Resistance Centralities Identify Local Vulnerabilities in Electric Power Grids.

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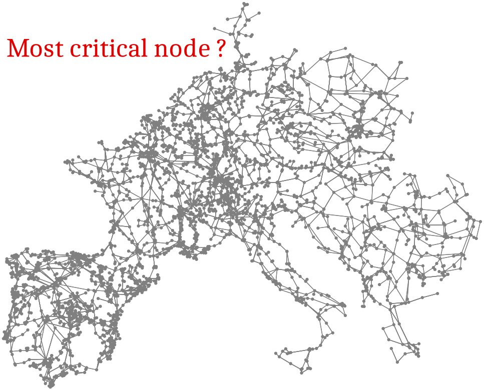
MT,Pagnier and Jacquod submitted (2018), arXiv:1810.09694.
MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018).
MT and Jacquod in preparation (2019).

Motivation

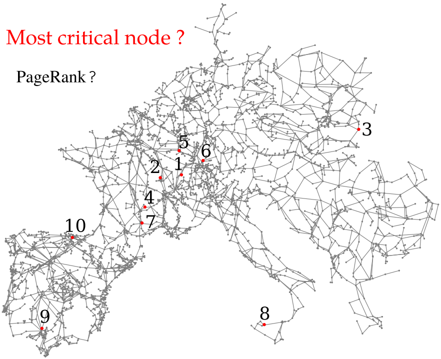
Coupled dynamical system → Most vulnerable component?

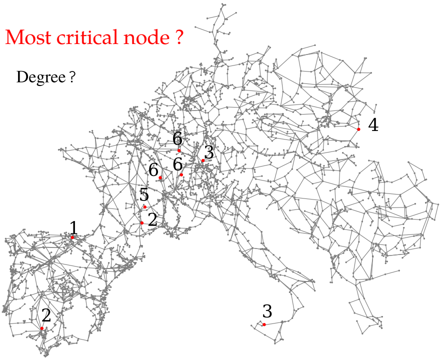


Most vulnerable component → Largest system's response when perturbed.

MT,Pagnier and Jacquod submitted (2018), arXiv:1810.09694.

Complex Networks Metric $\xrightarrow{?}$ Coupled Dynamical Systems





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Swing Equations in the lossless line limit:

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_j b_{ij}\sin(\theta_i - \theta_j) \ \ , \ i = 1, ..., n. \tag{1}$$

$b_{ij} = b_{ji} \geq 0.$

Steady-state solutions: Synchronous state $\{\theta_i^{(0)}\}$ such that:

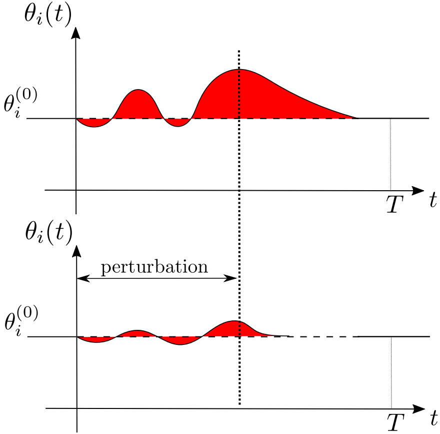
$$P_i = \sum_j b_{ij}\sin(\theta_i^{(0)} - \theta_j^{(0)}) \ \ , \ i = 1, ..., n. \tag{2}$$

$\sum_i P_i = 0.$

Perturbations: $P_i \rightarrow P_i^{(0)} + \delta P_i(t).$

Quantifying Robustness

Performance measures:



$$\mathcal{P}_1(T) = \sum_i \int_0^T |\theta_i(t) - \theta_i^{(0)}|^2 \mathrm{d}t \,,$$

$$\mathcal{P}_2(T) = \sum_i \int_0^T |\dot\theta_i(t) - \dot\theta_i^{(0)}|^2 \mathrm{d}t \,.$$

$$\mathcal{P}_{1,2}^{\infty} = \mathcal{P}_{1,2}(T \to \infty)\,.$$

Noisy disturbances $\rightarrow$ divide by T.

