Department of Computer Science

UCL

COMP0197: Applied Deep Learning

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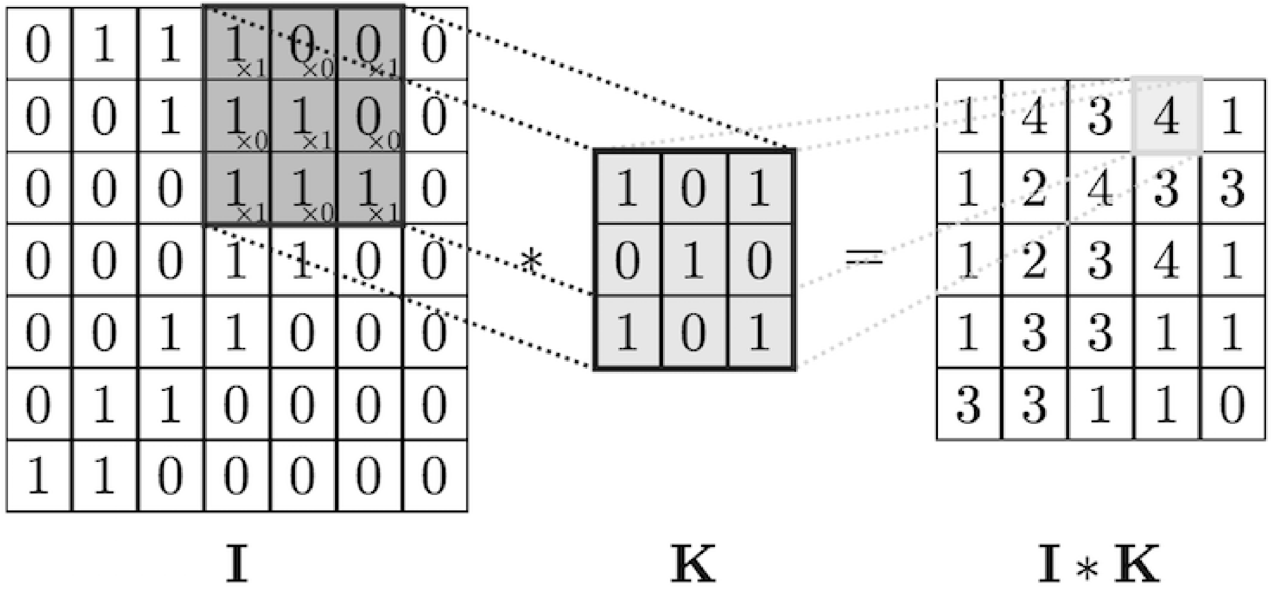
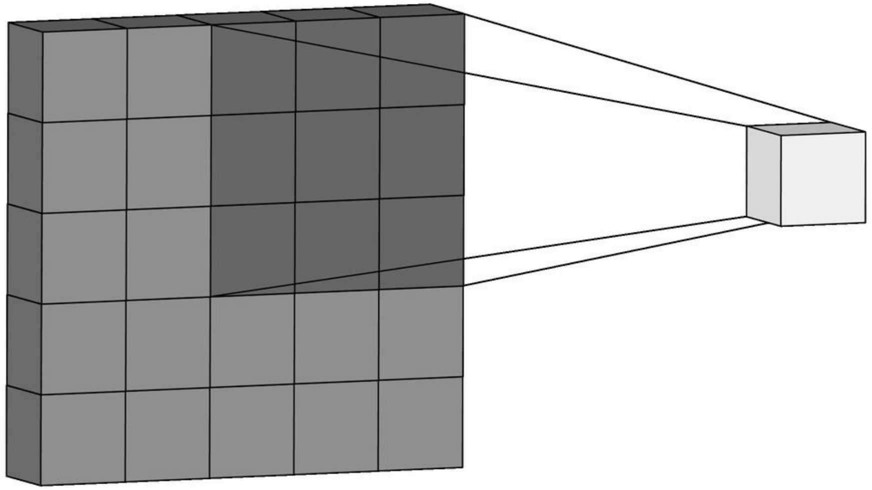
Convolutional Neural Networks

Discrete convolution

$$s(t) = (x * w)(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a)$$





$$\begin{bmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_8 \\ x_3 & x_6 & x_9 \end{bmatrix} * \begin{bmatrix} k_1 & k_3 \\ k_2 & k_4 \end{bmatrix} = \begin{bmatrix} \sum\sum\begin{pmatrix} x_1k_1 & x_4k_3 \\ x_2k_2 & x_5k_4 \end{pmatrix} & \sum\sum\begin{pmatrix} x_4k_1 & x_7k_3 \\ x_5k_2 & x_8k_4 \end{pmatrix} \\ \sum\sum\begin{pmatrix} x_2k_1 & x_5k_3 \\ x_3k_2 & x_6k_4 \end{pmatrix} & \sum\sum\begin{pmatrix} x_5k_1 & x_8k_3 \\ x_6k_2 & x_9k_4 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} x_1k_1 + x_2k_2 + x_4k_3 + x_5k_4 & x_4k_1 + x_5k_2 + x_7k_3 + x_8k_4 \\ x_2k_1 + x_3k_2 + x_5k_3 + x_6k_4 & x_5k_1 + x_6k_2 + x_8k_3 + x_9k_4 \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} x_1k_1 + x_2k_2 + x_4k_3 + x_5k_4 \\ x_2k_1 + x_3k_2 + x_5k_3 + x_6k_4 \\ x_4k_1 + x_5k_2 + x_7k_3 + x_8k_4 \\ x_5k_1 + x_6k_2 + x_8k_3 + x_9k_4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_8 \\ x_3 & x_6 & x_9 \end{bmatrix} * \begin{bmatrix} k_1 & k_3 \\ k_2 & k_4 \end{bmatrix} = \begin{bmatrix} \sum\sum\begin{pmatrix} x_1k_1 & x_4k_3 \\ x_2k_2 & x_5k_4 \end{pmatrix} & \sum\sum\begin{pmatrix} x_4k_1 & x_7k_3 \\ x_5k_2 & x_8k_4 \end{pmatrix} \\ \sum\sum\begin{pmatrix} x_2k_1 & x_5k_3 \\ x_3k_2 & x_6k_4 \end{pmatrix} & \sum\sum\begin{pmatrix} x_5k_1 & x_8k_3 \\ x_6k_2 & x_9k_4 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} x_1k_1 + x_2k_2 + x_4k_3 + x_5k_4 & x_4k_1 + x_5k_2 + x_7k_3 + x_8k_4 \\ x_2k_1 + x_3k_2 + x_5k_3 + x_6k_4 & x_5k_1 + x_6k_2 + x_8k_3 + x_9k_4 \end{bmatrix}$$

$$\begin{bmatrix} k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} x_1k_1 + x_2k_2 + x_4k_3 + x_5k_4 \\ x_2k_1 + x_3k_2 + x_5k_3 + x_6k_4 \\ x_4k_1 + x_5k_2 + x_7k_3 + x_8k_4 \\ x_5k_1 + x_6k_2 + x_8k_3 + x_9k_4 \end{bmatrix}$$

Convolution and cross-correlation

Intuition

Translation-invariance, Parameter sharing, Sparse weights, Infinitely strong prior

Sampling

Pooling, Strides, Interpolation, Un-pooling, Transpose convolution

Anatomy of a convolutional layer

Kernels

Dimensionality, receptive field, shape and size, kernel constraints, padding, depthwise separable convolution

