

R Notebook

Code ▼

Hide

```
# Wk= X1+...Xk (Binomial Random Variable)
Wk<-2*rbinom(200,1,0.5)-1
Wk
```

```
[1]  1 -1 -1 -1 -1 -1 -1 -1  1  1  1 -1  1  1  1 -1  1  1  1 -1 -1  1 -1 -1 -1 -1
[27]  1  1 -1 -1 -1 -1  1  1  1 -1  1 -1 -1  1  1 -1  1 -1  1  1 -1  1  1 -1  1
[53] -1  1 -1 -1 -1  1 -1  1 -1  1  1  1  1 -1 -1  1  1 -1  1  1  1 -1 -1  1 -1 -1
[79]  1 -1  1  1  1  1  1  1  1  1 -1 -1 -1  1  1 -1 -1 -1 -1  1 -1 -1  1 -1  1  1 -1
[105] -1 -1  1  1  1  1  1  1 -1 -1  1  1 -1 -1  1  1 -1  1 -1  1  1 -1  1 -1 -1 -1
[131] -1 -1  1  1  1  1  1  1 -1  1  1  1 -1 -1  1 -1 -1  1  1 -1 -1 -1 -1  1  1  1
[157]  1  1 -1  1 -1 -1 -1  1 -1 -1 -1  1 -1 -1 -1 -1  1 -1 -1  1  1  1 -1 -1 -1  1
[183]  1 -1 -1  1  1 -1 -1  1  1  1  1  1 -1 -1 -1 -1  1 -1
```

Hide

```
w<-sum(Wk)
w
```

```
[1] 2
```

Hide

```
#Another method would be by using the sample function.
Wk1<-sample(c(1,-1),200,replace=T,prob=c(0.5,0.5))
Wk1
```

```
[1] -1  1 -1  1  1 -1  1 -1  1 -1 -1  1 -1 -1 -1  1  1 -1  1  1 -1 -1 -1 -1 -1
[27] -1 -1 -1  1  1 -1  1 -1 -1  1  1  1 -1  1 -1  1  1  1 -1  1  1 -1 -1  1  1
[53] -1 -1 -1  1 -1  1 -1  1  1 -1  1 -1 -1 -1  1  1 -1  1  1  1 -1  1  1 -1 -1  1
[79] -1 -1 -1  1  1  1 -1  1  1  1  1 -1  1  1 -1  1 -1 -1  1 -1 -1 -1  1  1  1 -1
[105]  1  1 -1 -1  1 -1  1 -1 -1  1 -1  1 -1  1  1 -1 -1 -1  1  1 -1 -1 -1 -1 -1 -1
[131]  1 -1 -1 -1 -1 -1  1 -1  1  1  1  1 -1  1 -1  1 -1 -1  1  1 -1  1 -1  1  1  1
[157]  1 -1 -1 -1 -1 -1 -1  1  1 -1  1 -1  1 -1  1  1  1 -1  1 -1 -1 -1  1  1 -1 -1
[183] -1  1 -1 -1  1  1 -1  1  1  1  1  1 -1 -1  1 -1 -1  1
```

