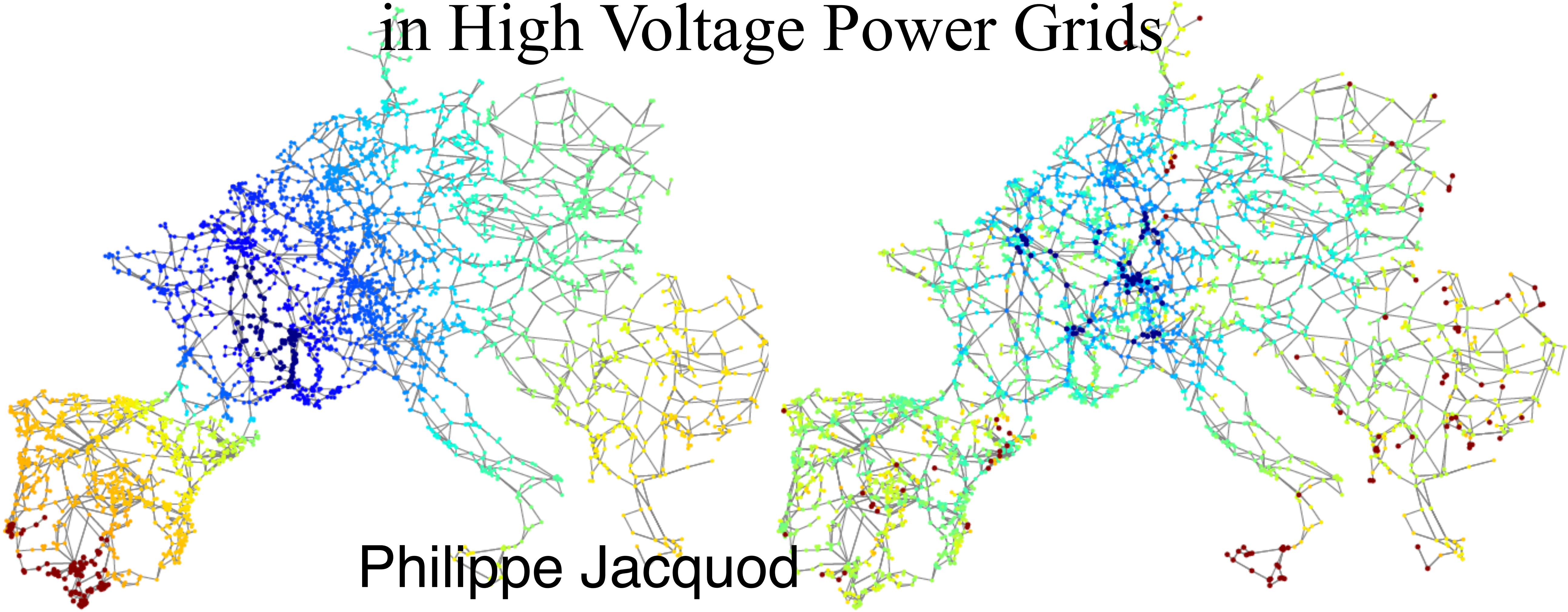
Optimal Placement of Inertia and Primary Droop Control vs. Slow Network Modes in High Voltage Power Grids



Philippe Jacquod

CISS19 - Baltimore 20.03.2019

(with L. Pagnier)

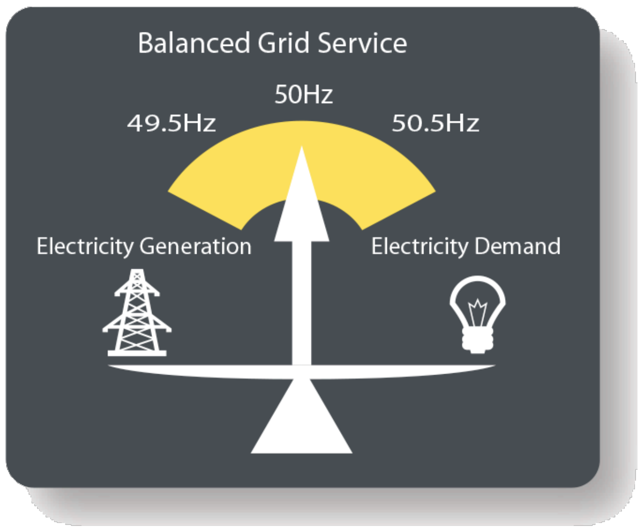
UNIVERSITÉ DE GENÈVE

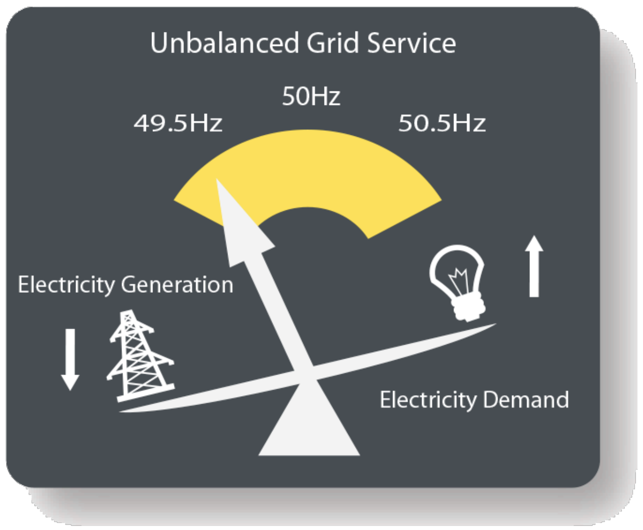
FNSNF

Hes·so // VALAIS WALLIS School of Engineering

Today's (and yesterday's) AC power grids

- Built around rotating synchronous machines
- Coupling between power balance and frequency





figs. Taken from http://northernutilities.co.uk

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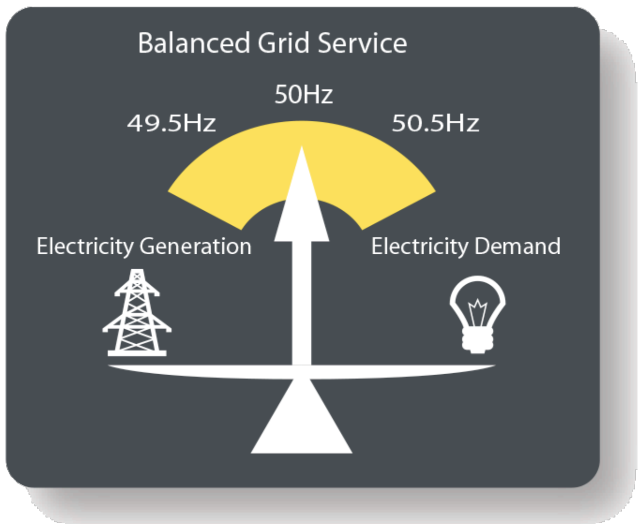

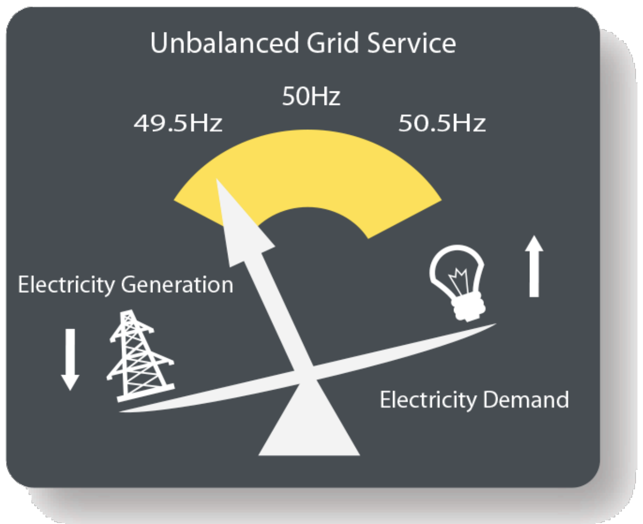

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- Dynamics given by swing equations
  ~ conservation of energy/power

$$\boxed{M\frac{d\omega}{dt} = P_{\text{gen}} - P_{\text{cons}}}$$

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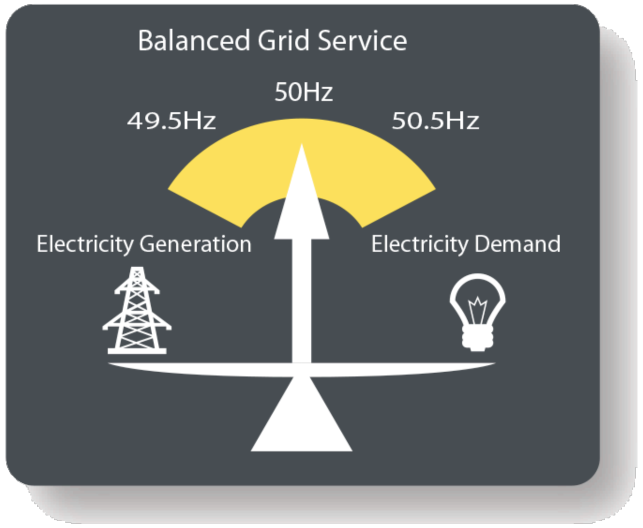

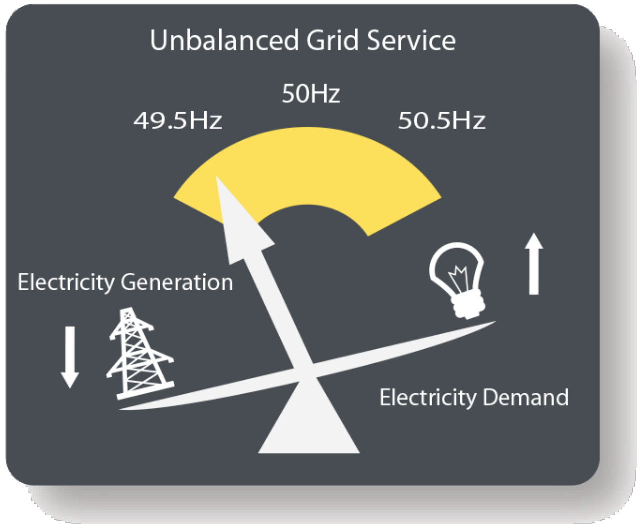

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Change in kinetic energy = balance of power

Frequency vs. Electric Power Quality

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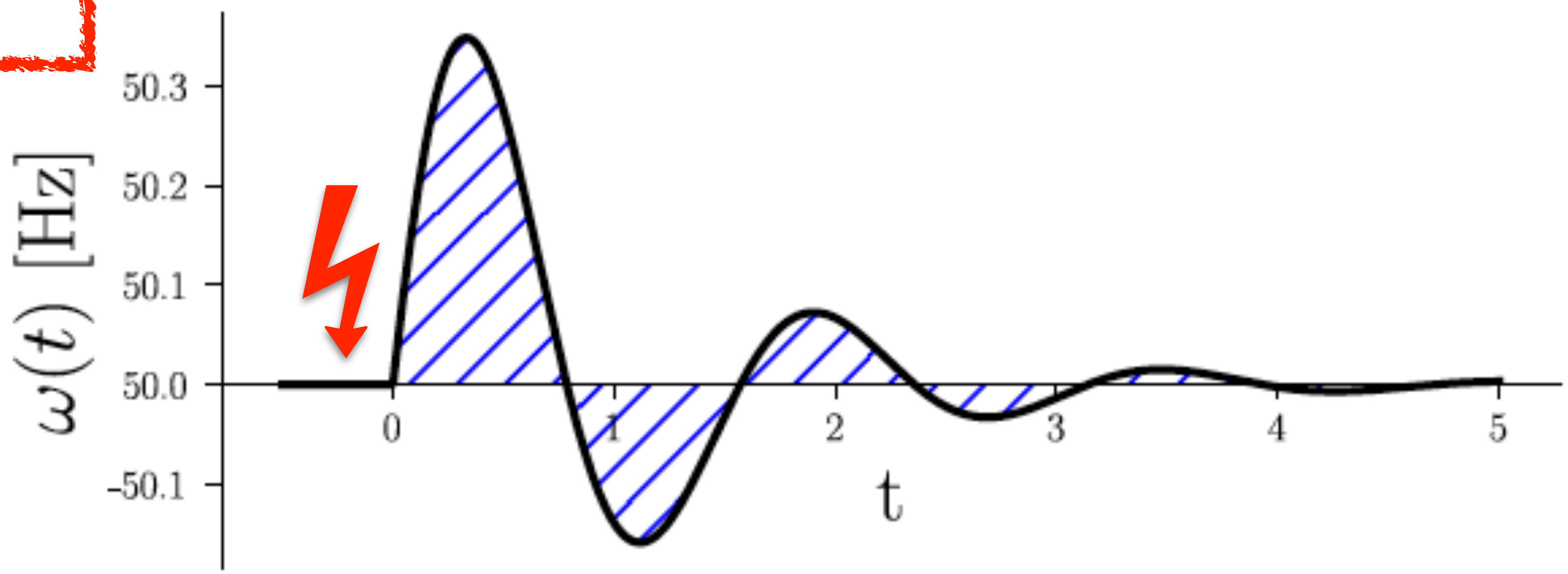

- Frequency transient as indicator of fault magnitude

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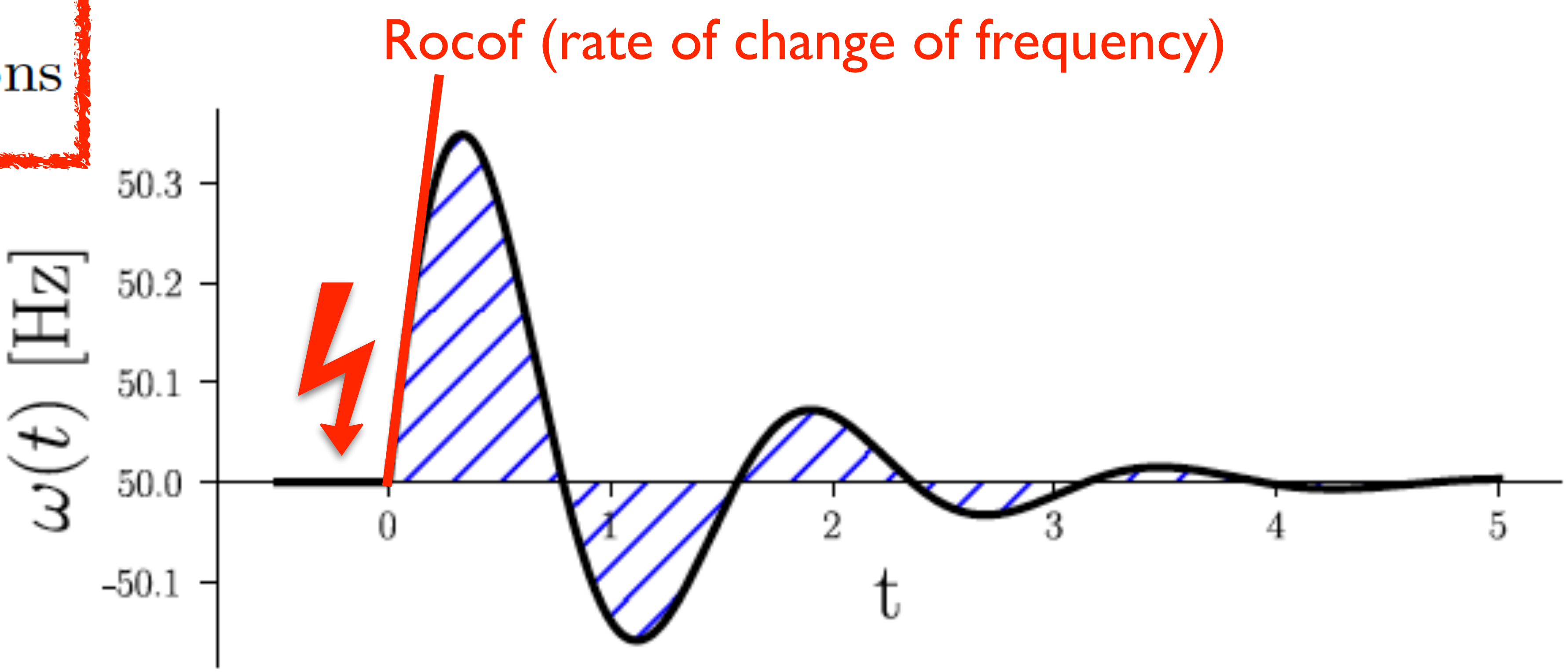

