two solids have weakly conducting layers of thickness $d_1 = 1$ mm (lower solid) and $d_2 = 100$ μm (upper solid) on top of perfectly conducting semi-infinite solids. The bottom solid is rigid while the upper solid surface layer has the modulus $E = 1$ MPa (as typical for the SC of wet skin) and the solid above it $E = 20$ kPa. We use $\epsilon_1^0 = 5$, $\epsilon_2^0 = 10^4$, $\kappa_1 = 10^{-11}$ $(\Omega\mathrm{m})^{-1}$ and $\kappa_2 = 10^{-5}$ $(\Omega\mathrm{m})^{-1}$. The frequency $f = 10^{-2}$ Hz.

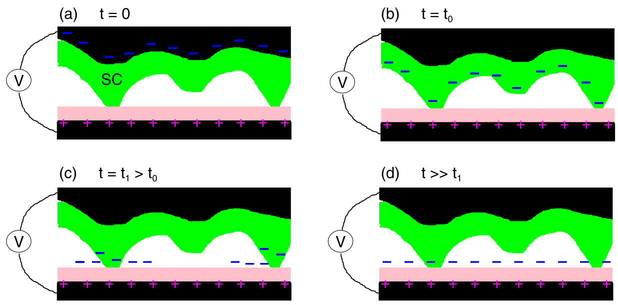

Figure 3. Some (insulating or semiconducting) solids have surfaces with conducting surface states. An example is the 7×7 reconstructed Si(111) surface (see references [25, 26]). In this case charges from the skin could migrate from the area of real contact out on the substrate surface. This would result in a vanishing electroadhesive force after long enough time.

(non-vanishing surface conductivity) (as is the case e.g. for the 7×7 reconstructed Si(111) surface [33, 34]) on the substrate (solid 1) and if the bulk conductivity of solid 1 can be neglected, one would in the DC limit (i.e. $\omega \to 0$) expect the whole potential drop to occur over the insulating film of solid 1 (see figure 3). In this case the electroadhesion force would vanish completely.

Finally we note that if a fluid with ions (e.g. sweat) occur in some regions between the finger and the insulating cover of the substrate, the charges can move in such a way that the electric field again occur just over the substrate insulating film and in this case too one would expect vanishing electroadhesion in the DC limit (see figure 4). If the fluid occupies only a fraction of the finger-substrate contact region, and if the interface is hydrophilic, there will be a contribution to the adhesion

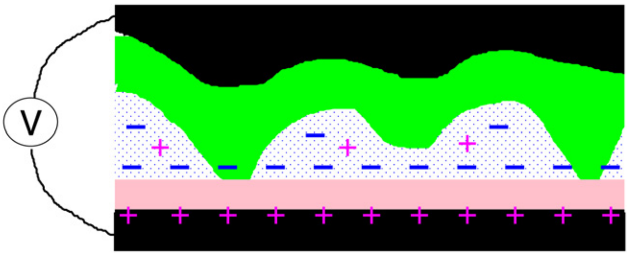

Figure 4. A fluid with ions between a finger and a touch screen. In response to a DC applied electric potential charges moves through the stratum corneum and are screened by ions in the fluid in such a way that the whole potential drop occur over the insulating film on the touch screen.

force from capillary bridges which depends on the fluid-solid contact angles θ_1 and θ_2. In an applied electric field the contact angles are modified resulting in a change in the adhesive fore. This electrowetting force was recently estimated and found to be important [8].

The importance of sweat for electroadhesion was indicated in the study by Shultz *et al* in reference [9]. They measured the current–voltage relation for the finger-touch screen contact and presented results for the electric impedance of the contact. They found that in going from sliding to stopped, the gap impedance is essential shorted out, and all that remains is the series combination of the skin and coating impedance. They proposed that this is due to the build up of sweat in the air gap, which is highly conductive and has a much higher dielectric constant than air, each of which dramatically lower the gap impedance.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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