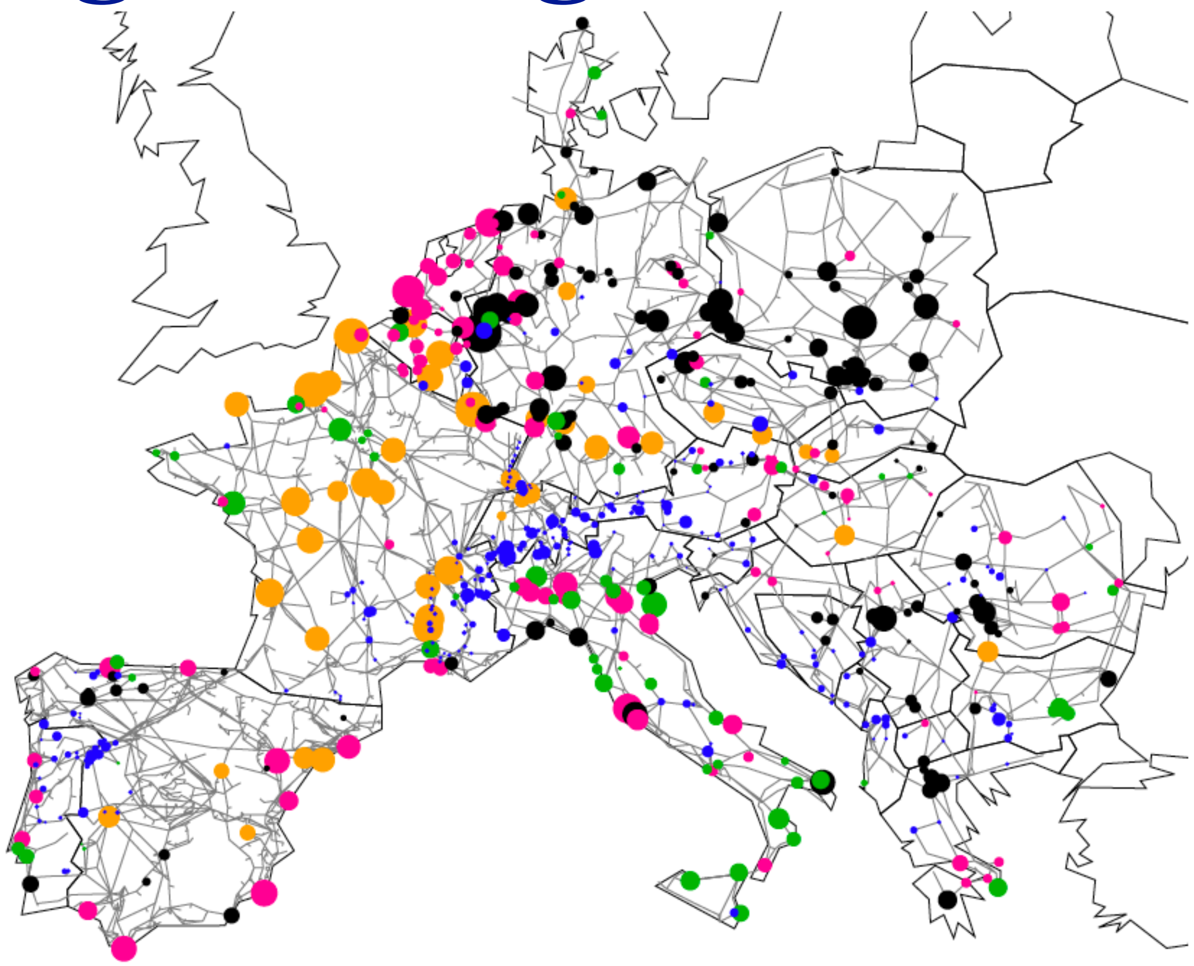
Optimal Placement of Inertia and Primary Control in High Voltage Power Grids



Philippe Jacquod

Dynamics Days - Rostock 03.09.2019

(with L. Pagnier and M. Tyloo)

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

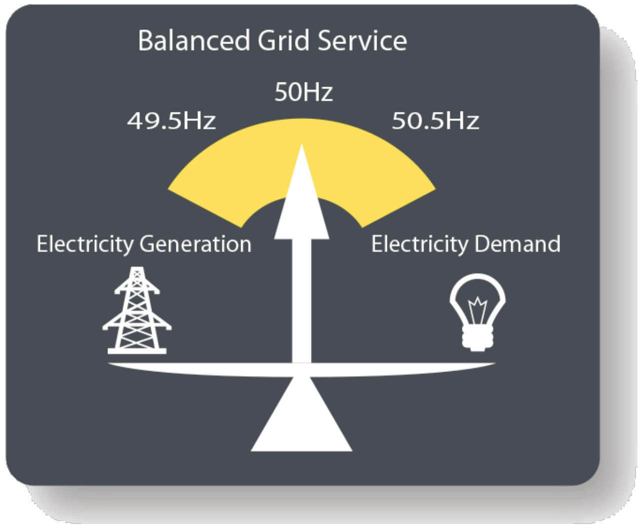
Hes·so VALAIS WALLIS
School of Engineering

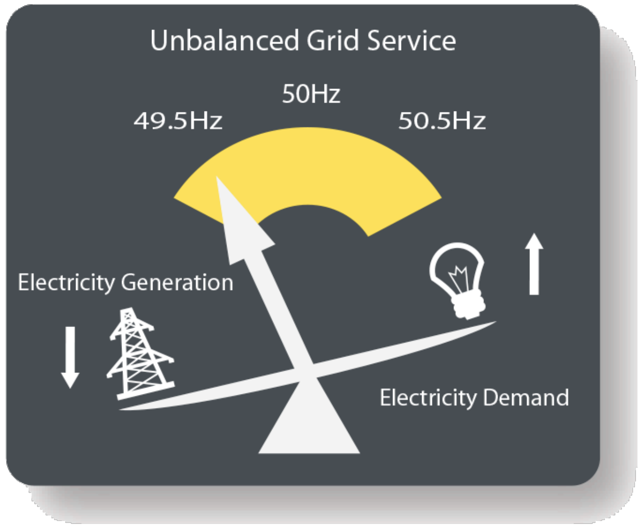
FNSNF

swissgrid

Today’s (and yesterday’s) AC power grids

- Built around rotating synchronous machines
- Coupling between power balance and frequency





figs. Taken from http://northernutilities.co.uk

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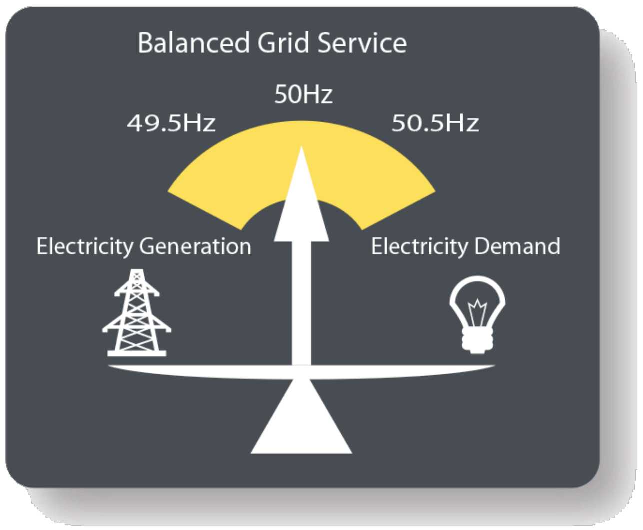

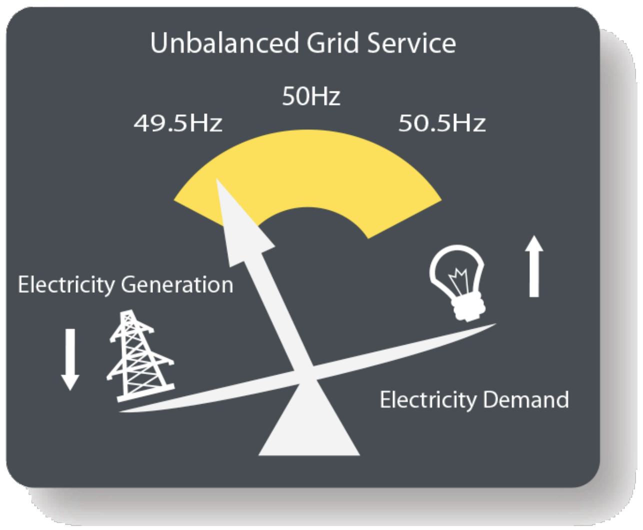

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- Dynamics given by swing equations ~ conservation of energy/power

$$\boxed{M\frac{d\omega}{dt} = P_{\text{gen}} - P_{\text{cons}}}$$

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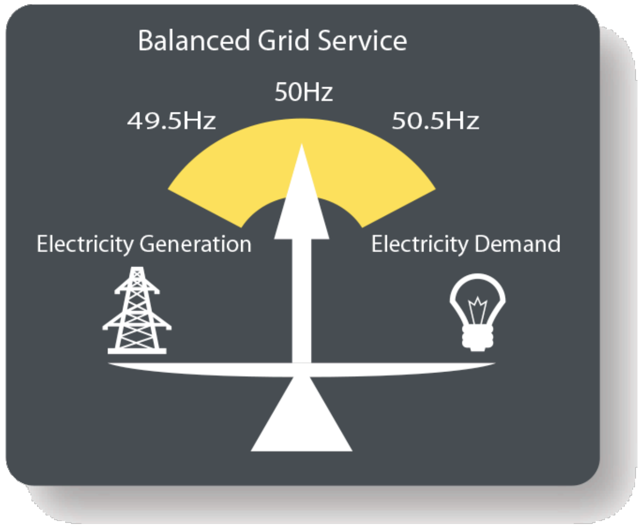

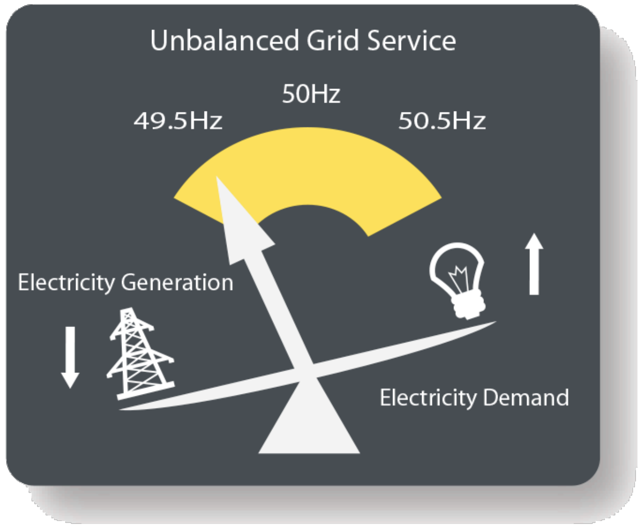

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- Dynamics given by swing equations
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Change in kinetic energy = balance of power

Frequency vs. Electric Power Quality

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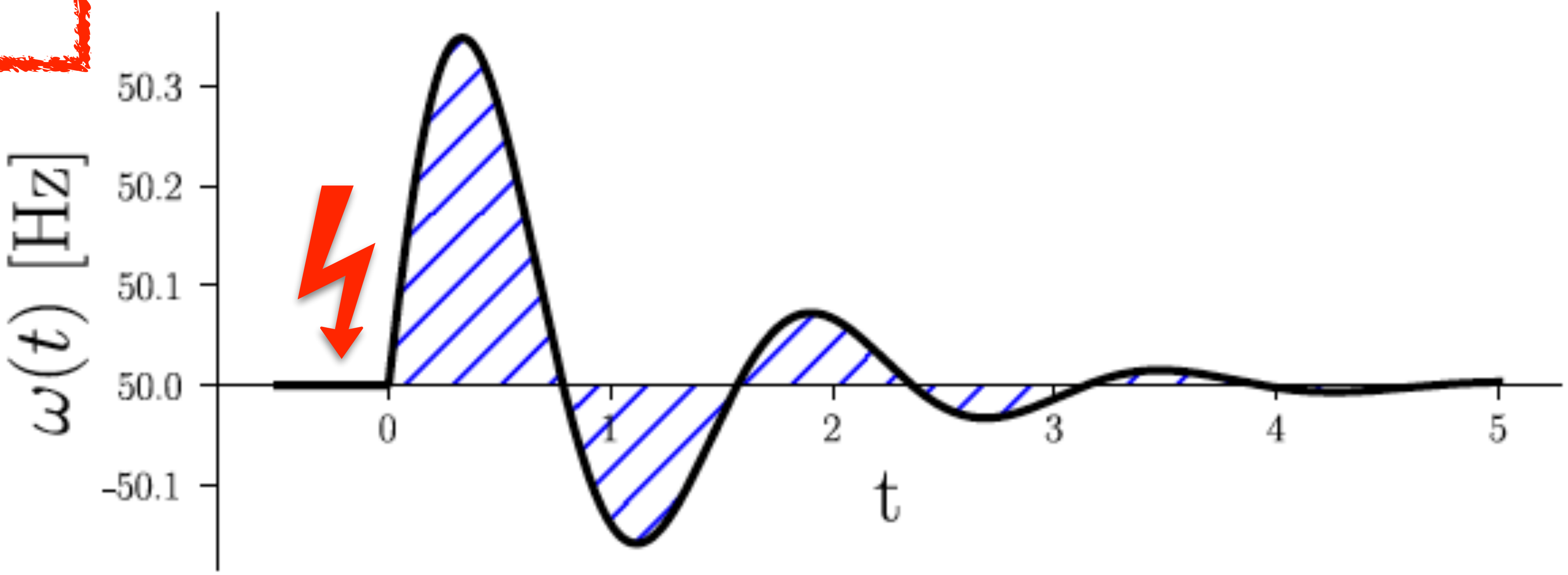

- Frequency transient as indicator of fault magnitude

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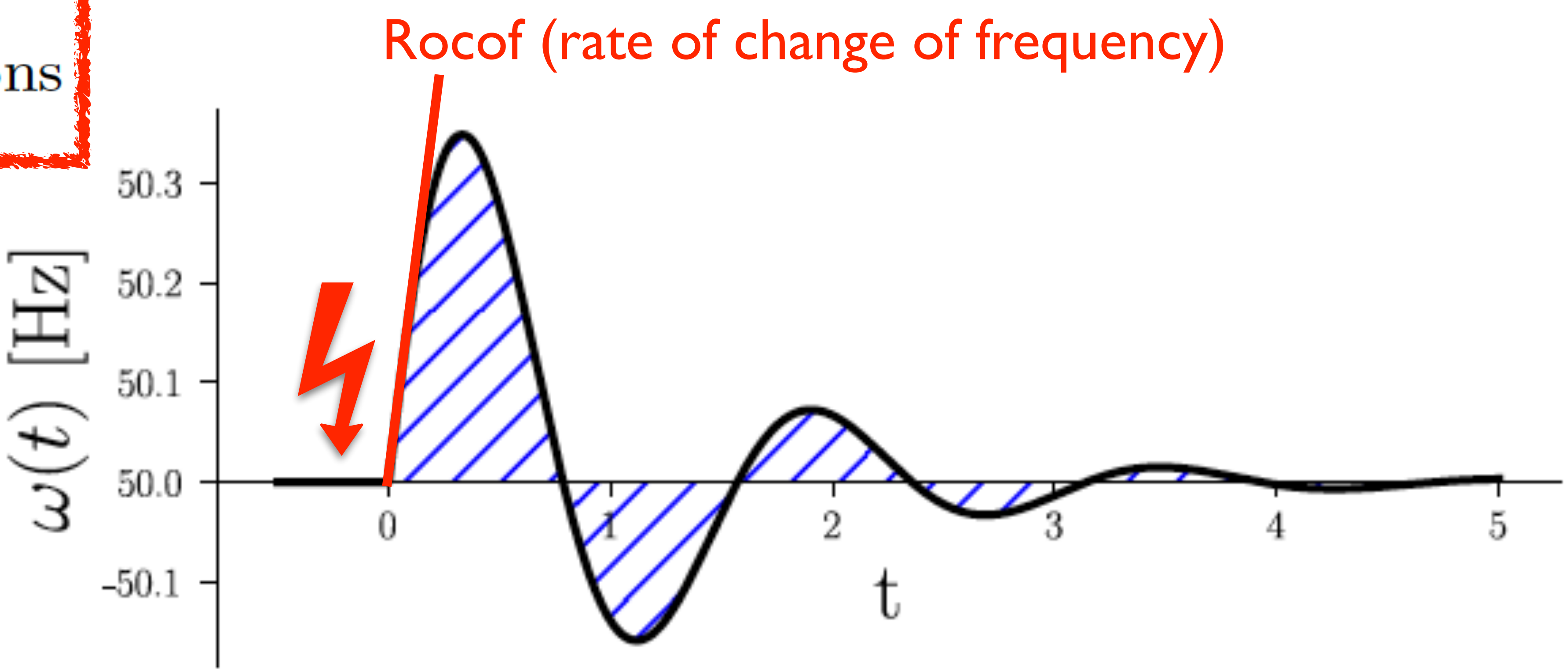

