Análisis Factorial En Diferentes Escalas

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1.- Lectura de la matriz de datos

```
x<-as.data.frame(state.x77)
```

2.- Quitar los espacios de los nombres

```
colnames(x)[4]="Life.Exp"
colnames(x)[6]= "HS.Grad"
```

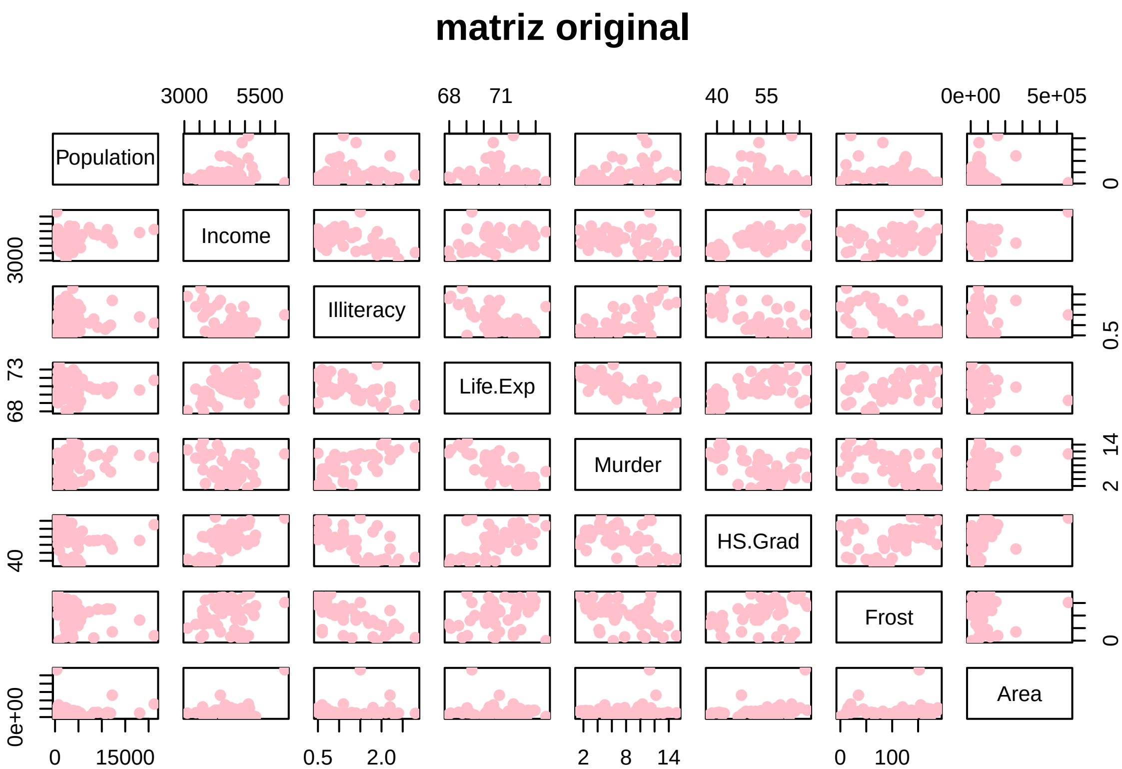
3.- Separa n (estados) y p (variables)

```
n<-dim(x)[1]
p<-dim(x)[2]
```

4.- Generacion de un scater plot

Visualización de variables originales

```
pairs(x, col="pink", pch=19, main="matriz original")
```



Transformación de alguna varibles

1.- Aplicamos logaritmo para las columnas 1,3 y 8

```
x[,1]<-log(x[,1])
colnames(x)[1]<-"Log-Population"
```