Convolution arithmetic

Applied architectures

Convolutional Neural Networks | Convolution and Cross-Correlation

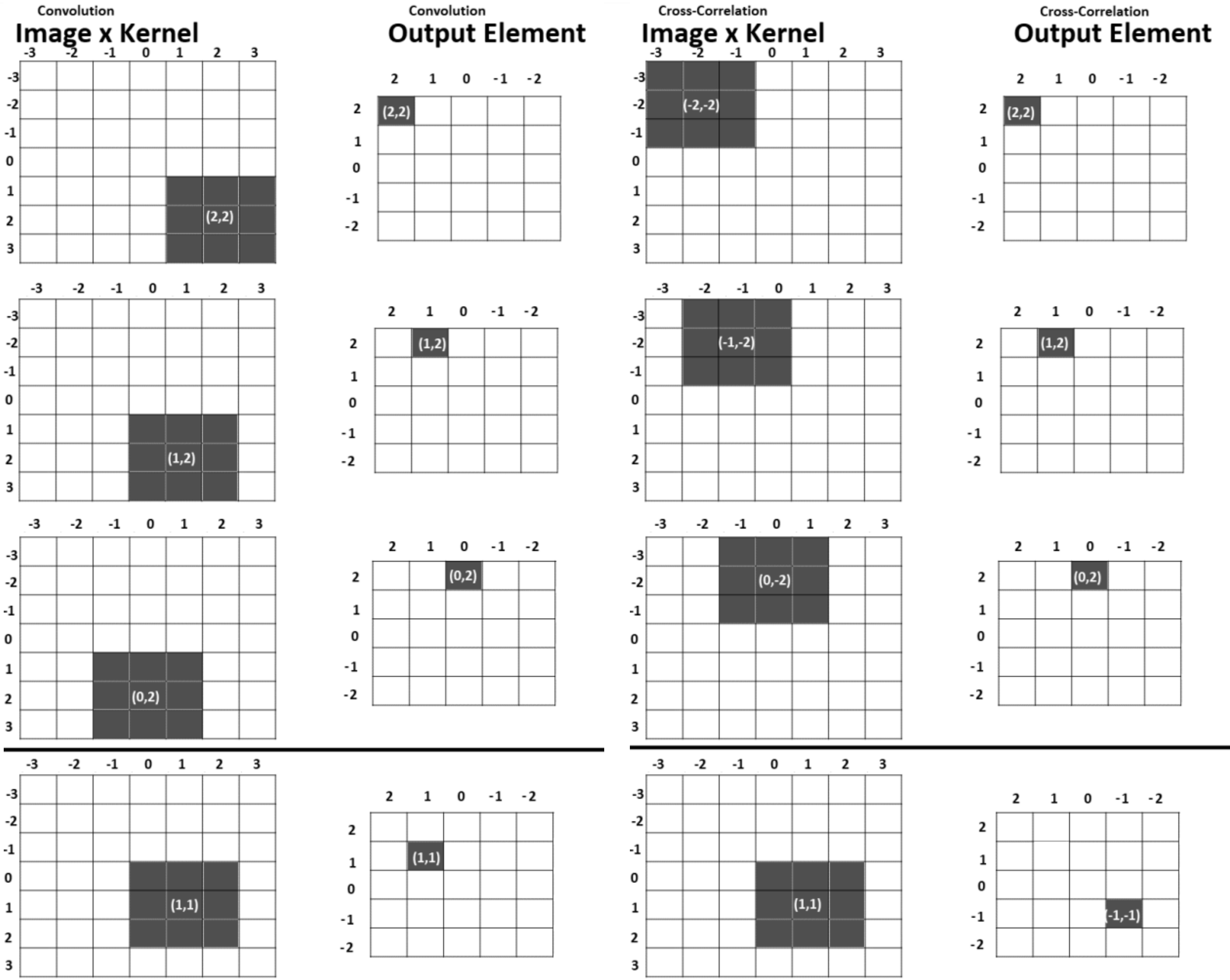
Convolution

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(m,n)K(i-m, j-n).$$

$$S(i,j) = (K * I)(i,j) = \sum_m \sum_n I(i-m, j-n)K(m,n).$$

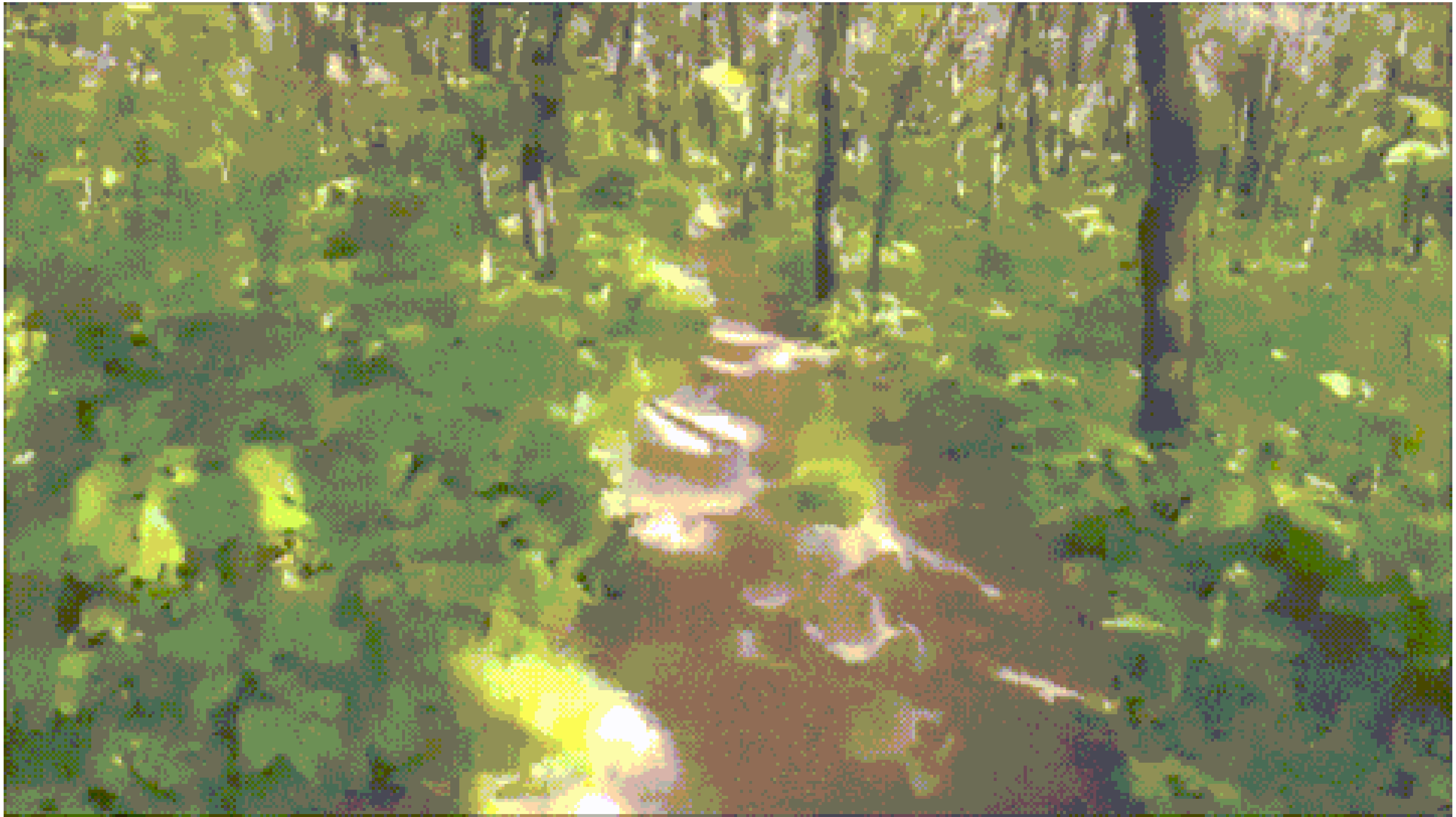
Cross-correlation

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(i+m, j+n)K(m,n).$$

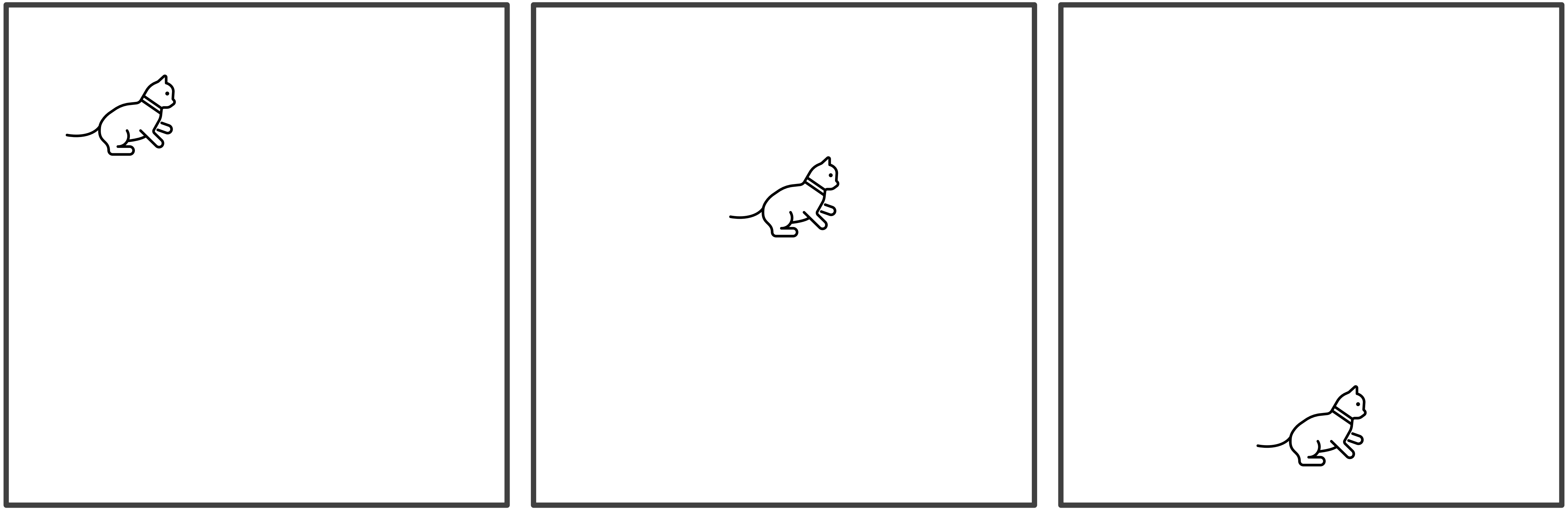


Convolutional Neural Networks | Intuition

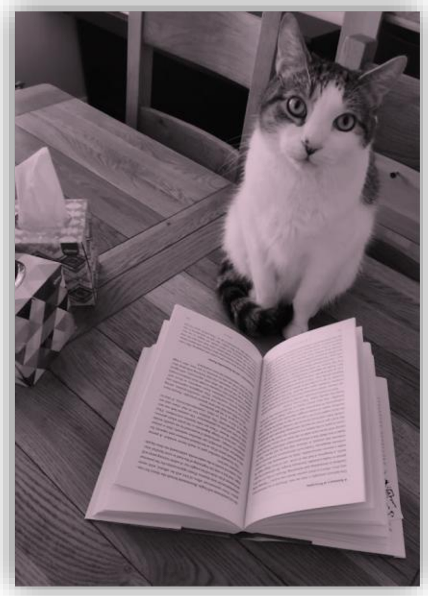
Translation invariance



Translation invariance



Translation invariance



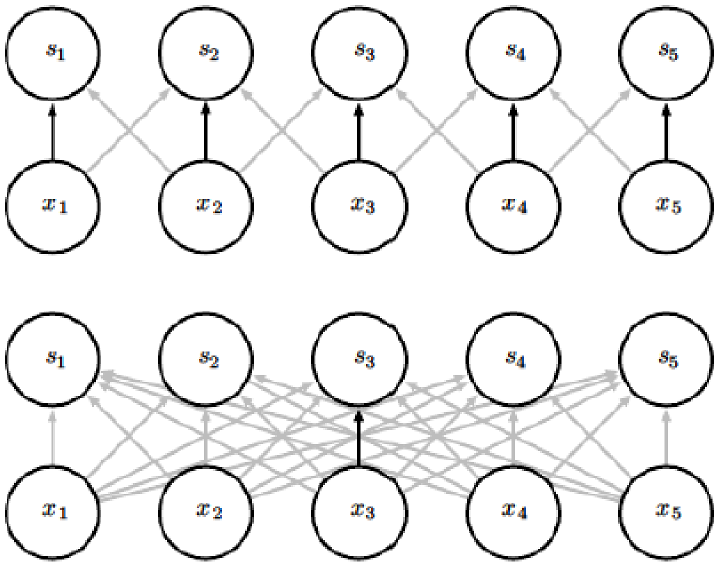
Parameter sharing

Efficiency

Regularisation effect*

$$\begin{bmatrix} w_1 & w_2 & 0 & w_3 & w_4 & 0 & 0 & 0 & 0 \\ 0 & w_1 & w_2 & 0 & w_3 & w_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_1 & w_2 & 0 & w_3 & w_4 & 0 \\ 0 & 0 & 0 & 0 & w_1 & w_2 & 0 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} x_1 w_1 + x_2 w_2 + x_4 w_3 + x_5 w_4 \\ x_2 w_1 + x_3 w_2 + x_5 w_3 + x_6 w_4 \\ x_4 w_1 + x_5 w_2 + x_7 w_3 + x_8 w_4 \\ x_5 w_1 + x_6 w_2 + x_8 w_3 + x_9 w_4 \end{bmatrix}$$

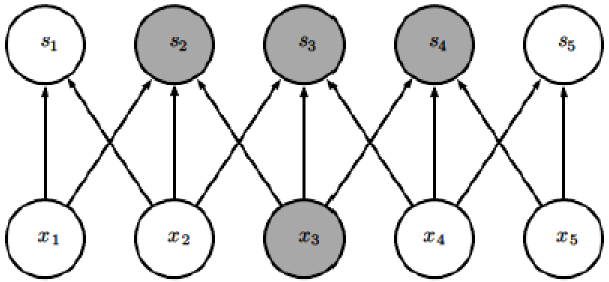
$$\begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,5} & w_{1,6} & w_{1,7} & w_{1,8} & w_{1,9} \\ w_{2,1} & w_{2,2} & w_{2,3} & w_{2,4} & w_{2,5} & w_{2,6} & w_{2,7} & w_{2,8} & w_{2,9} \\ w_{3,1} & w_{3,2} & w_{3,3} & w_{3,4} & w_{3,5} & w_{3,6} & w_{3,7} & w_{3,8} & w_{3,9} \\ w_{4,1} & w_{4,2} & w_{4,3} & w_{4,4} & w_{4,5} & w_{4,6} & w_{4,7} & w_{4,8} & w_{4,9} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}$$



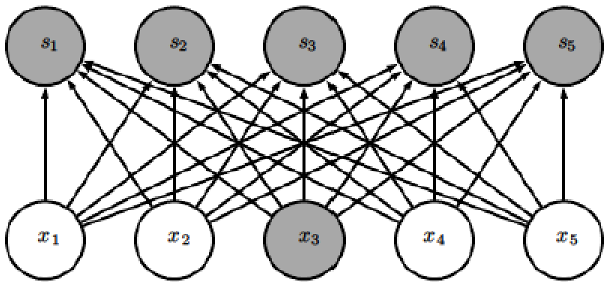
Sparse weights

Memory
Computation
Statistical efficiency

$$\begin{bmatrix} w_1 & w_2 & 0 & w_3 & w_4 & 0 & 0 & 0 & 0 \\ 0 & w_1 & w_2 & 0 & w_3 & w_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_1 & w_2 & 0 & w_3 & w_4 & 0 \\ 0 & 0 & 0 & 0 & w_1 & w_2 & 0 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} x_1 w_1 + x_2 w_2 + x_4 w_3 + x_5 w_4 \\ x_2 w_1 + x_3 w_2 + x_5 w_3 + x_6 w_4 \\ x_4 w_1 + x_5 w_2 + x_7 w_3 + x_8 w_4 \\ x_5 w_1 + x_6 w_2 + x_8 w_3 + x_9 w_4 \end{bmatrix}$$



$$\begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,5} & w_{1,6} & w_{1,7} & w_{1,8} & w_{1,9} \\ w_{2,1} & w_{2,2} & w_{2,3} & w_{2,4} & w_{2,5} & w_{2,6} & w_{2,7} & w_{2,8} & w_{2,9} \\ w_{3,1} & w_{3,2} & w_{3,3} & w_{3,4} & w_{3,5} & w_{3,6} & w_{3,7} & w_{3,8} & w_{3,9} \\ w_{4,1} & w_{4,2} & w_{4,3} & w_{4,4} & w_{4,5} & w_{4,6} & w_{4,7} & w_{4,8} & w_{4,9} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}$$

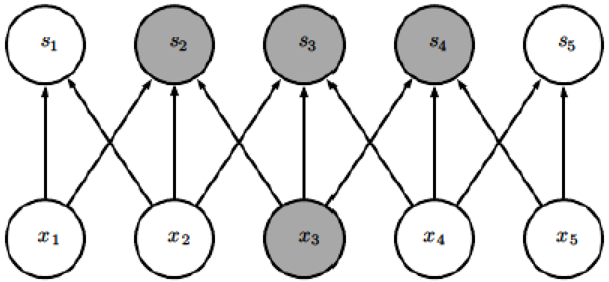


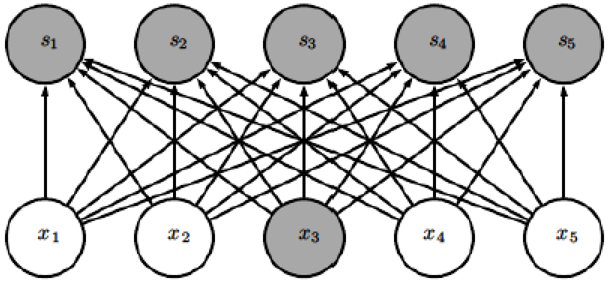
“Infinitely strong” prior

Underfitting

$$\begin{bmatrix} w_1 & w_2 & 0 & w_3 & w_4 & 0 & 0 & 0 & 0 \\ 0 & w_1 & w_2 & 0 & w_3 & w_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_1 & w_2 & 0 & w_3 & w_4 & 0 \\ 0 & 0 & 0 & 0 & w_1 & w_2 & 0 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} x_1 w_1 + x_2 w_2 + x_4 w_3 + x_5 w_4 \\ x_2 w_1 + x_3 w_2 + x_5 w_3 + x_6 w_4 \\ x_4 w_1 + x_5 w_2 + x_7 w_3 + x_8 w_4 \\ x_5 w_1 + x_6 w_2 + x_8 w_3 + x_9 w_4 \end{bmatrix}$$

$$\begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,5} & w_{1,6} & w_{1,7} & w_{1,8} & w_{1,9} \\ w_{2,1} & w_{2,2} & w_{2,3} & w_{2,4} & w_{2,5} & w_{2,6} & w_{2,7} & w_{2,8} & w_{2,9} \\ w_{3,1} & w_{3,2} & w_{3,3} & w_{3,4} & w_{3,5} & w_{3,6} & w_{3,7} & w_{3,8} & w_{3,9} \\ w_{4,1} & w_{4,2} & w_{4,3} & w_{4,4} & w_{4,5} & w_{4,6} & w_{4,7} & w_{4,8} & w_{4,9} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}$$





