

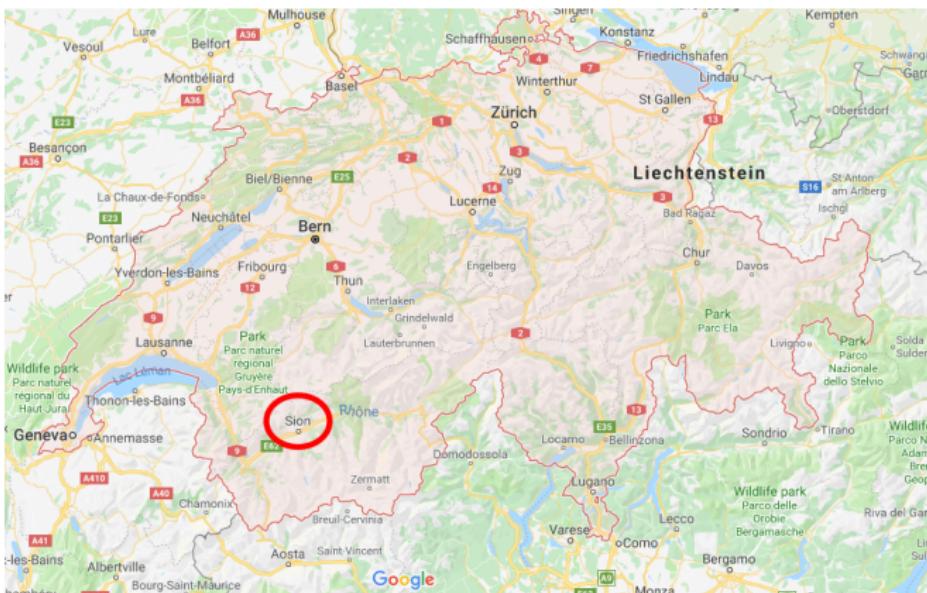
# Rate of Change of Frequency under line contingencies

*Robin Delabays*

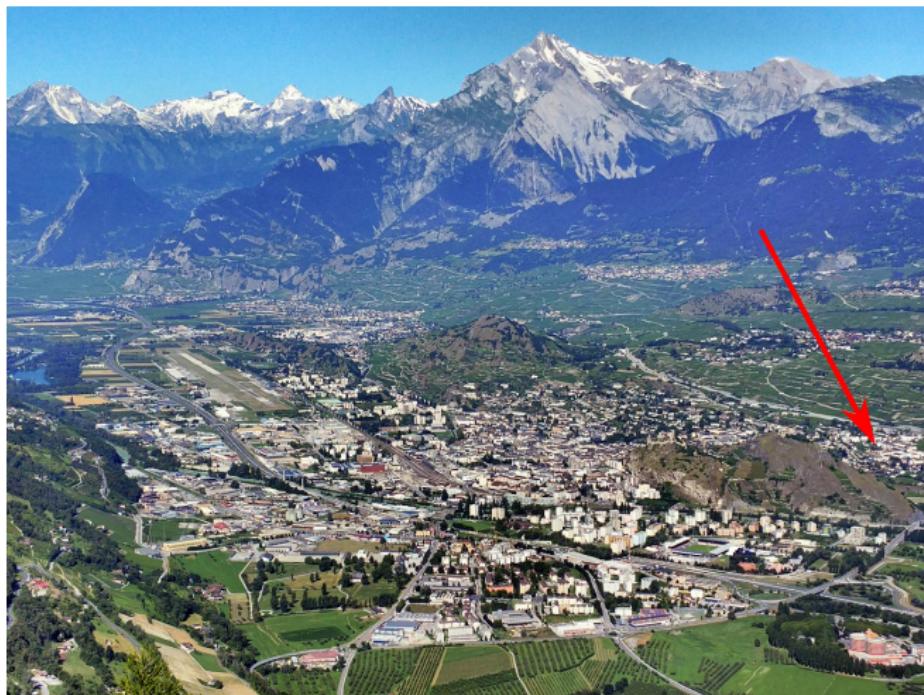
[robin.delabays@hevs.ch](mailto:robin.delabays@hevs.ch)