Perturbations: $P_i \rightarrow P_i^{(0)} + \delta P_i(t)$.

MT, Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)

Another Way to Quantify the Response: ROCOF

Disturbance propagation, inertia location and slow modes in large-scale high voltage power grids

Laurent Pagnier, *Member, IEEE* and Philippe Jacquod, *Member, IEEE*

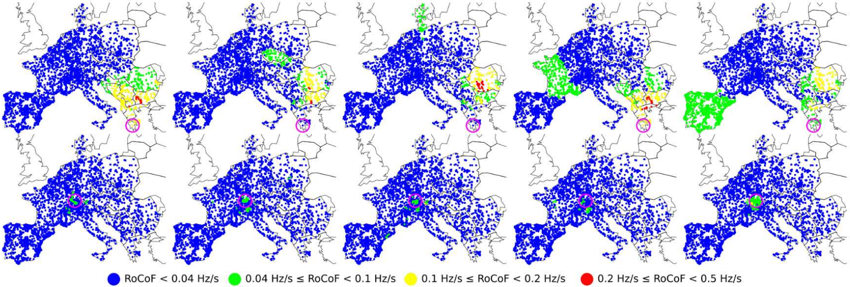


Fig 1. Spatio-temporal evolution of local RoCoFs for two different power losses of $\Delta P = 900$ MW in a moderate load (typical of a standard summer evening) configuration of the synchronous grid of continental Europe of 2018. The top five panels correspond to a fault in Greece and the bottom five to a fault in Switzerland. In both cases, the fault location is indicated by a purple circle. Panels correspond to snapshots over time intervals 0-0.5[s], 0.5-1[s], 1-1.5[s], 1.5-2[s] and 2-2.5[s] from left to right.

Pagnier and Jacquod submitted (2018), arXiv:1810.04982.

Response to Perturbations: Linearization

Linear response: Perturbation of inj./cons. power.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$

$$m\delta\ddot{\boldsymbol{\theta}} + d\delta\dot{\boldsymbol{\theta}}(t) = \delta\boldsymbol{P}(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t)\ , \tag{3}$$

$\mathbb{L}(\{\theta_i^{(0)}\})$: the weighted Laplacian matrix,

$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})\ , & i \neq j\ , \\ \sum_k b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)})\ , & i = j\ . \end{cases} \tag{4}$$

Topology $\rightarrow b_{ij}$.
Steady state $\rightarrow \{\theta_i^{(0)}\}$.
Expanding on the eigenvectors $\mathbf{u}_\alpha$ of $\mathbb{L}$, we have $\delta\boldsymbol{\theta}(t) = \sum_\alpha c_\alpha(t)\mathbf{u}_\alpha$.
$\rightarrow \mathcal{P}_1(T)$, $\mathcal{P}_2(T)$ for specific perturbations

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MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)
MT,Pagnier and Jacquod submitted (2018), arXiv:1810.09694.

Response to Large Perturbations

Change of fixed point!

Noise-Induced Desynchronization and Stochastic Escape from Equilibrium in Complex Networks

M. Tyloo[1,4], R. Delabays[2,4], and Ph. Jacquod[3,4]

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[2] *Automatic Control Laboratory, Swiss Federal Institute of Technology, CH-8092 Zürich, Switzerland.*
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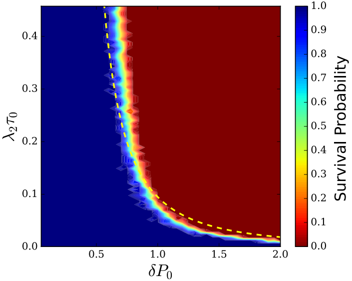
[4] *School of Engineering, University of Applied Sciences of Western Switzerland HES-SO, CH-1951 Sion, Switzerland.*

(Dated: December 27, 2018)



MT,Delabays and Jacquod submitted (2018), arXiv:1812.09497.

Response to Perturbations: Time Scales

Intrinsic Time Scales

- Individual elements: m/d.
- Network relaxation: d/λ_α with $\{\lambda_\alpha\}$ the eigenvalues of $\mathbb{L}$.

