Novosad Ivan 231

3. (HW) Find the following limits:

(a) $\lim\limits_{x\to+\infty}\left(\frac{x+1}{x+2}\right)^{1-x}$; (b) $\lim\limits_{x\to 0}\left(\frac{2-3x}{5-4x}\right)^{\frac{1}{x^2}}$; (c) $\lim\limits_{x\to 0}\left(\frac{1+2x}{1-3x}\right)^{\frac{3}{\sin 2x}}$.

$$3.\ a)\ \lim_{x\to+\infty}\left(\frac{x+1}{x+2}\right)^{1-x} = \lim_{x\to\infty^+}\left(1+\frac{-1}{x+2}\right)^{1-x} = \lim_{x\to\infty^+}\left(1+\frac{1}{2-x}\right)^{1-x} =$$

$$= \lim_{x\to\infty^+}\left(\left(1+\frac{1}{2-x}\right)^{2-x}\right)^{\frac{1-x}{2-x}} = \lim_{x\to\infty^+}\left(e^{\frac{1-x}{2-x}}\right) = e$$

$$\frac{-1}{-1}$$

$$b)\ \lim_{x\to 0}\left(\left(\frac{2-3x}{5-4x}\right)^{\frac{1}{x^2}}\right) = \lim_{x\to 0}\left(\frac{2}{5}\right)^{\frac{1}{x^2}} = 0$$

$$\to 2/5$$

$$c)\ \lim_{x\to 0}\left(\frac{1+2x}{1-3x}\right)^{\frac{3}{\sin(2x)}} = \lim_{x\to 0}\left(\left(1+\frac{5x}{1-3x}\right)^{\frac{1-3x}{5x}}\right)^{\frac{(5x)(3)}{\sin(2x)(1-3x)}} =$$

$$= \lim_{x\to 0}\left(e^{\frac{15x}{\sin(2x)(1-3x)}}\right) = \lim_{x\to 0}\left(e^{\frac{15}{2(1\cdot 0)}}\right) = e^{\frac{15}{2}} = e^{7}\sqrt{e}$$

$$\to \frac{15}{2}$$

4. (HW) Find the limit $\lim\limits_{x\to+\infty}(3x-2)\Big(\ln(5x+1)-\ln(5x-7)\Big)$.

$$\lim_{x\to\infty^+}(3x-2)\left(\ln(5x+1)-\ln(5x-7)\right) =$$

$$\lim_{x\to\infty^+}(3x-2)\left(\ln\left(\frac{5x+1}{5x+7}\right)\right) = \lim_{x\to\infty^+}\left(\ln\left(1+\frac{-6}{5x+7}\right)^{3x-2}\right) =$$

$$= \ln\left(\lim_{x\to\infty^+}\left(1+\frac{-6}{5x+7}\right)^{3x-2}\right) = \ln\left(\lim_{x\to\infty^+}\left(\left(1+\frac{-6}{5x+7}\right)^{\frac{5x+7}{-6}}\right)^{\frac{6(3x-2)}{5x+7}}\right) =$$

$$= \ln\left(\lim_{x\to\infty^+}\left(e^{\frac{6(3x-2)}{5x+7}}\right)\right) = \ln\left(\lim_{x\to\infty}\left(e^{\frac{18x-12}{5x+7}}\right)\right) = \ln\left(e^{\frac{18}{5}}\right) = \frac{18}{5}$$

5. (HW) Compute $\lim\limits_{x\to a}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$ if $a=0$, $a=1$, and $a=+\infty$.

$$\lim_{x\to a}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}} = \lim_{x\to a}\left(1+\frac{1}{-(x+2)}\right)^{\frac{1-\sqrt{x}}{1-x}} = \lim_{x\to a}\left(\left(1+\frac{1}{-x-2}\right)^{-x-2}\right)^{\frac{1-\sqrt{x}}{(x-1)(x+2)}} =$$

for $a=0$:

$$\lim_{x\to 0}\left(\left(1+\frac{1}{-x-2}\right)^{-x-2}\right)^{\frac{1-\sqrt{x}}{(x-1)(x+2)}} = \left(\left(1+\frac{1}{-2}\right)^{-2}\right)^{\frac{1}{-1\cdot 2}} = \left(\frac{1}{2}\right)^{\frac{-2}{-2}} = \frac{1}{2}$$

for $a=1$:

$$\lim_{x\to 1}\left(\left(1+\frac{1}{-x-2}\right)^{-x-2}\right)^{\frac{1-\sqrt{x}}{(x-1)(x+2)}}$$

$$\frac{1-\sqrt{x}}{(x-1)(x+2)} = \frac{-\cancel{(\sqrt{x}-1)}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)(x+2)} = \frac{-1}{(\sqrt{x}+1)(x+2)}$$

$$= \lim_{x\to 1}\left(\left(1+\frac{1}{-x-2}\right)^{-x-2}\right)^{\frac{-1}{(\sqrt{x}+1)(x+2)}} = \left(1+\frac{1}{-3}\right)^{\frac{-3\cdot(-1)}{(1+1)(1+2)}} = \left(\frac{2}{3}\right)^{\frac{3}{2\cdot 3}} = \left(\frac{2}{3}\right)^{\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

for $a=\infty$:

$$\lim_{x\to\infty^+}\left(\left(1+\frac{1}{-x-2}\right)^{-x-2}\right)^{\frac{1-\sqrt{x}}{(x-1)(x+2)}} = \lim_{x\to\infty}\left(e^{\frac{1-\sqrt{x}}{(x-1)(x+2)}}\right) = \lim_{x\to+\infty}(e)^{\frac{-1}{(\sqrt{x}+1)(x+2)}} =$$

$$e^0 = 1$$

7. (HW) Find vertical and oblique asymptotes of the following functions:

(a) $f(x) = \dfrac{x+1}{x^2+3x-4}$; (b) $f(x) = \sqrt{\dfrac{x^3}{x-2}}$; (c) $f(x) = \sqrt{x^2-1} - x$;

(d) $f(x) = \dfrac{\sqrt{4x^4+1}}{|x|}$; (e) $f(x) = 2x + \operatorname{arccot} x$.

a) $f(x) = \dfrac{x+1}{x^2+3x-4} = \dfrac{x+1}{(x-1)(x+4)} \Rightarrow \begin{cases} x=1 & \text{- asymptota} \\ x=-4 & \text{- assymptota} \end{cases}$

$\left(\lim_{x\to -4}\left(\frac{x+1}{(x-1)(x+4)}\right) \wedge \lim_{x\to -4}\left(\frac{x+1}{(x-1)(x+4)}\right)\right.$ does not exist.$\left.\right)$

b) $f(x) = \sqrt{\dfrac{x^3}{x-2}}$ $x=2$ is a vertical asymplote

'cause $\lim_{x\to 2}\left(\sqrt{\frac{x^3}{x-2}}\right) = \infty^+$

c) $f(x) = \sqrt{x^2 - 1} - 1$ Domain $(-\infty; -1] \cup [1; \infty)$ there aren't vertical or oblique asympt.

d) $f(x) = \frac{\sqrt{4x^4+1}}{|x|}$ Domain: $(-\infty; 0)(0; \infty)$

asymplote: $x=0$, 'cause $\lim_{x \to 0} \left(\frac{\sqrt{4x^4+1}}{|x|} \right)$ does $\frac{\text{not exist}}{\infty}$

e) $f(x) = 2x + \operatorname{arccot}(x)$

asymplote. $y = 2x$ 'cause $\lim_{x \to \infty} (\operatorname{arccot}(x)) = 0$