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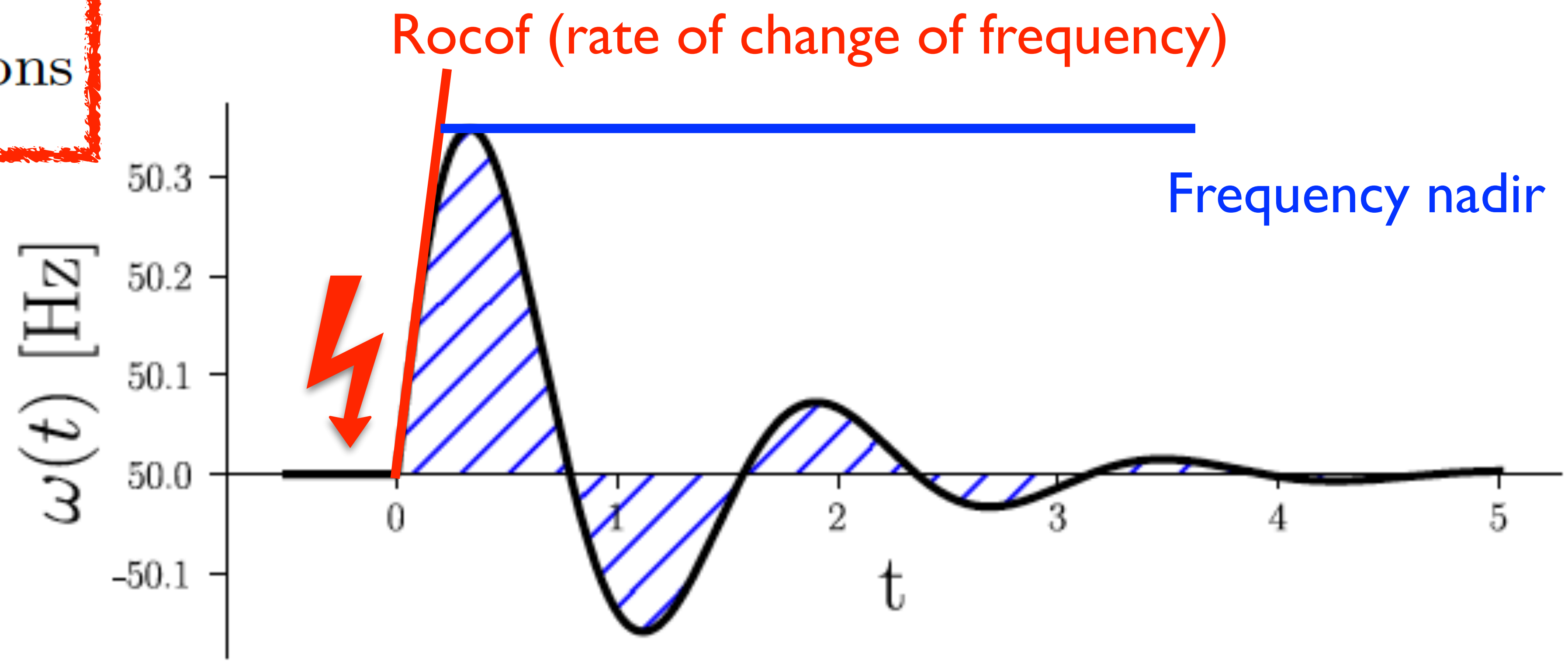

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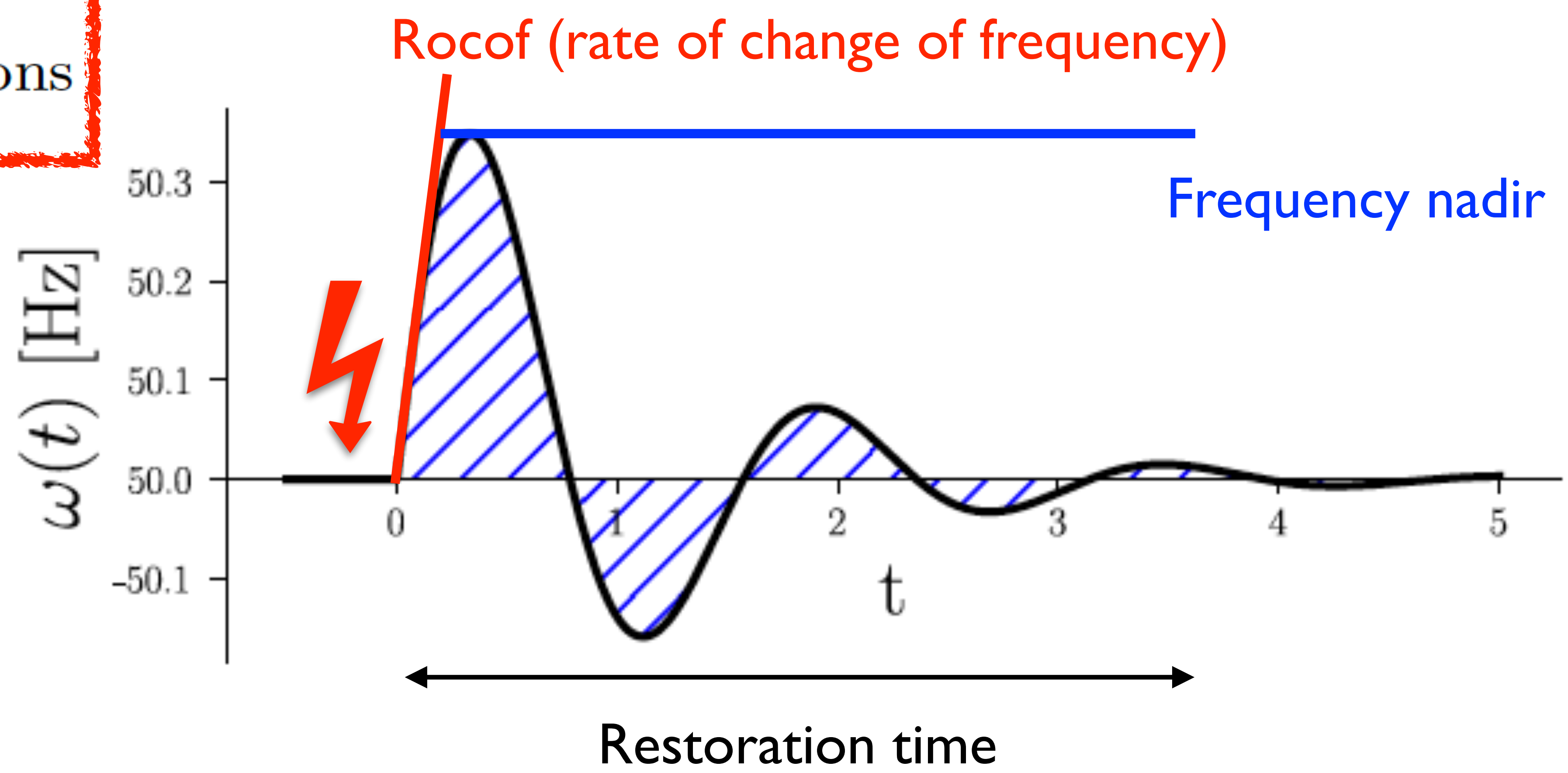

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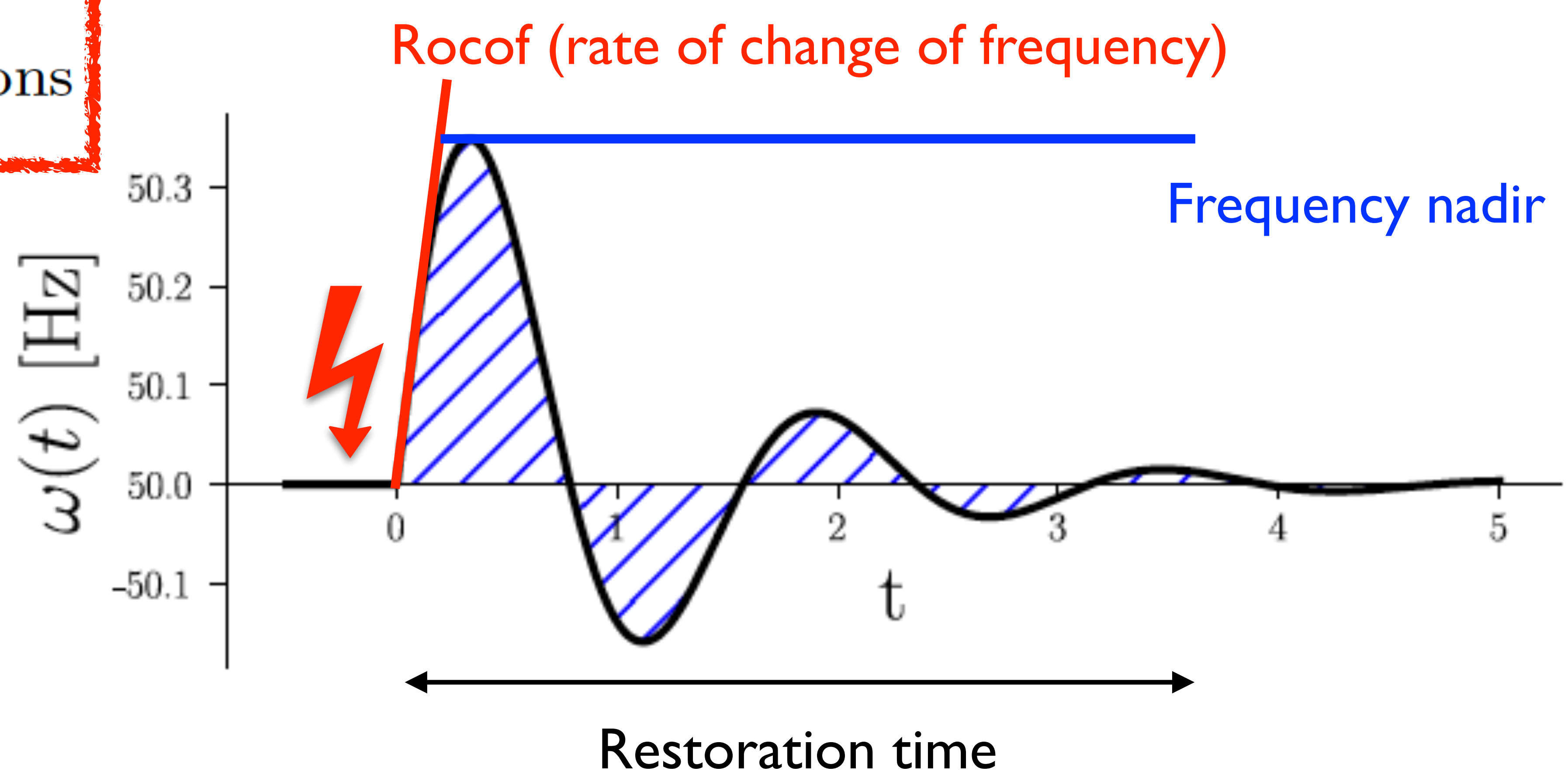

- Frequency transient as indicator of fault magnitude
- Frequency fluctuations need to be minimized
  … but how ?

The swing equations (linearized)

- Frequency vs. voltage angles $\omega(t) = \dot{\varphi}(t)$

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} = -\boldsymbol{D}\dot{\boldsymbol{\varphi}} - \boldsymbol{L}\boldsymbol{\varphi} + \boldsymbol{p}(t)$$

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Power in (generation)

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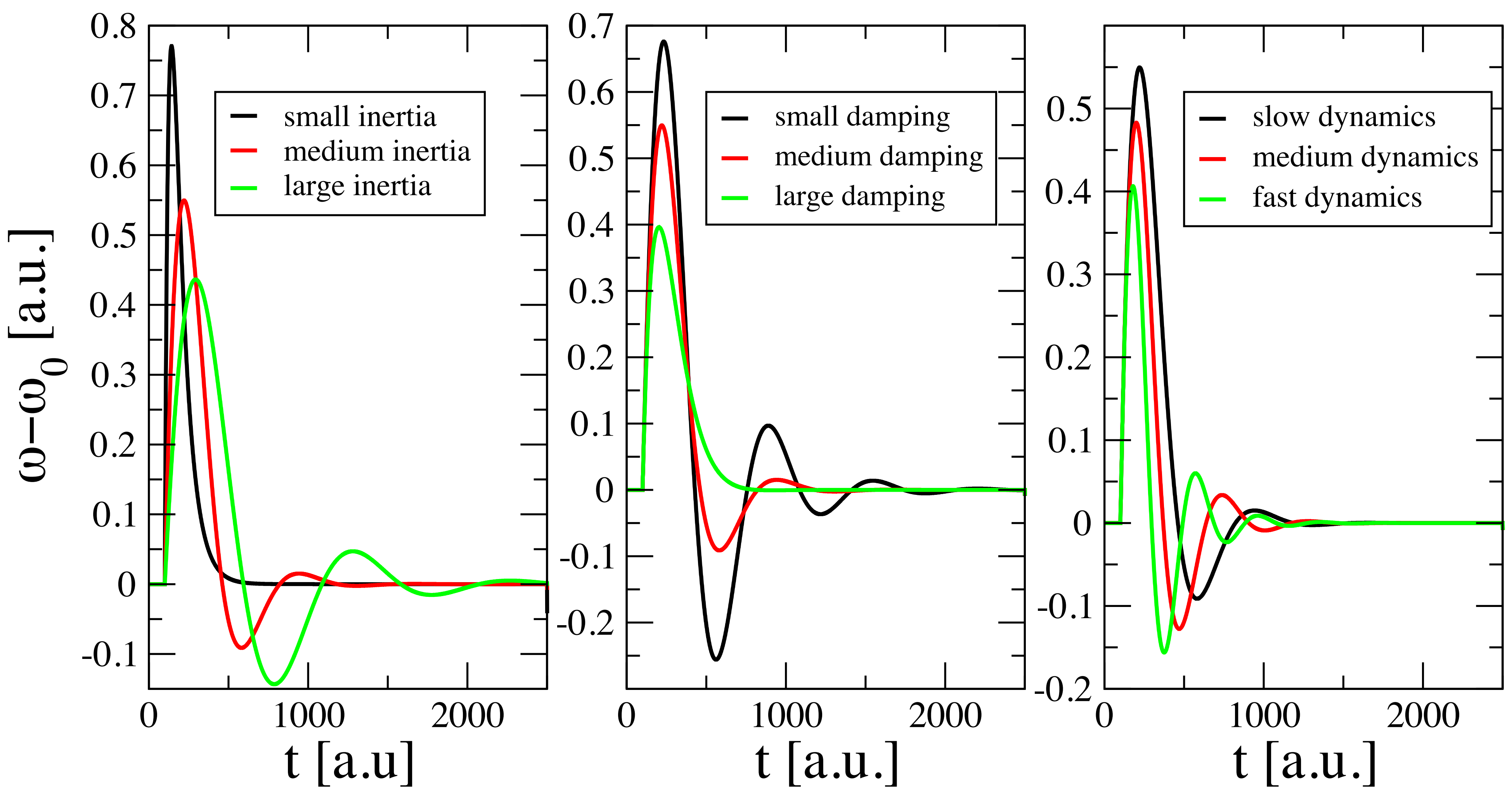
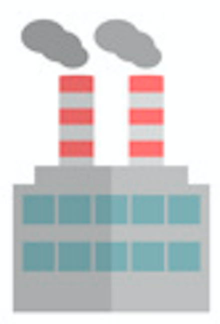
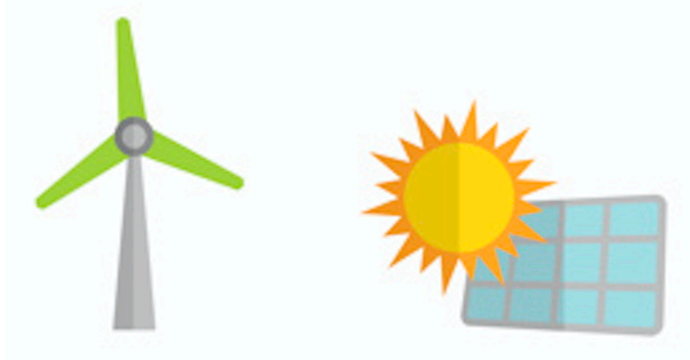


