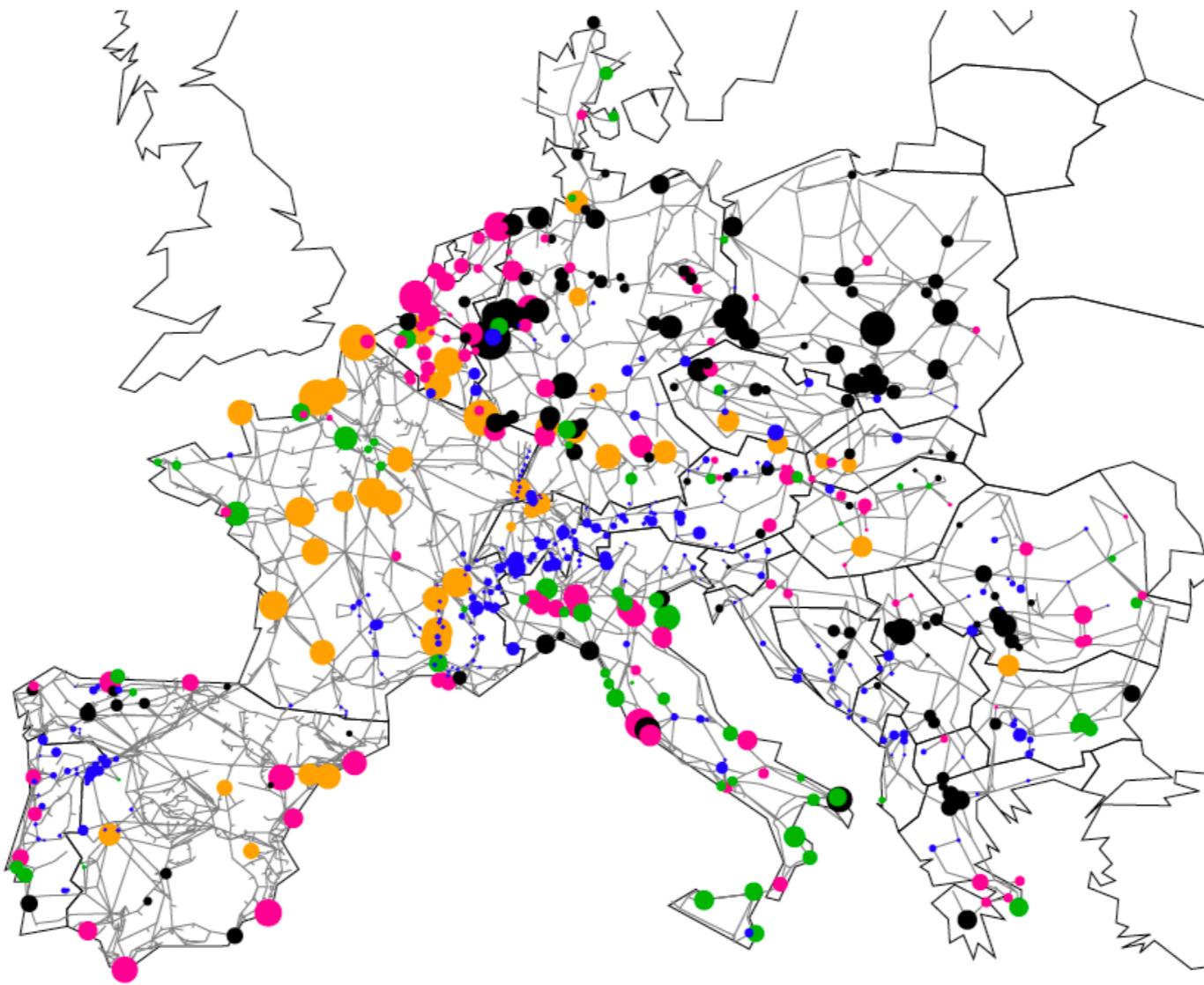


# The Physics of Electric Power Systems



Philippe Jacquod

Physics Colloquium - U Basel - 10.04.2019



UNIVERSITÉ  
DE GENÈVE  

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FACULTÉ DES SCIENCES

**Hes-SO** VALAIS WALLIS  
School of Engineering  $\pi$

**FNSNF**

**swissgrid**

# The team



Philippe



Laurent



Tommaso  
(now with Sophia genetics)

Robin,  
Melvyn  
André



Glory      Koen

**swissgrid**

**FNSNF**

# Outline

- Electric power systems - past and present
- Operating steady-state : power flow equations
- Electric power systems - future
- Power system dynamics : swing equations
  - frequency wave propagation
  - optimal placement of inertia

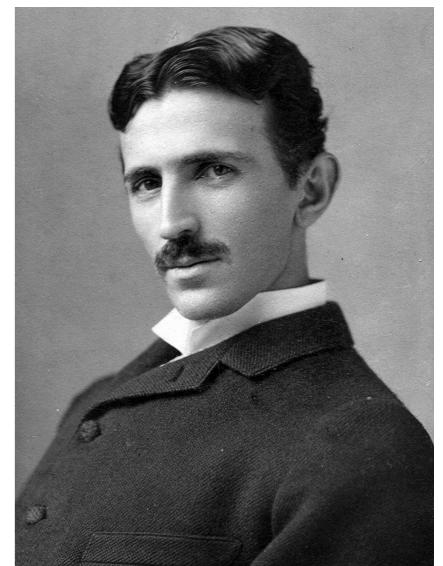
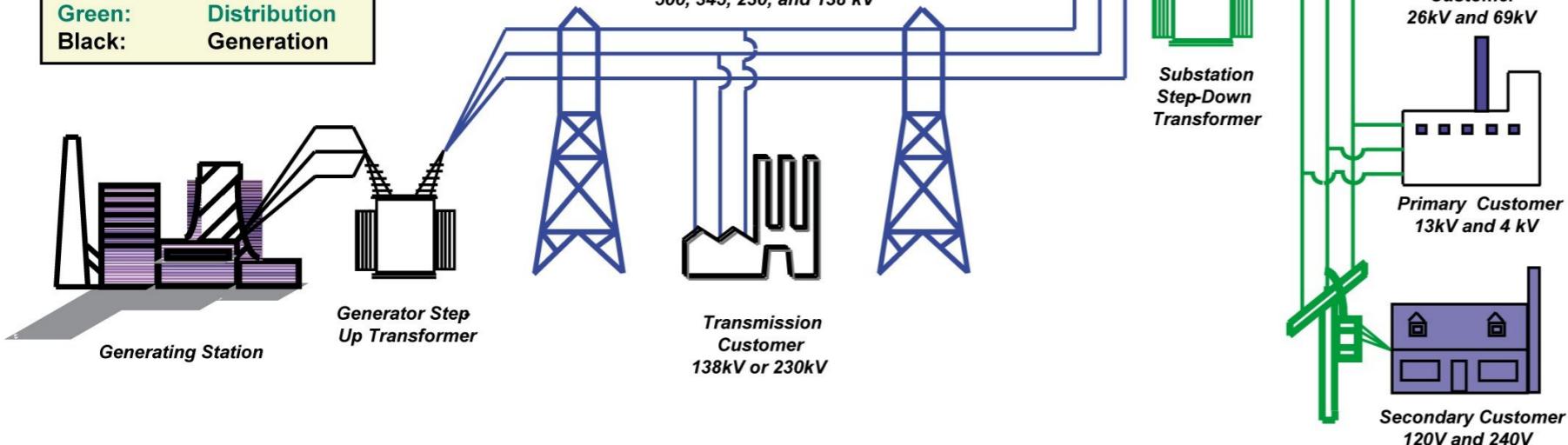
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# What are electric power systems ?

## Basic Structure of the Electric System

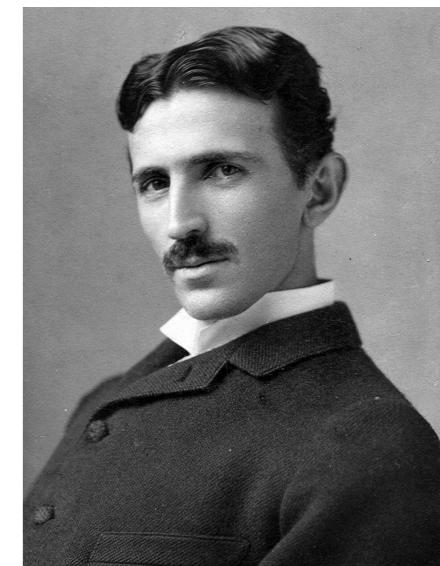
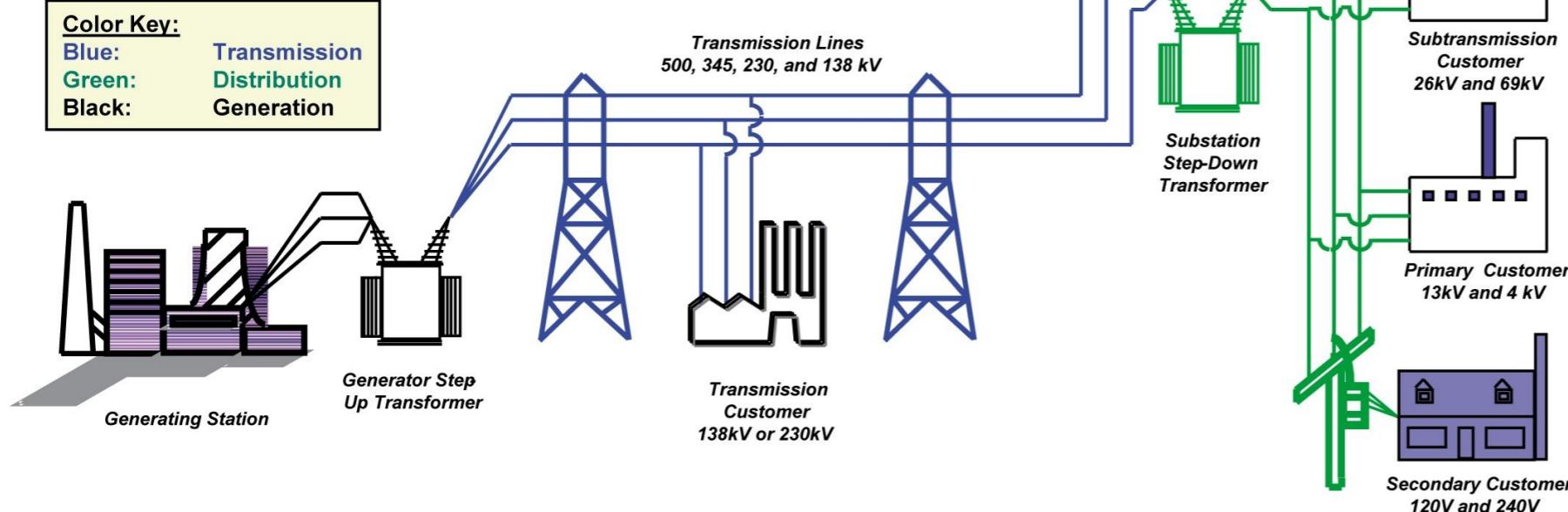
Color Key:	
Blue:	Transmission
Green:	Distribution
Black:	Generation



N Tesla 1856-1943

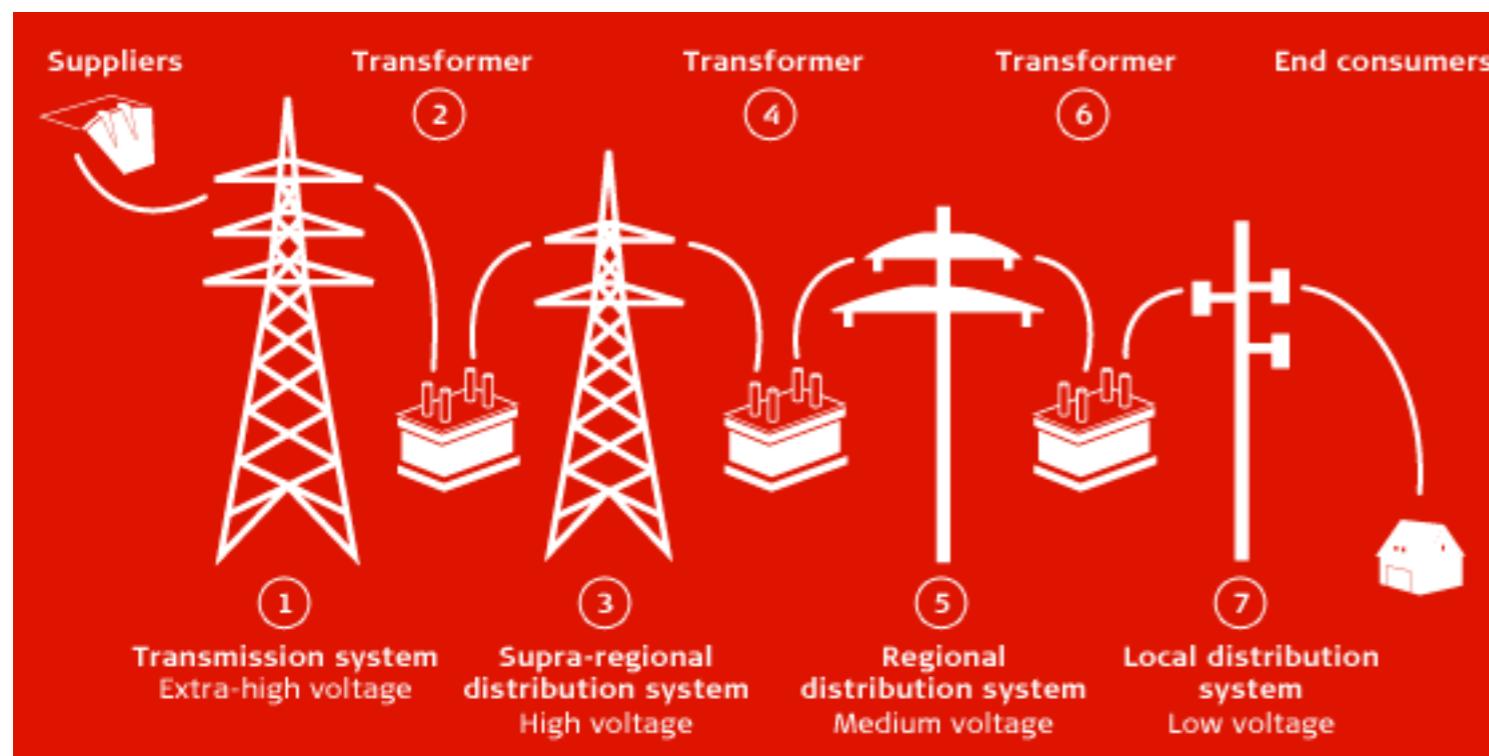
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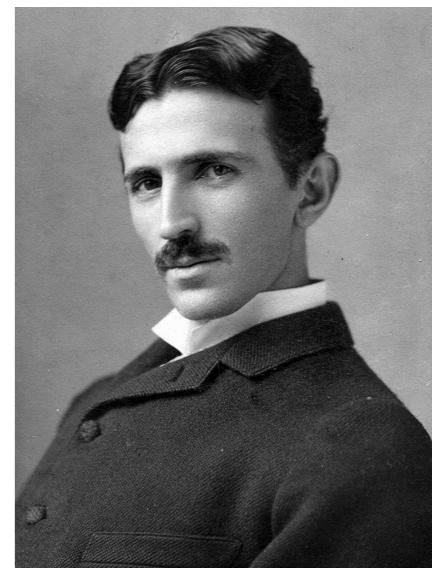
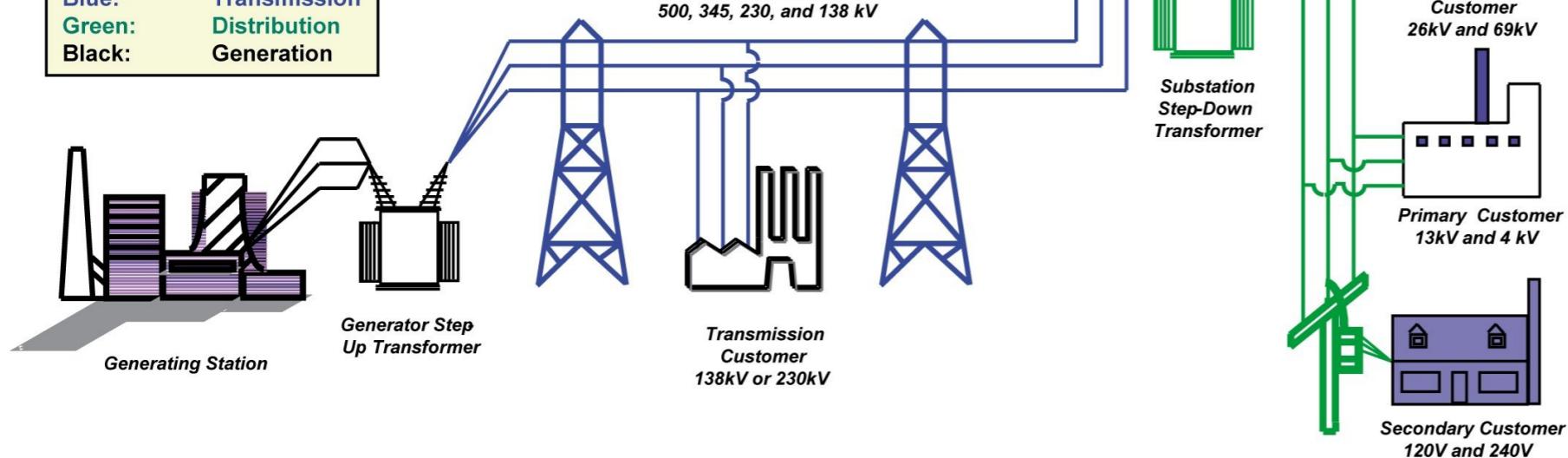
## Seven levels of electric power systems



