

a) prove that $\frac{2}{\sqrt{1+\pi}} < \int_0^{\pi} \frac{\sin(x)}{\sqrt{1+x}} dx < 2$

since $\min(\sqrt{1+x})$ on $[0, \pi]$ is 1:

$$\int_0^{\pi} \frac{\sin(x)}{\sqrt{1+x}} dx < \int_0^{\pi} \frac{\sin(x)}{1} dx = 2$$

respectively $\max(\sqrt{1+x})$ is $\sqrt{1+\pi}$:

$$\int_0^{\pi} \frac{\sin(x)}{\sqrt{1+\pi}} dx < \int_0^{\pi} \frac{\sin(x)}{\sqrt{1+x}} dx$$

$\frac{2}{\sqrt{1+\pi}}$

6) $xy = 5$; $x+y=6$

\Updownarrow

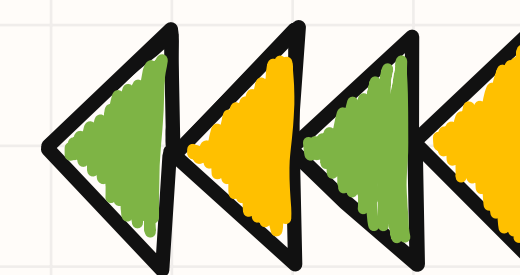
low $\nearrow y = \frac{5}{x}$; $y = 6-x$ \nwarrow up

$$\frac{5}{x} = 6-x \Leftrightarrow 5 - 6x + x^2 = 0$$

$$(x-1)(x-5) = 0$$

$$S = \int_1^5 \left((6-x) - \frac{5}{x} \right) dx = \int_1^5 (x^2 - 6x + 5) dx =$$

$$= \left. \frac{1}{3}x^3 - 3x^2 + 5x \right|_1^5 = \frac{125}{3} - 75 + 25 - \frac{1}{3} - 3 + 5 = \frac{124}{3} + 55 = \frac{289}{3}$$



3) a) $\int (x^2+3x-4)e^x dx = \left\{ \begin{array}{l} u = x^2+3x-4 \quad dw = e^x dx \\ du = (2x+3)dx \quad w = e^x \end{array} \right\} = (x^2+3x-4)e^x - \int (2x+3)e^x dx = \left\{ \begin{array}{l} u = 2x+3 \quad dw = e^x \\ du = 2dx \quad w = e^x dx \end{array} \right\}$

$$(x^2+3x-4)e^x - (2x+3)e^x + 2\int e^x dx = e^x(x^2+x-5) + C$$

b) $\int \frac{x^2-x+1}{(x+1)(x^2+1)} dx = \frac{3}{2} \int \frac{dx}{x+1} + -\frac{1}{2} \int \frac{x+1}{x^2+1} dx = \frac{3}{2} \ln|x+1| - \frac{1}{4} \ln(x^2+1) - \frac{1}{2} \arctan(x) + C$

draft: $\frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{x^2-x+1}{(x+1)(x^2+1)} \Leftrightarrow ax^2+a+bx^2+bx+cx+c$

$$\Rightarrow \begin{cases} a+b=1 \\ b+c=-1 \\ a+c=1 \end{cases} \Rightarrow \begin{cases} b-c=0 \\ b+c=-1 \\ a=1-c \end{cases} \Rightarrow (a,b,c) = (3/2; -1/2; -1/2)$$

$$- \frac{1}{2} \int \frac{x+1}{x^2+1} dx = - \frac{1}{4} \int \frac{2x}{x^2+1} dx \stackrel{u=x^2+1}{=} - \frac{1}{2} \int \frac{dx}{x^2+1} = - \frac{1}{4} \ln(x^2+1) - \frac{1}{2} \arctan(x) + C$$

4) a) $\int \sqrt{25-x^2} dx \left| \begin{array}{l} x = 5 \sin(\theta) \\ dx = 5 \cos(\theta) d\theta \end{array} \right| = \int 25 \cos^2(\theta) d\theta = \frac{25}{2} \int (1 + \cos(2\theta)) d\theta = \frac{25\theta}{2} + \frac{25 \sin(2\theta)}{4} + C =$

$$= \frac{25\theta}{2} + \frac{25 \sin(\theta) \cos(\theta)}{2} + C = \frac{25 \arcsin(\frac{x}{5})}{2} + \frac{25x\sqrt{25-x^2}}{2} + C$$

b) $\int \tan^3(x) dx = \int (\sec^2(x) - 1) \tan(x) dx = \int \sec^2(x) \tan(x) dx - \int \frac{\sin(x)}{\cos(x)} dx = \frac{1}{2} \tan(x)^2 + \ln|\cos(x)| + C$

5) a) $\int_1^6 \frac{x+3}{\sqrt{x+1}} dx = \left\{ u = x+1 \right\} = \int_2^7 \frac{u+2}{\sqrt{u}} du = \left. \frac{2}{3} u^{3/2} + 4\sqrt{u} \right|_2^7 = \frac{14}{3} \sqrt{7} + 4\sqrt{7} - \frac{4}{3} \sqrt{2} - 4\sqrt{2}$

b) $\int_0^1 \arctan(x) dx = \left\{ \begin{array}{l} u = \arctan(x) \quad dw = dx \\ du = \frac{dx}{1+x^2} \quad w = x \end{array} \right\} = x \arctan(x) \Big|_0^1 - \int_0^1 \frac{x}{1+x^2} dx = x \arctan(x) - \frac{1}{2} \ln(1+x^2) \Big|_0^1 = \arctan(1) - \frac{\ln(2)}{2} = \frac{\pi - \ln(2)}{4}$

