2. (HW) Describe the domain and range of the function:

(a) $z = \dfrac{xy}{x-y}$; (b) $z = \sqrt{9 - x^2 - 9y^2}$; (c) $z = \arcsin(y/x)$; (d) $z = \ln(xy - 6)$.

a) $D(z): \{x \in \mathbb{R}, y \in \mathbb{R} \mid x - y \neq 0\} \Leftrightarrow D_z = \{(x,y) \mid x \neq y\}$

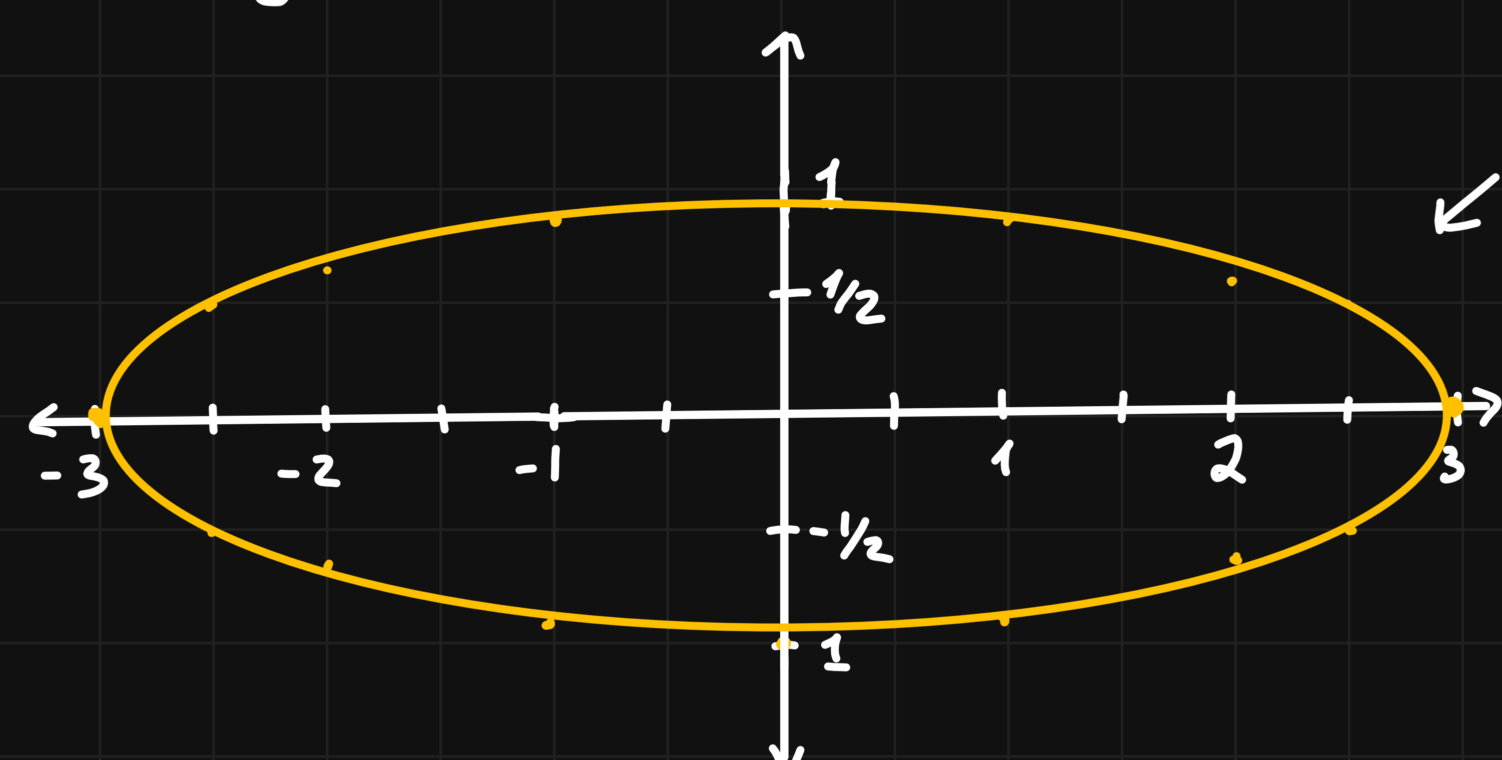
$R_z = \mathbb{R}$, since $\forall z$, we fix $y = 1$;

$z = \dfrac{x}{x+1} \Rightarrow x = -\dfrac{z}{z-1}, \ \forall z \neq 1$

for $z = 1$: $x = -2 \wedge y = 2$; $z(-2,2) = 1$

b) $z = \sqrt{9 - x^2 - 9y^2}$, since $\sqrt{\ } \geq 0$

$9 - x^2 - 9y^2 \geq 0 \Leftrightarrow x^2 + 9y^2 \leq 9$ is ellipse



$R_z = (x,y)$ s.t. (x,y) in ellipse

$-3 \leq x \leq 3 \Rightarrow -\frac{1}{3}\sqrt{9 - x^2} \leq y \leq \frac{1}{3}\sqrt{9 - x^2}$

$D_z = \{(x,y) \mid x^2 + 9y^2 \leq 9\}$

$0 \leq \sqrt{9 - x^2 - 9y^2} \leq 3$

since $x^2 + 9y^2 \geq 0$, $\max(-x^2 - 9y^2) = 0$

Hence $R_z = [0,3]$ $\forall z \in [0,3]$ we can fix $y = 0$

$\Rightarrow x = \pm\sqrt{9 - z^2}$

$z = \arcsin(y/x)$, since range of arcsin is $[-\frac{\pi}{2}; \frac{\pi}{2}]$

and we can fix $x = 1$, then $R_z = [-\frac{\pi}{2}; \frac{\pi}{2}]$

since domain of $\arcsin(m)$ is $[-1, 1]$

$x \in \mathbb{R} \setminus \{0\} \Rightarrow -x \leq y \leq x$

$y \in \mathbb{R} \Rightarrow x \geq y \vee x \leq -y \Leftrightarrow x \in (-\infty; -y] \cup [y; \infty) \setminus \{0\}$

thus $D_z = \{(x,y) \mid |\frac{y}{x}| \leq 1 \wedge x \neq 0\}$

$\Updownarrow$

or $D_z = \{(x,y) \mid x \in (-\infty; -y] \cup [y; \infty) \setminus \{0\}\}$

$\Updownarrow$

or $D_z = \{(x,y) \mid -x \leq y \leq x \wedge x \neq 0\}$

d) $z = \ln(xy - 6)$, since $\ln(k)$ is def. for $k > 0$:

$xy - 6 > 0 \Leftrightarrow xy > 6$, if we consider $xy = 6$, we will obtain hyperbola :

