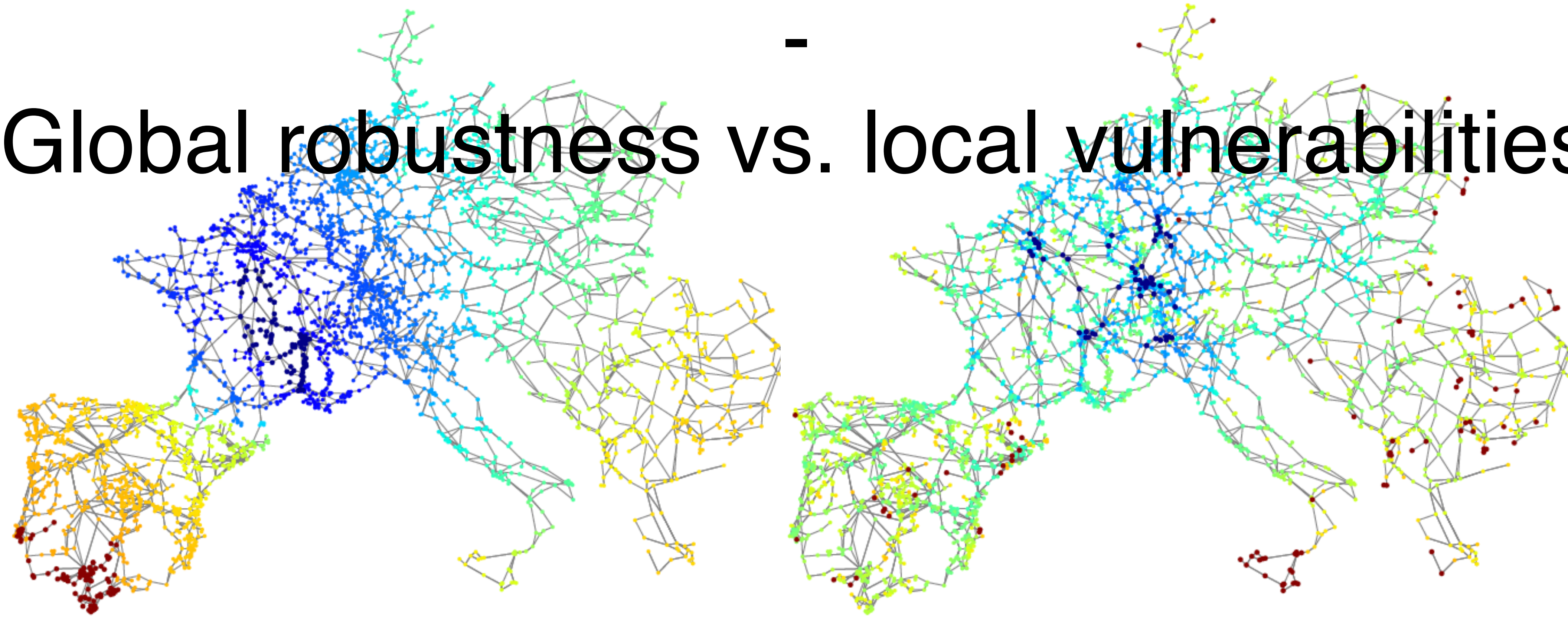
Transient Performance of Electric Power Networks under Colored Noise - Global robustness vs. local vulnerabilities



