R and Arxiv GR-QC Graph Analysis

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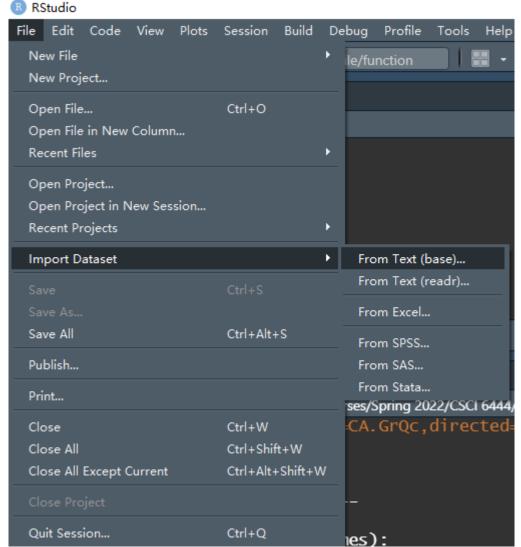
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Part 1 Dataset Loading

In order to learn how to use RStudio for graph analysis, I used the dataset of Arxiv GR-QC (General Relativity and Quantum Cosmology) collaboration network. This dataset contains 5242 nodes and 14496 edges in total and covers papers in the period from January 1993 to April 2003 (124 months).

a. Loading Approach 1: Import

File → Import Dataset → From Text (base)



b. Loading Approach 2: read.table() function

CA.GrQc = read.table(file="CA-GrQc.txt", header = TRUE, comment.char="#")

Then I got a dataframe variable named 'CA.GrQc'.

Part 2 Initial Graph Plotting

I installed the igraph package from CRAN mirrors using the install.package() function and loaded the package by library() function. Before continuing, I Ire required to convert the dataframe into an igraph format using graph_from_data_frame(). I named this object 'graph' and then used plot.igraph() to create the plot for the entire dataset as shown in Figure 1.

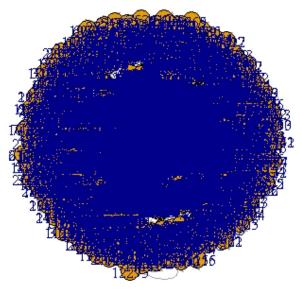


Figure 1. Raw Dataset

Due to the large amount of data, I can hardly observe the data structure, even though I set parameters like 'vertex.label.cex' and 'vertex.size', to view the graph more clearly (Figure 2).

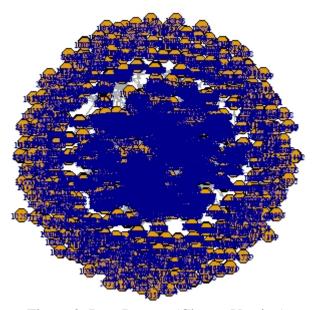


Figure 2. Raw Dataset (Clearer Version)

Part 3 Dataset Simplifying and Plotting

To solve the problem described above, I took the following steps towards graph simplification:

• Step 1. Choose the nodes with degree greater than or equal to 20 and plot the graph.

Code Implementation:

```
verticies_lt20 = V(graph)[igraph::degree(graph)<20]
graph1 = igraph::delete.vertices(graph,verticies_lt20) #remove nodes (degree <=20)
plot.igraph(graph1, edge.arrow.size=0.3, vertex.size=10, vertex.label.cex=0.5)</pre>
```

Plot Result as Figure 3:

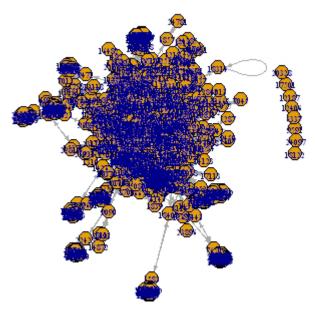


Figure 3. Simplify-Step 1

It is still hard to see the internal structure, so I continue to step 2.

• Step 2. Choose the first 200 edges and plot the graph.

Code Implementation:

```
graph1_df = as_data_frame(graph1, what="edges") #convert igraph object to dataframe graph2_df = graph1_df[1:200,] #choose the first 200 edges graph2 = graph_from_data_frame(graph2_df) plot.igraph(graph2, edge.arrow.size=0.3, vertex.size=10, vertex.label.cex=0.5)
```

Plot Result as Figure 4:

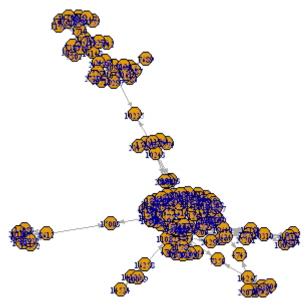


Figure 4. Simplify-Step 2

Now the structure is become clearer, but I can still simplify some more.

• Step 3. Choose the nodes with degree larger than 2 and plot the graph.