Old vs. new productions





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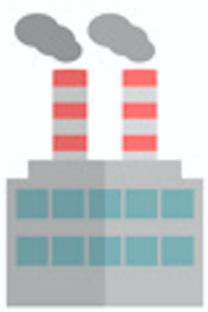
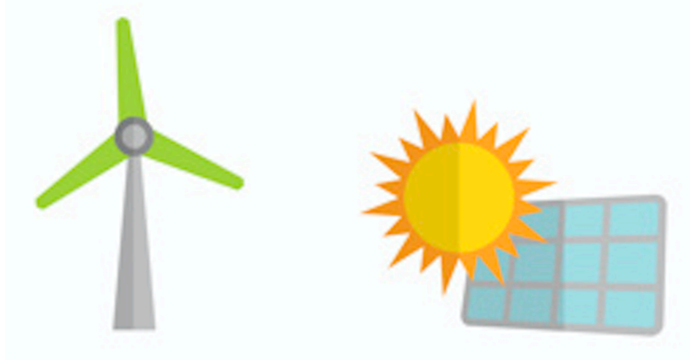


- Centralized



- Decentralized
  -issues at distribution level
  -easier at transmission level
  (=reduction in load) up to
  some level…

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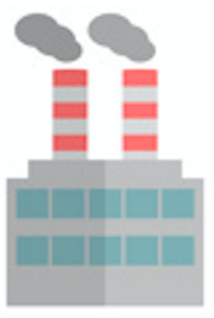
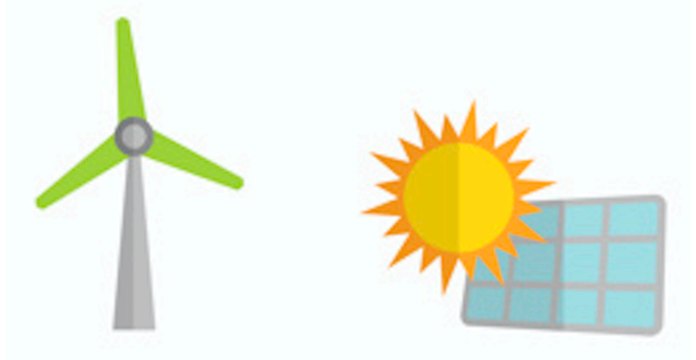


- Centralized

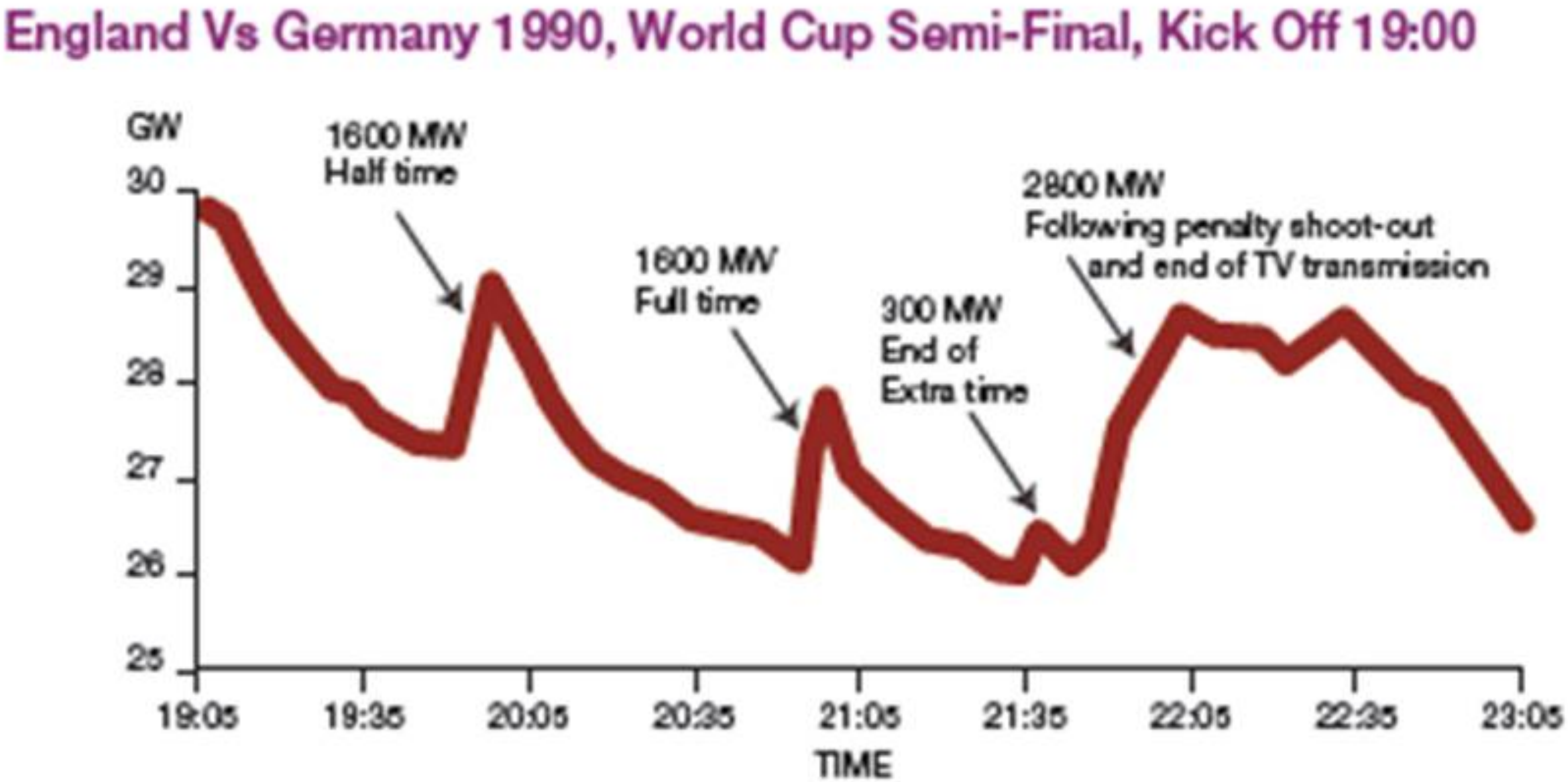
- Dispatchable/predictable



- Decentralized
  - -issues at distribution level
  - -easier at transmission level (=reduction in load) up to some level…

- Uncertain
  - -power / energy reserve

Variability and uncertainty are nothing new for grid operators



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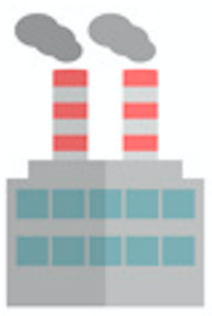
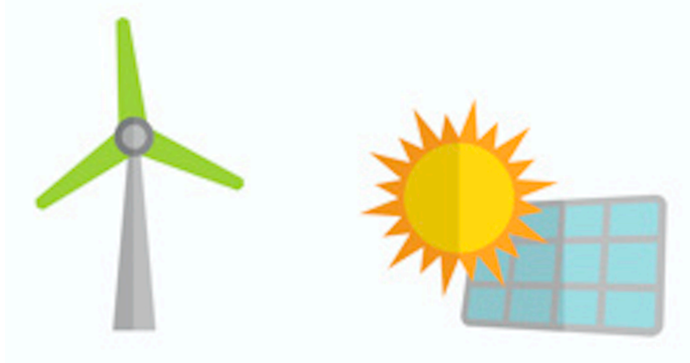
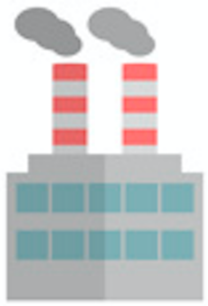


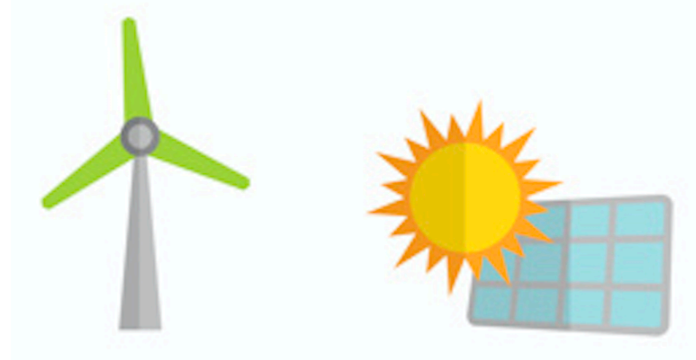
- Rotating machine with inertia
  - -frequency definition
  - -grid stability :
    - *energy reserve
    - *droop control



- No inertia
  (But : converters
  +local storage
  = synthetic inertia)

Old vs. new productions





?? Where is it problematic to substitute old productions with new renewables ??

- Ro
  inertia
  - -frequency definition
  - -grid stability :
    - *energy reserve
    - *droop control

(But : converters
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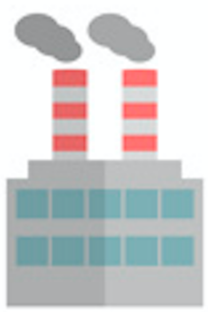
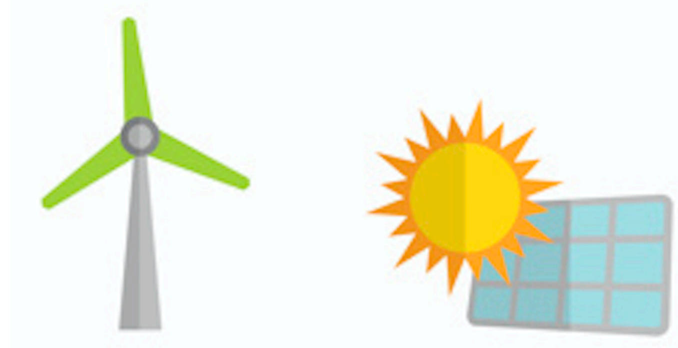
Old vs. new productions





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?? Where is it not ??

Content

- Disturbance propagation
  -vs. inertia
  -vs. Laplacian eigenmodes

- Optimal placement of inertia and droop control

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  -vs. inertia

  -vs. Laplacian eigenmodes

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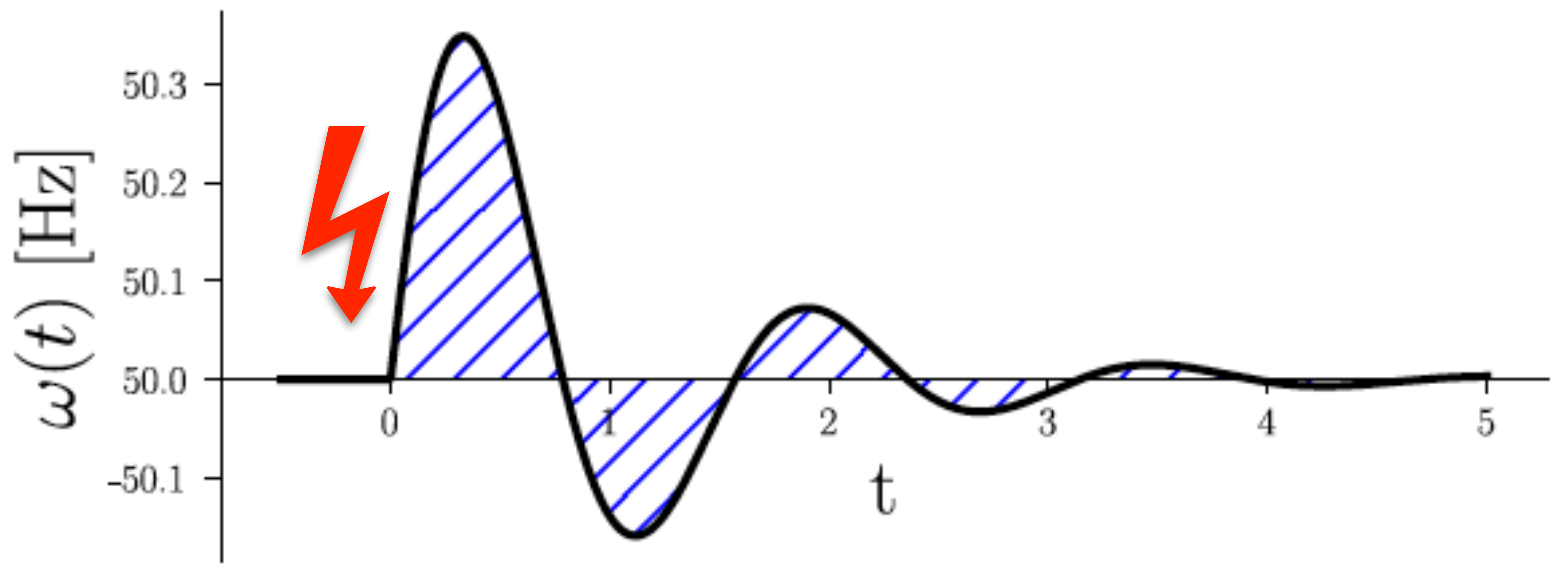
Fault : sudden power loss

$$P_b = P_b^{(0)} - \Delta P$$

$$\Delta P = 900 \text{ MW}$$



- Power injected/extracted @ bus #i :

$$P_i^{\mathrm{e}} = \sum_{j \in \mathcal{V}} B_{ij} \, V_i V_j \, \sin\left(\theta_i - \theta_j\right)$$

- Dynamics given by swing equations :

$$m_i \dot{\omega}_i + d_i \omega_i = P_i^{(0)} - P_i^{\mathrm{e}} \,, \text{ if } i \in \mathcal{V}_{\mathrm{gen}}$$

$$d_i \omega_i = P_i^{(0)} - P_i^{\mathrm{e}} \,, \text{ if } i \in \mathcal{V}_{\mathrm{load}}$$