# What are electric power systems ?

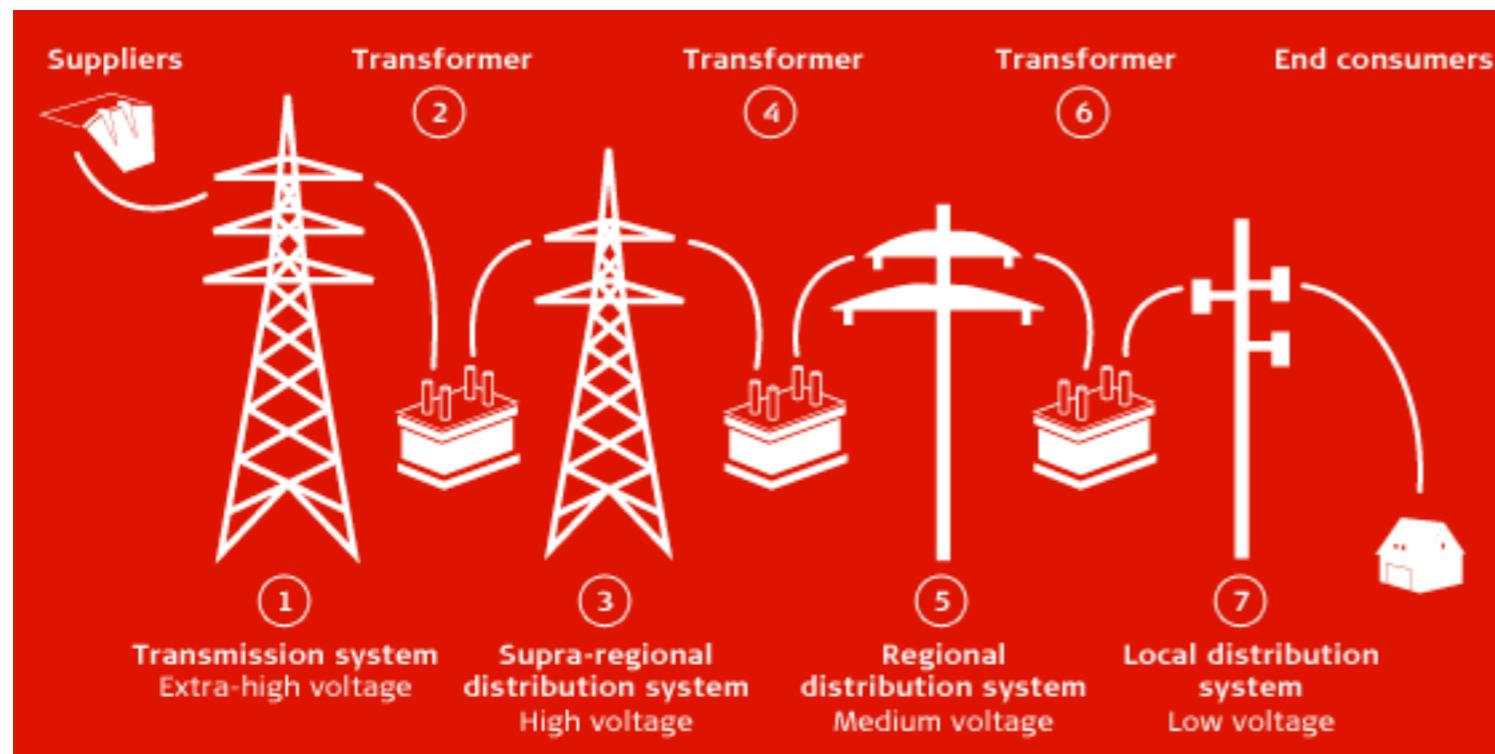
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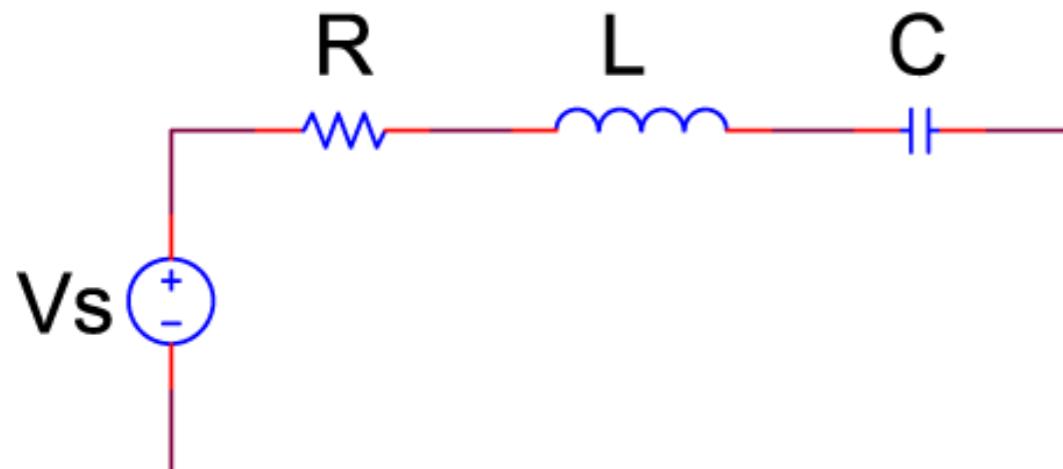
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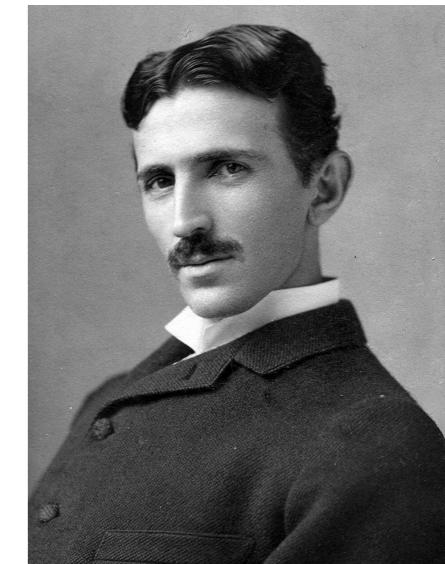
Power :  
\* conserved ( $\sim$ )  
\* control parameter  
“write Eq. for power”

# What are electric power systems ?

- AC electric current/voltages  
(minimize losses ~ high voltages,  
but then need transformers)
- current and voltage not in phase



$$u(t) = u_0 \exp[i\omega t]$$
$$i(t) = i_0 \exp[i(\omega t + \phi)]$$
$$\tan(\phi) = (\omega L - 1/\omega C)/R$$



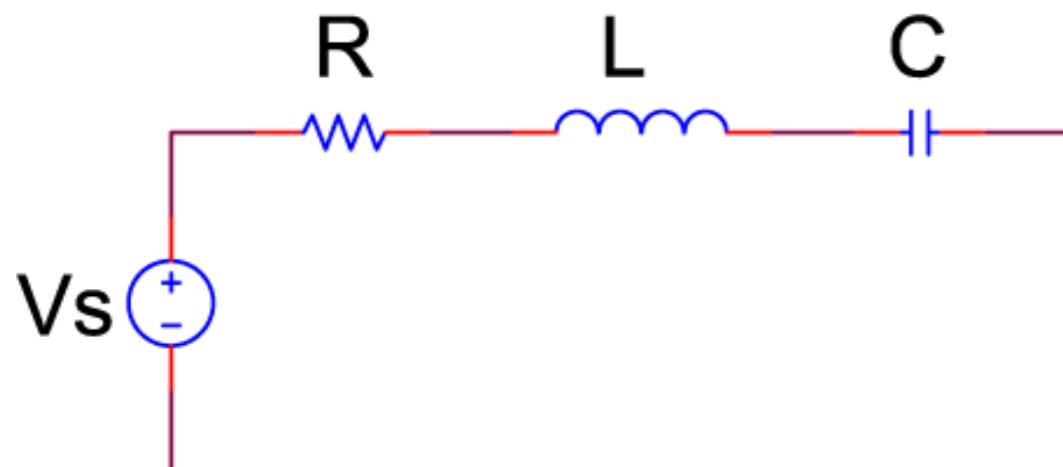
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- complex impedance  $u(\omega) = Z(\omega) i(\omega)$        $Z(\omega) = R + i\omega L - i/\omega C$
- AC ~ frequency coupled to power balance

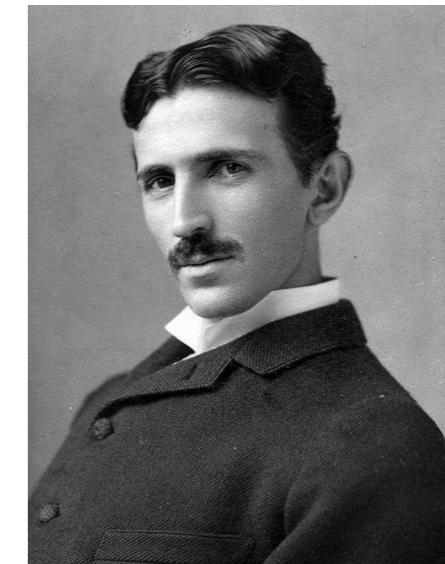


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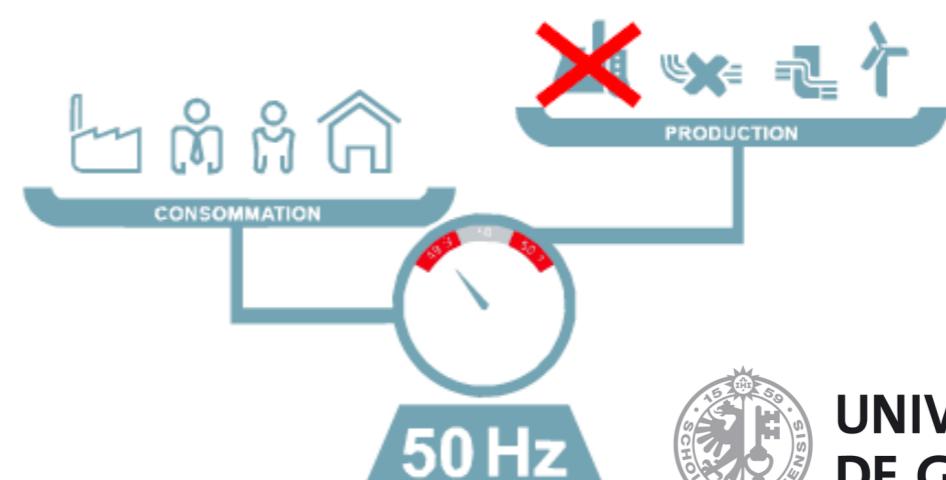


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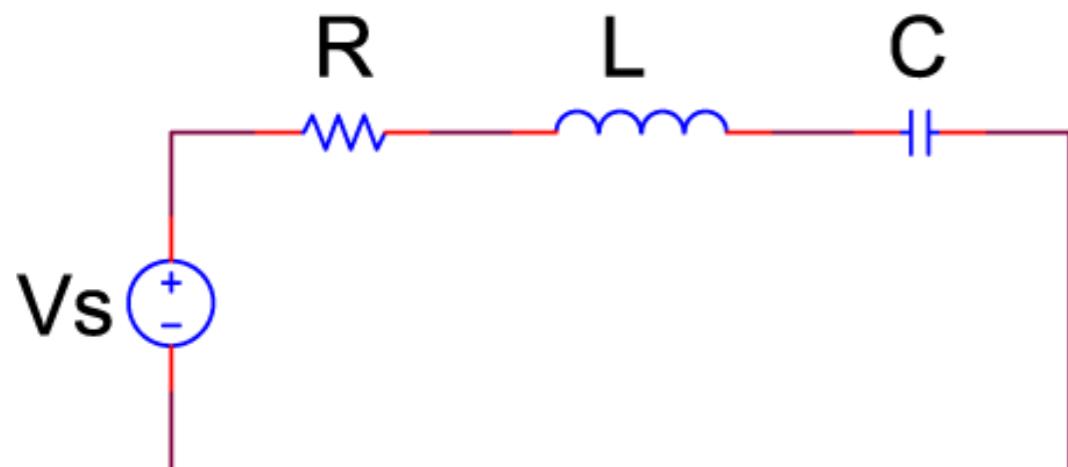
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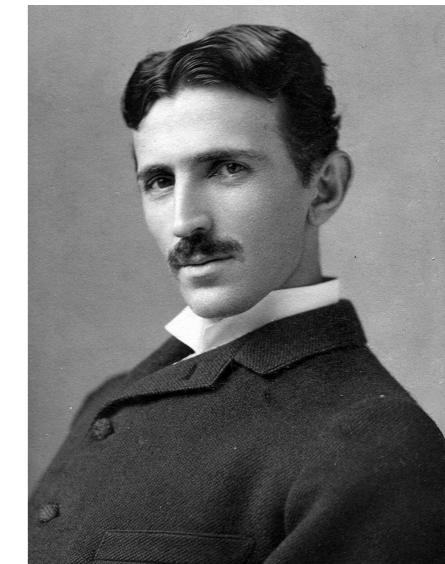


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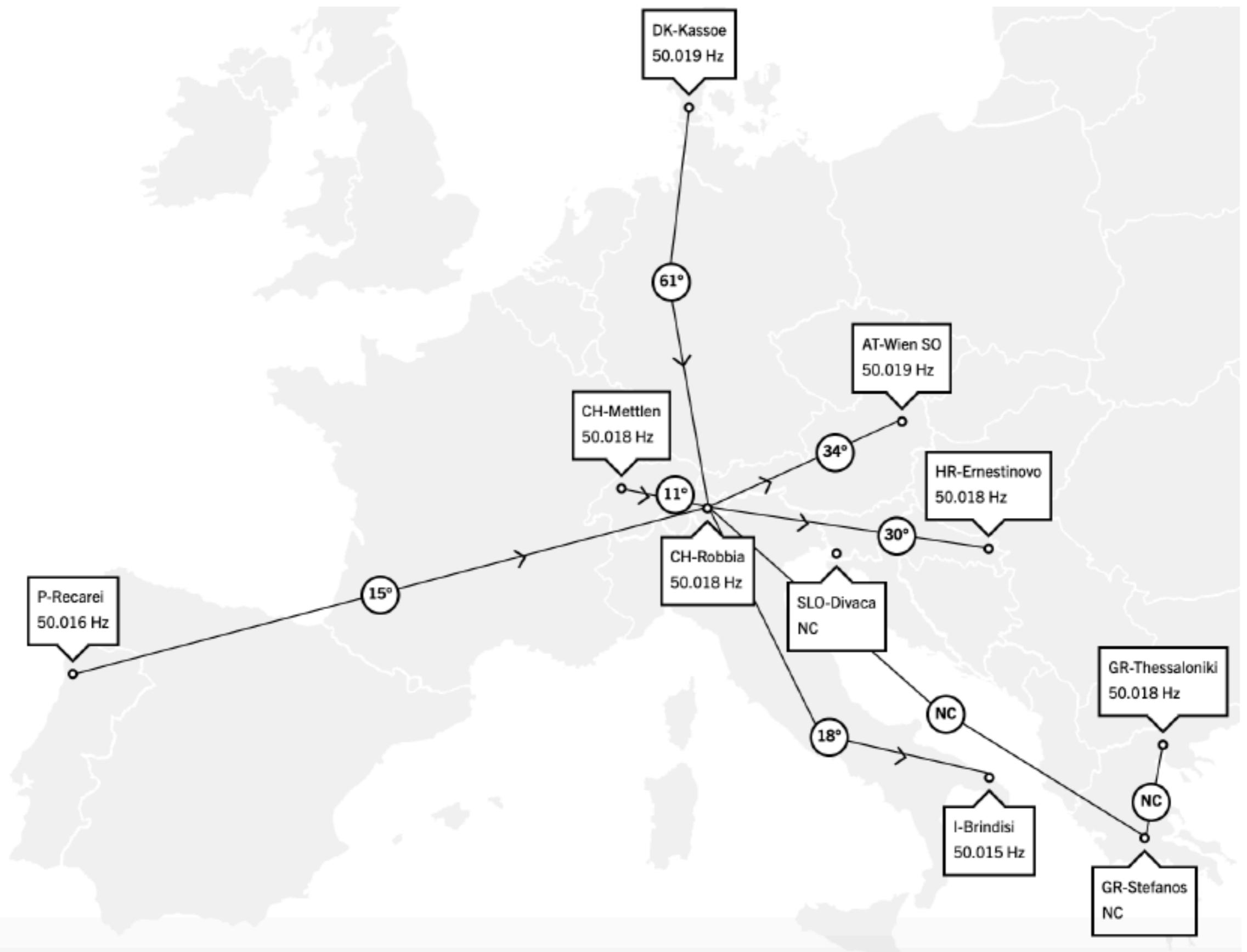
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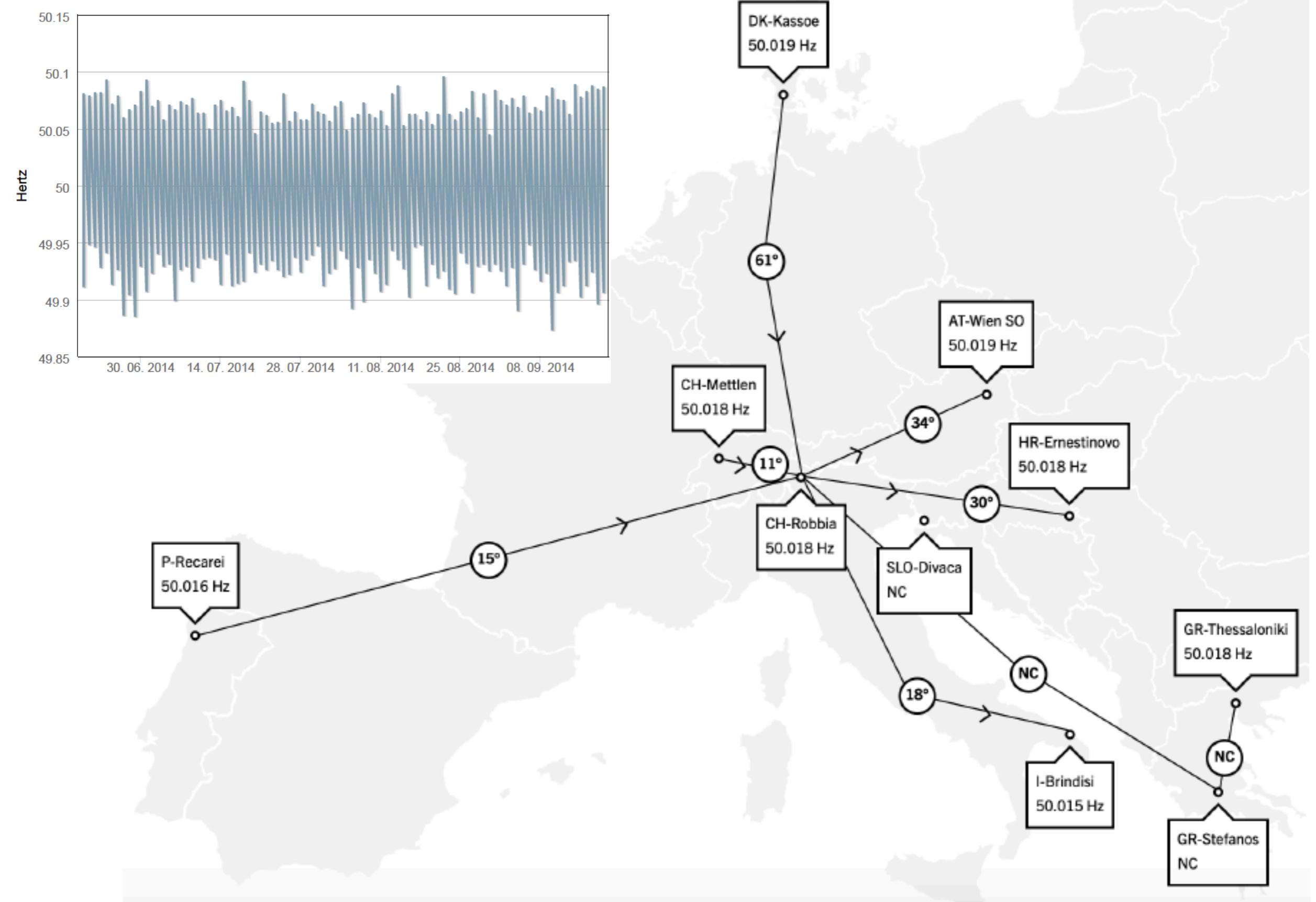
Very high voltage grid (220kV, 380 kV)  
 $R \ll \omega L$



