Course Code: EEE 4101
Course Title: Electrical Engineering

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Ref. Book: Fundamental of Electrical Circuit: Alexander Sadiku

Sinusoids

A sinusoid is a signal that has the form of the sine or cosine function.

- A sinusoidal current is usually referred to as *alternating current (ac)*.
- Such a current reverses at regular time intervals and has alternately positive and negative values.
- Circuits driven by sinusoidal current or voltage sources are called *ac circuits*.

$f = \frac{1}{T}$

$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi N}{t}$

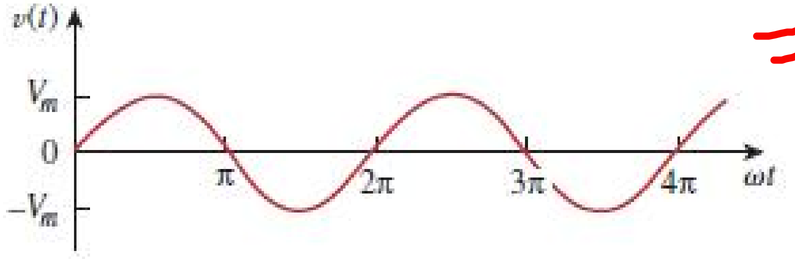
Consider the sinusoidal voltage $v(t) = v_m Sin\omega t$ [V]

where

V_m = the *amplitude* of the sinusoid [V]

ω= Angular frequency of the sinusoid [rad s^{-1}]

ωt= the argument of the sinusoid



* The argument of a function is **the "input" to the function**. In the case of sin(x) consider this: f(x) = sin(x) x is the argument.

Sinusoids

Let us now consider a more general expression for the sinusoid,

$$v(t) = v_m Sin(\omega t + \varphi)$$

or, $v(t) = V_m \cos(\omega t + \varphi)$

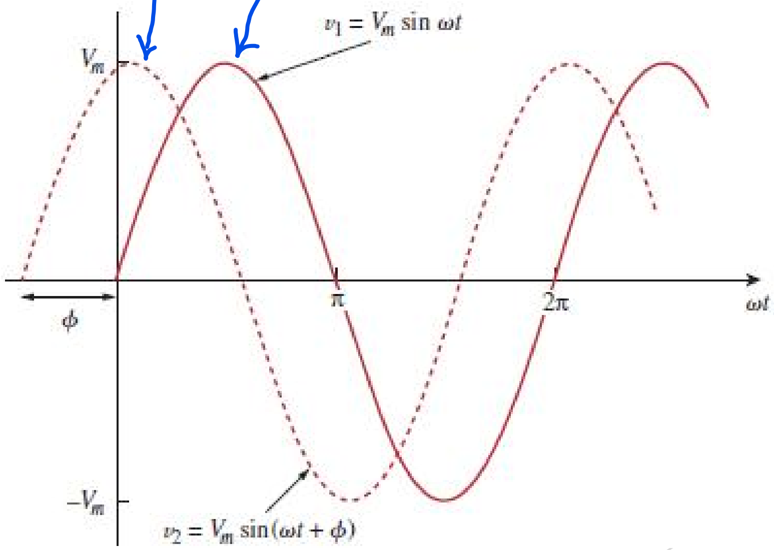
Where $(\omega t + \varphi)$ is the argument and φ is the *phase*. Both argument and phase can be in radians or degrees.

Let us examine the two sinusoids,

$v_1(t) = v_m Sin\omega t$ and $v_2(t) = v_m Sin(\omega t + \varphi)$

lead

lag



Sinusoids and Phasors

$\omega = 2\pi f$ $\quad\therefore f = \frac{25}{\pi}$ Hz

Q. Find the amplitude, phase, period, and frequency of the sinusoid $v(t) = 12\cos(50t + 10^o)$

$V_m \quad \varphi \quad T = \frac{1}{f} = \frac{\pi}{25}$ s $\quad f$

$\Delta\varphi = 50 - 80 = -30$

Q. Calculate the phase angle between v_1 =-10 cos(ωt +50) and v_2 = 12 Sin(ωt -10). State which sinusoid is leading.

= 10 cos (wt+50-180)
= 10 cos (wt-130)

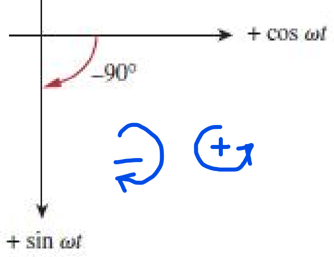
=12 cos(wt-10-90) = 12 cos(wt-100)