L. Pagnier & PJ, arXiv:1810.04982 to appear in PLOS ONE

Disturbance propagation : (ii) analytics

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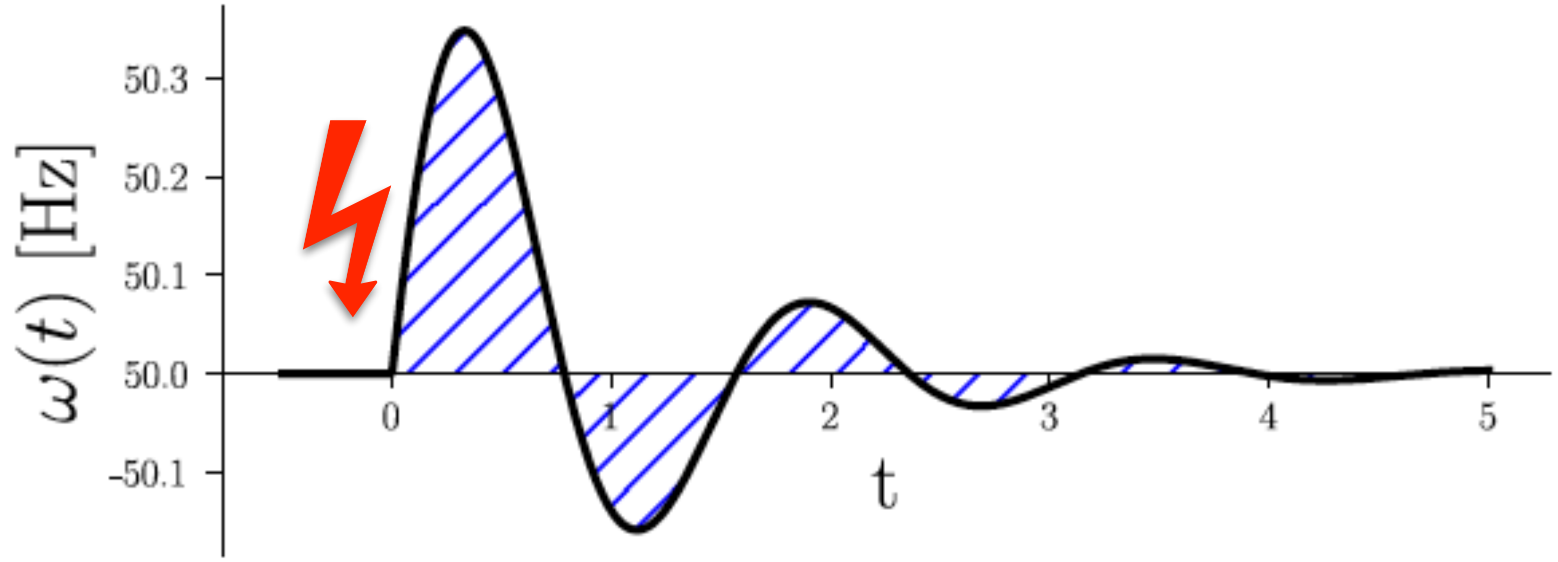

Assume m_i=m, d_i=d

Linearize dynamics about steady-state solution

$$\boldsymbol{M}\dot{\boldsymbol{\omega}} + \boldsymbol{D}\boldsymbol{\omega} = \boldsymbol{P} - \boldsymbol{L}\,\boldsymbol{\theta}$$

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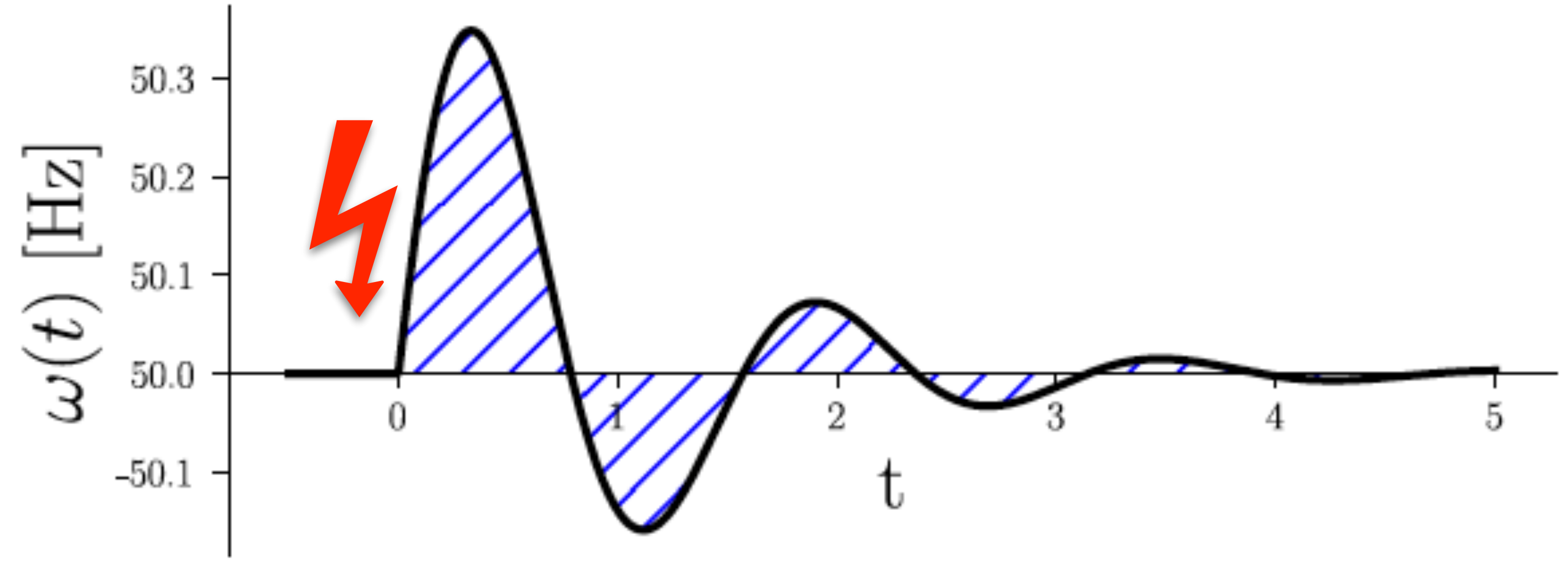

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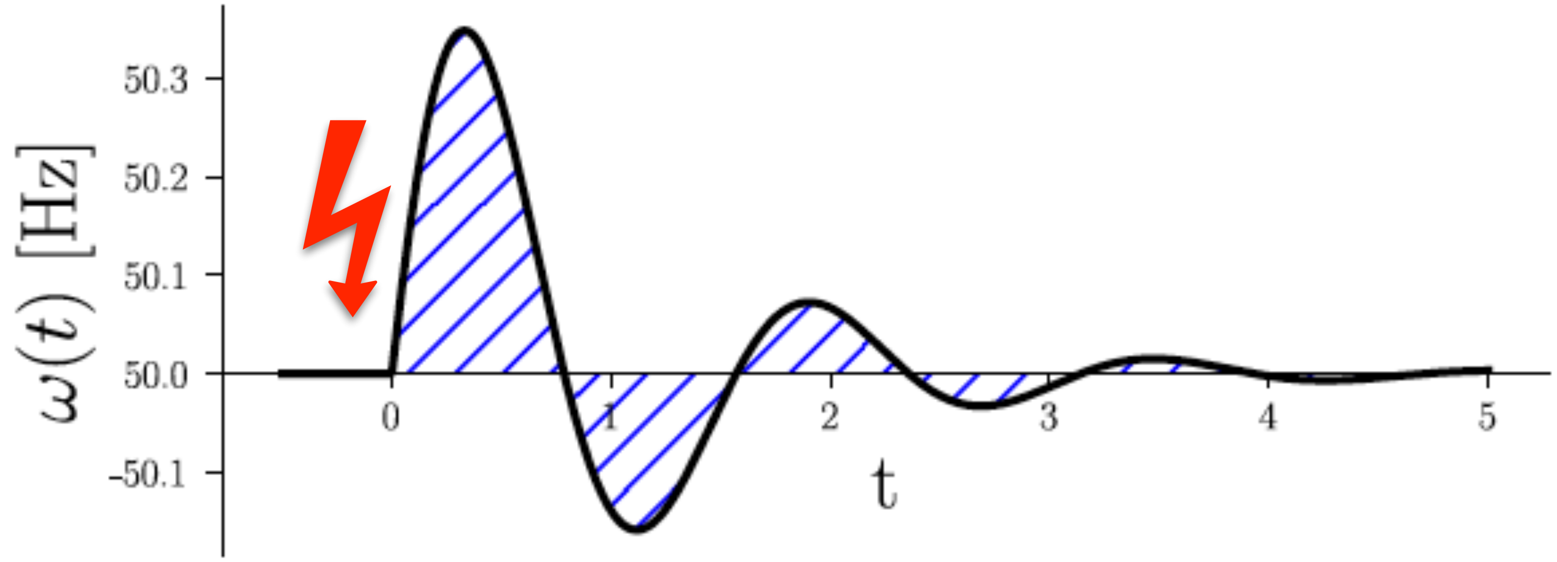

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Network Laplacian

$$(\boldsymbol{L})_{ij} = -B_{ij}V_i^{(0)}V_j^{(0)}$$

$$(\boldsymbol{L})_{ii} = \sum_k B_{ik}V_i^{(0)}V_k^{(0)}$$

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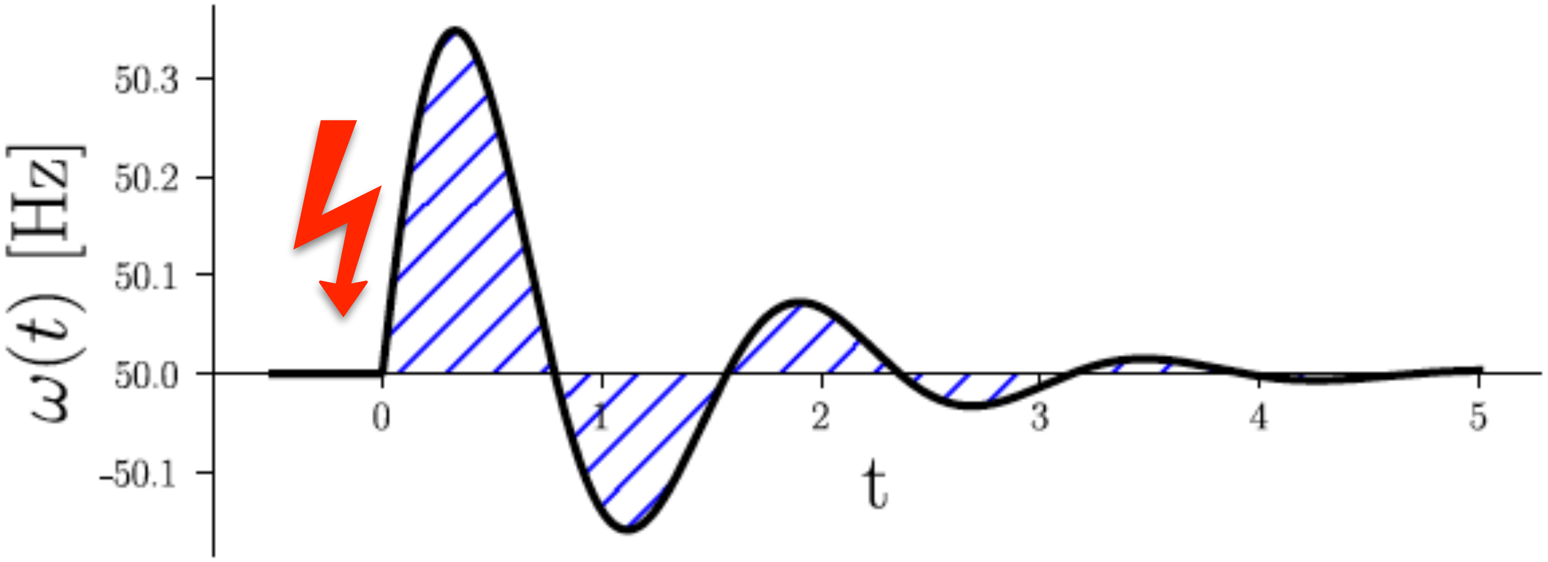

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With eigenvectors and -values

$$\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N\}$$

$$\{\lambda_1, \ldots \lambda_N\}$$

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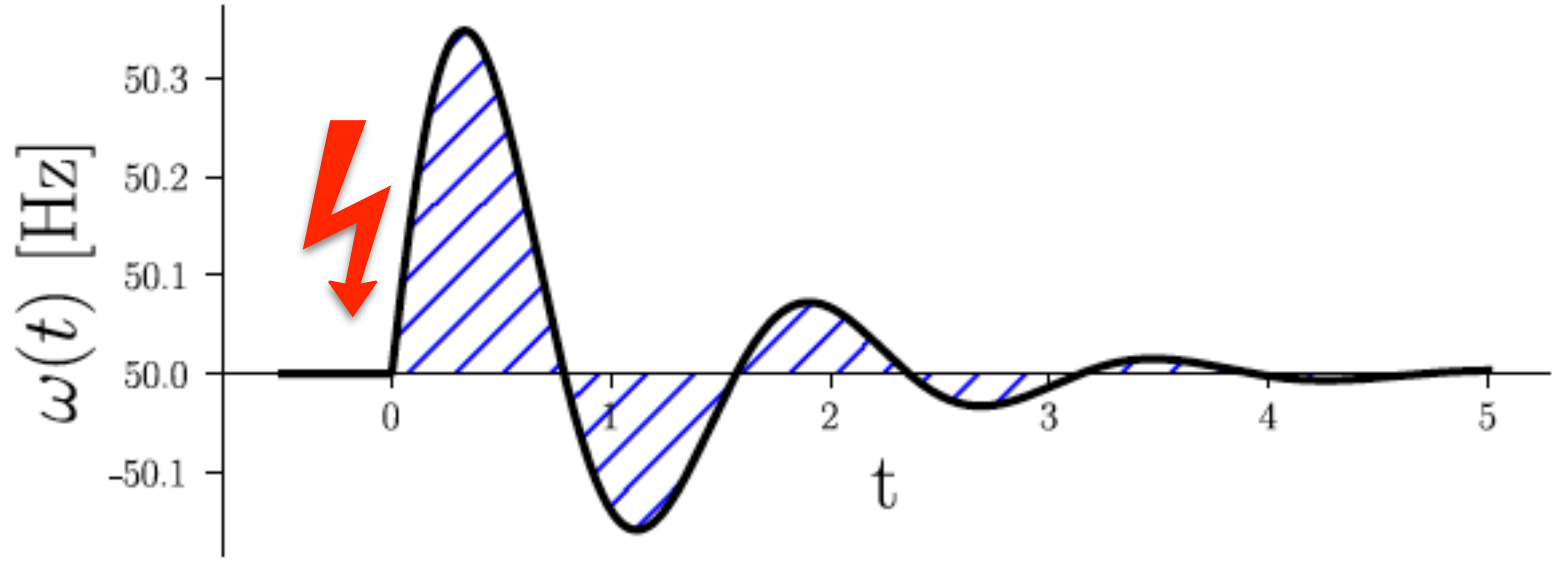