Code Implementation:

verticies_lt2 <- V(graph2)[igraph::degree(graph2)<2]
graph.sim = igraph::delete.vertices(graph2, verticies_lt2)
plot.igraph(graph.sim, edge.arrow.size=0.3, vertex.size=10, vertex.label.cex=0.5)</pre>

Plot Result as Figure 5:

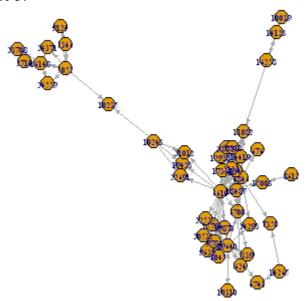


Figure 5. Simplify-Step 3 Finally, I arrive at a three-cluster igraph object for deeper analysis and plotting.

Using the graph from figure 5, I applied the functions from *Introduction to Graph Analytics* and received the following results:

Vertices of the Graph

```
> V(graph.sim)
+ 46/46 vertices, named, from 4df0ba4:
[1] 10310 5052 5346 19640 10243 14265 8612 16258 14123 2710 16148 4846 824 2133 6610 6700
[17] 9124 10235 24371 24559 339 624 4743 8045 9639 9785 15784 19870 20532 22527 24293 19423
[33] 21012 22691 7350 15003 10039 11082 23708 676 12928 14419 17330 18487 22779 23382
```

Edges of the Graph

```
> E(graph.sim)
+ 105/105 edges from 4df0ba4 (vertex names):

[1] 10310->19640 5052 ->5346 5052 ->9124 5052 ->10235 5052 ->16148 5052 ->24371 5052 ->24559
[8] 5346 ->5052 5346 ->9124 19640->339 19640->624 19640->4743 19640->6610 19640->6700
[15] 19640->8045 19640->9639 19640->9785 19640->10310 19640->15784 19640->19870 19640->20532
[22] 19640->22527 19640->24293 10243->10235 10243->19423 10243->21012 10243->22691 14265->4743
[29] 14265->7350 8612 ->15003 16258->10039 16258->11082 16258->14123 14123->10039 14123->16258
[36] 2710 ->16148 2710 ->23708 16148->2710 16148->5052 16148->23708 16148->24371 16148->24559
[43] 4846 ->676 4846 ->824 4846 ->2133 4846 ->6610 4846 ->6700 4846 ->7350 4846 ->1082
[50] 4846 ->12928 4846 ->14419 4846 ->17330 4846 ->18487 4846 ->22779 4846 ->23382 4846 ->24293
[57] 824 ->676 824 ->2133 824 ->4846 824 ->6610 824 ->6700 824 ->11082 824 ->1298
[64] 824 ->14419 824 ->17330 824 ->18487 824 ->23382 2133 ->824 ->1298
[64] 824 ->14419 824 ->17330 824 ->18487 824 ->22779 824 ->23382 2133 ->824 ->1298
[64] 824 ->14419 824 ->17330 824 ->18487 824 ->22779 824 ->23382 2133 ->824 ->133 ->4846
+ ... omitted several edges
```

Adjacency Matrix: Indicates which vertices are connected by edges

<pre>> as.matrix(get.adjacency(graph.sim)) 10310 5052 5346 19640 10243 14265 8612 16258 14123 2710 16148 4846 824 2133 6610 6700 9124 102</pre>																		
	10310	5052	5346	19640	10243	14265	8612	16258	14123	2710	16148	4846	824	2133	6610	6700	9124	10235
10310	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5052	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
5346	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
19640	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
10243	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
14265	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8612	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16258	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
14123	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
2710	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
16148	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
4846	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
824	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0
2133	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0
6610	0	0	0	1	0	0	0	0	0	0	0	1	1	1	0	1	0	0
6700	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
9124	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10235	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24371	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24559	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
339	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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8612	ø	ø	ø	ø	0	0	0	0	ø	ø	ø	ø	ø	ø	ø	ø	o o
16258	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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2710	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16148	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4846	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
824	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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9124	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10235	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24371	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24559	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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• Density: Gives us the ratio of edges in the graph vs the maximum number of possible edges in a graph. When loops = T this means that the maximum counts self-loops. Note: I use as.matrix() for compatibility with the sna functions.

```
> graph.sim.adj = as.matrix(get.adjacency(graph.sim))
> graph.sim.density = gden(graph.sim.adj)
> graph.sim.density
[1] 0.05072464
> edge_density(graph.sim)
[1] 0.05072464
> edge_density(graph.sim, loops=T)
[1] 0.04962193
```

• Egocentric Network and Degrees: Creates a subgraph of a given node and its immediate neighbors. At the bottom there is the number of degrees per node, which is a measurement of the number of edges connected to it

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$`8612`
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                                  676 12928 14419 17330 18487 22779 23382
```

• Betweenness Centrality: The betweenness centrality is measured through the number of times a node is present in the shortest path. The centralization is the graph level centrality, and the theoretical max is the maximum possible centrality for a graph with these parameters

```
igraph::centr_betw(graph.sim)
$res
[1] 0.000000 12.000000 0.000000 40.500000
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[11] 10.000000 11.166667 29.333333 0.000000 86.333333
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$centralization
[1] 0.0423382
$theoretical max
[1] 89100
```

• Closeness Centrality: The closeness centrality is similar to betlenness but is calculated using the average shortest path.