Hide

```
#a) create a vector below to simulate the probability of Wk >= 10
```

```
Forward <- rep(0,200)
for (i in 1:100000){Forward[i] <- as.numeric(sum(2*rbinom(200,1,0.5)-1) >= 10)}
#Simulation Result
Forward
```

```
[1] 0 1 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0  
[40] 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 1 0 1 1 1 0 1 0 0 0 0 1 1 0 0 1 0 0  
[79] 1 0 0 1 1 1 0 1 0 1 1 0 0 0 1 1 0 0 1 1 0 0 0 0 0 1 0 0 1 0 1 1 0 0 0 0 0 0 0 0  
[118] 0 1 0 0 0 0 0 1 0 0 0 1 0 0 1 1 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0  
[157] 0 0 0 0 1 0 0 1 0 0 0 1 1 0 0 0 0 0 0 1 0 1 0 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0  
[196] 0 0 1 1 0 0 1 1 1 0 0 0 0 1 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0  
[235] 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1  
[274] 0 1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0  
[313] 1 0 1 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 1 0 1 0 0 1 0 0 1 0 0 0 0 1 1  
[352] 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 1 0 1  
[391] 0 0 0 0 0 1 0 0 0 1 0 0 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0  
[430] 0 0 0 1 0 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 1 1 1  
[469] 1 1 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 0 1 0 1 0 0  
[508] 0 1 0 1 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 1 1 0 1 0 1 1 0 0 0 0 0  
[547] 0 0 0 1 0 0 0 0 0 0 1 0 0 0 1 1 0 0 0 1 0 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0 0 0  
[586] 0 0 1 0 1 0 0 0 1 1 0 1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 1 0 1 0 0 0 0 1 0 0 0  
[625] 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 1 1  
[664] 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 1 1 1 0 0 1 0 0 0 1 0 1 0 0  
[703] 0 1 0 0 1 1 0 0 1 0 0 0 1 0 0 1 1 0 0 1 0 0 0 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0 1  
[742] 0 1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 1 0 1  
[781] 0 0 0 0 1 0 0 0 1 0 0 0 0 1 0 0 1 0 1 0 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0  
[820] 0 0 0 0 0 1 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0  
[859] 0 0 0 1 0 0 1 0 0 1 1 1 0 1 0 0 0 1 0 0 1 1 1 0 0 0 0 0 0 1 0 0 0 0 1 0 0  
[898] 0 0 0 0 1 1 0 0 1 0 0 1 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 1 1  
[937] 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1 0 1 0 0 0 1  
[976] 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0  
[ reached getopt("max.print") -- omitted 99000 entries ]
```