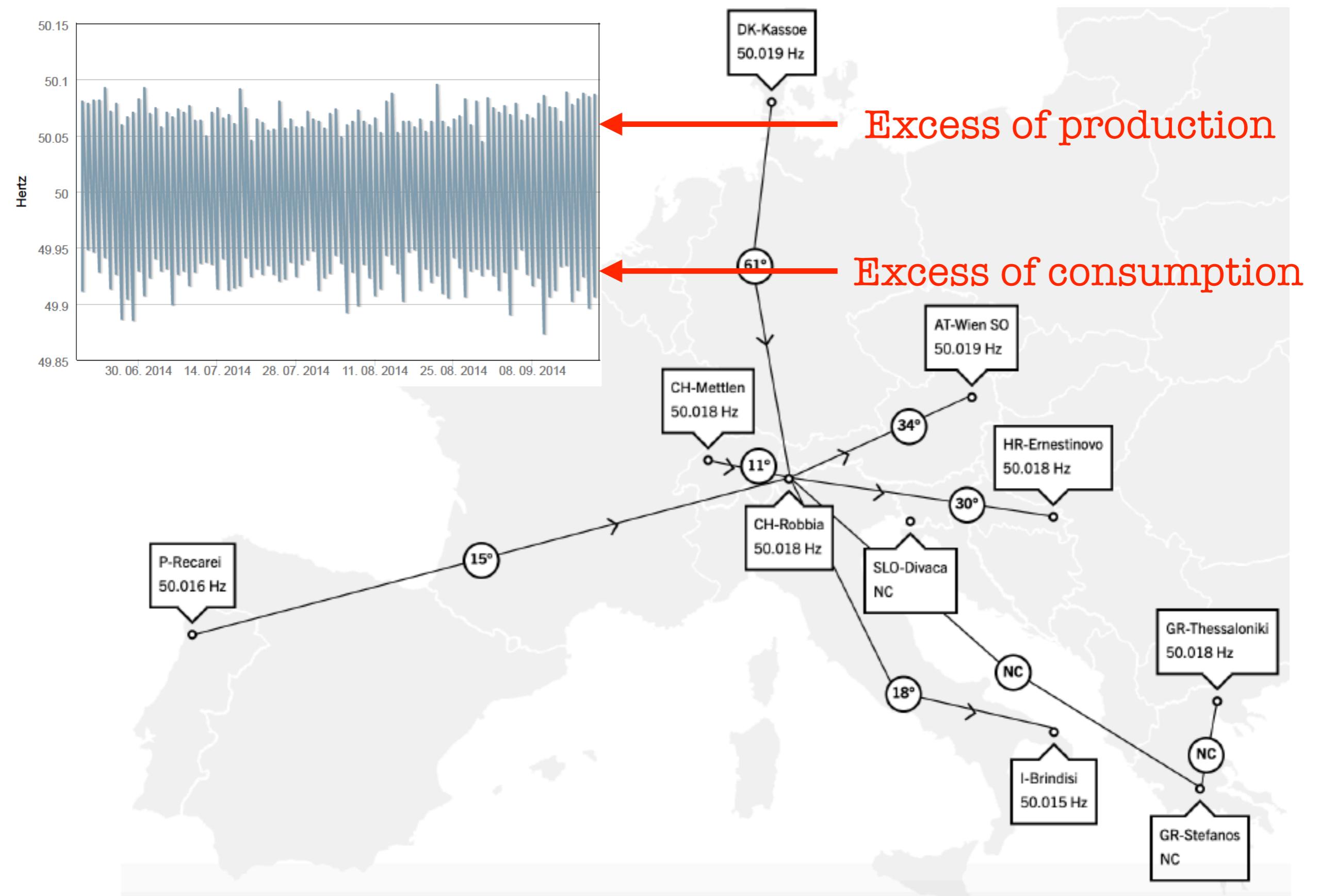
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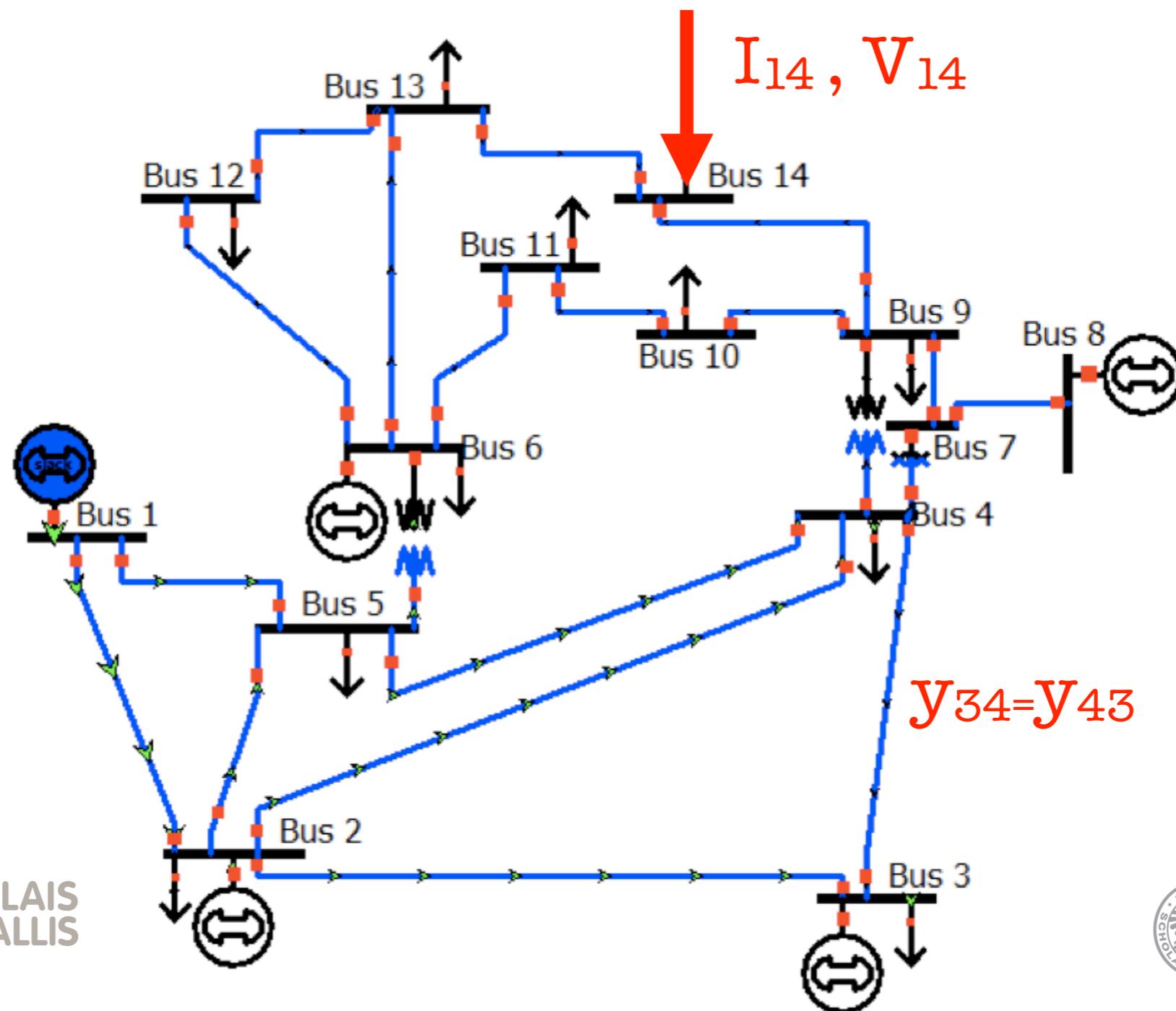
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# Operating steady-state of AC power grids

- Ohm's law

$$I_i = \sum_j Y_{ij} V_j$$

$I_i$  : current injected/collected at node i  
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 $Y_{ij}$  : admittance matrix



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- Active power vs.

$$P = Re(S)$$

## finite time-average

# “transmitted power”

(injected and consumed)

vs.

: current injected/collected at node i

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## reactive power

$$Q = \text{Im}(S)$$

## zero time-average

“oscillating in the circuit”

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Voltages  
at buses i and j

Conductance  
matrix

Voltage angles  
at buses i and j

Susceptance  
matrix

# Approximated power flow equations : very high voltage

- Admittance dominated by its imaginary part  
for large conductors  $\sim$  high voltage  
 $G/B < 0.1$  for 200kV and more

**neglect conductance**



$$P_i \simeq \sum_j |V_i V_j| B_{ij} \sin(\theta_i - \theta_j)$$
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- No conductance  $\sim$  no voltage drop

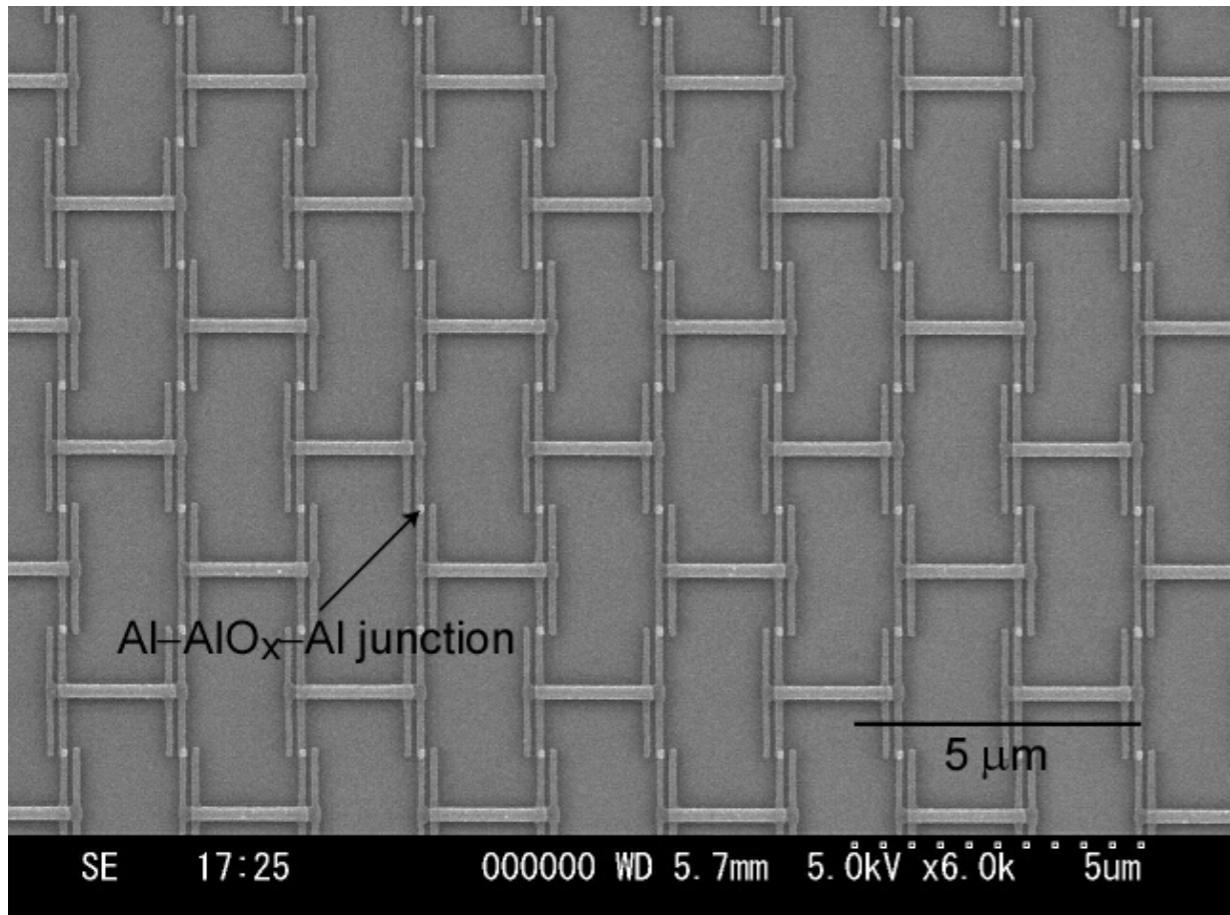
**consider constant voltage**

- \*decoupling between P and Q
- \*consider P only  
(a.k.a. lossless line approx.)



$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

# Synchronous fixed points vs. Josephson junctions



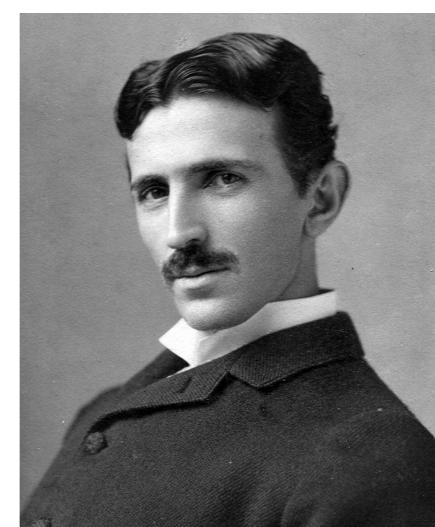
Josephson current

$$I_{ij} = I_c \sin(\theta_j - \theta_i)$$

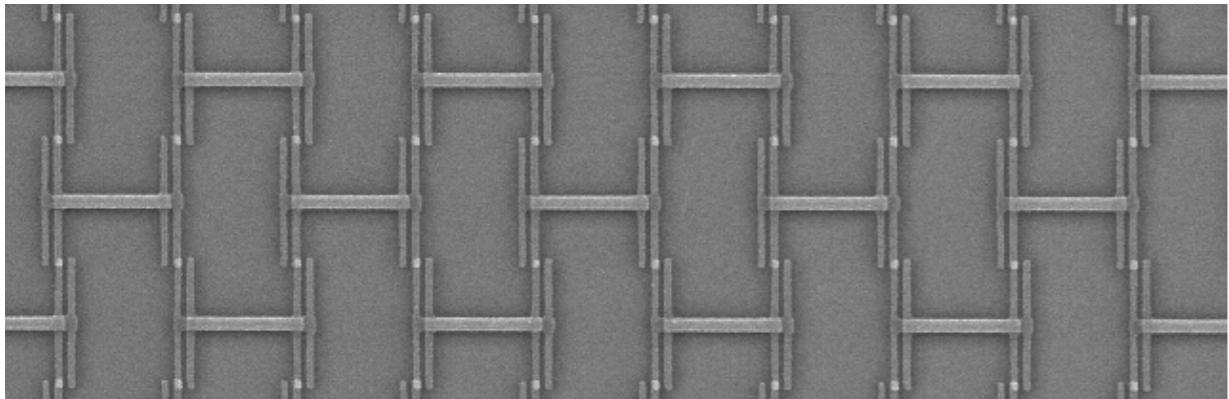


High voltage  
AC transmitted power

$$P_{ij} = B_{ij} \sin(\theta_i - \theta_j)$$



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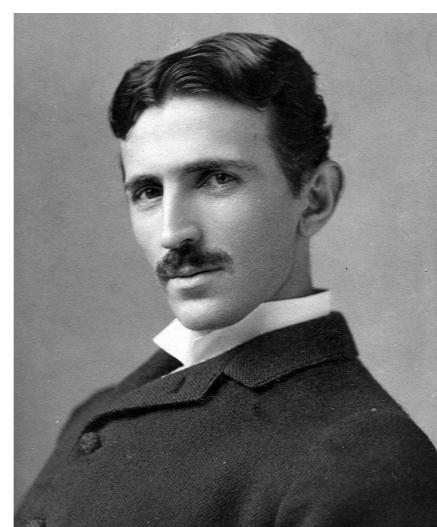
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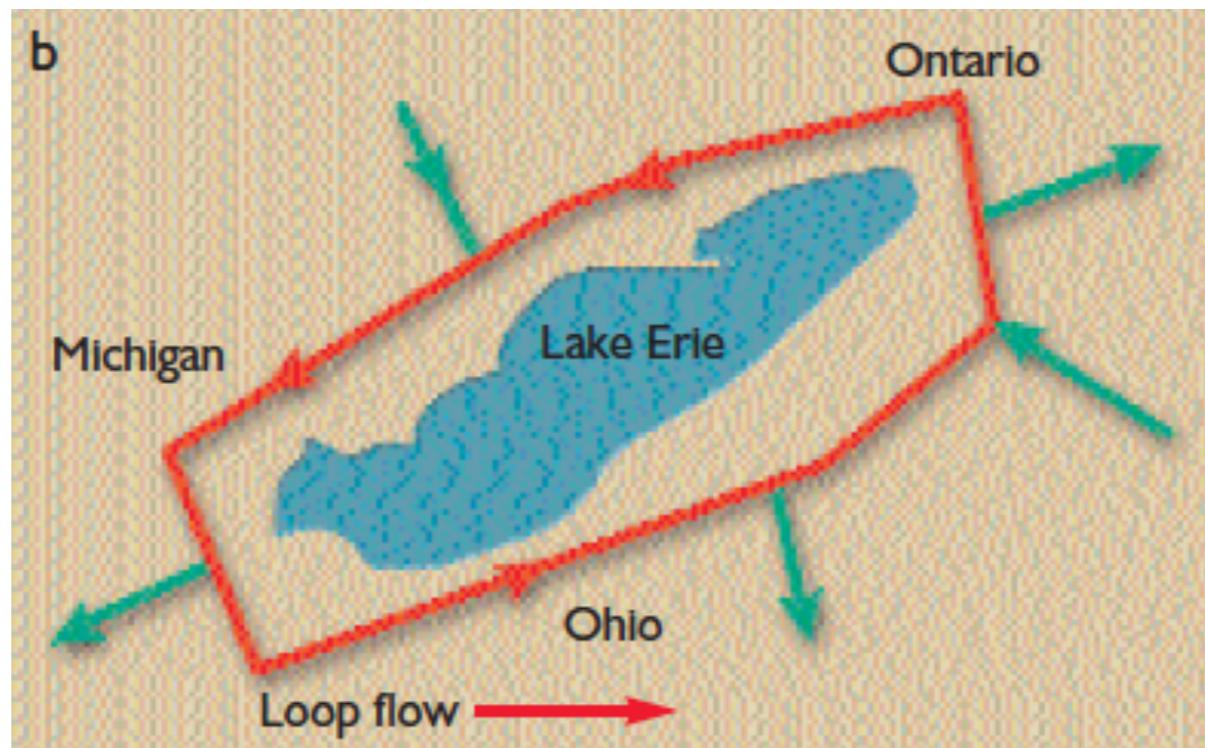


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# Circulating loop flows



“Where the network jogs around large geographical obstacles, such as the Rocky Mountains in the West or the Great Lakes in the East, loop flows around the obstacle are set up that can drive as much as 1 GW of power in a circle, **taking up transmission line capacity without delivering power to consumers.**”

# Circulating loop flows



“Where the network is interrupted by large geographical obstacles, such as the Rocky Mountains in the West or the Great Lakes in the East, loop flows can be set up that can drive as much as 1 GW of power in a circle around the obstacle. This transmission line capacity without delivering power to consumers is called a ‘persistent (power) current?’”

# Circulating loop flows

\*Thm: Different solutions to the simplified power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16

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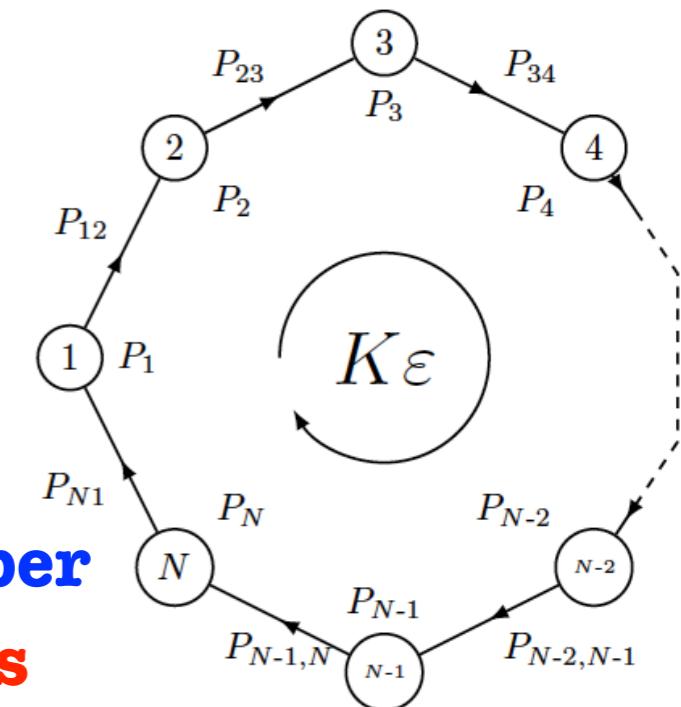
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\*Voltage angle  $V_i = |V_i|e^{i\theta_i}$  uniquely defined

→  $q = \sum_i |\theta_{i+1} - \theta_i|_{2\pi}/2\pi \in \mathbb{Z}$  **~topological winding number**