```
> igraph::centr_clo(graph.sim)
$res
  [1] 0.05867014 0.02628505 0.02619325 0.06097561 0.02380952 0.02272727 0.02222222 0.02325581 0.02324380
[10] 0.02617801 0.02626970 0.06097561 0.06081081 0.06072874 0.06189821 0.05882353 0.02173913 0.02173913
[19] 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913
[28] 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913
[37] 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913 0.02173913
[46] 0.02173913

$centralization
[1] 0.03525705

$theoretical_max
[1] 44.02174
```

• Shortest Paths: The length of the shortest path between any 2 nodes in the graph. I also show all the shortest paths for node 6610.

> igra	aph::sh	ortes	st.pat	hs(gra	aph.sim	n)												
	10310	5052	5346	19640	10243	14265	8612	16258	14123	2710	16148	4846	824	2133	6610	6700	9124	10235
10310	0	6	7	1	4	3	4	4	5	8	7	3	3	3	2	2	7	5
5052	6	0	1	5	2	7	6	6	7	2	1	5	5	5	4	5	1	1
5346	7	1	0	6	3	8	7	7	8	3	2	6	6	6	5	6	1	2
19640	1	5	6	0	3	2	3	3	4	7	6	2	2	2	1	1	6	4
10243	4	2	3	3	0	5	4	4	5	4	3	3	3	3	2	3	3	1
14265	3	7	8	2	5	0	5	4	5	9	8	2	3	3	3	3	8	6
8612	4	6	7	3	4	5	0	4	5	8	7	3	3	3	2	3	7	5
16258	4	6	7	3	4	4	4	0	1	8	7	2	2	2	2	3	7	5
14123	5	7	8	4	5	5	5	1	0	9	8	3	3	3	3	4	8	6
2710	8	2	3	7	4	9	8	8	9	0	1	7	7	7	6	7	3	3
16148	7	1	2	6	3	8	7	7	8	1	0	6	6	6	5	6	2	2
4846	3	5	6	2	3	2	3	2	3	7	6	0	1	1	1	1	6	4
824	3	5	6	2	3	3	3	2	3	7	6	1	0	1	1	1	6	4
2133	3	5	6	2	3	3	3	2	3	7	6	1	1	0	1	1	6	4
6610	2	4	5	1	2	3	2	2	3	6	5	1	1	1	0	1	5	3
6700	2	5	6	1	3	3	3	3	4	7	6	1	1	1	1	0	6	4
9124	/	1	1	6	3	8			8	3	2	6	6	6	5	6	0	2
10235	5	1	2	4	1	6	5	5	6	3	2	4	4	4	3	4	2	0
24371	7	1	2	6	3	8	7	7	8	2	1	6	6	6	5	6	2	2
24559	7	1	2	6	3	8	7	7	8	2	1	6	6	6	5	6	2	2
339	2	6	7	1	4	3	4	4	5	8	7	2	2	2	2	1	7	5

```
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$vpath
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+ 3/46 vertices, named, from 4df0ba4:
[1] 6610 19640 10310
$vpath[[2]]
+ 0/46 vertices, named, from 4df0ba4:
$vpath[[3]]
+ 0/46 vertices, named, from 4df0ba4:
$vpath[[4]]
+ 2/46 vertices, named, from 4df0ba4:
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[1] 6610 19640

+ 0/46 vertices, named, from 4df0ba4:

\$vpath[[5]]