R. D., M. Tyloo, and P. Jacquod, *arXiv preprint 1906.05698* (2019)

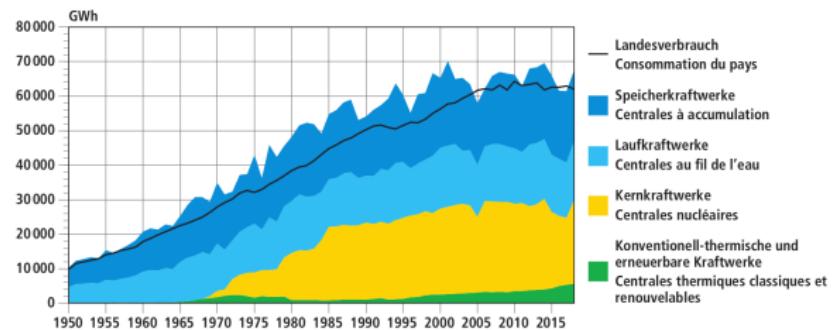
# Where is HES-SO?



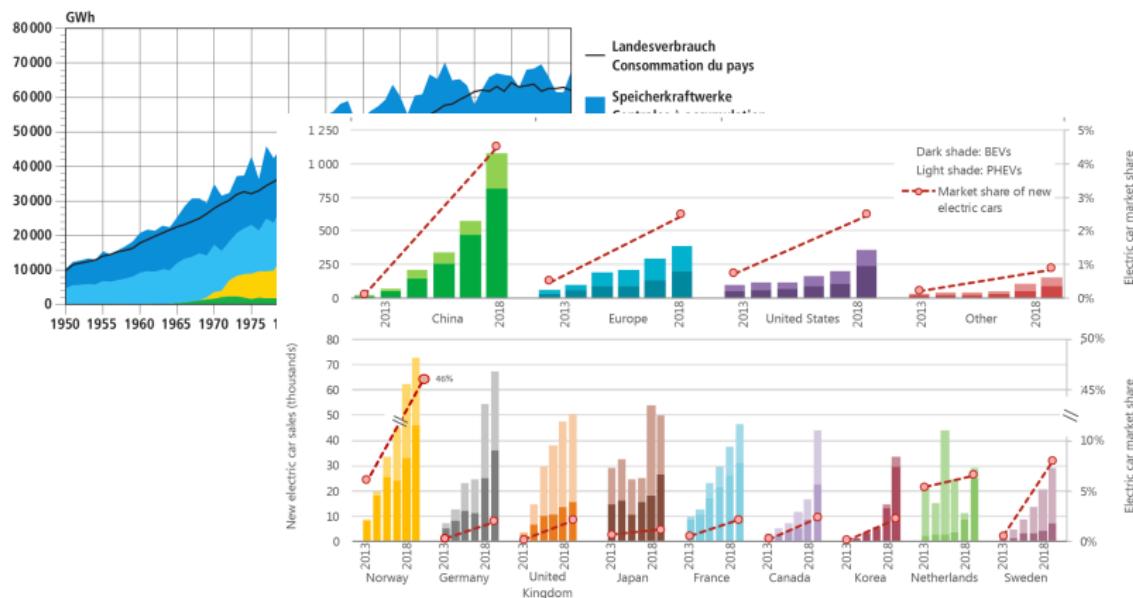
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# Motivation



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OFEN, Statistique Suisse de l'électricité 2018.

IEA, Global EV Outlook 2019 ([www.iea.org/publications/reports/globalevoutlook2019/](http://www.iea.org/publications/reports/globalevoutlook2019/)).

## Motivation

What is the impact of a given contingency?

What are the critical elements in a grid?

How to identify (efficiently) critical operating states?

# The Swing Equations

We consider:

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = P_i - \sum_j b_{ij}(\theta_i - \theta_j), \quad i \in \{1, \dots, n\},$$

$m_i$ : inertia,  $d_i$ : damping,  $b_{ij}$ : susceptance,  $P_i$ : generation/load.

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$$M \ddot{\theta} + D \dot{\theta} = \mathbf{P} - \mathbb{L} \theta,$$

$M = \text{diag}(\mathbf{m})$ ,  $D = \text{diag}(\mathbf{d})$ ,  $\mathbb{L}$  Laplacian matrix.

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Shorthand notation:  $\omega_i := \dot{\theta}_i$ .

