Course Code: EEE 4101
Course Title: Electrical Engineering

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Ref. Book: Fundamental of Electrical Circuit: Alexander Sadiku

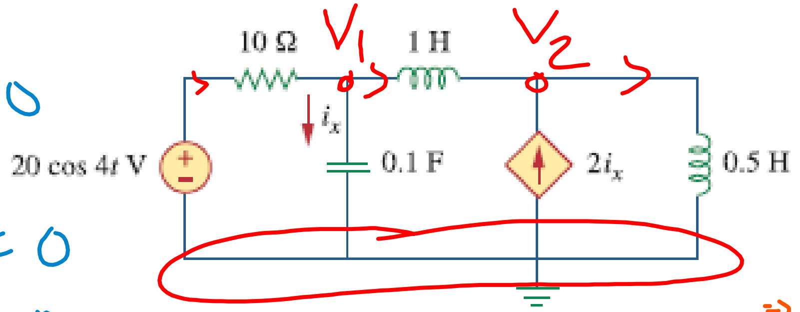
AC Circuit Analysis

Steps to Analyze AC Circuits:

1. Transform the circuit into phasor or frequency domain
2. Solve the problem using nodal, mesh, superposition, etc.
3. Transform the resulting circuit into time domain

Nodal Analysis

Q. Find I_x in following circuit using nodal analysis.



$$\frac{20 - V_1}{10} - \frac{V_1 - V_2}{j4} - \frac{V_1}{\frac{1}{j0.4}} = 0$$

$$16V_1 + j40 - j4V_1 + 10V_2 = 0 \quad — (1)$$

$$j0.8V_1 + \frac{V_1 - V_2}{j4} - \frac{V_2}{j2} = 0$$

$$\Rightarrow -j6.4V_1 + 2V_1 j - j\, V_2 = 0$$

$$\begin{bmatrix} 16 - j4 & -j6.4 + 2 \\ & \end{bmatrix}$$

$$(1 + j15)V_1 + j25V_2 = 20$$

$$j0.55V_1 + j0.75V_2 = 0$$

$$\begin{bmatrix} 1 + j15 & j2.5 \\ j0.55 & j0.75 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta} = \boxed{}\ V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \boxed{}\ V$$

$$\Delta = \begin{vmatrix} 1 + j15 & j25 \\ j0.55 & j0.75 \end{vmatrix} = j0.75 - 1.125 - (-1.375)$$

$$= j0.75 + 0.250$$

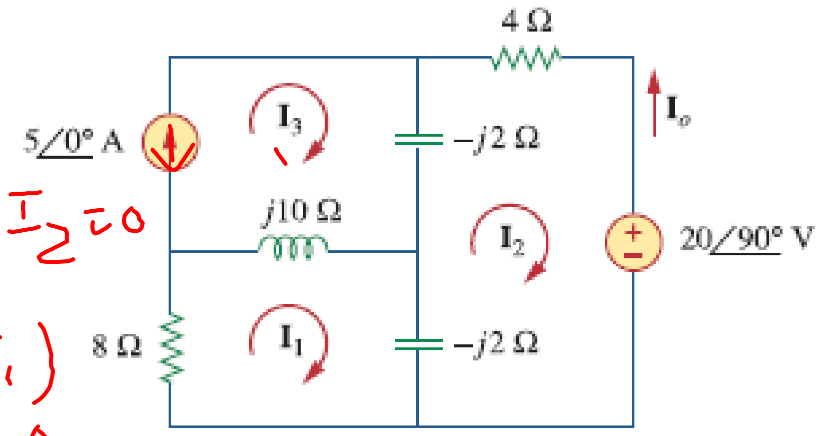
$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & j0.75 \end{vmatrix}$$

