R Notebook

Code

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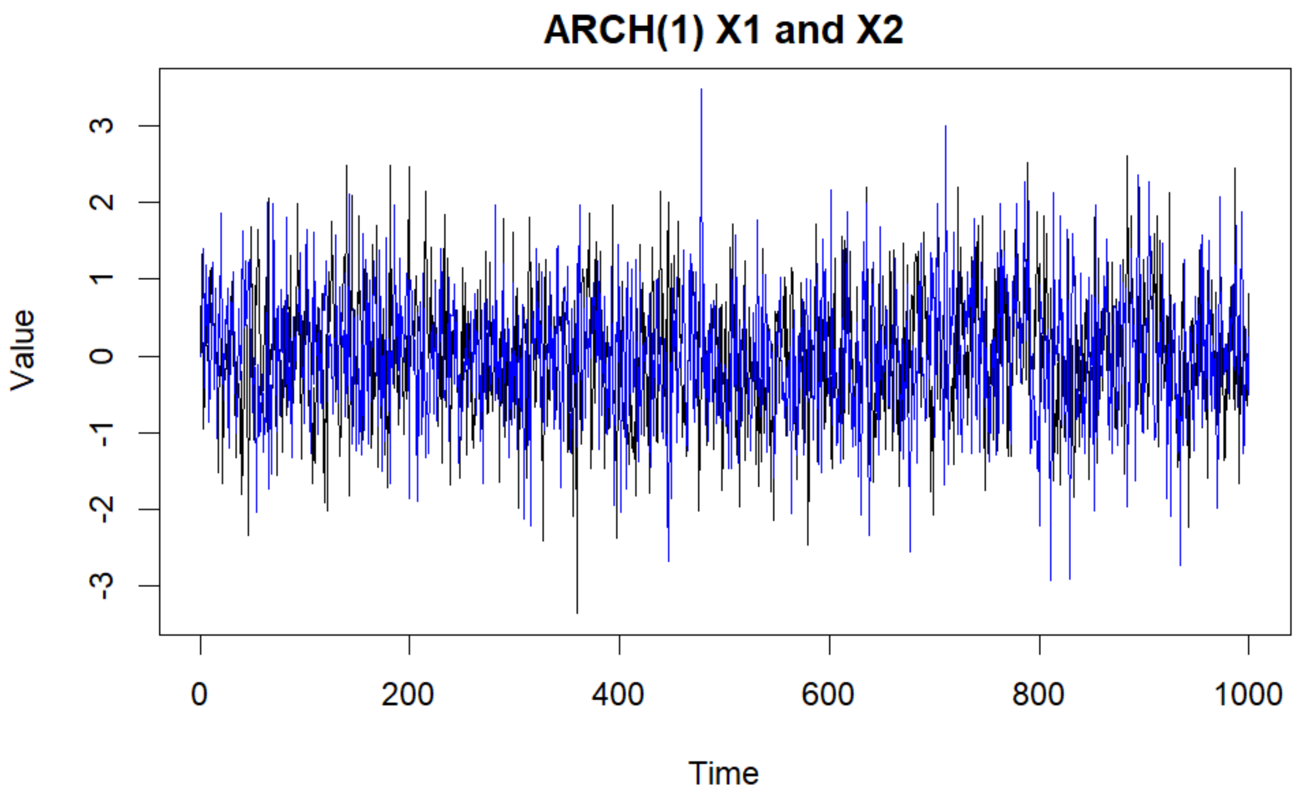
```
#1a- Set the parameters
n <- 1000
alpha <- 0.5

X1 <- numeric(n)
X2 <- numeric(n)

# Simulate the realizations
for (i in 2:n) {
  white_noise1 <- rnorm(1)
  white_noise2 <- rnorm(1)

  X1[i] <- white_noise1 / sqrt(1 + alpha * X1[i-1]^2)
  X2[i] <- white_noise2 / sqrt(1 + alpha * X2[i-1]^2)
}
# Plot the time series realizations
ts.plot(cbind(X1,X2), col = c("black", "Blue"), main = "ARCH(1) X1 and X2", ylab = "Value")
```