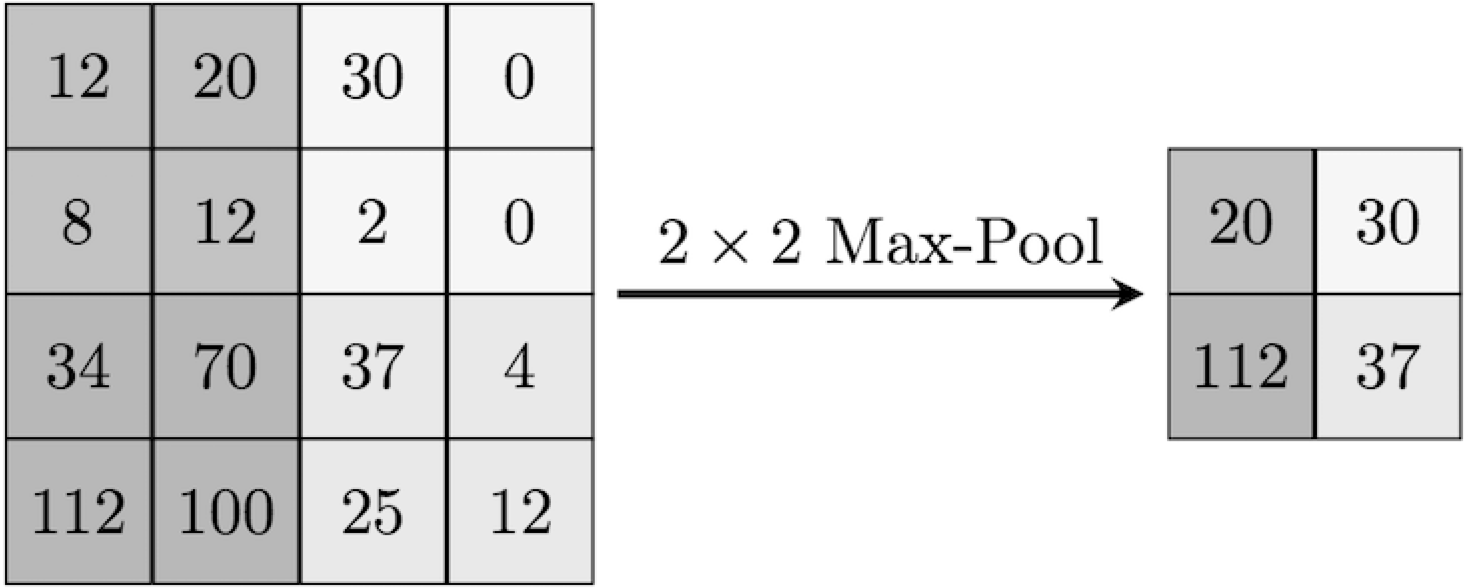
Convolutional Neural Networks | Sampling

Convolutional Neural Networks | Sampling

Pooling

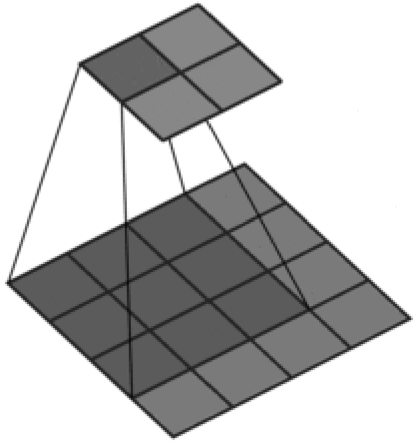
- Max pooling
- Average pooling

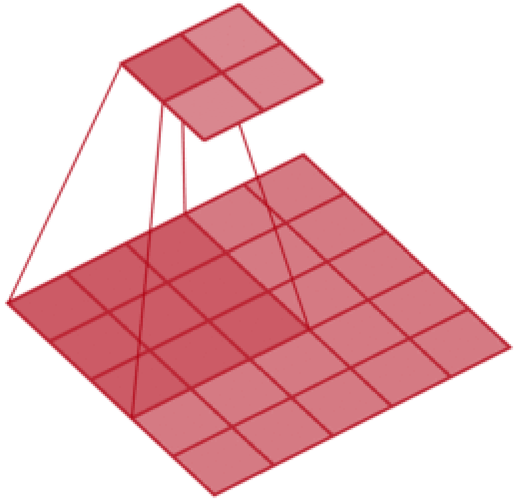
Invariance
No learnable parameters
As one of the de facto trio for down-sampling
    Conv weight -> activation -> pooling
    Down-sampling due to moving windows with strides



Strides for down-sampling

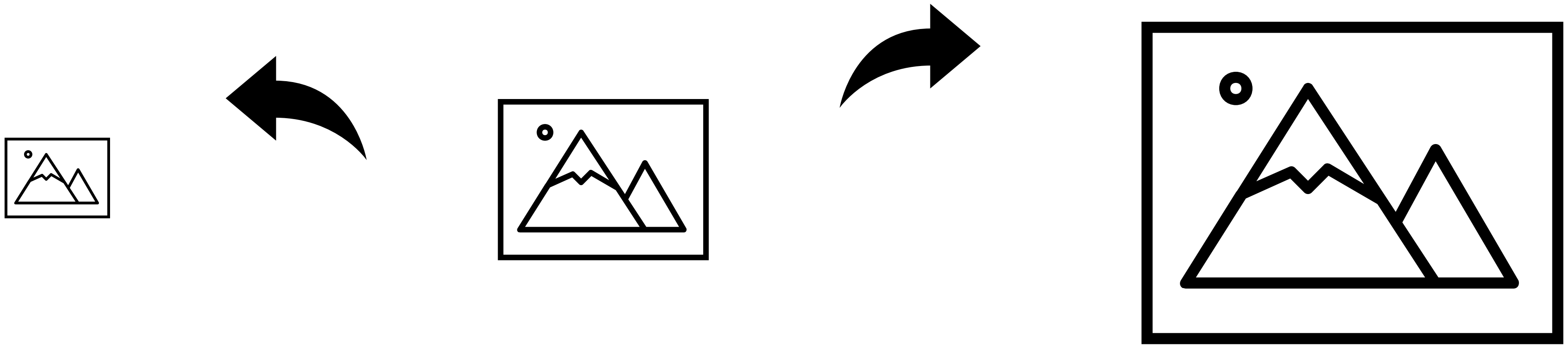
- Strides > 1
- Pooling
- Convolution with strides also down-samples (with learnable parameters)





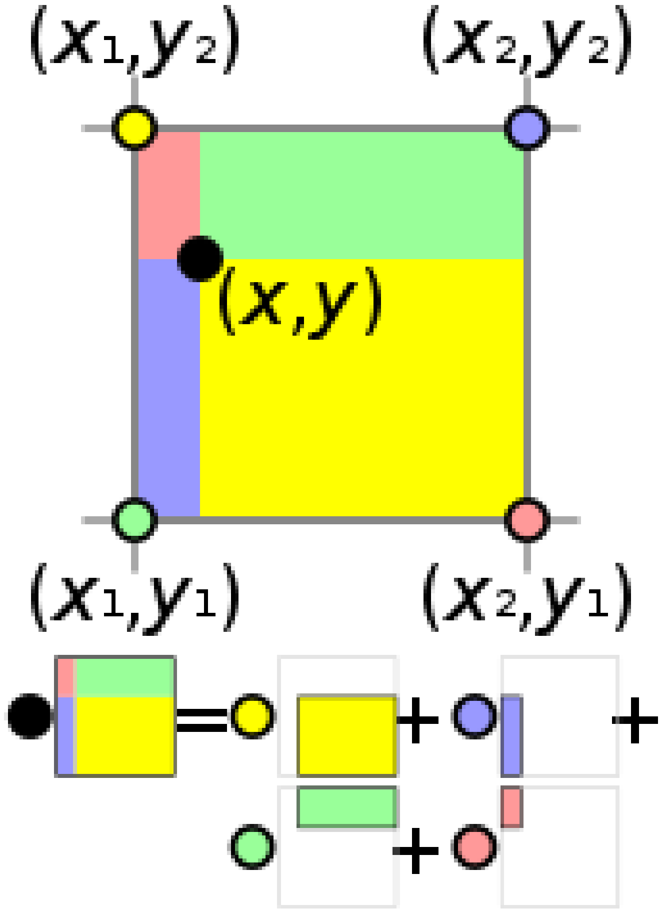
Interpolation for down-sampling and up-sampling

Bilinear, bicubic, spline.



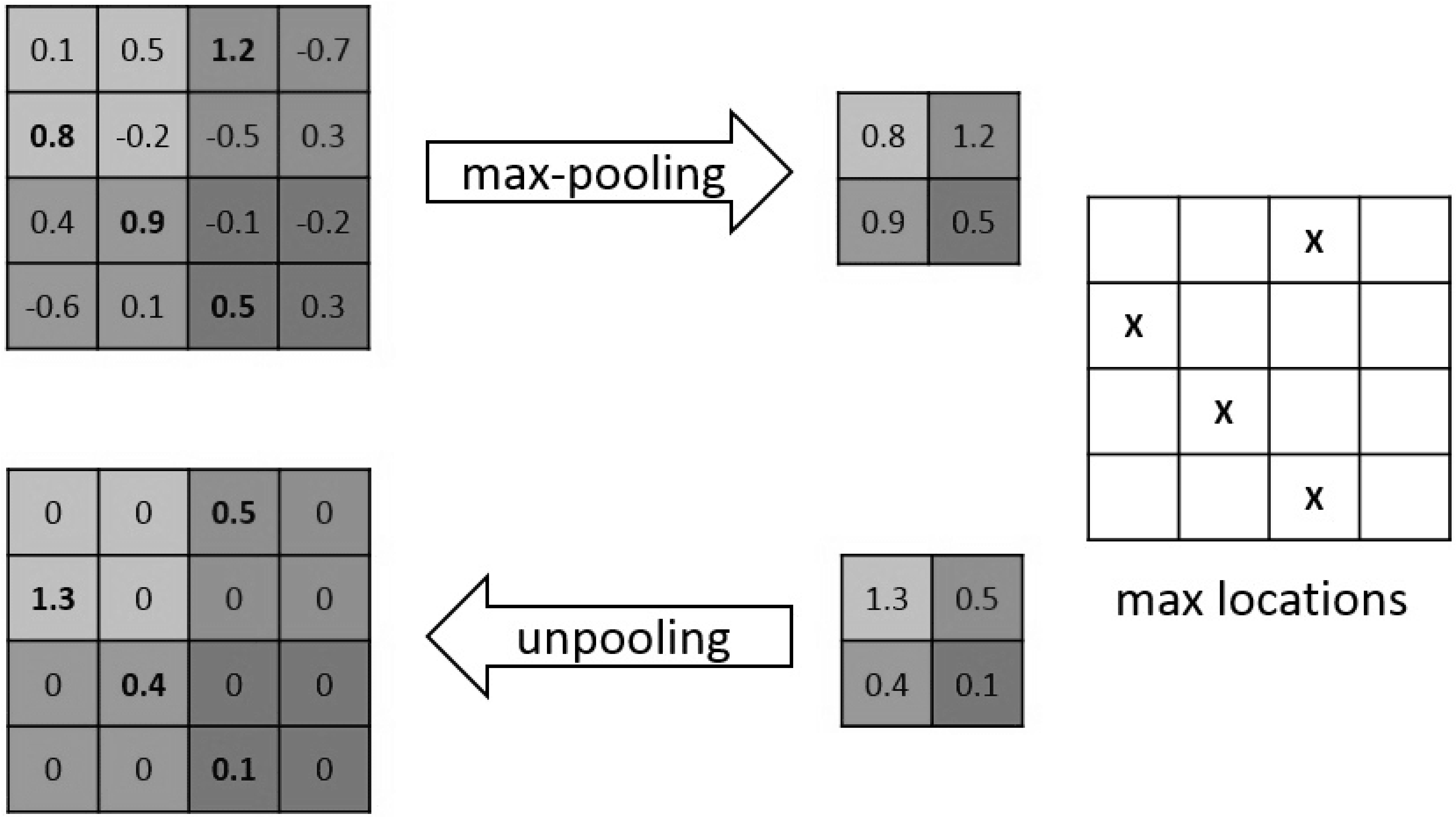
Interpolation for down-sampling and up-sampling

Is bilinear interpolation a linear operation?