$\varphi_{v_2} > \varphi_{v_1}$ $\therefore V_2$ leads

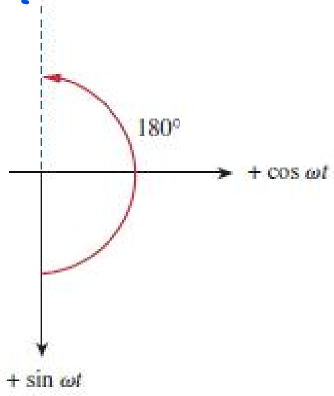
bring both of them to (+)cos while rotating in the same direction

phase=φv1-φv2 = -130+100 = -30^o [(-)(v1-v2) -> v2 leads v1 by φ^o
(+)(v1-v2) -> v1 leads v2 by φ^o]

$\varphi_{v_1} - \varphi_{v_2} < 0 \quad \therefore \varphi_{v_1} < \varphi_{v_2}$

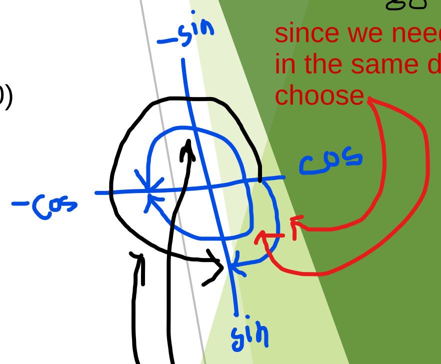


(a)



(b)

since we need to rotate both in the same direction, .'. choose



you can also choose

Phasors

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

$j = \sqrt{-1}$

A complex number z can be written in rectangular form as_ $z = x + jy$

The complex number z can also be written in polar or exponential form as_ $z = r\angle\varphi = re^{j\varphi}$

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$z = x + jy$	Rectangular form
$z = r\angle\phi$	Polar form
$z = re^{j\phi}$	Exponential form

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

Phasors

$v = V_m \cos(\omega t + \varphi)\, V$ the phasor form of this sinusoids is_ **V**=$V_m\ \angle\varphi$ A. V

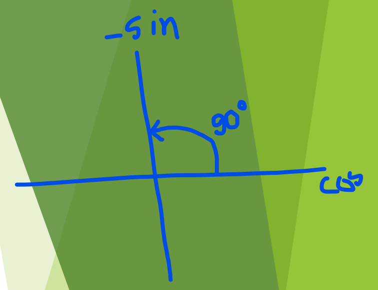
i= $I_m \cos(\omega t + \varphi)\, A$ the phasor form of this sinusoids is_ **I**=$I_m\ \angle\varphi$ A.

Q. Express these sinusoids as phasors: $(a) v = 7\cos(2t + 40^o)\, V,\ (b) i = -4\sin(10t + 10^o)$

$V = 7\angle 40^\circ\ V$

$= 4\cos(10t + 10^\circ + 90^\circ)$

$I = 4\angle 100^\circ$



Impedance

The impedance **Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (Ω).

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{ZI}$$

Impedances and admittances of passive elements.

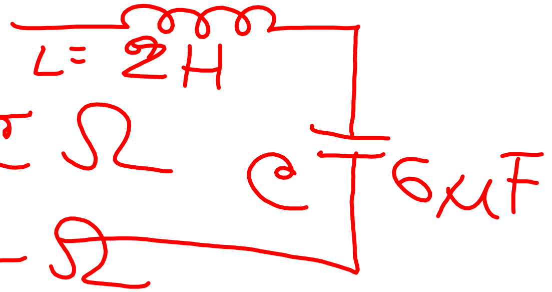
Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

As a complex quantity, the impedance may be expressed in rectangular form as $\boxed{Z} = R + jX$

where R= Re Z is the *resistance* and X= Im Z is the *reactance*. The reactance X may be positive or negative. We say that the impedance is inductive when X is positive or capacitive when X is negative.