→ discretization of these loop currents **~vortex flows**

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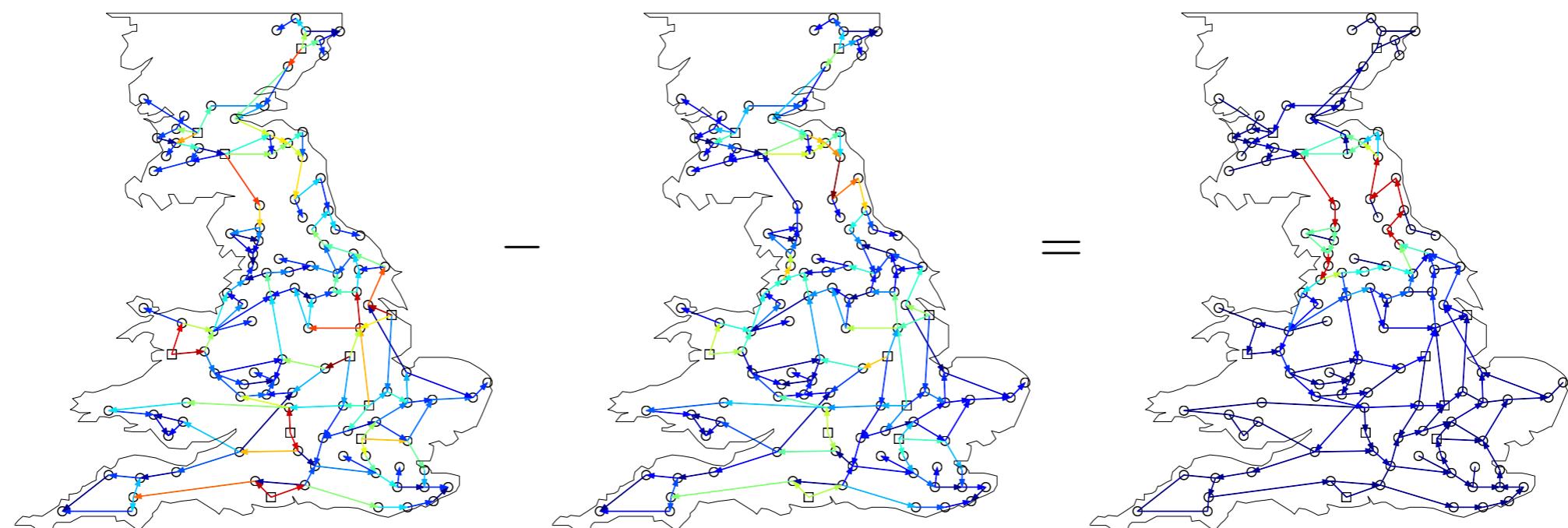
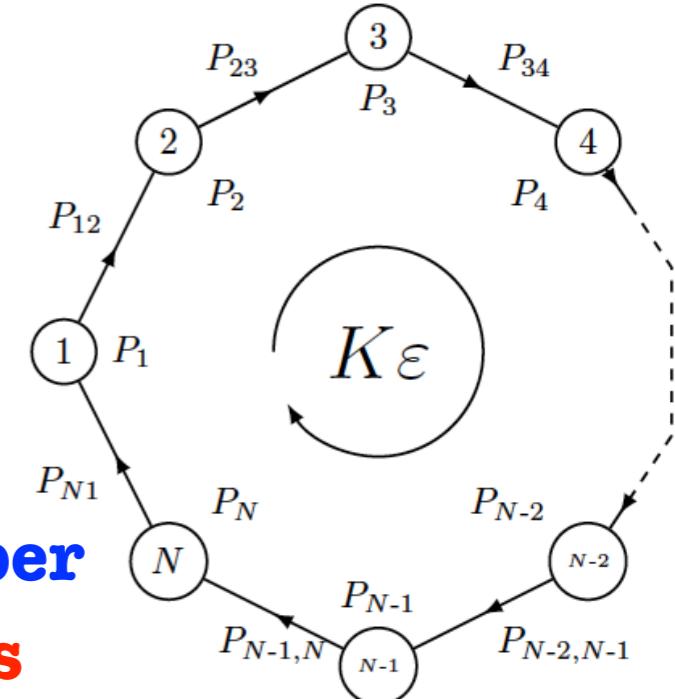
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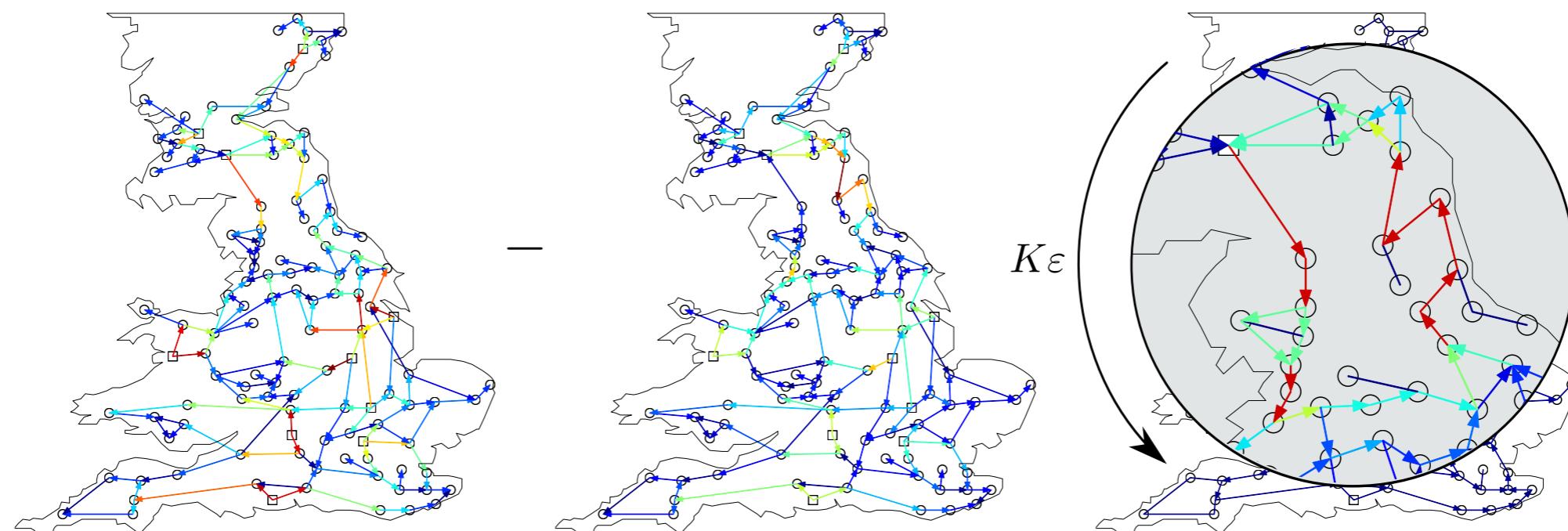
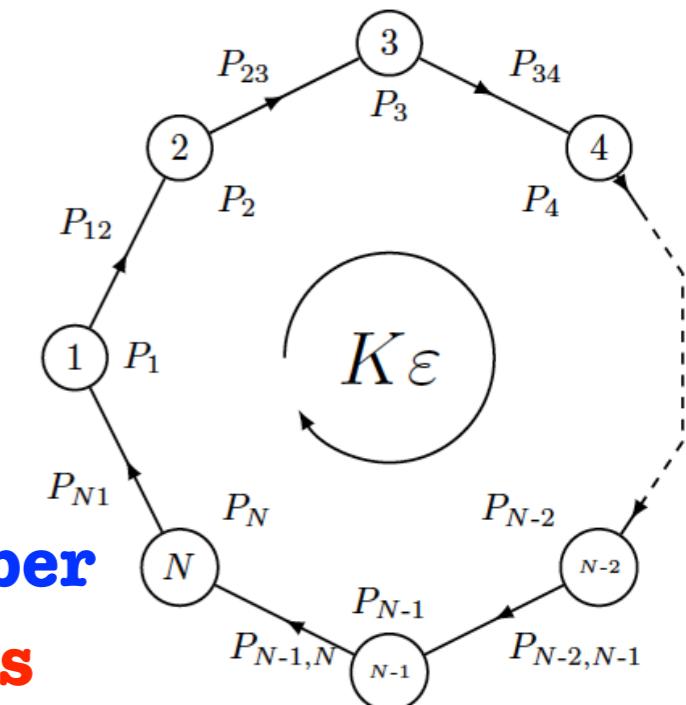
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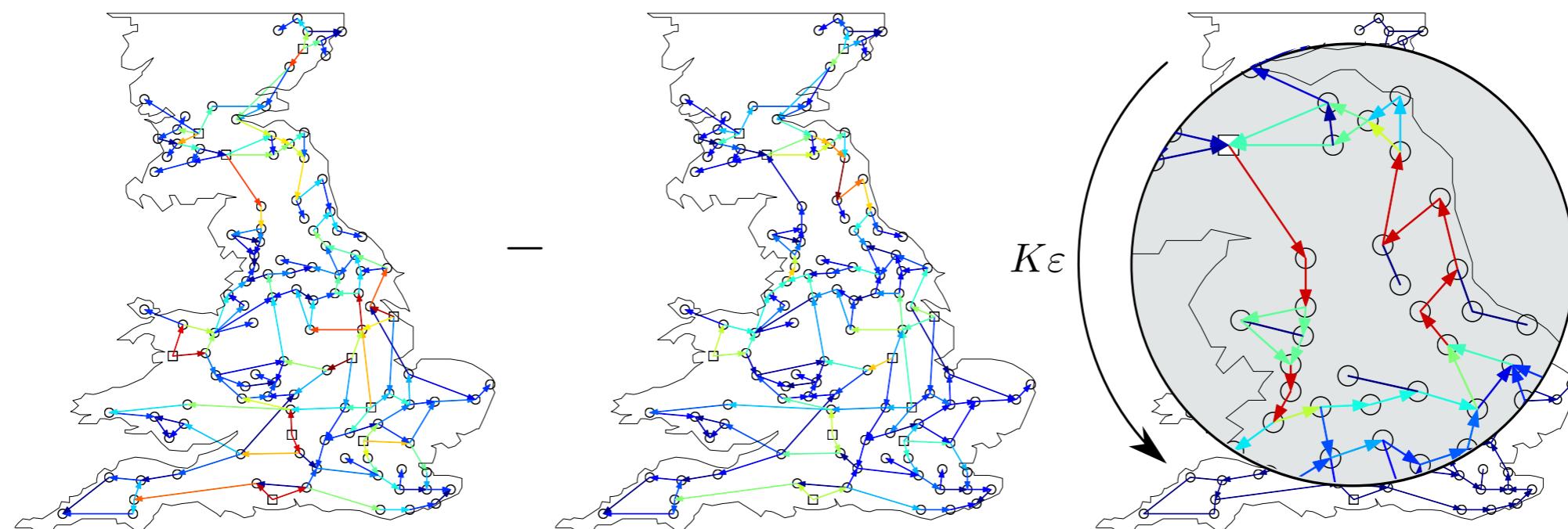
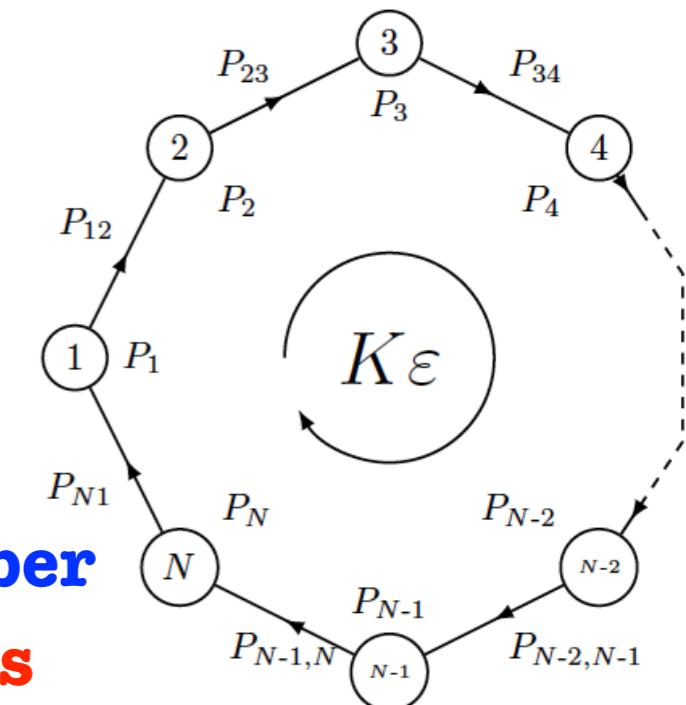
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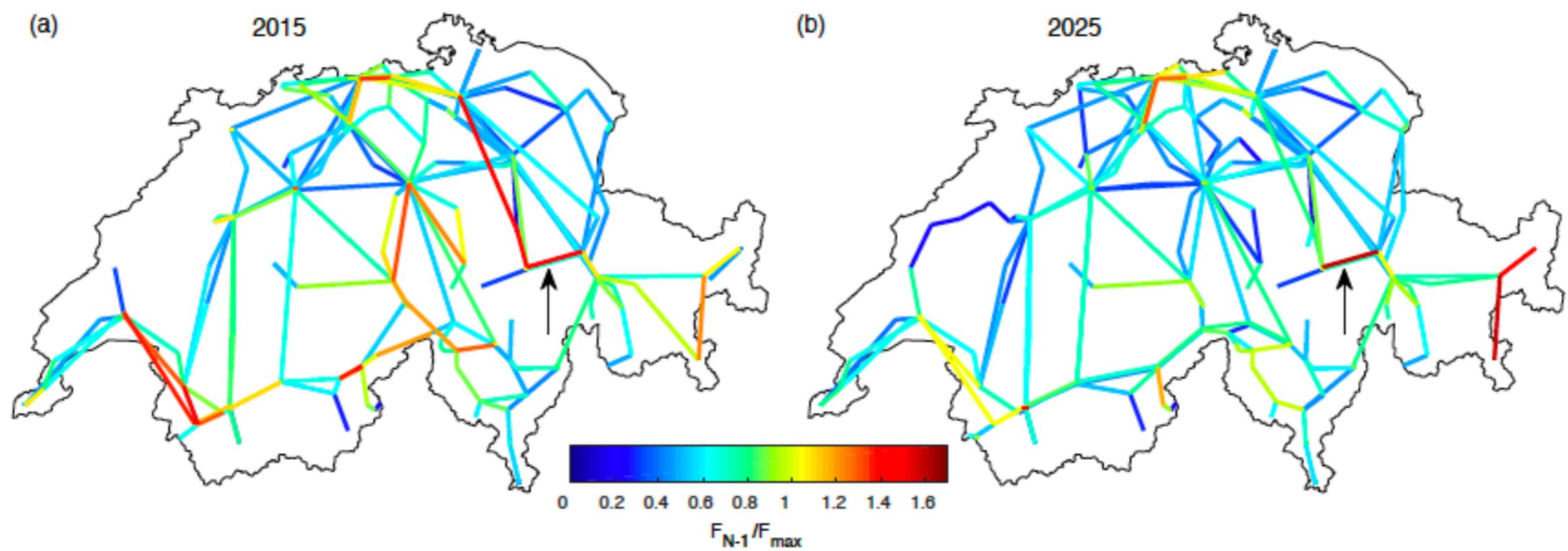
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They are used for :

- (i) planning future expansions/upgrade



- [www.swissgrid.ch/en/home/projects/strategic-grid.html](http://www.swissgrid.ch/en/home/projects/strategic-grid.html)
- L Pagnier and PJ, "Swissgrid's strategic grid 2025 : an independent analysis"

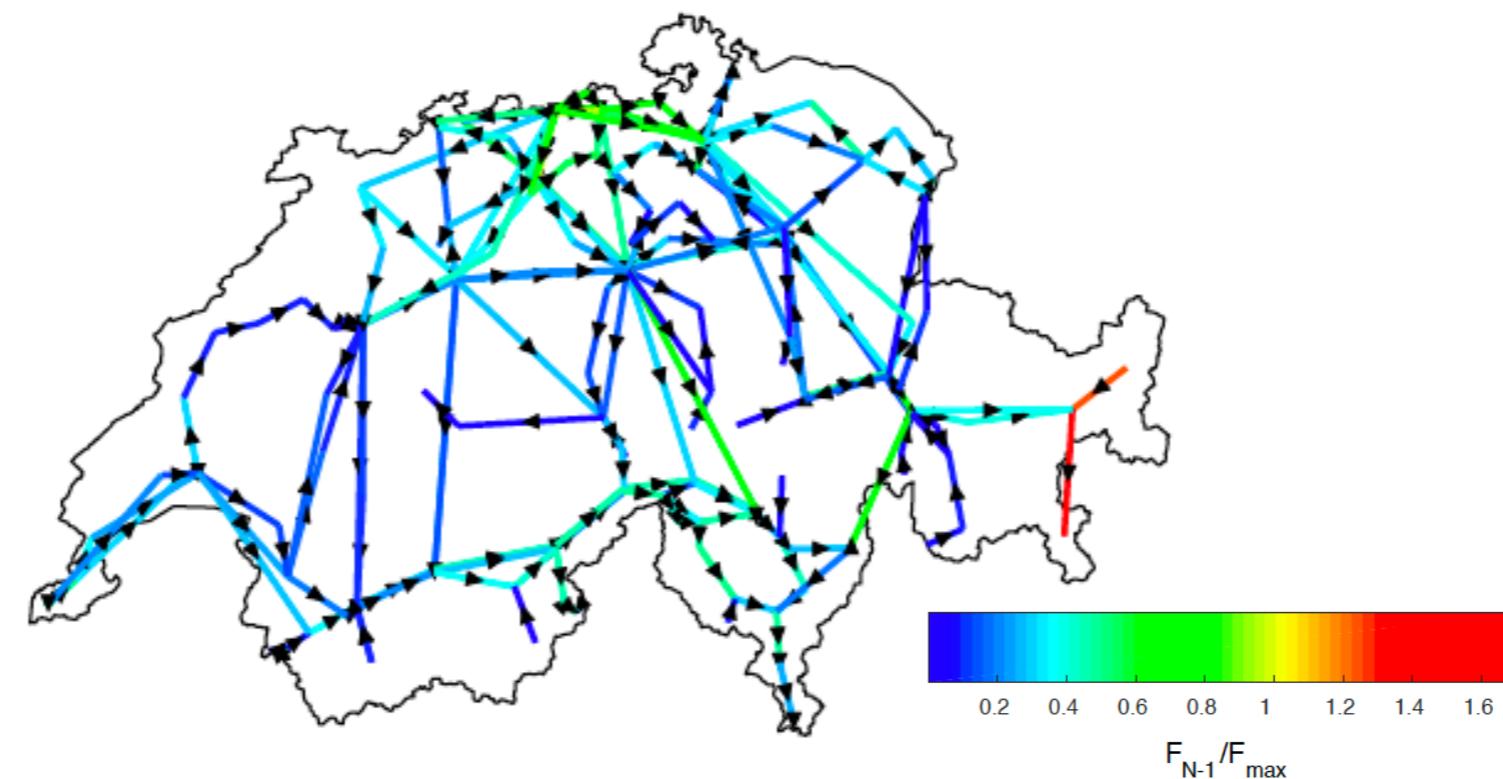
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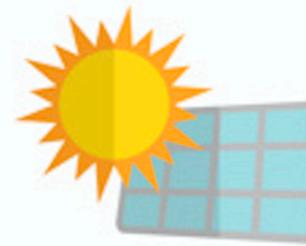
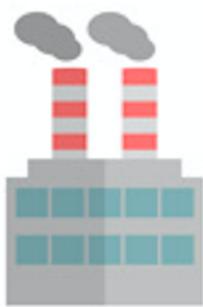
- (ii) running the existing system as well as possible
  - \* is the current state of the grid problematic ?
  - \* can it tolerate one fault (N-1 assessment) ?
  - \* is the scheduled state (say in 5 mins) problematic ?



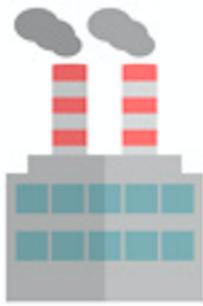
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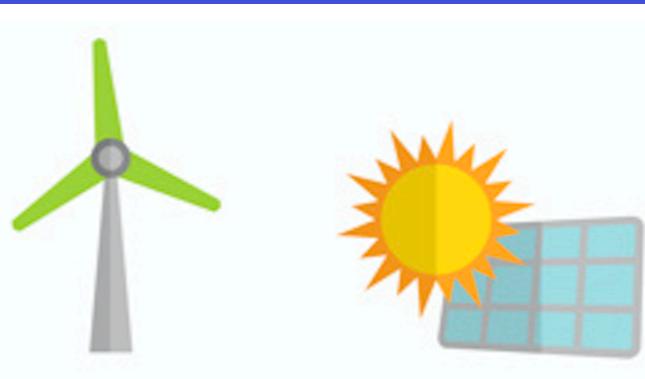
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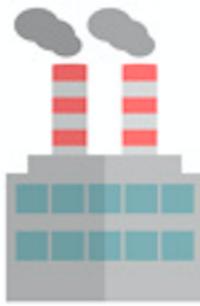


- Centralized

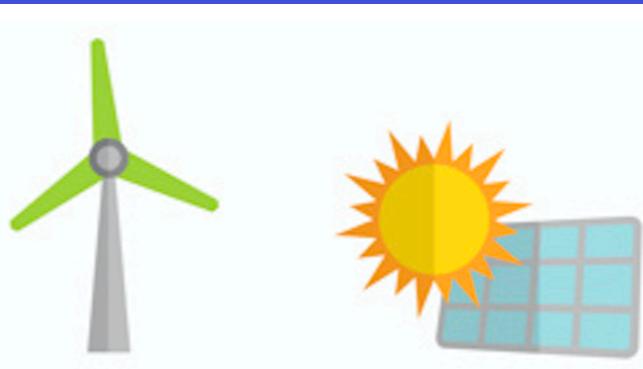


- Decentralized
  - issues at distribution level
  - easier at transmission level  
(=reduction in load)

# Today's vs. tomorrow's power grids



- Centralized
- Dispatchable/predictable



- Decentralized
  - issues at distribution level
  - easier at transmission level  
(=reduction in load)
- Uncertain
  - power / energy reserve

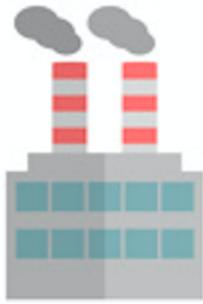
Today's



Aaron Rupar

@atrupar

Suivre



• Centr

• Dispa

TRUMP: "If Hillary got in... you'd be doing wind. Windmills. Weeeee. And if it doesn't blow, you can forget about television for that night. 'Darling, I want to watch television.' 'I'm sorry! The wind isn't blowing.' I know a lot about wind."

