2. (HW) Evaluate the trigonometric and hyperbolic integrals:

✓(a) $\int \cos^5(7x+3)\,dx$; ✓(b) $\int \cosh^4 x\,dx$; ✓(c) $\int \sin^4(2x)\cos^2(2x)\,dx$.

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✓ a) $\int \cos^5(7x+3)\,dx = \left| \begin{matrix} t = 7x+3 \\ dt = 7dx \end{matrix} \right| = \frac{1}{7}\int \cos^5(t)\,dt = \frac{1}{7}\int \cos(t)\left(1-\sin^2(t)\right)^2 dt =$

$= \left| \begin{matrix} a = \sin(t) \\ da = \cos(t)dt \end{matrix} \right| = \frac{1}{7}\int (1-a^2)^2 da = \frac{1}{7}\int [1-2a^2+a^4]\,da = \frac{1}{7}a - \frac{2}{21}a^3 + \frac{1}{35}a^5 + C$

$= \frac{1}{7}\sin(t) - \frac{2}{21}\sin^3(t) + \frac{1}{35}\sin^5(t) + C = \frac{1}{7}\sin(7x+3) - \frac{2}{21}\sin^3(7x+3) + \frac{1}{35}\sin^5(7x+3) + C$

✓ b) $\int \cosh^4(x)dx = \frac{1}{4}\int (\cosh(2x)+1)^2 dx = \frac{1}{4}\int [\cosh^2(2x) + 2\cosh(2x) + 1]\,dx$ ⊜

$\frac{1}{4}\int \cosh^2(2x)\,dx = \frac{1}{8}\int [\cosh(4x)+1]dx = \frac{\sinh(4x)}{32} + \frac{x}{8} + C$

$\frac{1}{2}\int \cosh(2x)\,dx = \frac{1}{4}\sinh(2x) + C$

$\frac{1}{4}\int dx = \frac{x}{4} + C$

⊜ $\frac{\sinh(4x)}{32} + \frac{3x}{8} + \frac{\sinh(2x)}{4} + C$

✓ $\int \sin^4(2x)\cos^2(2x)\,dx = \frac{1}{8}\int (1-\cos(4x))^2(1+\cos(4x))\,dx = \frac{1}{8}\int (1-\cos(4x))(1-\cos^2(4x))\,dx$ ⊜

⊜ $\frac{1}{8}\int (1-\cos(4x))\sin^2(4x)dx = \frac{1}{8}\int \sin^2(4x)\,dx - \frac{1}{8}\int \cos(4x)\sin^2(4x)dx = \frac{x}{16} - \frac{\sin(8x)}{128} - \frac{\sin^3(4x)}{96} + C$

$\frac{1}{8}\int \sin^2(4x)\,dx = \frac{1}{16}\int (1-\cos(8x))dx = \frac{x}{16} - \frac{\sin(8x)}{128} + C$

Belive me, that it's a correct solution;
If you don't, I can defend it:)

$-\frac{1}{8}\int \cos(4x)\sin^2(4x)dx = \{u = \sin(4x)\} = -\frac{1}{32}\int u^2 du = -\frac{1}{96}u^3 + C = -\frac{\sin^3(4x)}{96} + C$

4. (HW) Evaluate the trigonometric integrals:

(a) $\displaystyle\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$; (b) $\displaystyle\int \coth^3(2x+3)\,dx$; (c) $\displaystyle\int \cot^4(2-x)\,dx$;

(d) $\displaystyle\int \sin 5x \sin 7x\,dx$; (e) $\displaystyle\int \frac{\sin x}{1+5\cos x}\,dx$; (f) $\displaystyle\int \frac{dx}{\cos^3 x}$.

a) $\displaystyle\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = \int \frac{\cos(x)(1-\sin^2(x))}{\sqrt{\sin(x)}}\,dx = \{t=\sin(x)\} = \int \frac{1-t^2}{\sqrt{t}}\,dt \ominus$