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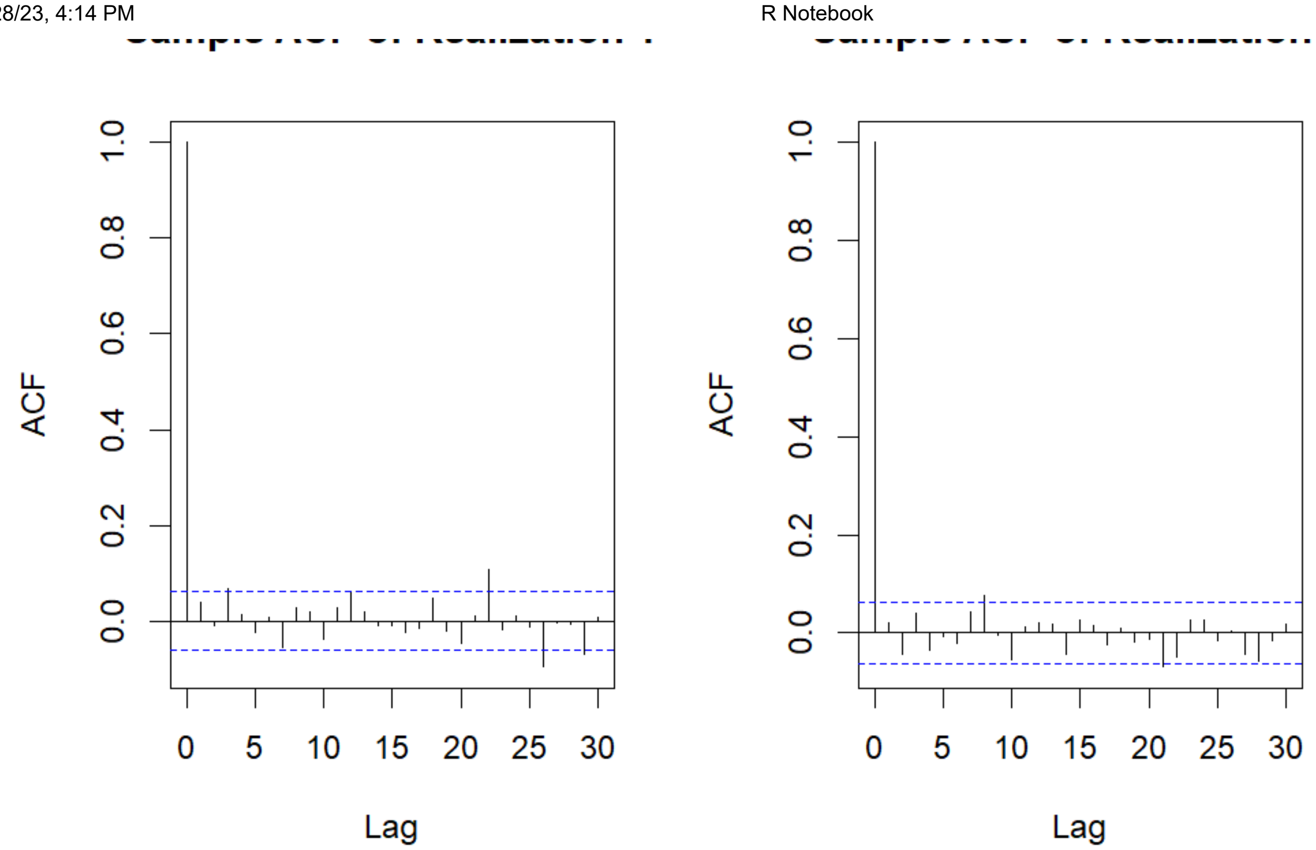
```
# Sample autocorrelation functions of both runs
par(mfrow = c(1, 2))
acf(X1, main = "Sample ACF of Realization 1")
acf(X2, main = "Sample ACF of Realization 2")
```



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```
# it seems that the realizations are consistent for arch and acf, it may be because most of t
he autocorrelations are close to zero for all lags so the weak white noise has no significant
autocorrelation. Another important thing to observe is their confidence interval, and the cor
relations lie within the interval making it consistent.
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#1b Lag parameter
M <- 10

# Apply the Ljung-Box test to both time series
test1 <- Box.test(X1, lag = M, type = "Ljung-Box")
test2 <- Box.test(X2, lag = M, type = "Ljung-Box")

# Print the test results
cat("Ljung-Box Test for X1:\n")
```

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Ljung-Box Test for X1:
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```
print(test1)
```

```
    Box-Ljung test

data:  X1
X-squared = 6.1389, df = 10, p-value = 0.8035
```