Traduire le Tweet

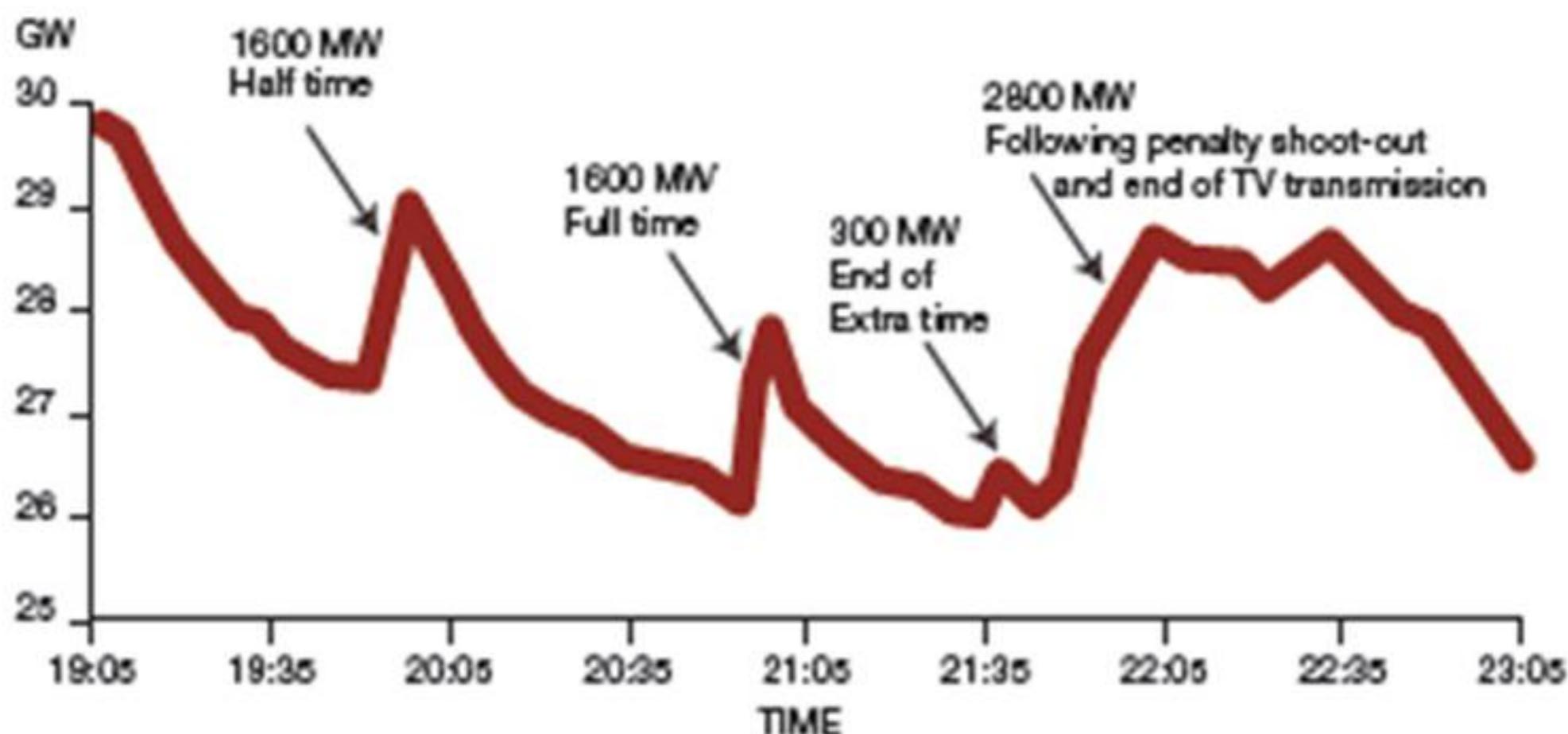


n level  
on level

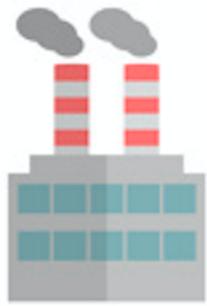
erve

# Variability and uncertainty are nothing new for grid operators

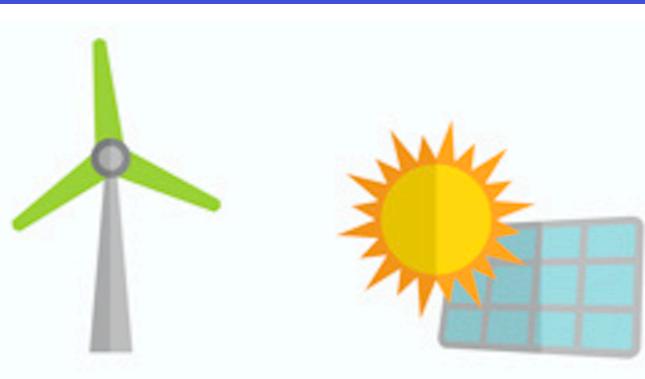
England Vs Germany 1990, World Cup Semi-Final, Kick Off 19:00



# Old vs. new productions

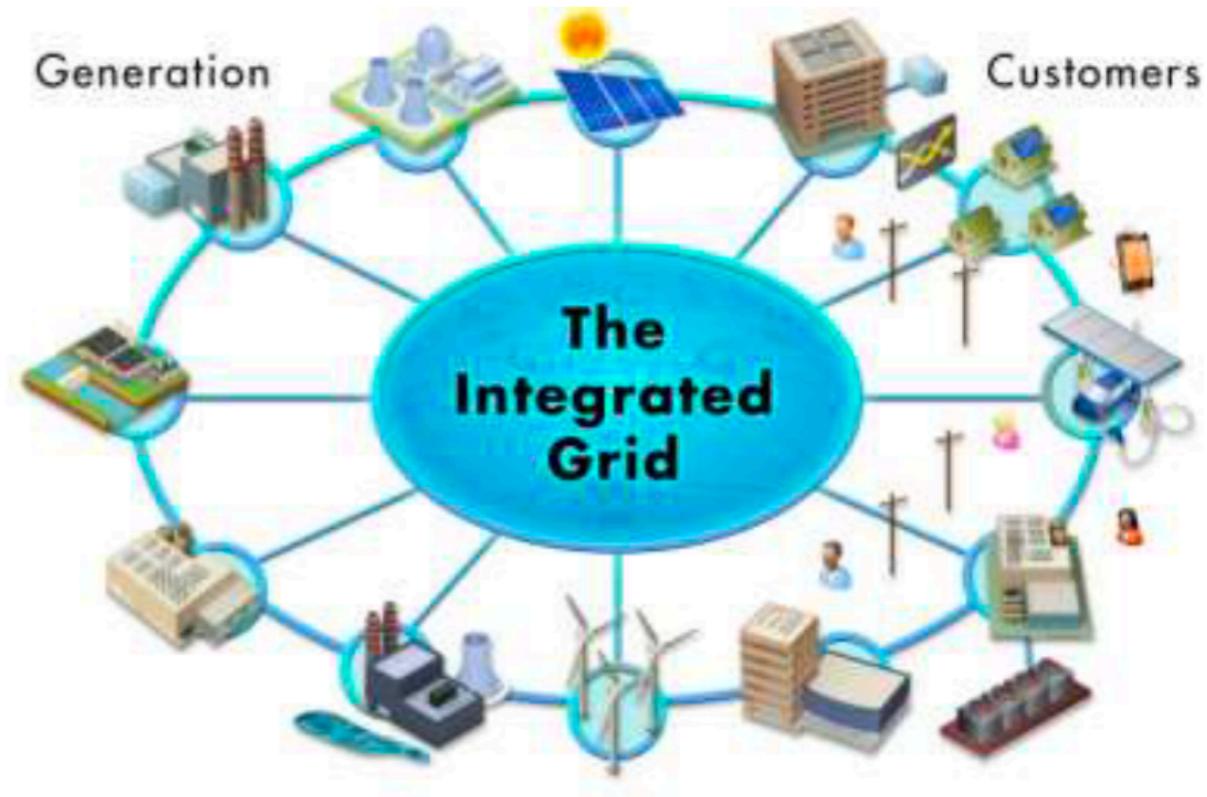
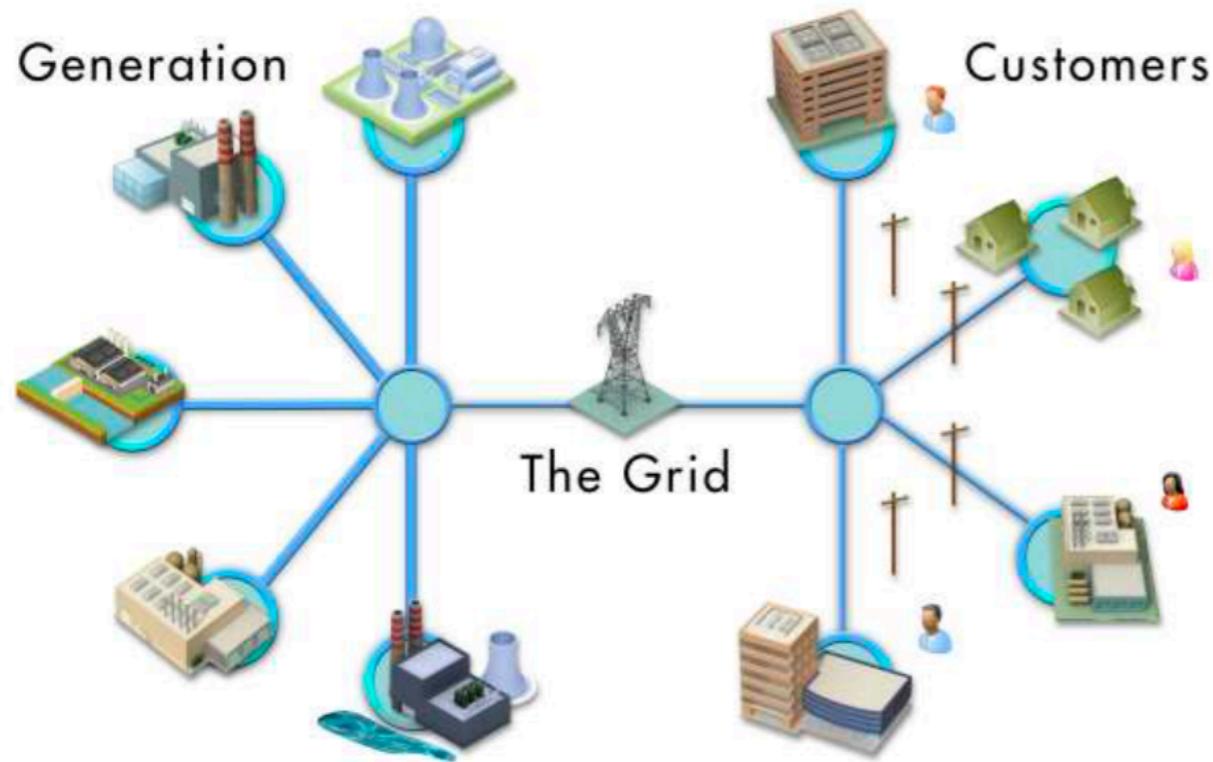


- Rotating machine with inertia
  - frequency definition
  - grid stability :
    - \* energy reserve
    - \* droop control



- No or much less inertia (inverter-connected)

# Today's vs. tomorrow's electric power systems



Tomorrow's grid will be :

- less dispatchable
- less forecastable
- more dynamic (less inertia)
- bi-directional
- based on power electronics

# Today's vs. tomorrow's electric power systems



?? Would substituting traditional plants  
with new renewables jeopardize grid  
stability ??

Tomorrow's grid will be :

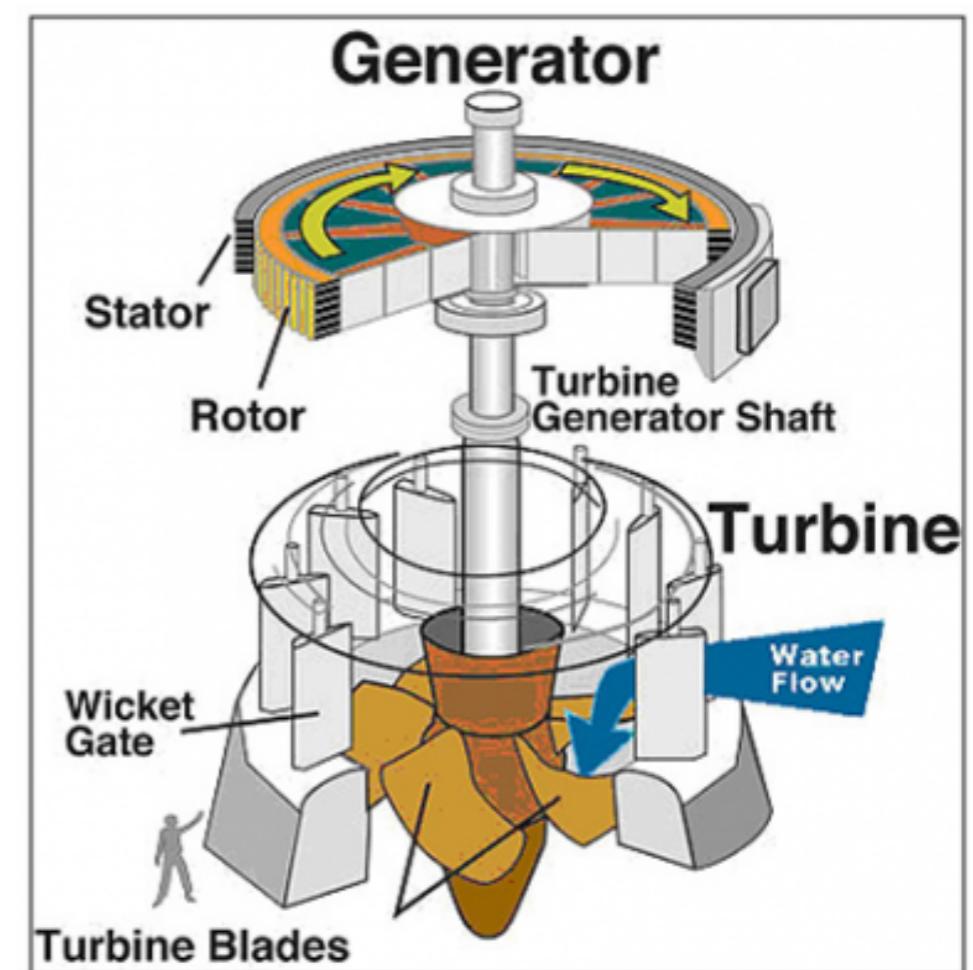
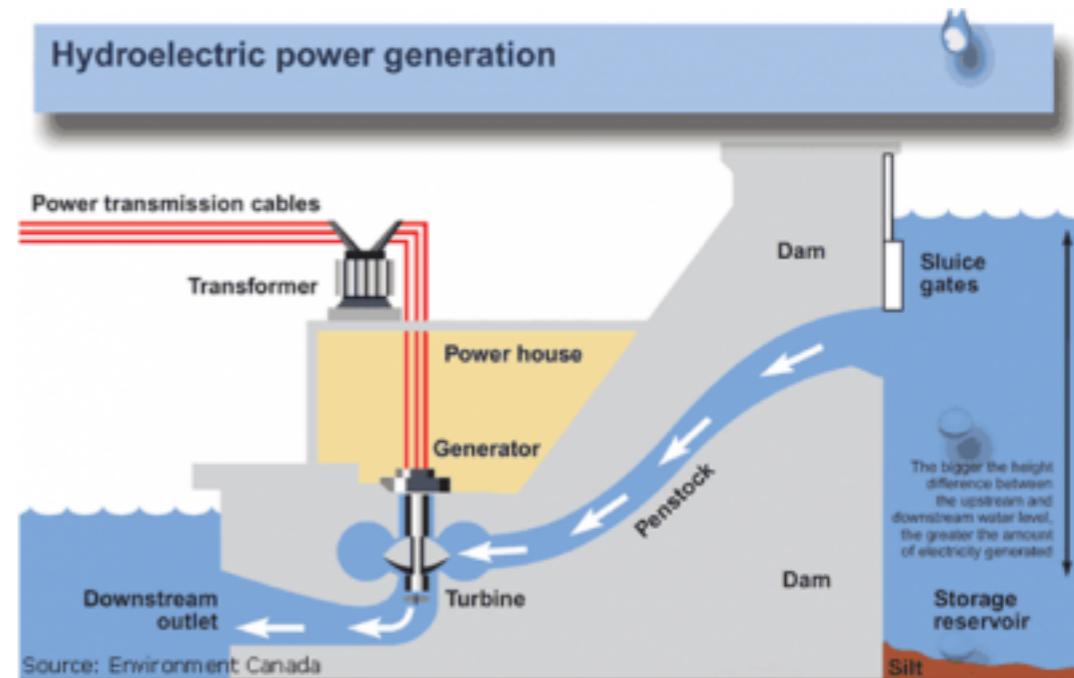
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# Outline

- Electric power systems - past and present
- Operating steady-state : power flow equations
- Electric power systems - future
- Power system dynamics : swing equations
  - frequency wave propagation
  - optimal placement of inertia+control

# Dynamics of AC power grids

- Electricity production with rotating machines
- Potential, chemical, nuclear or thermal energy converted into mechanical energy (rotation)
- Mechanical energy converted into electric energy
- Time-dependence : think power instead of energy
- Balance between power in, power out and energy change in rotator : **SWING EQUATIONS**



# Dynamics of AC power grids

## Dynamics: Swing Equations.

- from few AC cycles to ~30 secs.
- In rotating frame @ 50/60 Hz

$$I_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

Bergen and Hill, 1981

- I : inertia  $\sim$  rot. kinetic energy
- D : damping from primary control + frequency-dependence of loads