$\ominus \displaystyle\int t^{-1/2}\,dt - \int t^{3/2}\,dt = 2\sqrt{t} - \frac{2}{5}t^{5/2} + c = \frac{2}{5}\sqrt{t}\left(5-t^2\right) + c =$

$= \dfrac{2}{5}\sqrt{\sin(x)}\left(5-\sin^2(x)\right) + c$

b) $\displaystyle\int \coth^3(2x+3)\,dx = \{t=2x+3\} = \frac{1}{2}\int \coth^3(t)\,dt =$

$= \displaystyle\frac{1}{2}\int \coth(t)(1+\operatorname{csch}^2(t))\,dt = \frac{1}{2}\int \coth(t)\,dt + \frac{1}{2}\int \operatorname{csch}^2(t)\coth(t)\,dt \ominus$

$\displaystyle\frac{1}{2}\int \coth(t)\,dt = \frac{1}{2}\int \cosh(t)\operatorname{csch}(t)\,dt = \{m=\sinh(t)\} = \frac{1}{2}\int \frac{dm}{m} = \frac{1}{2}\ln(|\sinh(t)|) + c$

$\displaystyle\frac{1}{2}\int \operatorname{csch}^2(t)\coth(t)\,dt = \{m=\coth(t)\} = -\frac{1}{2}\int m\,dm = -\frac{1}{4}m^2 + c = -\frac{\coth^2(t)}{4} + c$

$\ominus \displaystyle\frac{1}{2}\ln(|\sinh(t)|) - \frac{\coth^2(t)}{4} + c = \frac{1}{2}\ln(|\sinh(2x+3)|) - \frac{\coth^2(2x+3)}{4} + c$

c) $\displaystyle\int \cot^4(2-x)\,dx = \{t=2-x\} = -\int \cot^4(t)\,dt = -\int \cot^2(t)(\csc^2(t)-1)\,dt \ominus$

$\ominus \displaystyle -\int \cot^2(t)\,dt + \int \cot^2(t)\csc^2(t)\,dt = \frac{\cot^3(t)}{3} + \cot(t) + t + c \ominus$

$\displaystyle\int \cot^2(t)\,dt = \int 1-\csc^2(t)\,dt = t + \cot(t) + c$

$\displaystyle -\int \cot^2(t)\csc^2(t)\,dt = \{m=\cot^2(t)\} = \int m^2\,dm = \frac{1}{3}m^3 + c = \frac{\cot^3(t)}{3} + c$

$\ominus \dfrac{\cot^3(2-x)}{3} + \cot(2-x) + 2 - x + c$

4. (HW) Evaluate the trigonometric integrals:

(a) $\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$; (b) $\int \coth^3(2x+3)\,dx$; (c) $\int \cot^4(2-x)\,dx$;

(d) $\int \sin 5x \sin 7x\,dx$; (e) $\int \frac{\sin x}{1+5\cos x}\,dx$; (f) $\int \frac{dx}{\cos^3 x}$.

using $\sin(a)\sin(\beta) = \frac{1}{2}\cos(a-\beta) - \cos(a+\beta))$

d) $\int \sin(5x)\sin(7x)\,dx = \frac{1}{2}\int[\cos(-2x) - \cos(12x)]dx = \frac{1}{2}\int[\cos(2x) - \cos(12x)]dx =$

$= \frac{1}{4}\sin(2x) - \frac{1}{24}\sin(12x) + c$ It's well-know formula, but If you want invate me to defense, and I'll show you how to obtain this :)

e) $\int \frac{\sin(x)}{1+5\cos(x)}\,dx = \{t = 1+5\cos(x)\} = -\frac{1}{5}\int \frac{1}{t}\,dt = -\frac{1}{5}\ln(|t|) + c = -\frac{1}{5}\ln(|5\cos(x)+1|) + c$

f) $\int \sec^3(x)\,dx = \int \sec(x)(1+\tan^2(x))\,dx = \int \sec(x)\,dx + \int \sec(x)\tan^2(x)\,dx$