"Un-pooling" for up-sampling

e.g. a fixed position for the max
and zeros everywhere else



Transpose convolution

- Fractionally strided convolution
- Deconvolution
- Up-sampling with parameters

$$\mathbf{y}^{(N\times 1)} = \mathbf{W}^{(N\times M)}\mathbf{x}^{(M\times 1)}$$

$$\mathbf{y} = \mathbf{W}\mathbf{x}$$

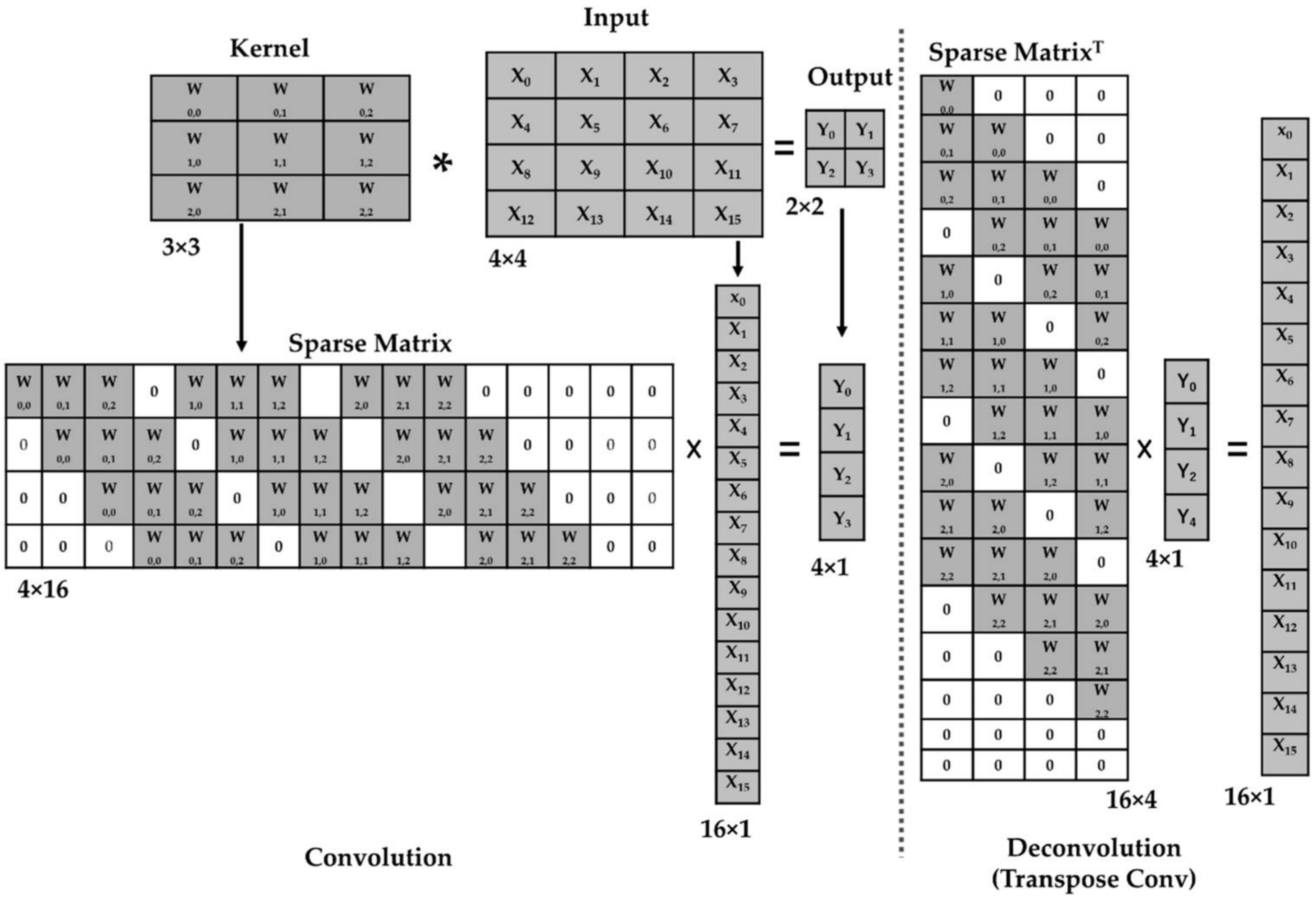
Diagonal-constant Toeplitz matrix

$$W_{i,j} = Const_{i,j}$$

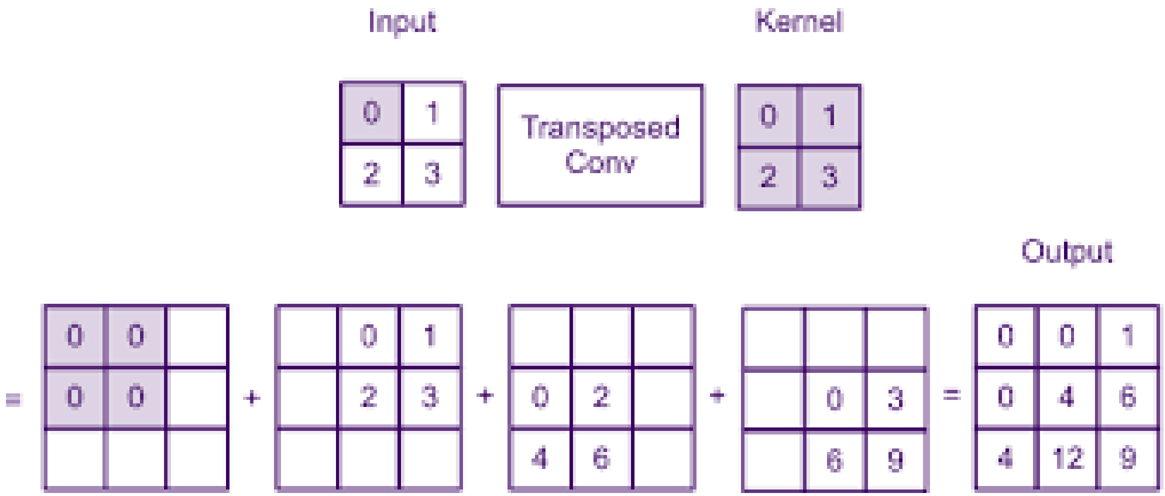
$$\widehat{W}_{j,i} = Const_{i,j}$$

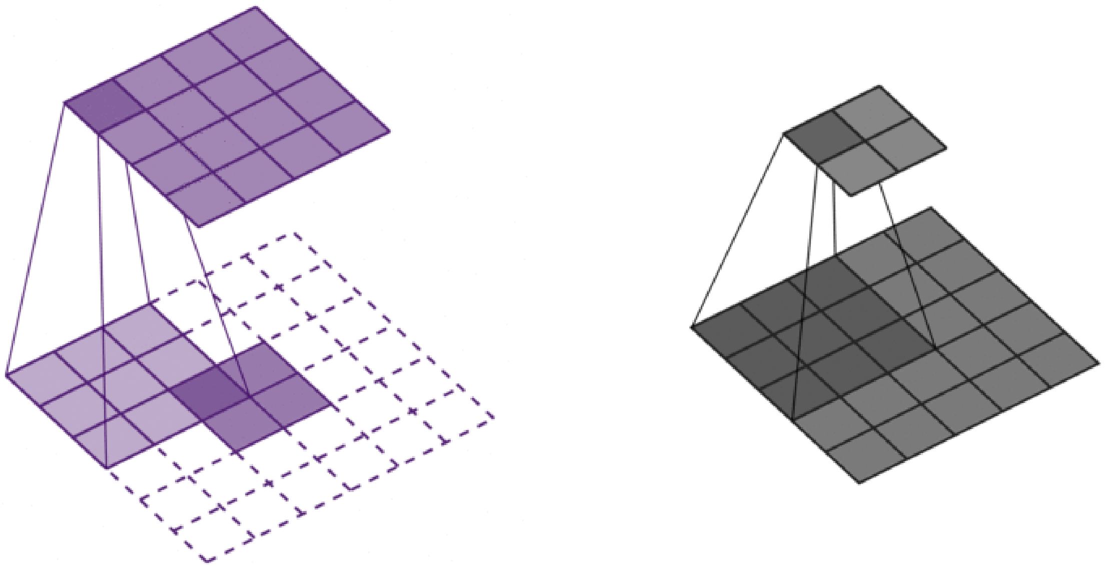
The reverse of convolution

$$\widehat{\mathbf{W}}^{\mathrm{T}}\mathbf{y} = \mathbf{x}$$



Transpose convolution





Transpose convolution vs. conv + interpolation, for up-sampling?

Convolutional Neural Networks | Anatomy of a Convolutional Layer

Tensors

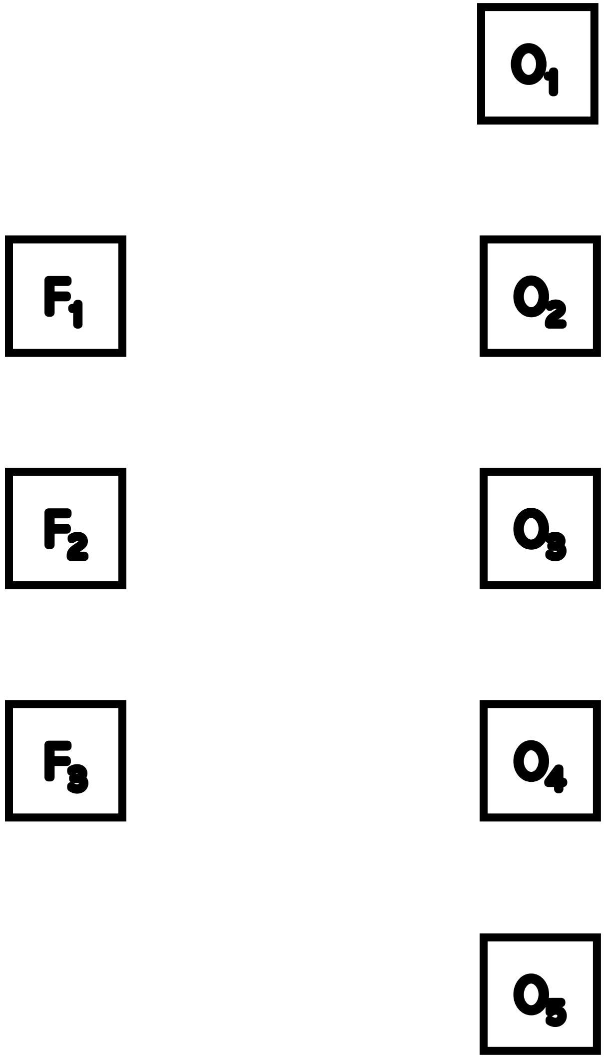
Input/output feature maps

Tensor for 2D feature maps
N **x** H **x** W **x** C (TensorFlow)
N **x** C **x** H **x** W (PyTorch)

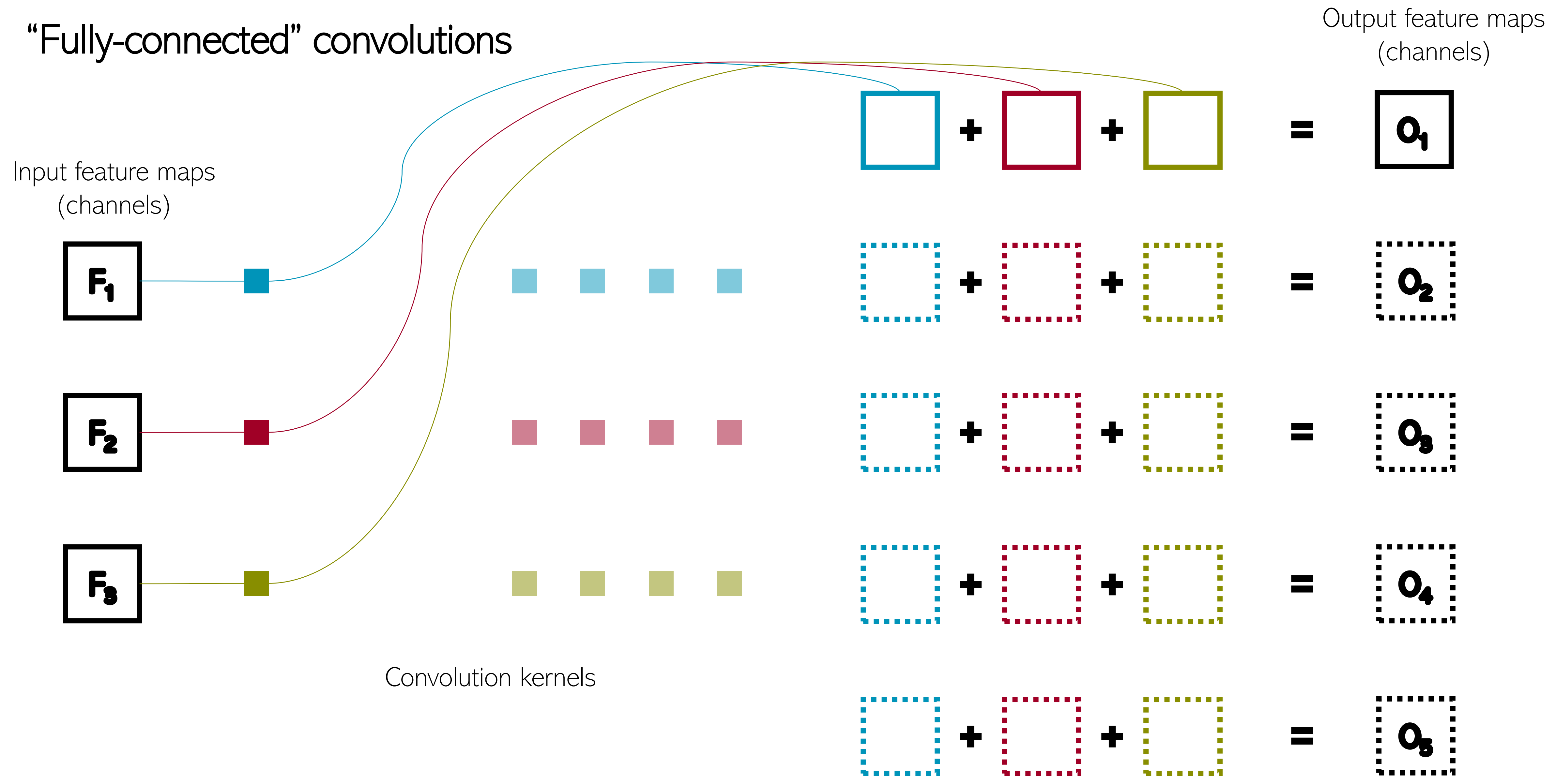
N: no. of data in a minibatch (batch size)
H: height of feature map
W: width of feature map

Tensor for 1D feature vector
N **x** L **x** C (TensorFlow)
N **x** C **x** L (PyTorch)
L: length of feature vector

Tensor for 3D feature tensor
N **x** D **x** H **x** W **x** C (TensorFlow)
N **x** C **x** D **x** H **x** W (PyTorch)
D: depth of feature tensor