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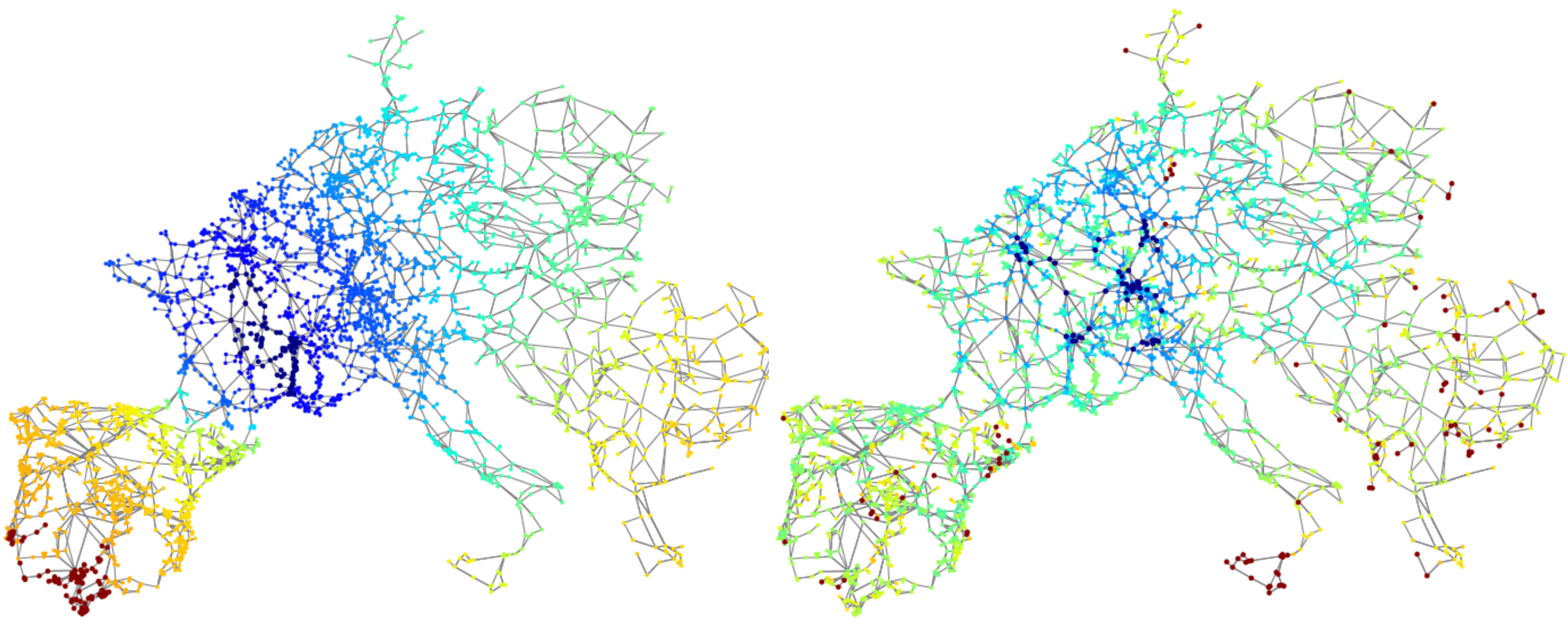
The questions of interest

Given a set of dynamical systems with couplings between them defined on a certain graph :



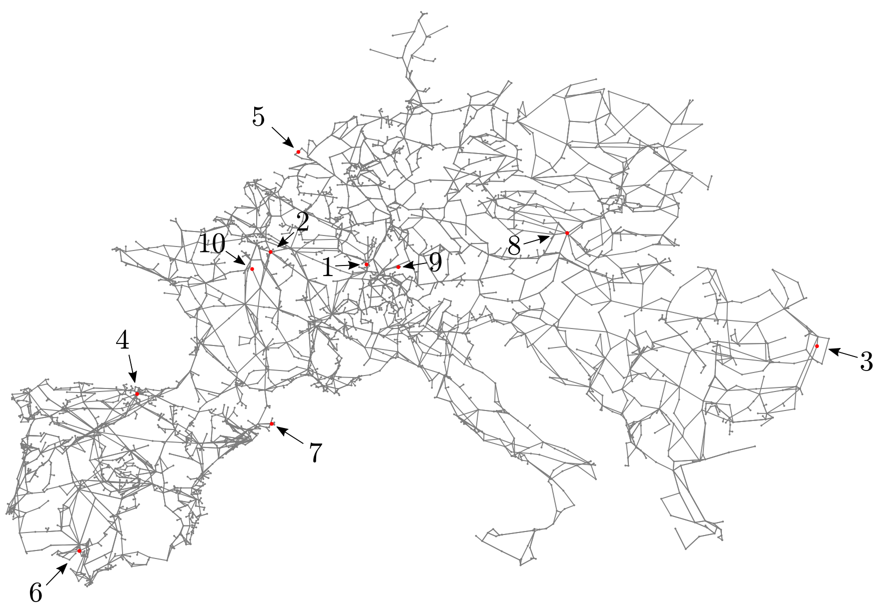
1. Is this network-coupled system globally robust against disturbances ?
2. Where should disturbances act to lead to the worst response ?

Outline

- The model : Electric power grid
  Equilibrium and dynamics
- How we disturb the model :
  Noisy disturbance
- How we assess the model's response .
  Performance measures
- Average vs. nodal disturbance
  From graph topological indices
  to resistance centralities