Hide

```
mean(Forward)
```

```
[1] 0.25825
```

Hide

```
#b) create a vector below to simulate the probability of  $\max(1 \leq k \leq 200) \text{ WK} \geq 10$ 

Forward1 <- rep(0,200)
for (i in 1:100000){Forward1[i] <- as.numeric(max(cumsum(2*rbinom(200,1,0.5)-1)) >= 10)}
Forward1
```

```

[1] 1 1 0 0 0 0 0 0 1 1 1 0 1 1 1 1 1 0 1 0 1 1 0 1 1 1 1 0 1 1 1 0 1 0 1 0 0 1 0
[40] 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 1 0 0 1 1 1 0 0 1 0
[79] 0 1 0 1 0 1 0 0 1 1 1 1 1 0 1 1 1 0 1 0 0 1 0 1 0 0 0 1 0 0 0 1 0 1 0 1 1 0 1
[118] 1 1 1 1 0 0 0 0 0 1 1 0 0 0 1 0 0 1 0 1 1 1 1 0 0 1 1 0 1 0 0 0 0 0 0 0 1 0 1
[157] 1 1 0 1 0 0 0 1 0 0 1 0 1 1 0 0 1 0 0 0 1 0 1 1 0 1 0 1 0 1 1 0 0 1 0 1 0 1 1
[196] 0 1 0 1 0 1 1 0 1 1 1 0 0 1 0 0 1 0 1 0 1 1 1 0 1 0 0 1 0 1 0 0 0 0 1 0 0 0 0
[235] 1 0 0 1 0 1 1 1 1 0 0 1 1 1 1 0 0 0 1 0 1 0 1 1 0 0 1 0 1 1 0 1 0 1 0 0 0 0 1
[274] 0 1 1 0 0 1 0 0 0 1 0 0 1 1 0 0 0 0 1 1 1 0 0 1 0 0 1 1 0 1 0 0 0 0 1 0 0 0 0
[313] 0 0 0 0 1 1 1 1 1 0 0 1 1 0 0 0 0 1 1 1 0 1 1 1 0 1 0 0 1 1 0 0 0 0 0 1 0 0 1
[352] 0 1 1 1 0 0 0 1 0 0 0 1 1 1 0 0 0 1 1 0 1 0 0 0 0 1 0 0 1 0 1 0 0 1 1 1 0 1 1
[391] 0 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 0 1 1 1 1 1 0 1 0 0 0 1 1 1 1 1 1 1 1 1 0 1 0 0
[430] 1 1 1 1 0 0 0 1 1 0 1 1 0 1 0 1 0 0 0 1 0 0 1 0 1 1 1 0 0 0 1 1 1 1 0 0 0 0 0
[469] 1 1 1 0 0 0 1 1 0 1 0 0 1 0 0 0 0 0 0 0 1 0 0 0 1 0 1 0 0 0 1 1 1 0 0 0 0 0 1 1
[508] 1 0 1 1 1 1 0 1 1 1 1 1 0 1 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 1 0 1 0 1 0 1 1 1 1
[547] 0 1 0 1 0 1 1 0 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 0 1 0 1 1 0 0 1 0 1 0 1 0 0 0 0
[586] 0 0 0 0 0 0 1 0 1 1 1 0 0 1 1 1 0 0 1 0 0 0 0 0 0 1 1 1 1 0 0 1 1 0 0 1 1 0 1
[625] 1 0 1 1 0 0 1 1 0 1 0 0 1 0 1 1 1 0 1 0 1 1 1 0 1 1 1 1 0 0 1 1 0 1 0 0 1 1 0
[664] 0 1 1 0 0 1 1 0 1 0 1 1 1 0 1 0 1 1 0 0 0 0 0 0 1 1 0 1 1 0 1 1 0 1 1 1 1 1 0
[703] 0 1 1 1 0 0 0 0 1 0 1 1 1 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1 0 1 1 1 1
[742] 1 0 1 1 0 1 1 0 0 0 1 1 1 1 0 0 0 0 0 1 1 1 1 0 0 0 1 1 1 1 0 0 1 0 0 1 0 0 1
[781] 0 1 1 0 1 1 1 1 1 1 0 1 0 1 0 1 1 1 1 1 0 0 1 1 0 1 0 0 1 0 0 0 0 1 1 0 0 1 0
[820] 0 1 0 0 1 0 1 0 1 0 0 0 0 1 1 0 0 1 1 0 0 0 0 1 0 0 0 0 1 0 0 1 0 1 1 0 0 0
[859] 0 0 1 1 1 1 1 1 0 0 1 1 1 0 1 1 0 0 1 1 0 1 1 0 0 1 1 0 1 0 0 1 1 1 1 0 1 1 0
[898] 0 1 1 0 0 1 1 1 1 1 1 0 0 0 0 1 0 1 0 0 1 1 0 1 0 0 1 0 1 1 0 0 0 1 0 0 1 1
[937] 1 0 0 1 0 0 1 0 0 0 1 1 0 0 0 1 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 1 0 0 1 0 0 1 1
[976] 1 1 1 1 0 0 0 0 1 0 0 1 0 0 1 1 0 1 0 1 1 0 1 1 0
[ reached getOption("max.print") -- omitted 99000 entries ]

```

Hide

```
mean(Forward1)
```

```
[1] 0.48127
```

Hide

```

#c)

# Theoretical

# pbinom
#Note: 2*sum(Bj) >=1*200+10
pbinom(104, 200, 0.5, lower.tail = FALSE)

```

```
[1] 0.2623112
```

Hide

```

# pbinom is calculating the cumulative probability of getting a result equal to or bigger tha
n 10 and for the simulation result runned 100 000 times, 26.4% of the times the values were b
igger or equal to 10, whereas theoretically 26.23% of the times the values where bigger or eq
ual to 10, but it can vary.

```

Hide

```
#2-Vk= Y1+...Yk (Binomial Random Variable)
Vk<-2*rbinom(200,1,0.55)-1
Vk
```

```
[1] -1  1  1  1  1  1  1  1 -1  1 -1 -1  1 -1 -1  1 -1 -1 -1  1  1 -1  1
[27] -1  1  1 -1 -1 -1  1  1 -1  1 -1 -1 -1  1  1 -1  1  1 -1 -1  1  1 -1  1
[53]  1 -1  1 -1  1 -1  1 -1 -1  1 -1  1  1 -1  1  1  1 -1 -1  1  1 -1 -1  1
[79]  1  1  1 -1  1 -1 -1  1  1  1  1 -1 -1  1  1 -1  1  1 -1 -1  1  1 -1 -1
[105] -1  1 -1 -1  1 -1  1 -1  1  1  1  1 -1 -1  1 -1  1  1  1 -1  1  1 -1 -1
[131]  1  1 -1 -1  1 -1  1  1 -1  1  1  1 -1  1  1 -1  1  1  1  1  1  1  1 -1
[157] -1  1 -1 -1 -1  1 -1  1  1 -1 -1 -1  1  1  1 -1  1  1 -1  1 -1 -1  1  1
[183]  1 -1 -1  1 -1  1 -1 -1 -1  1  1 -1  1 -1  1 -1  1 -1
```