$V_1 =$

Mesh Analysis

Q. Find I_o in following circuit using mesh analysis.

$I_o = -I_2$, $I_3 = 5\,A$



$-(8 + j8)I_1 - j10I_3 + j2I_2 = 0$

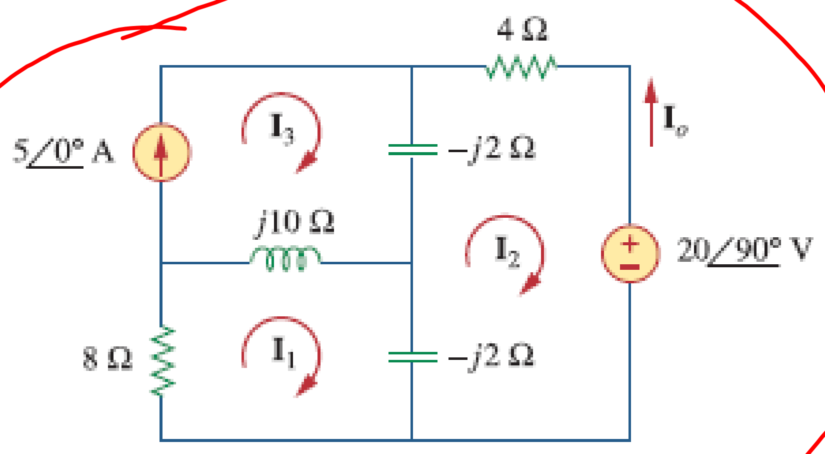
$+20\angle 90° + (4 - j4)I_2 - (-j2I_1) - (-j2I_3) = 0$