High Voltage Electric Power Grids

- Lossless line approximation
- Linearization in voltage angles
- Equilibrium : power flow (~Kirchhoff's current law)



$$P = L\theta^{\star}$$

Vector of active power

Network Laplacian ~topology

voltage angles at equilibrium

High Voltage Electric Power Grids

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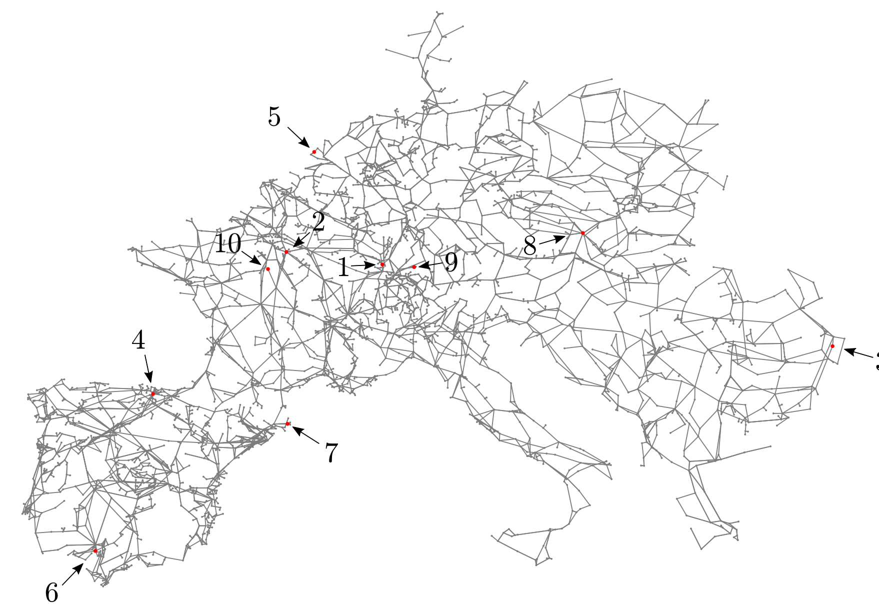


$$\boldsymbol{P} \;=\; \boldsymbol{L}\boldsymbol{\theta}^{\star}$$

- Under disturbance :

$$\boldsymbol{\theta}(t) \;=\; \boldsymbol{\theta}^{\star} + \boldsymbol{\varphi}(t) \qquad\qquad \boldsymbol{\omega}(t) \;=\; \dot{\boldsymbol{\varphi}}(t)$$

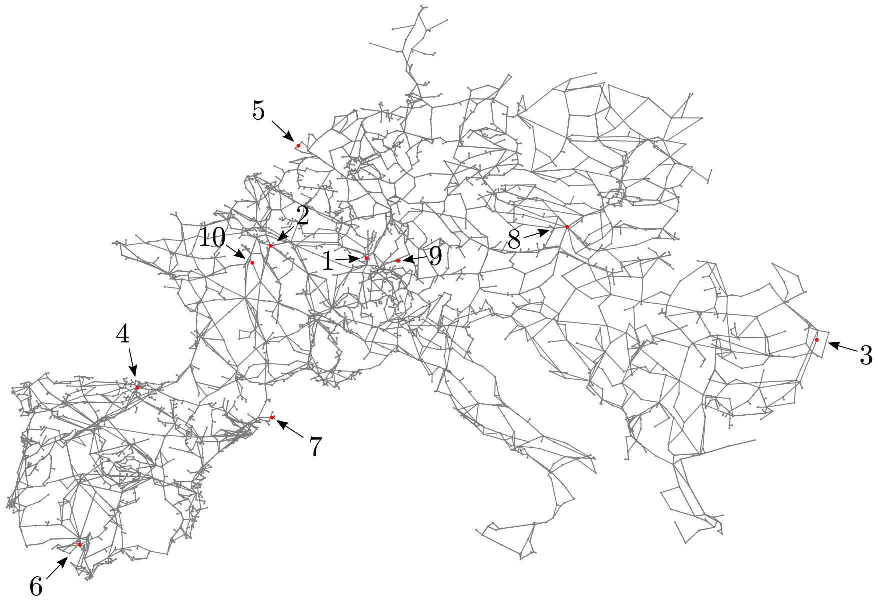
- Dynamics : swing equations (conservation of power)

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} = -\boldsymbol{D}\dot{\boldsymbol{\varphi}} - \boldsymbol{L}\boldsymbol{\varphi} + \boldsymbol{p}(t)$$

Change in kinetic energy	damping (droop)	flow in/out	power in/out

High Voltage Electric Power Grids

- Lossless line approximation
- Linearization in voltage angles
- Equilibrium : power flow



$$\boldsymbol{P} = \boldsymbol{L}\boldsymbol{\theta}^\star$$

- Under disturbance :

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}^\star + \boldsymbol{\varphi}(t) \qquad \boldsymbol{\omega}(t) = \dot{\boldsymbol{\varphi}}(t)$$

- Dynamics : swing equations (conservation of power)

