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2. (HW) Find the absolute maximum and minimum values of $f(x,y) = xy(9-x-y)$ on the closed triangular region R with vertices $(0,0)$, $(12,0)$, and $(0,12)$.

$$f_x = 9y - 2yx - y^2 = 0, \quad f_y = 9x - x^2 - 2yx = 0 \Rightarrow \begin{cases} p_1\ (0,0) \\ p_2\ (9,0) \\ p_3\ (0,9) \\ p_4\ (3,3) \end{cases} \in \text{Region}$$

$$f_{xx} = -2y \qquad f_{yx} = 9 - 2x$$
$$f_{yy} = -2x$$

$$p_1 : \begin{pmatrix} 0 & 9 \\ 9 & 0 \end{pmatrix}$$

$$\boxed{p_4 : \begin{pmatrix} -6 & 3 \\ 3 & -6 \end{pmatrix} : \begin{matrix} q_1 < 0 \\ q_2 > 0 \end{matrix} \Big\} \Rightarrow \text{abs. rel. maxima.} \quad f(p_4) = 27}$$

$$p_2 : \begin{pmatrix} 0 & x \\ x & x \end{pmatrix}$$

$$p_3 : \begin{pmatrix} -18 & 9 \\ 9 & 0 \end{pmatrix} \quad \begin{matrix} q_1 < 0 \\ q_2 < 0 \end{matrix} \Big\} \Rightarrow \text{saddle.}$$

To check if 27 is the absolute maxima in region, we need to check boundaries

$\Rightarrow$ along $x=0 \Rightarrow f(0,y) = 0$

along $y=0 \Rightarrow f(x,0) = 0$

along $\frac{x}{12} - \frac{y}{12} = 1$: $f(12-y, y) = (12-y)(y)(9-12+y-y) = 3y(y-12)$

Thus we need to find max value of $f(12-y,y)$, where $0 \le y \le 12$

$$g(y) = 3y^2 - 36y$$

$$g(y)' = 6y - 36$$

$g(y)' = 0 \Leftrightarrow y = 6 \Rightarrow g(6) = -108 \leftarrow$ absolute minima of $f(x,y)$ within triangle.

$g(y)'' = 6 > 0 \Rightarrow$ local minimum.

Thus maxima at the ends: $g(0) = 0 = g(12) \Rightarrow 0$ is maxima of $\frac{x}{12} - \frac{y}{12} = 1$ bouder.

So <u>27 is absolute maxima</u> of $f(x,y)$ with this boundaries

(b) (HW) $f(x, y) = 3x - 2y + 5$ subject to the constraint $9x^2 + 4y^2 = 36$.

$$\mathcal{L}(x, y, \lambda) = 3x - 2y + 5 + \lambda(9x^2 + 4y^2 - 36)$$

$$\begin{array}{l} \mathcal{L}_x \\ \mathcal{L}_y \\ \mathcal{L}_\lambda \end{array} \begin{cases} 3 + 18x\lambda = 0 \\ -2 + 8y\lambda = 0 \\ 9x^2 + 4y^2 - 36 = 0 \end{cases} \Rightarrow \begin{cases} p_1 : (x, y) = \left(\sqrt{2} ; -\frac{3\sqrt{2}}{2}\right) \\ p_2 (x, y) = \left(-\sqrt{2} ; \frac{3\sqrt{2}}{2}\right) \end{cases}$$

Thus $f(p_1) = 5 + 6\sqrt{2}$ ← absolute maximum

$f(p_2) = 5 - 6\sqrt{2}$ ← absolute minimum

5. (HW) Find the absolute extrema of the function $u = x - 2y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

$$\mathcal{L}(x, y, \lambda) = (x - 2y + 2z) + \lambda(x^2 + y^2 + z^2 - 1)$$

$$\begin{array}{l} \mathcal{L}_x : \\ \mathcal{L}_y : \\ \mathcal{L}_z : \\ \mathcal{L}_\lambda : \end{array} \begin{cases} 2\lambda x + 1 = 0 \\ 2\lambda y - 2 = 0 \\ 2\lambda z + 2 = 0 \\ x^2 + y^2 + z^2 - 1 = 0 \end{cases} \Rightarrow \left[\begin{array}{l} p_1 \left(-\frac{1}{3} ; \frac{2}{3} ; -\frac{2}{3}\right) \\ p_2 \left(\frac{1}{3} ; -\frac{2}{3} ; \frac{2}{3}\right) \end{array} \right.$$