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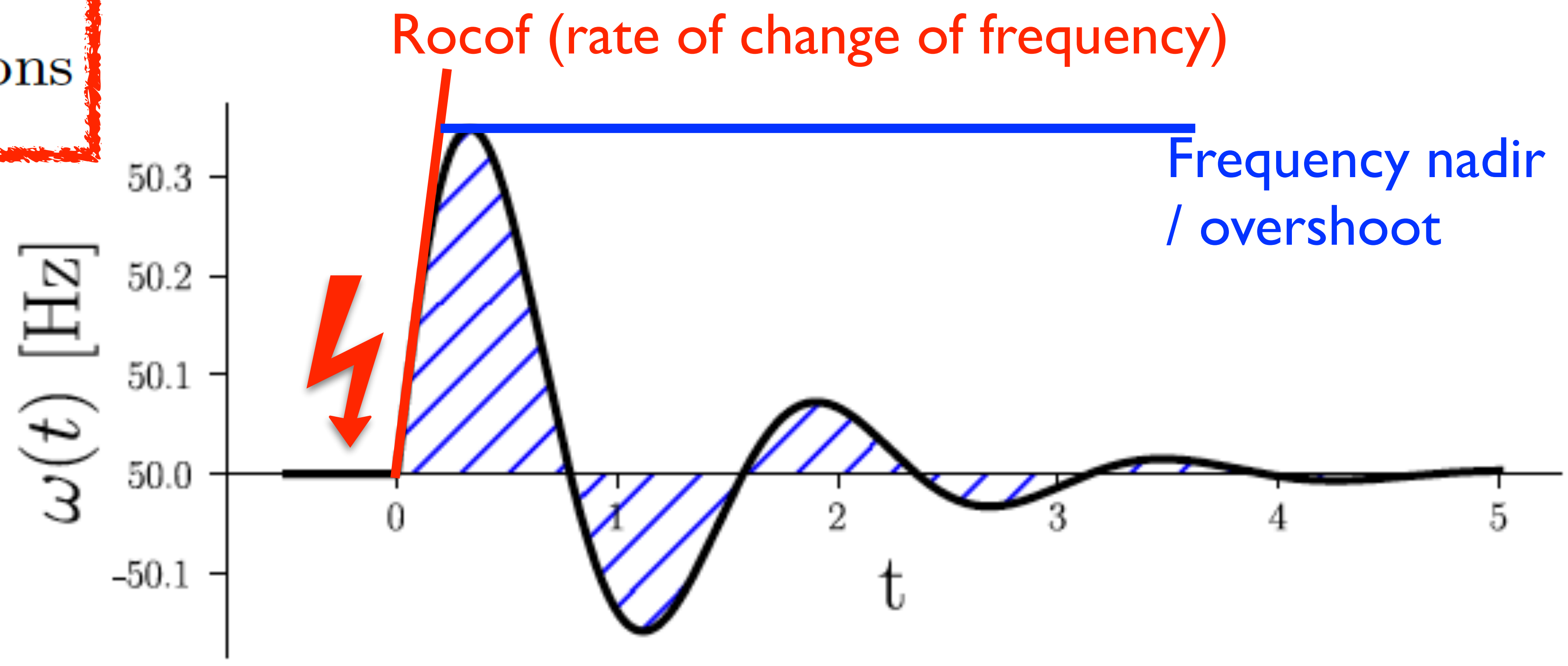

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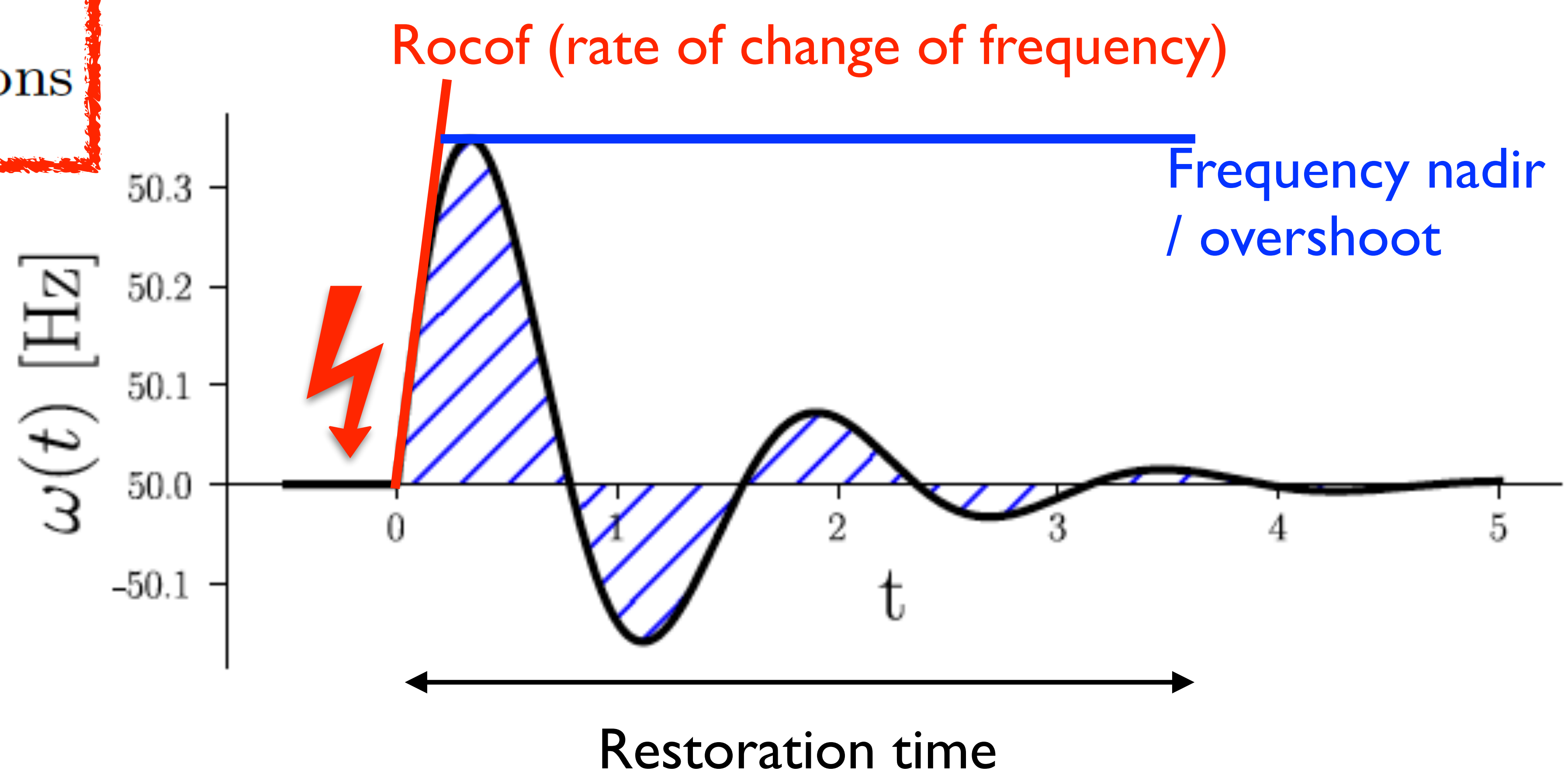

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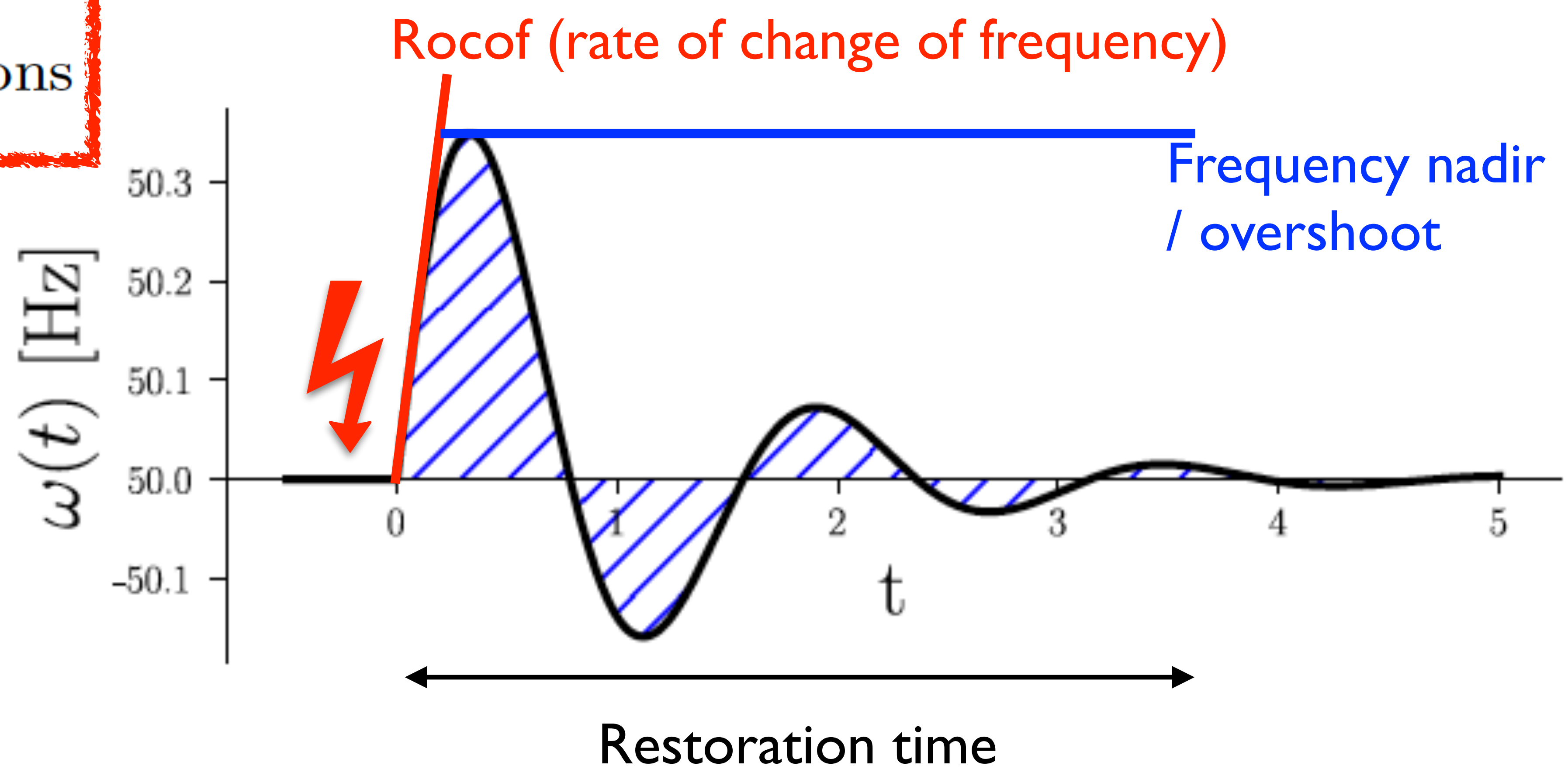


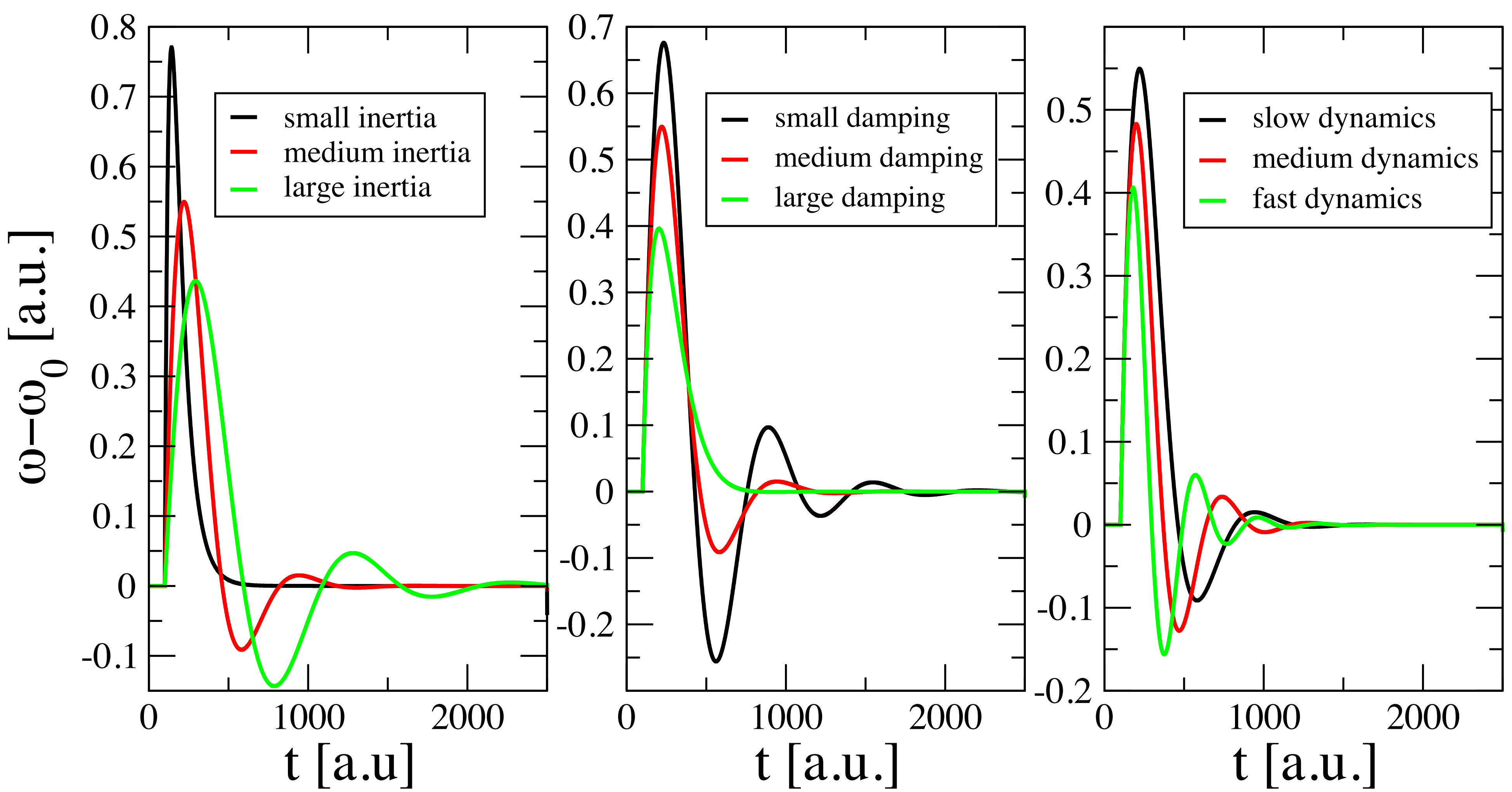
- Frequency transient as indicator of fault magnitude
- Frequency fluctuations need to be minimized

  … but how ?