$(I_1 - I_3) - j2(I_1 - I_2) = 0$

$1 - j10I_3 - j2I_1 + j2I_2 = 0$

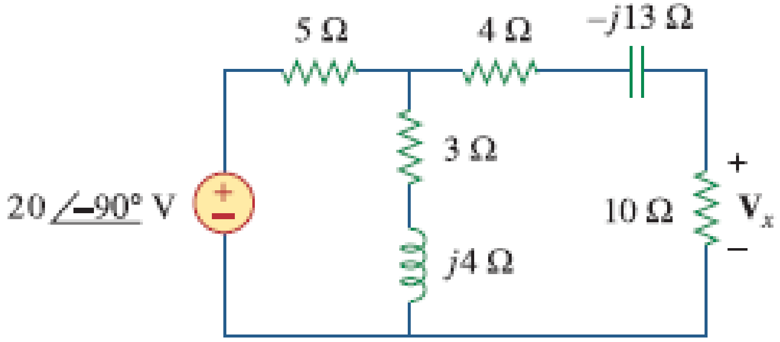
Superposition Theorem

Q. Use superposition theorem to find I_o in following circuit.



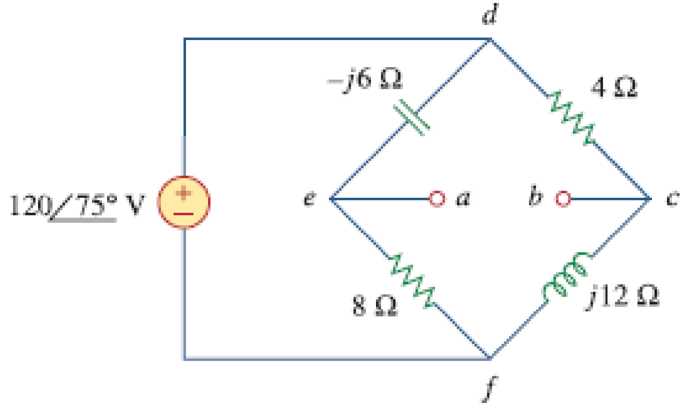
Source Transformation

Q. Calculate v_x in following circuit using source transformation technique.



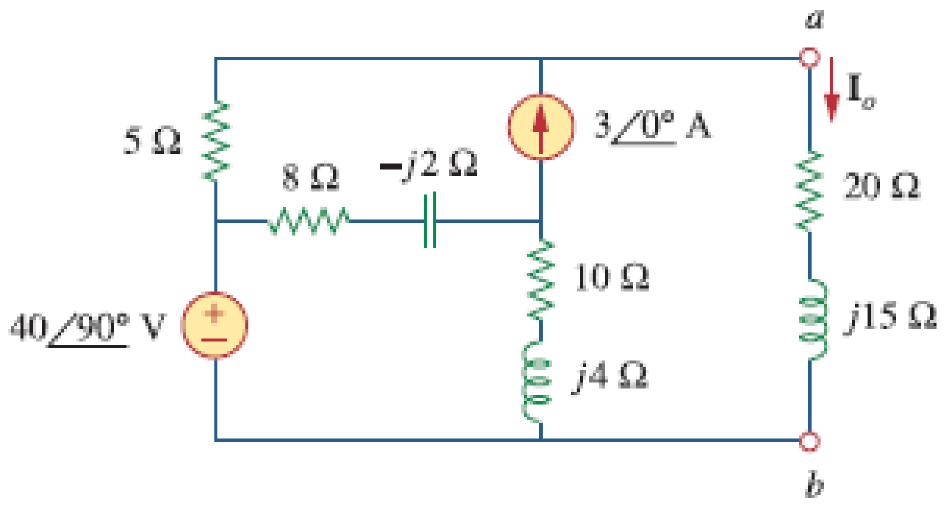
Thevenin’s Theorem

Q. Obtain the Thevenin’s equivalent at terminal a-b of the following circuit.



Norton’s Theorem

Q. Obtain current i_o in the following figure using Norton's theorem.



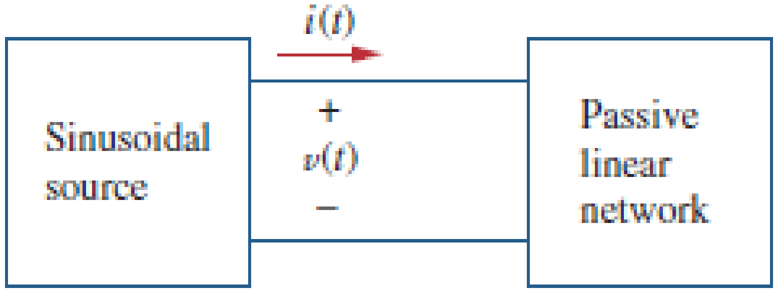
AC Power Analysis

- Power analysis is of paramount importance.
- Power is the most important quantity in electric utilities, electronic, and communication systems, because such systems involve transmission of power from one point to another.
- Also, every industrial and household electrical device—every fan, motor, lamp, pressing iron, TV, personal computer— has a power rating that indicates how much power the equipment requires; exceeding the power rating can do permanent damage to an appliance.
- The most common form of electric power is 50- or 60-Hz ac power..

Instantaneous Power

The instantaneous power (in watts) is the power at any instant of time.

1. $p(t) = v(t) * i(t)$



Let the voltage and current at the terminals of the circuit be

$v(t) = V_m Cos(\omega t + \Theta_v)$ and $i(t) = I_m Cos(\omega t + \Theta_i)$

Where V_m and I_m are the amplitudes (or peak values), Θ_v and Θ_i are the phase angles of the voltage and current, respectively. The instantaneous power absorbed by the circuit is

2. $p(t) = V_m\, I_m Cos(\omega t + \Theta_v) Cos(\omega t + \Theta_i)$

3. $$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$\left[\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos A-B\right)\right)$

This shows us that the instantaneous power has two parts. The first part is constant or time independent. Its value depends on the phase difference between the voltage and the current. The second part is a sinusoidal function whose frequency is which is twice the angular frequency of the voltage or current

Average Power

- The instantaneous power changes with time and is therefore difficult to measure.
- The *average* power is more convenient to measure.
- In fact, the wattmeter, the instrument for measuring power, responds to average power.

The average power, in watts, is the average of the instantaneous power over one period.

Thus, the average power is given by

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$\begin{aligned} P &= \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt \\ &\quad + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt \\ &= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt \\ &\quad + \frac{1}{2}V_m I_m \underbrace{\frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt}_{} = 0 \end{aligned}$$

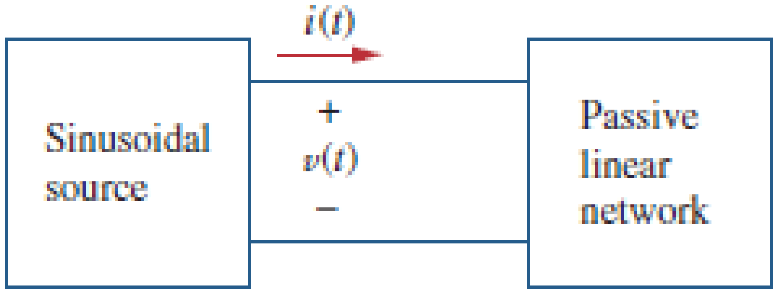
The first integrand is constant, and the average of a constant is the same constant. The second integrand is a sinusoid. We know that the average of a sinusoid over its period is zero because the area under the sinusoid during a positive half-cycle is canceled by the area under it during the following negative half-cycle.