# Analytical solution

Assume  $m_i \equiv m$ ,  $d_i \equiv d$ , and consider angle deviations

$$\delta\theta(t) = \theta(t) - \theta^*, \quad \theta^* = \mathbb{L}^\dagger \mathbf{P}_0, \quad \mathbf{P}(t) = \mathbf{P}_0 + \delta\mathbf{P}(t).$$

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Expanding on the eigenmodes of  $\mathbb{L}$ :

$$\mathbb{L}\mathbf{u}^{(\alpha)} = \lambda_\alpha \mathbf{u}^{(\alpha)}, \quad \delta\theta(t) = \sum_{\alpha=1}^n c_\alpha(t) \mathbf{u}^{(\alpha)}.$$

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Analytical solution:

$$c_\alpha(t) = m^{-1} e^{-(\gamma + \Gamma_\alpha)t/2} \int_0^t e^{\Gamma_\alpha t_1} \int_0^{t_1} \delta\mathbf{P}(t_2) \cdot \mathbf{u}^{(\alpha)} e^{(\gamma - \Gamma_\alpha)t_2/2} dt_2 dt_1.$$

# Contingencies

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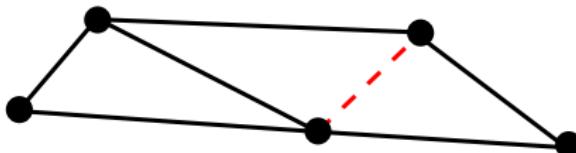
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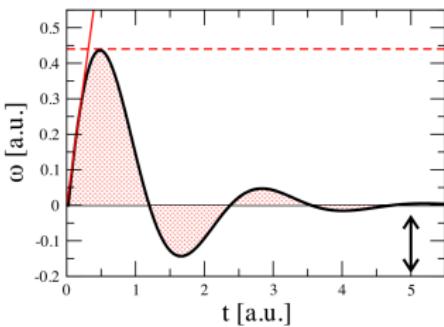


**Line perturbations:** multiplicative,  $\mathbb{L} \rightarrow \mathbb{L} - \beta \mathbf{e}_{ij} \mathbf{e}_{ij}^\top$ .



# Measures of the impact

**Transmission losses:**  $\mathcal{L}_2$ -norm of angle deviations.



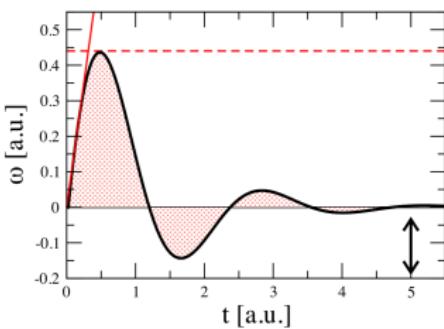
$$\int_0^{\infty} \delta\theta^2(t) dt$$

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- E. Tegling, B. Bamieh, and D. F. Gayme, *IEEE Trans. Control Netw. Syst.* **2** 254 (2015).  
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**Primary control effort:**  $\mathcal{L}_2$ -norm of frequency deviations.



$$\int_0^{\infty} \omega^2(t) dt$$

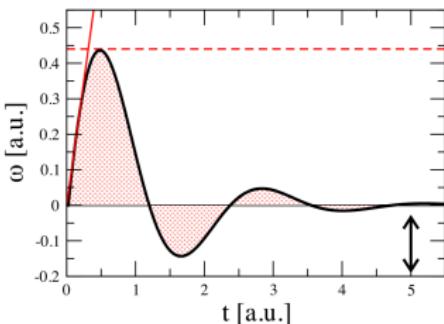
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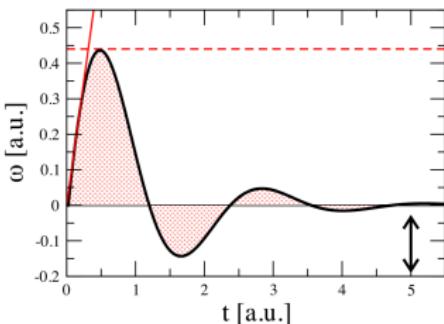
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**RoCoF:**  $\mathcal{L}_{\infty}$ -norm of the time derivative of the frequency.



