Iterative Constant Pitch Guidance

ICPG is an ascent guidance scheme aiming to provide flexibility at orbital insertion conditions, similar to PEG. Being an iterative scheme, the guidance makes some assumptions whose impact is reduced as the solution is improved through iteration.

Assumptions

- Constant mass flow rate.
- Constant exhaust velocity.
- Constant thrust.
- Constant gravitational field.

Main loop

1. Warm up ICPG during the last seconds of first stage burn by finding a workable starting solution.
2. Once upper stage is up and running, leave controls to ICPG.
3. Solve guidance equations for the best solution given boundary conditions and constraints.
4. Use current solution for some time.
5. Loop through 3-4 until close enough to insertion point.
6. Once close enough to injection point, use last solution to avoid the equations becoming indeterminate.

Boundary conditions

For a given orbital insertion point:

$\dot{x}_f =$ Insertion tangential velocity

$\dot{y}_f =$ Insertion radial velocity

$y_f =$ Insertion point's height over ground

Guidance equations

$$m = m_0 - \dot{m}t \tag{1}$$

$$\tau = \frac{m}{T_h \dot{m}} \tag{2}$$

$$a = \frac{F}{m_0 - \dot{m}t} = \frac{V_{ex}}{\tau - t} \tag{3}$$

$$\ddot{x} = \frac{V_{ex}}{\tau - t}\cos(\theta) \tag{4}$$

$$\ddot{y} = \frac{V_{ex}}{\tau - t}\sin(\theta) + \frac{\dot{x}_0^{\ 2}}{R} - \frac{GM}{R^2} \tag{5}$$

$$\dot{x} = \dot{x}_0 + V_{ex}\log\left(\frac{\tau}{\tau - t}\right)\cos(\theta) \tag{6}$$

$$\dot{y} = \dot{y}_0 + V_{ex}\log\left(\frac{\tau}{\tau - t}\right)\sin(\theta) + \left(\frac{\dot{x}_0^{\ 2}}{R} - \frac{GM}{R^2}\right)t \tag{7}$$

$$y = y_0 + \dot{y}_0 t - V_{ex}\left((\tau - t)\log\left(\frac{\tau}{\tau - t}\right) - t\right)\sin(\theta) + \left(\frac{\dot{x}_0^{\ 2}}{R} - \frac{GM}{R^2}\right)t \tag{8}$$

Constraints

$T_h \in (0,1)$

$\theta \in (-45^{\mathrm{o}}, 45^{\mathrm{o}})$

$t \in (0, \tau)$

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[1] T_h: Normalized thrust level (thrust from 0 to 1).

Objective function

$$\mathbf{w} = \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \mathbf{w}_3 \end{pmatrix} \tag{9}$$

$$\dot{x}_{err} = \mathbf{w}_1 \left(\dot{x}_f - \dot{x}_{tgt} \right) \tag{10}$$

$$\dot{y}_{err} = \mathbf{w}_2 \left(\dot{y}_f - \dot{y}_{tgt} \right) \tag{11}$$

$$y_{err} = \mathbf{w}_3 \left(y_f - y_{tgt} \right) \tag{12}$$

$$\mathrm{C}(x) = \dot{x}_{err}{}^2 + \dot{y}_{err}{}^2 + y_{err}{}^2 \tag{13}$$

Optimization algorithm

Any constrained optimization algorithm suitable for convex systems will suffice.

As of right now, the solver is using Nesterov Accelerated Gradient descent (NAG), as the small learning rates are needed for numerical stability reasons. The constraints are fulfilled by adding a penalty term to the objective function (will need improvement here).