Dynamics of a simple 2-bus system

- Frequency vs. voltage angles $\boldsymbol{\omega}(t) = \dot{\boldsymbol{\theta}}(t)$

$$\boldsymbol{M}\ddot{\boldsymbol{\theta}} = -\boldsymbol{D}\dot{\boldsymbol{\theta}} - \boldsymbol{L}\boldsymbol{\theta} + \boldsymbol{P}(t)$$



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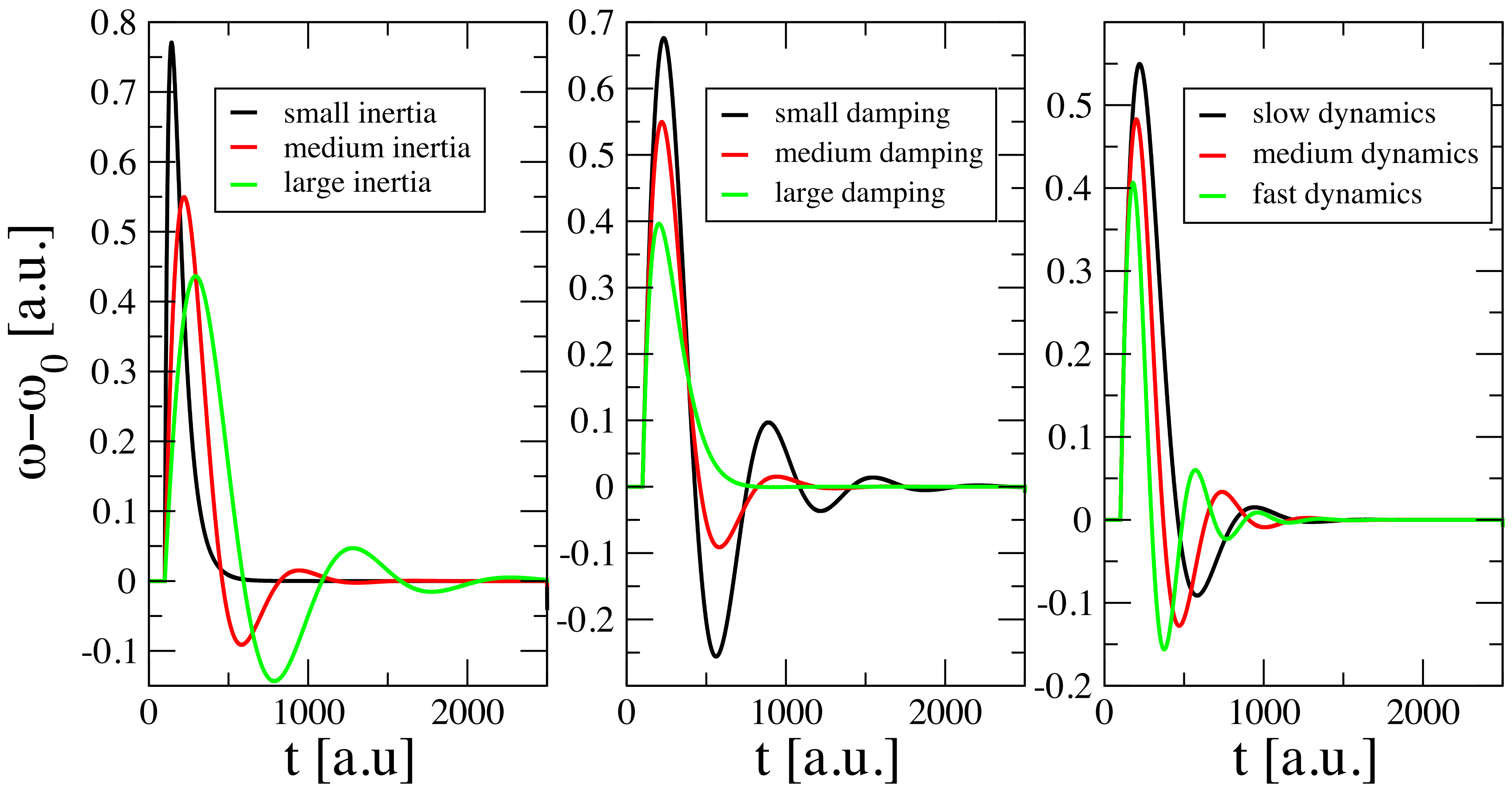

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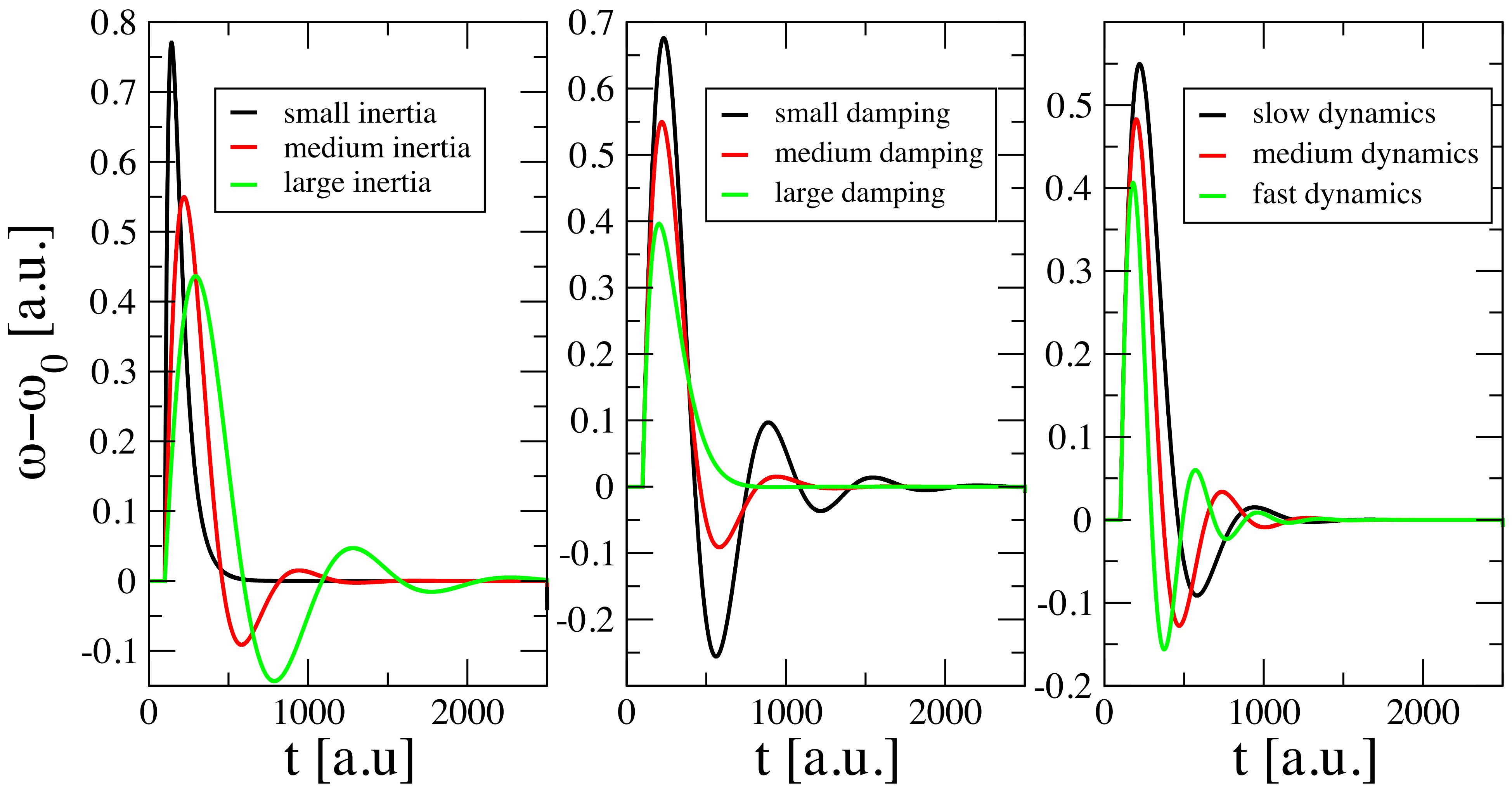
Damping (droop/ primary control)



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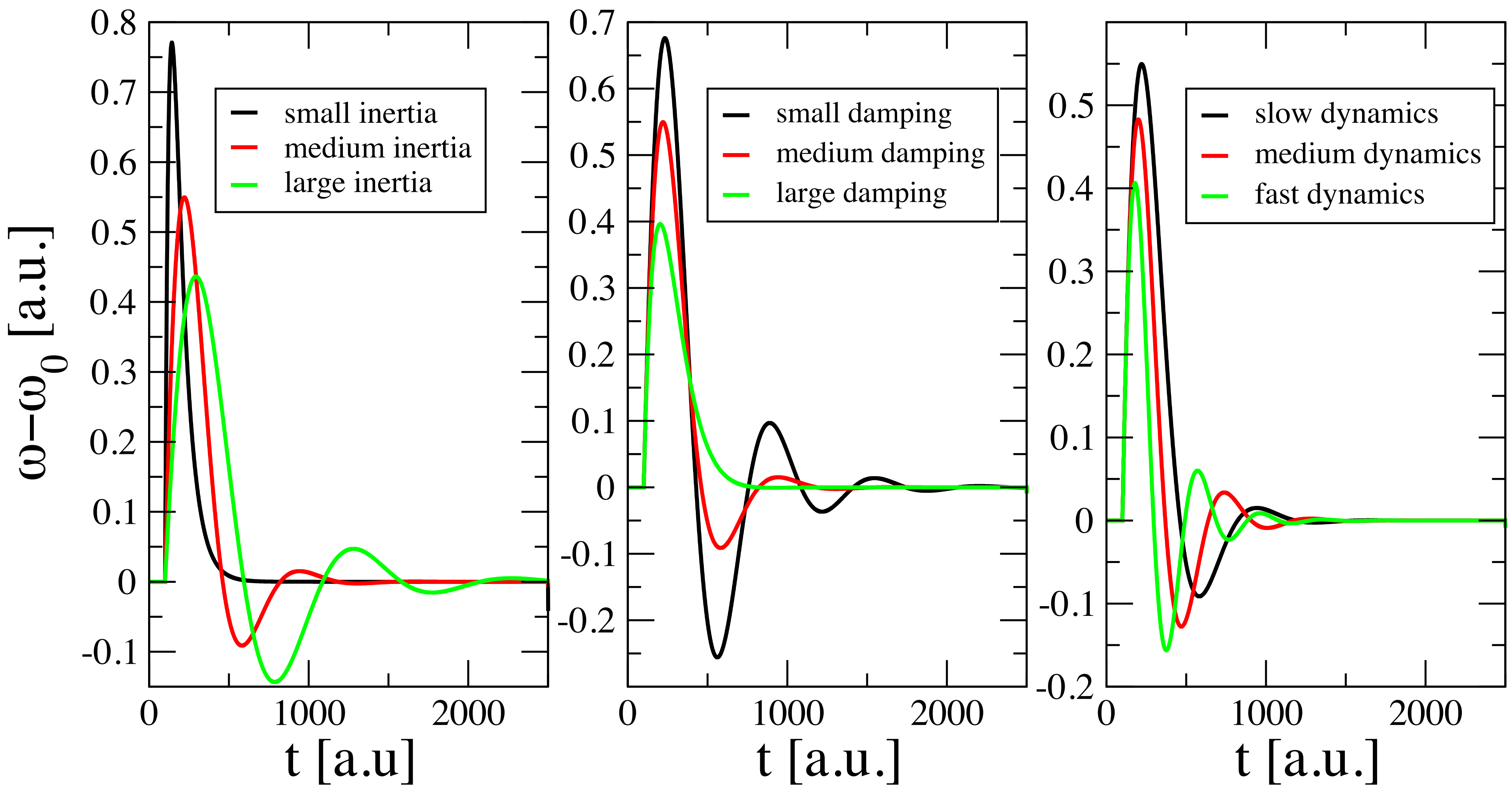
Power in/out (from/to network)



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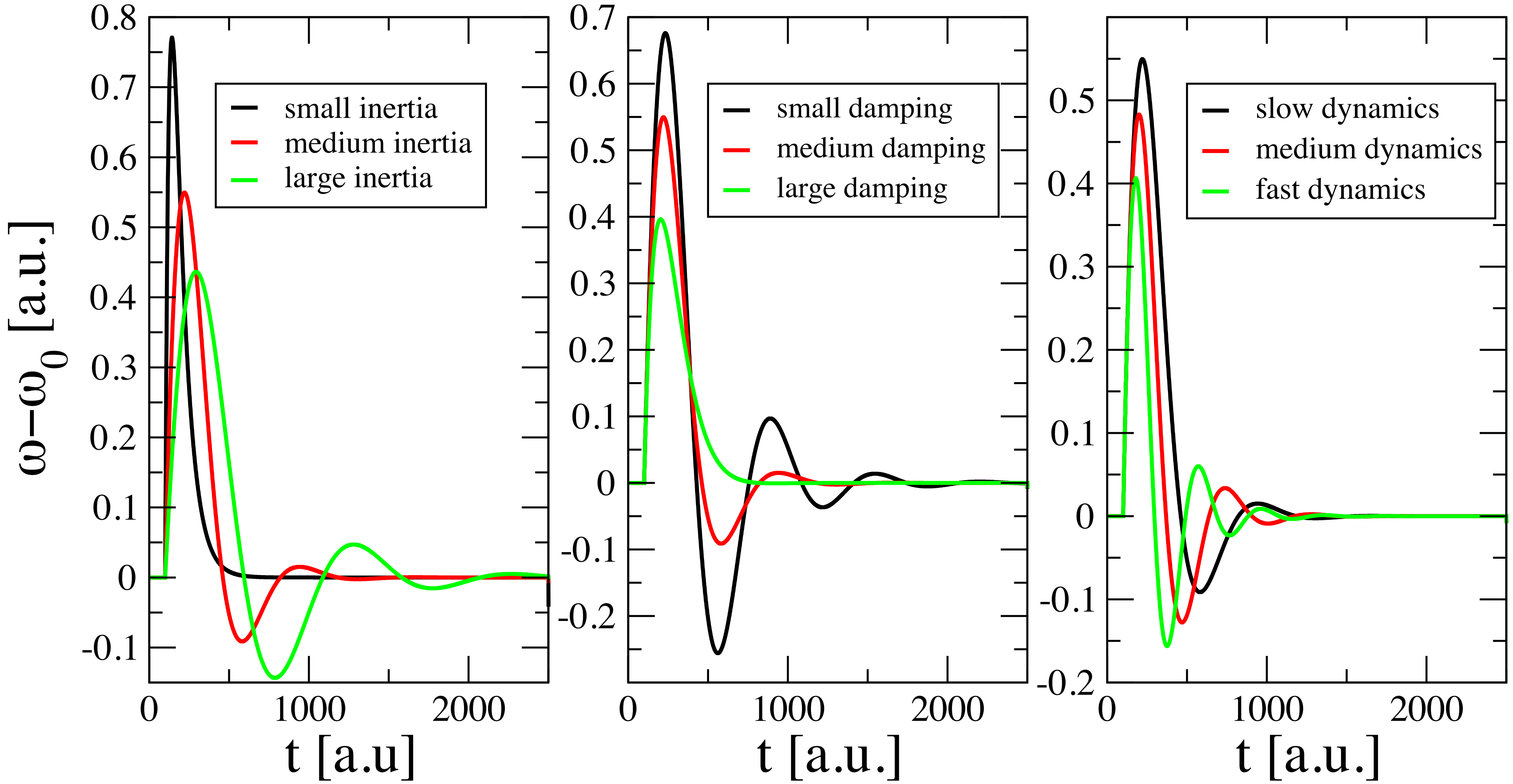