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$vpath[[20]]
$vpath[[6]]
                                        + 0/46 vertices, named, from 4df0ba4:
+ 0/46 vertices, named, from 4df0ba4:
                                        $vpath[[21]]
$vpath[[7]]
                                        + 3/46 vertices, named, from 4df0ba4:
+ 0/46 vertices, named, from 4df0ba4:
                                        [1] 6610 19640 339
$vpath[[8]]
+ 0/46 vertices, named, from 4df0ba4:
                                        $vpath[[22]]
                                        + 3/46 vertices, named, from 4df0ba4:
                                        [1] 6610 19640 624
$vpath[[9]]
+ 0/46 vertices, named, from 4df0ba4:
                                        $vpath[[23]]
$vpath[[10]]
                                        + 3/46 vertices, named, from 4df0ba4:
+ 0/46 vertices, named, from 4df0ba4:
                                        [1] 6610 19640 4743
$vpath[[11]]
                                        $vpath[[24]]
+ 0/46 vertices, named, from 4df0ba4:
                                        + 2/46 vertices, named, from 4df0ba4:
                                        [1] 6610 8045
$vpath[[12]]
+ 2/46 vertices, named, from 4df0ba4:
                                        $vpath[[25]]
[1] 6610 4846
                                        + 2/46 vertices, named, from 4df0ba4:
                                        [1] 6610 9639
$vpath[[13]]
+ 2/46 vertices, named, from 4df0ba4:
                                        $vpath[[26]]
[1] 6610 824
                                        + 2/46 vertices, named, from 4df0ba4:
                                        [1] 6610 9785
$vpath[[14]]
+ 2/46 vertices, named, from 4df0ba4:
                                        $vpath[[27]]
[1] 6610 2133
                                        + 2/46 vertices, named, from 4df0ba4:
                                        [1] 6610 15784
$vpath[[15]]
+ 1/46 vertex, named, from 4df0ba4:
                                        $vpath[[28]]
[1] 6610
                                        + 2/46 vertices, named, from 4df0ba4:
                                        [1] 6610 19870
$vpath[[16]]
+ 2/46 vertices, named, from 4df0ba4:
                                        $vpath[[29]]
[1] 6610 6700
                                        + 2/46 vertices, named, from 4df0ba4:
                                        [1] 6610 20532
$vpath[[17]]
+ 0/46 vertices, named, from 4df0ba4:
                                        $vpath[[30]]
                                        + 2/46 vertices, named, from 4df0ba4:
$vpath[[18]]
                                        [1] 6610 22527
+ 0/46 vertices, named, from 4df0ba4:
                                        $vpath[[31]]
$vpath[[19]]
                                        + 3/46 vertices, named, from 4df0ba4:
+ 0/46 vertices, named, from 4df0ba4:
                                        [1] 6610 19640 24293
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$vpath[[41]]
$vpath[[32]]
                                         + 2/46 vertices, named, from 4df0ba4:
+ 2/46 vertices, named, from 4df0ba4:
                                         [1] 6610 12928
[1] 6610 19423
                                         $vpath[[42]]
$vpath[[33]]
                                         + 2/46 vertices, named, from 4df0ba4:
+ 2/46 vertices, named, from 4df0ba4:
                                         [1] 6610 14419
[1] 6610 21012
                                         $vpath[[43]]
$vpath[[34]]
                                         + 2/46 vertices, named, from 4df0ba4:
+ 2/46 vertices, named, from 4df0ba4:
                                         [1] 6610 17330
[1] 6610 22691
                                         $vpath[[44]]
$vpath[[35]]
                                         + 2/46 vertices, named, from 4df0ba4:
+ 3/46 vertices, named, from 4df0ba4:
                                         [1] 6610 18487
[1] 6610 4846 7350
                                         $vpath[[45]]
$vpath[[36]]
                                         + 2/46 vertices, named, from 4df0ba4:
+ 2/46 vertices, named, from 4df0ba4:
                                         [1] 6610 22779
[1] 6610 15003
                                         $vpath[[46]]
$vpath[[37]]
                                         + 2/46 vertices, named, from 4df0ba4:
+ 0/46 vertices, named, from 4df0ba4:
                                         [1] 6610 23382
$vpath[[38]]
+ 2/46 vertices, named, from 4df0ba4:
                                         $epath
[1] 6610 11082
                                         NULL
$vpath[[39]]
                                         $predecessors
+ 0/46 vertices, named, from 4df0ba4:
                                         NULL
$vpath[[40]]
                                         $inbound edges
+ 3/46 vertices, named, from 4df0ba4:
                                         NULL
[1] 6610 4846 676
```

Geodesic: The shortest path between any two nodes in the network

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$counts
       [,1,] [,1,] [,14] [,14] [,15] [,16] [,9] [,9] [,19] [,19] [,13] [,14] [,15] [,16] [,17]
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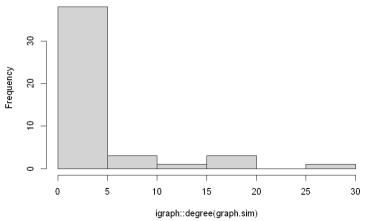
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 [ reached getOption("max.print") -- omitted 25 rows ]
```

Number of Paths Between Nodes: Gives the total number of possible paths betIen nodes

	Houck	-																		
> gra	aph.sim.																			
	10310	5052	2 5346	5 1964	0 10243	14265	8612	16258	14123	2710	16148	4846	824	2133	6610	6700	9124	10235	24:	371
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6610	0	2	2	1		1 1		1				2	0	0	0			0	0	3
6700	0	0	0	0		0 0		0				0	0	0	0			0	0	1
9124	0	0	0	0		0 0		0				0	0	0	0			0	0	0
10235	0	0	0	0	0 (0 0	0	0	0)	0	0	0	0	0	0		0	0	0
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9124	0	0	0			0	0	0												
10235	0	0	0			0	0	0												
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• Histogram: Shows the distribution of the number of degrees per node > hist(igraph::degree(graph.sim))

