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1 Introduction

2 Preliminaries

Definition 2.1 (Markov chains) *A Markov Chain C is a tuple (Q, δ) where Q is a finite set of states and $\delta : Q \to \mathbb{D}(Q)$ is a probabilistic transition function, where $\mathbb{D}(Q)$ is the set of all probabilistic distribution on a finite set Q. A probabilistic distribution on Q, is a function $f : Q \to \mathbb{Q}_{\geq 0}$ such that $\sum_{q \in Q} f(q) = 1$.*

A non-empty finite word $\varrho = p_1 p_2 ... p_n$ over Q is defined as a run of Markov Chain. The probability of the run is $\prod_{0 \leq i < n} \delta(p_i, p_{i+1})$. ϱ reaches q, if $q = p_i$ for some $0 \leq i \leq n$.

The probability of eventually reaching a set of state $T \subseteq Q$ in C starting from q_0 can be denoted as $\mathbb{P}^C_{q_0}[\Diamond T]$. $\Diamond T$ is equivalent to $Q \bigcup T$, Q until T. Since not necessary every state in Q is reached before T, the probability $\mathbb{P}^C_{q_0}[S \bigcup T]$ with $S \subseteq Q$ is created. If $q_0 \in T$, the probability is 1. Otherwise, the probability is calculated as follows:

$$\sum \{ \prod_{0 \leq i < n} \delta(q_i, q_{i+1}) \mid q_0 ... q_{n-1} \in (S \setminus T) \ \& \ q_n \in T \quad \text{for } n \geq 1 \}$$

Lemma 1 *Consider a Markov Chain $C = (Q, \delta)$ set of states $U, T \subseteq Q$, and a state $q_0 \in Q \backslash U$. If $\mathbb{P}^C_{q_0}[(Q \backslash U) \cup T] = 0$, then*

$$\mathbb{P}^C_{q_0}[\Diamond T] = \sum_{u \in U} \mathbb{P}^C_{q_0}[(Q \backslash U) \bigcup u] \mathbb{P}^C_{q_0}[\Diamond T]$$

Definition 2.2

Theorem 2 () *Let A*

Proof 2.1 *Here you write the proof of the theorem.*

3 Temporal logics

4 Conclusion

References