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- A.k.a. second-order Kuramoto model  $\sim$ model for synchronization

$$\frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$



C Huygens 1629-1695

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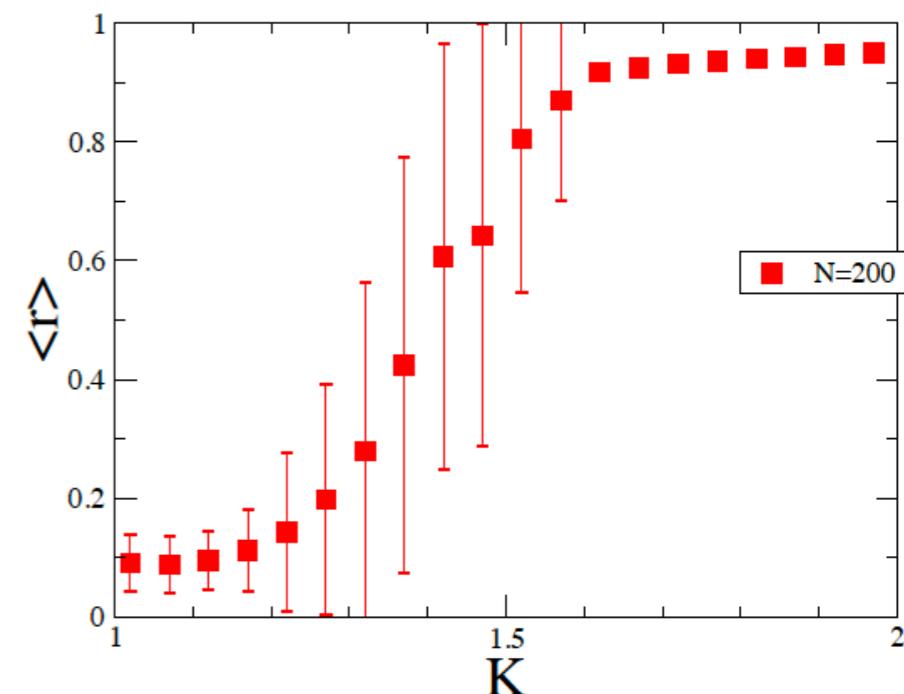
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- Winfree '67
- Kuramoto '84 :  
mfa + introduce order parameter

$$r(t) = \left| \frac{1}{N} \sum_j \exp[i\theta_j(t)] \right| = |r(t)| \exp[i\Psi(t)]$$

$K > K_c = 4/\pi \sim$  stable synchronous fixed point



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- Coupling between power balance and frequency



Change in kinetic energy = balance of power

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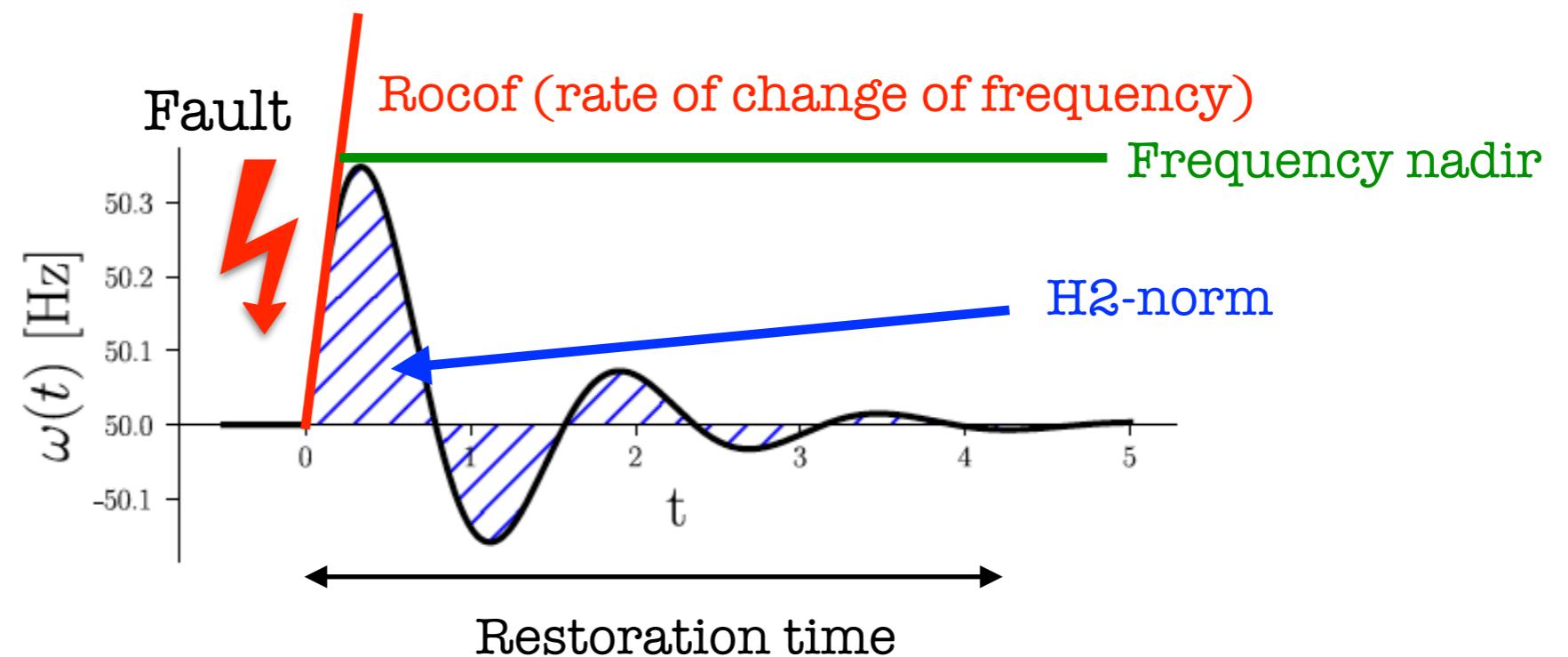
# Dynamics of AC power grids

## Dynamics: Swing Equations.

- from few AC cycles to ~10-20 secs.
- In rotating frame @ 50/60 Hz

$$I_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

- I : inertia  $\sim$  rot. kinetic energy
- D : damping from primary control



- Frequency transient as indicator of fault magnitude
- Frequency fluctuations need to be minimized... but how ?

# The questions of interest - and some answers

1. Is the power grid globally robust against disturbances ?
2. Is the power grid robust against specific, targeted disturbances ?
3. What specific disturbance(s) would lead to the worst response of the grid ?
4. Where should these specific disturbances act to lead to the worst response ?

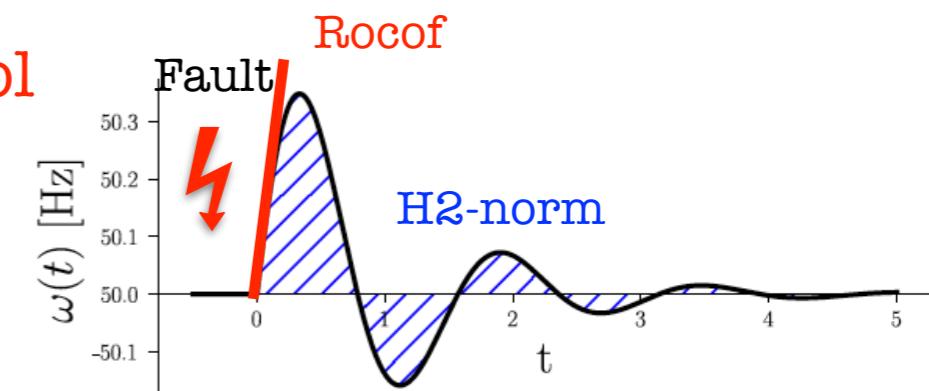
**Global robustness vs. local vulnerabilities**

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## Global robustness vs. local vulnerabilities

- 1) Global robustness vs. graph topological indices  
Tyloo, Coletta and PJ, PRL '18
- 2) Local vulnerabilities vs. network centralities  
Tyloo, Pagnier and PJ, Science Adv. '19; Tyloo and PJ, PRE '19
- 3) Disturbances vs. line faults  
Coletta and PJ, IEEE TCNS '19
- 4) Disturbance propagation vs. Network eigenmodes  
Pagnier and PJ, PLoS ONE '19
- 5) Optimal placement of inertia and primary control  
Pagnier and PJ, IEEE Access '19



# Outline

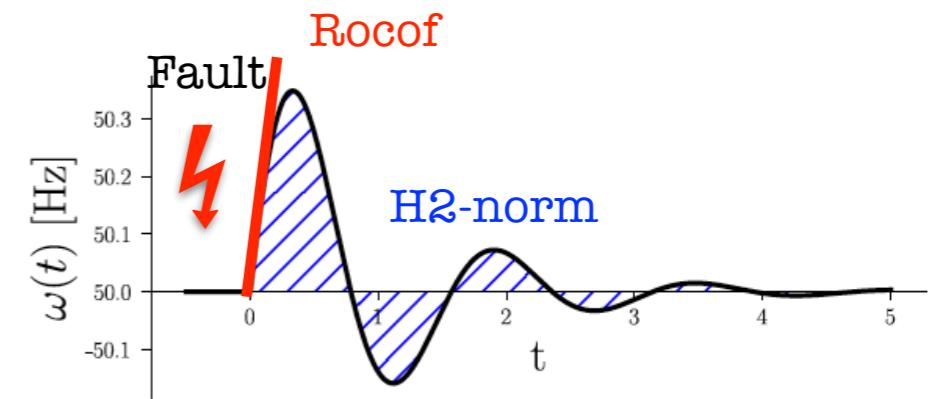
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frequency wave propagation  
optimal placement of inertia+control

# Frequency wave propagation

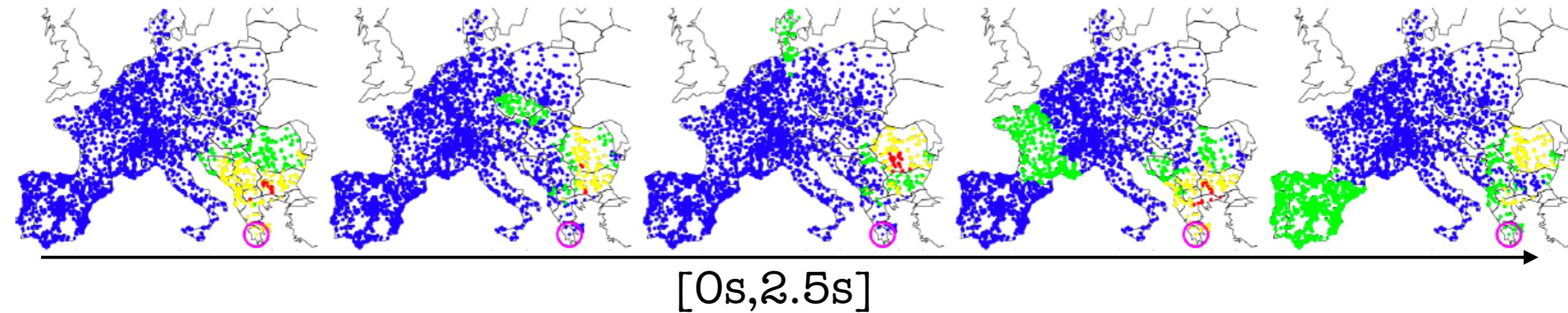
Fault : sudden power loss



$$P_b = P_b^{(0)} - \Delta P$$



- $\text{RoCoF} < 0.04 \text{ Hz/s}$
- $0.04 \text{ Hz/s} \leq \text{RoCoF} < 0.1 \text{ Hz/s}$
- $0.1 \text{ Hz/s} \leq \text{RoCoF} < 0.2 \text{ Hz/s}$
- $0.2 \text{ Hz/s} \leq \text{RoCoF} < 0.5 \text{ Hz/s}$

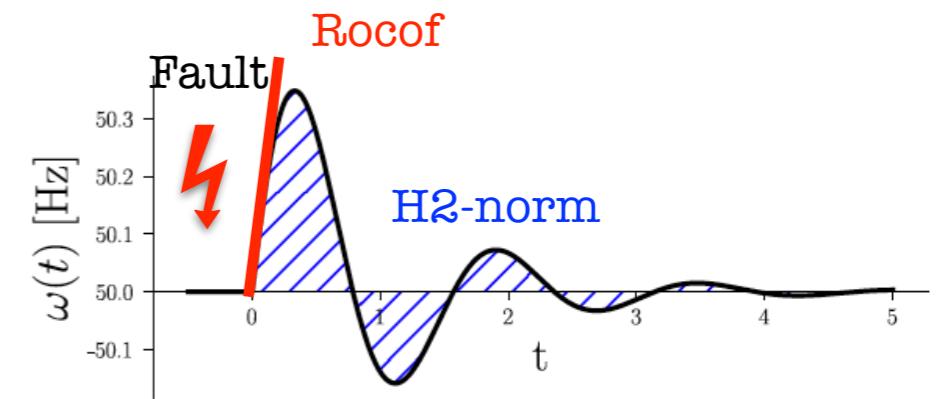


# Frequency wave propagation

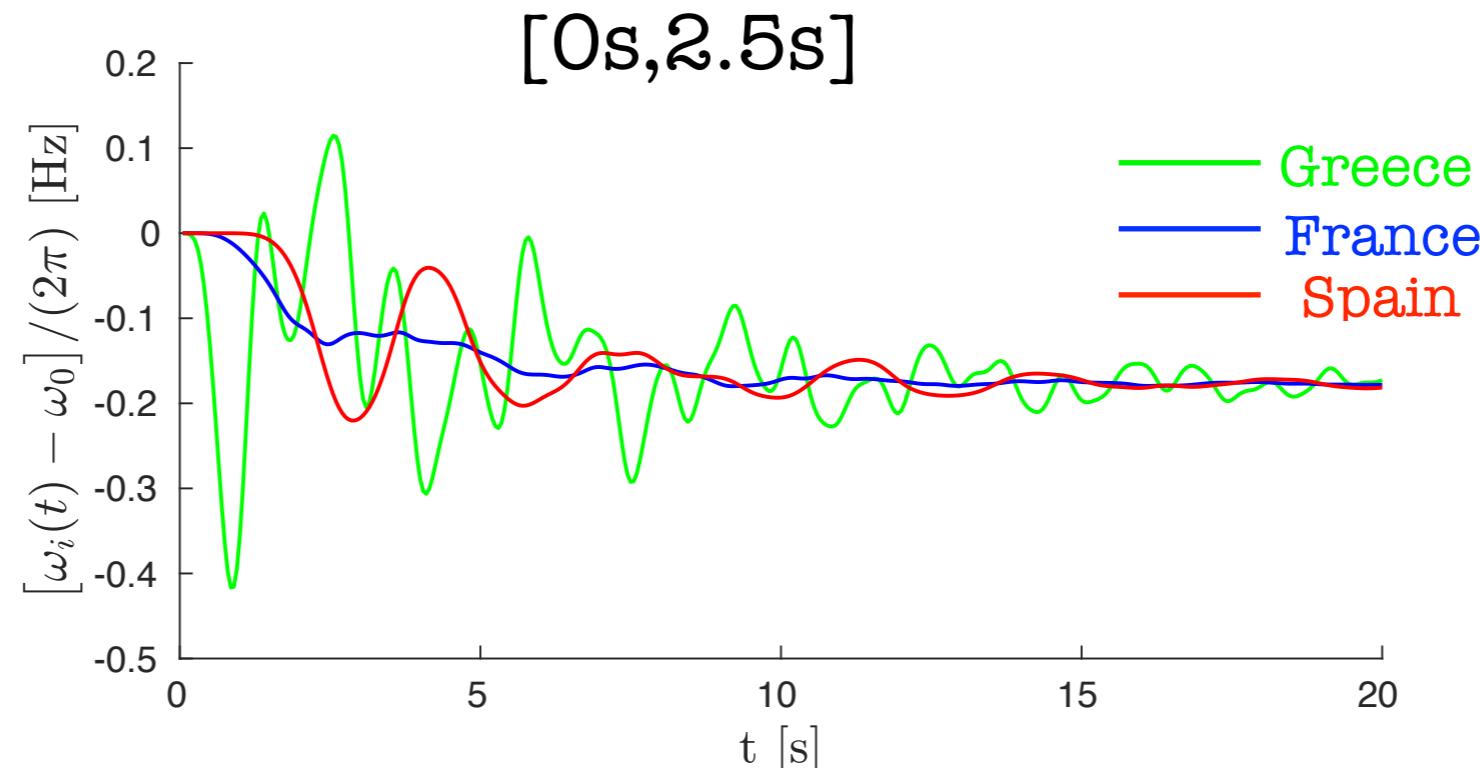
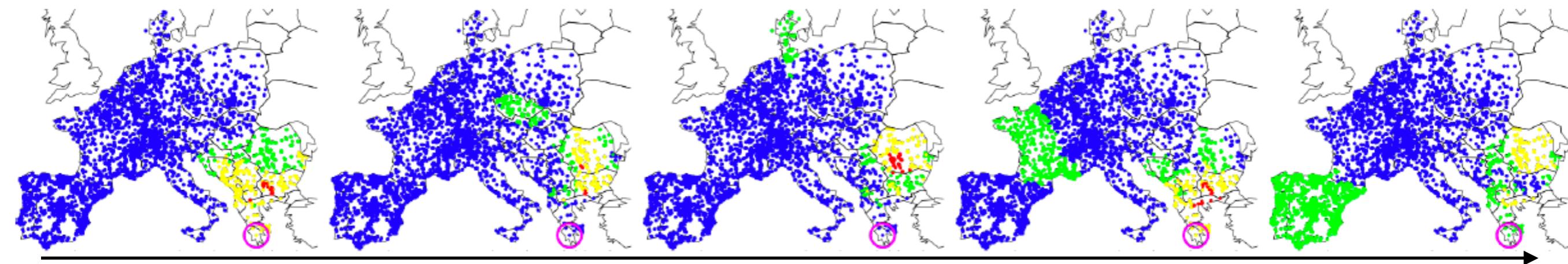
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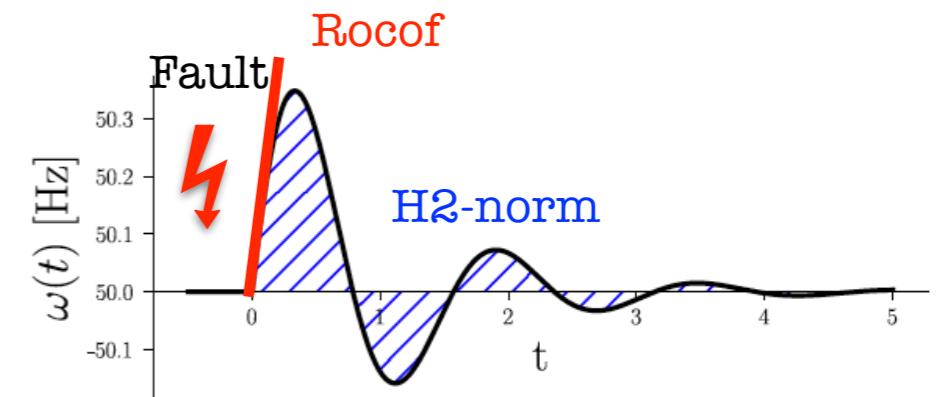