Perturbation Time Scale

- Correlation time of the external perturbation $\delta \boldsymbol{P}(t)$.

In the following:

Noisy perturbations on a single node k

- $\langle \delta P_k(t) \rangle = 0$,
- $\langle \delta P_k(t) P_k(t') \rangle = \delta P_0^2 \exp[-|t - t'|/\tau_0]$.

Correlation time $\rightarrow \tau_0$.

MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)
MT,Pagnier and Jacquod submitted (2018), arXiv:1810.09694.

Specific Local Vulnerabilities and C_m's

Performance Measures for Noisy Perturbations

$$\mathcal{P}_1^\infty = \sum_{\alpha\geq 2} \frac{\delta P_0^2 u_{\alpha,k}^2 (\tau_0 + m/d)}{\lambda_\alpha(\lambda_\alpha \tau_0 + d + m/\tau_0)} \,. \tag{5}$$

Short time correlated: $\tau_0 \ll d/\lambda_\alpha, m/d$

$$\mathcal{P}_1^\infty \simeq \frac{\delta P_0^2 \tau_0}{d} \sum_{\alpha\geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha} \,. \tag{6}$$

Long time correlated: $\tau_0 \gg d/\lambda_\alpha, m/d$

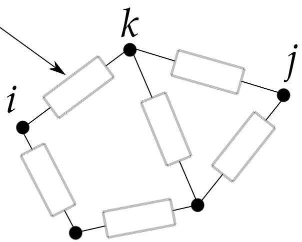
$$\mathcal{P}_1^\infty \simeq \delta P_0^2 \sum_{\alpha\geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha^2} \,. \tag{7}$$

Resistance Distance

$$\Omega_{ij} = \mathbb{L}^\dagger_{ii} + \mathbb{L}^\dagger_{jj} - \mathbb{L}^\dagger_{ij} - \mathbb{L}^\dagger_{ji} = \sum_{\alpha\geq 2} \frac{(u_{\alpha,i} - u_{\alpha,j})^2}{\lambda_\alpha} \,. \tag{8}$$

$\mathbb{L}^\dagger$: pseudo inverse of $\mathbb{L}$ (because of $\lambda_1 = 0$).

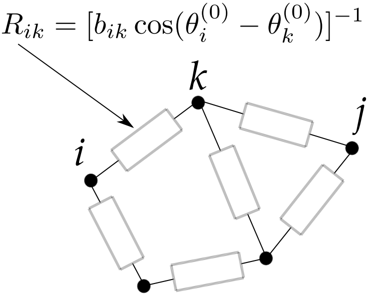
$$R_{ik} = [b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$$



Klein and Randić, *J. Math. Chem.* **12**, 81 (1993).

Kirchhoff Index

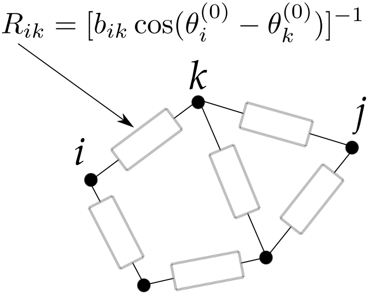
$$Kf_1 = \sum_{i<j} \Omega_{ij} = n \sum_{\alpha \geq 2} \lambda_\alpha^{-1} \, . \tag{9}$$



Klein and Randić, *J. Math. Chem.* **12**, 81 (1993).

Resistance Centrality

$$C_1(k) = \left[n^{-1}\sum_j \Omega_{kj}\right]^{-1} = \left[\sum_{\alpha\geq 2}\frac{u^2_{\alpha,k}}{\lambda_\alpha} + n^{-2}Kf_1\right]^{-1}. \tag{10}$$



