

1.a) Given the differential equation that models the behavior of the motorcycle suspension system,

$$m x'' + b x' + k x = 0$$

$$\therefore 12 x'' + 240 x' + 1152 x = 0$$

$$\therefore x'' + 20 x' + 96 x = 0$$

is the differential equation.

Given,

Weight of motorcycle with rider,
 $mg = (204 + 180)$

$$= 384 = mg \quad [m = \text{mass}]$$

Gravitational acceleration,

$$g = 32 \text{ ft s}^{-2}$$

$$\therefore \text{mass, } m = (384 / 32) \\ = 12$$

$$\text{Displacement, } s = 4/12 \quad [\text{converting inch to ft}]$$

$$\therefore \text{spring force, } k = \frac{mg}{s} \\ = \frac{384}{4/12} \\ = 1152$$

$$\text{damping} = b x' = 240 x' \quad [b = \text{damping force coefficient}]$$

$$\therefore b = 240$$

1. b) Finding the equation of motion, $x(t)$, of the motorcycle after jump,

Gotter DE from '1. a)'

$$x'' + 20x' + 96x = 0$$

which is a 2nd order, linear, homogenous with constant coefficient differential equation.

let, the solution, $x = e^{mt}$

$$\therefore m^2 + 20m + 96 = 0$$

$$\therefore m = -8, -12$$

\therefore The general solution is, $x(t) = C_1 e^{-8t} + C_2 e^{-12t}$

Given the initial conditions,

$$x(0) = \frac{4}{12} = \frac{1}{3}$$

$$x'(0) = 10$$

Now,

$$x'(t) = -8C_1 e^{-8t} - 12C_2 e^{-12t}$$

$$x'(0) = -8C_1 - 12C_2 = 10$$

$$\text{and, } \Rightarrow -4C_1 - 6C_2 = 5 \quad \dots \dots \textcircled{1}$$

$$x(0) =$$

and,

$$x(0) = C_1 + C_2 = \frac{1}{3}$$

$$\Rightarrow 3C_1 + 3C_2 = 1 \quad \dots \dots \textcircled{II}$$

solving \textcircled{I} & \textcircled{II} ,

$$C_1 = \frac{7}{2}$$

$$C_2 = -\frac{19}{6}$$

$$\therefore \text{Equation of motion, } x(t) = \frac{7}{2} e^{-8t} - \frac{19}{6} e^{-12t}$$

c) Gotler equation of motion from "1.b)",

$$x(t) = \frac{7}{2} e^{-8t} - \frac{19}{6} e^{-12t}$$

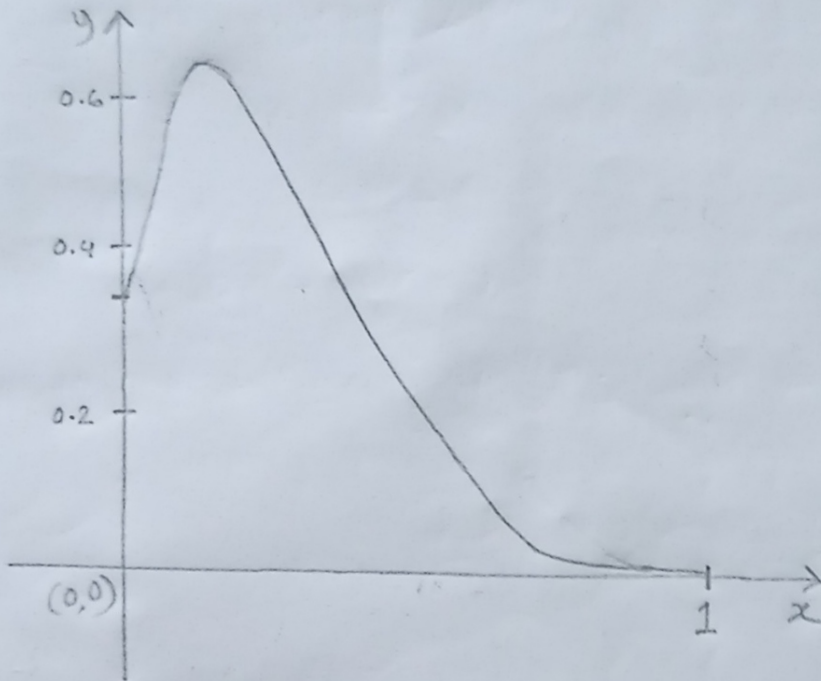


Figure1. : Graph.

table

x	$x(t)$
0	0.33333
0.1	0.61886
0.2	0.41936
0.3	0.23098
0.4	0.11660
0.5	0.05625
0.6	0.02643
0.7	0.01223
0.8	0.00560
0.9	0.00254
1	0.00115

Figure2 : Table