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} = -\boldsymbol{D}\dot{\boldsymbol{\varphi}} - \boldsymbol{L}\boldsymbol{\varphi} + \boldsymbol{p}(t)$$

$$\boldsymbol{M} = \mathrm{diag}(\{m_i\}) \qquad \boldsymbol{D} = \mathrm{diag}(\{d_i\})$$

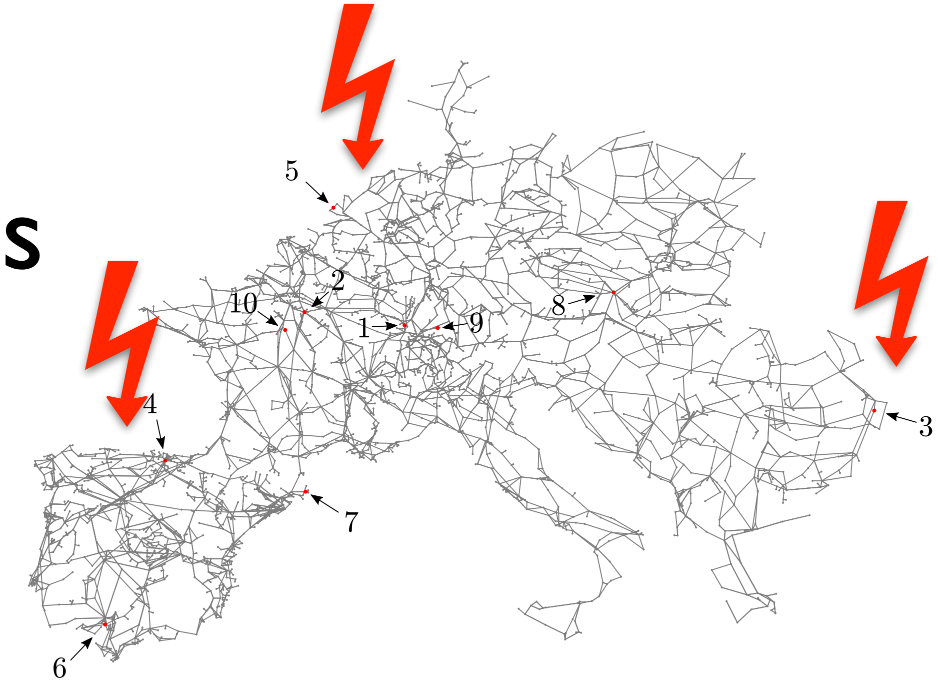
Nodal noise disturbance

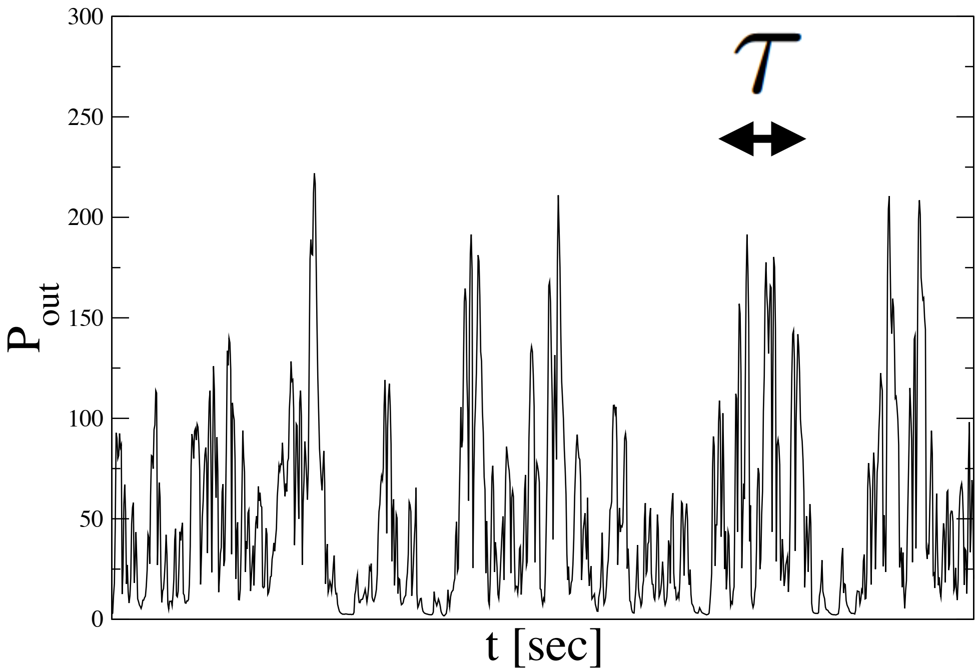
$$M\ddot{\varphi} = -D\dot{\varphi} - L\varphi + p(t)$$