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Zero mode

$$(\boldsymbol{u}_1)^\top = (1, \ldots, 1)/\sqrt{N}$$

$$\lambda_1 = 0$$

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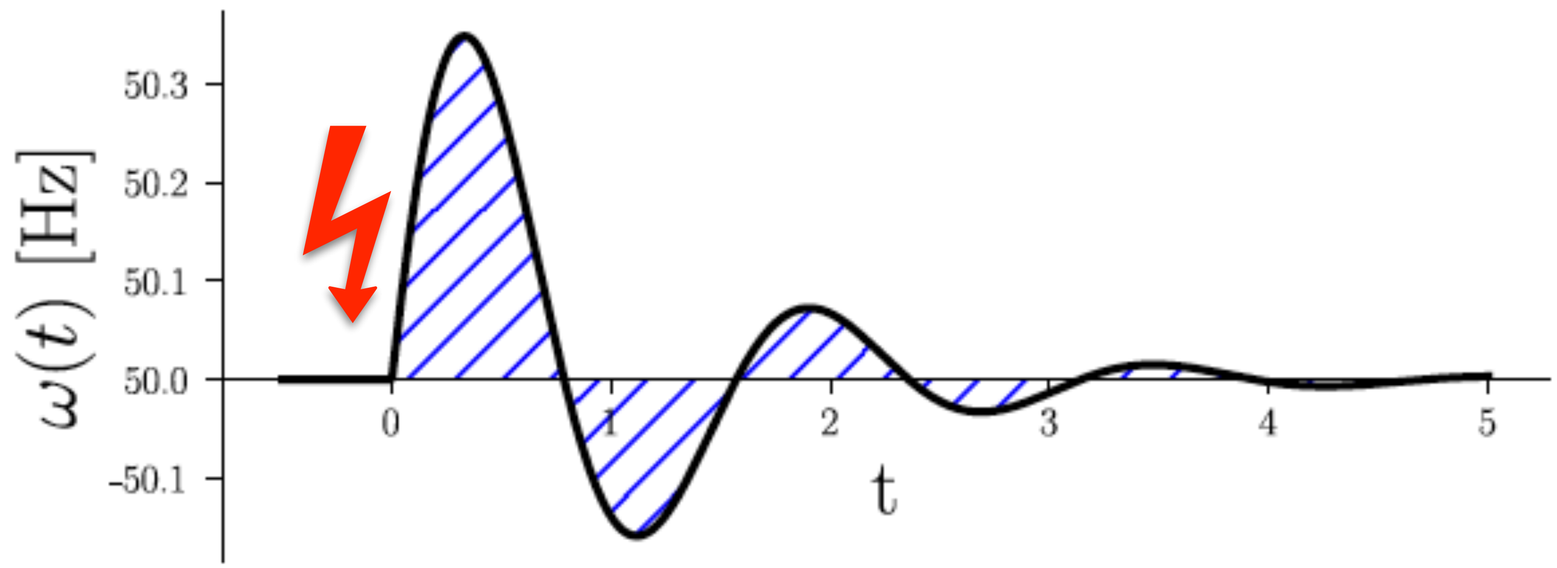

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Higher modes

$$\sum_i u_{\alpha i} = 0$$ (orthog. to zero mode)

$$\lambda_\alpha > 0$$ (stability)

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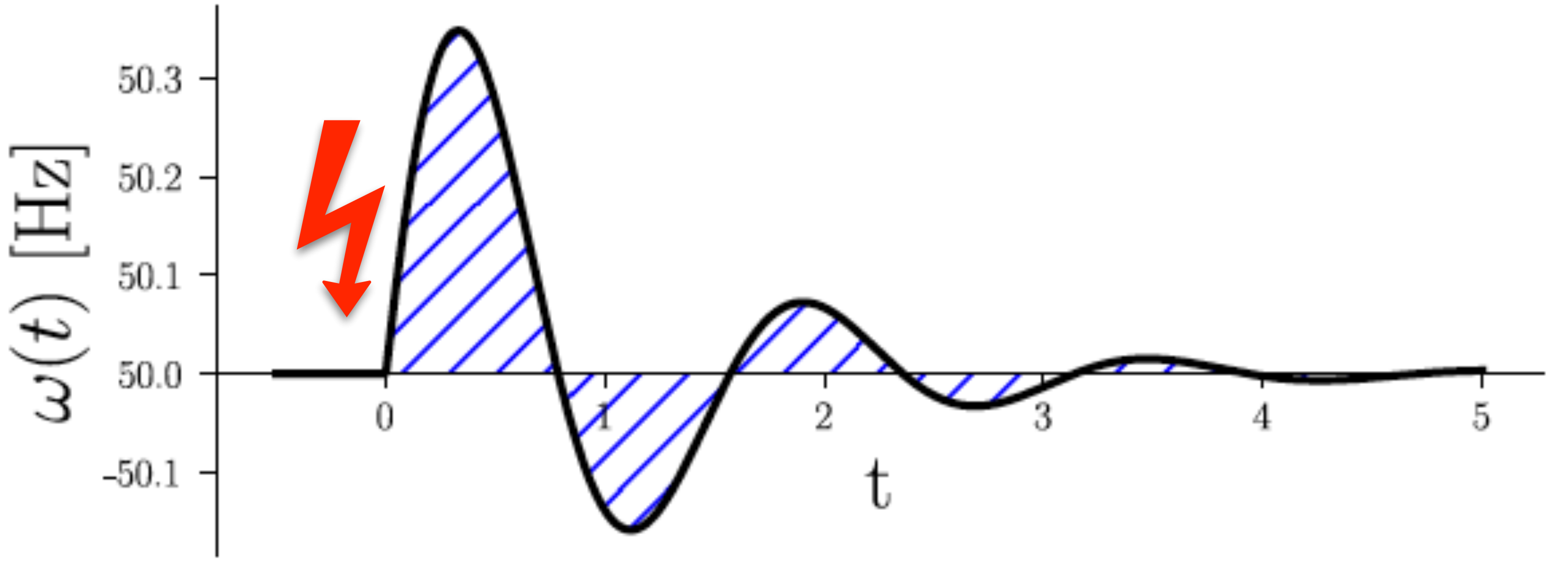

$$\boldsymbol{M}\dot{\boldsymbol{\omega}} + \boldsymbol{D}\boldsymbol{\omega} = \boldsymbol{P} - \boldsymbol{L}\,\boldsymbol{\theta}$$

Spectral decomposition over modes of network Laplacian (γ =d/m)

$$\boxed{\delta\omega_i(t) = \frac{\Delta P\, e^{-\gamma t/2}}{m} \sum_{\alpha=1}^{N} u_{\alpha i} u_{\alpha b} \frac{\sin\left(\sqrt{\lambda_\alpha/m - \gamma^2/4}\, t\right)}{\sqrt{\lambda_\alpha/m - \gamma^2/4}}}$$

Average over i or b

$$\langle \delta\omega_i \rangle = \frac{2\Delta P\, e^{-\gamma t/2}}{mN\gamma} \sinh(\gamma t/2) \qquad \langle \delta\omega_i \rangle (t \to \infty) = \frac{\Delta P}{dN}$$

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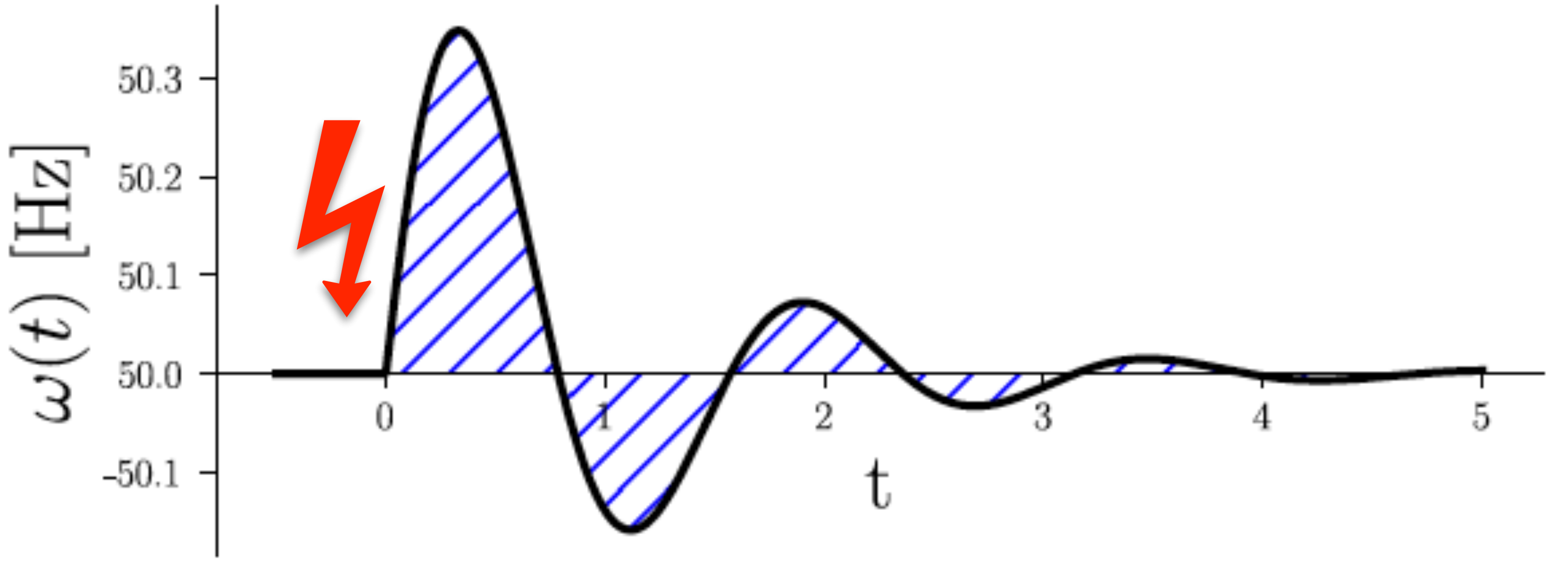

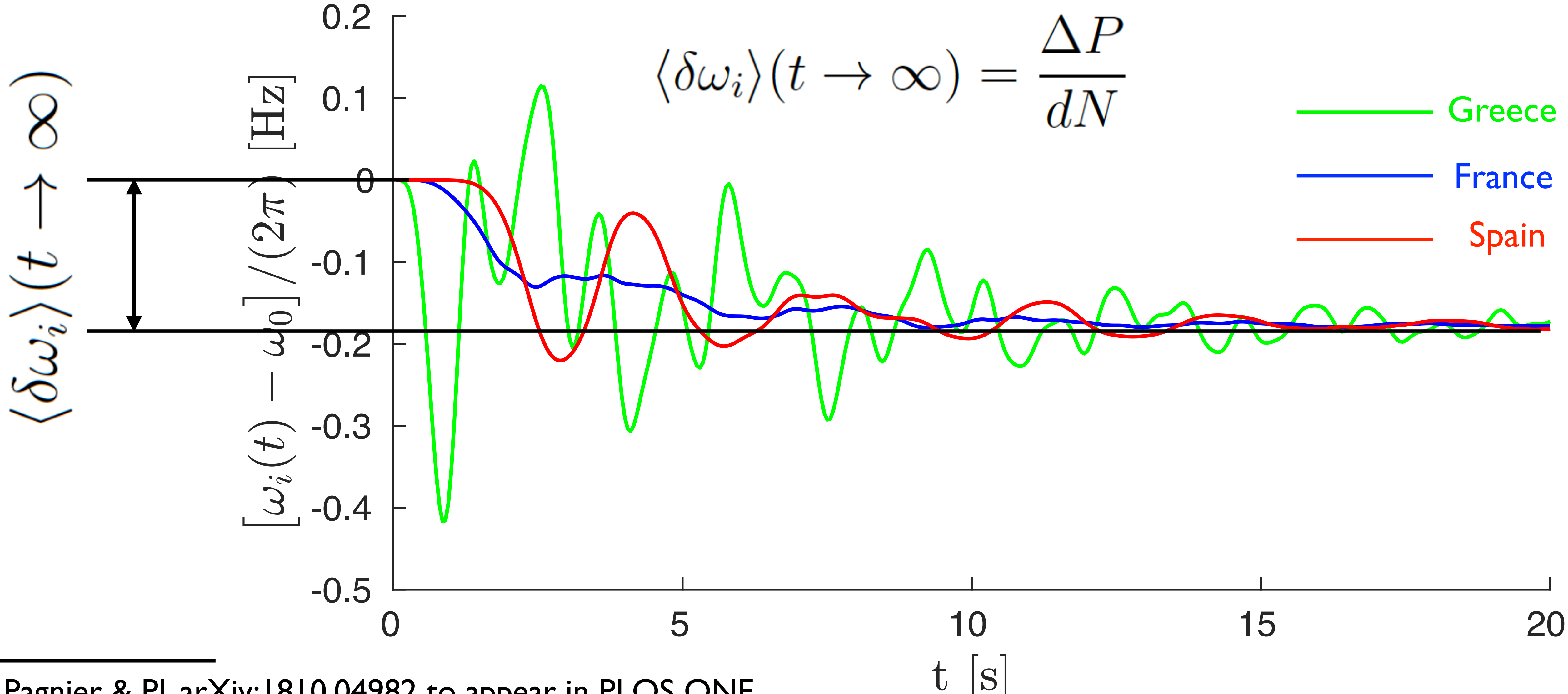

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Disturbance propagation : (iii) numerics

Today's Europe

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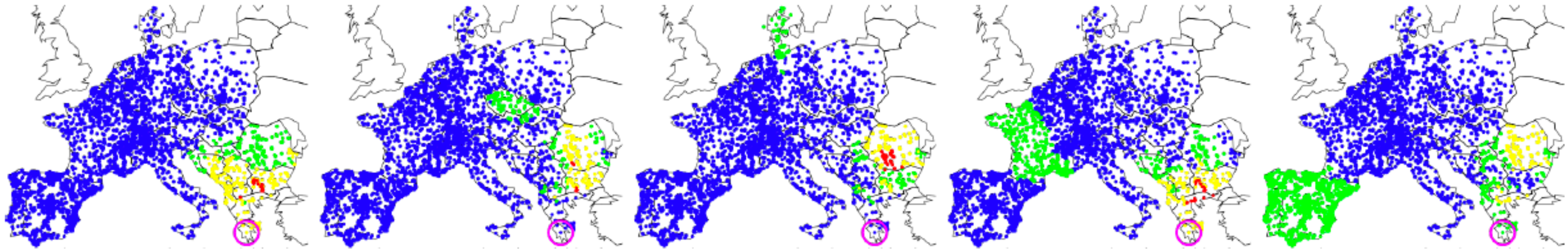
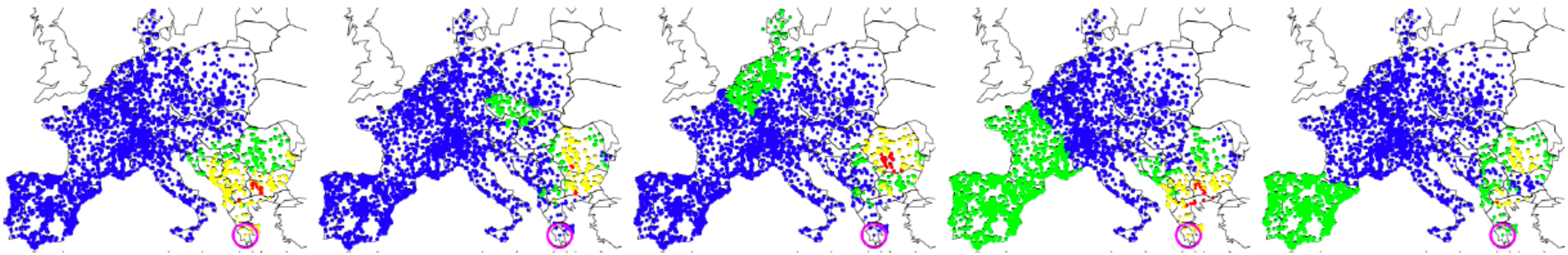