, so (x,y) in is in Domain

$D_z = \{(x,y) \mid xy > 6\}$

or $D_z = \{(x,y) \mid x \in \mathbb{R} \setminus \{0\} \wedge \begin{cases} y < \frac{6}{x}, x > 0 \\ y > \frac{6}{x}, x < 0 \end{cases}\}$

obv $R_z = \mathbb{R}$, since we can fix $y = 1$, so

$\forall z: x = e^z + 6$

$z = \ln(x - 6) \Leftrightarrow e^z = x - 6 \Leftrightarrow e^z + 6$

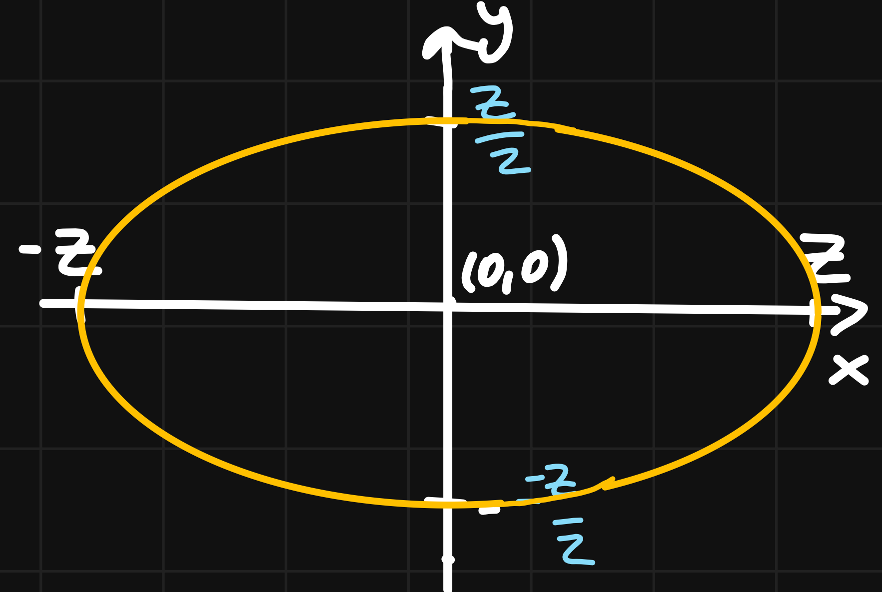
4. (HW) Describe the level curves of the function. Sketch the level curves for the given C-values:

(a) $z = x^2 + 4y^2$, $C = 0, 1, 2, 3, 4$; (b) $z = \dfrac{x}{x^2+y^2}$, $C = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$;

(c) $z = 3^{xy/2}$, $C = 1, \frac{1}{3}, 3, 9, \frac{1}{9}$; (d) $z = \ln(1 - xy)$, $C = 0, \pm 1, \pm 2, \pm 3$.

a) $z = x^2 + 4y^2 \Rightarrow \begin{cases} x = \pm\sqrt{4y^2 - z} \\ y = \pm\frac{1}{2}\sqrt{x^2 - z} \end{cases}$

in other words: it's ellipse with senter at $(0,0)$ $R = \sqrt{z}$ and coef $1/2$



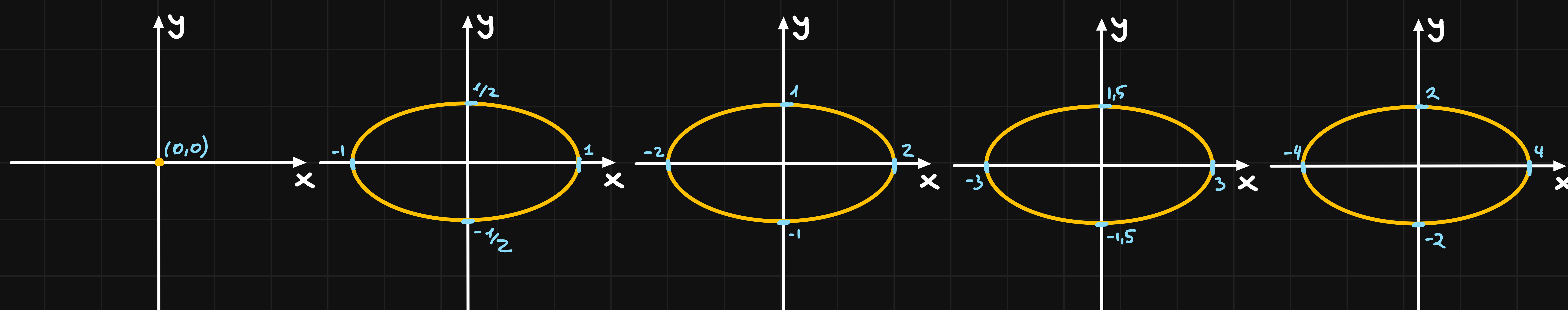
, if we fix $y=0$, then x^2 is any non-negative num also note that $x^2+9y^2 \geq 0$ Hence $R_z = [0, \infty)$

$D_z = \left\{ (x,y) \,\middle|\, y = \frac{\sqrt{z - x^2}}{3} \vee y = \frac{-\sqrt{z - x^2}}{3} \right\} \vee D_z = \left\{ (x,y) \,\middle|\, x = \sqrt{z - 9y^2} \vee x = -\sqrt{z - 9y^2} \right\}$

So $D_z = \mathbb{R}^2$, since for $\forall (x,y) \in \mathbb{R}^2$, we can find such z:

$z = x^2 + 4y^2$, just that.

$C = 0$ $\quad$ $C = 1$ $\quad$ $C = 2$ $\quad$ $C = 3$ $\quad$ $C = 4$



4. (HW) Describe the level curves of the function. Sketch the level curves for the given C-values:

(a) $z = x^2 + 4y^2$, $C = 0, 1, 2, 3, 4$; (b) $z = \frac{x}{x^2+y^2}$, $C = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$;

(c) $z = 3^{xy/2}$, $C = 1, \frac{1}{3}, 3, 9, \frac{1}{9}$; (d) $z = \ln(1-xy)$, $C = 0, \pm 1, \pm 2, \pm 3$.

