Calculus HW 8 Novosad Ivan

7. (HW) Find the points of discontinuity of the function and determine their types:

(a) $f(x) = \dfrac{x^2+x}{|x|(x-1)}$; (b) $f(x) = \begin{cases} -\dfrac{4}{x-5}, & x \leqslant 1 \\ 3x^2+5x-7, & 1 < x \leqslant 9 \\ \dfrac{9}{x-9}, & x > 9 \end{cases}$

(a) $f(x) = \dfrac{x^2+x}{|x|(x-1)}$; function is not define for:

$x = 0$
$x = 1$

then it's our pretendents

check $x=0$:

$$\lim_{x\to 0^+}\left(\frac{x(x+1)}{|x|(x-1)}\right) = \underbrace{\lim_{x\to 0^+}\left(\frac{x(x+1)}{x(x-1)}\right)}_{\text{'cause } x\to 0^{+}} = \lim_{x\to 0^+}\left(\frac{x+1}{x-1}\right) = \frac{1}{-1} = -1$$

$$\lim_{x\to 0^-}\left(\frac{x(x+1)}{|x|(x-1)}\right) = \lim_{x\to 0^-}\left(\frac{x(x+1)}{-x(x-1)}\right) = \frac{1}{-(-1)} = 1$$

then $x=0$ is a jump discontinuity

check $x=1$:

$$\lim_{x\to 1^-}\left(\frac{x(x+1)}{|x|(x-1)}\right) = \lim_{x\to 1^-}\left(\frac{x(x+1)}{x(x-1)}\right) = \lim_{x\to 1^-}\left(\frac{x+1}{x-1}\right) = \frac{2}{-0} = -\infty$$

$$\lim_{x\to 1^+}\left(\frac{x(x+1)}{|x|(x-1)}\right) = \lim_{x\to 1^+}\left(\frac{x(x+1)}{x(x-1)}\right) = \lim_{x\to 1^+}\left(\frac{x+1}{x-1}\right) = \frac{2}{+0} = +\infty$$

then $x=1$ is an essential discontinuity

(b) $f(x) = \begin{cases} -\dfrac{4}{x-5}, & x \leqslant 1 \\ 3x^2+5x-7, & 1 < x \leqslant 9 \\ \dfrac{9}{x-9}, & x > 9 \end{cases}$

b) $f(x) = \begin{cases} \dfrac{-4}{x-5} & x \leq 1 \\ 3x^2+5x-7, & 1 < x \leq 9 \\ \dfrac{9}{x-9}, & x > 9 \end{cases}$

check $x = 1$:

$$\lim_{x \to 1^-} \left(\frac{-4}{x-5} \right) = \frac{-4}{1-5} = \frac{-4}{-4} = 1$$

$$\lim_{x \to 1^+} \left(3x^2+5x-7 \right) = 3+5-7 = 1$$

then $x = 1$ is a continuity.

check $x = 9$:

$$\lim_{x \to 9^-} \left(3x^2+5x-7 \right) = 3 \cdot 81 + 5 \cdot 9 - 7 = 281$$

$$\lim_{x\to 9^+}\left(\frac{9}{x-9}\right) = \frac{9}{+0} = +\infty$$

then $x=9$ is an essentinal discontinuity.

8. (HW) Find the points of discontinuity of the following functions and determine types of discontinuity:

(a) $\dfrac{1}{\sin x - \cos x}$; (b) $\cos\dfrac{1}{x}$; (c) $x\sin\dfrac{1}{x^2}$.

a) $\dfrac{1}{\sin(x)-\cos(x)}$ function is not define for $\sin(x)-\cos(x)=0 \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} + \pi k, k \in \mathbb{Z}$

$$\left.\begin{aligned}\lim_{x\to \pi/4^-}\left(\frac{1}{\sin(x)-\cos(x)}\right) &= \frac{1}{-0} = -\infty\\ \lim_{x\to \pi/4^+}\left(\frac{1}{\sin(x)-\cos(x)}\right) &= \frac{1}{+0} = +\infty\end{aligned}\right\} \Rightarrow$$

then $x=\frac{\pi}{4}+2\pi k$ is an essentinal discontinuity

b) $\cos\left(\frac{1}{x}\right)$; function in not define for $x=0$

$\lim_{x \to 0} \left(\cos\left(\frac{1}{x}\right)\right)$ is not define $\Rightarrow$ $x=0$ is an essentinal discontinuity

c) $x \sin\left(\frac{1}{x^2}\right) = \lim_{x \to 0^+}(x) \cdot \lim_{x \to 0^+}\left(\sin\left(\frac{1}{x^2}\right)\right) =$

$= 0 \cdot \underbrace{\lim_{x \to 0^+}\left(\sin\left(\frac{1}{x^2}\right)\right)}_{\text{bounded}} \Rightarrow 0$

$\lim_{x \to 0^-}(x) \cdot \lim_{x \to 0^-}\left(\sin\left(\frac{1}{x^2}\right)\right) = 0 \cdot \text{bounded} \Rightarrow 0$

then. $x \cdot \sin\left(\frac{1}{x^2}\right)$ has a removeble discon.

9. (HW) For which value of k will the function $f(x) = \dfrac{x^2 - (k-2)x + 8}{x - k}$ have a removable discontinuity at $x = k$?

let's find roots of $x^2 - (k-2)x + 8 = 0$

$$\begin{cases} x_1 + x_2 = k - 2 \\ x_1 x_2 = 8 \end{cases} \quad \text{suppose } x_2 = k \Rightarrow \begin{cases} x_1 + k = k - 2 \\ x_1 k = 8 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x_1 = -2 \\ x_1 k = 8 \end{cases} \Rightarrow k = -4$$

check $k = -4$:

$$\lim_{x \to -4^-} \left(\frac{x^2 + 6x + 8}{x + 4} \right) = \lim_{x \to -4^-} \left(\frac{(x+2)(x+4)}{x+4} \right) = \lim_{x \to -4^-} (x - 2) = -2$$

$$\lim_{x \to -4^{+}} (x+2) = -2$$