- Time-dependent perturbation with zero average and correlations

$$\mathbb{E}[p_i(t_1)p_j(t_2)] = \delta_{ij}p_i^2 e^{-|t_1-t_2|/\tau}$$

- No spatial correlation
- Characteristic time τ



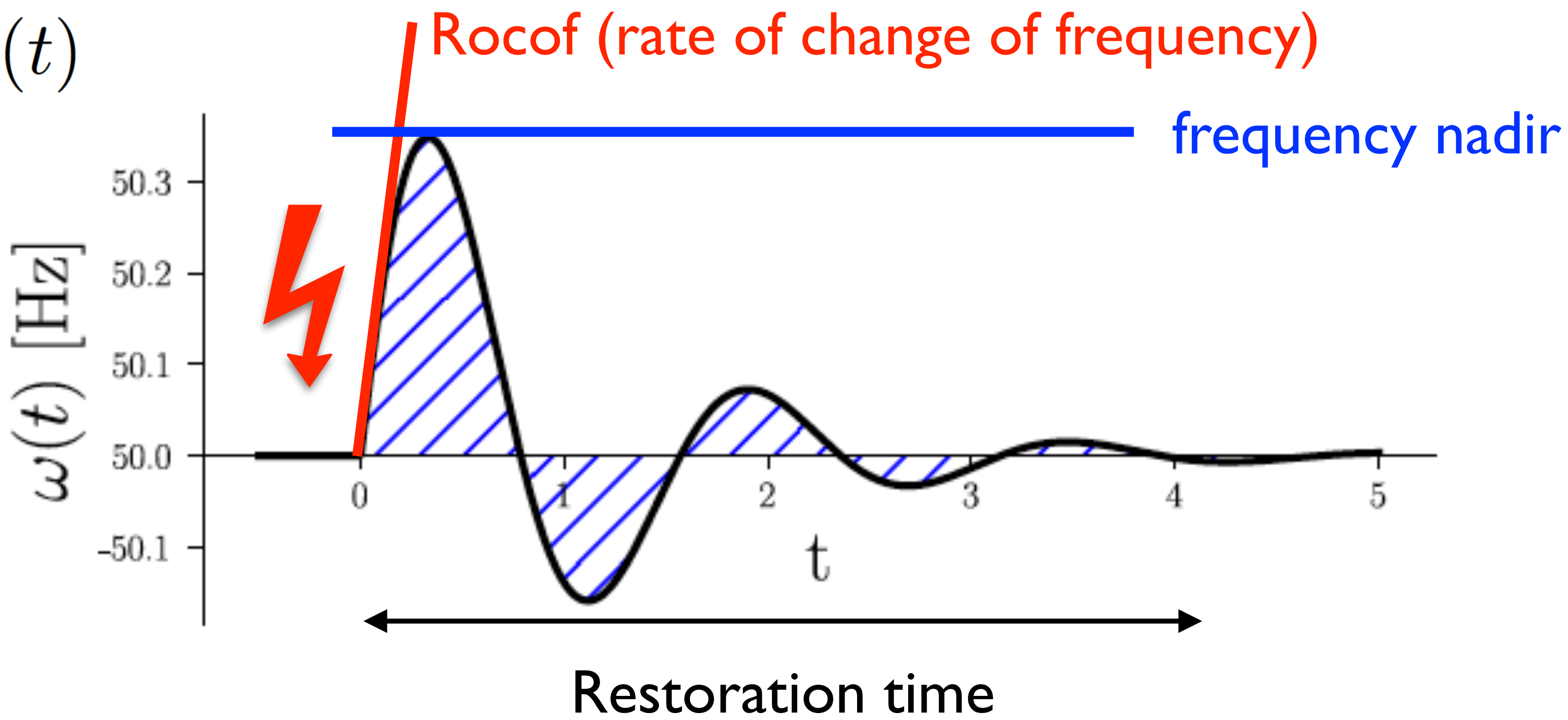


(i) single-node disturbance
(ii)average over all positions

Performance measures

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} = -\boldsymbol{D}\dot{\boldsymbol{\varphi}} - \boldsymbol{L}\boldsymbol{\varphi} + \boldsymbol{p}(t)$$

$$\omega(t) = \dot{\varphi}(t)$$



*Measure magnitude of whole excursion

- Phase coherence $$\mathcal{P}_{\varphi} = \lim_{T\to\infty} T^{-1} \int_0^T \sum_{i=1}^N (\varphi_i(t) - \tilde{\varphi}(t))^2 \mathrm{d}t$$
- Kinetic energy losses $$\mathcal{P}_{\omega} = \lim_{T\to\infty} T^{-1} \int_0^T \sum_{i=1}^N m_i \omega_i^2(t)/2 \,\mathrm{d}t$$