Inertia in France reduced to 50%



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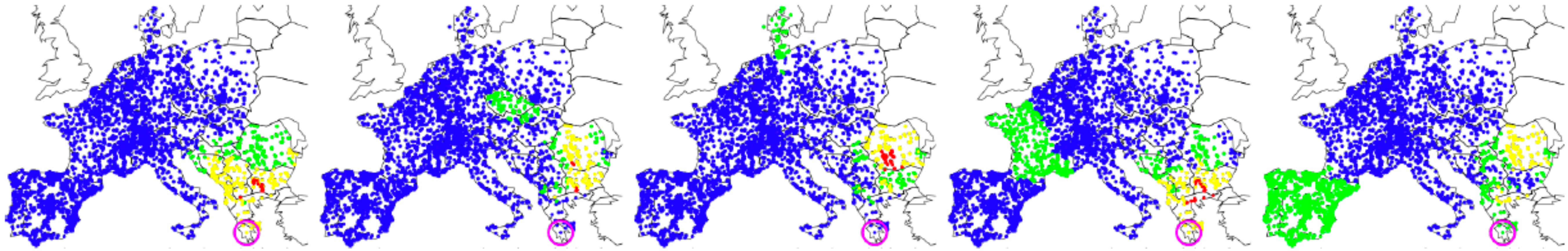

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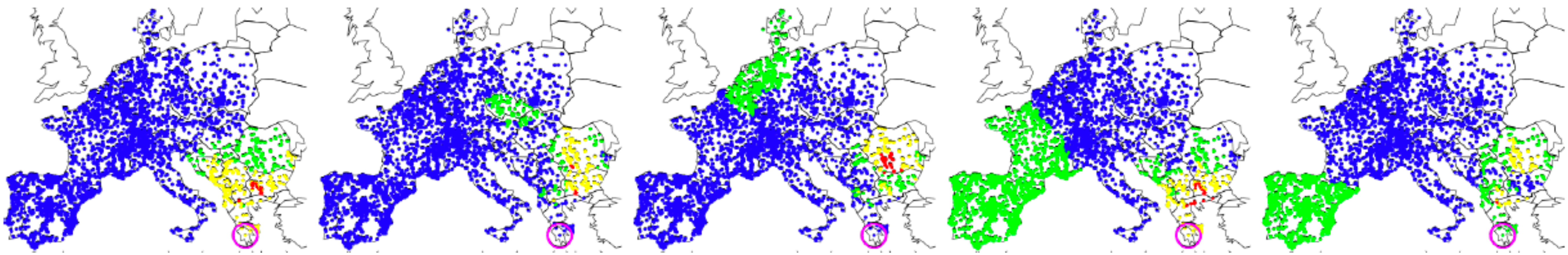
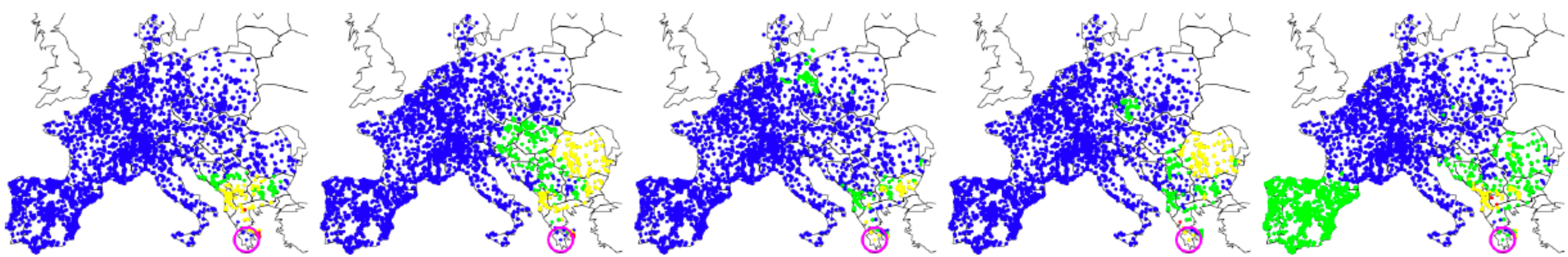


Inertia in Balkans doubled



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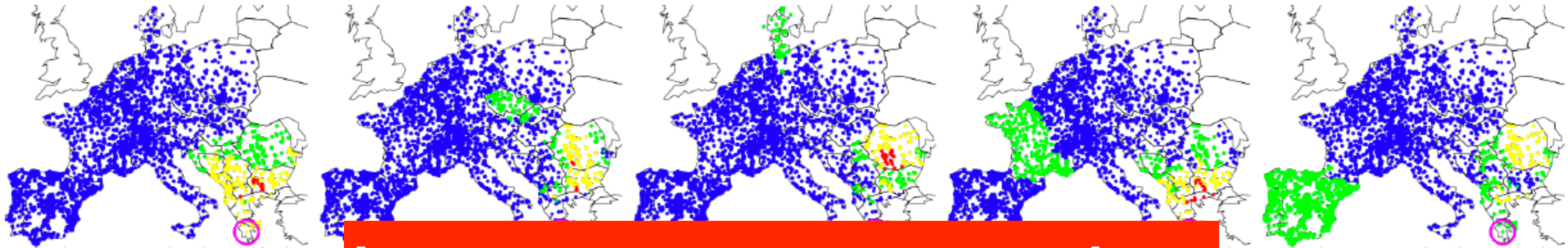
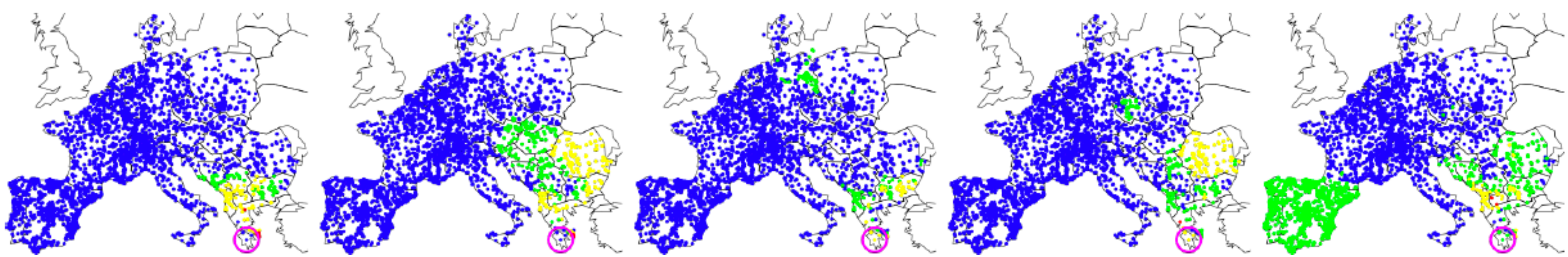


Inertia in France

Inertia matters mostly :
*locally
*at short times

Inertia in Balkans doubled



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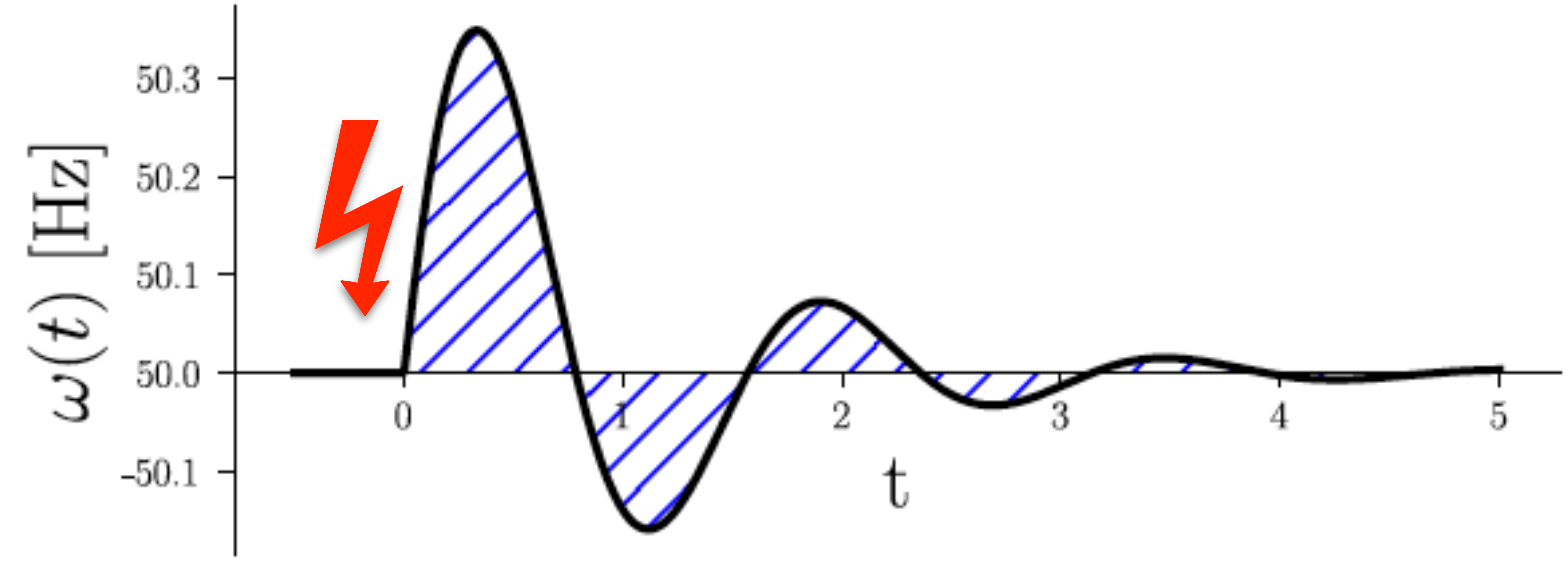
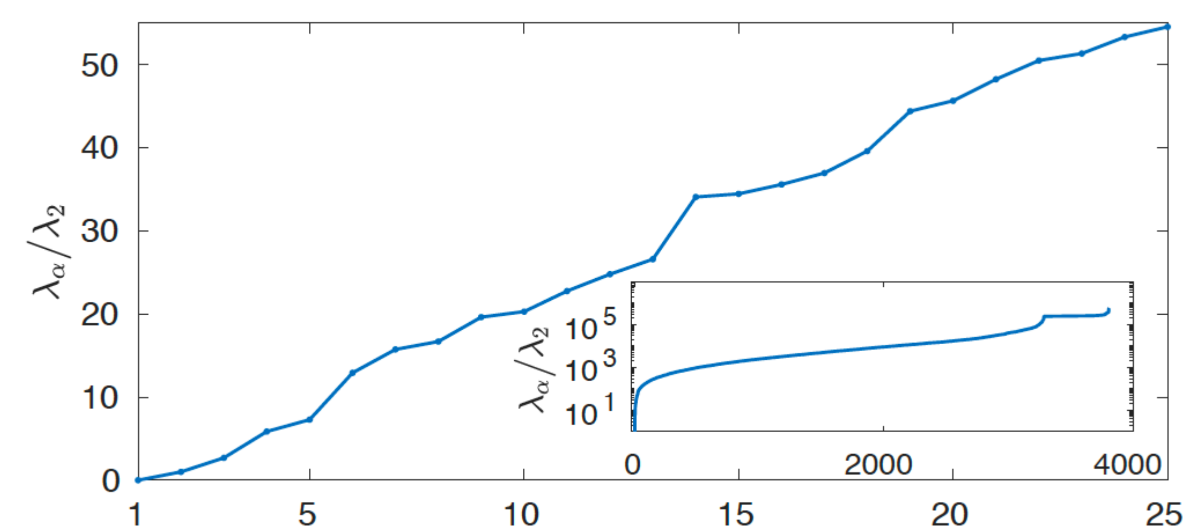


$$\delta\omega_i(t) = \frac{\Delta P\, e^{-\gamma t/2}}{m} \sum_{\alpha=1}^{N} u_{\alpha i} u_{\alpha b} \frac{\sin\left(\sqrt{\lambda_\alpha/m - \gamma^2/4}\, t\right)}{\sqrt{\lambda_\alpha/m - \gamma^2/4}}$$

Wave-propagation from higher modes $\alpha > 1$

Oscillations + denominator

→ only few, low modes matter ?



Disturbance propagation : (iii) numerics

Disturbance magnitude :