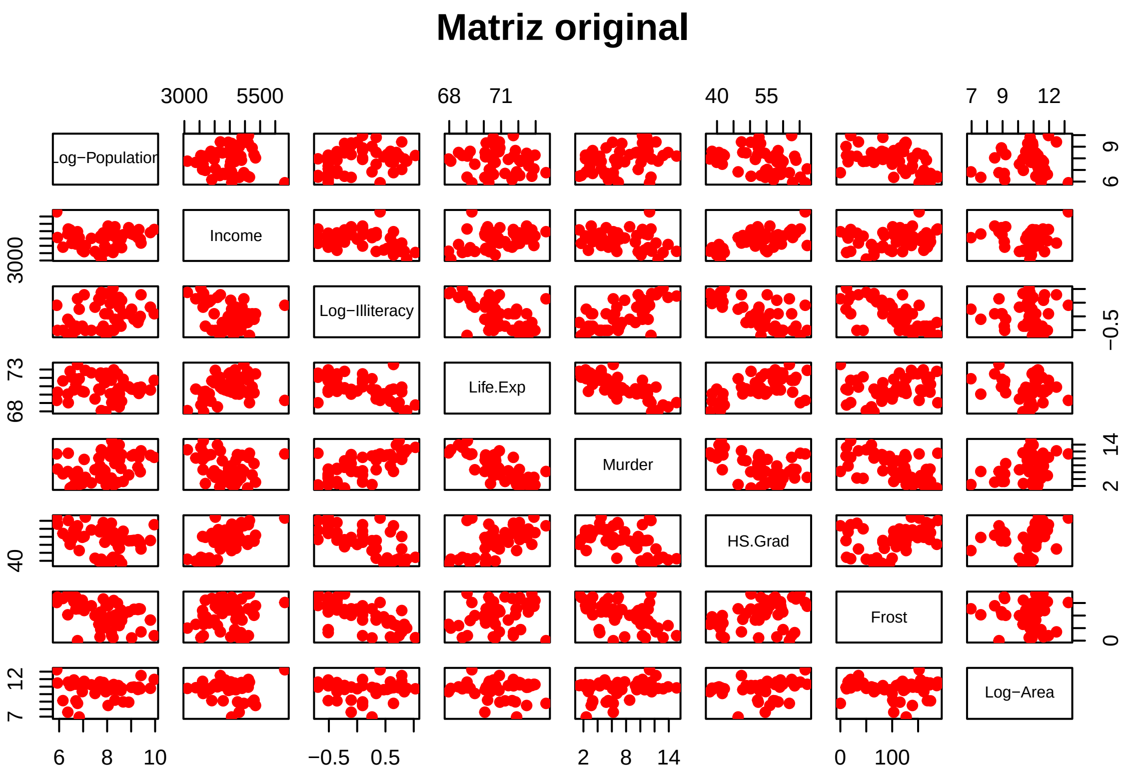
```
x[,3]<-log(x[,3])
colnames(x)[3]<-"Log-Illiteracy"
```

```
x[,8]<-log(x[,8])
colnames(x)[8]<-"Log-Area"
```

Grafico scater para la visualización de la matriz original

Con 3 variables que se incluyerón

```
pairs(x,col="red", pch=19, main="Matriz original")
```



Nota:

Como las variables tiene diferentes unidades de medida, se va a implementar la matriz de correlaciones para estimar la matriz de carga

Reduccion de la dimensionalidad

Análsis Factorial de componentes principales (PCFA)

1.- Calcular la matriz de medias y de correlaciones

Matriz de medias

```r
mu<-colMeans(x)
mu
```

```
## Log-Population         Income Log-Illiteracy       Life.Exp         Murder
##   7.863443e+00   4.435800e+03   3.128251e-02   7.087860e+01   7.378000e+00
##        HS.Grad          Frost       Log-Area
##   5.310800e+01   1.044600e+02   1.066237e+01
```

Matriz de correlaciones

```r
R<-cor(x)
R
```

```
##                Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population     1.00000000  0.034963788     0.28371749 -0.1092630  0.3596542
## Income             0.03496379  1.000000000    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy     0.28371749 -0.351477726     1.00000000 -0.5699943  0.6947320
## Life.Exp          -0.10926301  0.340255339    -0.56999432  1.0000000 -0.7808458
## Murder             0.35965424 -0.230077610     0.69473198 -0.7808458  1.0000000
## HS.Grad           -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost             -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area           0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                   HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         1.0000000  0.36677970  0.196743429
## Frost           0.3667797  1.00000000 -0.021211992
## Log-Area        0.1967434 -0.02121199  1.000000000
```

2.- Reducción de la dimensionalidad mediante

Análisis factorial de componentes principales (PCFA).