$z = \frac{x}{x^2+y^2}$; $x^2+y^2 \neq 0 \Leftrightarrow (x,y) \neq (0,0)$

There are no more restr., so $D_z = \mathbb{R}^2 \setminus \{0,0\}$

If we fix $y=0$, then $z = \frac{1}{x}$, so $\forall z \neq 0$ $x = \frac{1}{z}$

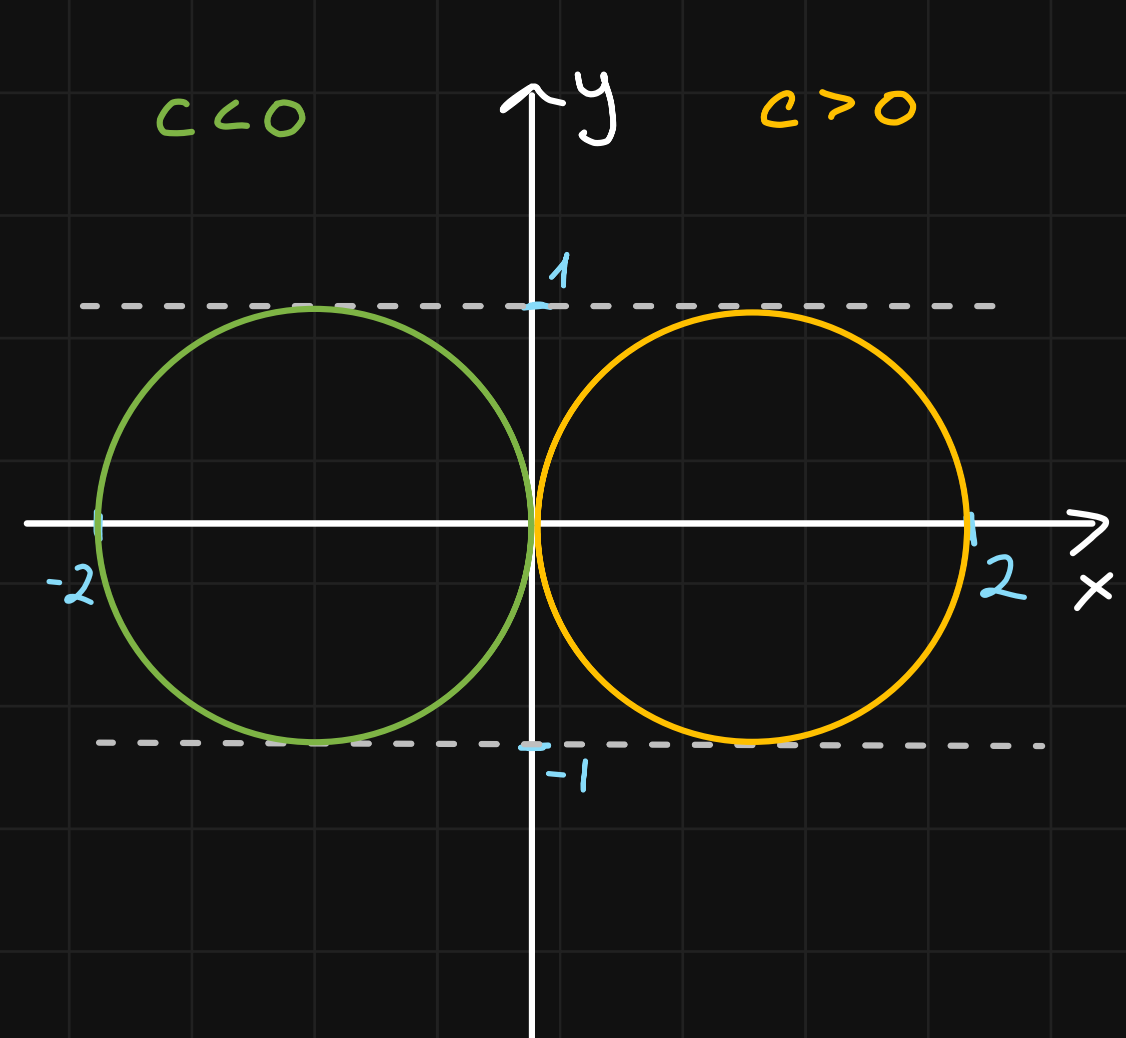
Thus we can obtain any $z \neq 0$, for $z=0$: $(x,y) = (0,1)$

Hence $R_z = \mathbb{R}$

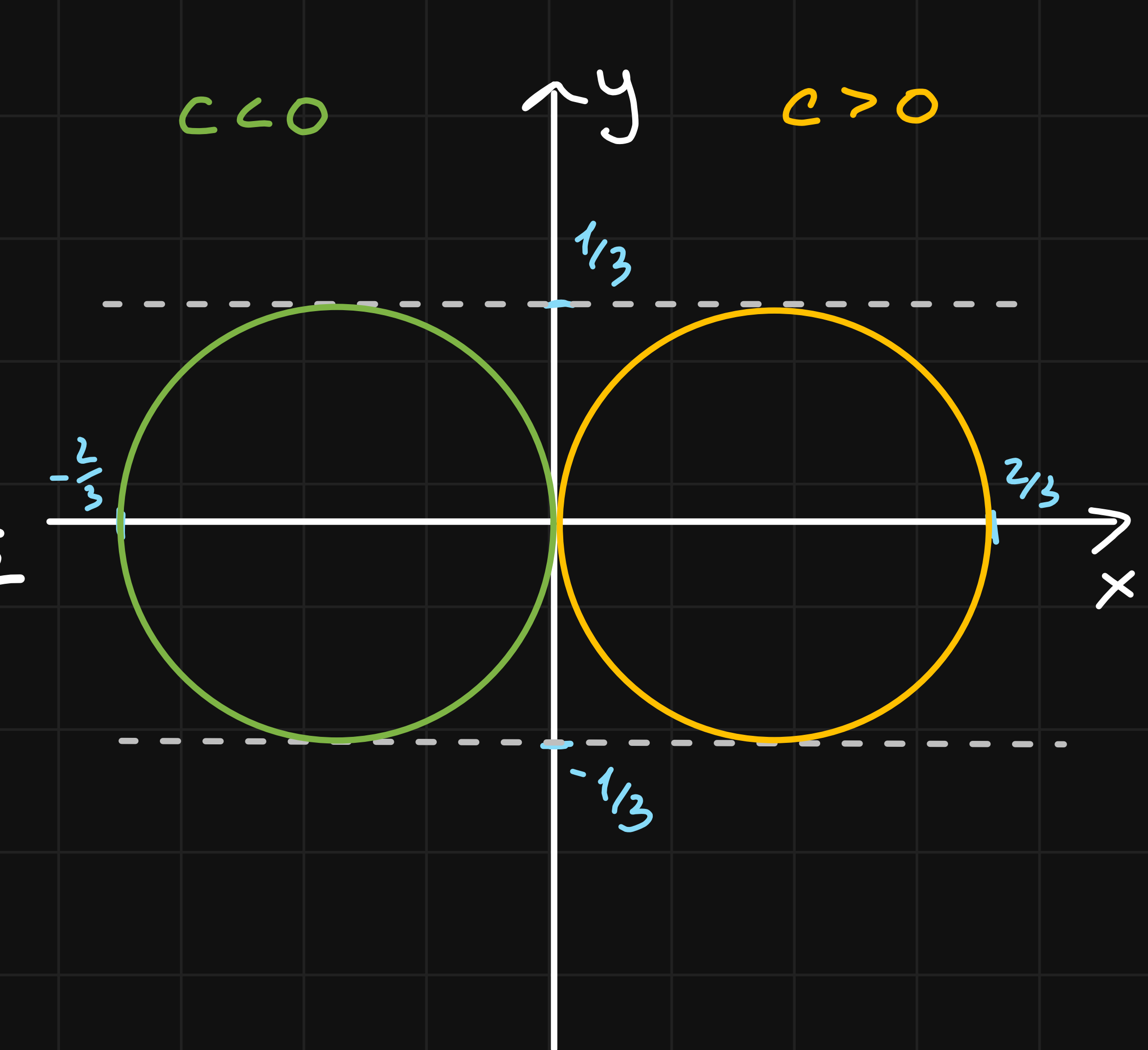
$z = \frac{x}{x^2+y^2} \Leftrightarrow x^2+y^2 = zx$