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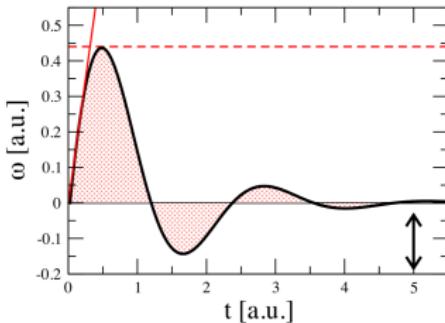
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# The RoCoF

Maximal local RoCoF:

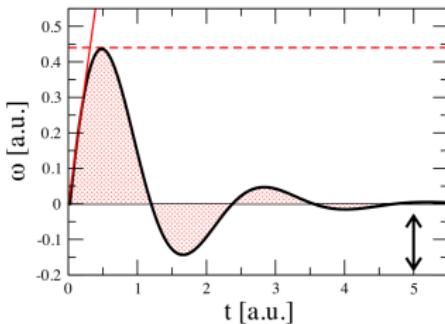
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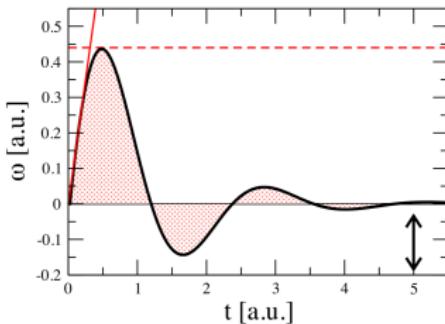
RoCoF is maximal at  $t = 0^+$ .

$$\omega(0) = 0, \quad \mathbf{P} = \mathbb{L} \theta(0), \quad \mathbb{L}^* = \mathbb{L} - b_{ij} \mathbf{e}_{ij} \mathbf{e}_{ij}^\top,$$

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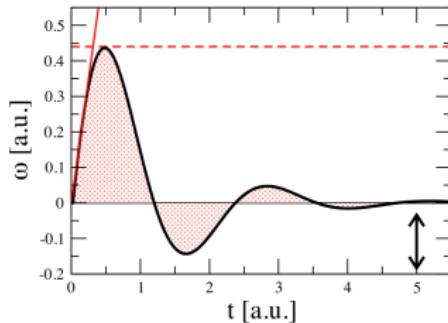
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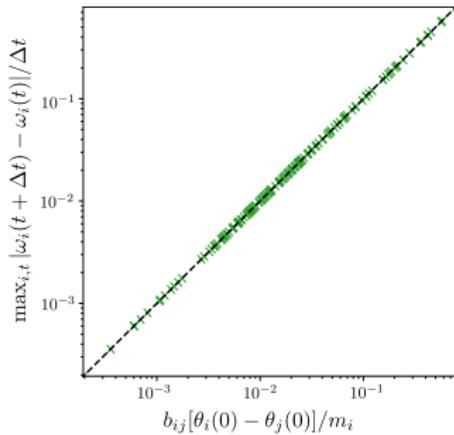
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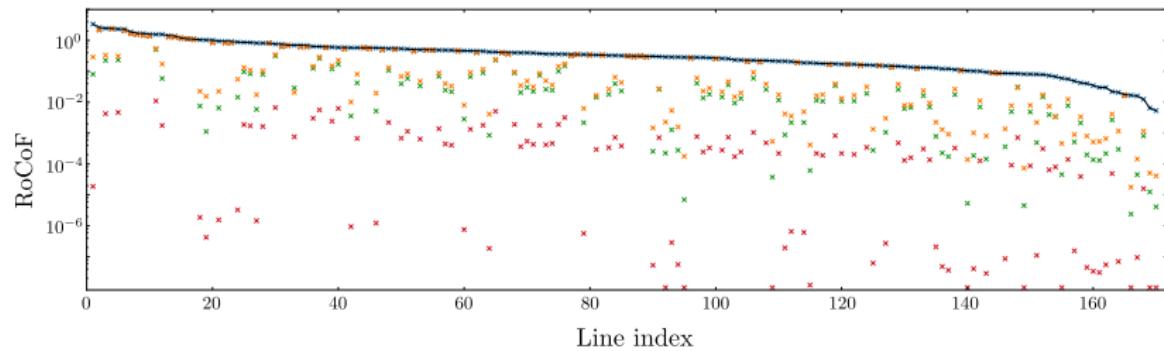
$$M\dot{\omega}(0) + D\omega(0) = \mathbf{P} - \mathbb{L}^* \theta(0),$$

$$\implies \dot{\omega}_k = (\delta_{ik} - \delta_{jk}) \frac{b_{ij}(\theta_i - \theta_j)}{m_k}. \rightarrow \text{RoCoF at nodes } i \text{ and } j.$$

# Numerics (IEEE 118-Bus)



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Black line: theory.