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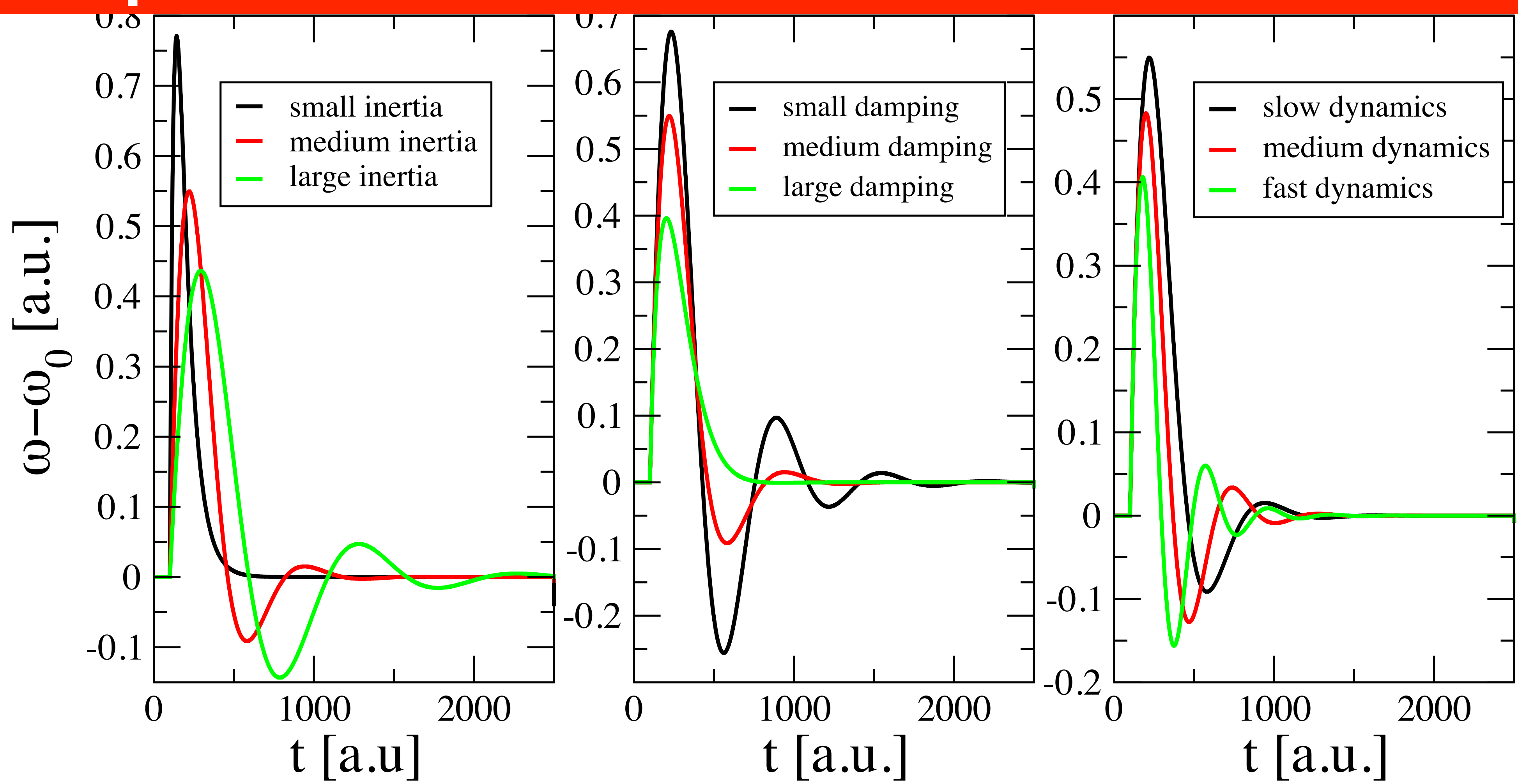
Power out/in

Ch
En

?? Where is it problematic to substitute old productions with new renewables ??



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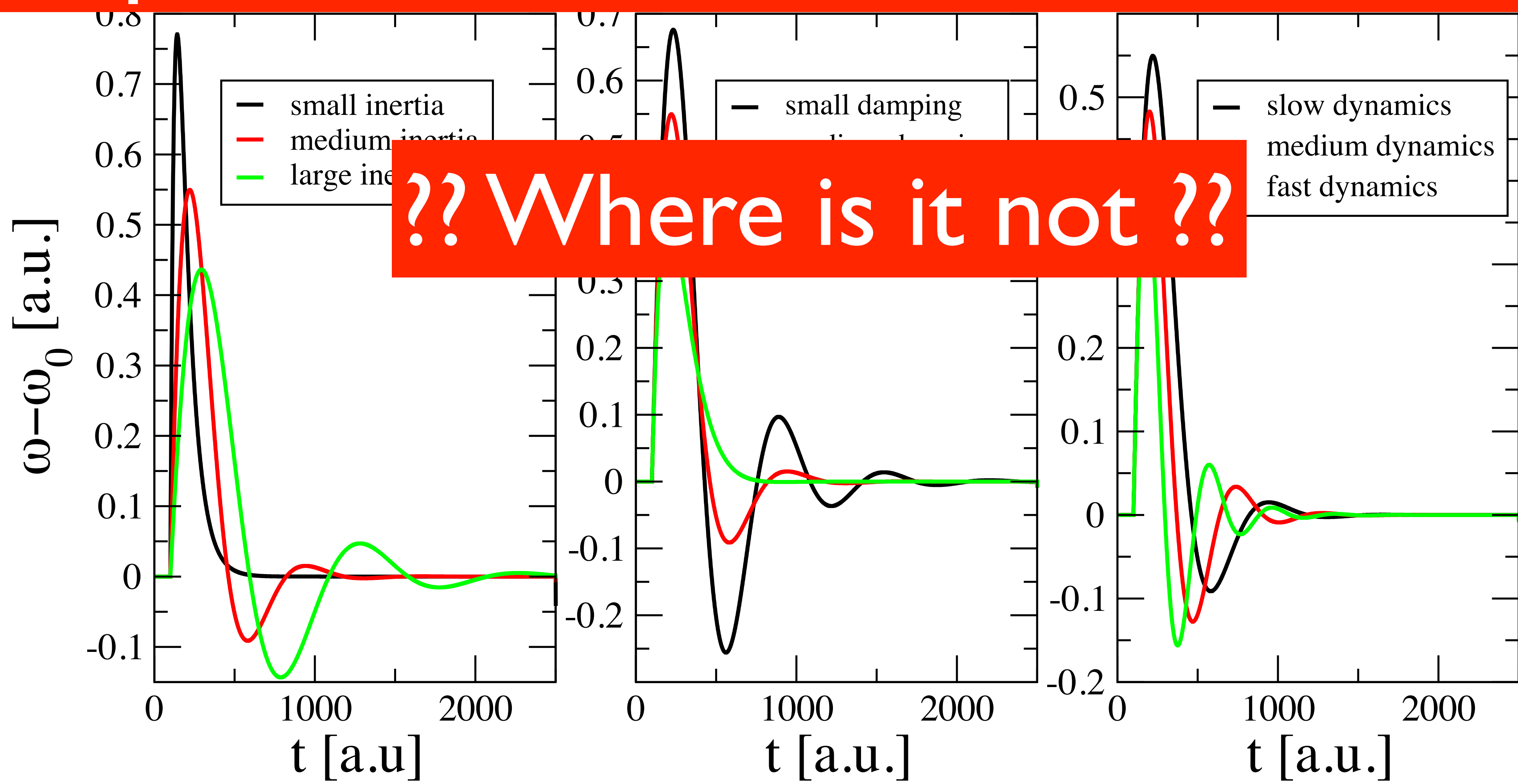

?? Where is it not ??

Problem definition

(1) Consider faults/disturbances

-line fault

-power loss

-fluctuation/noise in power generation

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(III) Quantify transient response : performance measures

-RoCoF, frequency overshoot

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-line fault

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-fluctuation/noise in power generation

(II) Investigate transient response - swing equations

(III) Quantify transient response : performance measures

-RoCoF, frequency overshoot

-quadratic performance measures

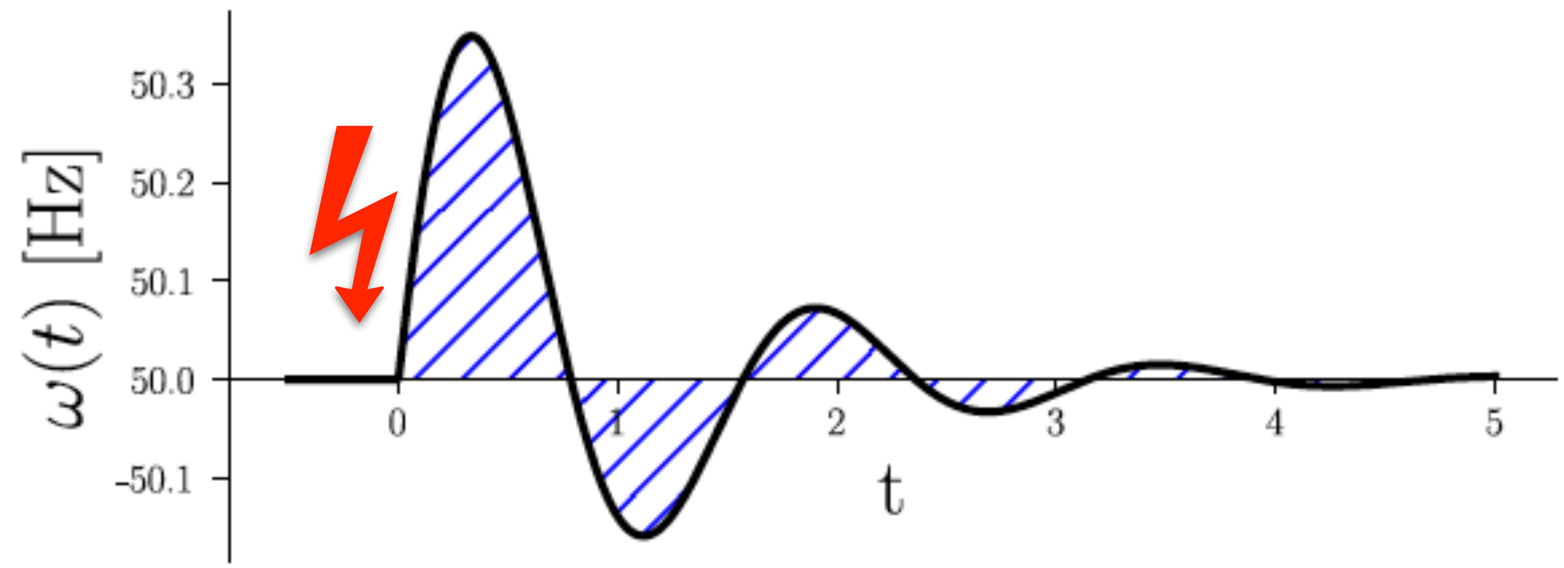
$$\boxed{\mathcal{M} = \int_0^\infty (\omega^\top - \bar{\omega}^\top) M (\omega - \bar{\omega}) \mathrm{d}t}$$

Bamieh, Jovanovic, Mitra, Patterson '12
Bamieh and Gayme '13
Siami and Motee '14
Grunberg and Gayme '16
Poolla, Bolognani, Dörfler '17
Paganini and Mallada '17
Tyloo, Coletta, PJ '18
Coletta, Bamieh, PJ '18

Problem definition

Fault : sudden power loss

$$P_b = P_b^{(0)} - \delta P$$



Linearize dynamics about steady-state solution (=initial operational state)

$$\boldsymbol{M}\dot{\boldsymbol{\omega}} + \boldsymbol{D}\boldsymbol{\omega} = \delta \boldsymbol{P} - \boldsymbol{L}\boldsymbol{\delta\theta}$$

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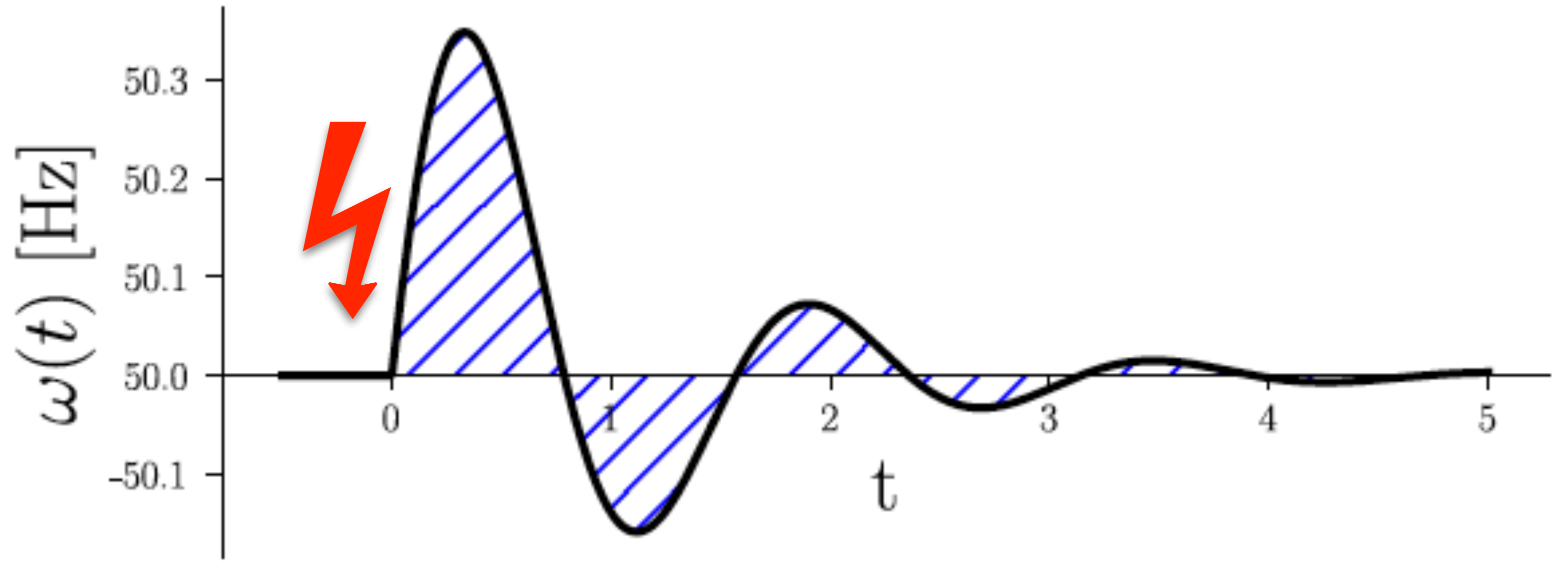

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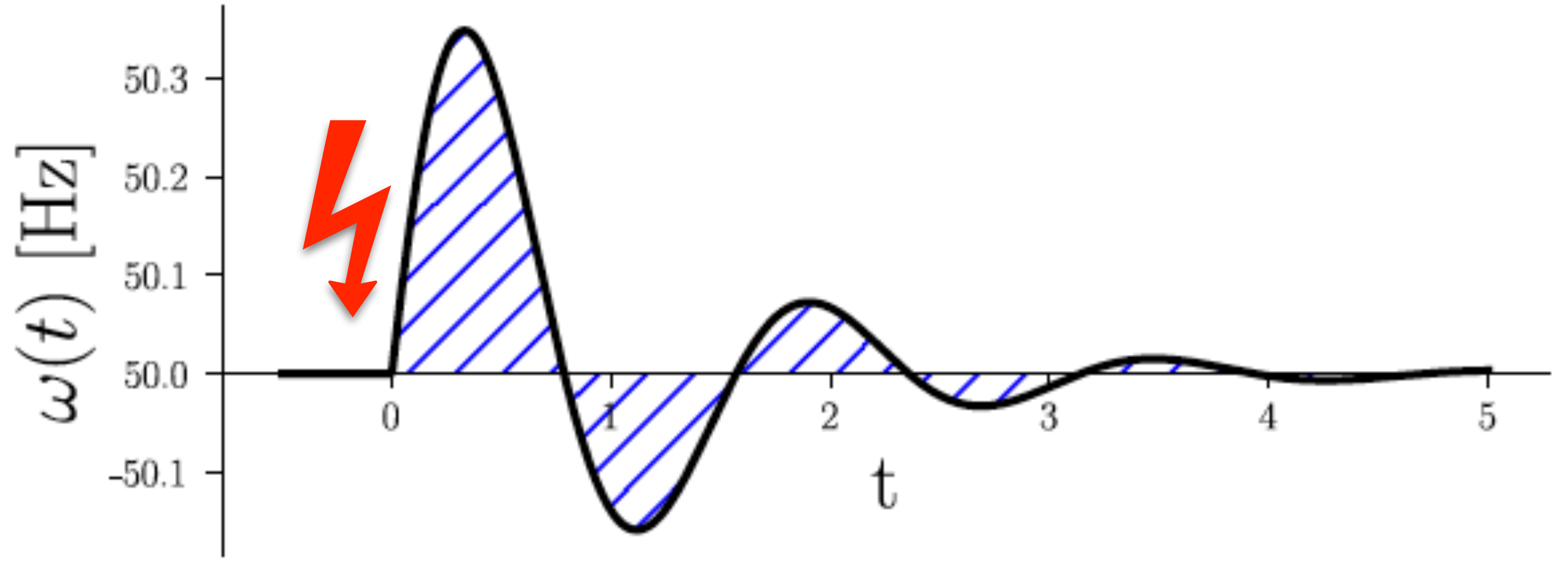