$\int \sec(x)\,dx = \int \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x)+\tan(x)}\,dx = \{\phi = \sec(x)+\tan(x)\} = \int \frac{1}{\phi}\,d\phi = \ln(|\sec(x)+\tan(x)|) + c$

$\int \sec(x)\tan^2(x)\,dx =$ Hell, I can not avoid Integration by parts :((

callenge faild.

f) $\int \sec^3(x)\,dx = \left\{\begin{matrix} u = \sec(x) & dw = \sec^2(x)\,dx \\ du = -\sec(x)\tan(x)\,dx & w = \tan(x) \end{matrix}\right\} = \sec(x)\tan(x) - \int \sec(x)\tan^2(x)\,dx =$

$= \sec(x)\tan(x) - \int[\sec^3(x) - \sec(x)]\,dx = \sec(x)\tan(x) + \int \sec(x)\,dx - \int \sec^3(x)\,dx$

$\Rightarrow \int \sec^3(x)\,dx = \frac{1}{2}\left(\sec(x)\tan(x) + \int \sec(x)\,dx\right) = \frac{1}{2}\left(\sec(x)\tan(x) + \ln(|\tan(x)+\sec(x)|)\right) + c$

note:

$\int \sec(x)\,dx = \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx = \{m = \sec(x)+\tan(x)\} = \int \frac{1}{m}\,dm = \ln(|\tan(x)+\sec(x)|) + c$

6. (HW) Evaluate the trigonometric integrals:

(a) $\int \frac{dx}{2\sin x + \cos x + 3}$; (b) $\int \frac{dx}{3\sin x - 4\cos x}$; (c) $\int \frac{2\sinh x + 3\cosh x}{4\sinh x + 5\cosh x}\,dx$.

a) $$\int \frac{dx}{2\sin(x)+\cos(x)+3} = \left| \begin{array}{l} t=\tan(x/2) \\ \cos(x) = \frac{1-t^2}{1+t^2} \\ \sin(x) = \frac{2t}{1+t^2} \\ dx = \frac{2}{1+t^2}\,dt \end{array} \right| = \int 2\left((1+t^2)\left(\frac{4t+1-t^2+3t^2+3}{1+t^2}\right)\right)^{-1} dt \ominus$$

$$\ominus \int \frac{2\,dt}{2t^2+4t+4} = \int \frac{dt}{(t+1)^2+1} = \{k=t+1\} = \int \frac{dk}{k^2+1} = \arctan(k)+c = \arctan(t+1)+c \ominus$$

$$\ominus \arctan\left(\tan(x/2)+1\right)+c$$

b) $$\int \frac{dx}{3\sin(x)-4\cos(x)} = \left| \begin{array}{l} t=\tan(x/2) \\ \sin(x) = \frac{2t}{1+t^2} \\ \cos(x) = \frac{1-t^2}{1+t^2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right| = \int 2\left((1+t^2)\left(\frac{6t-4+4t^2}{1+t^2}\right)\right)^{-1} dt = \int \frac{dt}{2t^2+3t-2} =$$

using "mind tecnique"

$$= \int \frac{dt}{(t+2)(2t-1)} = -\frac{1}{5}\int \frac{dt}{t+2} + \frac{2}{5}\int \frac{dt}{2t-1} = -\frac{1}{5}\ln(|t+2|) + \frac{1}{5}\ln(|2t-1|)+c =$$

$$= -\frac{1}{5}\ln(|\tan(x/2)+2|) + \frac{1}{5}\ln(|2\tan(x/2)-1|)+c$$

Draft

$$\frac{a}{t+2} + \frac{b}{2t-1} = \frac{1}{(t+2)(2t-1)} \iff 2at - a + bt + 2b = 1$$