1.- Calcular los valores y vectores propios.

```r
eR<-eigen(R)
```

2.- Valores propios

```r
eigen.val<-eR$values
eigen.val
```

```
## [1] 3.6796976 1.3201021 1.1357357 0.7517550 0.6168266 0.2578511 0.1366186
## [8] 0.1014132
```

3.- Vectores propios

```
eigen.vec<-eR$vectors
eigen.vec
```

```
##             [,1]        [,2]        [,3]       [,4]        [,5]       [,6]
## [1,] -0.23393451 -0.41410075  0.50100922  0.2983839  0.58048485  0.0969034
## [2,]  0.27298977 -0.47608715  0.24689968 -0.6449631  0.09036625 -0.3002708
## [3,] -0.45555443  0.04116196  0.12258370 -0.1824471 -0.32684654 -0.6084112
## [4,]  0.39805075 -0.04655529  0.38842376  0.4191134 -0.26287696 -0.3565095
## [5,] -0.44229774 -0.27640285 -0.21639177 -0.2610739  0.02383706  0.1803894
## [6,]  0.41916283 -0.36311753 -0.06807465 -0.1363534 -0.34015424  0.3960855
## [7,]  0.36358674  0.21893783 -0.37542494 -0.1299519  0.59896253 -0.3507630
## [8,] -0.03545293 -0.58464797 -0.57421867  0.4270918 -0.06252285 -0.3012063
##            [,7]        [,8]
## [1,] -0.1777562 -0.23622413
## [2,]  0.3285840  0.12483849
## [3,] -0.3268997 -0.39825363
## [4,] -0.3013983  0.47519991
## [5,] -0.4562245  0.60970476
## [6,] -0.4808140 -0.40675672
## [7,] -0.4202943 -0.06001175
## [8,]  0.2162424 -0.05831177
```

4.- Calcular la proporcion de variabilidad

```
prop.var<-eigen.val/sum(eigen.val)
prop.var
```

```
## [1] 0.45996220 0.16501277 0.14196697 0.09396938 0.07710332 0.03223139 0.01707733
## [8] 0.01267665
```

5.- Calcular la proporcion de variabilidad acumulada

```
prop.var.acum<-cumsum(eigen.val)/sum(eigen.val)
prop.var.acum
```

```
## [1] 0.4599622 0.6249750 0.7669419 0.8609113 0.9380146 0.9702460 0.9873233
## [8] 1.0000000
```

Estimacion de la matriz de carga

Nota:

Se estima la matriz de carga usando los autovalores y autovectores.

Se aplica la rotación varimax

Primera estimación de Lamda mayuscula se calcula multiplicando la matriz de los 3 primeros autovectores por la matriz diagonal formada por la raiz cuadrada de los primeros 3 autovalores.

```
L.est.1<-eigen.vec[,1:3] %*% diag(sqrt(eigen.val[1:3]))
L.est.1
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.44874575 -0.47578394  0.53393005
## [2,]  0.52366367 -0.54700365  0.26312322
## [3,] -0.87386900  0.04729332  0.13063856
## [4,]  0.76356236 -0.05349003  0.41394671
## [5,] -0.84843932 -0.31757498 -0.23061066
## [6,]  0.80406070 -0.41720642 -0.07254777
## [7,]  0.69745163  0.25155014 -0.40009375
## [8,] -0.06800771 -0.67173536 -0.61195003
```

Rotación varimax

```
L.est.1.var<-varimax(L.est.1)
L.est.1.var
```

```
## $loadings
##
## Loadings:
##      [,1]   [,2]   [,3]
## [1,]                0.840
## [2,]  0.785 -0.106  0.121
## [3,] -0.665         0.583
## [4,]  0.763  0.384 -0.168
## [5,] -0.573 -0.528  0.517
## [6,]  0.825 -0.202 -0.323
## [7,]  0.281        -0.794
## [8,]        -0.906
##
##                 [,1]  [,2]  [,3]
## SS loadings    2.744 1.300 2.091
## Proportion Var 0.343 0.163 0.261
## Cumulative Var 0.343 0.506 0.767
##
## $rotmat
##            [,1]      [,2]       [,3]
## [1,]  0.7824398 0.1724744 -0.5983649
## [2,] -0.5274231 0.6944049 -0.4895169
## [3,]  0.3310784 0.6986089  0.6342970
```