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Network Laplacian

$$(\boldsymbol{L})_{ij} = -B_{ij}V_i^{(0)}V_j^{(0)}$$

$$(\boldsymbol{L})_{ii} = \sum_k B_{ik}V_i^{(0)}V_k^{(0)}$$

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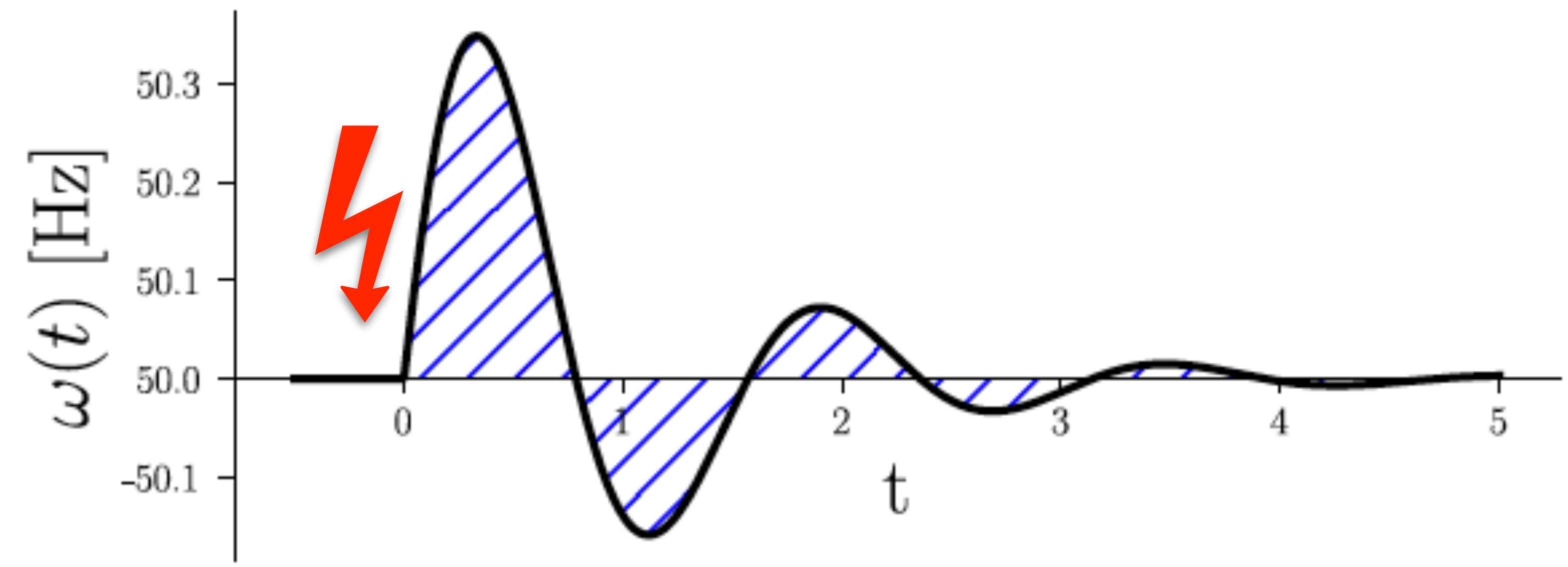

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With eigenvectors and -values

$$\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N\}$$

$$\{\lambda_1, \ldots \lambda_N\}$$

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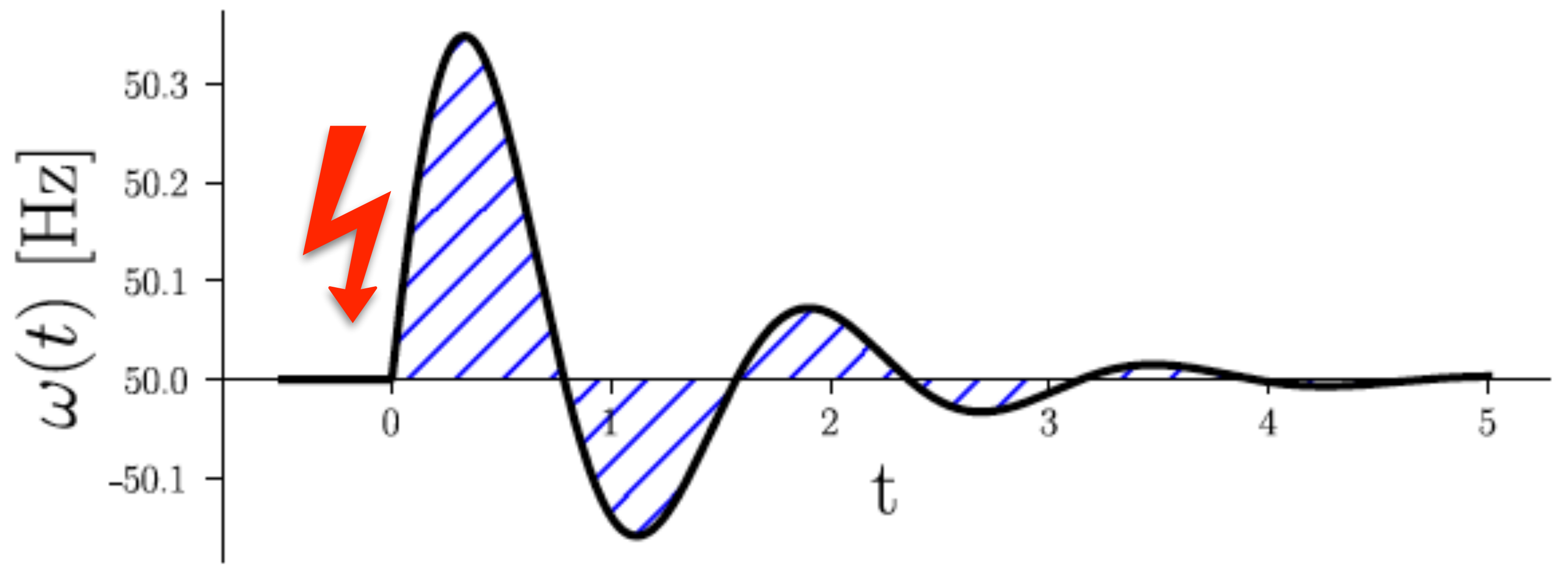

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Zero mode

$$(\boldsymbol{u}_1)^\top = (1,\ldots,1)/\sqrt{N}$$

$$\lambda_1 = 0$$

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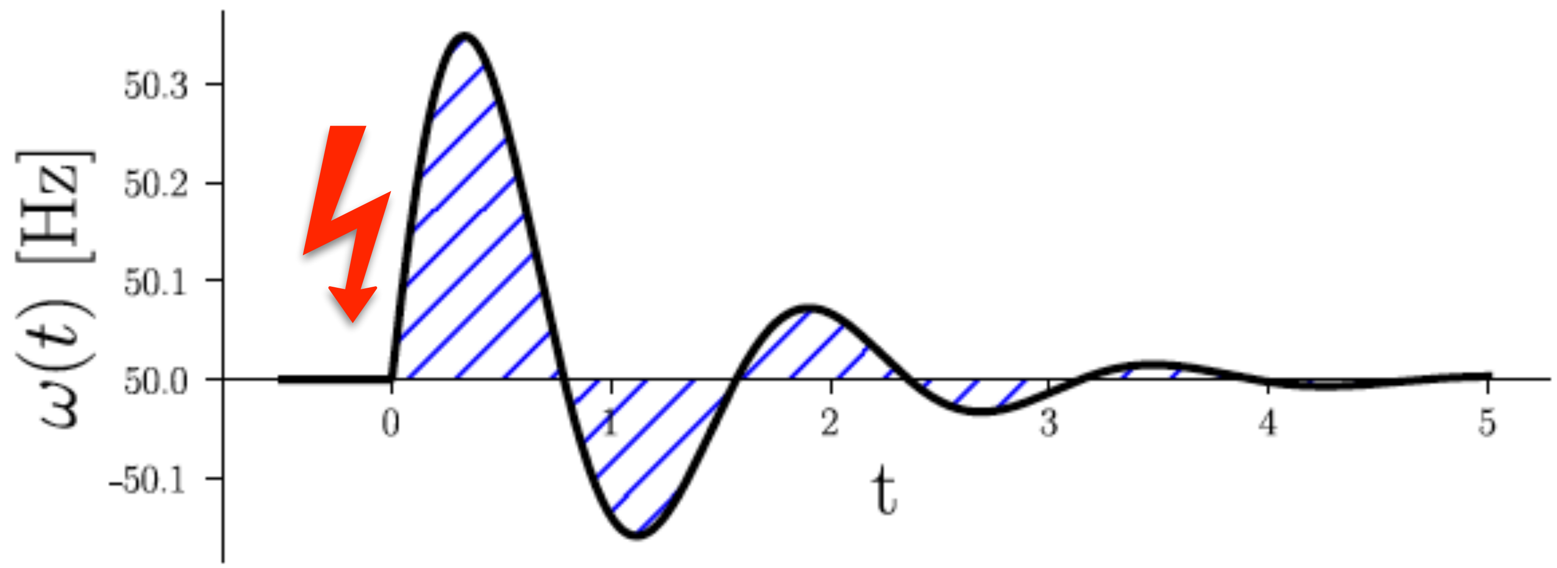

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Higher modes

$$\sum_i u_{\alpha i} = 0 \qquad \text{(orthog. to zero mode)}$$

$$\lambda_\alpha > 0 \qquad \text{(stability)}$$

Diagonalized solution - starting point

- Starting point : when can one diagonalize the swing eqs. ? (linearized about steady-state)

(Paganini and Mallada '17)

$$\boxed{\boldsymbol{M}\dot{\boldsymbol{\omega}} + \boldsymbol{D}\boldsymbol{\omega} = \boldsymbol{\delta P} - \boldsymbol{L}\boldsymbol{\delta\theta}}$$

$$\begin{aligned} \boldsymbol{M} &= \mathrm{diag}(\{m_i\}) \\ \boldsymbol{D} &= \mathrm{diag}(\{d_i\}) \end{aligned}$$

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$$\boldsymbol{\Gamma} = \mathrm{diag}(\{d_i/m_i\})$$

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$$\boxed{M\dot{\omega} + D\omega = \delta P - L\delta\theta}$$

$$M = \mathrm{diag}(\{m_i\})$$
$$D = \mathrm{diag}(\{d_i\})$$

$$\boxed{\delta\theta = M^{-1/2}\delta\theta_M}$$

$$\boxed{\dot{\omega}_M + \underbrace{M^{-1}D}_{\Gamma}\,\omega_M + \underbrace{M^{-1/2}LM^{-1/2}}_{L_M}\,\delta\theta_M = M^{-1/2}\delta P}$$

$$\Gamma = \mathrm{diag}(\{d_i/m_i\})$$

When can this eq. be reduced to a set of scalar diff. eqs. ?

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A. : when d_i/m_i=constant

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$$\boxed{L_M = U^\top \Lambda U}$$

$$\boxed{\delta\theta_M = U^\top \xi}$$

$$\boxed{\Gamma = \gamma\,\mathbb{1}}$$

$$\boxed{\ddot{\xi} + \gamma\dot{\xi} + \Lambda\xi = U M^{-1/2}\delta P}$$

$$\Lambda = \mathrm{diag}(\{\lambda_1 = 0, \lambda_2, \cdots, \lambda_N\})$$

~diagonal/scalar linear differential equation

Quadratic performance measure

$$\boxed{\ddot{\boldsymbol{\xi}} + \gamma\dot{\boldsymbol{\xi}} + \boldsymbol{\Lambda}\boldsymbol{\xi} = \boldsymbol{U}\boldsymbol{M}^{-1/2}\boldsymbol{\delta P}}$$

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Pagnier and PJ, PLoS ONE '19

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Solution with $\boldsymbol{\xi}$ =0 at t=0 : $\boxed{\dot{\xi}_\alpha(t) = \frac{2\mathcal{P}_\alpha}{f_\alpha} e^{-\gamma t/2} \sin\left(\frac{f_\alpha t}{2}\right)}$

$\mathcal{P}_\alpha = \sum_i u_{\alpha i}\,\delta P_i/m_i^{1/2}$ $\qquad$ $f_\alpha = \sqrt{4\lambda_\alpha - \gamma^2}$

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Performance measure : $$\mathcal{M}_b^{(0)} = \int_0^\infty (\boldsymbol{\omega}^\top - \bar{\boldsymbol{\omega}}^\top)\boldsymbol{M}(\boldsymbol{\omega} - \bar{\boldsymbol{\omega}})\mathrm{d}t$$

$$= \int_0^\infty \sum_{\alpha>1} \dot{\xi}_\alpha^2(t)\mathrm{d}t = \frac{\delta P^2}{2\gamma m_b} \sum_{\alpha>1} \frac{u_{\alpha b}^2}{\lambda_\alpha}$$

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(0)=unperturbed solution

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Power loss on bus #b

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Pagnier and PJ, PLoS ONE '19

Quadratic performance measure

$$\boxed{\ddot{\boldsymbol{\xi}} + \gamma\dot{\boldsymbol{\xi}} + \boldsymbol{\Lambda}\boldsymbol{\xi} = \boldsymbol{U}\boldsymbol{M}^{-1/2}\boldsymbol{\delta P}}$$

$$\boldsymbol{\Lambda} = \mathrm{diag}(\{\lambda_1 = 0, \lambda_2, \cdots, \lambda_N\})$$

Solution with ξ =0 at t=0 : $\boxed{\dot{\xi}_\alpha(t) = \frac{2\mathcal{P}_\alpha}{f_\alpha} e^{-\gamma t/2} \sin\left(\frac{f_\alpha t}{2}\right)}$

$$\mathcal{P}_\alpha = \sum_i u_{\alpha i}\, \delta P_i / m_i^{1/2} \qquad f_\alpha = \sqrt{4\lambda_\alpha - \gamma^2}$$

Performance measure : $\mathcal{M}_b^{(0)} = \int_0^\infty (\boldsymbol{\omega}^\top - \bar{\boldsymbol{\omega}}^\top)\boldsymbol{M}(\boldsymbol{\omega} - \bar{\boldsymbol{\omega}})\mathrm{d}t$

Note : can be recast in terms of resistance distance centralities
See Melvyn Tyloo's talk this afternoon

$$= \frac{\delta P^2}{2\gamma m_b} \sum_{\alpha>1} \frac{u_{\alpha b}^2}{\lambda_\alpha}$$

Pagnier and PJ, PLoS ONE '19

Optimal placement of inertia and primary control

- Deviation from homogeneity

$$m_i = m + \delta m \, r_i \,,$$

$$d_i = m_i \gamma_i = (m + \delta m \, r_i)(\gamma + \delta\gamma \, a_i)$$

$$-1 \;\leq\; a_i, r_i \;\leq\; 1$$

Do perturbation theory with small parameters

$$\mu \equiv \delta m / m \ll 1$$

$$g \equiv \delta\gamma / \gamma \ll 1$$

Pagnier and PJ, to appear in IEEE Access '19

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- Perturbative performance measure - to be minimized

$$\mathcal{M}_b = \mathcal{M}_b^{(0)} + r_i \rho_i + a_i \alpha_i + \mathcal{O}(\delta m^2, \delta\gamma^2)$$

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with susceptibilities (this is what we calculate)

$$\rho_i \equiv \partial \mathcal{M}_b / \partial r_i \quad \alpha_i \equiv \partial \mathcal{M}_b / \partial a_i$$

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$$\rho_i \equiv \partial \mathcal{M}_b / \partial r_i \quad \alpha_i \equiv \partial \mathcal{M}_b / \partial a_i$$

- Goal : determine optimal distribution of r_i and a_i with constraints

$$-1 \;\leq\; a_i, r_i \;\leq\; 1$$ Local reduction/increase

$$\textstyle\sum_i r_i \;=\; \sum_i a_i \;=\; 0$$ Total limited resources

$$\mu \equiv \delta m / m \ll 1$$
$$g \equiv \delta\gamma / \gamma \ll 1$$

Pagnier and PJ, to appear in IEEE Access ’19

Optimal placement of inertia and droop - algorithmic solution

- Susceptibilities
  1) inhomogeneities in inertia

$$\rho_i = -\frac{\mu\delta P^2}{\gamma N}\sum_{\alpha>1}\frac{u_{\alpha b}^{(0)}u_{\alpha i}^{(0)}}{\lambda_\alpha^{(0)}}$$

  2) inhomogeneities in damping/primary control

$$\alpha_i \equiv \partial\mathcal{M}_b/\partial a_i = \quad -\frac{g\delta P^2}{2\gamma m_b}\left[\sum_{\alpha>1}\frac{u_{\alpha i}^2 u_{\alpha b}^2}{\lambda_\alpha} + \sum_{\alpha>1,\beta\neq\alpha}\frac{u_{\alpha i}\,u_{\alpha b}u_{\beta i}\,u_{\beta b}}{(\lambda_\alpha-\lambda_\beta)^2+2\gamma^2(\lambda_\alpha+\lambda_\beta)}\right]$$

Pagnier and PJ, to appear in IEEE Access'19

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  2) inhomogeneities in damping/primary control

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  3) overall performance - weighted average over fault location

$$\mathcal{V} = \sum_b \eta_b \, \mathcal{M}_b(\delta P_b)$$

Pagnier and PJ, to appear in IEEE Access'19

Optimal placement of inertia and droop - algorithmic solution

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3) overall performance - weighted average over fault location

$$\mathcal{V} = \sum_b \eta_b\, \mathcal{M}_b(\delta P_b) \qquad \boxed{\eta_b \equiv 1}$$

No change in inertia !