Average Power (Contd.)

Thus, the second term vanishes and the average power becomes

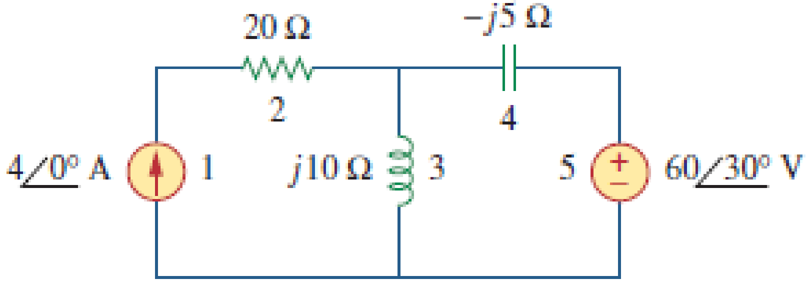
$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

Q. Given that, $v(t)$ = 120 cos($377t\ \ 45^o$) V and $i(t)$= 10 cos($377t\ 10^o$) A find the instantaneous power and the average power absorbed by the passive linear network of given Fig.



Average Power (Contd.)

Q. Determine the average power generated by each source and the average power absorbed by each passive element in the circuit of Fig shown bel~~·



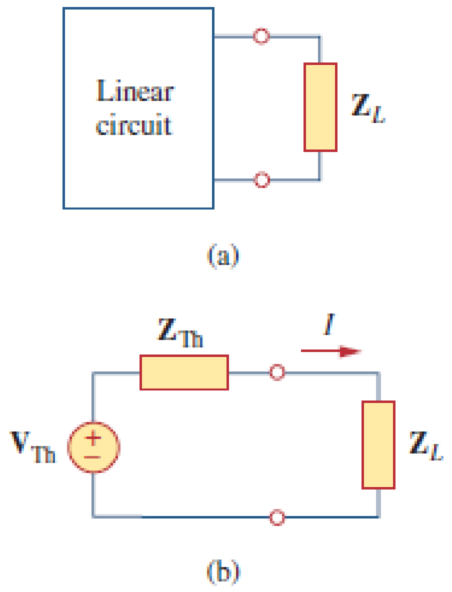
Maximum Average Power Transfer

For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{Th}$.

Consider the circuit, where an ac circuit is connected to a load, $\mathbf{Z}_L$ and is represented by its Thevenin equivalent. The load is usually represented by an impedance, which may model an electric motor, an antenna, a TV, and so forth. In rectangular form, the Thevenin impedance $\mathbf{Z}_{Th}$ and the load impedance $\mathbf{Z}_L$ are

$$Z_{th} = R_{th} + jX_{th}$$

$$Z_L = R_L + jX_L$$



$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \frac{\mathbf{V}_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)}$$

the average power delivered to the load is

$$P = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{Th}|^2 R_L/2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

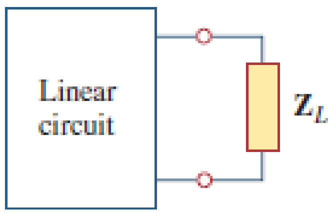
Maximum Average Power Transfer (Contd.)

Our objective is to adjust the load parameters R_L and X_L so that P is maximum. To do this we set

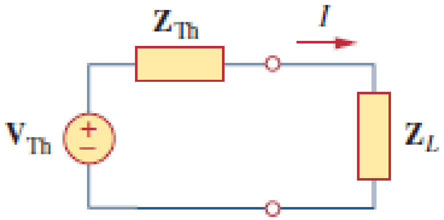
0

$$\frac{\partial P}{\partial X_L} = -\frac{|V_{th}|^2 R_L (X_{th} + X_L)}{[(R_{th} + R_L)^2 + (X_{th} + X_L)^2]^2}$$

$$\frac{\partial P}{\partial R_L} = -\frac{|V_{th}|^2 [R_L (R_{th} + R_L)^2 + (X_{th} + X_L)^2 - 2R_L (R_{th} + R_L)]}{2[(R_{th} + R_L)^2 + (X_{th} + X_L)^2]^2}$$