"Fully-connected" convolutions



Convolutional Neural Networks | Kernels

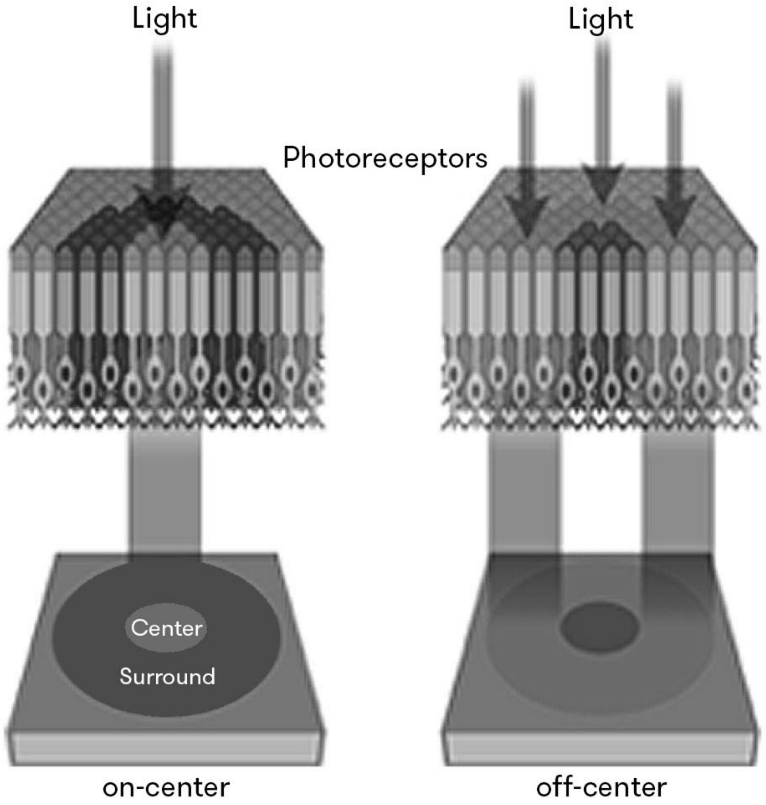
Dimensionality

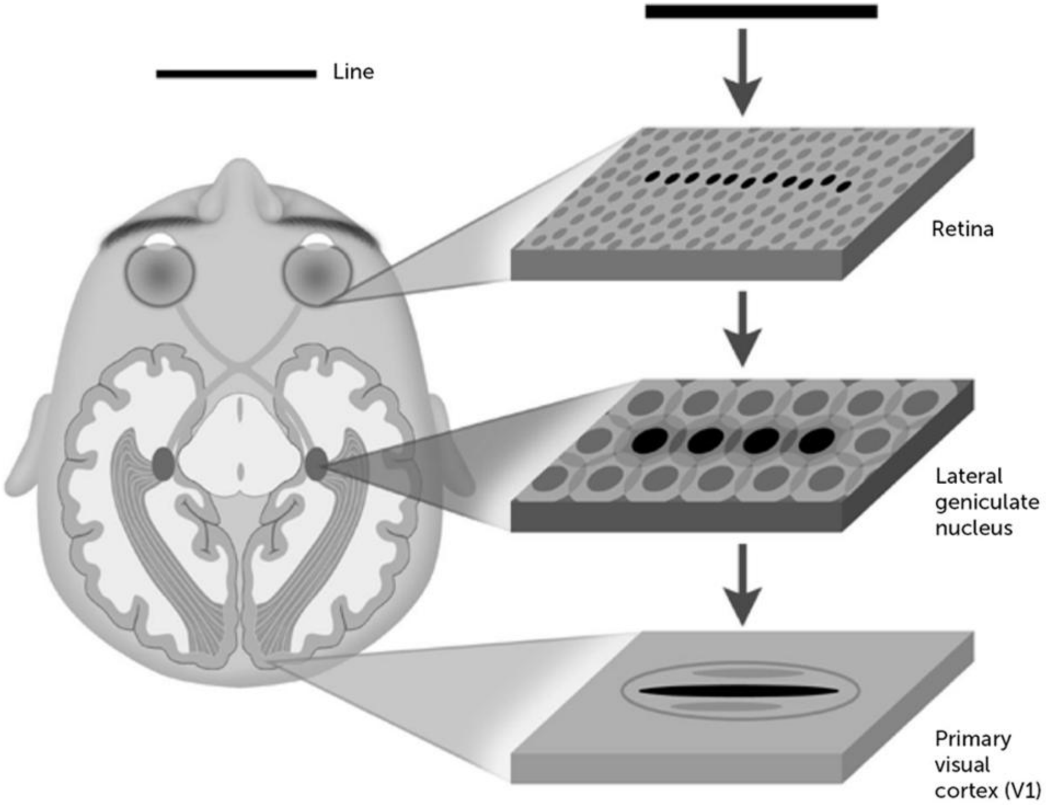
1D

2D

3D

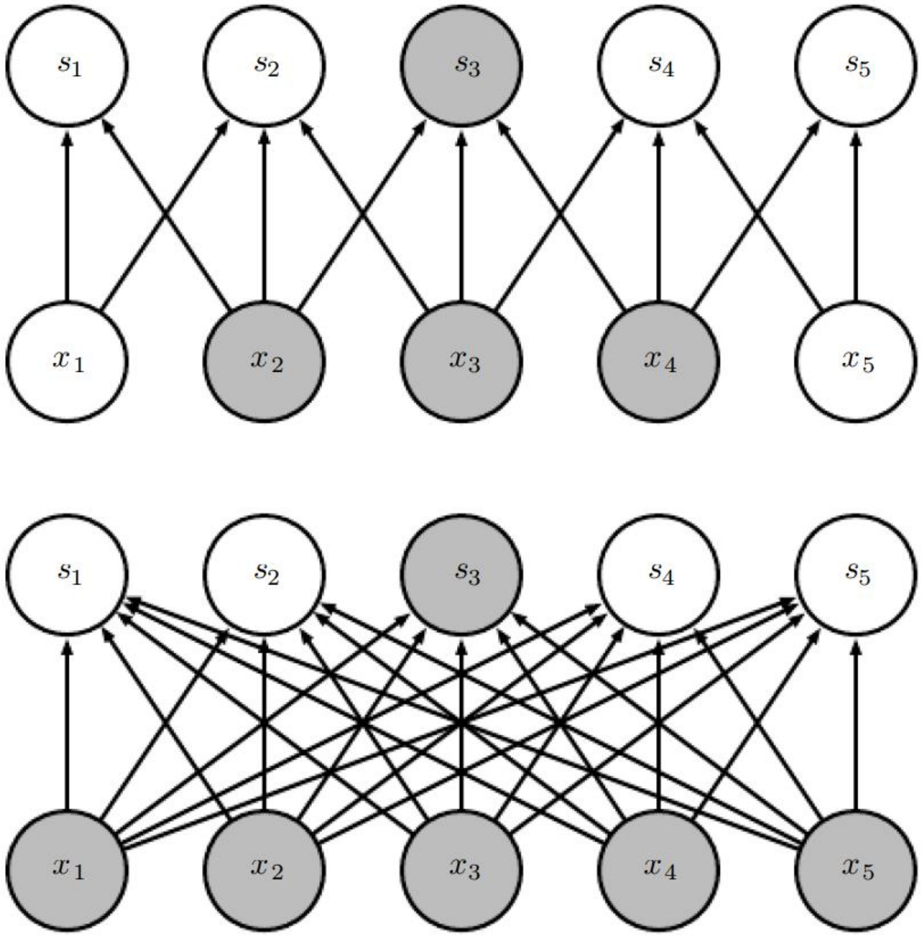
Receptive field
neuroscience / physics

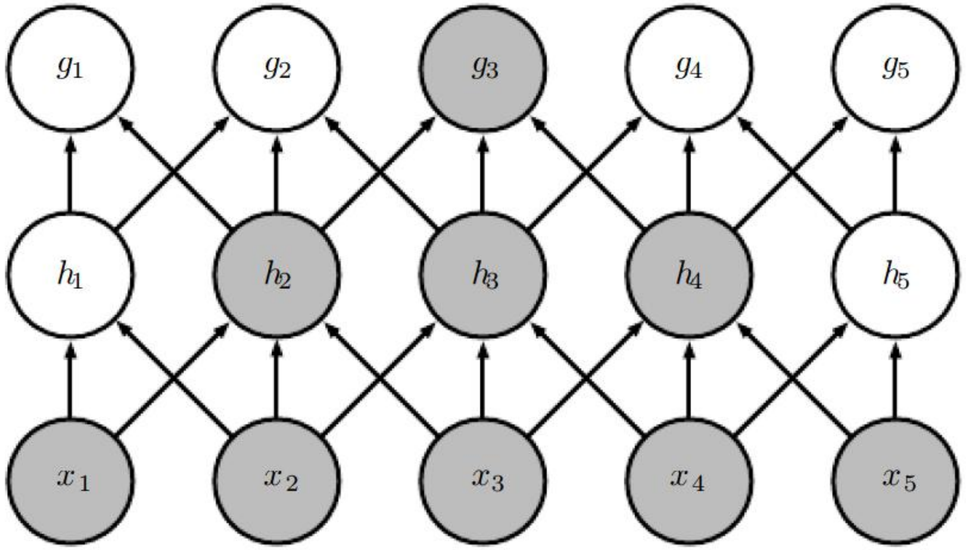




Receptive field

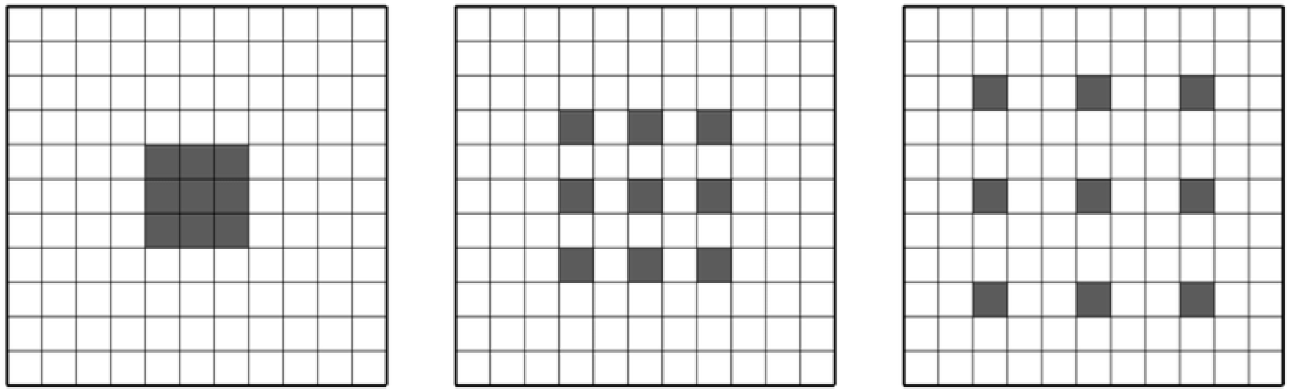
Fully-connected layers vs. convolutional layers vs. convolution with strides*

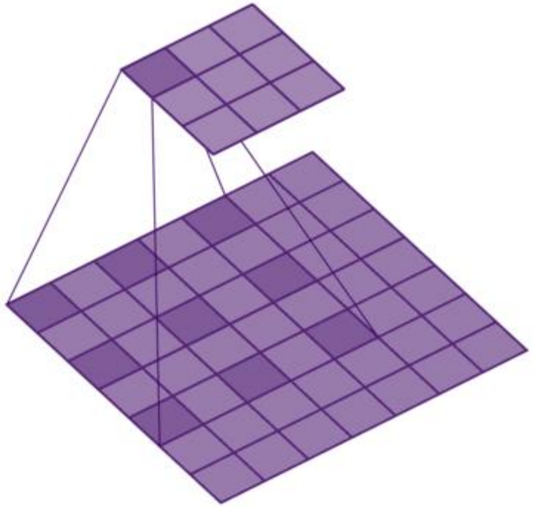




Receptive field

Dilated kernels (Atrous convolution)





Shape and size

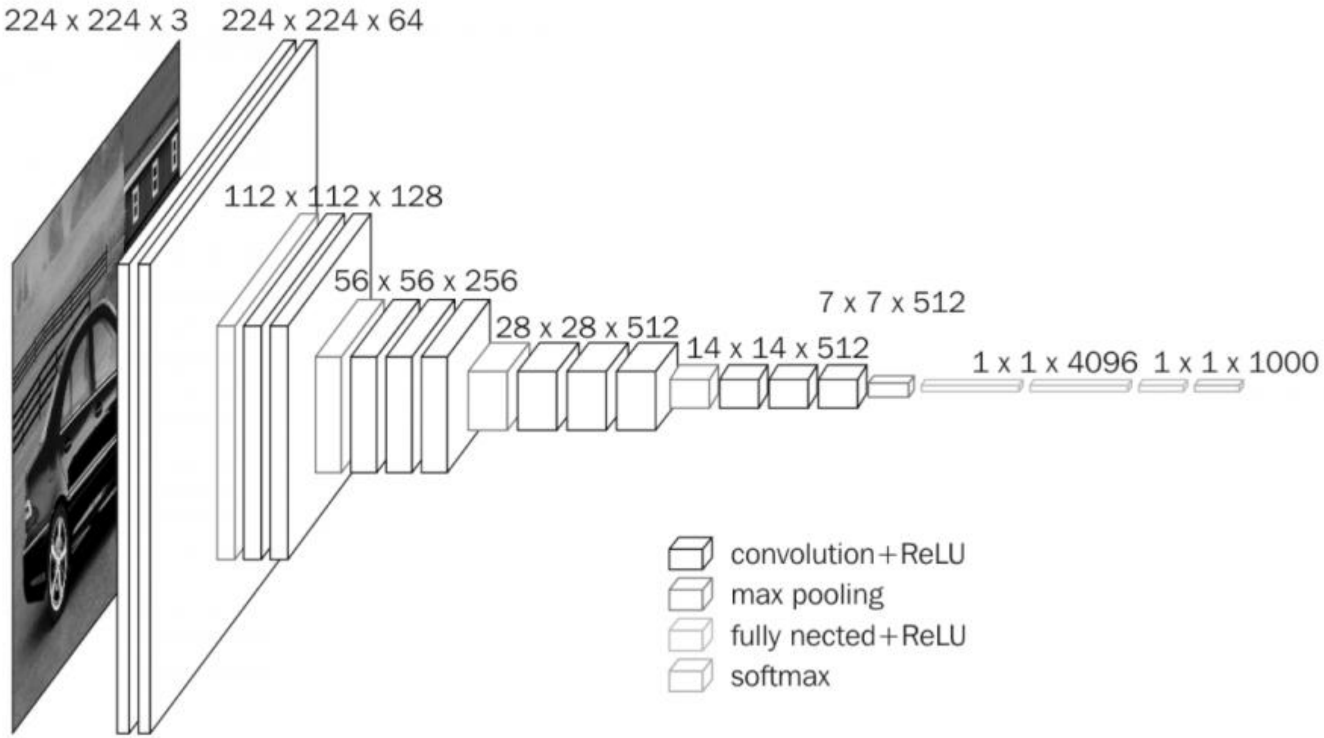
- Dilated
- Adaptive*
- Lower-dimensional 2.5D kernels*