$$\Updownarrow$$

$$\begin{cases} 2a+b = 0 \\ 2b-a = 1 \end{cases} \iff \begin{cases} a = -1/5 \\ b = 2/5 \end{cases}$$

c) $$\int \frac{2\sinh(x)+3\cosh(x)}{4\sinh(x)+5\cosh(x)}\,dx = \int \frac{-2\sinh(x)\operatorname{sech}^3(x) - 3\cosh(x)\operatorname{sech}^3(x)}{-4\sinh(x)\operatorname{sech}^3(x) - 5\cosh(x)\operatorname{sech}^3(x)}\,dx = \int \frac{-3\operatorname{sech}^2(x) - 2\tanh(x)\operatorname{sech}^2(x)}{-5\operatorname{sech}^2(x) - 4\tanh(x)\operatorname{sech}^2(x)}\,dx \ominus$$

$$\ominus \int \frac{\operatorname{sech}^2(x)(2\tanh(x)+3)}{\operatorname{sech}^2(x)(4\tanh(x)+5)}\,dx = \int \frac{\operatorname{sech}^2(x)(2\tanh(x)+3)}{(1-\tanh^2(x))(4\tanh(x)+5)}\,dx = \{t=\tanh(x)\} = \int \frac{2t+3}{(1-t^2)(4t+5)}\,dt =$$

$$\int \frac{2t+3}{(1-t)(1+t)(4t+5)} = \frac{5}{18}\int \frac{dt}{1-t} + \frac{1}{2}\int \frac{dt}{1+t} - \frac{8}{9}\int \frac{dt}{4t+5} = -\frac{5}{18}\ln(|1-t|) + \frac{1}{2}\ln(|1+t|) - \frac{2}{9}\ln(|4t+5|)+c \ominus$$

$$\ominus -\frac{5}{18}\ln(|-\tanh(x)+1|) + \frac{1}{2}\ln(|1+\tanh(x)|) - \frac{2}{9}\ln(|4\tanh(x)+5|)+c$$

Draft:

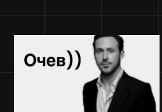
$$\frac{a}{1+t} + \frac{b}{1-t} + \frac{c}{4t+5} = a(4t+5)(1-t) + b(1+t)(4t+5) + c(1-t^2) = a(-4t^2-t+5) + b(4t^2+9t+5) + c(1-t^2) \Rightarrow \begin{cases} -4a+4b-c = 0 \\ -a+9b = 2 \\ 5a+5b+c = 3 \end{cases} \iff \begin{cases} a = 1/2 \\ b = 5/18 \\ c = -\frac{8}{9} \end{cases}$$

(having one:)

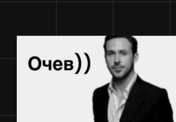
Another Approach:

c) $$\int \frac{2\sinh(x)+3\cosh(x)}{4\sinh(x)+5\cosh(x)}\,dx \left| \begin{array}{l} t=\tanh(x/2) \\ \sinh(x) = \frac{2t}{1-t^2} \\ \cosh(x) = \frac{1+t^2}{1-t^2} \\ dx = \frac{2dt}{1-t^2} \end{array} \right| = 2\int \frac{\frac{4t+3+3t^2}{1-t^2}}{\frac{4t+5+5t^2}{1-t^2}}\,dt = 2\int \frac{3t^2+4t+3}{5t^2+4t+5}\,dt = \frac{6}{5}\int dt + \frac{16}{5}\int \frac{t}{5t^2+4t+5}\,dt \ominus$$

$$\frac{16}{5}\int \frac{t}{5t^2+4t+5}\,dt = \frac{16}{50}\int \frac{10t+4}{5t^2+4t+5}\,dt - \frac{64}{50}\int \frac{dt}{5t^2+4t+5} = \frac{8}{25}\ln(5t^2+4t+5) - \frac{32}{25\sqrt{21}}\arctan\left(\frac{5t+2}{\sqrt{21}}\right)+c$$