Quadratic performance measures

- Linear system with output

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} = -\boldsymbol{D}\dot{\boldsymbol{\varphi}} - \boldsymbol{L}\boldsymbol{\varphi} + \boldsymbol{p}(t)$$

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{Q}^{(1,1)} & 0 \\ 0 & \boldsymbol{Q}^{(2,2)} \end{bmatrix}^{1/2} \begin{bmatrix} \boldsymbol{\varphi} \\ \boldsymbol{\omega} \end{bmatrix}$$

- Quadratic performance measures

$$\mathcal{P}_{\varphi}, \mathcal{P}_{\omega} = \lim_{t\to\infty} \mathbb{E}[\boldsymbol{y}(t)^{\top}\boldsymbol{y}(t)]$$

- Q-matrices

	$\boldsymbol{Q}^{(1,1)}$	$\boldsymbol{Q}^{(2,2)}$
$\mathcal{P}_{\varphi}$	$\mathbb{I} - \boldsymbol{u}^{(1)}\boldsymbol{u}^{(1)\top}$	0
$\mathcal{P}_{\omega}$	0	$\boldsymbol{M}/2$

Quadratic performance measures - some literature

- Quadratic performance measures / H_2 - L_2 metrics
  - White-noise disturbance
    Bamieh, Jovanovic, Mitra, Patterson '12
    Bamieh and Gayme '13
    Siami and Motee '14
    Grunberg and Gayme '16
    Poolla, Bolognani, Dörfler '17
    Paganini and Mallada '17
  - Colored noise
    Tyloo, Coletta, PJ '18 (no inertia)

Quadratic performance measures for colored noise

How to include colored noise ?

Introduce additional degrees of freedom !

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{L} & -\boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix} \begin{bmatrix} \varphi \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

generates colored noise

couples frequency to it

- Quadratic performance measures

$$\mathcal{P} = \int_0^\infty [\varphi^\top \omega^\top \eta]\, \boldsymbol{Q} \begin{bmatrix} \varphi \\ \omega \\ \eta \end{bmatrix} \mathrm{d}t\,, \quad \boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}^{(1,1)} & 0 & 0 \\ 0 & \boldsymbol{Q}^{(2,2)} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Quadratic performance measures - Grammian calculation

1. Changes of variables - symmetrize Laplacian

$$\overline{\boldsymbol{\varphi}} = \boldsymbol{M}^{1/2}\boldsymbol{\varphi} \qquad \overline{\boldsymbol{\omega}} = \boldsymbol{M}^{1/2}\boldsymbol{\omega}$$

$$\begin{bmatrix} \dot{\overline{\boldsymbol{\varphi}}} \\ \dot{\overline{\boldsymbol{\omega}}} \\ \dot{\eta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \mathbb{I} & 0 \\ -\boldsymbol{L}_{\mathrm{M}} & -\boldsymbol{M}^{-1}\boldsymbol{D} & \boldsymbol{M}^{-1/2}\boldsymbol{p} \\ 0 & 0 & -\tau^{-1} \end{bmatrix}}_{\boldsymbol{A}} \begin{bmatrix} \overline{\boldsymbol{\varphi}} \\ \overline{\boldsymbol{\omega}} \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta(t)\eta_0 \end{bmatrix}$$

$$\boldsymbol{L}_{\mathrm{M}} = \boldsymbol{M}^{-1/2}\boldsymbol{L}\boldsymbol{M}^{-1/2}$$

2. Formal solution

$$\begin{bmatrix} \overline{\boldsymbol{\varphi}}(t) \\ \overline{\boldsymbol{\omega}}(t) \\ \eta \end{bmatrix} = e^{\boldsymbol{A}t} \underbrace{\begin{bmatrix} 0 \\ 0 \\ \eta_0 \end{bmatrix}}_{\boldsymbol{B}}$$

Quadratic performance measures - Grammian calculation

3. Performance measure

$$\mathcal{P} = \boldsymbol{B}^\top \boldsymbol{X} \boldsymbol{B}$$

4. Observability Grammian

$$\boldsymbol{X} \;=\; \int_0^\infty e^{\boldsymbol{A}^\top t} \boldsymbol{Q}_{\mathrm{M}} e^{\boldsymbol{A} t} \, \mathrm{d}t$$

$$\boldsymbol{Q}_{\mathrm{M}} = \begin{bmatrix} \boldsymbol{M}^{-1/2} \boldsymbol{Q}^{(1,1)} \boldsymbol{M}^{-1/2} & 0 & 0 \\ 0 & \boldsymbol{M}^{-1/2} \boldsymbol{Q}^{(2,2)} \boldsymbol{M}^{-1/2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. X determined as solution of Lyapunov equation

$$\boldsymbol{A}^\top \boldsymbol{X} + \boldsymbol{X} \boldsymbol{A} = -\boldsymbol{Q}_{\mathrm{M}}$$

Quadratic performance measures - average vs. localized dist.