- VGG: 3x3 kernels
- Bottleneck 1x1 kernels vs. fully-connected layers

Published as a conference paper at ICLR 2015

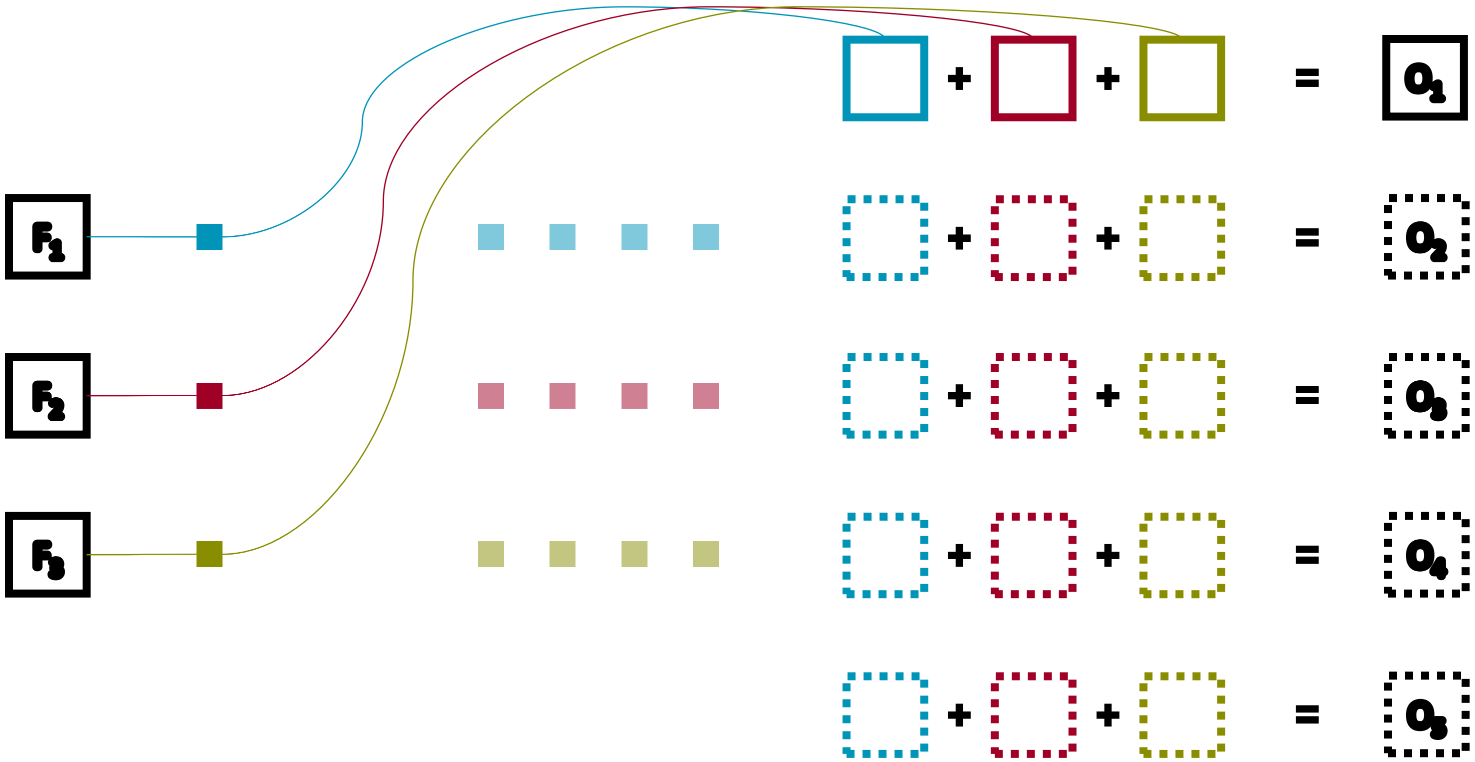
VERY DEEP CONVOLUTIONAL NETWORKS FOR LARGE-SCALE IMAGE RECOGNITION

Karen Simonyan* **& Andrew Zisserman**+
Visual Geometry Group, Department of Engineering Science, University of Oxford
{karen,az}@robots.ox.ac.uk



Shape and size

- 1x1 kernels
- "Channel-wise" linear projection (fully-connected layer)
- Dimensionality reduction for information "bottleneck" layer
- Upsampling



Kernel constraints

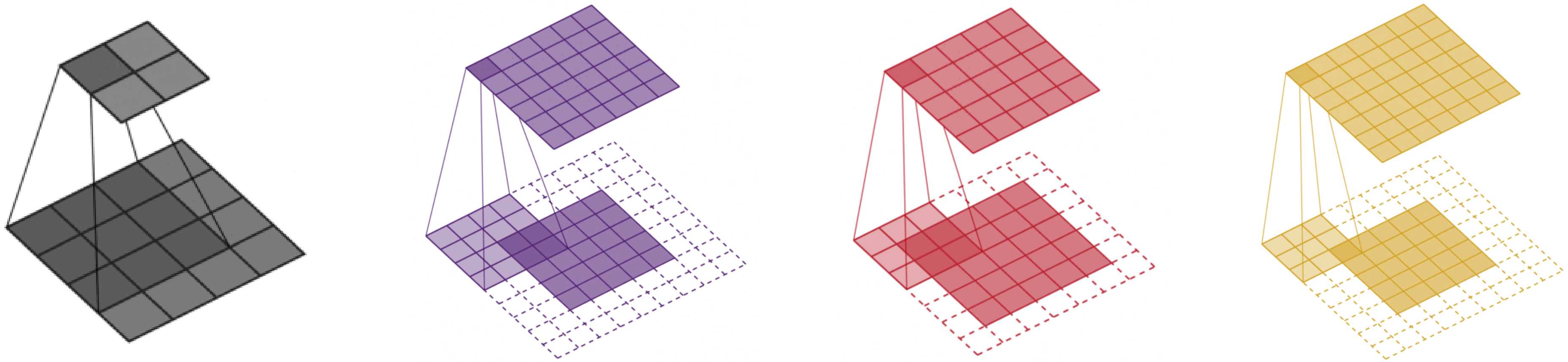
- Soft constraints: Regularise, e.g. L^2-norm*

- Hard constraint: MaxNorm*

Padding

No padding, arbitrary padding, half padding, full padding

- No strides
- With strides*
- Transpose*
- Dilated*



https://github.com/vdumoulin/conv_arithmetic

Depthwise separable kernels

- Separable convolution

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$\mathbf{K} = \mathbf{u} * \mathbf{v}$

$\mathbf{I} * \mathbf{K} = \mathbf{I} * \mathbf{u} * \mathbf{v}$ (associativity)

Computational complexity

$O(M \times N \times 3 \times 3) \rightarrow O\big(M \times N \times (3 + 3)\big)$

$O(M \times N \times m \times n) \rightarrow O(M \times N \times (m + n))$

Singular value decomposition

$$\mathbf{K} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3] \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]^{\mathrm{T}}$$

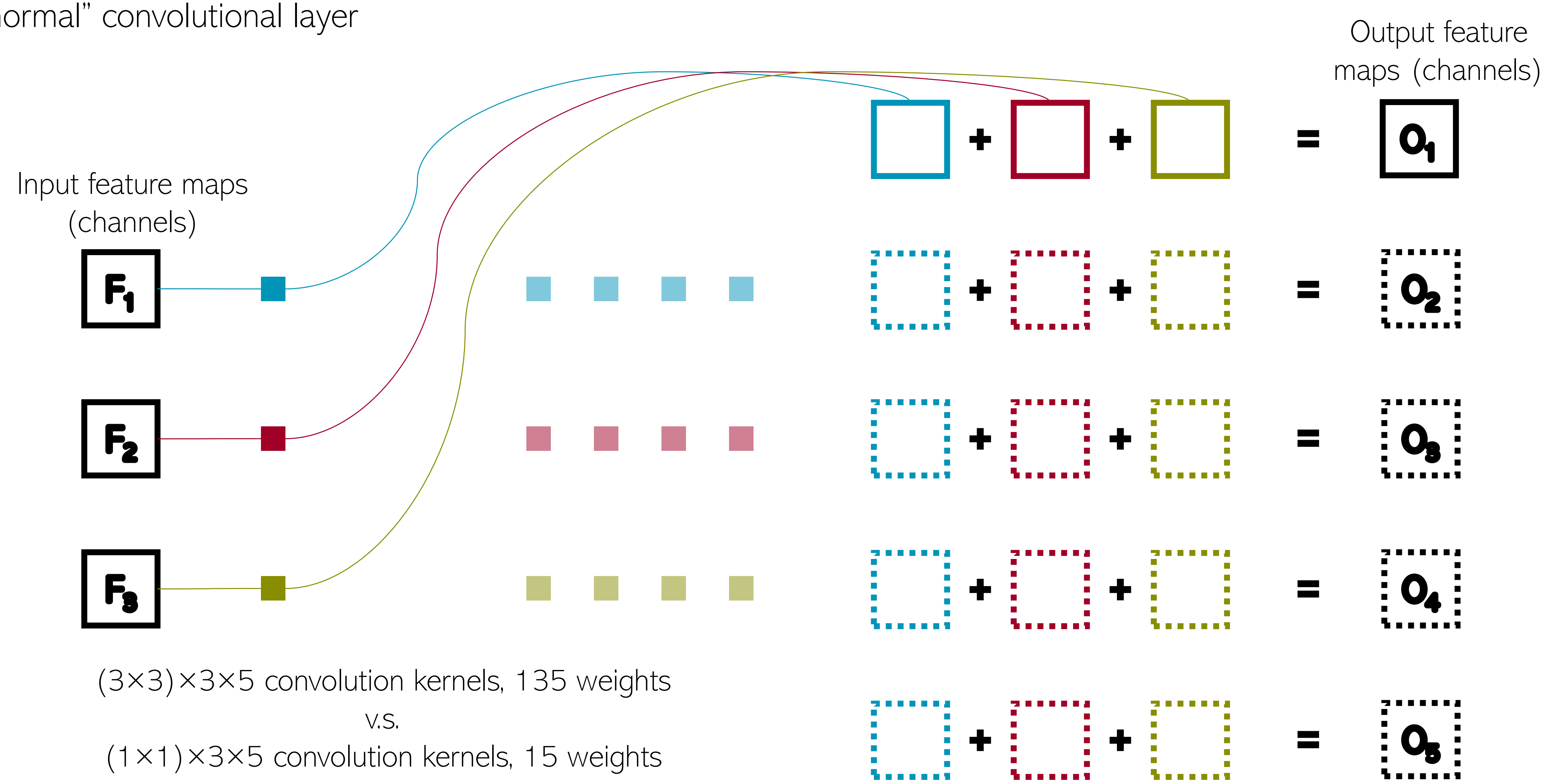
If $\mathbf{rank(K) = 1}$ (test the number of non-singular values / linearly-independent vectors)

$\mathbf{K} = S_1 \mathbf{u}_1 \mathbf{v}_1^{\mathrm{T}} = S_1 \mathbf{u}_1 * \mathbf{v}_1^{\mathrm{T}}$ (definition of convolution)

Reduced degrees of freedom from 9 to 6

Depthwise separable kernels

- Revisit the "normal" convolutional layer



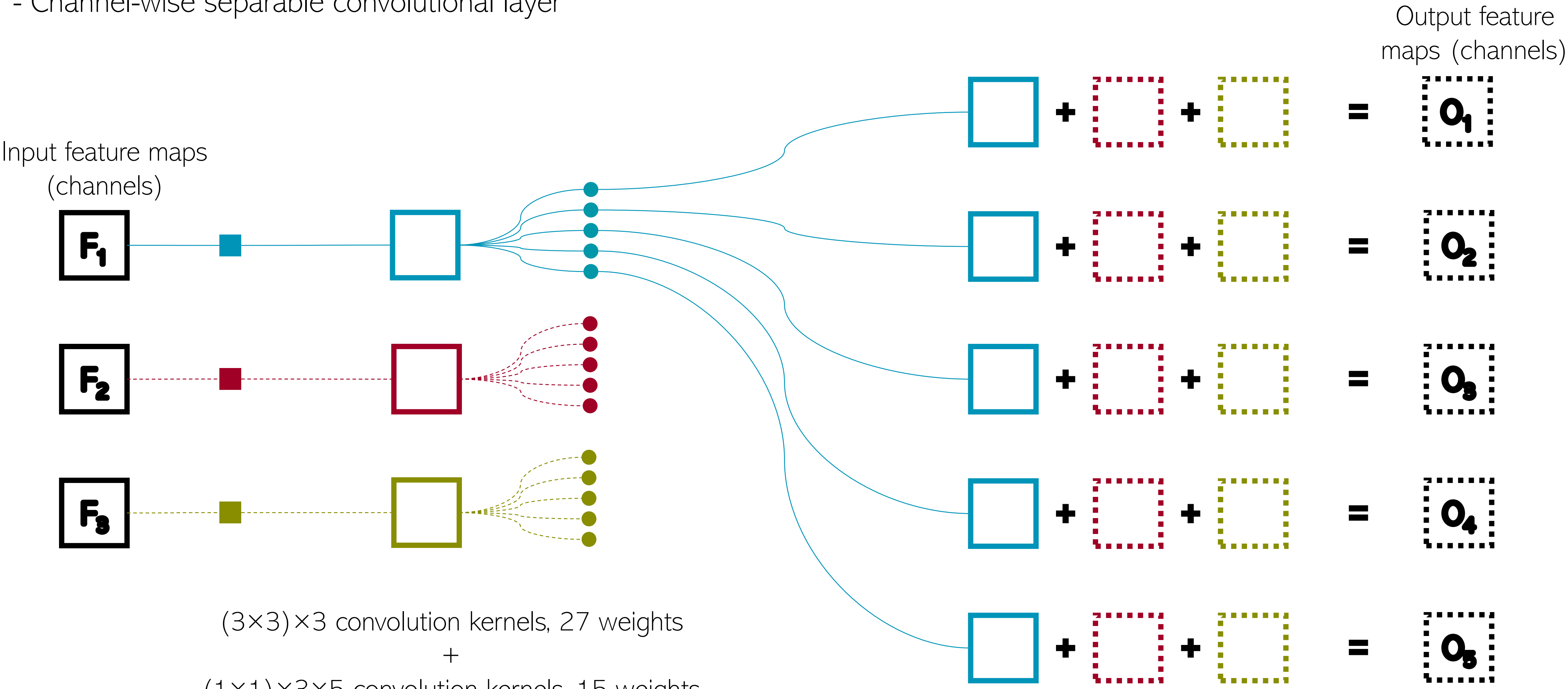
(3×3)×3×5 convolution kernels, 135 weights

v.s.