$$\frac{16}{50}\int \frac{10t+4}{5t^2+4t+5}\,dt = \{m = 5t^2+4t+5\} =$$ 

$$= \frac{8}{25}\ln(5t^2+4t+5)+c$$

$$-\frac{64}{50}\int \frac{dt}{5t^2+4t+5} = -\frac{64}{250}\int \frac{dt}{(t+\frac{2}{5})^2+\frac{21}{25}}\,dt = \{m = t+\tfrac{2}{5}\} =$$ 

$$= -\frac{32}{125}\int \frac{dm}{m^2+\frac{21}{25}} = -\frac{32}{125}\cdot\frac{1}{\sqrt{\frac{21}{25}}}\arctan\left(\frac{m}{\sqrt{\frac{21}{25}}}\right)+C = -\frac{32}{25\sqrt{21}}\arctan\left(\frac{t+2/5}{\sqrt{\frac{21}{25}}}\right)$$

$$\ominus \frac{6}{5}\tanh(x/2) + \frac{8}{25}\ln\left(5\tanh^2(x/2)+4\tanh(x/2)+5\right) - \frac{32}{25\sqrt{21}}\arctan\left(\frac{5\tanh(x/2)+2}{\sqrt{21}}\right)+C$$

7. (HW) Solve miscellaneous problems:

(a) $\int \frac{\cos(\log_8 5x + 8)}{x}\, dx$; (b) $\int (e^{-2x} + 5e^{-x}) \cos(e^{-x} + 8)\, dx$.

a) $\int \frac{\cos(\log_8(5x)+8)}{x} dx = \left| \begin{array}{l} t = \log_8(5x)+8 \\ dt = \frac{dx}{\ln(8)x} ;\ dx = \ln(8) x\, dt \end{array} \right| = \ln(8) \int \cos(t)\, dt = \ln(8) \sin(\log_8(5x)+8) + C$

b) $\int (e^{-2x} + 5e^{-x}) \cos(e^{-x}+8) dx = \int e^{-x}(e^{-x}+5)\cos(e^{-x}+8) dx = \{ t = e^{-x}+8 \} = -\int (t-3)\cos(t)\, dt \ominus$

$\ominus -t\sin(t) - \cos(t) + 3\sin(t) + C = -(e^{-x}+8)\sin(e^{-x}+8) - \cos(e^{-x}+8) + 3\sin(e^{-x}+8) + C \ominus$

$\int t\cos(t)\, dt = \left| \begin{array}{ll} u = t & dv = \cos(t)dt \\ du = dt & v = \sin(t) \end{array} \right| = t\sin(t) - \int \sin(t)\, dt = t\sin(t) + \cos(t) + C$

$\ominus -\sin(e^{-x}+8)(e^{-x}+5) - \cos(e^{-x}+8)$

$f(x) = -\sin(e^{-x}+8)(e^{-x}+5) - \cos(e^{-x}+8)$

8*. (HW) For the integrals $J_{n,m} = \int \sin^n x \cos^m x \, dx$, $n,\ m \in \mathbb{N}$, prove the following recursive formulas:

✓ 1) $J_{n,m} = -\frac{\sin^{n-1} x \cos^{m+1} x}{n+m} + \frac{n-1}{n+m} J_{n-2,m}$, $\qquad J_{n,m} = \frac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \frac{m-1}{n+m} J_{n,m-2}$.

Use these formulas to find the integral $\int \sin^6 x \cos^4 x \, dx$.