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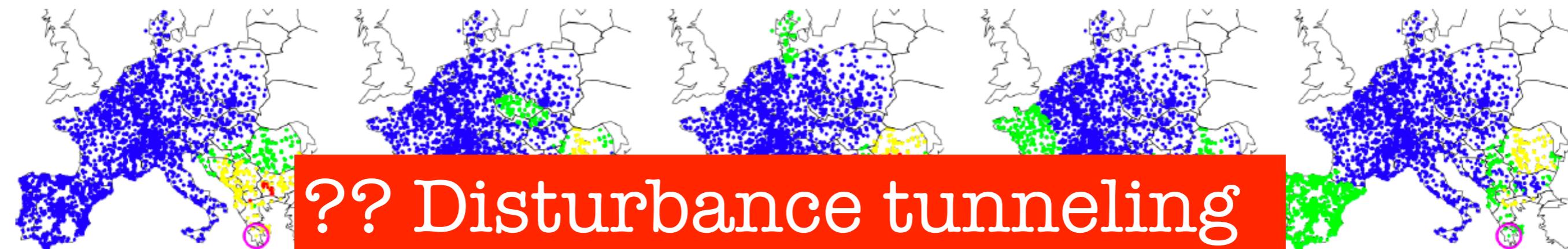
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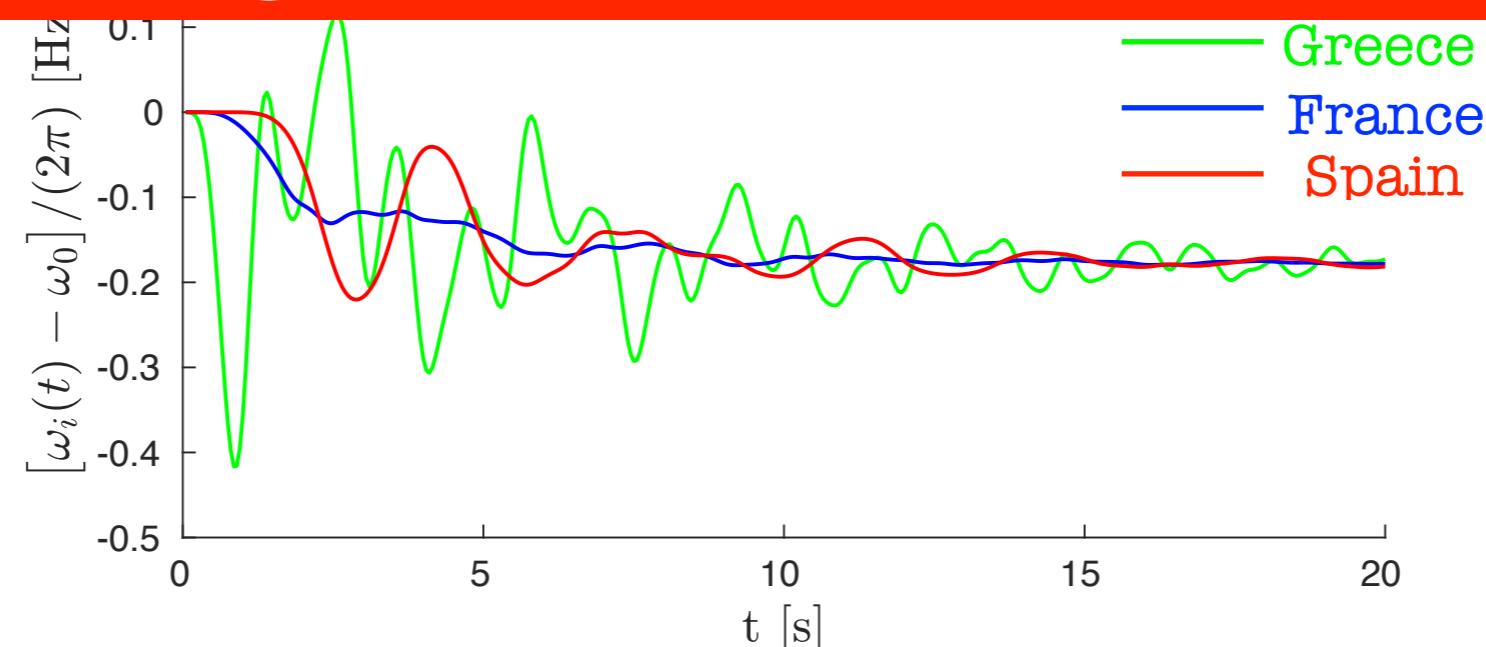
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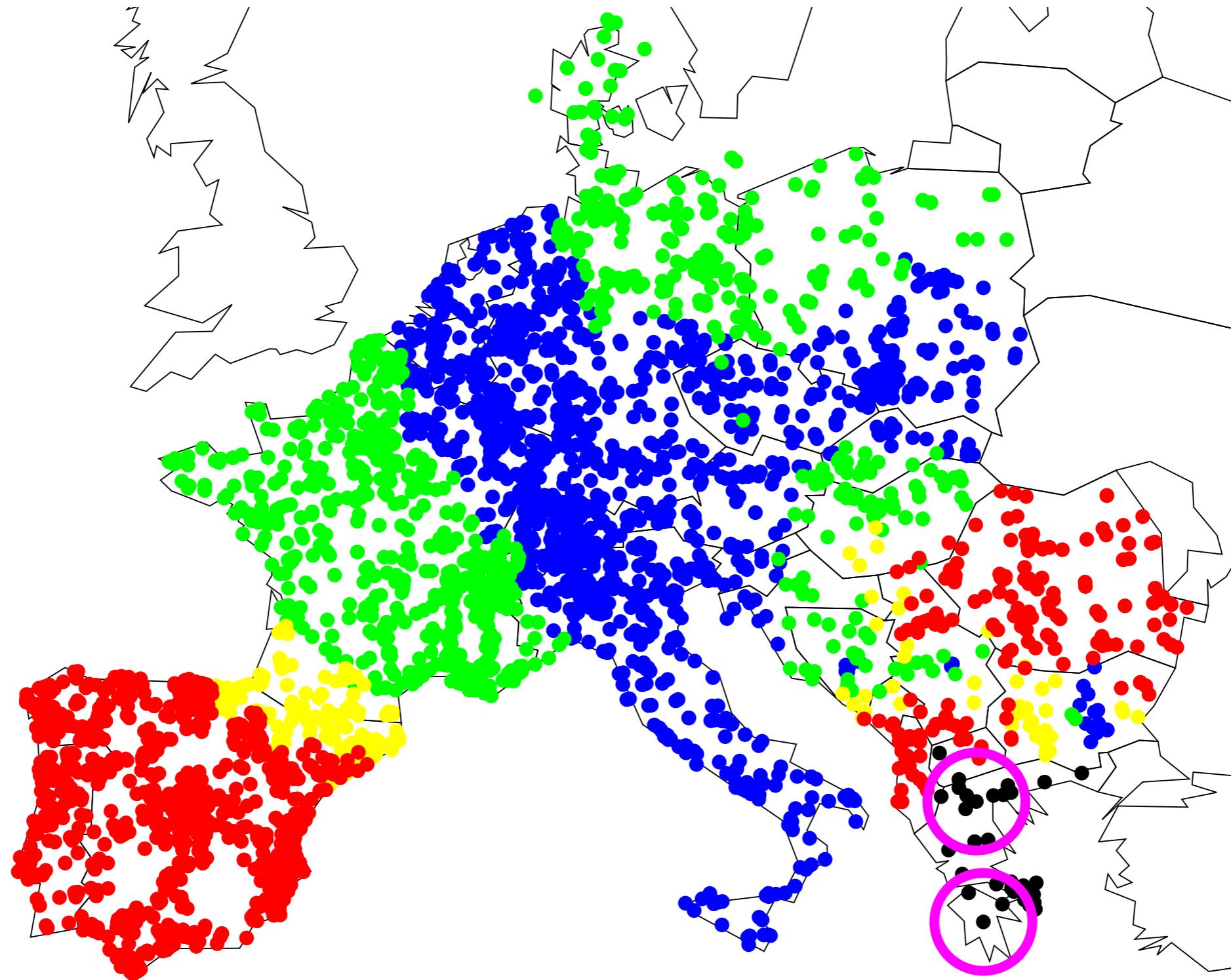


?? Disturbance tunneling  
through France ??



# Frequency wave propagation

● RoCoF < 0.04 Hz/s   ● 0.04 Hz/s ≤ RoCoF < 0.1 Hz/s   ● 0.1 Hz/s ≤ RoCoF < 0.2 Hz/s   ● 0.2 Hz/s ≤ RoCoF < 0.5 Hz/s   ● 0.5 Hz/s ≤ RoCoF < 1 Hz/s



# Frequency wave propagation vs. slow modes

Linearization of perturbation next to operating steady-state

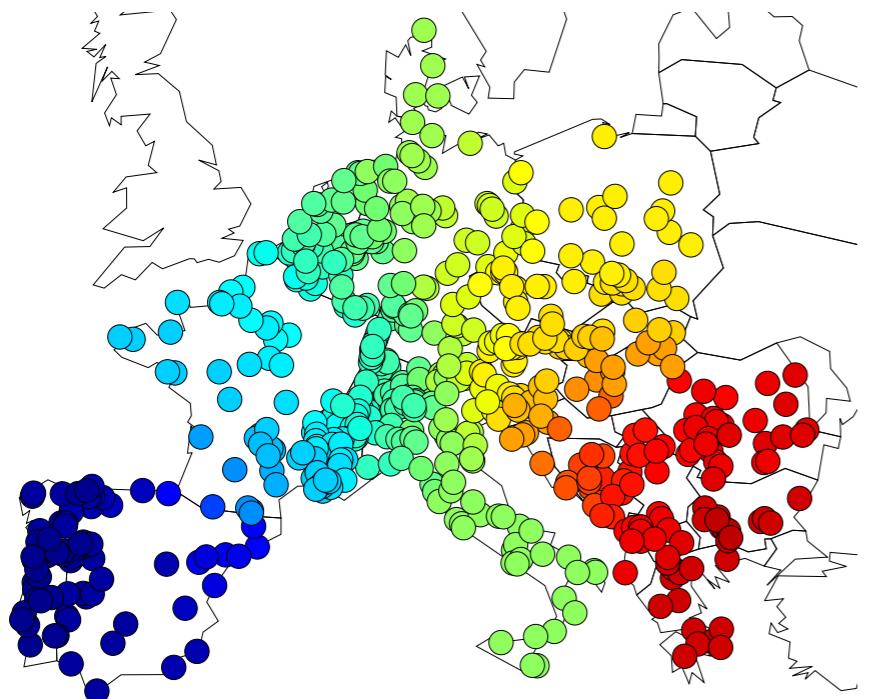
$$\textcolor{brown}{M}\delta\ddot{\theta} + \textcolor{blue}{D}\delta\dot{\theta} + \textcolor{teal}{L}\delta\theta = \delta P$$

# Frequency wave propagation vs. slow modes

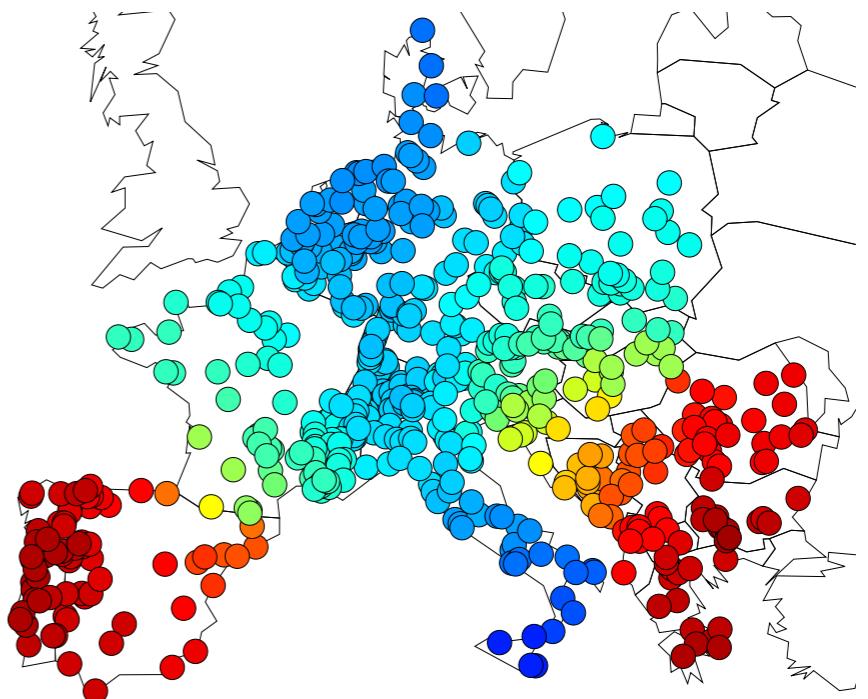
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$$M\delta\ddot{\theta} + D\delta\dot{\theta} + L\delta\theta = \delta P$$

Two slowest, nonzero modes of the Laplacian matrix



n=2 antisymmetric

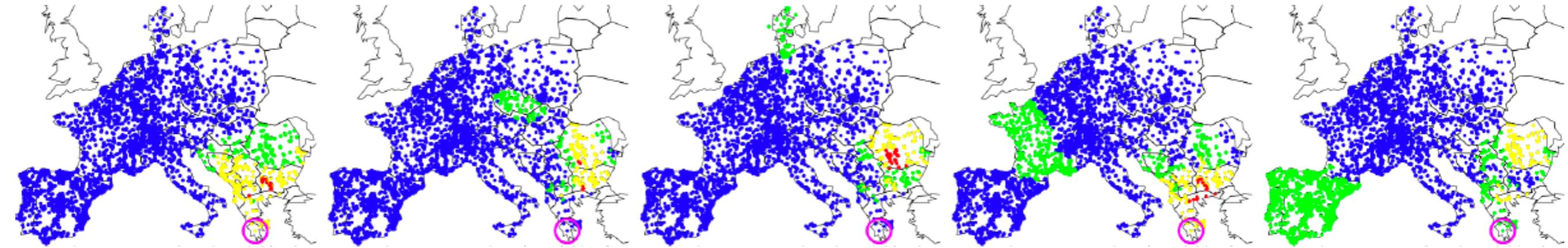


n=3 symmetric

...with the corresponding eigenvalues rather close  
~double-well potential vs. inter-area oscillations ?

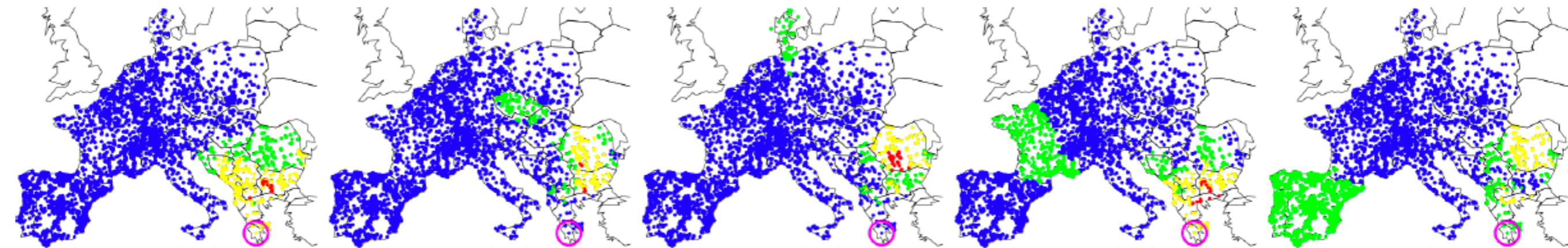
# Frequency wave propagation vs. inertia

## Today's Europe

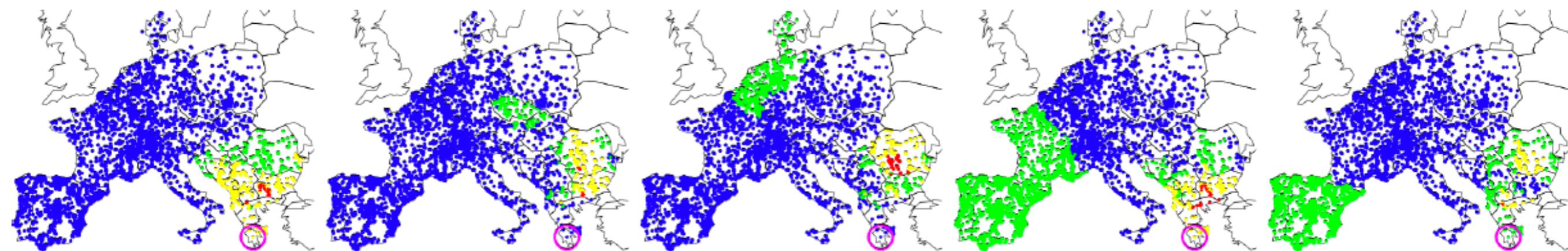


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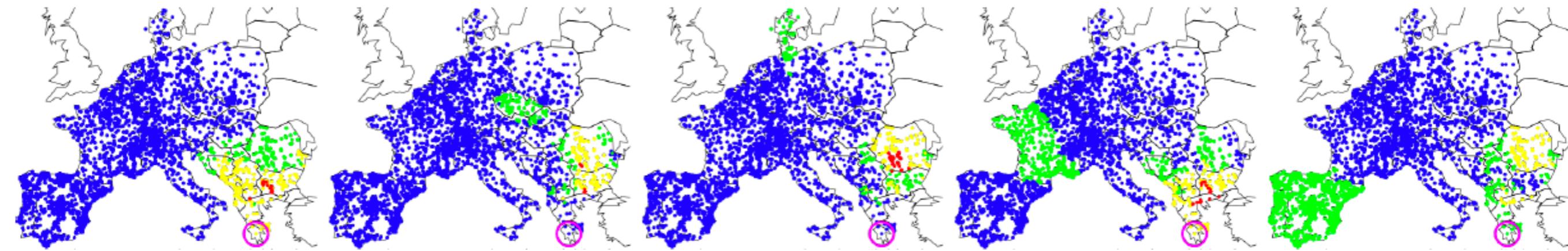


Inertia in France reduced to 50%

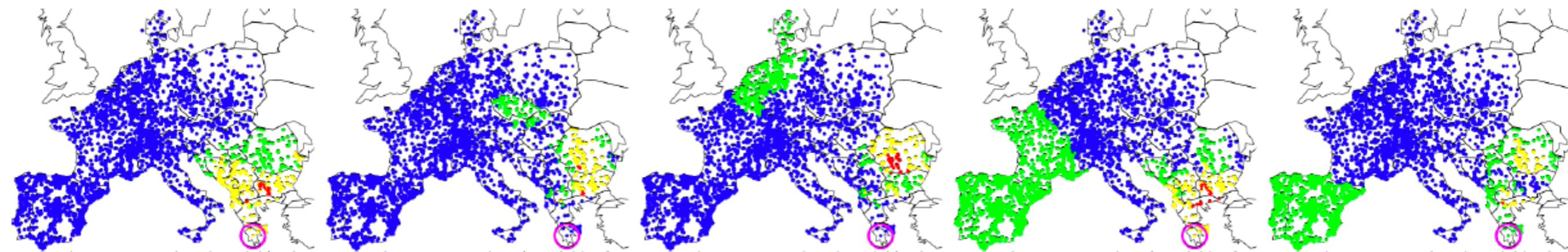


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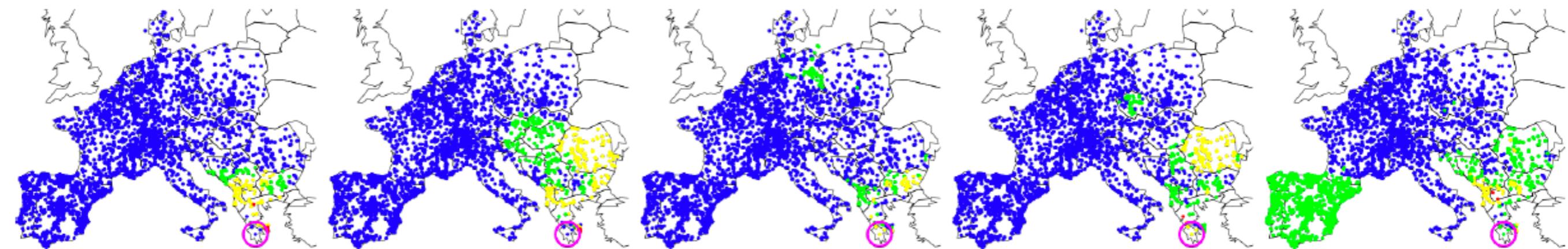
Today's Europe



Inertia in France reduced to 50%



Inertia in Balkans doubled



# Frequency wave propagation vs. inertia

## Today's Europe



# Outline

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Our approach : cast the problem into  
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- Starting point : when can one diagonalize the swing eqs. ? (linearized about steady-state) (Paganini and Mallada '17)

$$\boxed{M\dot{\omega} + D\omega = \delta P - L\delta\theta}$$

$$M = \text{diag}(\{m_i\})$$
$$D = \text{diag}(\{d_i\})$$

$$\delta\theta = M^{-1/2}\delta\theta_M$$

$$\dot{\omega}_M + \underbrace{M^{-1}D}_{\Gamma}\omega_M + \underbrace{M^{-1/2}LM^{-1/2}}_{L_M}\delta\theta_M = M^{-1/2}\delta P$$

$$\Gamma = \text{diag}(\{d_i/m_i\})$$

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When can this eq. be reduced to a set of scalar diff. eqs. ?