Estimación de la matriz de los errores

1.- Estimación de la matriz de perturbaciones

```
Psi.est.1<-diag(diag(R-as.matrix(L.est.1.var$loadings)%*% t(as.matrix(L.est.1.var$loadings))))
Psi.est.1
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]      [,7]
## [1,] 0.2871756 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [2,] 0.0000000 0.3573295 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2170499 0.0000000 0.0000000 0.000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.2427595 0.0000000 0.000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1261156 0.000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.174162 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.2902087
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.1696637
```

2.- Se utiliza el método Análisis de factor principal (PFA)

Para estimación de autovalores y autovectores

```
RP<-R-Psi.est.1
RP
```

```
##                Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population     0.71282441  0.034963788     0.28371749 -0.1092630  0.3596542
## Income             0.03496379  0.642670461    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy     0.28371749 -0.351477726     0.78295012 -0.5699943  0.6947320
## Life.Exp          -0.10926301  0.340255339    -0.56999432  0.7572405 -0.7808458
## Murder             0.35965424 -0.230077610     0.69473198 -0.7808458  0.8738844
## HS.Grad           -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost             -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area           0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                   HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         0.8258380  0.36677970  0.196743429
## Frost           0.3667797  0.70979126 -0.021211992
## Log-Area        0.1967434 -0.02121199  0.830336270
```

Calculo de la matriz de autovalores y autovectores

```
eRP<-eigen(RP)
```

Autovalores

```
eigen.val.RP<-eRP$values
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

Autovectores

```
eigen.vec.RP<-eRP$vectors
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

Proporcion de variabilidad

```
prop.var.RP<-eigen.val.RP/ sum(eigen.val.RP)
prop.var.RP
```

```
## [1]  0.564152306  0.180134556  0.143675179  0.079382934  0.057632455
## [6]  0.004585668 -0.011014811 -0.018548286
```

Proporcion de variabilidad acumulada

```
prop.var.RP.acum<-cumsum(eigen.val.RP)/ sum(eigen.val.RP)
prop.var.RP.acum
```

```
## [1] 0.5641523 0.7442869 0.8879620 0.9673450 1.0249774 1.0295631 1.0185483
## [8] 1.0000000
```

Estimación de la matriz de cargas con rotación varimax

```
L.est.2<-eigen.vec.RP[,1:3] %*% diag(sqrt(eigen.val.RP[1:3]))
L.est.2
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.42621819 -0.27609775  0.56228420
## [2,]  0.48528446 -0.36092954  0.32467098
## [3,] -0.84791581  0.08163995  0.10816670
## [4,]  0.73812189  0.02688907  0.36866093
## [5,] -0.84699944 -0.34227865 -0.12211117
## [6,]  0.78817342 -0.40399024  0.04935203
## [7,]  0.66112453  0.12457105 -0.40191996
## [8,] -0.06868291 -0.77165602 -0.36531090
```

Rotacion varimax

```r
L.est.2.var<-varimax(L.est.2)
```

Estimación de la matriz de covarianzas de los errores.

```r
Psi.est.2<-diag(diag(R-as.matrix(L.est.2.var$loadings)%*% t(as.matrix(L.est.2.var$loadings))))
Psi.est.2
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4259446 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [2,] 0.0000000 0.5288176 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2626737 0.0000000 0.0000000 0.0000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.3185422 0.0000000 0.0000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1505261 0.0000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.2131389 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.3858568
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.2663776
```