Increase of damping on few slowest modes !

Pagnier and PJ, to appear in IEEE Access'19

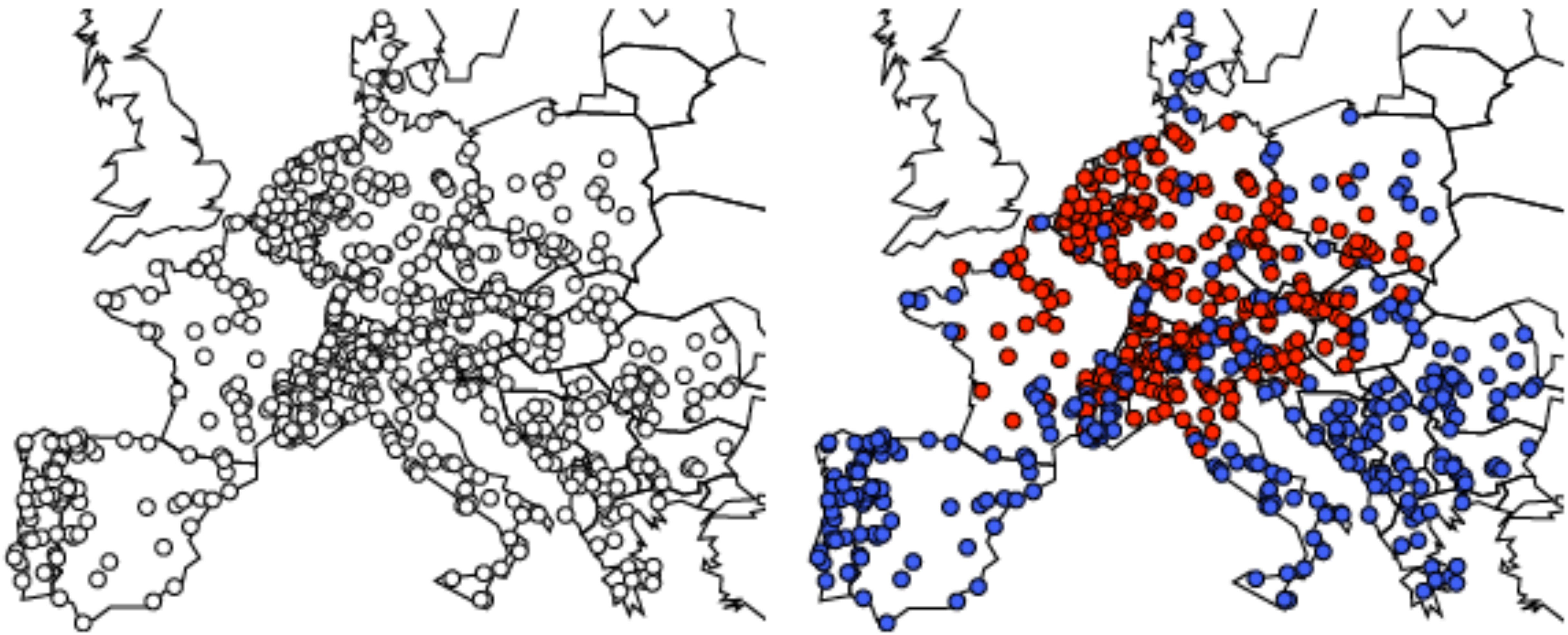
Optimal placement of inertia and droop - solution

Inertia. : r_i =0
Wants to keep it homogeneous

$$m_i = m + \delta m\, r_i\ ,$$
$$d_i = m_i \gamma_i = (m + \delta m\, r_i)(\gamma + \delta\gamma\, a_i)$$

Pagnier and PJ, to appear in IEEE Access'19

Optimal placement of inertia and droop - solution



Inertia. : r_i =0
Wants to keep it homogeneous

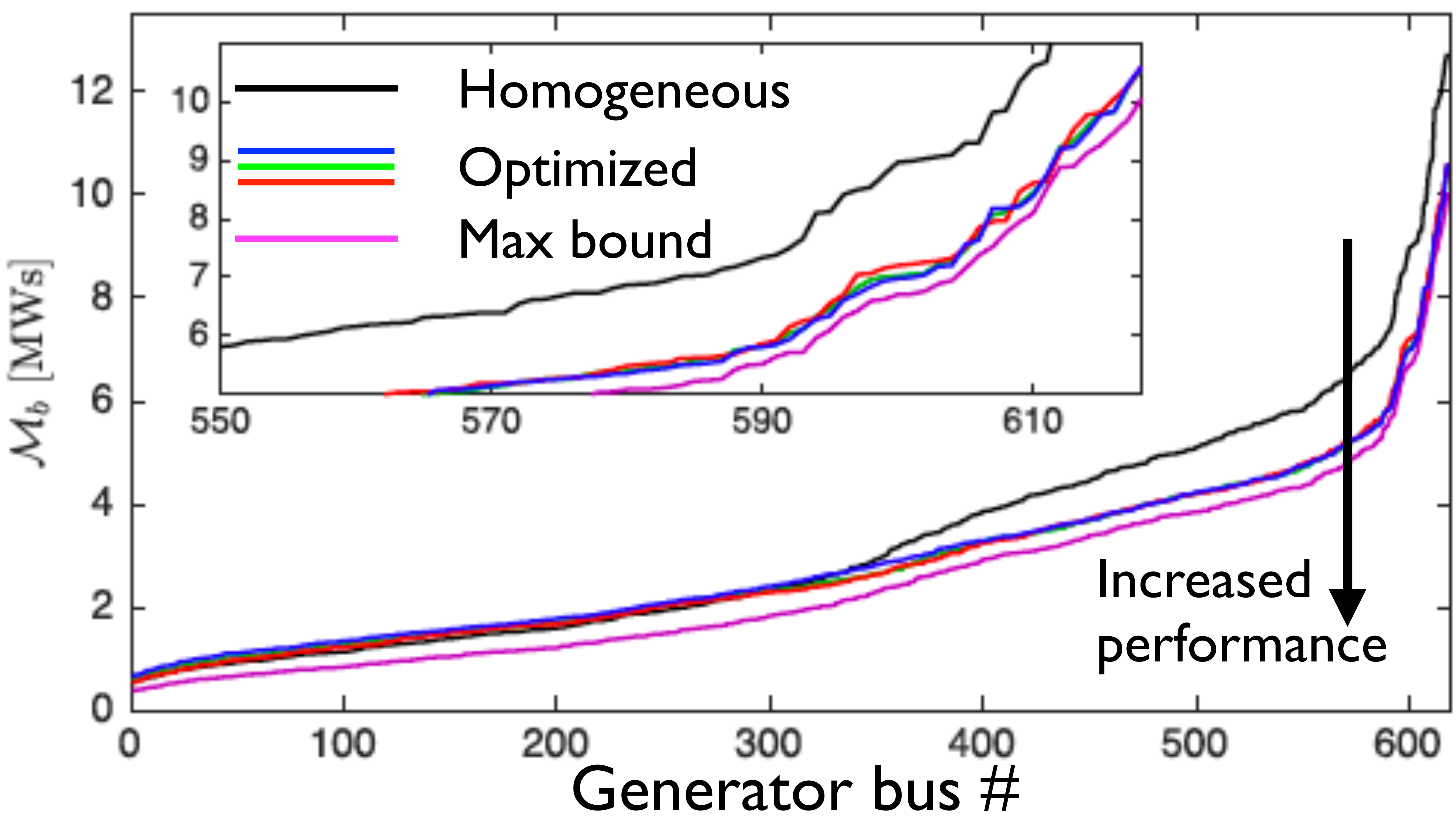
Damping/droop : r_i =1 (blue) r_i =-1 (red)
Wants to put it at periphery
(vs. resistance distance centrality)

$$m_i = m + \delta m \, r_i \, ,$$

$$d_i = m_i \gamma_i = (m + \delta m \, r_i)(\gamma + \delta\gamma \, a_i)$$

Pagnier and PJ, to appear in IEEE Access'19

Optimal placement of inertia and droop - solution



- Performance strongly increased especially at most sensitive buses

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Optimal placement of inertia and droop - solution

The method can be extended :

1) to optimize about another solution
   - -current European grid
   - -future European grid (less inertia)

Pagnier and PJ, to appear in IEEE Access'19

Optimal placement of inertia and droop - solution

The method can be extended :

1) to optimize about another solution
   - current European grid
   - future European grid (less inertia)

2) to keep cost of upgrade within a given budget B
   (instead of constraint of constant total resources)

$$\sum_i \left(c_i^{\mathrm{m}+}\, \delta m_i^+ + c_i^{\mathrm{d}+}\, \delta d_i^+ + c_i^{\mathrm{m}-}\, \delta m_i^- + c_i^{\mathrm{d}-}\, \delta d_i^- \right) \leq B$$

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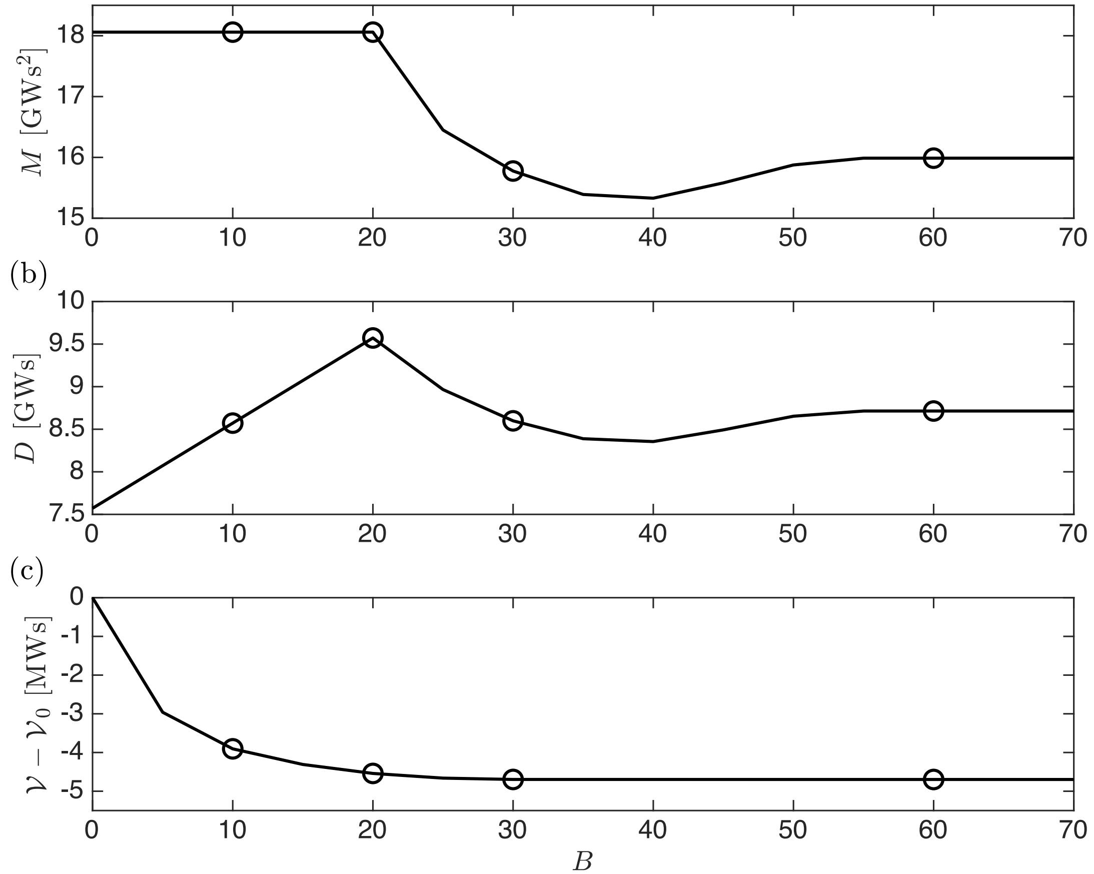
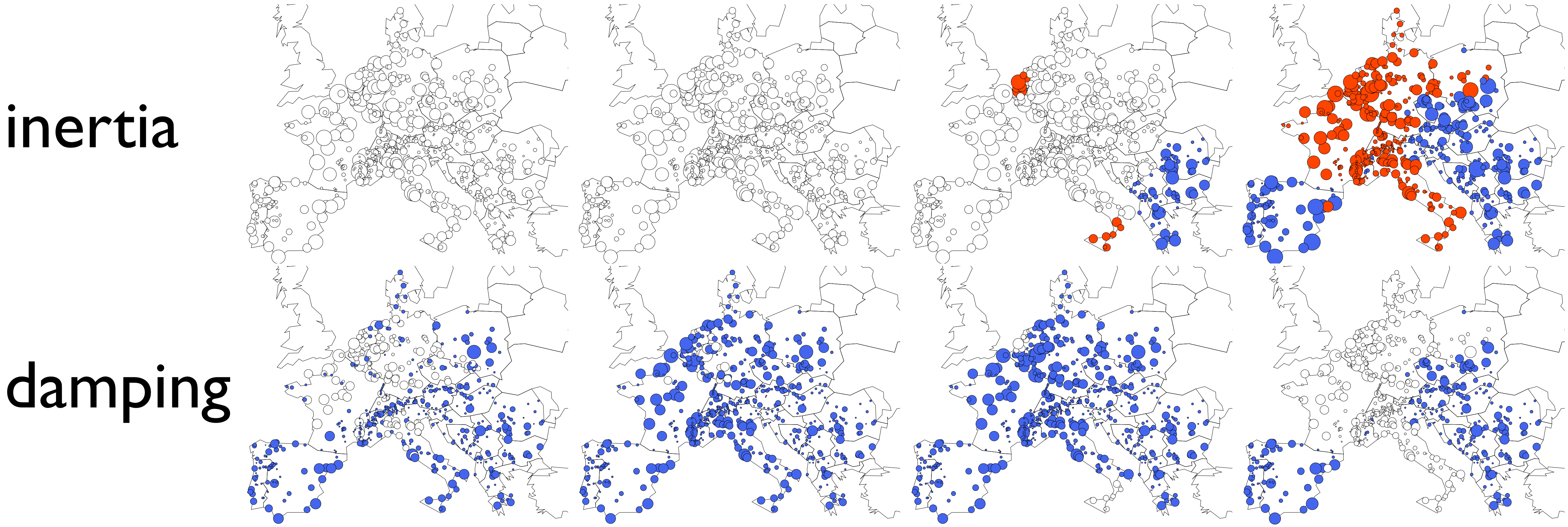
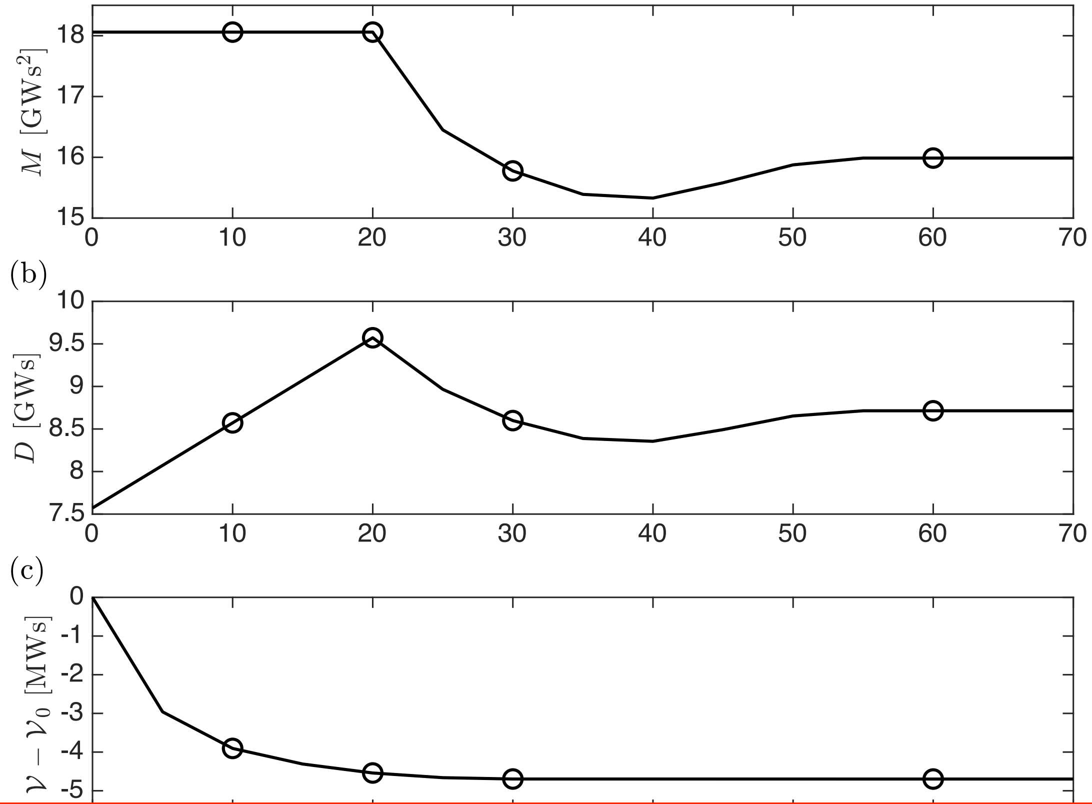
Optimal placement of inertia and droop - current European grid





