HW 15 Novosad Ivan 231

① $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ -1 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow U_1 = \underbrace{\left\langle \overset{\phi_1}{\begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}}, \overset{\phi_2}{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}} \right\rangle}_{\text{basis for } U_1}$

$\begin{bmatrix} -3 & 2 & 1 \\ -5 & 3 & 2 \\ 4 & -2 & -2 \\ -4 & 3 & 1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow U_2 = \underbrace{\left\langle \overset{\varphi_1}{\begin{bmatrix} -3 \\ -5 \\ 4 \\ -4 \end{bmatrix}}, \overset{\varphi_2}{\begin{bmatrix} 2 \\ 3 \\ -2 \\ 3 \end{bmatrix}} \right\rangle}_{\text{basis for } U_2}$

a) $\begin{array}{c} \quad U_1 \qquad\quad U_2 \\ \begin{bmatrix} 1 & 0 & -3 & 2 \\ 2 & 1 & -5 & 3 \\ -1 & 1 & 4 & -2 \\ 2 & 2 & -4 & 3 \end{bmatrix} \end{array} \to \begin{array}{c} \qquad\quad \text{"free" col} \\ \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \Rightarrow U_1 + U_2 = \underbrace{\left\langle \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -2 \\ 3 \end{bmatrix} \right\rangle}_{\text{basis for } U_1 + U_2}$

b) Using Zassenhaus algorithm and bases found in (a) we get

$\begin{bmatrix} [\phi_1^T] & [\phi_1^T] \\ [\phi_2^T] & [\phi_2^T] \\ [\varphi_1^T] & \\ [\varphi_2^T] & 0 \end{bmatrix} \xrightarrow{REF} \begin{bmatrix} [\ \in\ ] & [\ *\ ] \\ & [\ \$\ ] \\ [\ 0\ ] & [\ 0\ ] \end{bmatrix}$, where $\in^T_{(i)} \in U_1 + U_2$, $\$^T_{(i)} \in U_1 \cap U_2$

that's how algorithm works in general (google it) ❤

$\begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 & -1 & 2 \\ 0 & 1 & 1 & 2 & 0 & 1 & 1 & 2 \\ -3 & -5 & 4 & -4 & 0 & 0 & 0 & 0 \\ 2 & 3 & -2 & 3 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{REF} \begin{bmatrix} 1 & 2 & -1 & 2 & & & & \\ 0 & 1 & 1 & 2 & & [\ * \ ] & & \\ 0 & 0 & 1 & 1 & & & & \\ 0 & 0 & 0 & 0 & 3 & 5 & -4 & 4 \end{bmatrix} \Rightarrow U_1 \cap U_2 = \left\langle \begin{bmatrix} 3 \\ 5 \\ -4 \\ 4 \end{bmatrix} \right\rangle$

ans*

② $\mathcal{U}_1 = \langle (x+1)(x-2) ; (x+1)(x-2)x ; (x+1)(x-2)x^2 \rangle$

$\mathcal{U}_2 = \langle (x-1) ; (x-1)x ; (x-1)x^2 ; (x-1)x^3 \rangle$

$$\Downarrow$$

$$\mathcal{U}_1 \cap \mathcal{U}_2$$
$$\shortparallel$$
$$\langle (x-1)(x+1)(x-2) ; (x-1)(x+1)(x-2)x \rangle$$

③ Consider $\mathcal{U}_1 = \langle (x-a) ; (x-a)x , (x-a)x^2 \dots (x-a)x^{n-1} \rangle$

and $\mathcal{U}_2 = \langle (x-b) ; (x-b)x ; (x-b)x^2 \dots (x-b)x^{n-1} \rangle$

then take vectors from both bases with the same deg.

and subtract one from another.

i.e. $(x-a) - (x-b) = b - a \neq 0 = \lambda$ hence $\lambda \in \mathcal{U}_1 + \mathcal{U}_2 \Rightarrow \mathcal{U}_1 + \mathcal{U}_2 = \mathbb{R}(x;n)$

④ $25 \leq \dim(\mathcal{U}_1 + \mathcal{U}_2) \leq 33$ ✓

$0 \leq \dim(\mathcal{U}_1 \cap \mathcal{U}_2) \leq 14$ weak (note: 1st is trivial ; 2nd:

$\Downarrow$ suppose we have 33 chairs (basis vectors of V)

$6 \leq \dim(\mathcal{U}_1 \cap \mathcal{U}_2) \leq 14$ ✓ hence 25 was occupied by $\mathcal{U}_1$, hence

rest is 8 free chairs $\Rightarrow$ min 6 chairs is object of war!)