Histogram of igraph::degree(graph.sim)



• Diameter: The greatest distance between any pair of vertices

```
> igraph::diameter(graph.sim)
[1] 4
```

Max Cliques: Returns all the cliques for a given node in the graph

```
> node = c(1)
> graph.sim.1clique = igraph::max_cliques(graph.sim, min=NULL, max=NULL, subset=node)
Warning message:
In igraph::max_cliques(graph.sim, min = NULL, max = NULL, subset = node) :
   At maximal_cliques_template.h:261 :Edge directions are ignored for maximal clique calculation
> graph.sim.1clique
[[1]]
+ 2/46 vertices, named, from 4df0ba4:
[1] 15003 6610

[[2]]
+ 2/46 vertices, named, from 4df0ba4:
[1] 15003 8612
```

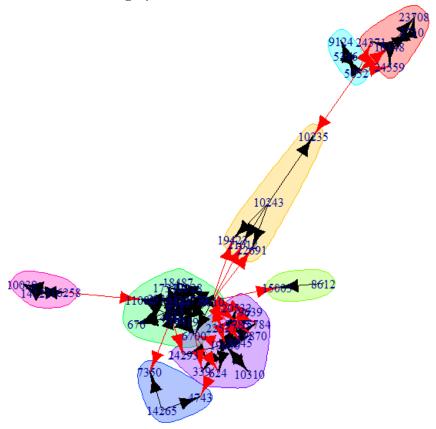
• Largest Cliques: Returns the largest clique in the graph

```
> igraph::clique_num(graph.sim)
[1] 5
```

 Alpha Centrality: Another measurement of centrality which is calculated similarly to eigenvector centrality

```
10310
                   5052
                               5346
                                            19640
                                                        10243
                                                                     14265
                                                                                   8612
26.6324435
             -2.6760563
                            4084507
                                      28.4804928
                                                    1.0000000
                                                                 1.0000000
                                                                              1.0000000
     16258
                  14123
                                2710
                                            16148
                                                         4846
                                                                        824
                                                                                   2133
10.0000000
            10.0000000
                         -1.4084507
                                       -2.6760563
                                                    -9.5893224
                                                                 -6.0780287
                                                                              -9.5893224
      6610
                   6700
                                9124
                                            10235
                                                        24371
                                                                      24559
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                                       -0.5084507
 3.9014374
              7.4127310
                          -2.6760563
                                                    -3.8169014
                                                                 -3.8169014
                                                                             33.3039014
                                             9639
                   4743
                               8045
                                                         9785
                                                                      15784
                                                                                  19870
       624
33.3039014
            27.5324435
                         30.1437372
                                      30.1437372
                                                   30.1437372
                                                                 30.1437372
                                                                             30.1437372
                  22527
     20532
                               24293
                                            19423
                                                        21012
                                                                      22691
                                                                                   7350
30.1437372
             30.1437372
                         18.0020534
                                          4112936
                                                    5.4112936
                                                                    4112936
                                                                                7303901
     15003
                  10039
                               11082
                                           23708
                                                          676
                                                                      12928
                                                                                  14419
   4112936
            19.0000000
                           9.2197125
                                          6760563
                                                  -13.1006160 -18.2197125
                                                                            -18.2197125
     17330
                  18487
                               22779
                                            23382
           -18.2197125
                        -18.2197125
18.2197125
                                      -18.2197125
```

• Walktrap Communities: Automatically clusters nodes in order to determine a structure to the graph

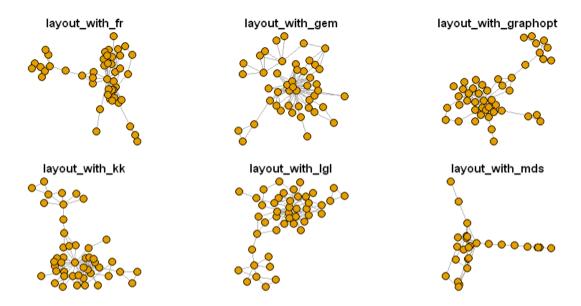


Part 4 Explore Functions

Igraph is a powerful package which contains over 700 functions for graph plotting. I chose to explore and exercise by 11 of them.