Hide

```
#Another method would be by using the sample function.
Vk1<-sample(c(1,-1),200,replace=T,prob=c(0.55,0.45))
Vk1
```

```
[1] -1 -1 -1 -1 -1  1 -1 -1  1  1 -1 -1  1  1  1 -1  1 -1 -1 -1  1 -1 -1  1 -1
[27]  1 -1 -1 -1 -1 -1 -1  1  1  1  1 -1 -1 -1 -1  1  1 -1  1  1 -1 -1 -1  1  1
[53]  1  1 -1 -1  1  1 -1 -1 -1 -1  1 -1 -1 -1  1  1 -1  1  1  1 -1  1 -1 -1  1 -1
[79] -1  1  1  1  1  1  1 -1  1 -1  1 -1 -1 -1  1  1 -1 -1 -1 -1  1 -1 -1  1 -1  1
[105]  1 -1 -1  1 -1  1 -1  1 -1 -1  1  1 -1  1  1 -1  1  1 -1  1  1  1  1  1  1
[131] -1 -1 -1 -1  1  1 -1 -1 -1 -1  1  1 -1 -1 -1  1 -1 -1 -1  1  1  1 -1 -1  1 -1
[157]  1 -1 -1  1 -1 -1  1 -1 -1  1 -1 -1 -1  1  1  1 -1  1 -1 -1  1  1  1 -1  1 -1
[183] -1 -1  1 -1  1  1  1 -1 -1 -1 -1  1  1  1  1  1 -1 -1
```

Hide

```
#a) create a vector below to simulate the probability of Wk >= 10

Forward2 <- rep(0,200)
for (i in 1:100000){Forward2[i] <- as.numeric(sum(2*rbinom(200,1,0.55)-1) >= 10)}
#Simulation Result
Forward2
```

```

[1] 0 1 0 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1
[40] 1 1 0 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 1 0 1
[79] 0 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0 0 1 1 1 1 0 1 0 0 0 1 1 1 1 1 1 1 1
[118] 1 0 0 1 0 1 0 1 1 1 0 1 1 1 1 1 1 0 1 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1
[157] 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 0 1 1 1 1 0 1 1 1 1 1 1 0 0 0 1 1 1 1
[196] 1 0 0 0 0 1 1 1 1 1 1 1 0 1 1 1 0 1 1 0 0 1 1 0 1 1 1 0 1 1 1 0 1 1 1 0 1 0
[235] 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 0 1 1
[274] 1 0 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1
[313] 1 0 1 0 0 1 1 0 1 1 1 0 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1
[352] 1 1 1 1 1 0 1 1 0 1 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1
[391] 1 1 1 1 0 1 1 1 1 0 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0
[430] 1 1 1 0 1 0 1 1 1 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 0 0 0 1 1 1 0 0 1 1
[469] 1 1 1 1 1 1 1 1 1 0 1 0 0 1 1 1 1 1 0 0 1 1 1 1 1 0 1 0 0 1 1 1 1 1 1 0 1 1 1 1
[508] 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 1 1 0 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 0 0 1 1 0 1
[547] 1 1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1
[586] 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 0 0 1 1 1 1 1 0 1 0 1 1 1 0 1 1 1 1 0 1 1 1 1 1
[625] 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 0
[664] 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 0 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 0 1 1 0 0
[703] 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 0 0
[742] 0 1 1 1 1 0 1 1 1 1 1 0 0 1 1 0 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 0 0 0 1 1 1 1 0
[781] 1 1 1 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1
[820] 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 0 1 1 0 1 1 0 0 1 1 0 1 1 1 1
[859] 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1
[898] 0 0 1 1 0 1 0 0 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 0 1 1 1 1 1 1 0
[937] 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 0 0 1 0 1 0 1 1 1 0 1 0 1 1 1
[976] 1 1 1 1 0 1 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1
[ reached getOption("max.print") -- omitted 99000 entries ]

