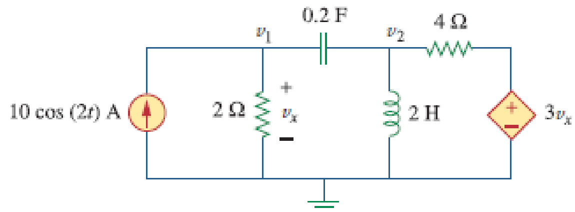
Question 1

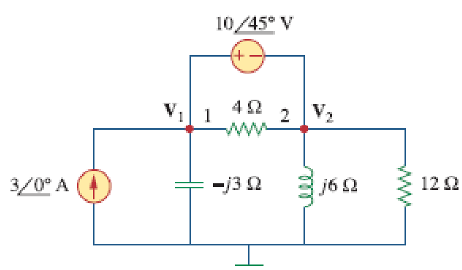
Using Nodal analysis find v_1 and v_2 in the given circuit.



Ans: $v_1(t) = 11.325\,cos(2t + 60.01^o)V\ and\ v_2(t) = 33.02\,cos(2t + 57.12^o)$ V

Question 2

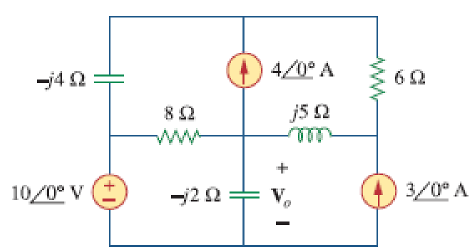
Compute v_1 and v_2 in the given circuit.



Ans: $v_1 = 25.78\angle{-70.48^o}\ V\ and\ v_2 = 31.41\angle{-87.18^o}$ V

Question 3

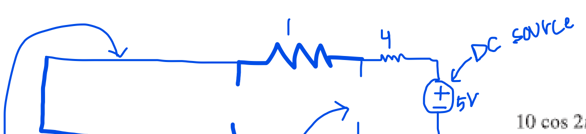
Solve for v_o in the circuit given below using mesh analysis.

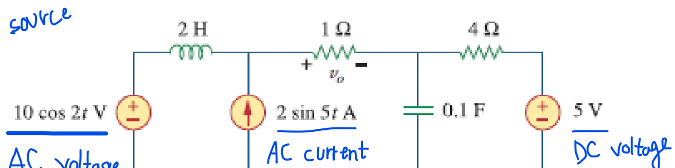


Ans: $v_o = 9.756\angle 222.32^o$ V

Question 4

Find v_o in the circuit given below using superposition.





AC voltage source

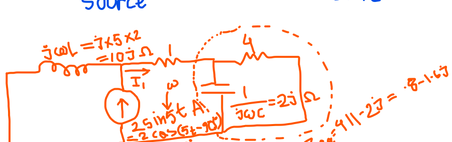
AC current source

DC voltage source

Under DC condition, Capacitors are replaced with open circuit

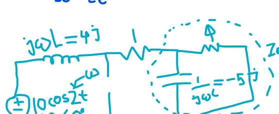
Inductors are replaced with short circuit

$V_{o_1} = 5\times\frac{1}{1+4} = 1$ V [voltage divider rule]



$Z_{eq} = 4||-2j = 0.8-1.6j$

$V_{o_2} = I_1\times1 = I\times\frac{10j}{10j+Z_{eq}+1}\times1$ [Current div. rule] $= 2.328\angle 72.9°$



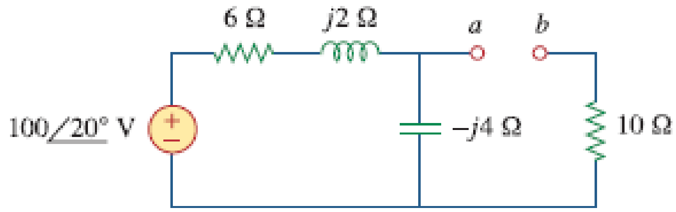
$Z_{eq} = -5j||4 = 2.4-1.8j$

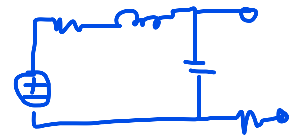
$V_{o_3} = V\times\frac{1}{4j+1+2.4-1.8j}$

Question 5

Obtain the Thevenin's equivalent at terminal a-b of the following circuit.