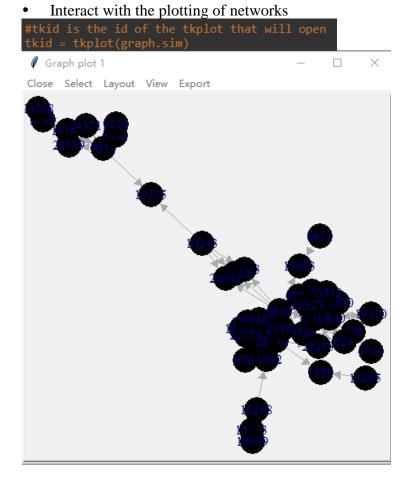
• Look at more available layouts in igraph

```
layouts = grep("^layout_", ls("package:igraph"), value=TRUE)[-1]
# Remove layouts that do not apply to our graph
layouts = layouts[!grepl("bipartite|merge|norm|sugiyama|tree", layouts)]
l = do.call(layout, list(graph.sim))
+     plot(graph.sim, edge.arrow.mode=0, layout=1, main=layout, vertex.label = NA) [
[1] "layout_as_star"
[1] "layout_components"
[1] "layout_in_circle"
[1] "layout_nicely"
[1] "layout_on_grid"
[1] "layout_on_sphere"
[1] "layout_randomly"
[1] "layout_with_dh"
[1] "layout_with_drl"
[1] "layout_with_fr"
[1] "layout_with_gem"
    "layout_with_graphopt"
"layout_with_kk"
"layout_with_lgl"
"layout_with_mds"
     layout_as_star
                                                  layout_components
                                                                                                      layout_in_circle
      layout_nicely
                                                      layout_on_grid
                                                                                                    layout_on_sphere
    layout_randomly
                                                     layout_with_dh
                                                                                                      layout_with_drl
```



Reset the graphical parameters





```
# grab the coordinates from tkplot
           [,1]
                     [,2]
 [1,] 430.00000 157.28386
 [2,] 100.22101 360.74600
 [3,] 114.70852 376.62594
 [4,] 379.40899 151.92319
 [5,] 221.79991 243.92975
 [6,] 423.12877 77.64722
 [7,] 366.50639 252.82990
 [8,] 289.30982 38.90435
 [9,] 283.20887
                13.70336
[10.]
      25.59058 395.64810
[11,]
     55.52754 383.49962
[12,] 312.59872 124.28955
[13,] 301.66005 129.76949
[14,] 300.08468 136.47132
[15,] 316.92797 160.52278
[16,] 347.15917 132.89946
[17,] 116.24348 391.18442
[18,] 159.49564 303.56805
      78.85337 388.80169
      58.26132 364.65567
[20.]
[21,] 407.83971 135.08751
[22,] 396.73725 123.49188
[23,] 429.86180 111.19825
[24,] 371.38786 170.42204
[25,] 356.86588 185.18401
[26,] 340.65845 179.46412
[27,] 359.64920 150.96960
[28,] 388.56324 170.52054
[29,] 375.16029 183.08621
[30,] 351.97997 167.55718
[31,] 365.03762 116.66337
[32,] 273.70866 212.07237
[33,] 261.86309 206.77228
[34,] 250.71904 200.88233
[35,] 367.82546 85.38216
[36,] 342.43621 215.95485
[37,] 283.85098
                  0.00000
[38,] 301.05211 101.36903
[39,] 20.00000 410.00000
[40,] 274.73991 99.81228
[41,] 284.69661 122.63150
[42,] 270.30168 139.39674
[43,] 277.76006 147.24872
[44,] 291.98137 150.30532
[45,] 275.90481 131.27425
[46,] 320.62322 138.50694
```

Create a heatmap of the network matrix

```
22779
                                                                                     17330
                                                                                     12928
                                                                                     23708
                                                                                     10039
                                                                                     7350
                                                                                     21012
                                                                                     24293
                                                                                     20532
                                                                                     15784
                                                                                     9639
                                                                                     4743
                                                                                     339
                                                                                     24371
                                                                                     9124
                                                                                     6610
                                                                                     824
                                                                                     16148
                                                                                     14123
                                                                                     8612
                                                                                     10243
                                                                                     5346
                                                                                     10310
10310
5346
10243
8612
16148
824
6610
9124
24371
339
4743
9639
9639
9639
9639
270632
24293
21012
7360
10039
12928
12779
2779
```

Reciprocity: the proportion of reciprocated ties

```
> reciprocity(graph.sim)
[1] 0.247619
```

Transitivity: which is calculated according to two different ways: global and local

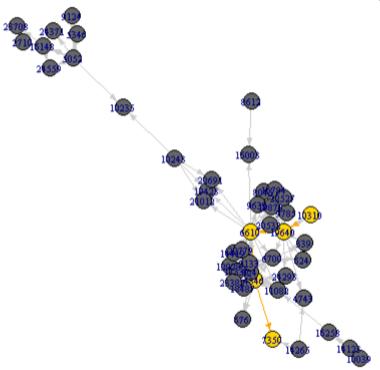
• get diameter()

#In igraph, diameter() returns the distance, while get_diameter() returns the nodes along the first found path of that distance.

```
#In igraph, diameter() returns the distance, while get_diameter() returns the nodes along the first found
+ 5/46 vertices, named, from 4df0ba4:
[1] 10310 19640 6610 4846 7350
```