- Coherence (assume m_i=m; d_i=d)

Dist. on node # α $$\mathcal{P}_\varphi = \sum_{l\geq 2}^{N} p_\alpha^2 \boxed{u_\alpha^{(l)^2}} \frac{(m+d\tau)}{\lambda_l d(\tau^{-1}m + d + \lambda_l \tau)}$$

Averaged dist.

component of eigenvector of Laplacian on dist. node

- Kinetic energy (assume $m_i/d_i = \gamma$)

Dist. on node # α $$\mathcal{P}_\omega = \frac{1}{2}\sum_{l=1}^{N} \frac{p_\alpha^2 \boxed{u_\alpha^{(l)^2}}}{\gamma(\tau^{-1}m + d + \lambda_l \tau)}$$

Averaged dist. $$\mathcal{P}_\omega = \frac{p^2}{2}\sum_{l=1}^{N} \frac{1}{\gamma(\tau^{-1}m + d + \lambda_l \tau)}$$

Quadratic performance measures - vs. Kirchhoff index

Averaged disturbance (over all nodes)

$$\boxed{\mathcal{P}_\omega = \frac{p^2}{2} \sum_{l=1}^{N} \frac{1}{\gamma(\tau^{-1} m + d + \lambda_l \tau)}}$$

τ large

$$\boxed{\mathcal{P}_\omega = \frac{p^2}{2N\gamma\tau} \boxed{Kf_1}}$$

"Kirchhoff index" (Klein and Randic '93)

$$Kf_1 = N \sum_{l \geq 2} \lambda_l^{-1}$$

Quadratic performance measures - vs. information centrality

Disturbance on node # α

$$\boxed{\mathcal{P}_\omega = \frac{1}{2}\sum_{l=1}^{N} \frac{p^2 {u_\alpha^{(l)}}^2}{\gamma(\tau^{-1}m + d + \lambda_l \tau)}}$$

τ large

$$\mathcal{P}_\omega = \frac{p^2}{2\gamma}\left[\frac{1}{Nd} + \tau^{-1}\left(\boxed{C_\alpha^{-1}} + \frac{Kf_1}{N^2}\right)\right]$$

"Resistance centrality"
"Information centrality"
"Closeness centrality"
(Klein '97)

$$C_\alpha^{-1} = \sum_{j=1}^{N} \Omega_{\alpha j}/N$$

Resistance distance between α and j

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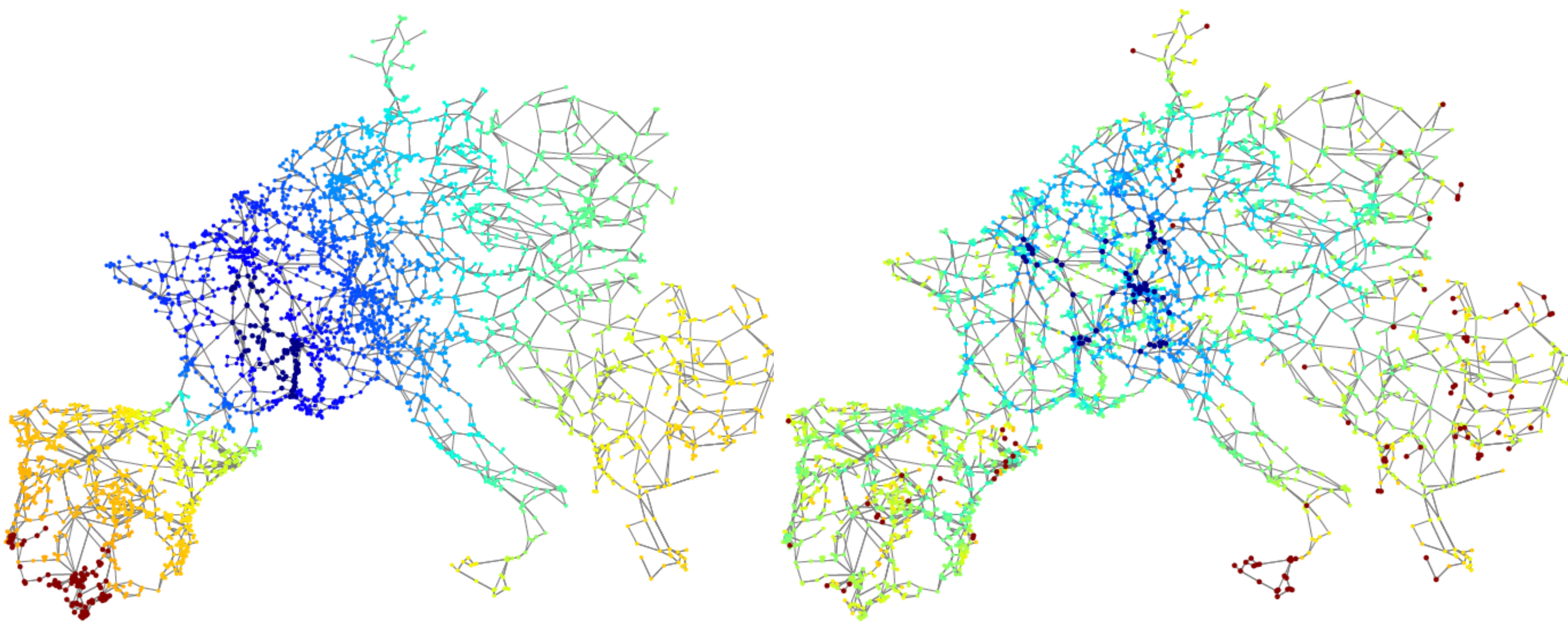