(a)



(b)

Setting $\frac{\partial P}{\partial X_L}$ to zero gives $X_L = -X_{th}$

$\frac{dP}{dR_L}$

and setting ~~and setting to zero results in~~ to zero results in

$$R_L = \sqrt{R_{th}^{\,2} + (X_{th} + X_L)^2}$$

$$=> R_L = R_{th}$$

$$\mathbf{Z}_L = R_L + jX_L = R_{Th} - jX_{Th} = \mathbf{Z}_{Th}^*$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}}$$

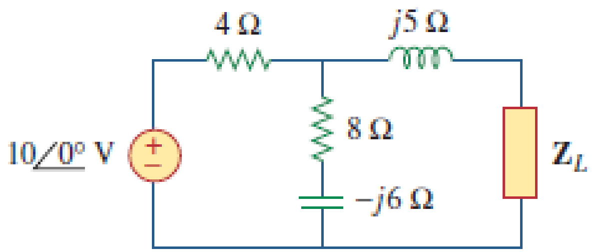
Maximum Average Power Transfer (Contd.)

In a situation in which the load is purely real, the condition for maximum power transfer is obtained from Eq. (11.18) by setting $X_L = 0$; that is,

$$\boxed{R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |\mathbf{Z}_{Th}|}$$

This means that for maximum average power transfer to a purely resistive load, the load impedance (or resistance) is equal to the magnitude of the Thevenin impedance.

Q. Determine the load impedance Z_L that maximizes the average power drawn from the circuit of Fig. 11.8. What is the maximum average power?



Effective or RMS value

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

In given Fig., the circuit in (a) is ac while that of (b) is dc. Our objective is to find I_{eff} that will transfer the same power to resistor R as the sinusoid i. The average power absorbed by the resistor in the ac circuit is

$$P = \frac{1}{T}\int_0^T i^2 R\, dt = \frac{R}{T}\int_0^T i^2\, dt$$

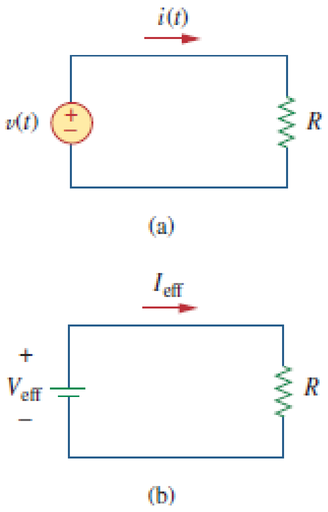
while the power absorbed by the resistor in the dc circuit is

$$P = I_{\text{eff}}^2 R$$

Equating the expressions above and solving for, we obtain

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

$$V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt}$$



Finding the effective current: (a) ac circuit, (b) dc circuit.

Effective or RMS value (Contd.)

This indicates that the effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal. Thus, the effective value is often known as the *root-mean-square* value, or *rms* value for short; and we write

$$I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}}$$

For the sinusoid $i(t) = I_m Cos(\omega t)$the effective or rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \, dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \boxed{\frac{I_m}{\sqrt{2}}}$$

Similarly, for $\text{v}(t) = V_m Cos(\omega t)$

$$\boxed{V_{\text{rms}} = \frac{V_m}{\sqrt{2}}}$$

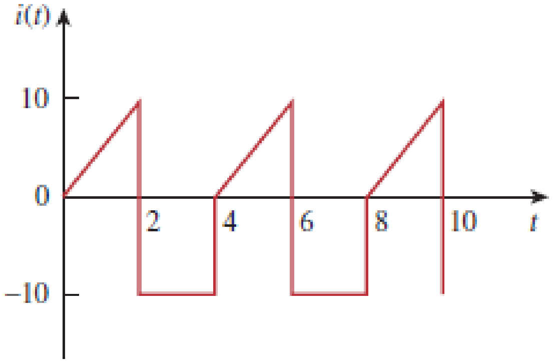
The average power

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

Effective or RMS value (Contd.)

Q. Determine the rms value of the current waveform in Fig. given. If the current is passed through a 2-ohm resistor, find the average power absorbed by the resistor.



The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165\text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3\text{ W}$$

Thank You