it's obv circle with $r = \frac{1}{2|z|}$, with center in $(0, \frac{1}{2z})$

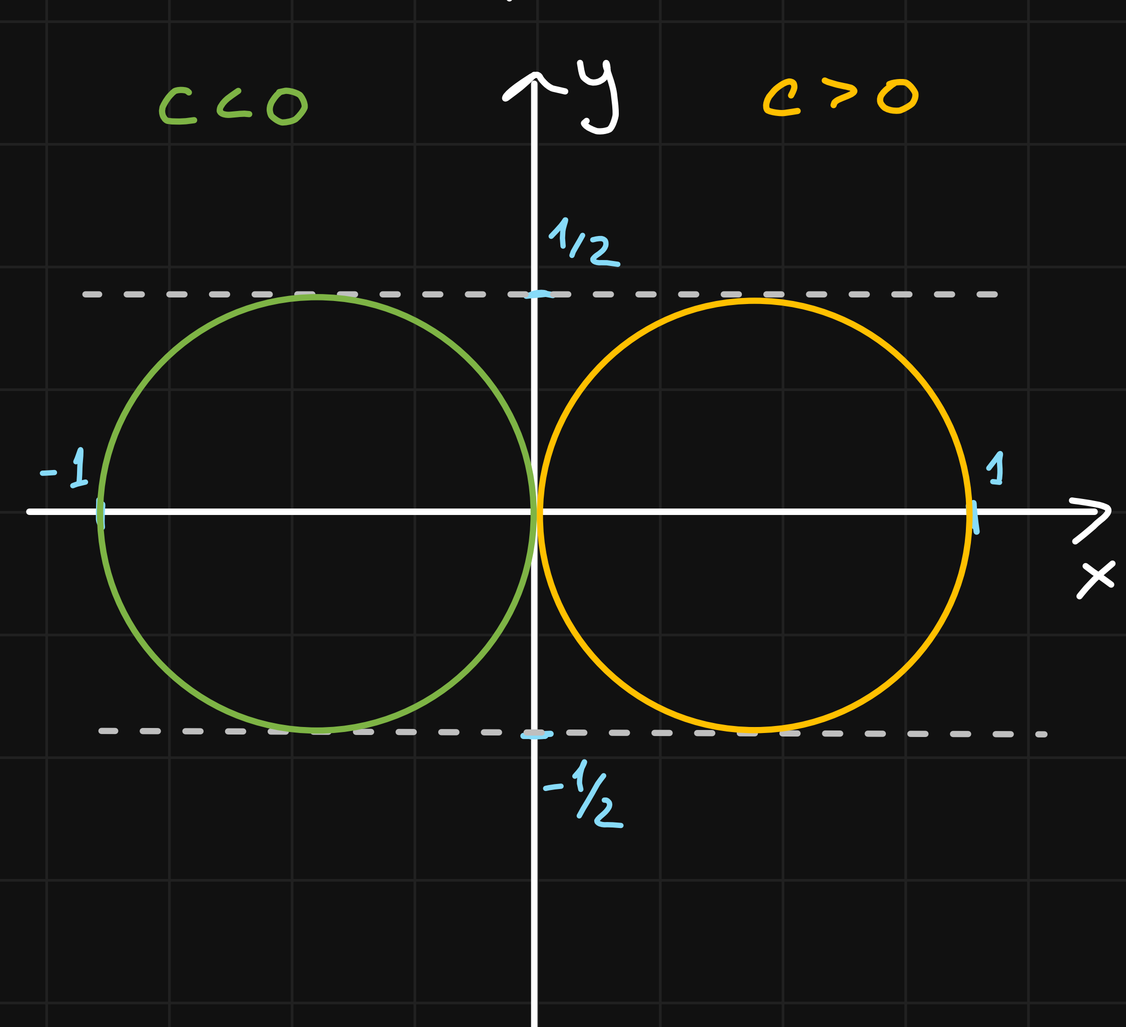
$C = \pm\frac{1}{2}$



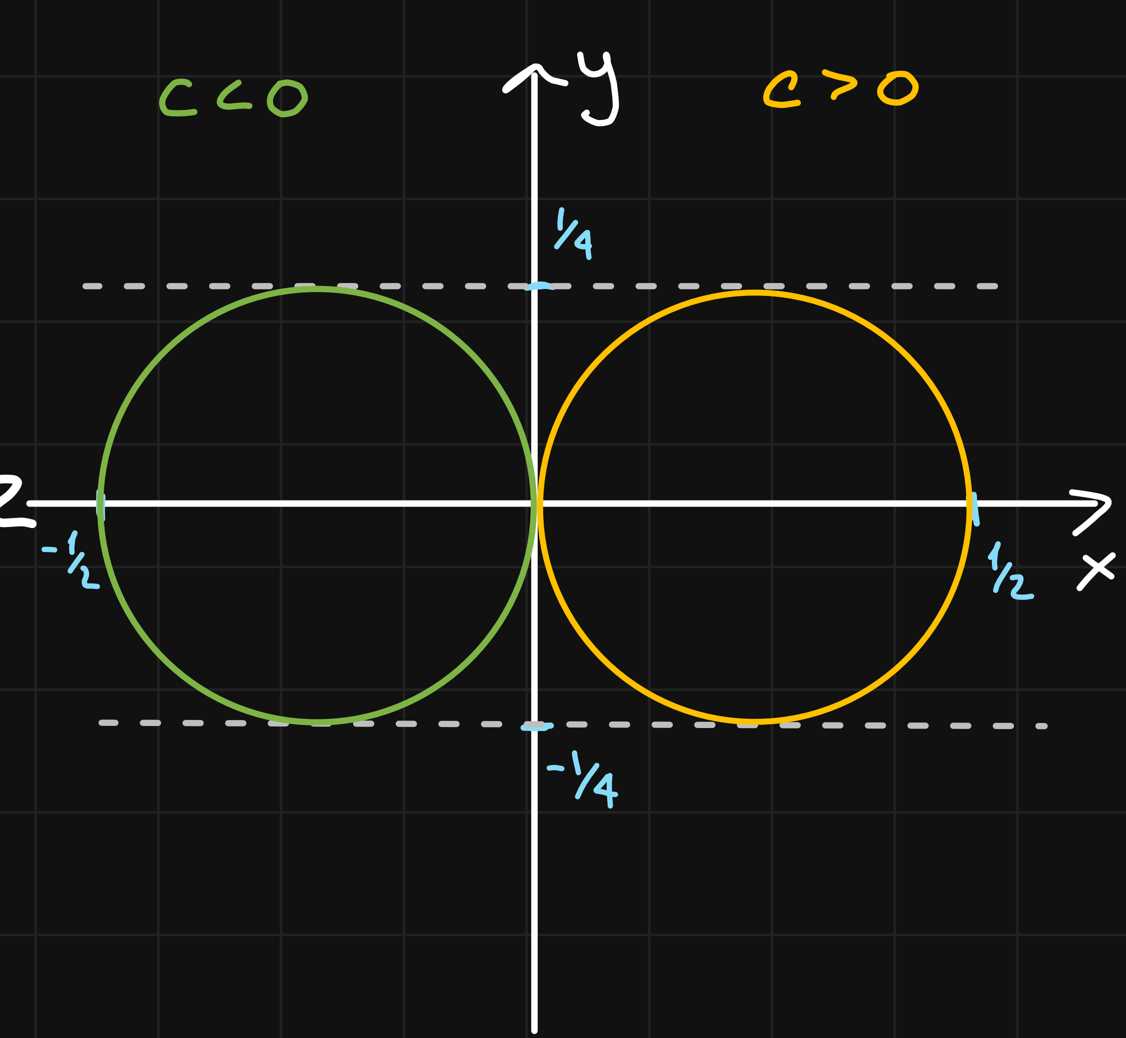
$C = \pm\frac{3}{2}$



$C = \pm 1$



$C = \pm 2$



4. (HW) Describe the level curves of the function. Sketch the level curves for the given C-values:

(a) $z = x^2 + 4y^2$, $C = 0, 1, 2, 3, 4$; (b) $z = \dfrac{x}{x^2+y^2}$, $C = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$;

