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CSC 206 Algorithms and Paradigms
CSC 140 Advanced Algorithm Design and Analysis
Spring 2022
Assignment 2: Recurrences
Due Monday, February 28, 2022

Solve **all** the following recurrences using the Master Method (if possible) and the **first five** of them using the recursion tree method. Each of the five recursion tree solutions is worth 12 points, and each of the eight Master Theorem solutions is worth 5 points. To get full credit, you must show all steps. You will lose points for any missing steps, especially on the recursion tree problems.

1. $T(n) = T(\frac{3n}{4}) + c$

Recursion Tree Method:

a=1
b=4
H=$\log_4 n$
L=$1^{\log_4 n}$ = $n^{\log_4 1}$=n^0 = 1
Base cost = d
# of operations = 1c
Internal cost = c * $\log_4 n$ = $c\log_4 n$
Total cost = d + $c\log_4 n$ = $\theta(\log n)$

Master's Method:

L= $n^{\log_4 1}$=n^0=1
F(n)=1

L is equal to F(n) so condition 2 holds. Therefore, its runtime is $\theta(\log n)$.

2. $T(n) = T(\frac{n}{4}) + cn$

Recursion Tree Method:

a=1

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b=4
H=$\log_4 n$
L=$1^{\log_4 n}$ = $n^{\log_4 1}$=n^0 = 1
Base cost = d
# of operations = 1cn
Internal cost = cn * $\log_4 n$ = cn$\log_4 n$
Total cost = d + cn$\log_4 n$ = θ(logn)

Master's Method:

L= $n^{\log_4 1}$=n^0=1
F(n) = n

1<n. So, L<F(n). Now to check the regularity condition which has no value that meets the requirement. Therefore, we cannot use this method.

3. $T(n) = 5T(\frac{n}{2}) + n^2$

Recursion Tree Method:

a=5
b=2
H=$\log_2 n$
L= $5^{\log_2 n}$ = $n^{\log_2 5}$
Base cost= $dn^{\log_2 5}$
# of operations = $(5/4)^i n^2$

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$$\sum \left(\frac{5}{4}\right)^i n^2 = n^2 \sum \left(\frac{5}{4}\right)^i$$

$$= n^2 \left[\frac{\left(\frac{5}{4}\right)^{\log_2 n} - 1}{\frac{5}{4} - 1} \right]$$

$$= \frac{1}{4} n^2 \left[1 - \frac{n^{\log_2 5}}{n^{\log_2 4}} \right]$$

$$= \frac{1}{4} n^2 - \frac{1}{4} n^{\log_2 5}$$

total cost $= d n^{\log_2 5} + \frac{1}{4} n^2 - \frac{1}{4} n^{\log_2 5}$

$$= \Theta(n^{\log_2 5})$$

Master's Method:

L= $n^{\log_2 5}$
F(n)=n^2

L>n. So, condition 1 is met. Therefore, the solution is $\theta(n^{\log_2 5})$.

4. $T(n) = 8T(\frac{n}{2}) + n^3$

Recursion Tree Method:

a=8
b=2
H=$\log_2 n$
L= $8^{\log_2 n} = n^{\log_2 8} = n^3$
Base cost= dn^3
# of operations= $(8/8)^i n^2$
Internal cost = $cn^3 \log_2 n$
Total cost = $dn^3 + cn^3 \log_2 n = \theta(n^3 \log n)$

Master's Method:

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L= $n^{\log_2 8}$= n^3
F(n) = n^3

L==F(n). So, condition 2 was met. Therefore, the solution is $\theta(n^3 \log n)$.

5. $T(n) = 8T(\frac{n}{3}) + n^2$

Recursion Tree Method:

a=8
b=3
H=$8^{\log_3 n}$ = $n^{\log_3 8}$
Base cost = d $n^{\log_3 8}$
# of operations= $(8/9)^i n^2$

$$\Sigma (\tfrac{8}{9})^i n^2 \Rightarrow n^2 \Sigma (\tfrac{8}{9})^i$$

$$= n^2 \left[\frac{(\frac{8}{9})^{\log_3 n} - 1}{\frac{8}{9} - 1} \right]$$

$$= 9n^2 \left[1 - \frac{n^{\log_3 8}}{n^{\log_3 9}} \right]$$

$$= 9n^2 - 9n^{\log_3 8}$$

total cost $= dn^{\log_3 8} + 9n^2 - 9n^{\log_3 8}$

$$= \theta(n^2)$$

Master's Method:

L= $n^{\log_3 8}$
F(n) = n^2

L<F(n). So, now we must check the regularity condition. Where there is a solution for k where k=(8/9). Therefore, we can use case 3 and say the solution is $\theta(n^2)$.

6. $T(n) = 7T(\frac{n}{6}) + n\log n$

Master's Method:

L=$n^{\log_6 7}$
F(n)=nlogn
L>F(n). The thing is L is not greater by a term of a polynomial degree so we cannot use the Master's Method on this problem.

7. $T(n) = 4T(\frac{n}{2}) + n^2 \log^3 n$

Master's Method:

L=$n^{\log_2 4} = n^2$
F(n) = $n^2\log^3 n$

L<F(n). The thing is F(n) is not greater by a term of a polynomial degree so we cannot use the Master's Method on this problem.

8. $T(n) = 5T(\frac{n}{6}) + n\log n$

Master's Method:

L=$n^{\log_6 5}$
F(n)= nlogn

L<F(n). So, now we must check the regularity conditions which is satisfied by the value of k=(5/6). Therefore, the solution must be θ(nlogn).