Generalized Resistance Distances

$$\Omega_{ij}^{(m)} = \mathbb{L}'^{\dagger}_{ii} + \mathbb{L}'^{\dagger}_{jj} - \mathbb{L}'^{\dagger}_{ij} - \mathbb{L}'^{\dagger}_{ji} \tag{11}$$

$$= \sum_{\alpha \geq 2} \frac{(u_{\alpha,i} - u_{\alpha,j})^2}{\lambda_\alpha^m} \,, \tag{12}$$

$$\mathbb{L}' = \mathbb{L}^m \,. \tag{13}$$

Generalized Kirchhoff Indices

$$Kf_m = \sum_{i<j} \Omega_{ij}^{(m)} = n \sum_{\alpha \geq 2} \lambda_\alpha^{-m} \,. \tag{14}$$

Generalized Resistance Centralities

$$C_m(k) = \left[n^{-1} \sum_j \Omega_{kj}^{(m)} \right]^{-1} = \left[\sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha^m} + n^{-2} Kf_m \right]^{-1} \,. \tag{15}$$

MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)
MT,Pagnier and Jacquod submitted (2018), arXiv:1810.09694.

Summary

Local Vulnerability:

$\tau_0 \ll d/\lambda_\alpha, m/d$

$$
\begin{aligned}
\mathcal{P}_1^\infty(k) &\simeq \frac{\delta P_0^2 \tau_0}{d} \left(C_1^{-1}(k) - n^{-2} K f_1 \right) \,, \\
\mathcal{P}_2^\infty(k) &\simeq \frac{\delta P_0^2 \tau_0}{dm} \frac{(n-1)}{n} \,.
\end{aligned}
$$

$\tau_0 \gg d/\lambda_\alpha, m/d$

$$
\begin{aligned}
\mathcal{P}_1^\infty(k) &\simeq \delta P_0^2 \left(C_2^{-1}(k) - n^{-2} K f_2 \right) \,, \\
\mathcal{P}_2^\infty(k) &\simeq \frac{\delta P_0^2}{d \tau_0} \left(C_1^{-1}(k) - n^{-2} K f_1 \right) \,.
\end{aligned}
$$

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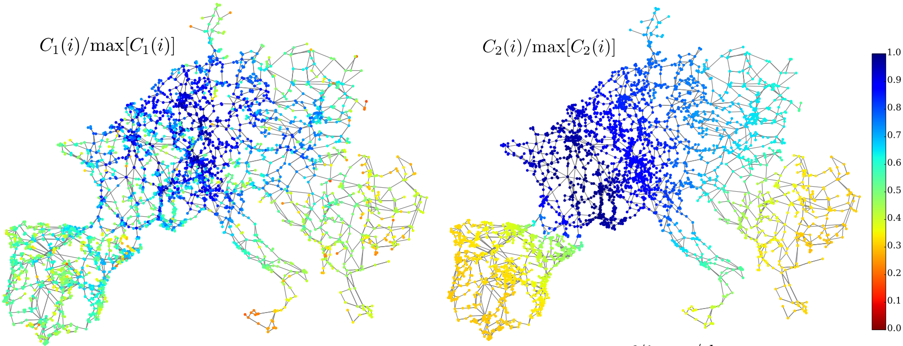
Physical Realization: European Electric Power Grid



$\tau_0 \ll d/\lambda_\alpha\,, m/d$

$$\mathcal{P}_1^\infty \simeq \frac{\delta P_0^2 \tau_0^2}{2}\left(C_1^{-1}(k) - n^{-2}Kf_1\right)$$

$$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2 \tau_0}{dm}\frac{(n-1)}{n}$$

$\tau_0 \gg d/\lambda_\alpha \;\;, m/d$

$$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2}{d\tau_0}\left(C_1^{-1}(k) - n^{-2}Kf_1\right)$$

$\tau_0 \gg d/\lambda_\alpha\,, m/d$

$$\mathcal{P}_1^\infty \simeq \delta P_0^2\left(C_2^{-1}(k) - n^{-2}Kf_2\right)$$

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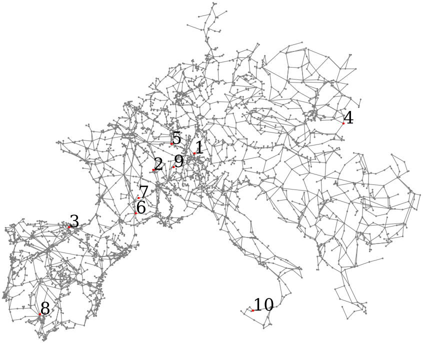
Conclusion

Specific Local Vulnerabilities

- Generalized Resistance Centralities, C_m's.

