2. (HW) Find Taylor expansion about $x_0 = -2$ for

$$f(x) = x^4 + 5x^3 + 11x^2 + 15x + 13.$$

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$$f(x) = \sum_{k=0}^{n} \frac{f^k(a)}{k!}(x-a)^k + \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

$$f(x) = \frac{f(-2)}{0!}(x+2)^0 + \frac{f'(-2)}{1!}(x+2)^1 + \frac{f''(-2)}{2!}(x+2)^2 + \frac{f'''(-2)}{3!}(x+2)^3$$

$f(-2) = 16 - 40 + 44 - 30 + 13 = 3$

$f'(x) = 4x^3 + 15x^2 + 22x + 15$

$f'(-2) = -32 + 60 - 44 + 15 = -1$

$f''(x) = 12x^2 + 30x + 22$

$f''(-2) = 48 - 60 + 22 = 10$

$f'''(x) = 24x + 30$

$f'''(-2) = -18$

$f''''(-2) = 24$

$f(x) = (x+2)^4 - 3(x+2)^3 + 5(x+2)^2 - (x+2) + 3$

3. Find Taylor expansion about the origin with $o(x^n)$ for

(a) $f(x) = e^{3x+1}$, (b) $f(x) = \sin^2 x$, (c) $f(x) = \ln(5 - 3x)$, (d) (HW) $f(x) = \dfrac{1}{2x+3}$.

d) $\frac{1}{2x+3} = \frac{1}{3} \cdot \frac{1}{1+\frac{2x}{3}} = \frac{1}{3} \sum_{k=0}^{n} \frac{(-2)^k x^k}{3^k} + o(x^n) = \frac{1}{3} + \sum_{k=1}^{n} \frac{(-2)^k x^k}{3^{k+1}} + o(x^n)$

4. Find Taylor expansion about the origin with $o(x^n)$ for

(a) $f(x) = \dfrac{x^2+5}{x^2+x-12}$, (b) $f(x) = \ln \dfrac{1+2x}{2-x}$, (c) (HW) $f(x) = \dfrac{2x+5}{x^2+5x+4}$.

$$f(x) = \frac{2x+5}{x^2+5x+4} \Leftrightarrow f(x) = \frac{2x+5}{(x+1)(x+4)} \Leftrightarrow f(x) = \frac{1}{x+1} + \frac{1}{x+4} \Leftrightarrow$$

$$f(x) = \frac{1}{1+x} + 4\frac{1}{1+\frac{x}{4}}$$

$$f(x) = \sum_{k=0}^{n} (-1)^k x^k + o(x^n) + \sum_{k=0}^{n} (-1)^k \left(\frac{x}{4}\right)^k + o\left(\left(\frac{x}{4}\right)^n\right)$$

$$f(x) = \frac{5}{4} \sum_{k=0}^{n} \left((-1)^k \cdot x^k \cdot \left(1 + \frac{1}{4^{k+1}}\right)\right) + o(x^n)$$

6. (HW) Find Taylor expansion about the origin with $o(x^3)$ for

(a) $f(x) = \ln(8-2x-x^2)$, (b) $f(x) = (2x+1)\sqrt{1-x}$, (c) $f(x) = \dfrac{\ln(1+2x)}{1-3x}$.

$f(x) =$

6. (HW) Find Taylor expansion about the origin with $o(x^3)$ for

(a) $f(x) = \ln(8 - 2x - x^2)$, (b) $f(x) = (2x+1)\sqrt{1-x}$, (c) $f(x) = \dfrac{\ln(1+2x)}{1-3x}$.

a) $f(x) = \ln(8-2x-x^2) \ominus$

$f(0) = \ln(8)$

$f'(x) = -\dfrac{2+2x}{-x^2-2x+8}$ $\quad f'(0) = -\frac{1}{4}$

$f''(x) = -\dfrac{2x^2+4x+20}{(-x^2-2x+8)^2}$ $\quad f''(0) = -\frac{20}{64} = -\frac{5}{16}$

$f'''(x) = -\dfrac{4x^3+12x^2+120x+112}{(-x^2-2x+8)^3}$ $\quad f'''(0) = -\frac{112}{512} = \frac{-7}{32}$

$\ominus \ln(8) - \frac{1}{4}x - \frac{5}{32}x^2 - \frac{7}{192}x^3 + o(x^3)$

b) $f(x) = (2x+1)\sqrt{(1-x)} \ominus$

$f(0) = 1$

$f'(x) = \dfrac{3-6x}{2\sqrt{1-x}}$ $\quad f'(0) = \frac{3}{2}$

$f''(x) = \dfrac{-9+6x}{2\sqrt{1-x}}$ $\quad f''(0) = \frac{-9}{4}$

$f'''(x) = \dfrac{6x-15}{8\sqrt{1-x}(1-x)^2}$ $\quad f'''(0) = -\frac{15}{8}$

$\ominus 1 + \frac{3}{2}x - \frac{9}{8}x^2 - \frac{5}{16}x^3 + o(x^3)$

c) $f(x) = \dfrac{\ln(1+2x)}{1-3x} \ominus$ $\quad t = 2x$

$f(0) = \ln(1)$

$f'(x) = \dfrac{2-6x+3\ln(1+2x)(1+2x)}{18x^3-3x^2-4x+1}$ $\quad f'(0) = \dfrac{2+3\cdot\ln(1)}{1} = 2$

$\ominus \ln(1+t)\cdot\dfrac{1}{1-3x} = \left(t - \frac{t^2}{2} + \frac{t^3}{3} + o(t^3)\right)\left(1+3x+9x^2-27x^3+o(x^3)\right) = 2x + 4x^2 + \frac{44}{3}x^3 + o(x^3)$