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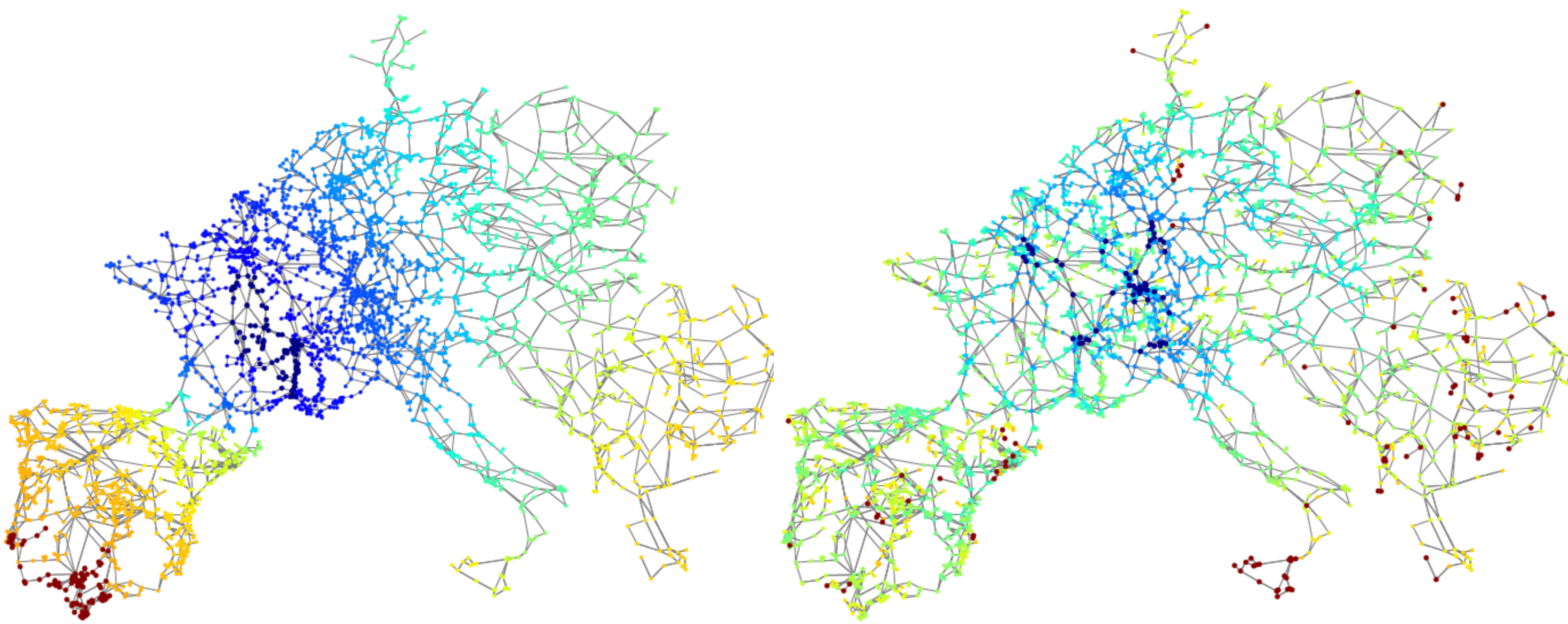

1. Global robustness : Kirchhoff index
2. Local vulnerability : Resistance centrality

Tyloo, Pagnier, PJ arXiv:1810.09694 (note : in HV transmission grid, $\mathcal{T}$ is largest time scale)