- blue 'x': 100% inertia at loads, RoCoF at all nodes.
- red 'x': 100% inertia at loads, RoCoF at generators only.
- green 'x': 1% inertia at loads, RoCoF at generators only.
- orange 'x': 0% inertia at loads, RoCoF at generators only.

# Including uncertainties

Statistics on generation and loads:

$$\mathbb{E}[P_k] = \mu_k, \quad \mathbb{E}[(P_k - \mu_k)(P_\ell - \mu_\ell)] = \Pi_{k\ell}.$$

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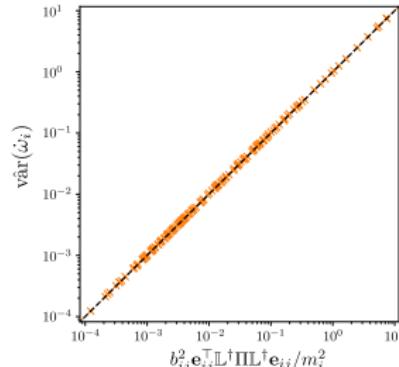
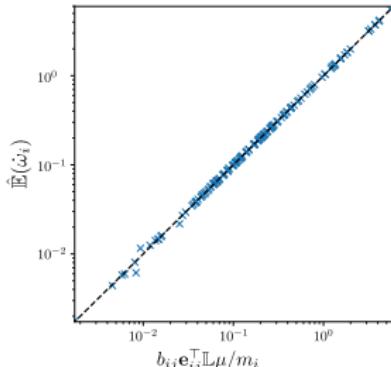
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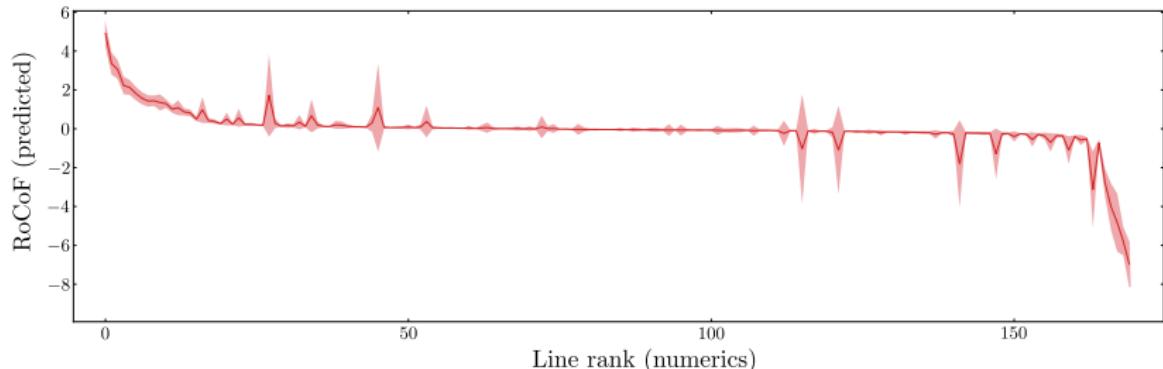
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# Conclusion

The RoCoF after a line loss is:

- ▶ proportional to the flow on the line;
- ▶ inversely proportional to the inertia of the node where it is measured.

If we have only statistics on the power injections, we derive statistics on the RoCoFs.

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**Caveat:** We assume inertia at every nodes, which is not true (yet...).

**Geometry of Complex Webs 2020**

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- Elisenda Feliu (Copenhagen)
- Kathryn Hess-Bellwald (EPFL)
- Philippe Jacquod (HES-SO Valais)
- Ivan Manolescu (Fribourg)
- Toshiyuki Nakagaki (Hokkaido)
- Alan Newell (Tucson)
- Gerd Schröder-Turk (Murdoch Perth)

\* to be confirmed

[sites.google.com/view/geocow2020](http://sites.google.com/view/geocow2020)

**Organizers:**  
Robin Delabays (HES-SO Valais and ETH Zurich)  
Matthieu Jacquemet (HES-SO Valais and Uni Fribourg)  
Christian Mazza (Uni Fribourg)

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