Obtencion de los scores de ambos métodos

PCFA

```
FS.est.1<-scale(x)%*% as.matrix(L.est.1.var$loadings)
FS.est.1
```

```
##                       [,1]          [,2]       [,3]
## Alabama        -5.84072356 -1.3993671511  4.0008109
## Alaska          2.12443806 -3.6163397014 -1.3435941
## Arizona        -0.77245459 -1.1030150088  1.7864181
## Arkansas       -4.26961555 -0.1287634469  1.8680205
## California      1.57843978 -1.6386262821  3.0959757
## Colorado        3.35619481 -0.5747409714 -1.9955520
## Connecticut     2.96609993  2.5265114588 -1.0120520
## Delaware        0.15111765  2.2707877284 -1.3473631
## Florida        -0.91278118 -0.8518787165  3.2141818
## Georgia        -5.10406769 -1.5374188978  3.5972606
## Hawaii          1.68679592  2.0782245763  0.6972161
## Idaho           1.93931571  0.0374520725 -2.6403015
## Illinois        0.36572803 -0.9730363911  1.3246992
## Indiana         0.69870165  0.1740586327 -0.1660034
## Iowa            3.77325852  0.8634090197 -2.4308546
## Kansas          3.22079390  0.2206198504 -1.7333568
## Kentucky       -3.97957229 -0.1711842990  1.8581455
## Louisiana      -6.15095874 -1.1449716511  4.2193388
## Maine           0.38912287  0.9352663421 -2.8385772
## Maryland        0.54556931  0.6481615589  0.7313943
## Massachusetts   1.95531363  1.9508870989 -0.0699601
## Michigan        0.06109118 -0.8995742724  1.1610156
## Minnesota       3.83625590  0.7199310360 -2.2609012
## Mississippi    -6.73875213 -1.1336057288  3.0124928
## Missouri       -0.63621057 -0.5673516660  0.5606479
## Montana         1.70022911 -0.7530855537 -2.9827203
## Nebraska        3.31393569  0.5702899251 -2.6630094
## Nevada          1.83953234 -2.1624547546 -2.8632403
## New Hampshire   1.76672303  1.8835104424 -3.2522623
## New Jersey      1.23076573  1.5154423999  0.6483326
## New Mexico     -2.42369795 -1.2184859435  0.1095350
## New York       -0.55160991 -0.8431042602  2.9025469
## North Carolina -4.53932589 -0.7126552652  2.8168209
## North Dakota    3.26810535  1.0664889529 -3.5180166
## Ohio            0.67643704 -0.0394642439  0.5816740
## Oklahoma       -0.43628926  0.0293430043  0.2108486
## Oregon          2.64633236 -0.0126633017 -0.6563722
## Pennsylvania   -0.06313819  0.0425262164  0.8538298
## Rhode Island    0.25059508  4.0533333045 -1.3779994
## South Carolina -6.20030464 -0.7067780563  3.0142562
## South Dakota    2.51505516  0.8539599931 -3.9694575
## Tennessee      -3.75602365 -0.3764569265  2.4225536
## Texas          -2.74825842 -2.0176142597  4.0126966
## Utah            3.40911641  0.2638533973 -3.0642167
## Vermont         1.26368503  1.7670538099 -3.5748058
## Virginia       -1.45435214 -0.4332714574  1.8388594
```

```
## Washington      2.95298764  0.0002978623 -0.1436737
## West Virginia  -3.41599674  0.5649932020  0.5132111
## Wisconsin       2.58972274  0.8701285803 -1.5397225
## Wyoming         1.92267355 -0.8906222579 -3.6087703
```

PFA

```
FS.est.2<-scale(x)%*% as.matrix (L.est.2.var$loadings)
FS.est.2
```

```
##                       [,1]         [,2]       [,3]
## Alabama        -5.69766092 -1.133005866  3.9030908
## Alaska          1.77921500 -3.310049553 -1.2425530
## Arizona        -0.80948635 -1.007423566  1.6833688
## Arkansas       -4.04451164 -0.036340306  1.8899610
## California      1.28900772 -1.589528660  2.7938220
## Colorado        3.21256763 -0.645092519 -1.9103448
## Connecticut     2.85639977  2.291700954 -1.1152442
## Delaware        0.22491218  2.168332191 -1.3109174
## Florida        -1.04778981 -0.760012075  2.9630979
## Georgia        -5.04193484 -1.243399542  3.4848855
## Hawaii          1.64548810  1.848120424  0.5487863
## Idaho           1.99602286 -0.067186945 -2.4442739
## Illinois        0.17329771 -0.870927790  1.1838509
## Indiana         0.66348403  0.140717116 -0.1900850
## Iowa            3.70915552  0.657976435 -2.3698485
## Kansas          3.13617617  0.071725764 -1.6894853
## Kentucky       -3.82119443 -0.051170443  1.8492550
## Louisiana      -5.97309240 -0.880509145  4.1021292
## Maine           0.58567717  0.845398887 -2.6098620
## Maryland        0.40855637  0.650876372  0.5867974
## Massachusetts   1.91021424  1.761365924 -0.1964750
## Michigan       -0.07208772 -0.823049544  1.0671998
## Minnesota       3.74953682  0.518054623 -2.2104937
## Mississippi    -6.45121865 -0.852611917  3.0320154
## Missouri       -0.64446964 -0.519762510  0.5472506
## Montana         1.72574501 -0.752576236 -2.7507980
## Nebraska        3.28773039  0.392513546 -2.5439122
## Nevada          1.69672312 -1.994626548 -2.6292009
## New Hampshire   1.87991014  1.704867403 -3.0632652
## New Jersey      1.10782292  1.425042094  0.4638907
## New Mexico     -2.26112419 -1.086582245  0.2653217
## New York       -0.72255151 -0.744949928  2.6624378
## North Carolina -4.42441540 -0.513264749  2.7372284
## North Dakota    3.22068093  0.897031063 -3.3556310
## Ohio            0.59453054 -0.051780182  0.4905274
## Oklahoma       -0.36512462  0.000708499  0.2244101
## Oregon          2.56050584 -0.129810062 -0.6934180
## Pennsylvania   -0.10451900  0.054229408  0.7553645
## Rhode Island    0.40356926  3.785456289 -1.3760426
## South Carolina -5.98815271 -0.435831413  2.9745853
## South Dakota    2.60764548  0.683975660 -3.7117087
## Tennessee      -3.63769564 -0.249263663  2.3593673
```