Color nodes along the diameter:

```
vcolor nodes along the drameter.
> vcol = rep("gray40", vcount(graph.sim))
> vcol[diam] = "gold"
> ecol = rep("gray80", ecount(graph.sim))
> ecol[E(graph.sim, path=diam)] = "orange"
> ew = rep(2, ecount(graph.sim))
> ew[E(graph.sim, path=diam)] = 4
> ew[E(graph.sim, vertex.color=vcol, edge.color=ecol, edge.arrow.size=0.3, vertex.size=10, vertex.label.cex=
```



Values of the first eigenvector of the graph matrix.

```
centr_eigen(graph.sim, directed=T, normalized=T)
$vector
[10] 0.00000000 0.00000000 0.71592474 0.94336314 0.71592474 0.77256160 1.00000000 0.00000000 0.000000000
[19] 0.00000000 0.00000000 0.36770565 0.36770565 0.07331048 0.30074888 0.30074888 0.30074888 0.30074888
[28] 0.30074888 0.30074888 0.30074888 0.28407526 0.22743840 0.22743840 0.22743840 0.21076478 0.22743840
[37] 0.00000000 0.92668952 0.00000000 0.48848634 0.92668952 0.92668952 0.92668952 0.92668952 [46] 0.92668952
$value
[1] 3.396795
```

```
$value
[1] 3.396795
$options
$options$bmat
[1] "I"
$options$n
[1] 46
$options$which
[1] "LR"
$options$nev
[1] 1
$options$tol
[1] 0
$options$ncv
[1] 0
$options$ldv
[1] 0
$options$ishift
[1] 1
$options$maxiter
[1] 1000
$options$nb
[1] 1
$options$mode
[1] 1
```

```
$options$start
[1] 1
$options$sigma
[1] 0
$options$sigmai
[1] 0
$options$info
[1] 0
$options$iter
[1] 1
$options$nconv
[1] 1
$options$numop
[1] 20
$options$numopb
[1] 0
$options$numreo
[1] 18
$centralization
[1] 0.6719116
$theoretical max
[1] 45
```

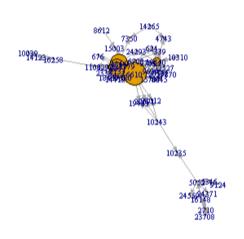
• Average path length

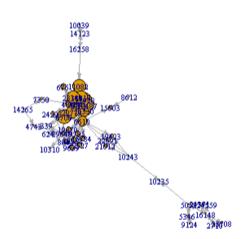
```
> mean_distance(graph.sim, directed=T)
[1] 1.780392
```

Hubs Score (contain a large number of outgoing links) and
 Authorities Score (contain many incoming links from hubs)

```
> plot(graph.sim, vertex.size=hs*25, main="Hubs", edge.arrow.size=0.3, vertex.label.cex=0.5)
> plot(graph.sim, vertex.size=as*15, main="Authorities", edge.arrow.size=0.3, vertex.label.cex=0.5)
> hs <- hub_score(graph.sim, weights=NA)$vector
> as <- authority_score(graph.sim, weights=NA)$vector
> par(mfrow=c(1,2))
> plot(graph.sim, vertex.size=hs*25, main="Hubs", edge.arrow.size=0.3, vertex.label.cex=0.5)
> plot(graph.sim, vertex.size=as*15, main="Authorities", edge.arrow.size=0.3, vertex.label.cex=0.5)
```

Hubs Authorities





Part 5 Determine Several Technical Values

(a) central node(s)

```
Code Implementation:
```

d = igraph::degree(graph.sim)
d = as.data.frame(d)

The central node is the nodes with the high number of degrees in the graph. For this question, I took the degrees of each node and used the sorting tool within R to find

the largest value. For our graph, the central node is 6610.

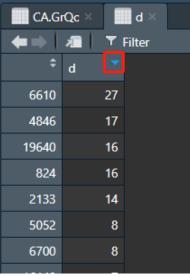


Figure 6. Dataframe of Degree

(b) longest path(s)

```
Code Implementation:
```

```
igraph::diameter(graph.sim)
diam = get_diameter(graph.sim, directed=T)
diam
```

I can get the length of the longest path simply by using the diameter function in igraph. I can also use get_diameter to get the nodes that follow this longest path. For this graph, I have a longest path of length 4 using the following sequence of nodes shown in Figure 7.

```
> igraph::diameter(graph.sim)
[1] 4
> diam = get_diameter(graph.sim, directed=T)
> diam
+ 5/46 vertices, named, from ca05217:
[1] 10310 19640 6610 4846 7350
```

Figure 7: Longest Path Calculation

(c) largest clique(s)

Code Implementation:

```
vcol = rep("grey80", vcount(graph.sim))
vcol[unlist(largest_cliques(graph.sim))] = "gold"
plot(as.undirected(graph.sim), vertex.color=vcol, vertex.label.cex=0.5, vertex.size=15)
```