```

Hide

```
mean(Forward2)
```

```
[1] 0.78305
```

Hide

```
#b) create a vector below to simulate the probability of  $\max(1 \leq k \leq 200) W_k \geq 10$ 
```

```

Forward3 <- rep(0,200)
for (i in 1:100000){Forward3[i] <- as.numeric(max(cumsum(2*rbinom(200,1,0.55)-1)) >= 10)}
Forward3

```

```

[1] 1 0 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1
[40] 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 0
[79] 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0
[118] 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1
[157] 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[196] 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 0 1
[235] 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[274] 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1
[313] 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0 1 1 1 0 1 0
[352] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 1 1
[391] 1 1 0 0 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 0 1 1 1 1
[430] 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1
[469] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 0 1
[508] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1
[547] 1 0 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[586] 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 0 1
[625] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1
[664] 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 0 0 1
[703] 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1
[742] 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 1 1 1 1 1 0
[781] 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1
[820] 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[859] 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1
[898] 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0
[937] 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1
[976] 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1
[ reached getOption("max.print") -- omitted 99000 entries ]

```

Hide

```
mean(Forward3)
```

```
[1] 0.8889
```

Hide

```

# Theoretical

# pbinom
#Note: 2*sum(Bj) >=1*200+10
pbinom(104, 200, 0.55, lower.tail = FALSE)

```

```
[1] 0.78305
```

Hide

```

# pbinom is calculating the cumulative probability of getting a result equal to or bigger than 10 and for the simulation result runned 100 000 times, 78.3% of the times the values were bigger or equal to 10, whereas theoretically 78.31% of the times the values where bigger or equal to 10, but it can vary.

```

Hide

```
#3- Uk= Z1+...Zk (Normal Random Variable)
```

```
Uk<-rnorm(200,mean=0, sd=1)
```

```
Uk
```

```
[1] -0.869127604 -2.020300768 -1.285650697 -0.326393481 -0.087500036 0.249767239
[7] 0.356693924 0.182340957 0.437081046 -0.412481818 -1.979036354 0.406420867
[13] 0.044685302 -1.406610792 2.599205007 1.040369149 2.030855803 0.337940269
[19] 2.238440970 1.126256353 0.426141308 1.715821099 0.006734780 1.115350850
[25] 0.191985217 -1.050180773 -0.007805107 -0.560920301 1.111927215 -0.542579865
[31] -0.172802541 0.541566836 0.286392109 -1.974744114 -0.642683631 -0.694615982
[37] 0.394270999 0.486367520 -1.662690556 -0.770712255 2.581219947 0.363621338
[43] 0.690701874 -2.950612702 -1.102123566 0.123677549 -0.880728729 0.437706832
[49] 1.117407778 -1.055084808 0.028939497 -0.562933648 0.734026747 -0.175986284
[55] 0.459950447 -1.695172276 -0.838864958 -0.463708743 -1.144437255 1.364497290
[61] -0.893480464 -1.329004306 -0.573795394 -0.881043228 -0.506268116 -0.922617566
[67] -1.797258635 -1.086255133 -0.506521379 0.555207584 -1.410592161 -1.330286671
[73] 1.822881236 -0.325560901 -0.046735685 0.392948955 -0.869001800 -0.979325216
[79] 0.357429049 -0.785561308 0.107853762 0.704790219 -0.540002898 -0.056695314
[85] -1.740317440 -0.488958209 1.475799046 0.470515006 0.888574680 0.001011775
[91] -0.936729929 0.152848770 -1.922397395 1.675913842 0.088970520 1.349154574
[97] -1.292249759 -1.487036974 0.660398039 0.925530089 -0.019795788 -0.158646656
[103] -0.188074865 -1.427963502 -0.894978742 -0.816198678 -0.515069632 0.595300387
[109] 0.548866030 0.117366914 0.170271072 -0.439981548 0.219280804 -0.614253024
[115] -0.219524208 -0.713776904 -0.245057008 0.570538559 0.856696823 -0.381232514
[121] -1.060901490 -0.695882233 -0.842275412 0.309310923 -0.459707428 -0.519805584
[127] -1.193648545 -1.301454459 -0.866412962 -1.127753550 -2.286410971 0.630110886
[133] 0.974269578 -1.636131314 0.410987194 1.013308232 1.360474169 0.696167278
[139] -0.379220647 -0.099377775 -2.512788325 -0.408544384 2.265503048 1.382585878
[145] 0.086741588 -1.591264297 1.069864490 -0.255661813 -0.582064819 0.227617972
[151] -0.415090817 0.550478048 0.143404285 -0.713768013 0.495659158 1.197113169
[157] 1.120591630 0.855748042 1.113502552 -0.992186598 0.215697566 -1.365558221
[163] 0.342391799 0.828804986 2.067183632 1.497159008 -1.642404132 -0.967888058
[169] -0.955438617 0.558787300 -1.144891258 -1.539764303 -1.488569654 0.398855370
[175] -0.308604992 -1.264105009 1.021337733 -0.179949376 -0.336202234 0.442611481
[181] -0.217111775 0.037576295 0.807411799 -0.183342727 1.108883727 -0.237071745
[187] 0.262104055 0.834387744 -0.355860204 0.256551626 -1.834036528 1.156058376
[193] -1.343437704 -0.595666033 0.403331982 -0.363511892 0.036138899 0.048143091
[199] 0.973880052 -0.411447919
```