(c) $z = 3^{xy/2}$, $C = 1, \frac{1}{3}, 3, 9, \frac{1}{9}$; (d) $z = \ln(1-xy)$, $C = 0, \pm 1, \pm 2, \pm 3$.

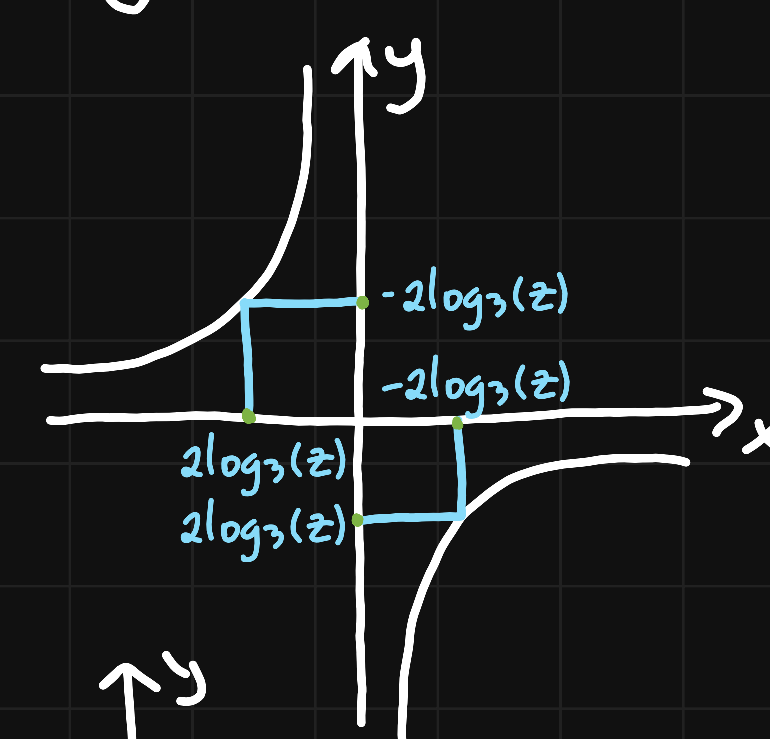
c) $z = 3^{\frac{xy}{2}}$, since no restr. for x and y: $D_z = \mathbb{R}^2$

since $3^k > 0 \ \forall k \in \mathbb{R}$, $R_z = (0, \infty)$, to obtain any $z > 0$

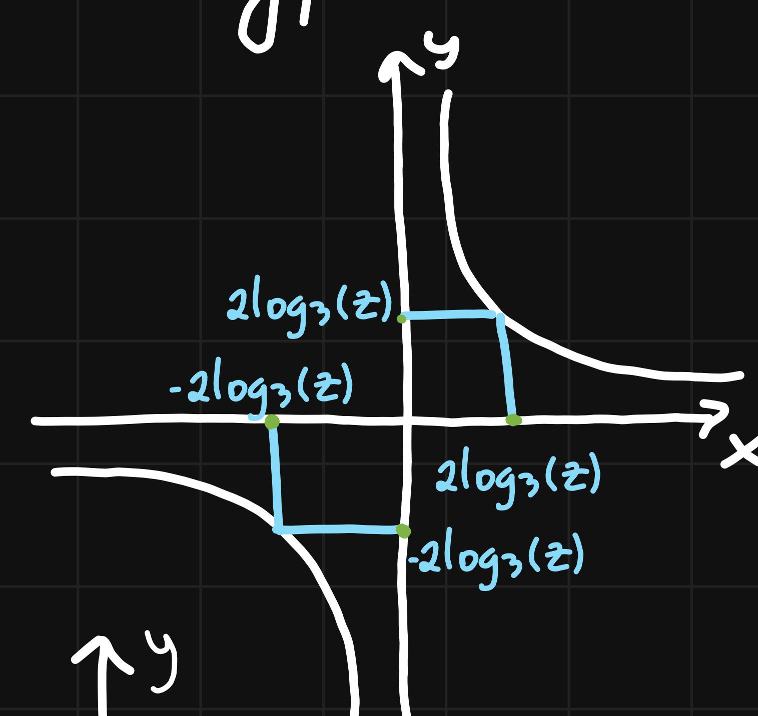
we can fix $y = 2$, then $z = 3^x \Rightarrow x = \log_3(z)$.