Plotting Result:

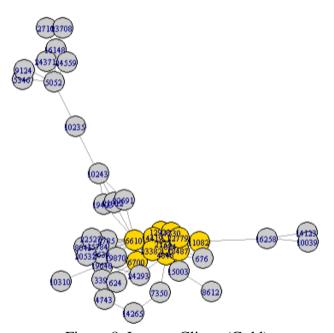


Figure 8: Largest Clique (Gold)

(d) ego(s)

I can find the biggest immediate neighborhood for all nodes by ego() function, here I only show the result for our central node: 6610.

```
> ego(graph.sim,nodes=V(graph.sim)["6610"])
[[1]]
+ 24/46 vertices, named, from 4df0ba4:
[1] 6610 19640 4846 824 2133 6700 8045 9639 9785 15784 19870 20532 22527 19423 21012 22691
[17] 15003 11082 12928 14419 17330 18487 22779 23382
```

Figure 9: Ego for node 6610

(e) power centrality

power_centrality takes a graph (data) and returns the Boncich power centralities of positions (selected by nodes).

Code Implementation:

```
pc = power_centrality(graph.sim, exponent = 0.9)
pc_df = as.data.frame(pc)
pc_df <- cbind(node = rownames(pc_df), pc_df)
rownames(pc_df) <- 1:nrow(pc_df)
pc_df[which.max(pc_df$pc),]
```

Using power_centrality() from the igraph package, I Ire able to calculate the power centrality for each node in the graph, as Ill as the central node. Note: I must set exponent = 0.9 to avoid an error

```
10310
                                                   19640
                                     5346
                                                                  10243
                                                                                 14265
                                            4.638441e+00
2710
4.202730e+00
             -2.763556e-01 -1.924528e-01
                                                          1.125344e-01
                                                                         5.626719e-02
                      16258
                                    14123
                                                                  16148
                                                                                  4846
              7.107435e-01
                             6.959363e-01
                                           -2.156824e-01
                                                          -3.021662e-01
                                                                         9.866892e-01
2.813360e-02
                                                    6700
1.016303e+00
              -1.031111e+00
                             1.343731e+00
                                           -8.302723e-01
                                                          0.000000e+00
                                                                         0.000000e+00
                      24559
                                                     624
       24371
                                      339
                                                                   4743
                                                                                  8045
0.000000e+00
              0.000000e+00
                             0.000000e+00
                                            0.000000e+00
                                                          0.000000e+00
                                                                         0.000000e+00
                                    15784
        9639
                       9785
                                                   19870
                                                                  20532
                                                                                 22527
0.000000e+00
              0.000000e+00
                             0.000000e+00
                                            0.00000e+00
                                                           0.000000e+00
       24293
                      19423
                                    21012
                                                   22691
                                                                   7350
                                                                                 15003
0.000000e+00
                             0.000000e+00
                                            0.000000e+00
                                                          0.000000e+00
              0.000000e+00
                                                                         0.000000e+00
       10039
                     11082
                                    23708
                                                    676
                                                                  12928
                                                                                 14419
                             2.922532e-17
22779
                                                          0.000000e+00
0.000000e+00
                                            0.000000e+00
                                                                         0.000000e+00
              0.000000e+00
       17330
                     18487
                                                   23382
0.000000e+00
              0.000000e+00
                             0.000000e+00
                                            0.000000e+00
```

```
> pc_df[which.max(pc_df$pc),]
node pc
4 19640 4.638441
```

Figure 10b: Central Node using Power Centrality

Part 6 Discussion and Conclusion

For this project, I loaded a large data set of authors and co-authors into a graph using the igraph package in R. Since this dataset contained over 5000 nodes and 14000 edges, it was clear I would need to implement some graph simplification strategies.

I applied multiple strategies before successfully completing our simplification. Firstly, I order my nodes based on frequency and selected the top 300 authors. This strategy helped simplify our graph, but I am concerned this would not have enough randomness within the sample. Instead, I selected 400 nodes randomly, and then reduced this number by mandating the nodes have at least one degree. This strategy performed better, but resulted in a graph containing two subgraphs, and I preferred to work with just one. The final strategy had me performing degree removal first, dropping all nodes with less than 20 degrees, then I took the first 200 nodes of the resulting graph and dropped the remaining nodes with less than 2 degrees. The result was the graph shown in Figure 5, which contains 46 vertices, 105 edges, and no subgraphs, this is the graph I used for the remainder of my analysis

The rest of our analysis includes practicing with the igraph functions mentioned in the rubric, as III as some others. This exploration helped me become much more comfortable working with, visualizing, and understanding graph. The functions made it easier for me to identify and label cliques that occurred within the dataset. I also learned how to use different measurements for centrality, including closeness, betweenness, alpha, and power centrality. Finally, I learned the difference between directional measurements, such as hubs and authorities, and how to represent them in my graph.