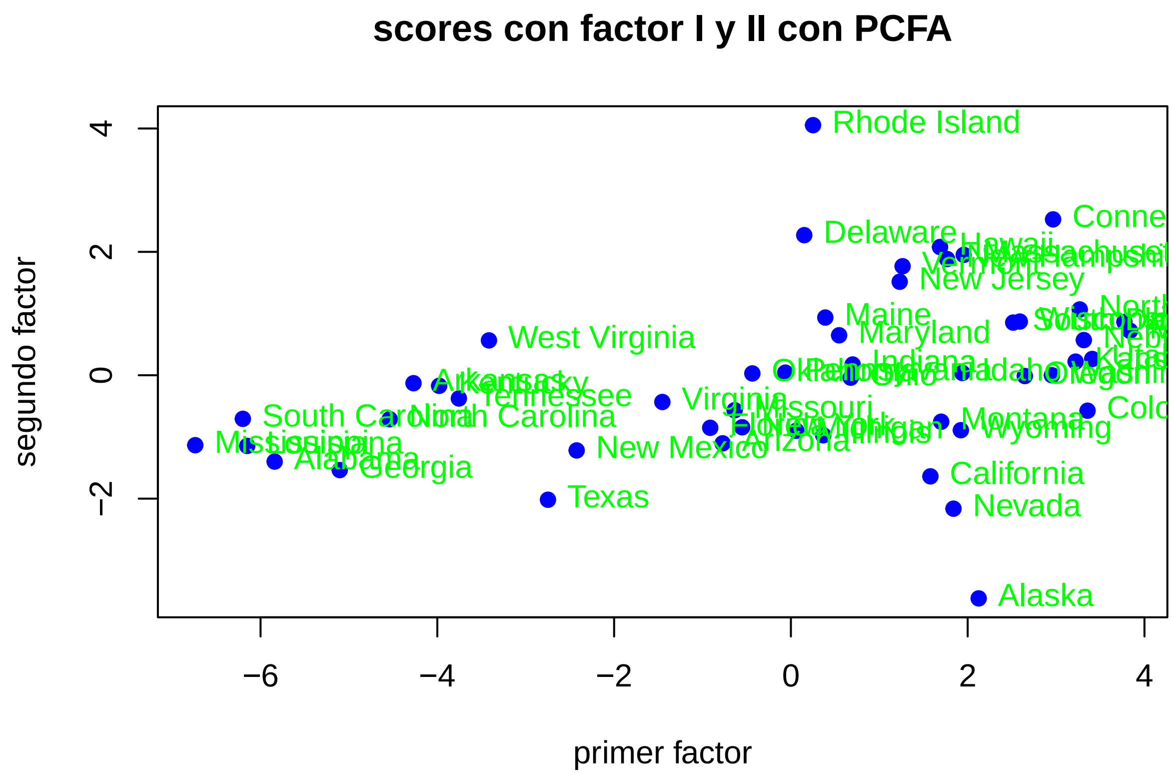
```
## Texas          -2.80670233 -1.827474308  3.8156526
## Utah            3.44131011  0.069209103 -2.8669774
## Vermont         1.44160727  1.580578146 -3.3086066
## Virginia       -1.50774364 -0.328200587  1.7151967
## Washington      2.81601549 -0.109025242 -0.2503494
## West Virginia  -3.18525955  0.632647668  0.5745805
## Wisconsin       2.55487697  0.699000994 -1.5141208
## Wyoming         1.92835024 -0.866073018 -3.3204601
```