Hide

```
sum(Uk)
```

```
[1] -24.21671
```

Hide

```
#a) create a vector below to simulate the probability of Wk >= 10

Forward4 <- rep(0,200)
for (i in 1:100000){Forward4[i] <- as.numeric(sum(rnorm(200,mean=0, sd=1)) >= 10)}

#Simulation Result
Forward4
```

```
[1] 0 1 0 1 0 0 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0
[40] 0 1 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
[79] 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 1 1 0 0 0 1 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
[118] 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 0
[157] 1 0 0 0 1 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 1
[196] 0 0 0 0 1 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 0 0 0
[235] 1 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0 1 0 0 0 0 1 1 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0
[274] 0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0
[313] 0 0 1 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 1 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 1 1 0
[352] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
[391] 1 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0
[430] 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 1 0 0 1 1 0 0 0 0 0 0 0
[469] 0 0 0 1 1 0 0 1 1 1 0 1 1 0 0 1 0 0 0 0 1 1 0 0 0 0 1 1 0 0 1 0 0 1 1 1 0 0 0 0 0 0
[508] 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 1 0
[547] 1 0 0 0 0 1 1 0 1 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0
[586] 0 1 0 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 1 1 1 1 0 0 1 0 0 0 0 0
[625] 0 1 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 1 0 0 1 1 0 0 0
[664] 0 0 0 0 0 0 0 1 0 0 0 0 1 0 1 1 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0
[703] 1 0 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1
[742] 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 1 0 1 0 1 0 0 0 1 1 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
[781] 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0
[820] 0 0 1 1 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 1 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
[859] 0 0 1 0 0 1 0 0 1 0 1 0 0 0 0 0 0 1 0 1 0 1 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1
[898] 1 0 0 1 1 0 1 0 1 0 1 0 0 0 1 1 1 0 1 0 1 0 0 1 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1 0 0
[937] 0 0 1 1 0 0 1 1 1 0 0 0 1 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
[976] 0 0 1 1 0 0 0 0 1 1 0 1 0 1 1 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
[ reached getopt("max.print") -- omitted 99000 entries ]
```