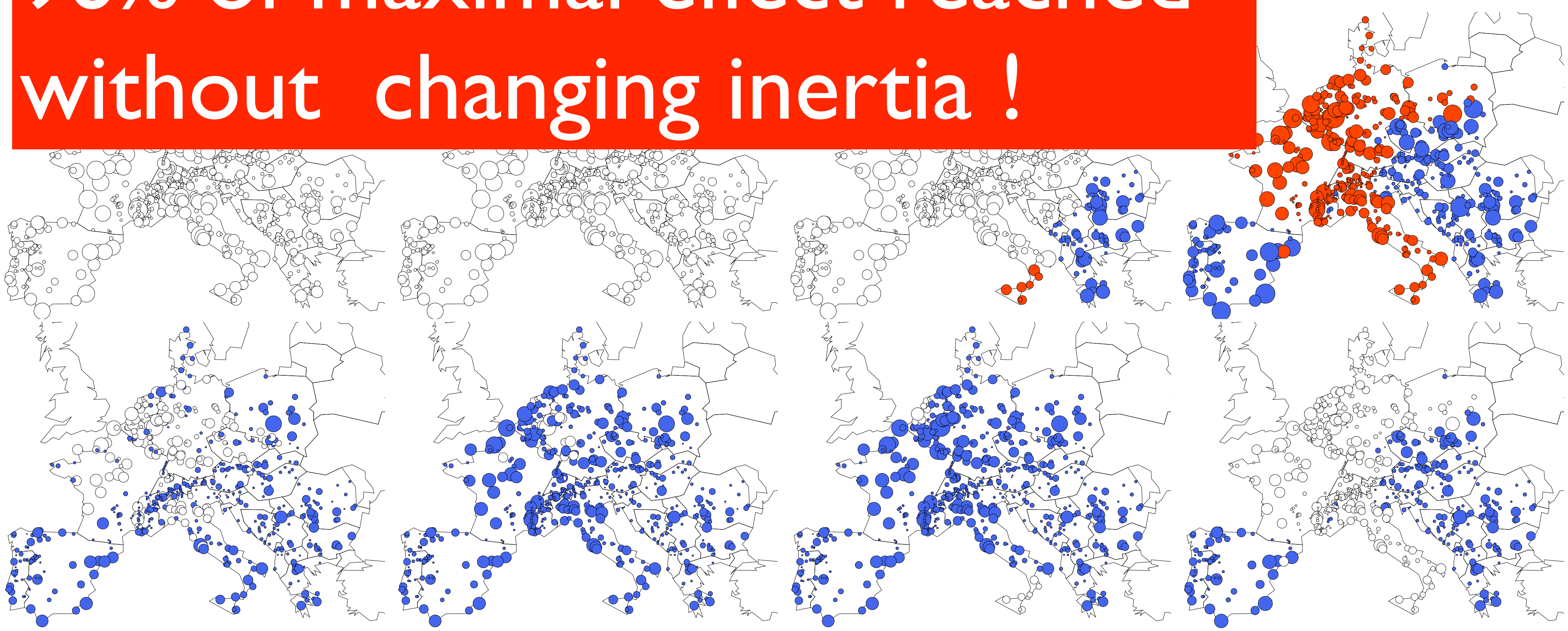
Optimal placement of inertia and droop - current European grid



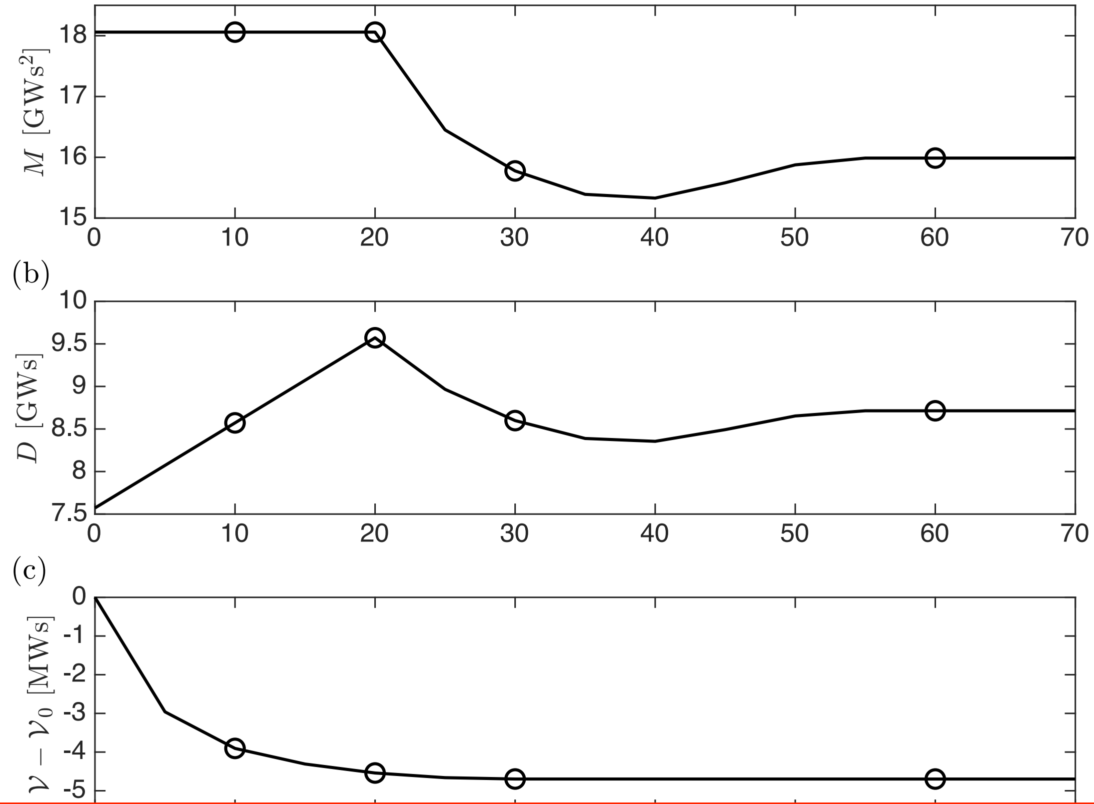
96% of maximal effect reached without changing inertia !

inertia

damping



Optimal placement of inertia and droop - current European grid



96% of maximal effect reached without changing inertia !

inertia

Add damping on slow modes !

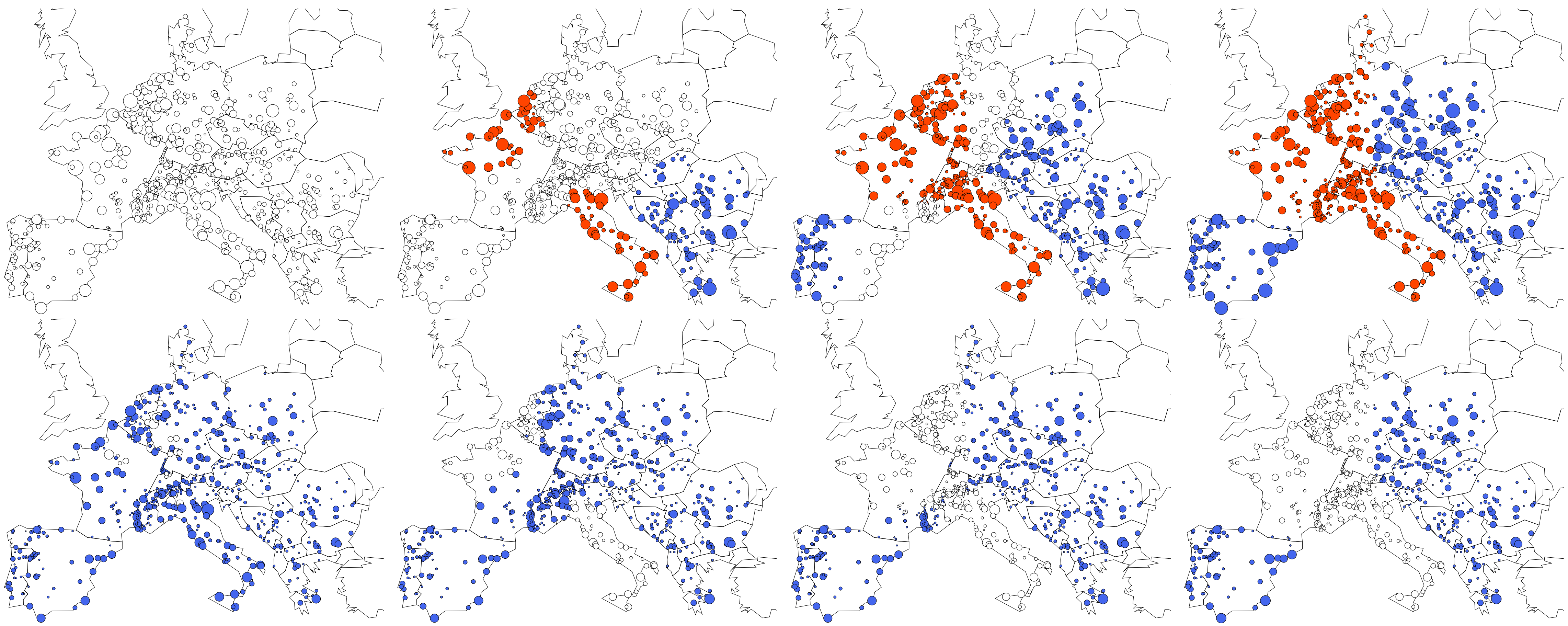
damping

Optimal placement of inertia and droop - future European grid

inertia divided by 3 (~converter-connected new renewables)

inertia

damping



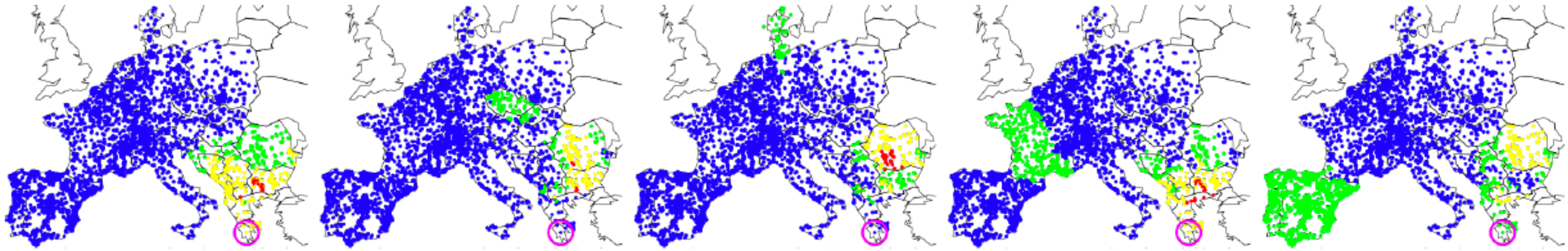
Interpretation

Today's Europe

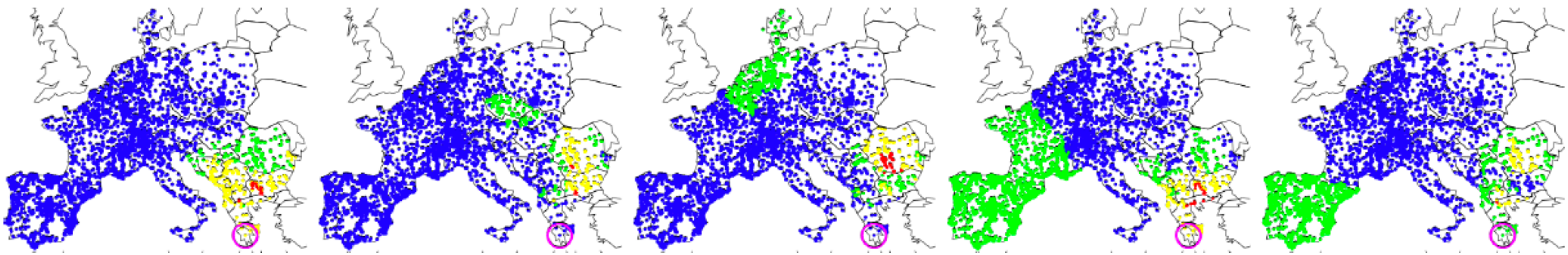
Pagnier and PJ, PLoS ONE '19

Interpretation

Today's Europe



Inertia in France reduced to 50%



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Interpretation

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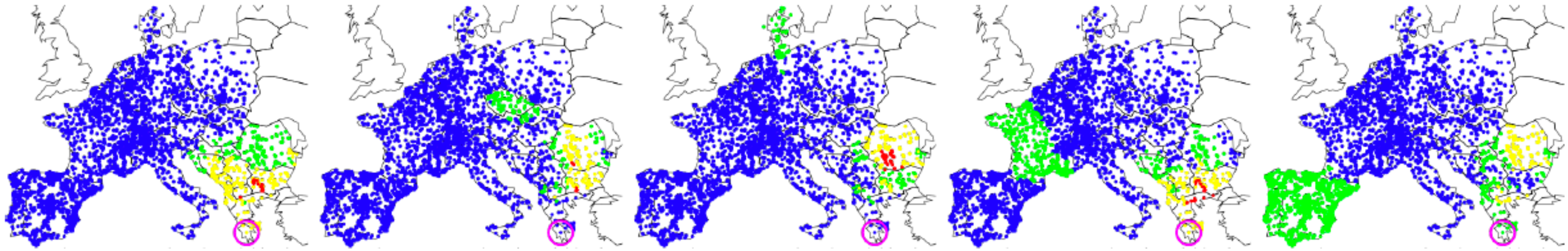