(1×1)×3×5 convolution kernels, 15 weights

Depthwise separable kernels

- Channel-wise separable convolutional layer



(3×3)×3 convolution kernels, 27 weights

+

(1×1)×3×5 convolution kernels, 15 weights

Depthwise separable kernels

- $(3\times3)\times3\times5=135$ vs. $(3\times3)\times3+(1\times1)\times3\times5=42$
- $(5\times5)\times64\times128=204{,}800$ vs. $(5\times5)\times64+(1\times1)\times64\times128=9{,}792$

- Model scaling
  * Width – number of (input and output) channels
  * Depth – number of layers
  * Resolution – feature map size

- “MobileNet”, scaling vs. performance
- “EfficientNet”, neural architecture search

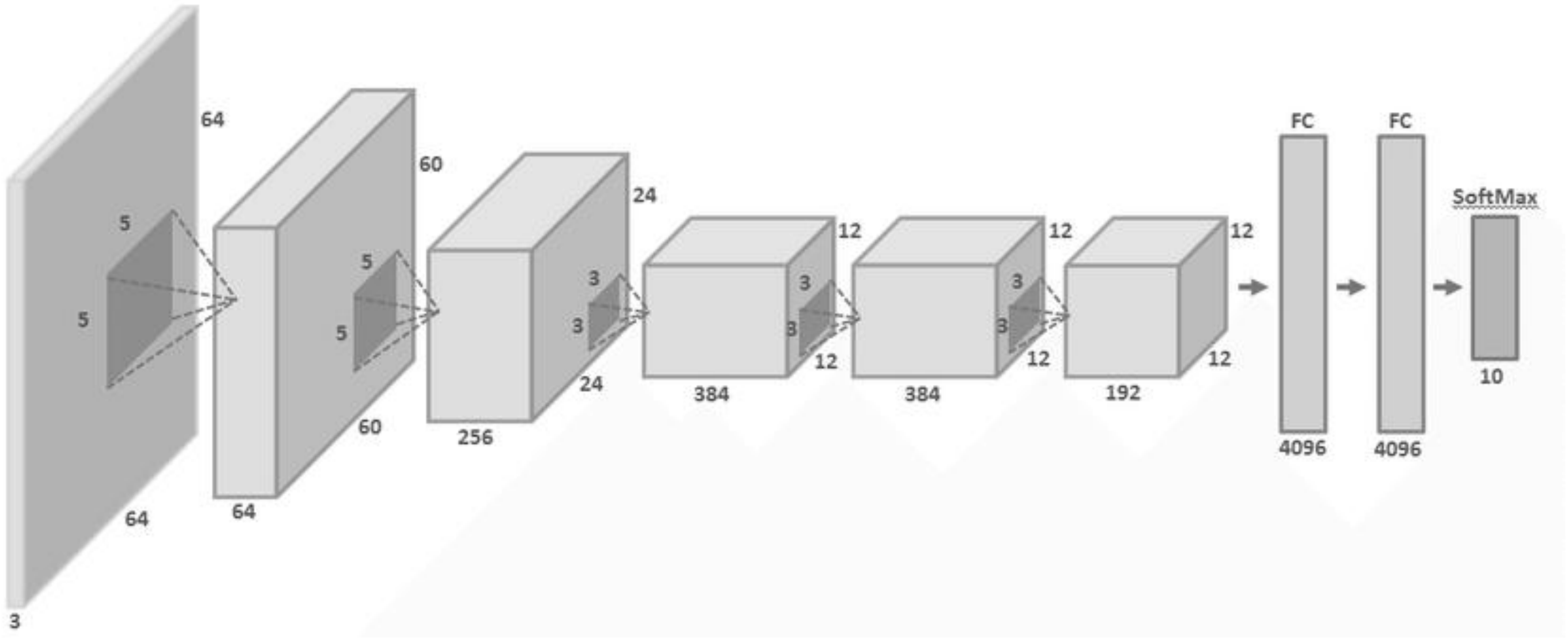
Convolutional Neural Networks | Convolution Arithmetic

Dumoulin, V. and Visin, F., 2016.
A guide to convolution arithmetic for deep learning.
arXiv preprint arXiv:1603.07285.
https://github.com/vdumoulin/conv_arithmetic

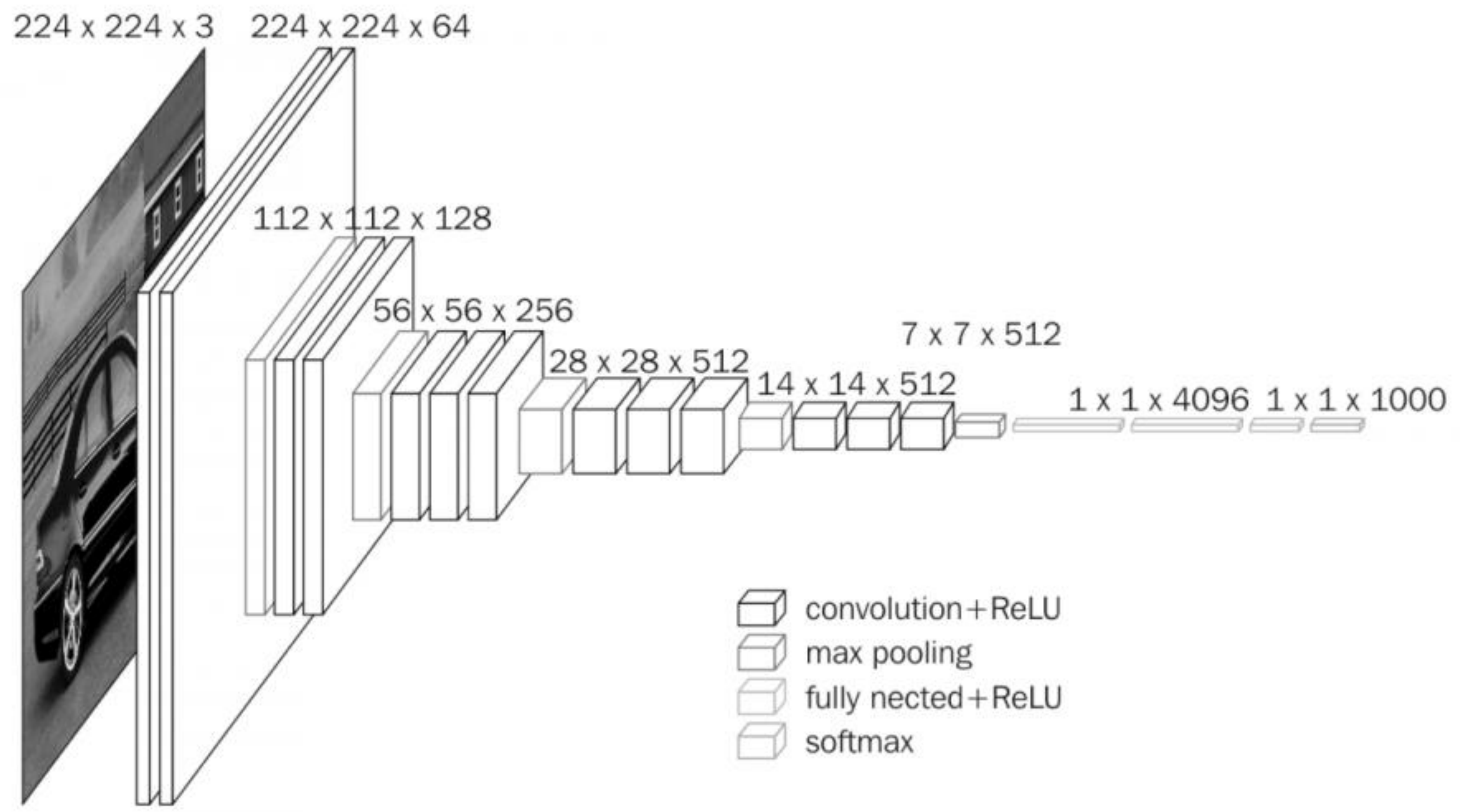
Things need to consider
- Input/output feature map size and even/odd
- Kernel size and even/odd
- Strides
- Padding
- Kernel dilation rate
- Transpose convolution
- Pooling

Convolutional Neural Networks | Applied Architectures

AlexNet

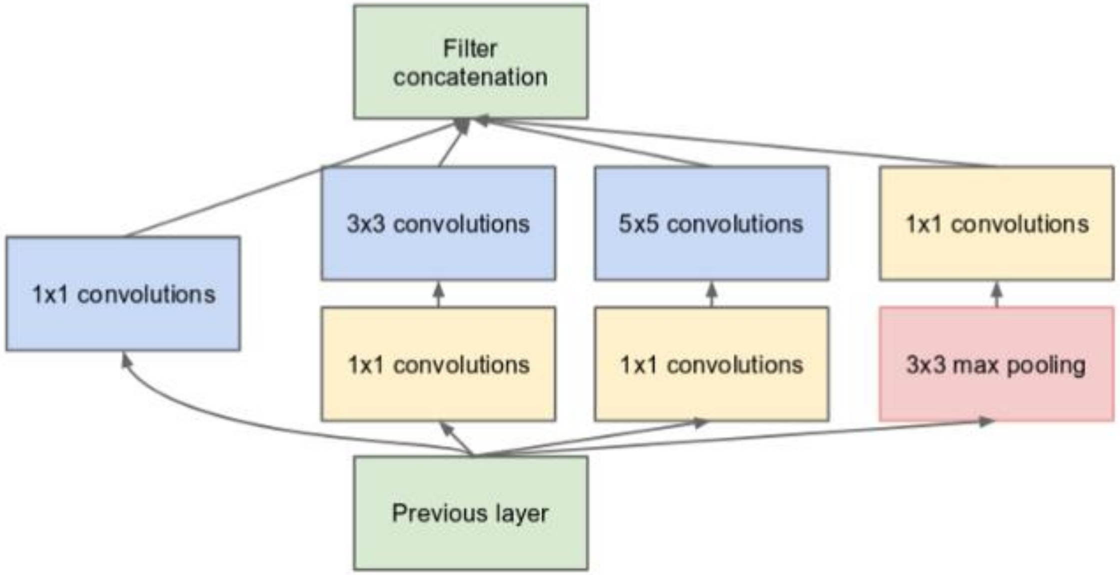

64
64
3
5
5
60
60
64
5
5
24
24
256
3
3
12
12
384
3
3
12
12
384
3
3
12
12
192
FC
4096
FC
4096
SoftMax
10