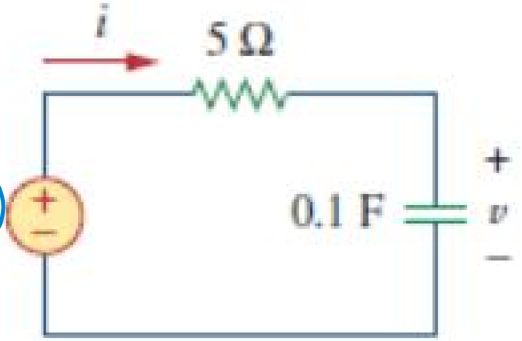
$X_C = \frac{1}{j\omega C}\ \Omega$

$X_L = j\omega L\ \Omega$



Impedance

Q. Find i(t) and v(t) in the circuit shown in the figure.



$v_s = V_m \cos(\omega t + \phi)$

$V_m = 10$, $\omega = 4$ rad/s, $\phi = 0^\circ$

time domain Signal

$$\boxed{V_s = 10\angle 0^\circ \ V}$$

$V = I Z$

$= I(R + jx)$

$= I(R + j(X_L - X_C))$

$X_L = j\omega L \ \Omega$

$X_C = \frac{1}{j\omega C} \ \Omega$

$= \frac{\angle -90^\circ}{\omega C}$

$j = \angle 90^\circ$

$= -j(\omega C)^{-1}$

$I = \frac{V_s}{Z}$

$= \frac{10\angle 0^\circ}{5 - j2.5}$ A

$= 1.6 + j0.8$

$= 1.78\angle$

$V =$

$V = V_m \cos(\omega t + \phi)$

$I = 1.78\angle 26.56^\circ$ A

$\Rightarrow 1.78 \cos(4t + 26.56^\circ)$

$V = I X_C$

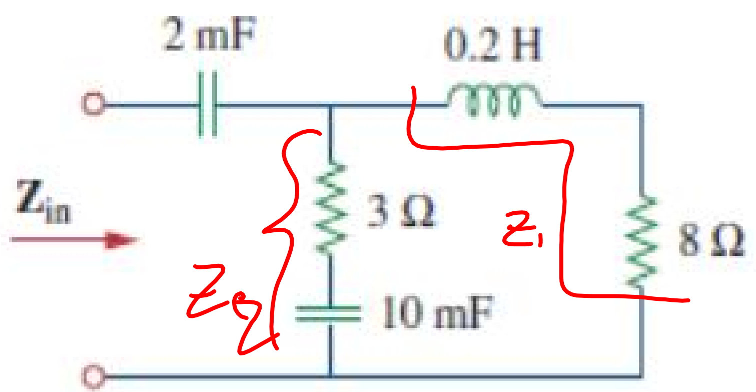
$= \frac{I}{j\omega C}$

$= \frac{I}{j0.4}$

$= -j2.5 \times I$

Impedance

Q. Find the input impedance of the circuit in the Fig. Assume that the circuit operate at ω=50 rad/s.



$$Z_1 = 8 + j\omega L$$
$$= (8 + j10)\,\Omega$$

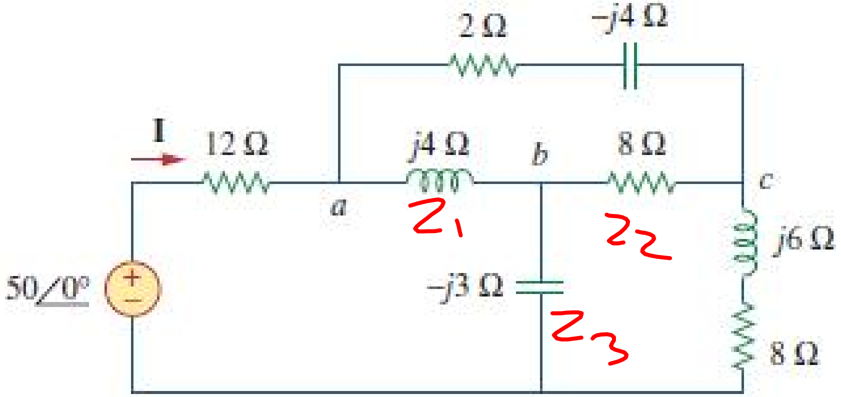
$$Z_2 = 3 + \frac{1}{j\omega C}$$
$$= (3 - j20)\,\Omega$$

$$Z_3 = \frac{1}{j\omega \times 2\times10^{-3}} = -j01$$

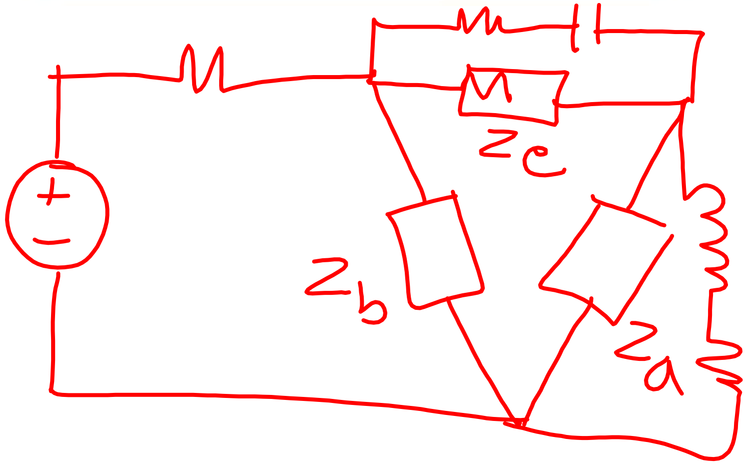
$$Z_{in} = Z_3 + (Z_1 \| Z_2)$$

Impedance

Q. Find current *I* in the circuit of the Fig.



$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$



Thank You