$f(p_1) = -3$ – absolute minimum

$f(p_2) = 3$ – absolute maximum

10. (HW) Find the absolute maximum and minimum values of $f(x, y) = xy$ subject to the constraint $x^2 + 4y^2 = 4$.

$$L(x, y, \lambda) = xy + \lambda(x^2 + 4y^2 - 4)$$

$$\begin{array}{l} L_x: \\ L_y: \\ L_\lambda: \end{array} \begin{cases} 2\lambda x + y = 0 \\ 8\lambda x + x = 0 \\ x^2 + 4y^2 - 4 = 0 \end{cases} \Rightarrow \left[\begin{array}{l} P_1\left(\sqrt{2}\ -\frac{\sqrt{2}}{2}\right) \\ P_2\left(\sqrt{2}\ \frac{\sqrt{2}}{2}\right) \\ P_3\left(-\sqrt{2}\ -\frac{\sqrt{2}}{2}\right) \\ P_4\left(-\sqrt{2}\ \frac{\sqrt{2}}{2}\right) \end{array} \right.$$

$f(P_1) = -1$ - absolute minimum

$f(P_2) = 1$ - absolute maximum

$f(P_3) = 1$ - absolute maximum

$f(P_4) = -1$ - absolute minimum

14*. (HW) For the following function, find the critical points and discuss their nature:

$$f(x,y) = (ax^2 + by^2)e^{-x^2-y^2} \qquad (b > a > 0).$$

$$f(x,y) = (ax^2+by^2)e^{-x^2-y^2} \quad (b>a>0)$$

$$\begin{cases} f_x: & e^{-x^2-y^2}(2ax - 2ax^3 - 2bxy^2) = 0 \\ f_y: & e^{-x^2-y^2}(2by - 2ax^2y - 2by^3) = 0 \end{cases} \Rightarrow \begin{cases} a \neq 0,\ b = a,\ y = \pm\sqrt{1-x^2} \\ a \neq 0, & \forall b & x = -1, & y = 0 \\ a \neq 0, & \forall b & x = 1, & y = 0 \\ \forall a, & b \neq 0, & x = 0, & y = \pm 1 \end{cases}$$

since $0<a<b$ $\Rightarrow$

$$\Rightarrow \begin{cases} 0<a<b & x=1 & y=0 & P_1 \\ 0<a<b & x=-1 & y=0 & P_2 \\ 0<a<b & x=0 & y=1 & P_3 \\ 0<a<b & x=0 & y=-1 & P_4 \end{cases}$$

P_1:
$$\left.\begin{aligned} f_{xx}(P_1) &= -\frac{4a}{e} < 0 \text{ since } a>0 \\ f_{xy}(P_1) &= 0 \\ f_{yy}(P_1) &= \frac{2(b-a)}{e} > 0 \text{ since } b>a \end{aligned}\right\} \Rightarrow \begin{aligned} q_1 &< 0 \\ q_2 &< 0 \end{aligned} \Rightarrow P_1 \text{ - is saddle point}$$

P_2:
$$\left.\begin{aligned} f_{xx}(P_2) &= -\frac{4a}{e} < 0 \text{ since } a>0 \\ f_{xy}(P_2) &= 0 \\ f_{yy}(P_2) &= \frac{2(b-a)}{e} > 0 \text{ since } b>a \end{aligned}\right\} \Rightarrow \begin{aligned} q_1 &< 0 \\ q_2 &< 0 \end{aligned} \Rightarrow P_2 \text{ - is saddle point}$$

К счастью эта ересь не имеет смысла.

P_3:
$$\left.\begin{aligned} f_{xx}(P_3) &= \frac{2(a-b)}{e} < 0 \text{ since } b>a \\ f_{xy}(P_3) &= 0 \\ f_{yy}(P_3) &= -\frac{4a}{e} < 0 \text{ since } a>0 \end{aligned}\right\} \Rightarrow \begin{aligned} q_1 &< 0 \\ q_2 &> 0 \end{aligned} \Rightarrow P_3 \text{ is rel. maxima}$$

P_4:
$$\left.\begin{aligned} f_{xx}(P_4) &= \frac{2(a-b)}{e} < 0 \text{ since } b>a \\ f_{xy}(P_4) &= 0 \\ f_{yy}(P_4) &= -\frac{4a}{e} < 0 \text{ since } a>0 \end{aligned}\right\} \Rightarrow \begin{aligned} q_1 &< 0 \\ q_2 &> 0 \end{aligned} \Rightarrow P_4 \text{ is rel. maxima}$$

Thx for an awesome year