Graficamos ambos scores

```
par(mfrow=c(2,1))
```

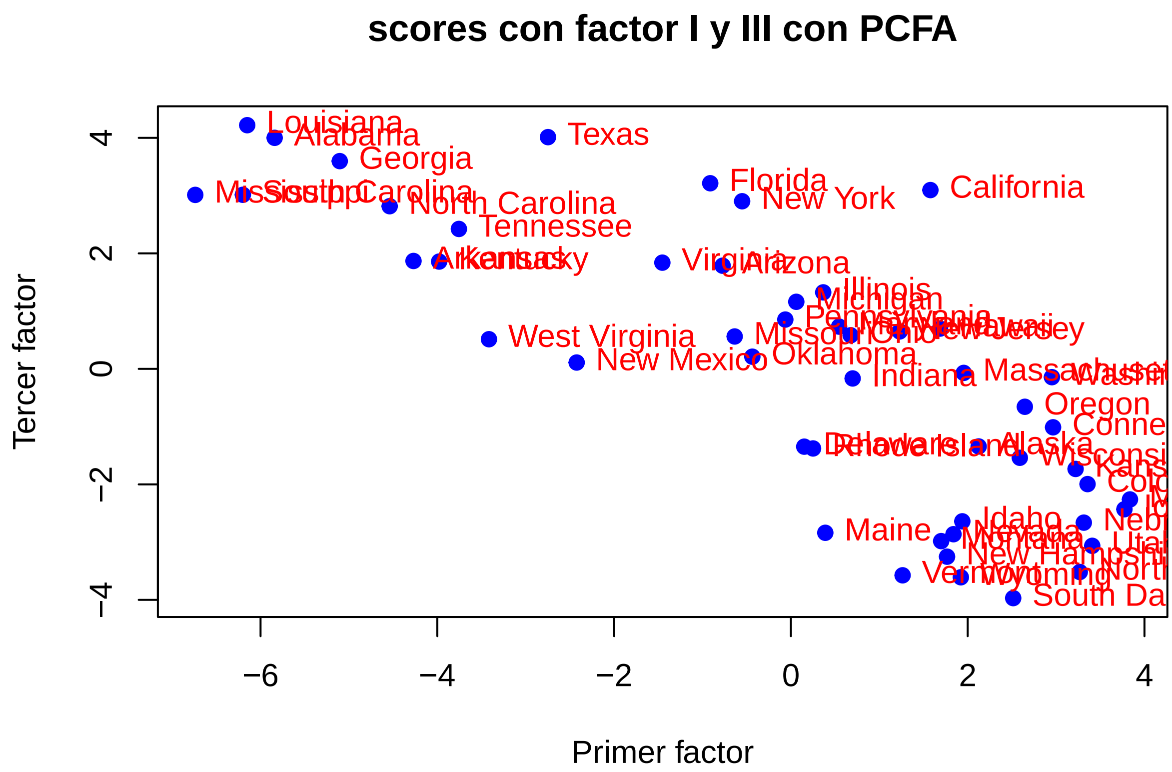
Factor I y II

```
pl1<-plot(FS.est.1[,1], FS.est.1[,2], xlab="primer factor",
          ylab="segundo factor", main="scores con factor I y II con PCFA",
          pch=19, col="blue")
text(FS.est.1[,1], FS.est.1[,2], labels = rownames(x), pos=4, col="green")
```



Factor I y III

```
pl2<-plot(FS.est.1[,1], FS.est.1[,3], xlab="Primer factor",
          ylab="Tercer factor", main="scores con factor I y III con PCFA",
          pch=19, col="blue")
text(FS.est.1[,1], FS.est.1[,3], labels = rownames(x), pos=4, col="red")
```



Factor II y III

```
pl3<-plot(FS.est.1[,2], FS.est.1[,3], xlab="Segundo factor",
          ylab="Tercer factor", main="scores con factor II y III con PCFA",
          pch=19, col="blue")
text(FS.est.1[,2], FS.est.1[,3], labels = rownames(x), pos=4, col="black")
```