$Z_{th} = \left((6+j2) \| -j4\right) + 10$

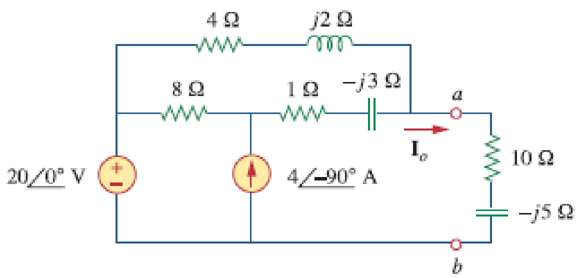




Ans: $\mathbf{Z}_{Th} = 12.4 - j3.2\ \Omega$, $\mathbf{V}_{Th} = 63.24\angle -51.57°$ V.

Question 6

Determine the Norton's equivalent of the circuit in the given figure as seen from the terminal a and b. Use equivalent to find I_o.



Ans: $\mathbf{Z}_N = 3.176 + j0.706\ \Omega$, $\mathbf{I}_N = 8.396\angle -32.68°$ A,

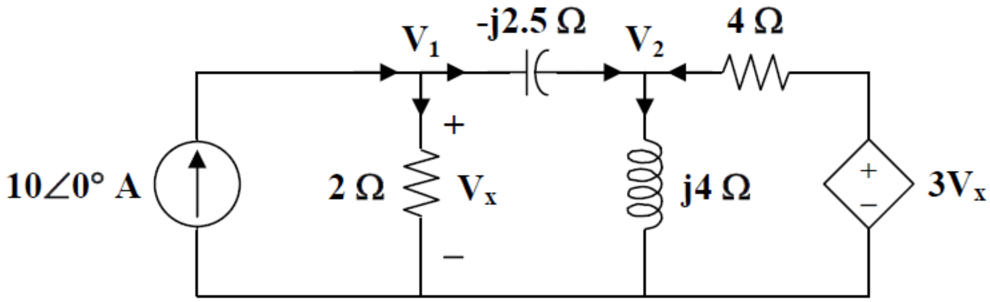
$\mathbf{I}_o = 1.9714\angle -2.10°$ A.

1.

$$10\sin(2t) \longrightarrow 10\angle 0°, \quad \omega = 2$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = -j2.5$$



At node 1, $10 = \dfrac{\mathbf{V}_1}{2} + \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{\text{- j2.5}}$

At node 2, $\dfrac{\mathbf{V}_2}{\text{j4}} = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{\text{- j2.5}} + \dfrac{3\mathbf{V}_x - \mathbf{V}_2}{4}$ where $\mathbf{V}_x = \mathbf{V}_1$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 5 + \text{j}4 & \text{- j}4 \\ \text{-}(7.5 + \text{j}4) & 2.5 + \text{j}1.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

where $\Delta = (5 + \text{j}4)(2.5 + \text{j}.15) - (\text{-j}4)(\text{-}(7.5 + \text{j}4)) = 22.5 - \text{j}12.5 = 25.74\angle\text{-}29.05°$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \frac{\begin{bmatrix} 2.5 + \text{j}1.5 & \text{j}4 \\ 7.5 + \text{j}4 & 5 + \text{j}4 \end{bmatrix}}{22.5 - \text{j}12.5} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = \frac{2.5 + \text{j}1.5}{22.5 - \text{j}12.5}(100) = \frac{2.915\angle 30.96°}{25.74\angle\text{-}29.05°}(100) = 11.32\angle 60.01°$$

$$\mathbf{V}_2 = \frac{7.5 + \text{j}4}{22.5 - \text{j}12.5}(100) = \frac{8.5\angle 28.07°}{25.74\angle\text{-}29.05°}(100) = 33.02\angle 57.12°$$

2.

Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2$$

But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45°}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\underline{/135°} = (1 + j2)\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 31.41\underline{/-87.18°}\text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45°} = 25.78\underline{/-70.48°}\text{ V}$$

3.

