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1 Introduction

2 Preliminaries

Definition 2.1 (Markov chains) *A Markov Chain $\mathcal{C}$ is a tuple (Q, δ) where Q is a finite set of states and $\delta : Q \to \mathbb{D}(Q)$ is a probabilistic transition function, where $\mathbb{D}(Q)$ is the set of all probabilistic distribution on a finite set Q. A probabilistic distribution on Q, is a function $f : Q \to \mathbb{Q}_{\geq 0}$ such that $\sum_{q \in Q} f(q) = 1$.*

A non-empty finite word $\varrho = p_1 p_2 ... p_n$ over Q is defined as a run of Markov Chain. The probability of the run is $\prod_{0 \leq i < n} \delta(p_i, p_{i+1})$. ϱ reaches q, if $q = p_i$ for some $0 \leq i \leq n$.

The probability of eventually reaching a set of state $T \subseteq Q$ in $\mathcal{C}$ starting from q_0 can be denoted as $\mathbb{P}^{q_0}_{\mathcal{C}}[\lozenge T]$. $\lozenge T$ is equivalent to $Q \bigcup T$, Q until T. Since not necessary every state in Q is reached before T, a set of states $S \subseteq Q$ is defined, where the path until T only consists of these states. The probability $\mathbb{P}^{q_0}_{\mathcal{C}}[S \bigcup T]$. If $q_0 \in T$, the probability is 1. Otherwise, the probability is calculated as follows:

$$\sum \left\{ \prod_{0 \leq i < n} \delta(q_i, q_{i+1}) \mid q_0 ... q_{n-1} \in (S \setminus T) \ \& \ q_n \in T \quad \text{for } n \geq 1 \right\}.$$

If there exists a set $U \subseteq Q$, where all runs from q_0 to T reaches, the probability of $\mathbb{P}^{q_0}_{\mathcal{C}}[\lozenge T]$ can be reduced to the following lemma:

Lemma 1 *Consider a Markov Chain $\mathcal{C} = (Q, \delta)$ set of states $U, T \subseteq Q$, and a state $q_0 \in Q \backslash U$. If $\mathbb{P}^{q_0}_{\mathcal{C}}[(Q \backslash U) \bigcup T] = 0$, then*

$$\mathbb{P}^{q_0}_{\mathcal{C}}[\lozenge T] = \sum_{u \in U} \mathbb{P}^{q_0}_{\mathcal{C}}[(Q \backslash U) \bigcup u] \mathbb{P}^{q_0}_{\mathcal{C}}[\lozenge T]$$

Definition 2.2 (Markov decision processes) *A (finite and discrete-time) Markov decision processes, MDP, M, is a tuple (Q, A, δ, T) where Q is a finite set of states, A a finite set of actions, $\delta : Q \times A \to \mathbb{D}(Q)$ a probabilistic transition function, and $T \subseteq Q$ a set of target state.*

The notation for the probability of the state p reaching q with the action a, $\delta(p, a)(q)$ will be changed to $\delta(q|p, a)$ for convenience.

Definition 2.3 (Strategies) *A (memoryless deterministic) strategy σ in an MDP $M = (Q, A, \delta, T)$ is a function $\sigma : Q \to A$.*

From MDPs to Chains

3 Temporal logics

4 Conclusion

References