1) Using MIP

✓ basis: $m, n = 1$;

$$\int \sin(x)\cos(x) = -\frac{\sin^0(x)\cos^2(x)}{2} + \frac{0}{2}\int \frac{\cos(x)}{\sin(x)}dx = -\frac{\cos^2(x)}{2} + c$$

$$f(x) = -\frac{\cos^2(x)}{2} \Rightarrow f'(x) = -\frac{1}{2}2\cos(x)(-\sin(x)) = \cos(x)\sin(x)$$

✓ $n = 2 \wedge m = 1$, then

$$J_{n,m} = J_{2,1} = \int \sin^2(x)\cos(x)dx$$

due to our formula:

$$\int \sin^2(x)\cos(x)dx = -\frac{\sin(x)\cos^2(x)}{3} + \frac{1}{3}\overset{\frac{1}{3}\sin(x)+c}{\overset{\|}{\int \sin^0(x)\cos(x)}} = \frac{1}{3}\left(\sin(x) - \sin(x)\cos^2(x)\right) + c$$

but also:

$$\frac{d}{dx}\left(\frac{1}{3}\left(\sin(x) - \sin(x)\cos^2(x)\right) + c\right) = \sin^2(x)\cos(x)$$

✓ Assume for $n-1, m-1$; $n-2, m-1$;

Consider φ of n, m:

$$\int \sin^n(x)\cos^m(x)\,dx = -\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2}(x)\cos^m(x)\,dx$$

$$\frac{d}{dx}\left(-\frac{\sin^{n-1}(x)\cos^{m+1}(x)}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2}(x)\cos^m(x)\,dx\right) =$$

$$\frac{\sin^n(x)\cos^m(x)(m+1) + (-n+1)\sin^{n-2}(x)\cos^{m+2}(x)}{n+m} + \frac{(n-1)\sin^{n-2}(x)\cos^m(x)}{n+m} =$$

$$\frac{\cos^m(x)\sin^{n-2}(x)}{n+m}\left(m\sin^2(x) - n\cos^2(x) + n + \sin^2(x) + \cos^2(x) - 1\right)$$

$$\frac{\cos^m(x)\sin^{n-2}(x)}{n+m}\left(m\sin^2(x) + n\left(1 - \cos^2(x)\right) + \sin^2(x) + \cos^2(x) - 1\right)$$

$$\frac{\cos^m(x)\sin^{n-2}(x)}{n+m}\left(m\sin^2(x) + n\sin^2(x) + 1 - 1\right)$$

$$\frac{\cos^m(x)\sin^{n-2}(x)}{n+m}\left(\sin^2(x)(m+n)\right) = \cos^m(x)\sin^n(x) \qquad \blacksquare$$

8*. (HW) For the integrals $J_{n,m} = \int \sin^n x \cos^m x \, dx$, $n, m \in \mathbb{N}$, prove the following recursive formulas:

1) $J_{n,m} = -\frac{\sin^{n-1} x \cos^{m+1} x}{n+m} + \frac{n-1}{n+m} J_{n-2,m}$, 2) $J_{n,m} = \frac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \frac{m-1}{n+m} J_{n,m-2}$.

Use these formulas to find the integral $\int \sin^6 x \cos^4 x \, dx$.

2) Using MIP

base: $m = 1 ; n = 1$:

$$J_{1,1} = \int \sin(x)\cos(x)\,dx = \frac{\sin^2(x)\cos^0(x)}{2} + \frac{0}{2}\int \tan(x)\,dx = \frac{\sin^2(x)}{2} + c$$

$$\frac{d}{dx} = \left(\frac{\sin^2(x)}{2}\right) = \frac{1}{2}\cdot 2\sin(x)\cos(x) = \sin(x)\cos(x) = J_{1,1}$$

$m = 2 ; n = 1$:

$$J_{1,2} = \int \sin(x)\cos^2(x) = \frac{\sin^2(x)\cos(x)}{3} + \frac{1}{3}\int \sin(x)\,dx = \frac{\sin^2(x)\cos(x)}{3} - \frac{1}{3}\cos(x) + c$$

$$\frac{d}{dx}\left(\frac{\cos(x)}{3}\left(\sin^2(x) - 1\right)\right) = \frac{d}{dx}\left(-\cos^3(x)\right) = (-1)\left(-\cos^2(x)\right)\sin(x) = \sin(x)\cos^2(x)$$