1) RoCoF at bus #i

$$r_i(t) = \frac{\delta\omega_i(t+\Delta t) - \delta\omega_i(t)}{2\pi\Delta t}$$

2) Performance measure
vs. Disturbance location #b

$$\mathcal{M}_b = \sum_{k=1}^{N_{\rm sim}} \sum_i \big| r_i(k\Delta t) \big|$$

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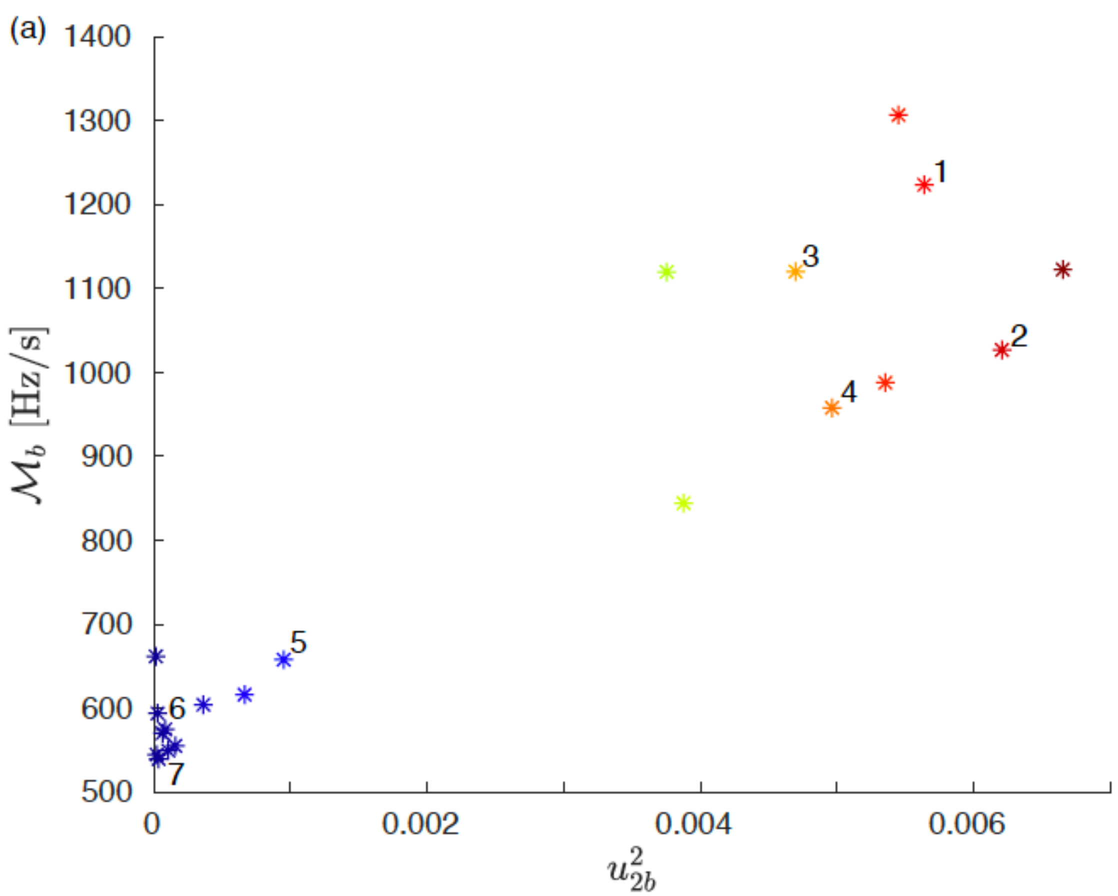
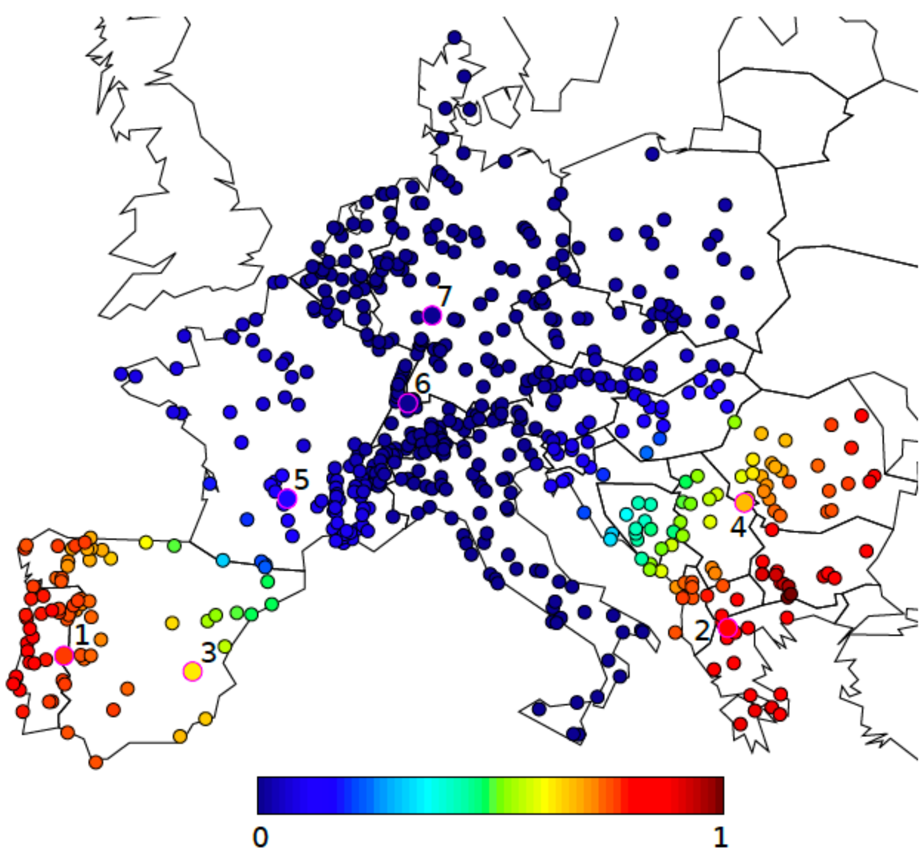
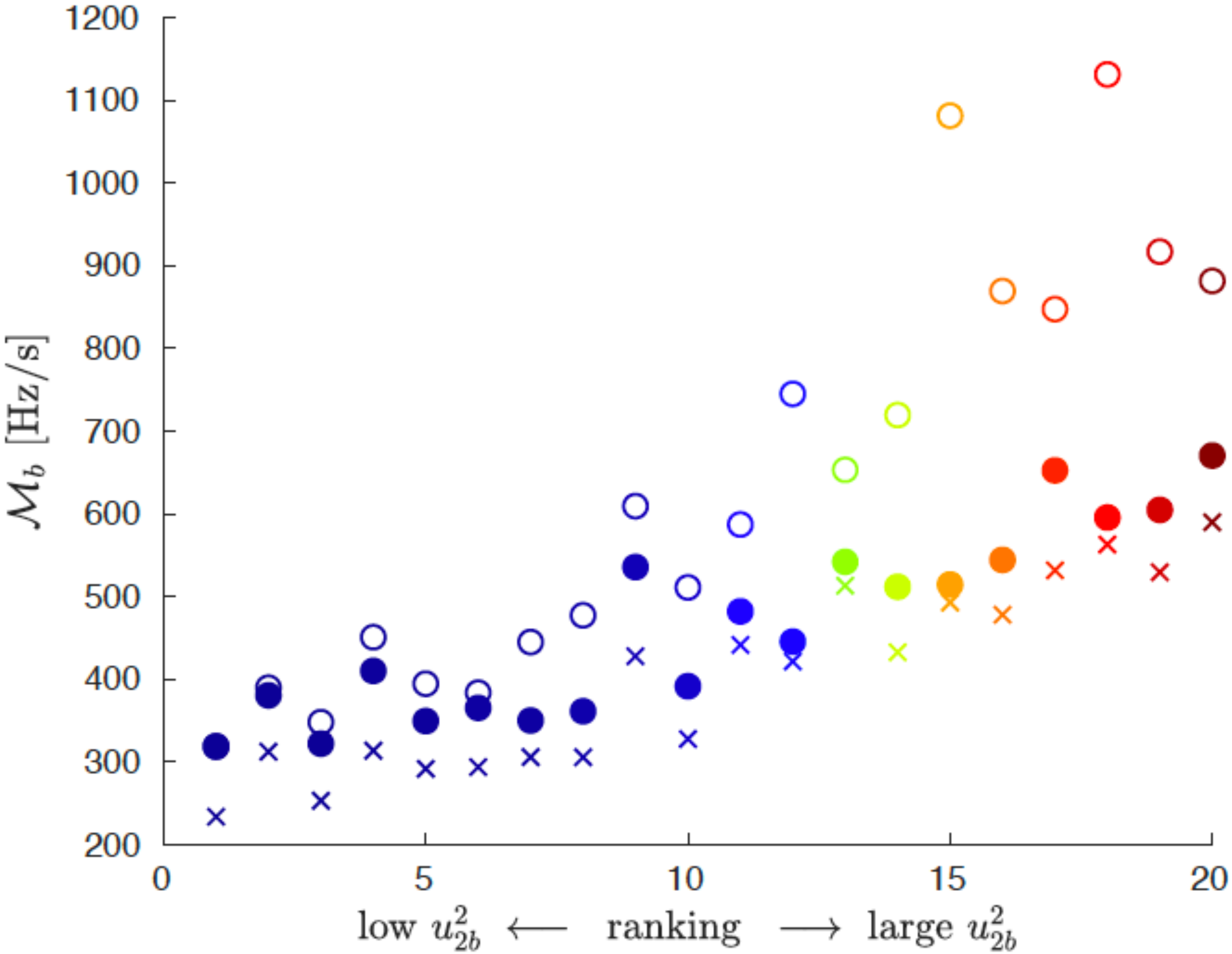




~amplitude of slowest, nonzero mode
(a.k.a. Fiedler mode) on disturbance location

Disturbance propagation : (iii) numerics

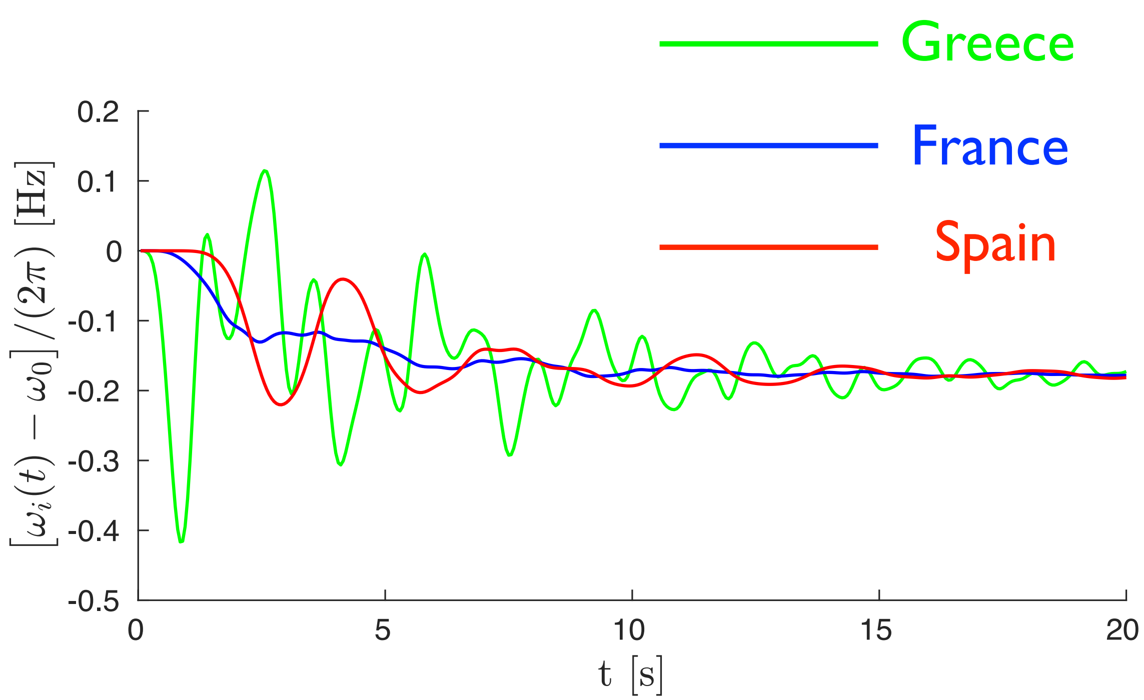


x : today
• : -60% inertia (outside Fiedler)
o : -60% inertia (on Fiedler)

Conclusion 1

- Inertia absorbs disturbances
  - -locally (close to the disturbance)
  - -on short times

- Inertia outside slow modes has very little effect on disturbances inside slow modes



- Inertia inside slow modes is key to mitigate RoCoF's

Content

- Disturbance propagation

  -vs. inertia

  -vs. Laplacian eigenmodes

- Optimal placement of inertia and droop control

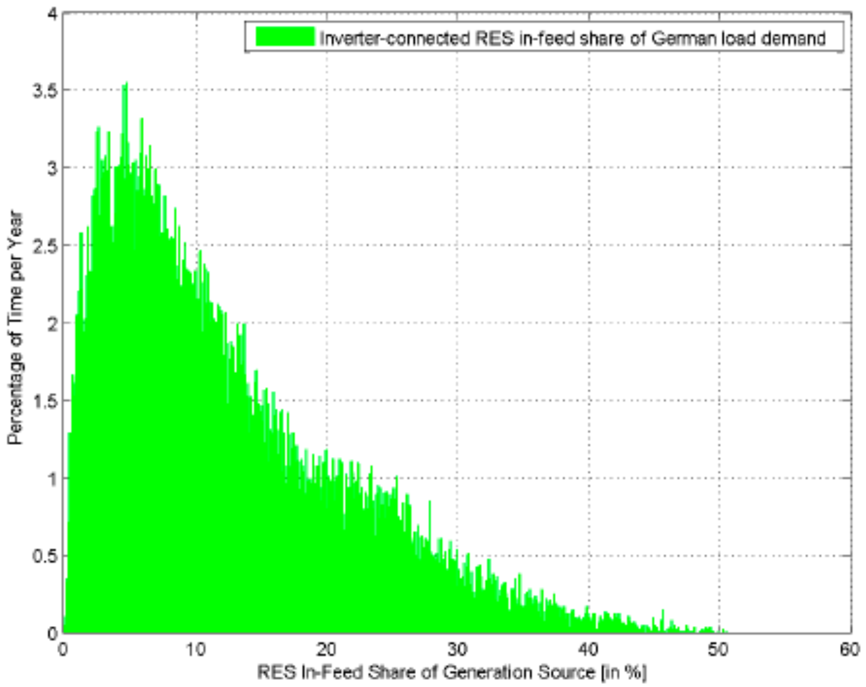
Context (i)

- Increased penetration of new renewables

- Decrease of inertia (+time dependent)

- Inertia can be replaced by

  *synthetic inertia (power electronics+storage)
  *flywheels
  *synchronous condensers



(see e.g. : Poolla, Bolognani, Dörfler '17)

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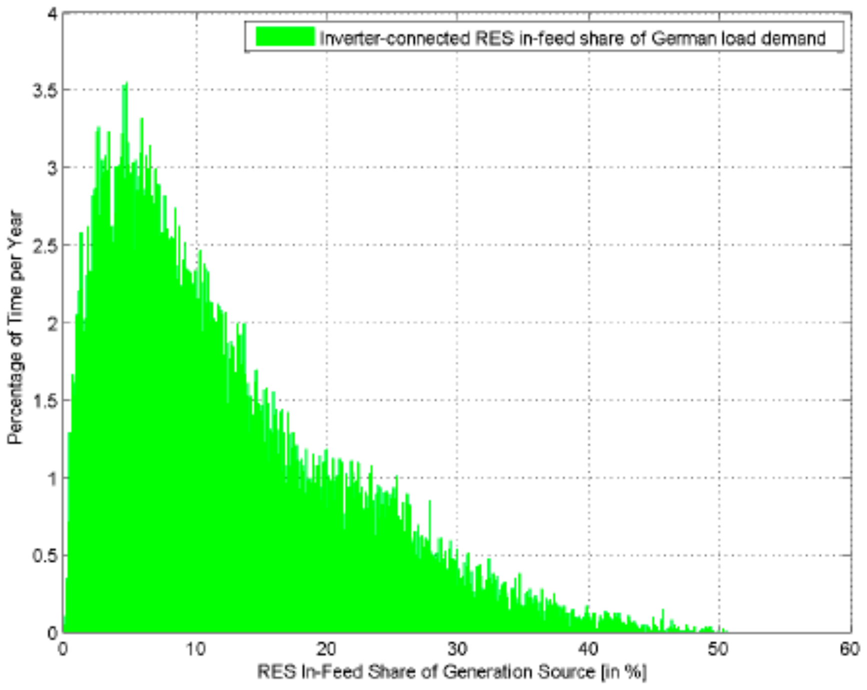

*synthetic inertia (power electronics+storage)
*flywheels
*synchronous condensers

Where is it best to deploy substitution inertia ?

(see e.g. : Poolla, Bolognani, Dörfler '17)

Context (ii)

- Frequency disturbance $\delta\omega_i(t) = \frac{\Delta P\, e^{-\gamma t/2}}{m} \sum_{\alpha=1}^{N} u_{\alpha i} u_{\alpha b} \frac{\sin\left(\sqrt{\lambda_\alpha/m - \gamma^2/4}\, t\right)}{\sqrt{\lambda_\alpha/m - \gamma^2/4}}$
- Quadratic performance measures / H_2-norms

$$\|\tilde{w}\|_2^2 = \sum_{k=1}^{n-1} \frac{(v_k^T u_0)^2}{2d\lambda_k}$$

$$E\left[\|\tilde{w}\|_2^2\right] = \frac{1}{2d} \sum_k \frac{1}{\lambda_k}$$

From : Paganini and Mallada '17

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Other works (list not complete) :
Bamieh, Jovanovic, Mitra, Patterson '12
Bamieh and Gayme '13
Siami and Motee '14
Grunberg and Gayme '16

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Performance measures vs. modes and spectrum of the network Laplacian

$\|\tilde{\omega}\|_2^2 = \ldots \sum_{k=1} \ldots \lambda_k \quad \ldots \sum_k \frac{1}{\lambda_k}$

From : Paganini and Mallada '17

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$\|\tilde{w}\|_2^2$ ... $k=1$ ... λ_k ... $\sum_k \frac{1}{\lambda_k}$

Performance measures vs. modes and spectrum of the network Laplacian

From : Paganini and Mallada '17

All these works considered either

$m_i = m$, $d_i = d$

$d_i / m_i = \text{constant}$

Other works (lis...

Bamieh, Jovanovi...