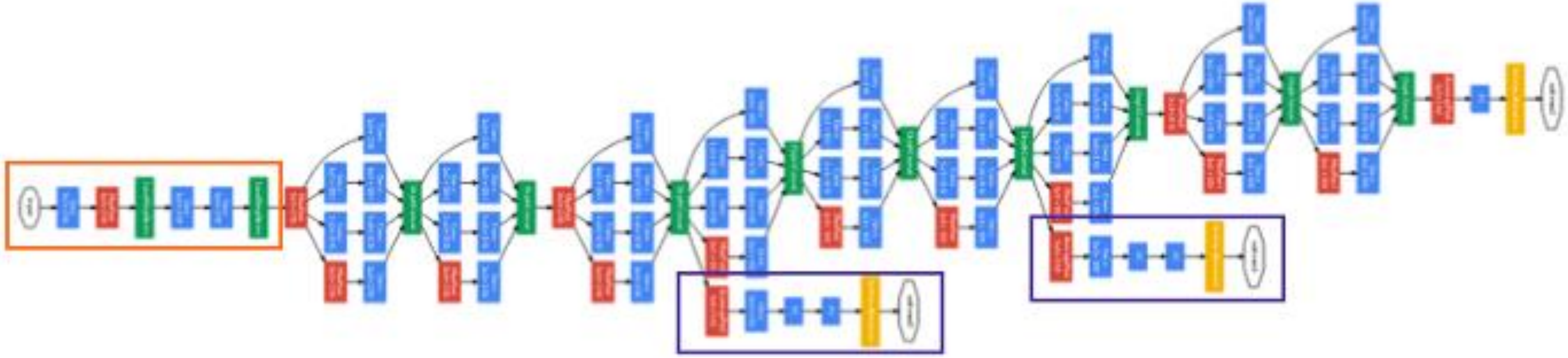
VGG



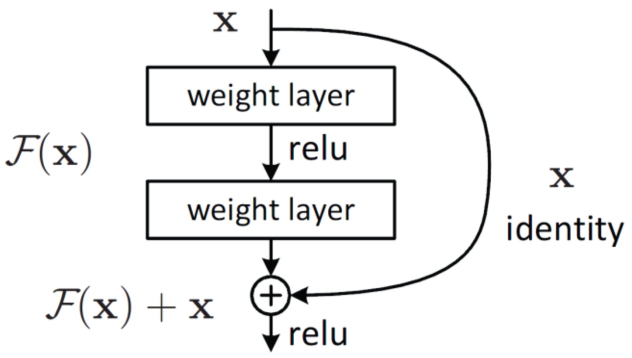
224 x 224 x 3
224 x 224 x 64
112 x 112 x 128
56 x 56 x 256
28 x 28 x 512
14 x 14 x 512
7 x 7 x 512
1 x 1 x 4096
1 x 1 x 1000
convolution+ReLU
max pooling
fully nected+ReLU
softmax

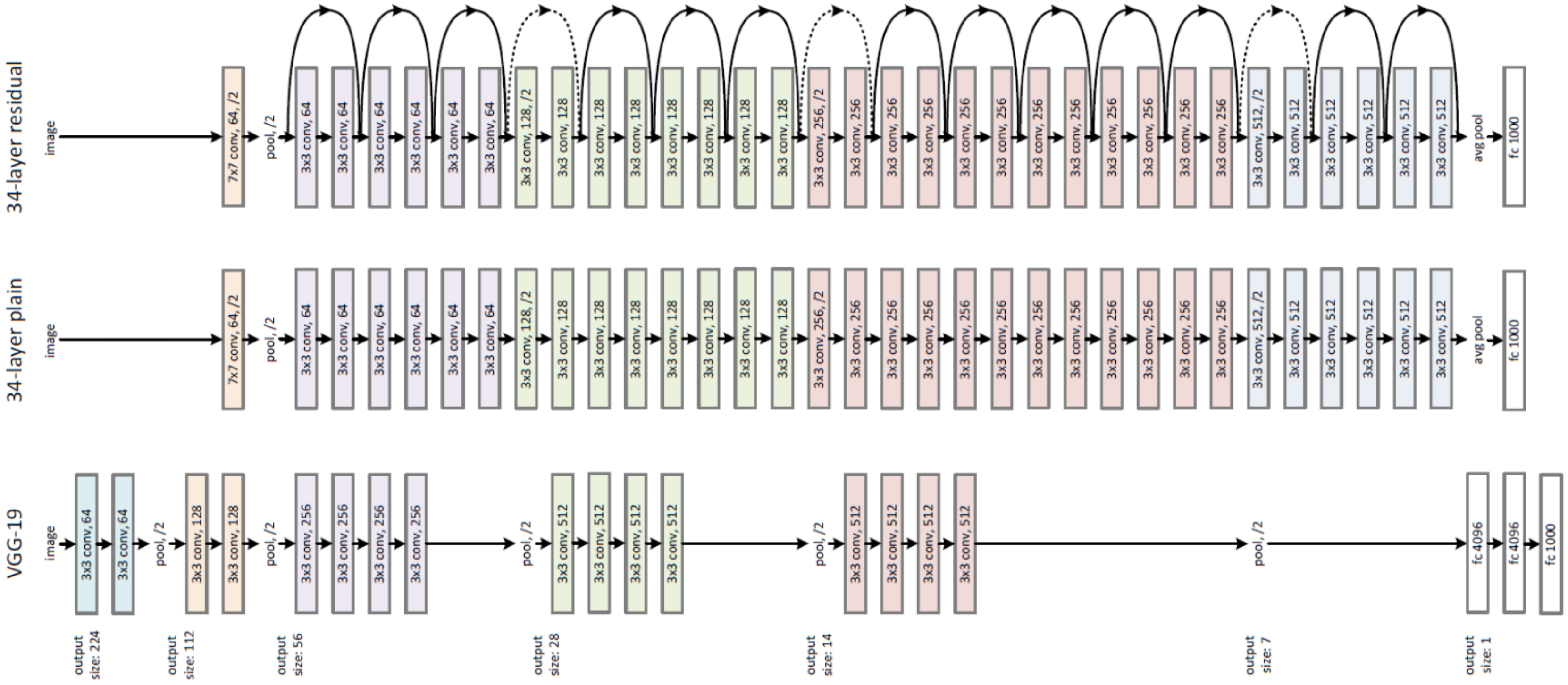
Inception


Filter concatenation
1x1 convolutions
3x3 convolutions
5x5 convolutions
1x1 convolutions
1x1 convolutions
1x1 convolutions
3x3 max pooling
Previous layer

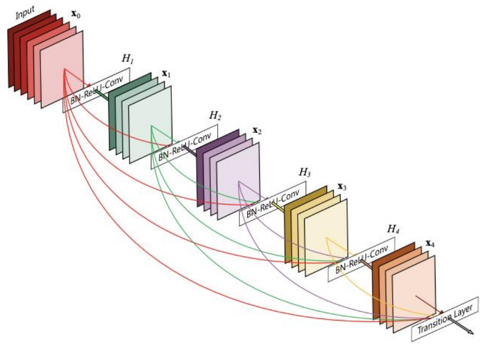


ResNet

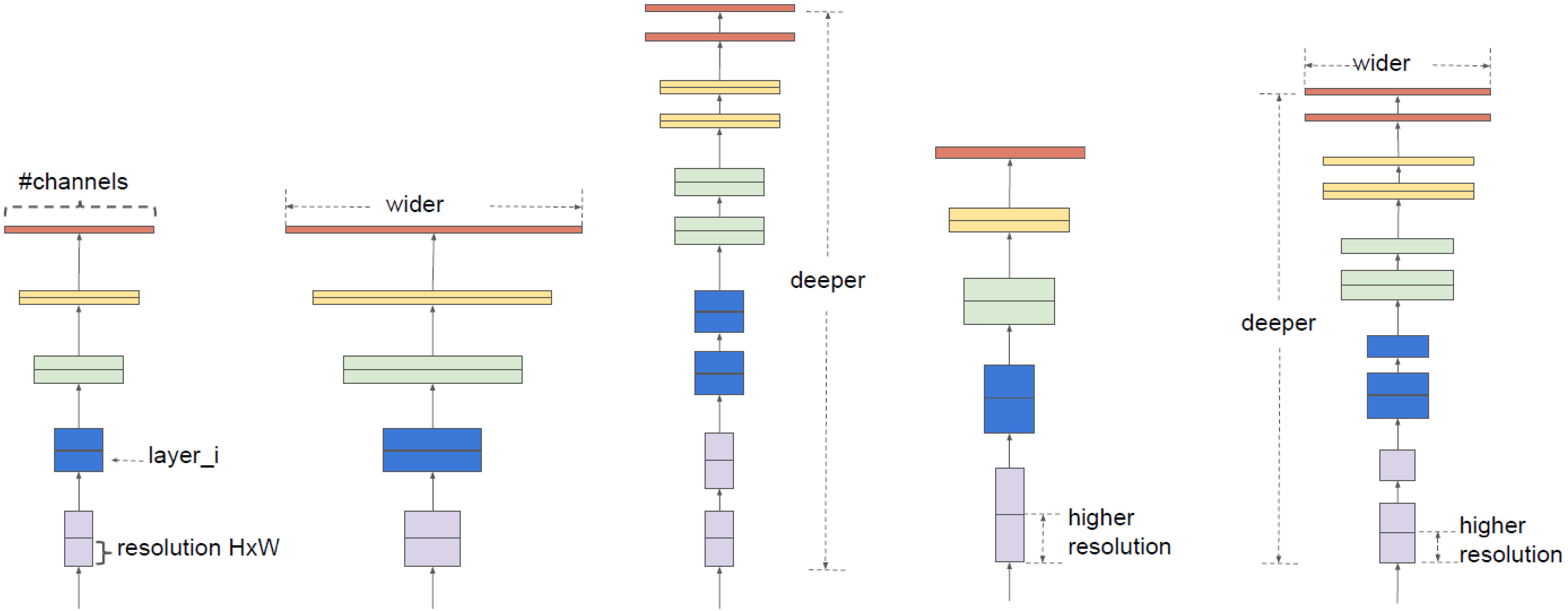




DenseNet

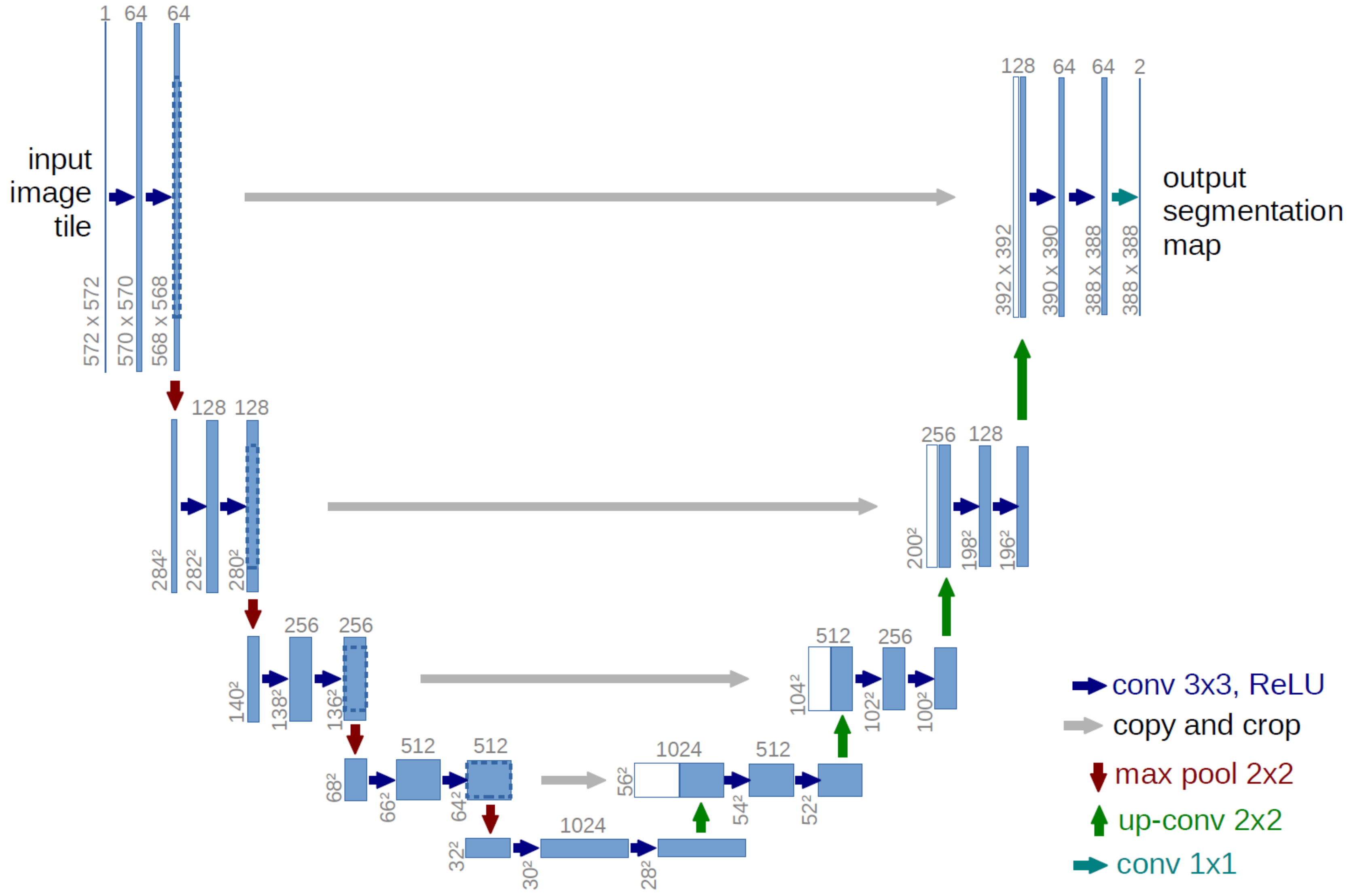


EfficientNet



(a) baseline (b) width scaling (c) depth scaling (d) resolution scaling (e) compound scaling

UNet



1 64 64
input image tile
572 x 572
570 x 570
568 x 568
128 128
284²
282²
280²
256 256
140²
138²
136²
512 512
68²
66²
64²
1024
32²
30²
28²
1024 512
56²
54²
52²
512 256
104²
102²
100²
256 128
200²
198²
196²
128 64 64 2
392 x 392
390 x 390
388 x 388
388 x 388
output segmentation map
conv 3x3, ReLU
copy and crop
max pool 2x2
up-conv 2x2
conv 1x1

Convolution and cross-correlation

Intuition

Translation-invariance, Parameter sharing, Sparse weights, Infinitely strong prior

Sampling

Pooling, Strides, Interpolation, Un-pooling, Transpose convolution

Anatomy of a convolutional layer

Kernels

Dimensionality, receptive field, shape and size, kernel constraints, padding, depthwise separable convolution

Convolution arithmetic

Applied architectures

Using a different CNN for the “image classification” tutorial
Adding/reducing a resolution level in the “image segmentation” tutorial