1. Give an example of sets A, B, C such that:

a) $A \in B$, $B \in C$, and $A \in C$;

b) $A \in B$, $B \in C$ but $A \notin C$;

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ⓐ $A = \{\emptyset\};\ B = \{\{\emptyset\}\},\ C = \{\{\{\emptyset\}\}, \{\emptyset\}\}$

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2. Give a set-builder specification for the set of all natural numbers which are either even or whose every natural divisor's sine is less than 9/10.

$\{x \in \mathbb{N} \mid \exists k \in \mathbb{N}\ 2k = x \ \vee \ \forall m \in \mathbb{N}\ \exists q \in \mathbb{N}\ mq = x \cdot \sin(m) < 9/10\}$

4. Is it always the case that $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$?

(1) Let $C \in P(A \cap B) \Leftrightarrow C \subseteq A \cap B \Leftrightarrow C \subseteq A \wedge C \subseteq B \Leftrightarrow C \in P(A) \wedge C \in P(B)$

(2) Let $C \in P(A) \cap P(B) \Leftrightarrow C \in P(A) \wedge C \in P(B) \Leftrightarrow C \subseteq A \wedge C \subseteq B \Leftrightarrow C \subseteq A \cap B \Leftrightarrow C \in P(A \cap B)$

Hence from $(1) \wedge (2) \Rightarrow P(X \cap Y) = P(X) \cap P(Y)$ ▲

5. Prove that for every sets A, B, C the following hold:

a) $(A \setminus B) \cup B = A \iff B \subseteq A$;

b) $A \subseteq B \cap C \iff A \subseteq B$ and $A \subseteq C$;

c) $A \subseteq B \cup C \iff A \cap \bar{B} \subseteq C$.

a) $(A \setminus B) \cup B = A \iff B \subseteq A$

Consider $A \setminus B$ as arbitrary set D

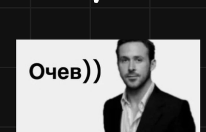
$\Rightarrow$ 1) then if $D \cup B = A \Rightarrow B \subseteq A\ (\forall D)$

$\{x \mid x \in B \vee x \in D\}$ $\quad$ $\forall x \in B\ x \in A$

$\Leftarrow$ 2) $B \subseteq A$ then $\forall x \in A \wedge x \notin B\ \ x \in A \setminus B$ and also

$\forall x \in A$ either $x \in A \wedge x \notin B$ or $x \in A \wedge x \in B$, hence

$A = A \setminus B \cup B \Leftrightarrow A = (A \setminus B) \cup B$

b) 

$A \subseteq B \cap C \Leftrightarrow \forall a \in A\ a \in (B \cap C) \Leftrightarrow \forall a \in A \cdot a \in B \wedge a \in C \Leftrightarrow$

$\Leftrightarrow \forall a \in A \cdot a \in B \wedge \forall a \in A \cdot a \in C \Leftrightarrow A \subseteq B \wedge A \subseteq C$

or, more formally.

$\Rightarrow$ 1) if $A \subseteq B \cap C \Rightarrow \forall a \in A \rightarrow a \in B \cap C,\ (a \in B \cap C \Leftrightarrow a \in B \wedge a \in C) \Leftrightarrow$

$\Leftrightarrow \forall a \in A \rightarrow x \in B \wedge x \in C \Rightarrow A \subseteq B \wedge A \subseteq C$

$\Leftarrow$ 2) if $A \subseteq B \wedge A \subseteq C$, then $\forall a \in A \rightarrow a \in B \wedge a \in C \Rightarrow$

(since $a \in B \wedge a \in C \Leftrightarrow x \in B \cap C$) $\Rightarrow \forall a \in A \rightarrow a \in B \cap C$ ■

c) $A \subseteq B \cup C \Leftrightarrow A \cap \bar{B} \subseteq C$

$\Rightarrow$ 1) $A \subseteq B \cup C \Leftrightarrow \forall a \in A \rightarrow a \in B \vee a \in C$ ⊛

$A \cap \bar{B} \Leftrightarrow \forall a \in A \rightarrow a \notin B$, but if $a \in A \wedge a \notin B$,

then due to ⊛ $x \in C \Rightarrow \forall x\,(x \in A \cap \bar{B} \rightarrow x \in C) \Rightarrow A \subseteq B \cup C \Rightarrow A \cap \bar{B} \subseteq C$

$\Leftarrow$ 2) $A \cap \bar{B} \subseteq C \Leftrightarrow \forall x((x \in A \wedge x \in \bar{B}) \rightarrow x \in C) \Rightarrow$ if $x \in A$

$\wedge\ x \in \bar{B}$, then $x \in C \Rightarrow x \in A \rightarrow x \in C \cup B \Rightarrow A \subseteq C \cup B \Rightarrow A \cup \bar{B} \subseteq C \Rightarrow A \subseteq C \cup B$

6. Give an example of sets A, B, C such that $A \times (B \times C) \neq (A \times B) \times C$. (You should *prove* your sets to be distinct.)

Suppose $A = \{1\}$ $B = \{2\}$ $C = \{3\}$

$A \times (B \times C) \neq (A \times B) \times C$

$\{1\} \times \{(2,3)\} \neq \{(1,2)\} \times \{3\}$

$\{(1,(2,3))\} \neq \{((1,2),3)\}$

Suppose they are equal, then:

$(1,(2,3)) = ((1,2),3) \Rightarrow 1 = (1,2) \wedge (2,3) = 3$

$\Updownarrow$

$[1 = \{\{1\},\{1,2\}\}]$ - False, hence ①

7. Suppose $A \subseteq C$ and $B \subseteq D$. Prove that $A \times B = (A \times D) \cap (C \times B)$.

given: $A \subseteq C \wedge B \subseteq D$

prove: $A \times B = (A \times D) \cap (C \times B)$

not formal → "$A \times B$ contain pairs of elements from each set, that is:"

if $(x,y) \in (A \times D) \cap (C \times B) \Leftrightarrow (x,y) \in (A \times D) \wedge (x,y) \in (C,B)$

we can change (x,y) to whatever "object"

$\Leftrightarrow (x \in A \wedge y \in D) \wedge (x \in C \wedge y \in B) \Leftrightarrow (x \in A \wedge x \in C) \wedge (y \in D \wedge y \in B)$

$\Leftrightarrow x \in A \cap B \wedge y \in D \cap B \Leftrightarrow x \in A \wedge y \in B$ (since $A \subseteq C \wedge B \subseteq D$)

$\Leftrightarrow (x,y) \in A \times B$ ■

since all statements are follows from eachother, it's to prove right hand from left in fact it's already prooved.

3*. Suppose that there exists a set S such that for each x, $x \in S$ iff $x = \{y\}$ for some y ("the set of all singletons"). Get a contradiction.

Assume that exist such S, set of all singletones, then it's power set ($P(S)$) would to contain all sets, including non-singletones. This leads to a contradiction with Russell's paradox.