$\rightarrow$ m depends on which performance measures you are interested in and on the correlation time of the perturbation.

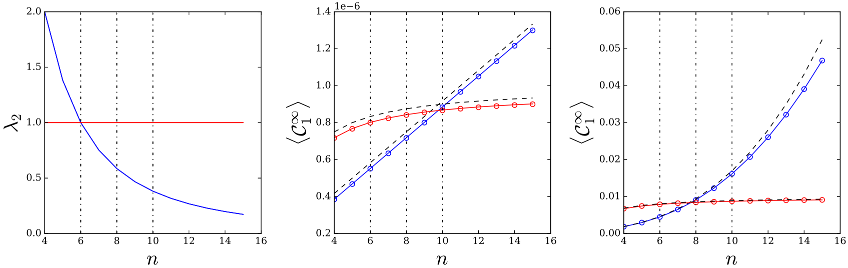
- Establish a ranking of the nodes.



node #	$C_{\rm geo}$	Degree	PageRank	C_1	C_2	$\mathcal{P}_1^{\rm num}$	$\mathcal{P}_2^{\rm num}$ $[\gamma^2]$
1	7.84	4	2782	31.86	5.18	0.047	0.035
2	6.8	1	199	22.45	5.68	0.021	0.118
3	5.56	10	3802	22.45	2.33	0.32	0.116
4	4.79	3	362	21.74	3.79	0.126	0.127
5	7.08	1	1217	21.74	5.34	0.026	0.125
6	4.38	6	3091	21.69	5.65	0.023	0.129
7	5.11	2	445	19.4	5.89	0.016	0.164
8	4.15	6	3648	19.38	1.83	0.453	0.172
9	5.06	1	8	10.2	5.2	0.047	0.449
10	2.72	4	3124	7.49	2.17	0.335	0.64

MT,Pagnier and Jacquod submitted (2018), arXiv:1810.09694.

Supplemental Material



blue : cycle graph

red : star graph

MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)

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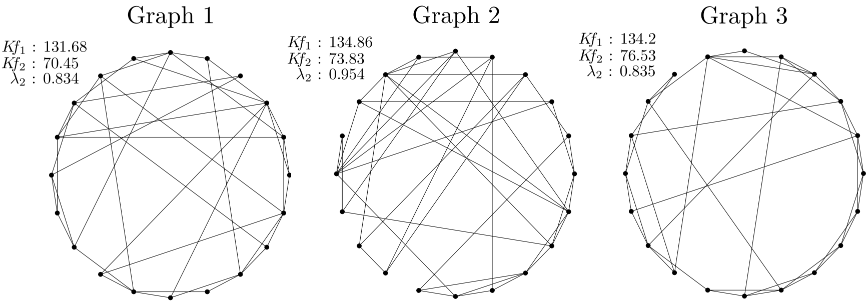
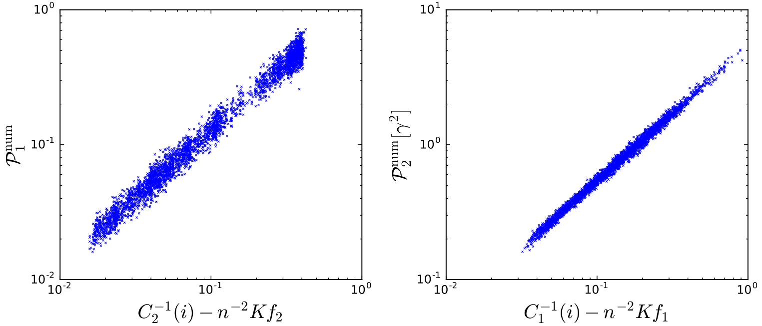


MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)

Supplemental Material

$d_i = d_0,\ m_i = m_0 + \delta m_i$



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