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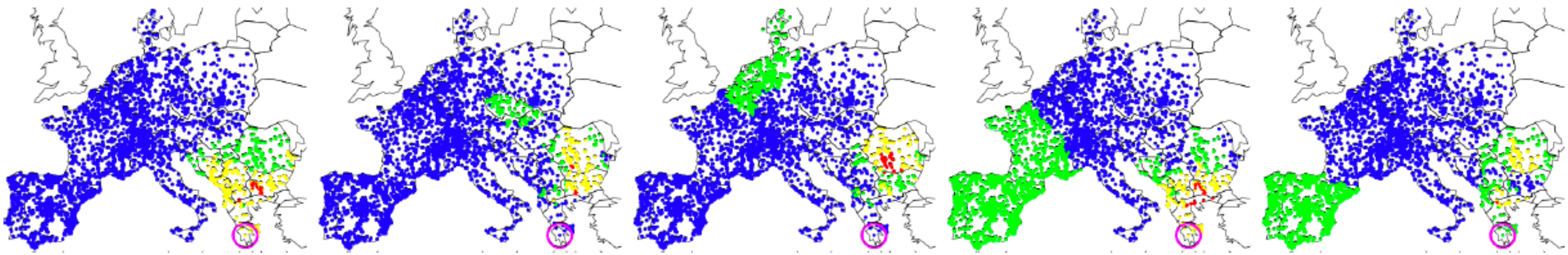
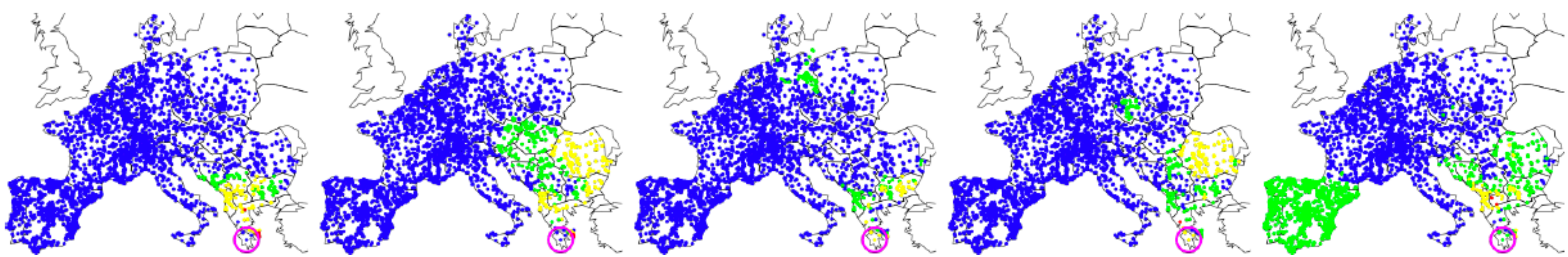


Inertia in Balkans doubled



Pagnier and PJ, PLoS ONE ’19

Interpretation

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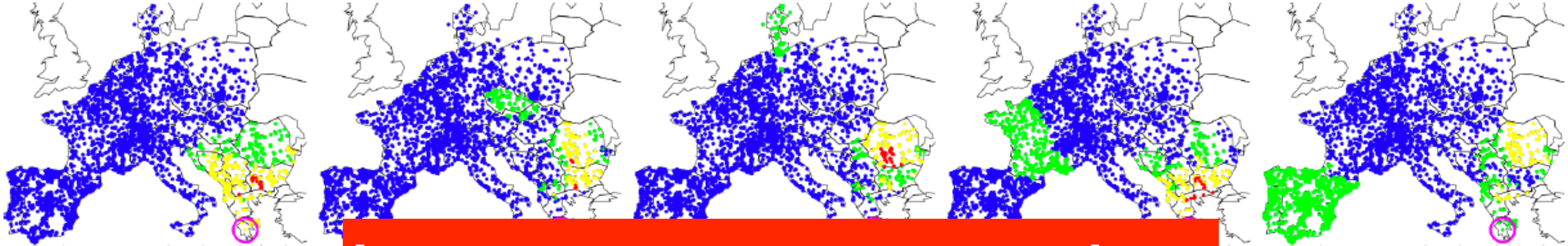


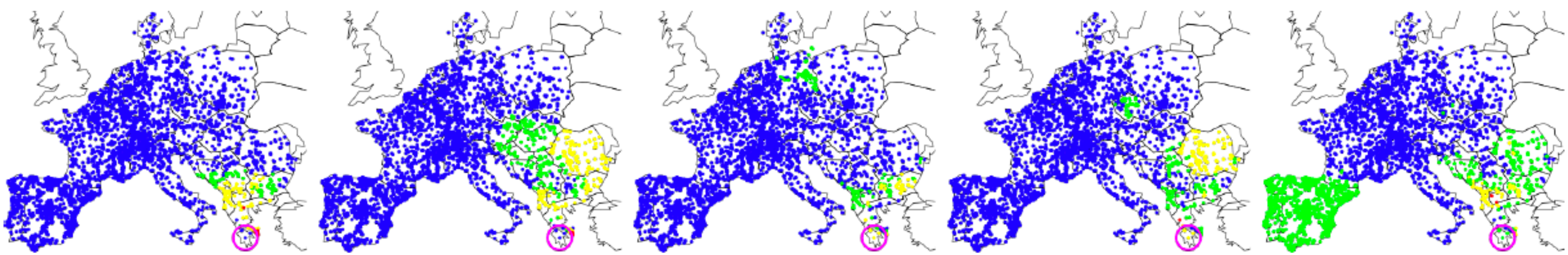
Inertia in France

Inertia matters mostly :

*locally

*at short times

Inertia in Balkans doubled

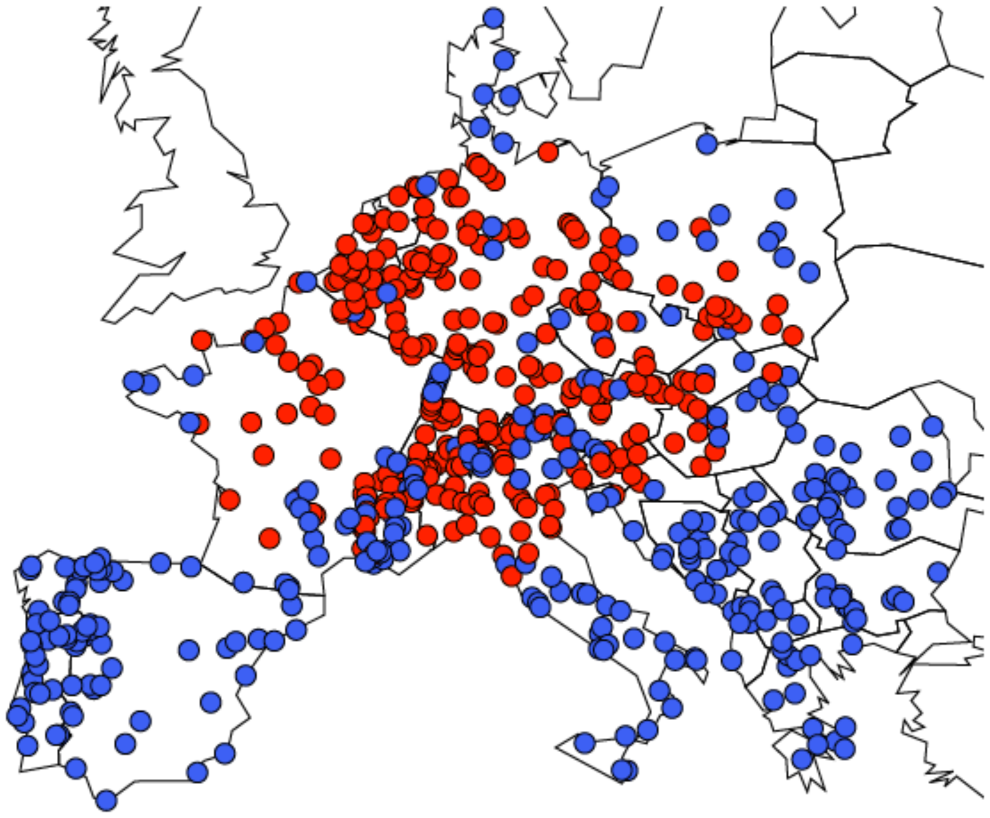


Pagnier and PJ, PLoS ONE '19

Conclusion

- Acting on control is much more crucial than on inertia
- Short time effect of inertia $\longrightarrow$ keep it homogeneous !
- Optimal droop / damping placement

  -on few slow modes ~vs. resistance distance centrality



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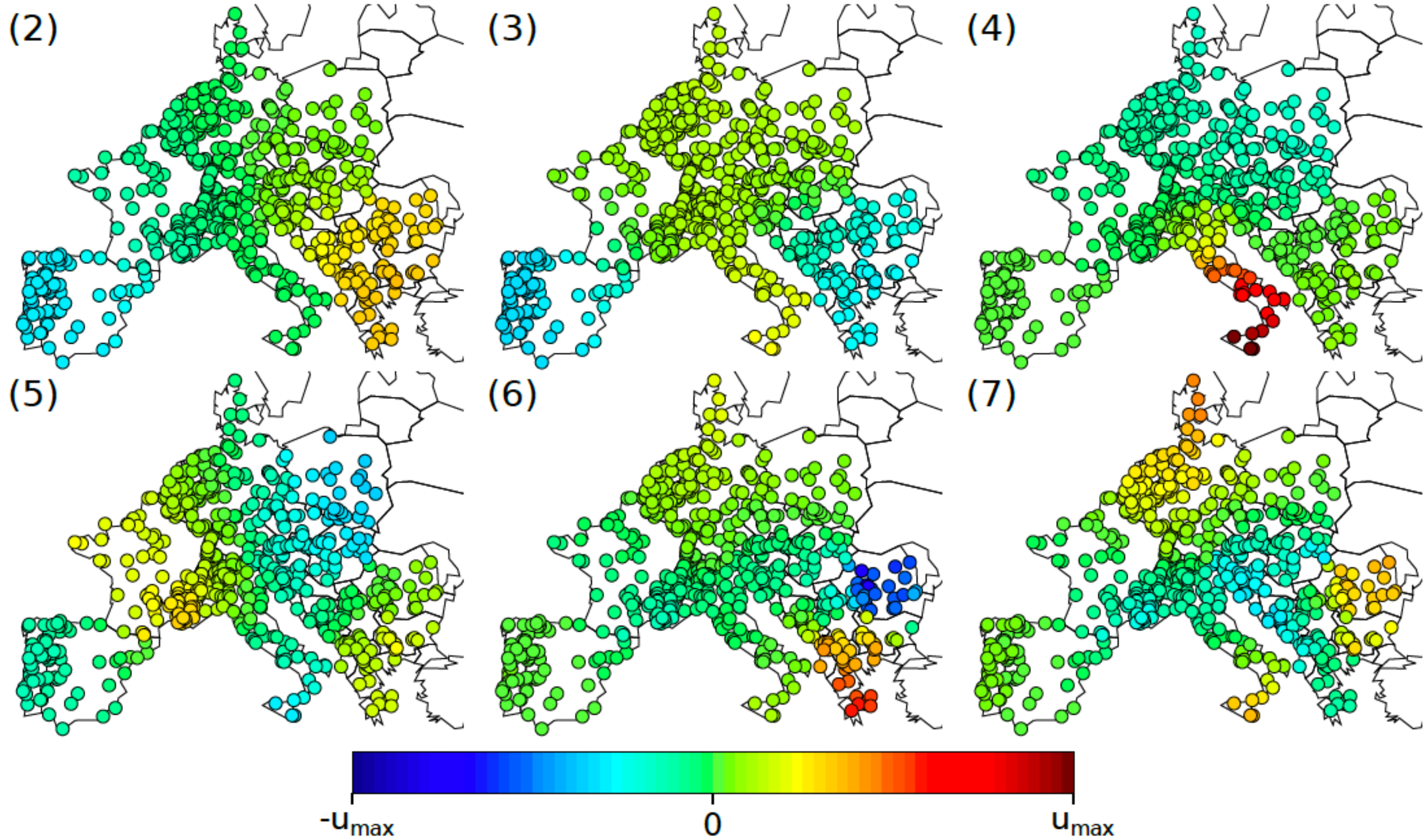

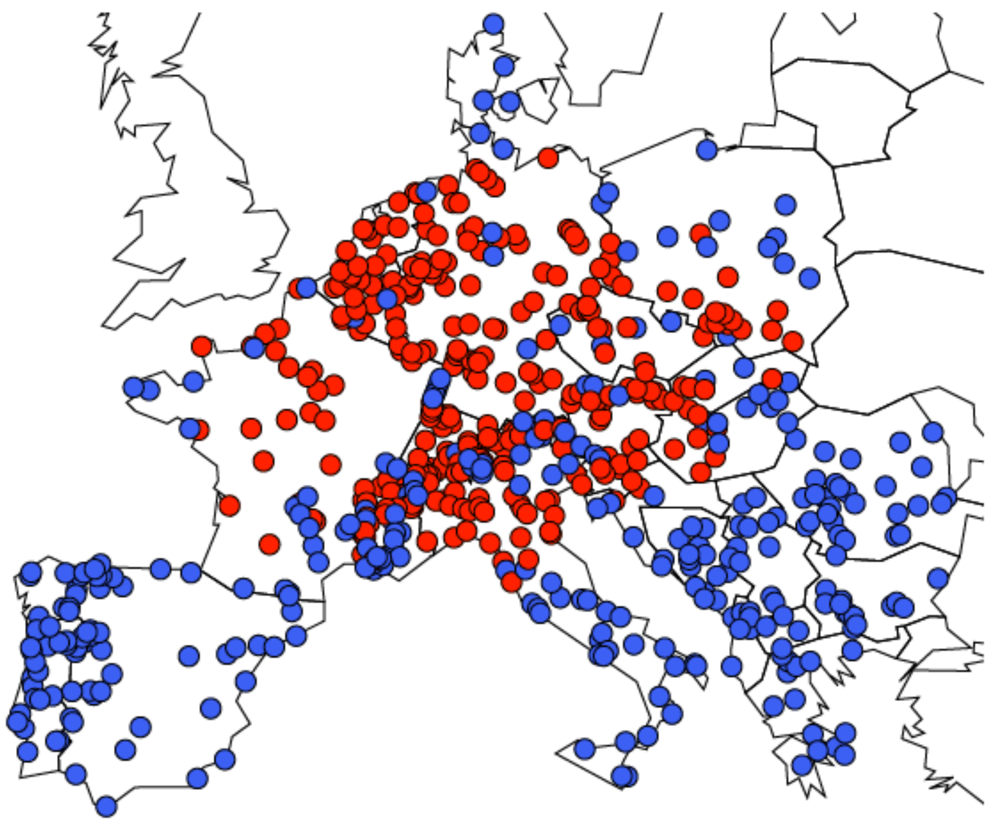

Thank you for your attention !

Laurent Pagnier

Melvyn Tyloo
(talks this afternoon)

M Tyloo, T Coletta and PJ, Phys. Rev. Lett. '18
L Pagnier and PJ, PLoS ONE '19
M Tyloo, L Pagnier and PJ, Sc. Advances '19
L Pagnier and PJ, IEEE Access '19
M Tyloo and PJ, Phys Rev E '19