Assume fou $(m-1 ; n-1), (m-2 ; n-1)$

$$J_{n,m} = \int \sin^n(x)\cos^m(x)\,dx = \frac{\sin^{n+1}(x)\cos^{m-1}(x)}{n+m} + \frac{m-1}{n+m}\int \sin^n(x)\cos^{m-2}(x)\,dx$$

$$\frac{d}{dx}\left(\frac{\sin^{n+1}(x)\cos^{m-1}(x)}{n+m} + \frac{m-1}{n+m}\int \sin^n(x)\cos^{m-2}(x)\,dx\right) =$$

$$= \frac{(n+1)\sin^n(x)\cos^m(x) - \sin^{n+2}(x)\cos^{m-2}(x)(m-1) + (m-1)\sin^n(x)\cos^{m-2}(x)}{n+m} =$$

$$= \frac{(n+1)\sin^n(x)\cos^m(x) + (m-1)\left(\sin^n(x)\cos^{m-2}(x) - \sin^{n+2}(x)\cos^{m-2}(x)\right)}{n+m} =$$

$$= \frac{\sin^n(x)\cos^{m-2}(x)}{n+m}\left((n+1)\cos^2(x) + (m-1)\left(1 - \sin^2(x)\right)\right) =$$

$$= \frac{\sin^n(x)\cos^{m-2}(x)}{n+m}\left((n+1)\cos^2(x) + (m-1)\cos^2(x)\right) =$$

$$= \frac{\sin^n(x)\cos^{m-2}(x)}{n+m}\left(\cos^2(x)(n+1+m-1)\right) = \cos^m(x)\sin^n(x)$$

8*. (HW) For the integrals $J_{n,m} = \int \sin^n x \cos^m x \, dx, \quad n,\, m \in \mathbb{N}$, prove the following recursive formulas:

1) $J_{n,m} = -\dfrac{\sin^{n-1} x \cos^{m+1} x}{n+m} + \dfrac{n-1}{n+m} J_{n-2,m}$, 2) $J_{n,m} = \dfrac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \dfrac{m-1}{n+m} J_{n,m-2}$.

Use these formulas to find the integral $\int \sin^6 x \cos^4 x \, dx$.

$$\int \sin^6(x)\cos^4(x)\,dx = -\frac{\sin^5(x)\cos^5(x)}{10} + \frac{5}{10}\int \sin^4(x)\cos^4(x)\,dx =$$

$$-\frac{\sin^5(x)\cos^5(x)}{10} - \frac{5\sin^3(x)\cos^5(x)}{80} + \frac{15}{80}\int \sin^2(x)\cos^4(x)\,dx =$$

$$-\frac{\sin^5(x)\cos^5(x)}{10} - \frac{5\sin^3(x)\cos^5(x)}{80} + \frac{15\sin^3(x)\cos^3(x)}{480} + \frac{45}{480}\int \sin^2(x)\cos^2(x)\,dx =$$

$$-\frac{\sin^5(x)\cos^5(x)}{10} - \frac{5\sin^3(x)\cos^5(x)}{80} + \frac{15\sin^3(x)\cos^3(x)}{480} + \frac{45\sin^3\cos(x)}{1920} + \frac{45}{1920}\int \sin^2(x)\,dx$$

$$-\frac{\sin^5(x)\cos^5(x)}{10} - \frac{5\sin^3(x)\cos^5(x)}{80} + \frac{15\sin^3(x)\cos^3(x)}{480} + \frac{45\sin^3\cos(x)}{1920} + \frac{45}{3840}\big(x - \sin(x)\cos(x)\big) + C$$

Thx for checking, I hope you enjoy;

If somethig isn't clear feel free to ask me for defence!

Have a nice day

by Novosad Ivan
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