As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10$$

For mesh 2,

$$\mathbf{I}_2 = -3$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4$$

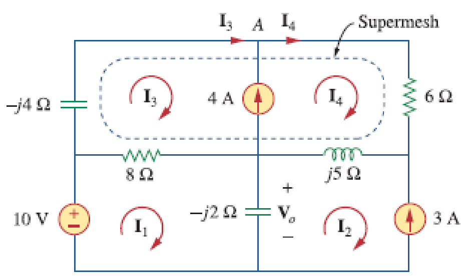
Instead of solving the above four equations, we reduce them to two by elimination.

combining above equations

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35$$



$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_0$ is

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618\underline{/274.5^\circ} + 3)$$

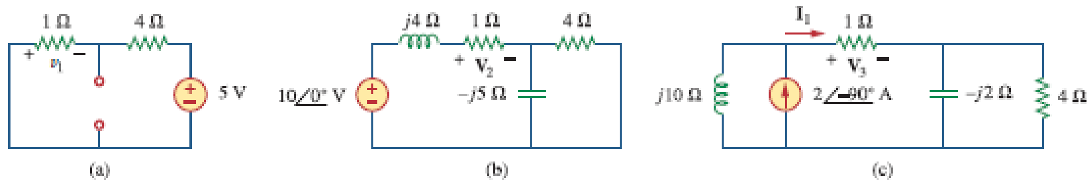
$$= -7.2134 - j6.568 = 9.756\underline{/222.32^\circ} \text{ V}$$

4.

$$10\cos 2t \quad\Rightarrow\quad 10\underline{/0^\circ}, \qquad \omega = 2 \text{ rad/s}$$

$$2\text{ H} \quad\Rightarrow\quad j\omega L = j4\ \Omega$$

$$0.1\text{ F} \quad\Rightarrow\quad \frac{1}{j\omega C} = -j5\ \Omega$$



$$-v_1 = \frac{1}{1+4}(5) = 1\text{ V}$$

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498\underline{/-30.79^\circ}$$

$$2\sin 5t \quad\Rightarrow\quad 2\underline{/-90^\circ}, \qquad \omega = 5 \text{ rad/s}$$

$$2\text{ H} \quad\Rightarrow\quad j\omega L = j10\ \Omega$$

$$0.1\text{ F} \quad\Rightarrow\quad \frac{1}{j\omega C} = -j2\ \Omega$$

$$\mathbf{Z}_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6\ \Omega$$

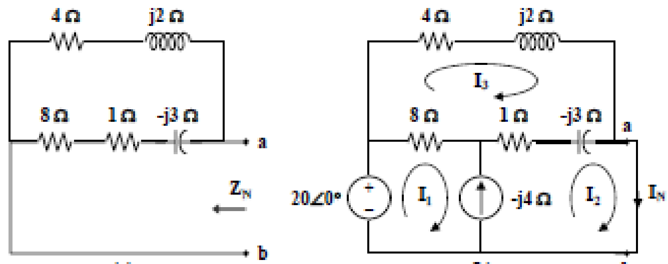
By current division,

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\underline{/-90^\circ})\text{ A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-80^\circ}\text{ V}$$

$$v_o(t) = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ)\text{ V}$$

5.



(a) (b)

$$\mathbf{Z}_N = (4 + j2) \parallel (9 - j3) = \frac{(4 + j2)(9 - j3)}{13 - j}$$

$$\mathbf{Z}_N = \underline{\mathbf{3.176 + j0.706\ \Omega}}$$

To find $\mathbf{I}_N$, short-circuit terminals a-b as shown in Fig. (b). Notice that meshes 1 and 2 form a supermesh.

For the supermesh, $\quad -20 + 8\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$

Also, $\quad \mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$

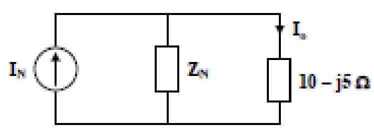
For mesh 3, $\quad (13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$

Solving for $\mathbf{I}_2$, we obtain

$$\mathbf{I}_N = \mathbf{I}_2 = \frac{50 - j62}{9 - j3} = \frac{79.65\angle -51.11°}{9.487\angle -18.43°}$$

$$\mathbf{I}_N = \underline{\mathbf{8.396\angle -32.68°\ A}}$$

Using the Norton equivalent, we can find $\mathbf{I}_o$ as in Fig. (c).



(c)

By current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + 10 - j5}\mathbf{I}_N = \frac{3.176 + j0.706}{13.176 - j4.294}(8.396\angle -32.68°)$$

$$\mathbf{I}_o = \frac{(3.254\angle 12.53°)(8.396\angle -32.68°)}{13.858\angle -18.05°}$$

$$\mathbf{I}_o = \underline{\mathbf{1.971\angle -2.10°\ A}}$$