Hide

```
mean(Forward4)
```

```
[1] 0.24056
```

Hide

```
#b) create a vector below to simulate the probability of  $\max(\text{cumsum}(1 \leq k \leq 200)) \text{ Wk} \geq 10$ 

Forward5 <- rep(0,200)
for (i in 1:100000){Forward5[i] <- as.numeric(max(cumsum(rnorm(200))) >= 10)}
Forward5
```



```

[1] 0 1 1 0 1 0 1 1 1 0 0 0 1 1 1 0 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1
[40] 0 0 0 1 0 1 0 1 1 0 0 0 1 0 1 0 0 0 1 0 0 1 1 1 1 0 1 0 1 1 0 1 1 0 1 0 0 1 1
[79] 0 0 0 1 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 1 0 1 0 0 1 1 0 0 1 1
[118] 1 1 0 1 0 0 1 1 0 1 0 1 0 0 1 0 0 1 1 0 0 1 1 0 0 0 1 0 1 0 1 0 1 1 0 1 1 0 0
[157] 1 1 0 1 0 1 0 0 1 1 0 1 1 0 0 0 1 0 0 0 0 1 0 1 1 0 0 0 1 1 0 0 1 1 0 1 1 1 0
[196] 1 1 0 0 1 0 1 1 0 0 1 0 1 1 1 0 1 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0 1 0 1 0 1 1 1
[235] 1 1 1 0 0 0 0 0 0 1 1 0 0 0 1 0 0 1 1 1 1 1 1 0 0 0 0 1 0 0 0 1 1 1 1 0 0 1 0
[274] 0 0 1 1 0 0 0 0 0 1 1 0 0 1 1 1 1 0 0 1 1 1 0 0 0 0 1 0 1 1 1 1 1 0 0 1 1 1 1
[313] 0 1 0 0 0 0 0 1 0 1 0 0 1 0 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 0 0 0 1
[352] 0 1 0 1 1 0 0 1 1 1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 1 1 1 0 0
[391] 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 1 1 1 1 0 1 1 0 1 1 0 1 1 0 0 0 0 0
[430] 0 1 0 0 0 1 1 0 0 1 1 1 0 1 0 0 1 1 1 0 1 1 1 0 0 1 1 0 0 0 1 1 1 0 1 0 0 0 0
[469] 1 0 1 1 1 0 1 0 0 1 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 1 1 1 0
[508] 0 1 0 1 0 1 1 1 1 0 1 1 0 1 0 1 1 0 1 1 0 1 0 1 0 1 0 1 0 1 1 0 0 1 0 0 0 0 1
[547] 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 1 0 0 1 0 0 1 0 0 1 1 1 1 1 0 0 1 0 1 1
[586] 1 1 1 1 1 0 0 1 0 0 1 0 1 1 0 1 0 1 0 0 0 1 0 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 0 1
[625] 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0
[664] 0 0 0 1 1 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1 1 0 0 1 0 1 1 1 1 1 0 1 1 1 0 1 1 0 1 1
[703] 1 1 1 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 1 1 1 1 0 0 1 0 1 0 0 1 1 0 0
[742] 1 1 1 1 0 1 1 0 1 1 0 0 0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 0 1 1 0 1 1 1 1 1 0 0 0
[781] 1 1 0 0 1 0 1 1 0 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1 0 0 0 1 1 1 0 0
[820] 0 0 0 1 0 0 1 0 0 0 0 1 0 0 0 0 1 1 0 1 0 1 0 0 1 0 0 0 0 1 0 1 0 1 0 1 0 0 0
[859] 1 0 1 0 0 1 1 0 0 0 0 0 1 0 0 1 1 0 1 0 1 0 0 1 1 1 0 1 0 1 0 1 0 1 1 1 1 0 0
[898] 1 0 1 0 0 0 0 0 0 0 1 1 0 0 0 0 1 0 1 1 0 0 1 1 1 1 0 0 1 0 1 1 0 0 1 0 1 1
[937] 0 0 1 0 1 1 0 1 0 0 0 0 1 1 1 1 0 1 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 0 0 0 1 0
[976] 0 1 0 1 1 0 1 1 1 0 0 0 0 0 0 0 1 1 0 0 1 0 1 0 0
[ reached getOption("max.print") -- omitted 99000 entries ]

```

Hide

```
mean(Forward5)
```

```
[1] 0.4541
```

Hide

```
#Theoretically
```

```
pnorm(10/(sqrt(200)), lower.tail = FALSE)
```

```
[1] 0.2397501
```

Hide

pnorm is calculating the cumulative probability of getting a result equal to or bigger than 10 and for the simulation result runned 100 000 times, 23.96% of the times the values were bigger or equal to 10, whereas theoretically 23.98% of the times the values where bigger or equal to 10, but it can vary.