Bamieh and Gayme '13

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Grunberg and Gayme '16

...17

Paganini and Mallada '17

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Matrix perturbation theory (a.k.a. Rayleigh-Schrödinger)

- Starting point : known, diagonalized matrix

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$$\lambda_\alpha^{(1)} = \boldsymbol{u}_\alpha^{(0)\top} \boldsymbol{V}_1 \boldsymbol{u}_\alpha^{(0)}$$

$$\boldsymbol{u}_\alpha^{(1)} = \sum_{\beta \neq \alpha} \frac{\boldsymbol{u}_\beta^{(0)\top} \boldsymbol{V}_1 \boldsymbol{u}_\alpha^{(0)}}{\lambda_\alpha^{(0)} - \lambda_\beta^{(0)}} \boldsymbol{u}_\beta^{(0)}$$

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We use matrix perturbation theory to free ourselves from the homogeneity assumption and calculate optimal distribution of m_i and d_i

$$\boxed{\begin{aligned}\lambda_\alpha &= \lambda_\alpha^{(0)} + \mu\lambda_\alpha^{(1)} + \mathcal{O}(\mu^2)\\ \boldsymbol{u}_\alpha &= \boldsymbol{u}_\alpha^{(0)} + \mu\boldsymbol{u}_\alpha^{(1)} + \mathcal{O}(\mu^2)\end{aligned}}$$

$$\boldsymbol{u}_\alpha^{(1)} = \sum_{\beta\neq\alpha} \frac{\boldsymbol{u}_\beta^{(0)\top}\boldsymbol{V}_1\boldsymbol{u}_\alpha^{(0)}}{\lambda_\alpha^{(0)} - \lambda_\beta^{(0)}}\boldsymbol{u}_\beta^{(0)}$$

Optimal placement of inertia and droop

- Starting point : when can one diagonalize the swing eqs. ? (linearized about steady-state)

(Paganini and Mallada '17)

$$\boxed{\boldsymbol{M}\dot{\boldsymbol{\omega}} + \boldsymbol{D}\boldsymbol{\omega} = \boldsymbol{\delta P} - \boldsymbol{L}\boldsymbol{\delta\theta}}$$

$$\boldsymbol{M} = \mathrm{diag}(\{m_i\})$$
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When can this eq. be reduced to a set of scalar eps. ?

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A. : when d_i/m_i=constant

Diagonalized solution - starting point

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$$\boxed{\Gamma = \gamma\,\mathbb{1}}$$

$$\boxed{\ddot{\xi} + \gamma\dot{\xi} + \Lambda\xi = U M^{-1/2}\delta P}$$

$$\Lambda = \mathrm{diag}(\{\lambda_1 = 0, \lambda_2, \cdots, \lambda_N\})$$

~diagonal/scalar linear differential equation

Optimal placement of inertia and droop - problem formulation

- Performance measure (to be minimized)

$$\mathcal{M} = \int_0^\infty (\boldsymbol{\omega}^\top - \bar{\boldsymbol{\omega}}^\top)\boldsymbol{M}(\boldsymbol{\omega} - \bar{\boldsymbol{\omega}})\mathrm{d}t$$

- Power loss at bus #b

$$\boxed{\mathcal{M}_b = \frac{\delta P^2}{2\gamma m_b}\sum_{\alpha>1}\frac{u_{\alpha b}^2}{\lambda_\alpha}}$$

(Similar expressions in :
Bamieh and Gayme '13, Siami and Motee '14
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- Deviation from homogeneity

$$m_i = m + \delta m\, r_i\,,$$

$$d_i = m_i\gamma_i = (m + \delta m\, r_i)(\gamma + \delta\gamma\, a_i)$$

$$-1 \;\leq\; a_i, r_i \;\leq\; 1$$

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Do perturbation theory with small parameters

$$\mu \equiv \delta m/m \ll 1$$

$$g \equiv \delta\gamma/\gamma \ll 1$$

Optimal placement of inertia and droop - problem formulation

- Perturbative performance measure - to be minimized

$$\mathcal{M}_b = \mathcal{M}_b^{(0)} + r_i \rho_i + a_i \alpha_i + \mathcal{O}(\delta m^2, \delta\gamma^2)$$

with susceptibilities (this is what we calculate)

$$\rho_i \equiv \partial \mathcal{M}_b / \partial r_i \quad \alpha_i \equiv \partial \mathcal{M}_b / \partial a_i$$

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- Goal : determine optimal distribution of r_i and a_i with constraints

$$-1 \leq a_i, r_i \leq 1$$ Local reduction/increase

$$\textstyle\sum_i r_i = \sum_i a_i = 0$$ Total limited resources

$$\mu \equiv \delta m / m \ll 1$$

$$g \equiv \delta\gamma / \gamma \ll 1$$

Optimal placement of inertia and droop - algorithmic solution

- Susceptibilities

  1) inhomogeneities in inertia

$$\rho_i = \frac{\partial \mathcal{M}_b}{\partial r_i} = -\frac{\mu \delta P^2}{\gamma} \sum_{\substack{\alpha>1 \\ \beta\neq\alpha}} \frac{u_{\alpha b}^{(0)} u_{\beta b}^{(0)} u_{\alpha i}^{(0)} u_{\beta i}^{(0)}}{\lambda_\alpha^{(0)} - \lambda_\beta^{(0)}}$$

  2) inhomogeneities in damping

$$\alpha_i \equiv \partial \mathcal{M}_b / \partial a_i = -\frac{g \delta P^2}{2\gamma m_b} \left[\sum_{\alpha>1} \frac{u_{\alpha i}^2 u_{\alpha b}^2}{\lambda_\alpha} + \sum_{\alpha>1, \beta\neq\alpha} \frac{u_{\alpha i} u_{\alpha b} u_{\beta i} u_{\beta b}}{(\lambda_\alpha - \lambda_\beta)^2 + 2\gamma^2 (\lambda_\alpha + \lambda_\beta)} \right]$$

Optimal placement of inertia and droop - algorithmic solution

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Algorithm for optimal placement

**rank susceptibilities in ascending order*

**a_i, r_i =-1 for the N/2 smallest susceptibilities*

**a_i, r_i =+1 for the N/2 largest susceptibilities*

**for simultaneous optimal inertia and damping placement,*
include constraint $\sum_i r_i a_i = 0$

**if all susceptibilities equal :* a_i, r_i *=0*

Optimal placement of inertia and droop - solution

Inertia. : r_i =0
Wants to keep it homogeneous

$$m_i = m + \delta m \, r_i \, ,$$
$$d_i = m_i \gamma_i = (m + \delta m \, r_i)(\gamma + \delta\gamma \, a_i)$$

Optimal placement of inertia and droop - solution

Inertia. : r_i =0
Wants to keep it homogeneous

Damping/droop : r_i =1 (blue) r_i =-1 (red)
Wants to keep it homogeneous

$$m_i = m + \delta m \, r_i \, ,$$
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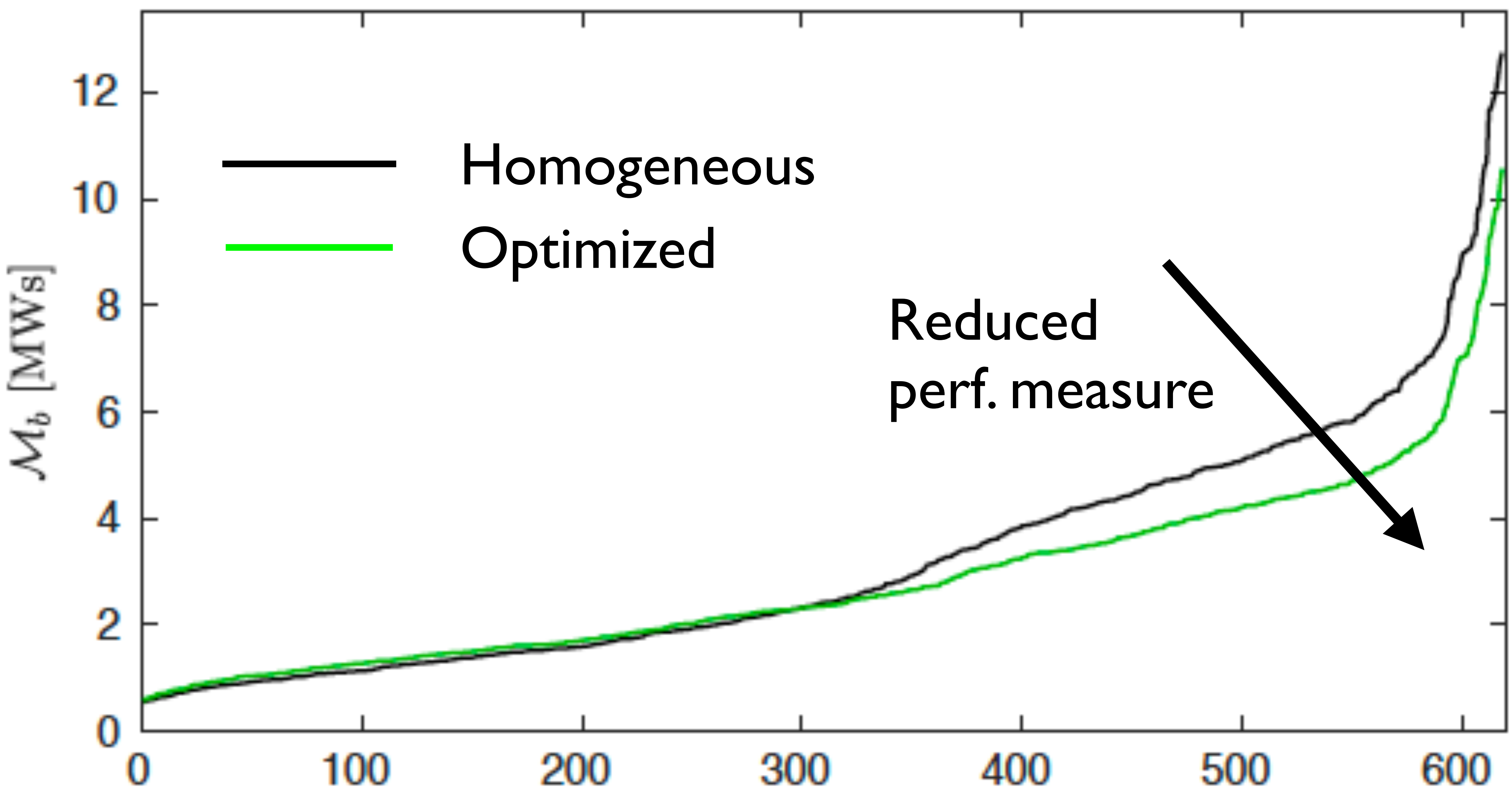
Optimal placement of inertia and droop - solution



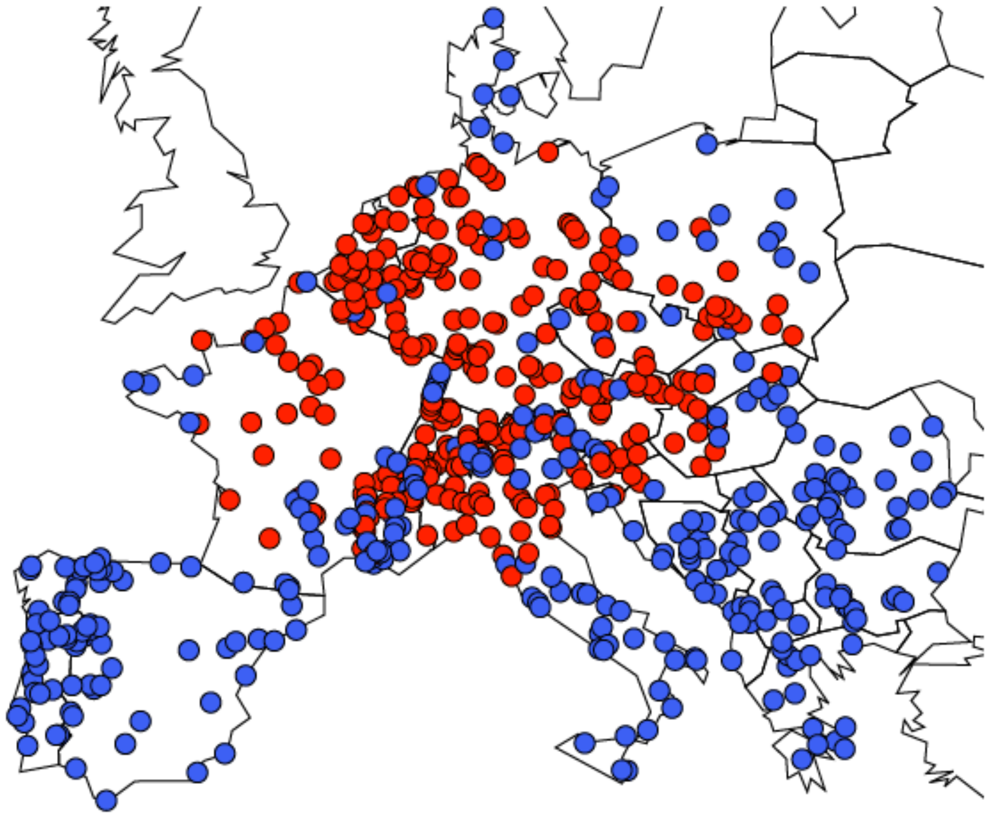
- Performance strongly increased especially at most sensitive buses

Conclusion 2

- Short time effect of inertia $\longrightarrow$ keep it homogeneous !

- Optimal droop / damping placement

  Seems correlated with location of few slow modes



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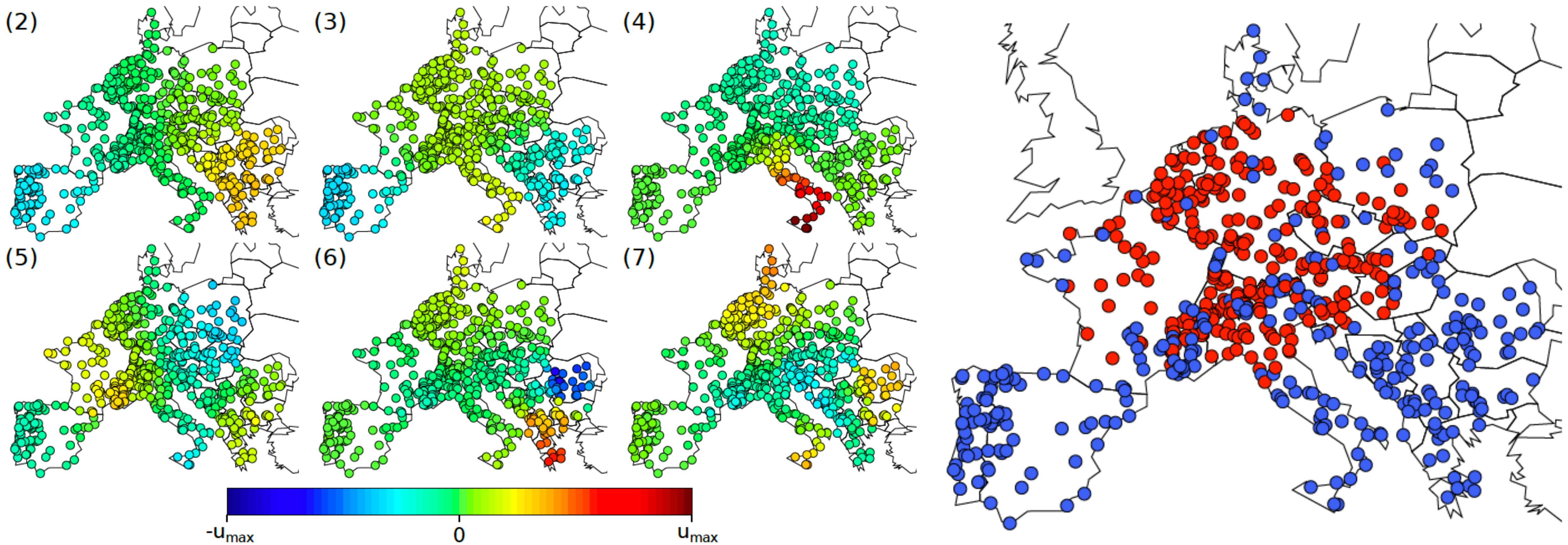

Conclusions - work in progress

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