$z = 3^{xy/2} \Rightarrow xy = 2\log_3(z)$

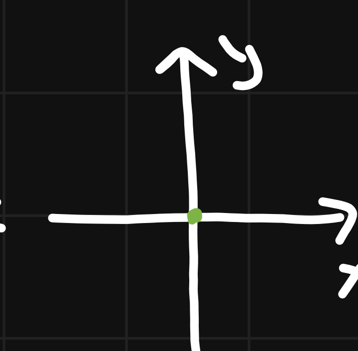
So, we have a hyperbola: for $z < 1$:



for $z > 1$

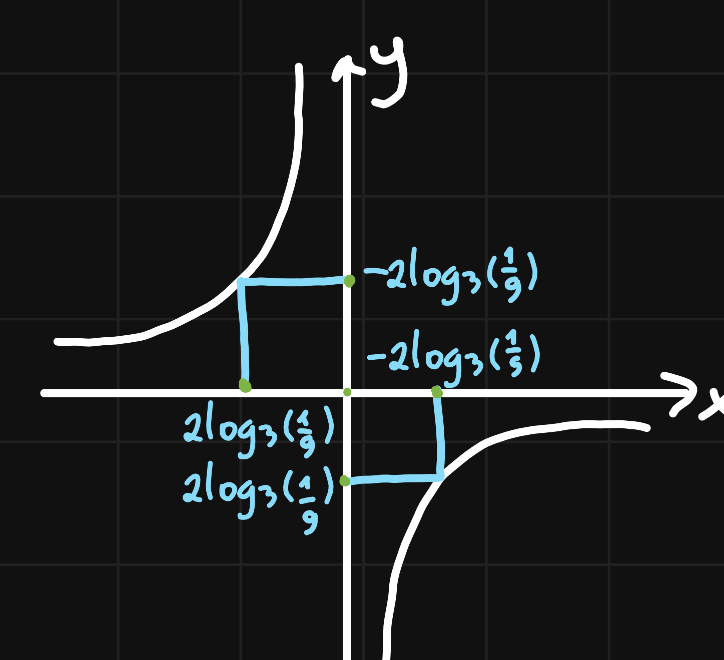


for $z = 0$:

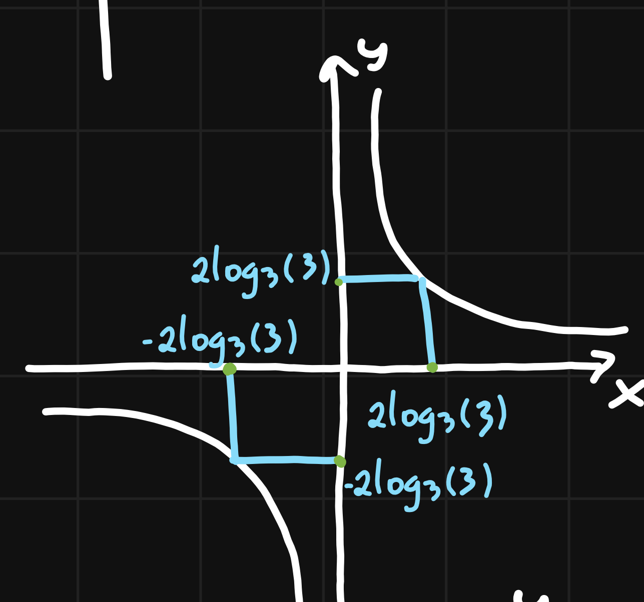


$C = 0$:

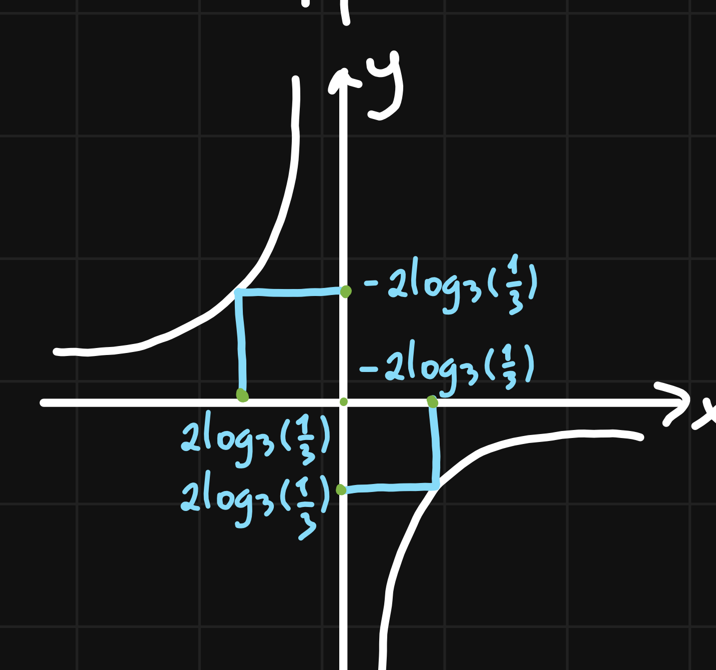
$C = \frac{1}{9}$:



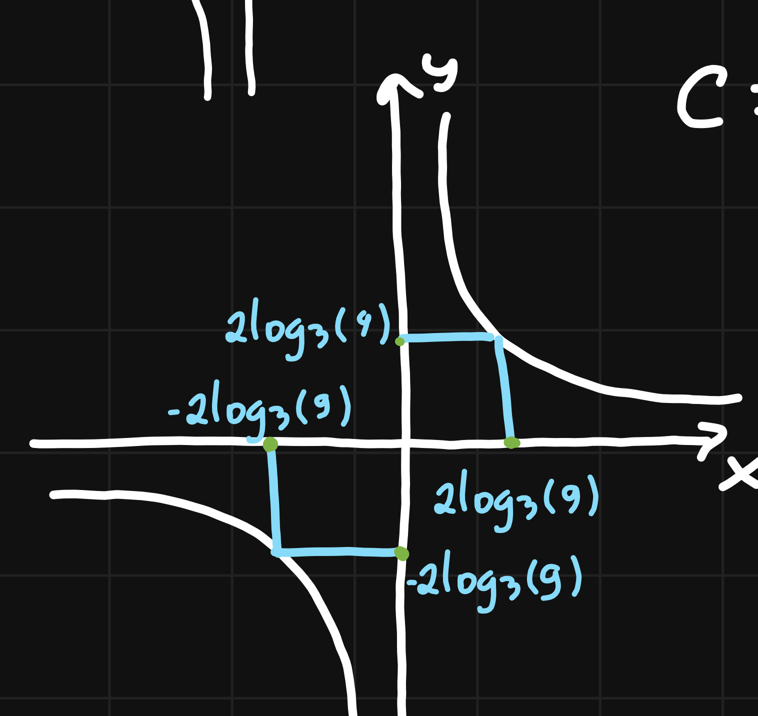
$C = 3$:



$C = \frac{1}{3}$:



$C = 9$:



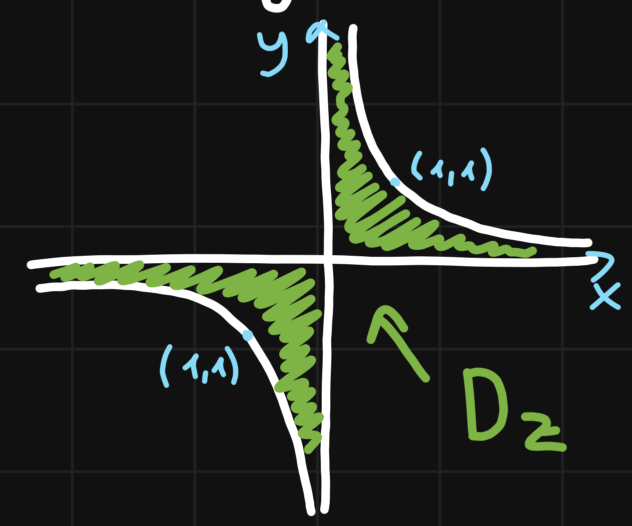
4. (HW) Describe the level curves of the function. Sketch the level curves for the given C-values:

(a) $z = x^2 + 4y^2$, $C = 0, 1, 2, 3, 4$; (b) $z = \dfrac{x}{x^2+y^2}$, $C = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2$;

(c) $z = 3^{xy/2}$, $C = 1, \frac{1}{3}, 3, 9, \frac{1}{9}$; (d) $z = \ln(1-xy)$, $C = 0, \pm 1, \pm 2, \pm 3$.

$z = \ln(1-xy)$, since $\ln(k)$ only for $k>0$:

$1-xy>0 \Rightarrow xy<1$

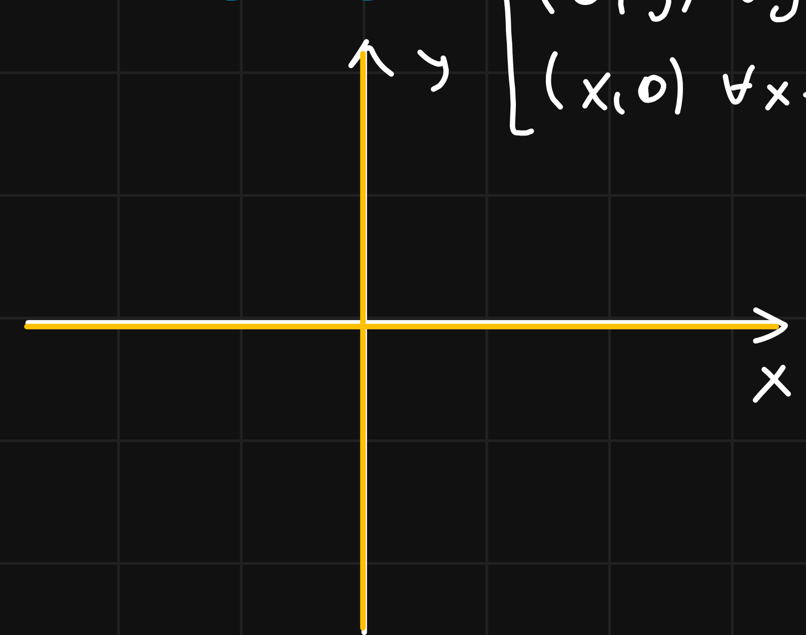


$D_z = \{(x,y) \mid xy<1\}$

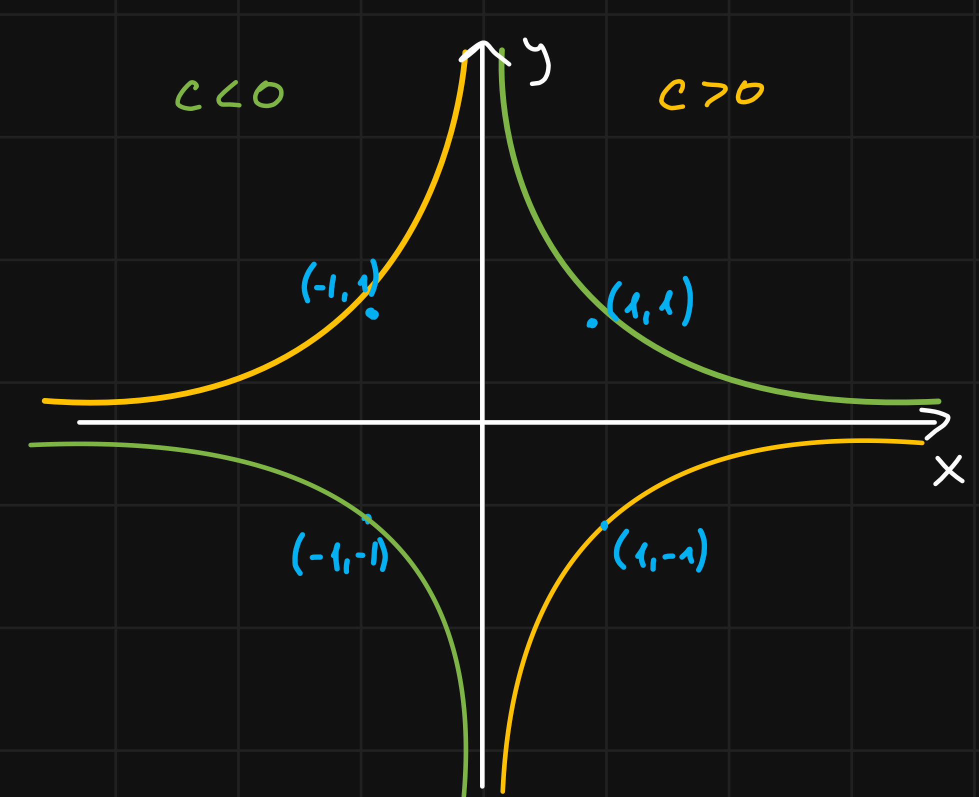
$R_z = \mathbb{R}$, since we can fix $y=1$, then $\forall z$: $x = 1-e^z$

