# Homework #14   Novosad Ivan 231

**1) a)**

$$B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \quad B_1 - \text{basis}, \quad 2B_1 = X \Rightarrow \underset{B_1}{\underline{\frac{2}{c}}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \underset{X}{\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}}$$

**b)**

$$V_2 = \left\langle \underset{b_1}{\underline{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}}, \underset{b_2}{\underline{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}}, \underset{b_3}{\underline{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}} \right\rangle$$

$$X = 2b_1 - b_2 - 3b_3$$

**c)**

$$V_3 = \left\langle \underset{a_1}{\underline{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}}, \underset{a_2}{\underline{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}}, \underset{a_3}{\underline{\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}}}, \underset{a_4}{\underline{\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}}} \right\rangle$$

$$\left\langle (x-1) \ (x-1)x \ (x-1)x^2 \ (x-1)x^3 \right\rangle$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & | & 1 \\ 1 & -1 & 0 & 0 & | & -3 \\ 0 & 1 & -1 & 0 & | & 1 \\ 0 & 0 & 1 & -1 & | & 1 \\ 0 & 0 & 0 & 1 & | & 0 \end{bmatrix} \xrightarrow[\text{calc}]{\text{RREF}} \begin{bmatrix} & & & & | & -1 \\ & I_4 & & & | & 2 \\ & & & & | & 1 \\ & & & & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix} \Longrightarrow \bar{X} = - \underset{a_1}{\underline{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}} + 2 \underset{a_2}{\underline{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}} + \underset{a_3}{\underline{\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}}} + 0a_4 = \underset{X}{\underline{\begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}}}$$

**d)**

$$\begin{bmatrix} 1 & -2 & 2 & 1 & 3 & 4 & | & 0 \\ -1 & 2 & -3 & -2 & -1 & -1 & | & 0 \\ 2 & -4 & 3 & 1 & 9 & 12 & | & 0 \\ -2 & 4 & -7 & -5 & 1 & 2 & | & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{matrix} \scriptstyle x_1 \ \scriptstyle x_2 \ \scriptstyle x_3 \ \ \scriptstyle x_4 \ \ \scriptstyle x_5 \ \scriptstyle x_6 \\ \begin{bmatrix} 1 & -2 & 0 & -1 & 0 & 3 & | & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix} \end{matrix} \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 2a+b-3c \\ a \\ 0a-b+c \\ b \\ 0a+0b-c \\ c \end{bmatrix}, \ a,b,c \in \mathbb{R}$$

$$a=0 \quad b=0 \ c=1$$
$$a=0 \quad c=0 \ b=1$$
$$b=0 \quad c=0 \ a=1$$

$$W_5 = \left\langle \underset{c}{\underline{\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}}}, \underset{b}{\underline{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}}, \underset{a}{\underline{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}} \right\rangle$$

$$\begin{bmatrix} \bar{c} & \bar{b} & \bar{a} & | & \bar{x} \\ -3 & 1 & 2 & | & 0 \\ 0 & 0 & 1 & | & 2 \\ 1 & -1 & 0 & | & 2 \\ 0 & 1 & 0 & | & -1 \\ -1 & 0 & 0 & | & -1 \\ 1 & 0 & 0 & | & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & | & 2 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\bar{x} = \bar{c} - \bar{b} + 2\bar{a}$$

② $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \cdot L' = \left[ f(x) \right]_L$ , where $L'$ — is a matrix s.t. $j$ column $= l_j(x)$

$$f(x) = \sum_{j=0}^{n} b_j \, l_j(x)$$

③ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

a) $C(A,B) = \begin{bmatrix} -2 & 3 & 2 \\ 1 & 1 & 1 \\ 1 & -2 & -1 \end{bmatrix} \Rightarrow C(B,A) = C(A,B)^{-1} = \begin{bmatrix} -1/2 & 1/2 & -1/2 \\ -1 & 0 & -2 \\ 3/2 & 1/2 & 5/2 \end{bmatrix}$
b)

c) $\overset{v_1 \quad v_2 \quad v_3}{\quad} \text{s.t. } (v_1 v_2 v_3) = A$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ coordinates $\Rightarrow 2v_1 - v_1 + 0v_3 = D$

d) $\overset{u_1 \quad u_2 \quad u_3}{\quad} \text{s.t. } (u_1 u_2 u_3) = B$

$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 2 \\ 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ coordinates $\Rightarrow -3/2\, u_1 - 2u_2 + 5/2\, u_3 = D$