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cat("\nLjung-Box Test for X2:\n")
```

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Ljung-Box Test for X2:
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print(test2)
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```
    Box-Ljung test

data:  X2
X-squared = 7.0974, df = 10, p-value = 0.7162
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# Since there is no p-value smaller than 0.05 we may not have enough evidence to reject the n
ull hypothesis, but we can conclude that the sample acfs for X1 and X2 are consistent with th
e weak white noise, as there is no significant evidence of autocorrelation, if one of them we
re less than 0.05, we could be suspicious of null hypothesis.
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```
#2d
# Function of Y^2 for given alpha and sample size n=1000
Y2_ <- function(alpha, n) {
  Y2 <- numeric(n)
  Y2[1] <- 1 / (1 - alpha)
  et <- rnorm(n)

  for (t in 2:n) {
    Y2[t] <- (1 / (1 - alpha)) + alpha * Y2[t - 1] + et[t]
  }

  return(Y2)
}

n <- 1000
alpha1 <- 0.2
alpha2 <- 0.8

# Generate realizations of Y^2
Y2_1 <- Y2_(alpha1, n)
Y2_2 <- Y2_(alpha2, n)
```

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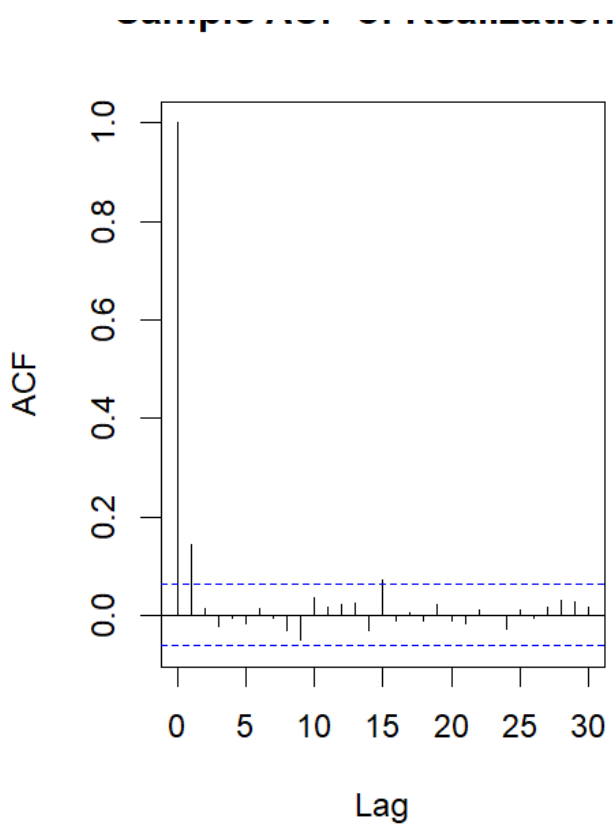
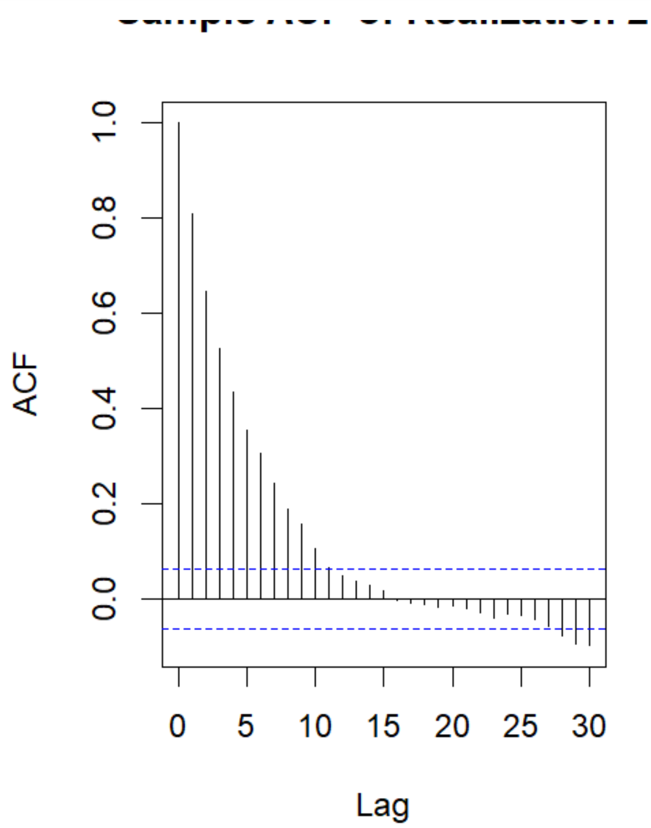
```
par(mfrow = c(1, 2))
acf(Y2_1, main = "Sample ACF of Realization 1")
acf(Y2_2, main = "Sample ACF of Realization 2")
```





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```
# For both alphas we can conclude that for an alpha closer than 0 the acf decays more rapidly
making the process more natural and less persistent whereas for an alpha closer to 1 the proc
ess decays more rapidly making the process more persistent meaning that its current values ar
e strongly influenced by past values.
```