$z = \ln(1-xy) \Leftrightarrow xy = 1-e^z$

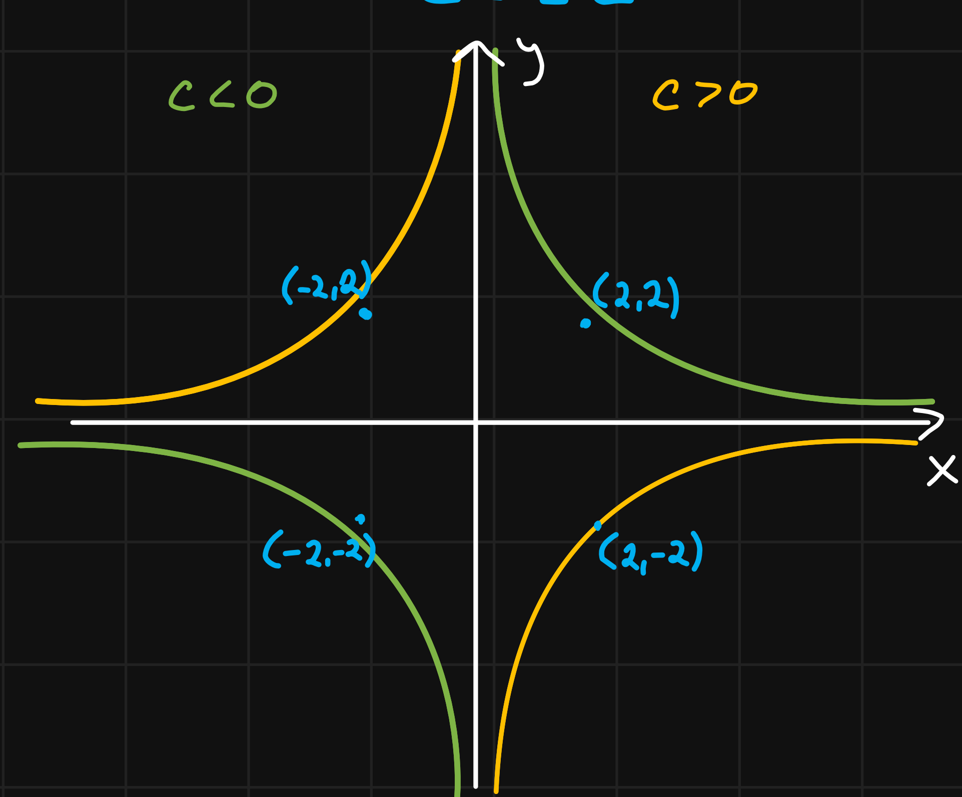
$C = 0$
$$\begin{cases}(0,y) & \forall y \in \mathbb{R} \\ (x,0) & \forall x \in \mathbb{R}\end{cases}$$



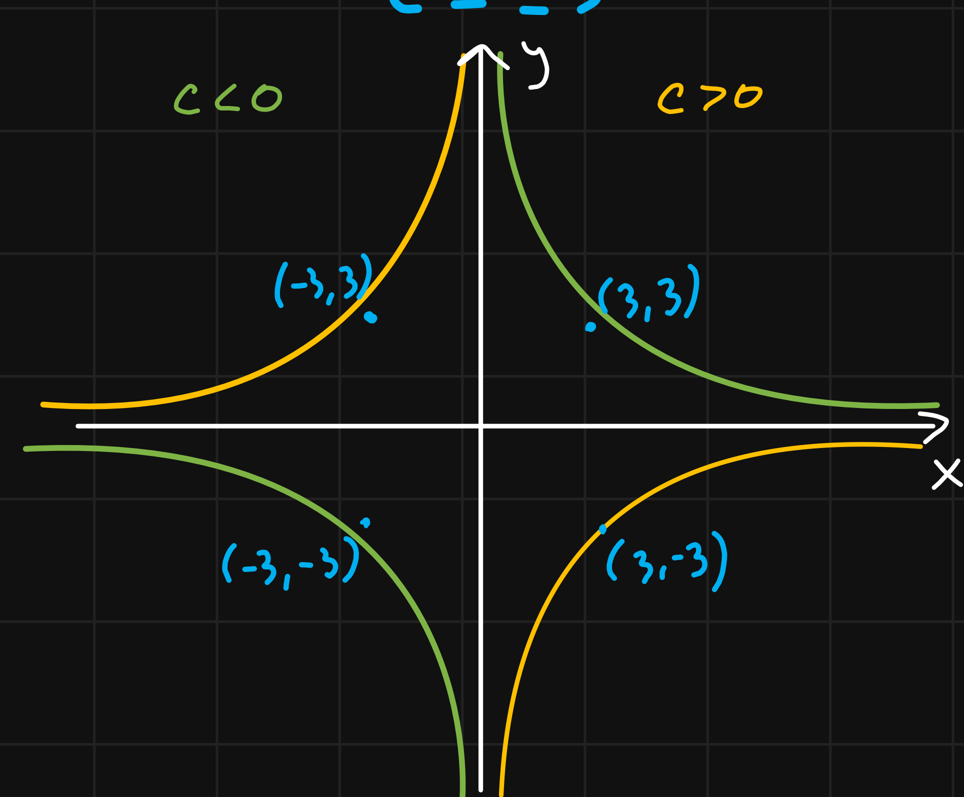
$C = \pm 1$



$C = \pm 2$



$C = \pm 3$



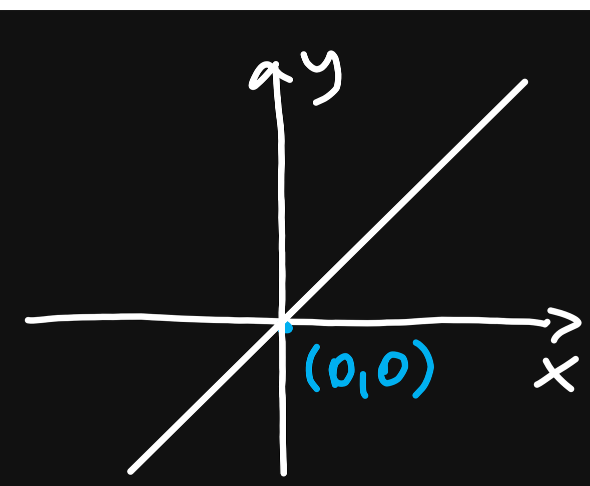
6. (HW) All of the level curves of the surface given by $z = f(x, y)$ are concentric circles. Does this imply that the graph of f is a hemisphere? If it is true, explain why. If it is false, give a counterexample.

No, it isn't; Consider a function $z = x^2 + y^2$;
So all level curves are circles with center at $(0, 0)$, so group of level curves construct a concentric circles, but at the other hand $f(x, y)$ is shaped by expending cone not hemisphere.

7. (HW) Construct a function whose level curves are lines passing through the origin (with the exception of the origin itself).

If I understand task correctly:

$z = x - y$; $R_z = \mathbb{R}$ $D_z = \mathbb{R}^2$, for $c = 0$: $0 = x - y \Rightarrow y = x$



If you wanna for all level-curves:

It's doesn't work this way, since there is no intersection for different level curves!
Proof: suppose $\exists c \in \mathbb{R}$, then $\exists x_0 \exists y_0 \wedge \exists x' \exists y'$ s.t. $(x_0, y_0) \neq (x', y')$, then $f(x_0, y_0) = c \wedge f(x', y') = c$, then $f(x_0, y_0) = f(x', y') \Rightarrow$ ⦶ (since so f is not functional)

8. (HW) Does a vertical line can intersect the graph of $z = f(x, y)$ at most once? If it is true, explain why. If it is false, give a counterexample.

8) No, $z = x^2 + y^2$, vertical line will intersect 2 times, i.e. $x = 0$: $(-\sqrt{z}, 0) \wedge (\sqrt{z}, 0)$
Vert line $|x| \leq \sqrt{z}$ will intersect graph twice, $\forall z \neq 0$. (also $R_z = [0, \infty)$, so $\forall z > 0$)
If you wanna vertical on 3D, then sphere is a counterexample.