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# Optimal placement of inertia and primary control

- Deviation from homogeneity

$$m_i = m + \delta m r_i,$$

$$d_i = m_i \gamma_i = (m + \delta m r_i)(\gamma + \delta \gamma a_i)$$

$$-1 \leq a_i, r_i \leq 1$$

Do perturbation theory with small parameters

$$\mu \equiv \delta m / m \ll 1$$

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- Perturbative performance measure - to be minimized

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$$\mathcal{M}_b = \sum_{k=1}^{N_{\text{sim}}} \sum_i |r_i(k\Delta t)|$$

“for loss of power at b, sum over RoCoF’s at all sites and integrate over time”

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with susceptibilities (this is what we calculate)

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$$\rho_i \equiv \partial \mathcal{M}_b / \partial r_i \quad \alpha_i \equiv \partial \mathcal{M}_b / \partial a_i$$

- Goal : determine optimal distribution of  $r_i$  and  $a_i$  with constraints

$$-1 \leq a_i, r_i \leq 1$$

Local reduction/increase

$$\sum_i r_i = \sum_i a_i = 0$$

Total limited resources

# Optimal placement of inertia and primary control



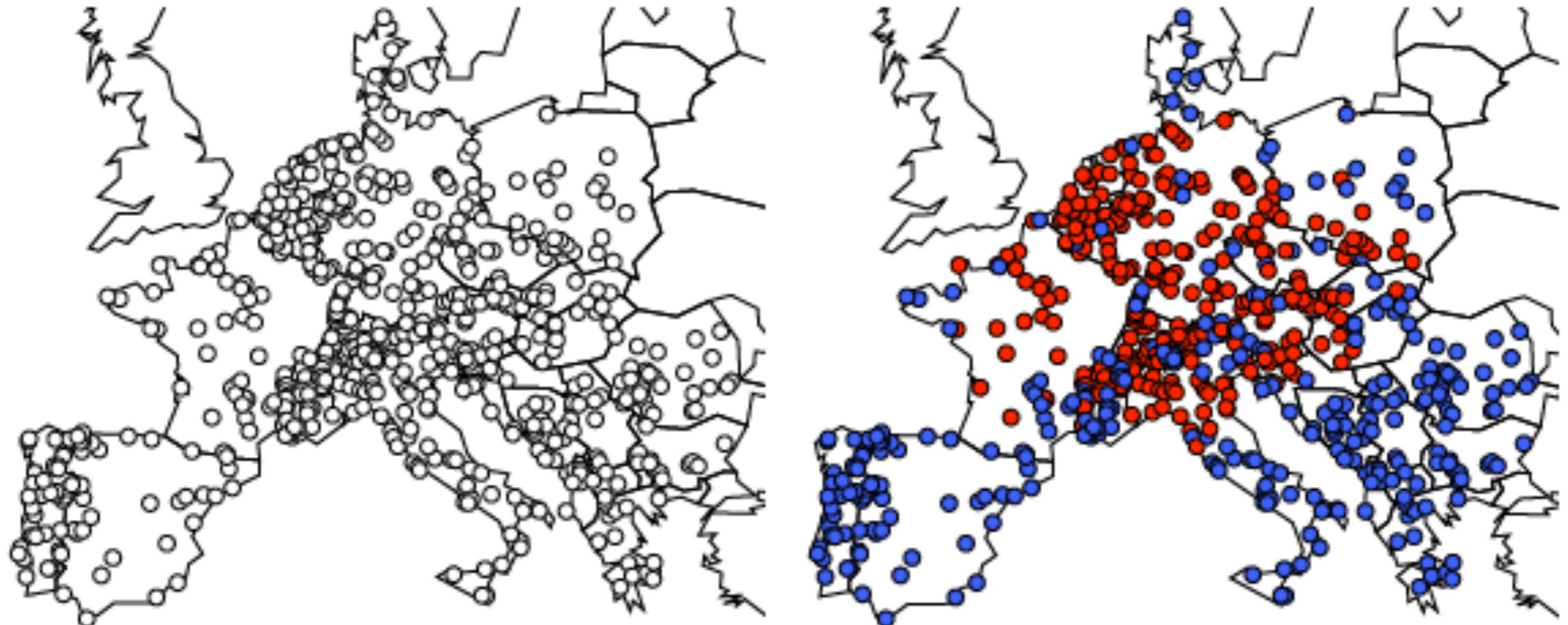
Inertia. :  $r_i = 0$

Wants to keep it homogeneous

$$m_i = m + \delta m r_i ,$$

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# Optimal placement of inertia and primary control



Inertia. :  $r_i = 0$

Wants to keep it homogeneous

Damping/droop :  $r_i = 1$  (blue)  $r_i = -1$  (red)

Wants to put it at periphery

(vs. resistance distance centrality)

$$m_i = m + \delta m r_i ,$$

$$d_i = m_i \gamma_i = (m + \delta m r_i)(\gamma + \delta \gamma a_i)$$

# Optimal placement of inertia and primary control

The method can be extended :

- 1) to optimize about another solution
  - current European grid
  - future European grid (less inertia)

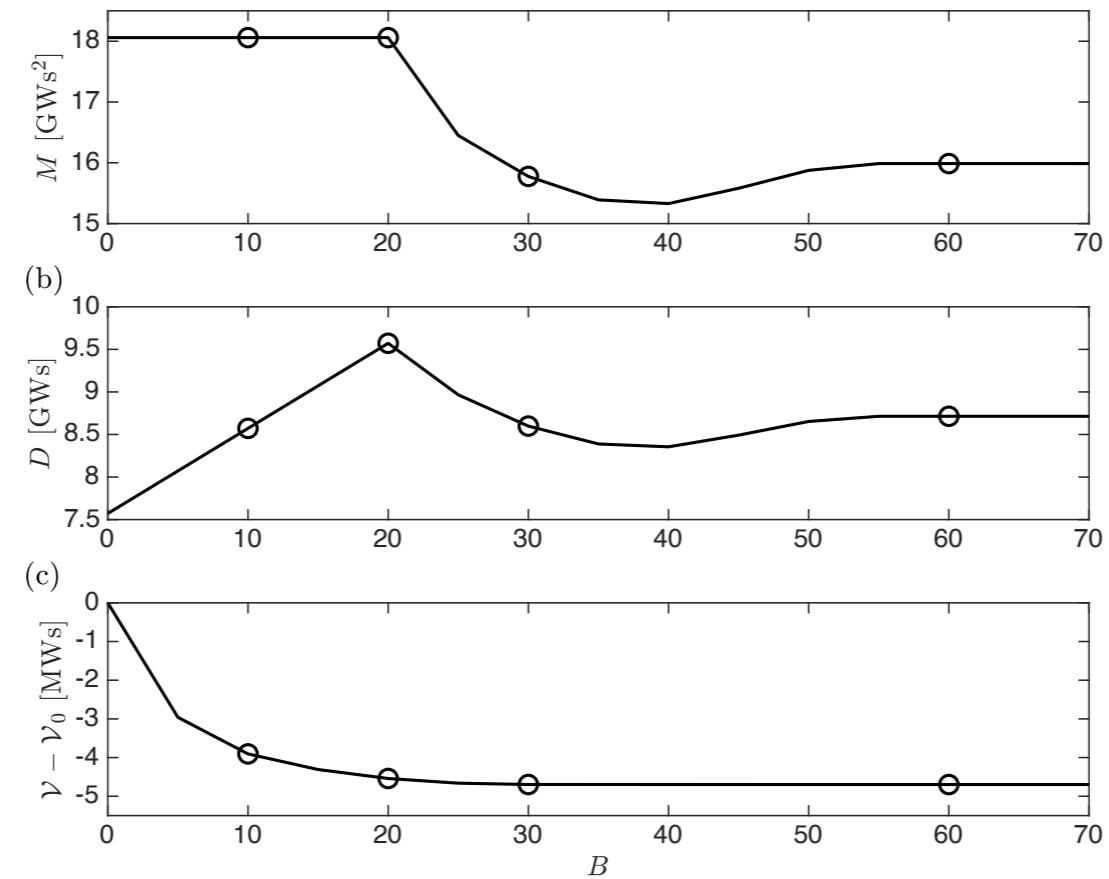
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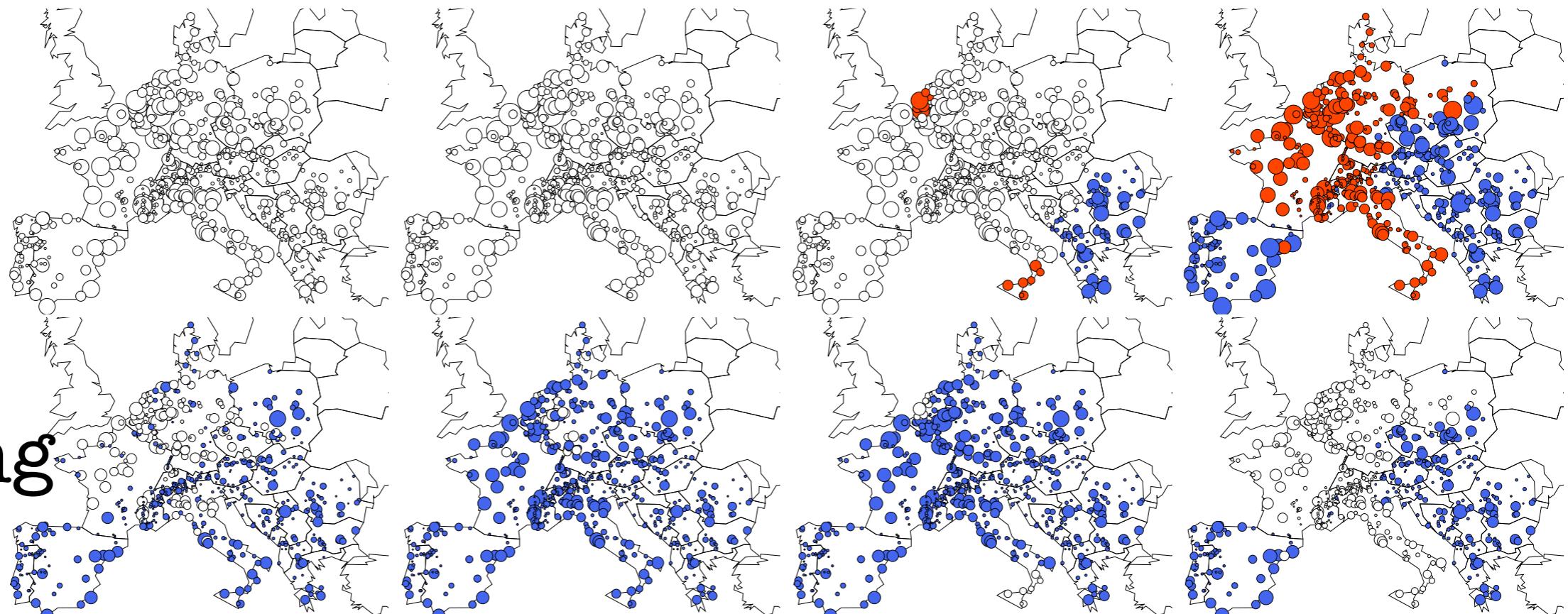
- 1) to optimize about another solution
  - current European grid
  - future European grid (less inertia)
- 2) to keep cost of upgrade within a given budget  $B$   
(instead of constraint of constant total resources)

$$\sum_i \left( c_i^{\text{m}+} \delta m_i^+ + c_i^{\text{d}+} \delta d_i^+ + c_i^{\text{m}-} \delta m_i^- + c_i^{\text{d}-} \delta d_i^- \right) \leq B$$

# Optimal placement of inertia and primary control

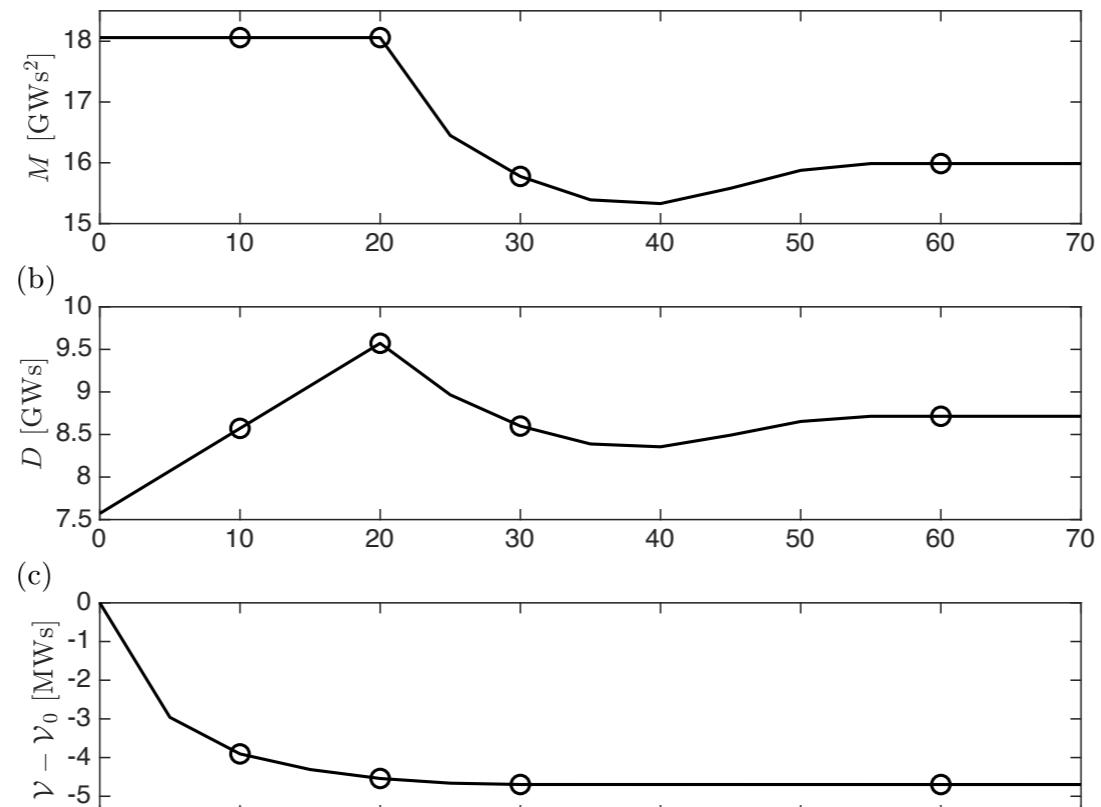


inertia



damping

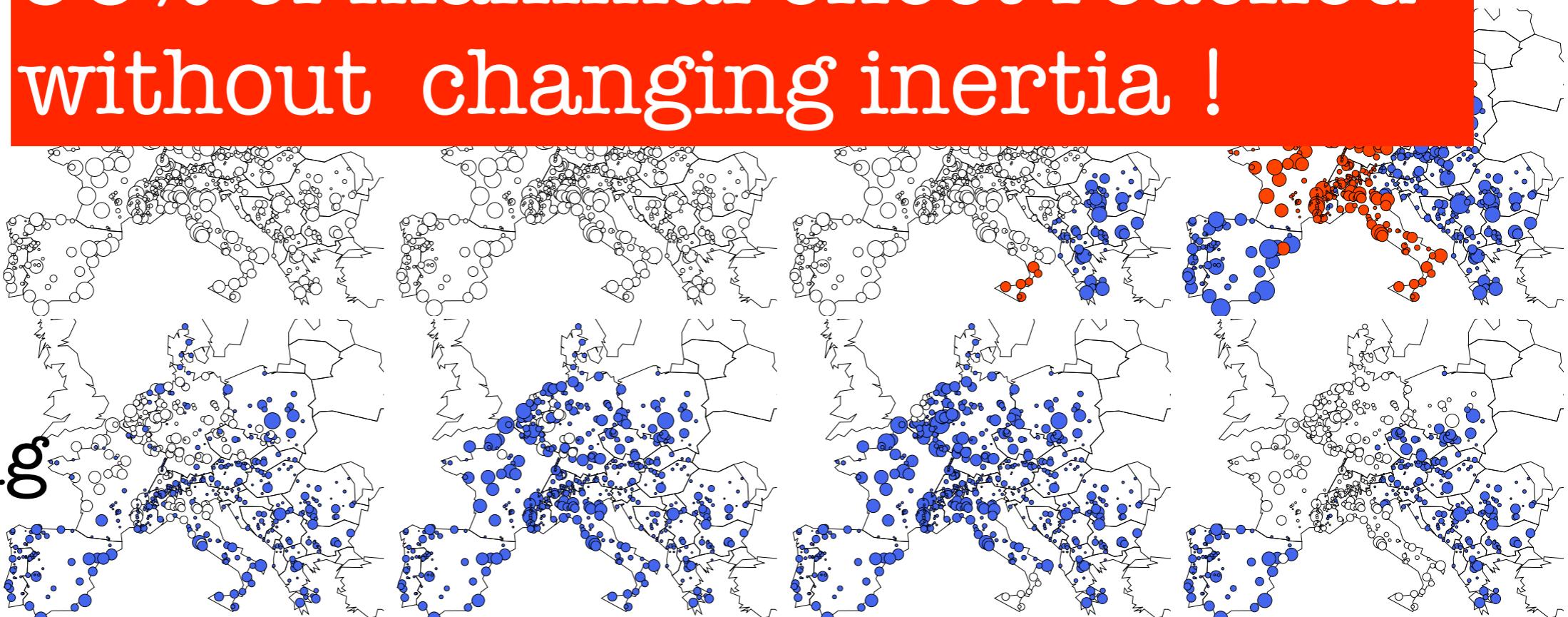
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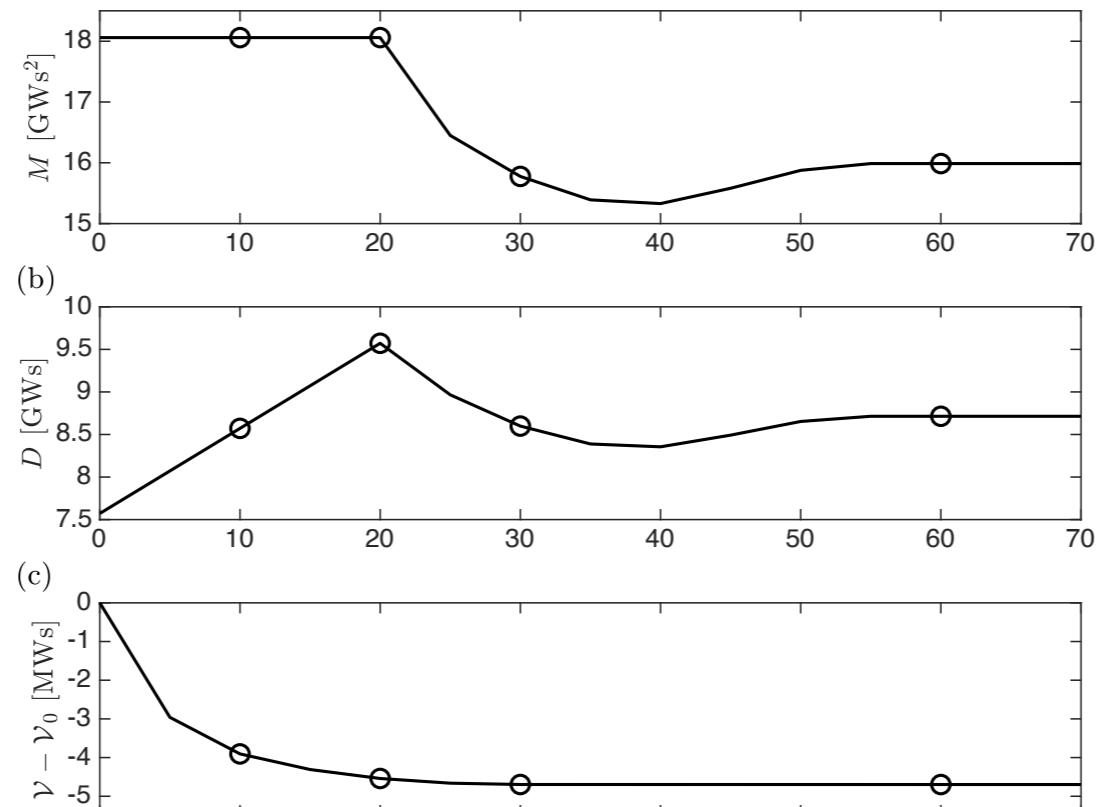
96% of maximal effect reached  
without changing inertia !

inertia

damping



# Optimal placement of inertia and primary control



96% of maximal effect reached  
without changing inertia !

inertia

Add damping on slow modes !

damping

# Conclusion

- Power grids are coupled dynamical systems on complex networks
- Analogies with superconductivity / Josephson junction arrays
- They are fast evolving - now is the time for physicists to work on them!
- They will need more primary control in the future, but not necessarily more inertia / inertia substitution

