CSC 206 Algorithms and Paradigms CSC 140 Advanced Algorithm Design and Analysis Spring 2022

Assignment 2: Recurrences Due Monday, February 28, 2022

Solve <u>all</u> the following recurrences using the Master Method (if possible) and the <u>first five</u> of them using the recursion tree method. Each of the five recursion tree solutions is worth 12 points, and each of the eight Master Theorem solutions is worth 5 points. To get full credit, you must show all steps. You will lose points for any missing steps, especially on the recursion tree problems.

1.
$$T(n) = T(\frac{3n}{4}) + c$$

Recursion Tree Method:

a=1 b=4 H=log₄n L=1^{log₄n} = n^{log_4} ¹= n^0 = 1 Base cost = d # of operations = 1c Internal cost = c * log₄n = clog₄n Total cost = d + clog₄n = θ (logn)

Master's Method:

L=
$$n^{\log_4 1} = n^0 = 1$$

F(n)=1

L is equal to F(n) so condition 2 holds. Therefore, its runtime is $\theta(logn)$.

$$2. \quad T(n) = T(\frac{n}{4}) + cn$$

Recursion Tree Method:

b=4 H=log₄n L=1^{log₄n} = n^{log_4} ¹= n^0 = 1 Base cost = d # of operations = 1cn Internal cost = cn * log₄n = cnlog₄n Total cost = d + cnlog₄n = θ (logn)

Master's Method:

L=
$$n^{\log_4 1} = n^0 = 1$$

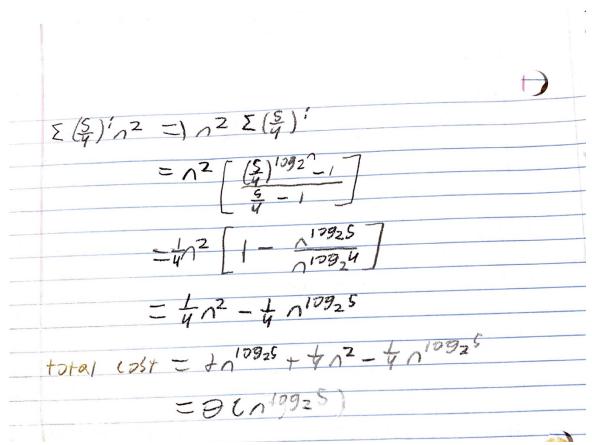
F(n) = n

1<n. So, L<F(n). Now to check the regularity condition which has no value that meets the requirement. Therefore, we cannot use this method.

3.
$$T(n) = 5T(\frac{n}{2}) + n^2$$

Recursion Tree Method:

a=5 b=2 H=log₂n L= $5^{\log_2 n} = n^{\log_2 5}$ Base cost= $dn^{\log_2 5}$ # of operations = $(5/4)^i n^2$



Master's Method:

$$L= n^{\log_2 5}$$

$$F(n)=n^2$$

L>n. So, condition 1 is met. Therefore, the solution is $\theta(n^{\log_2 5})$.

4.
$$T(n) = 8T(\frac{n}{2}) + n^3$$

Recursion Tree Method:

a=8
b=2
H=log₂n
L=
$$8^{\log_2 n}$$
 = $n^{\log_2 8}$ = n^3
Base cost= dn^3
of operations= $(8/8)^i n^2$
Internal cost = $dn^3 + dn^3 \log_2 n$
Total cost = $dn^3 + dn^3 \log_2 n$

Master's Method:

L=
$$n^{\log_2 8} = n^3$$

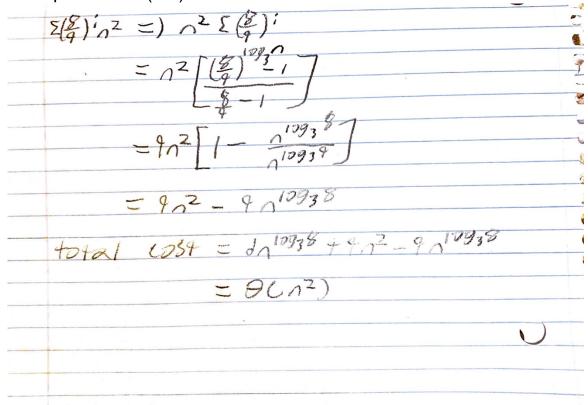
F(n) = n^3

L==F(n). So, condition 2 was met. Therefore, the solution is $\theta(n^3 \log n)$.

5.
$$T(n) = 8T(\frac{n}{3}) + n^2$$

Recursion Tree Method:

a=8 b=3 H= $8^{\log_3 n} = n^{\log_3 8}$ Base cost = d $n^{\log_3 8}$ # of operations= $(8/9)^i n^2$



Master's Method:

$$L= n^{\log_3 8}$$

$$F(n) = n^2$$

L<F(n). So, now we must check the regularity condition. Where there is a solution for k where k=(8/9). Therefore, we can use case 3 and say the solution is $\theta(n^2)$.

$$6. \quad T(n) = 7T(\frac{n}{6}) + n\log n$$

Master's Method:

L=n^{log}6⁷

F(n)=nlogn

L>F(n). The thing is L is not greater by a term of a polynomial degree so we cannot use the Master's Method on this problem.

7.
$$T(n) = 4T(\frac{n}{2}) + n^2 \log^3 n$$

Master's Method:

$$L=n^{\log_2 4} = n^2$$

F(n) = $n^2 \log^3 n$

L<F(n). The thing is F(n) is not greater by a term of a polynomial degree so we cannot use the Master's Method on this problem.

8.
$$T(n) = 5T(\frac{n}{6}) + n \log n$$

Master's Method:

L=n^{log}6⁵ F(n)= nlogn

L<F(n). So, now we must check the regularity conditions which is satisfied by the value of k=(5/6). Therefore, the solution must be $\theta(n\log n)$.