Course Code: EEE 4101
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Unit: VA unitless

Apparent Power and Power Factor

The apparent power (in VA) is the product of the rms values of voltage and current.

We know,

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\boxed{S = V_{rms} * I_{rms}}$$

The product $V_{rms} * I_{rms}$ is known as the *apparent power and the factor* $Cos\ (\theta_v - \theta_i)$ is called power factor.

- The apparent power is so called because it seems apparent that the power should be the voltage-current product, by analogy with dc resistive circuits.
- It is measured in volt-amperes or VA to distinguish it from the average or real power, which is measured in watts.
- The power factor is dimensionless, since it is the ratio of the average power to the apparent power,

$$\boxed{\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance. $\cos(\varphi_z)$

Apparent Power and Power Factor

Q. A series-connected load draws a current $i(t) = 4\cos(100\Pi t + 10^o)\ A$ when the applied voltage is $v(t) = 120\cos(100\Pi t - 20^o)\ V$. Find the apparent power and the power factor of the load. Determine the element values that form the series–connected load.

The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20^\circ - 10^\circ) = 0.866 \qquad \text{(leading)}$$

The pf is leading because the current leads the voltage. The pf may also be obtained from the load impedance.

$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}} = \frac{120\angle{-20^\circ}}{4\angle{10^\circ}} = 30\angle{-30^\circ} = 25.98 - j15\ \Omega$$

$$\text{pf} = \cos(-30^\circ) = 0.866 \qquad \text{(leading)}$$

The load impedance $\mathbf{Z}$ can be modeled by a 25.98-Ω resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C}$$

or

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

Complex Power

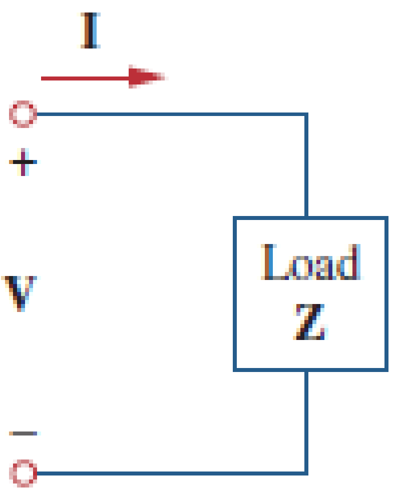
Consider the ac load in shown Fig. Given the phasor form $V = V_m\angle\theta_v$ and $I = I_m\angle\theta_i$. the *complex power* **S** absorbed by the ac load is the product of the voltage and the complex conjugate of the current,

$$\boxed{\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*}$$

$$\boxed{S = V_{rms} * I_{rms}^{\ *}}$$

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}}\underline{/\theta_v}$$

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}}\underline{/\theta_i}$$



$$\begin{aligned} \mathbf{S} &= V_{\text{rms}}I_{\text{rms}}\underline{/\theta_v - \theta_i} \\ &= V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) + jV_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i) \\ &= \boxed{P + jQ} \end{aligned}$$

We notice from above Eq. that the magnitude of the complex power is the apparent power; hence, the complex power is measured in volt-amperes (VA). Also, we notice that the angle of the complex power is the power factor angle.

where P and Q are the real and imaginary parts of the complex power; that is,

$$\boxed{P = V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i),} \qquad \boxed{Q = V_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i)}$$

Complex Power

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$

$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| \angle \theta_v - \theta_i$$

$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$

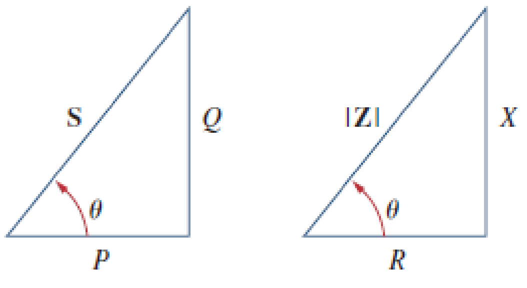
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Complex Power

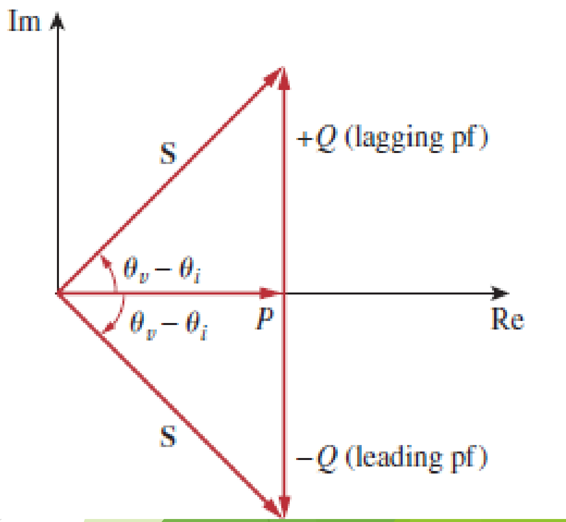
It is a standard practice to represent **S**, *P*, and *Q* in the form of a triangle, known as the ***power triangle***, shown in Fig. (a). This is similar to the impedance triangle showing the relationship between **Z**, *R*, and *X*, illustrated in Fig. (b).



(a) (b)

The power triangle has four items—the apparent/complex power, real power, reactive power, and the power factor angle.

when **S** lies in the first quadrant, we have an inductive load and a lagging pf. When **S** lies in the fourth quadrant, the load is capacitive and the pf is leading. It is also possible for the complex power to lie in the second or third quadrant. This requires that the load impedance have a negative resistance, which is possible with active circuits.



Complex Power

Q. The voltage across a load is $v(t) = 60\cos(\omega t - 10^o)\ V$ and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50^o$Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

(a) For the rms values of the voltage and current, we write

$$\mathbf{V}_{\text{rms}} = \frac{60}{\sqrt{2}}\angle -10^\circ, \qquad \mathbf{I}_{\text{rms}} = \frac{1.5}{\sqrt{2}}\angle +50^\circ$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \left(\frac{60}{\sqrt{2}}\angle -10^\circ\right)\left(\frac{1.5}{\sqrt{2}}\angle -50^\circ\right) = 45\angle -60^\circ \text{ VA}$$

The apparent power is

$$S = |\mathbf{S}| = 45 \text{ VA}$$

(b) We can express the complex power in rectangular form as

$$\mathbf{S} = 45\angle -60^\circ = 45[\cos(-60^\circ) + j\sin(-60^\circ)] = 22.5 - j38.97$$

Since $\mathbf{S} = P + jQ$, the real power is

$$P = 22.5 \text{ W}$$

while the reactive power is

$$Q = -38.97 \text{ VAR}$$

(c) The power factor is

$$\text{pf} = \cos(-60^\circ) = 0.5 \text{ (leading)}$$

It is leading, because the reactive power is negative. The load impedance is

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\angle -10^\circ}{1.5\angle +50^\circ} = 40\angle -60^\circ\ \Omega$$

which is a capacitive impedance.

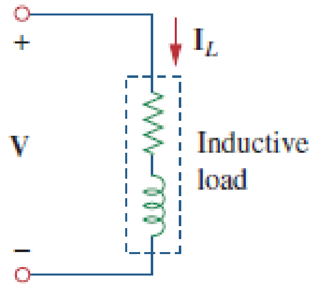
Power Factor Correction

- Most domestic loads (such as washing machines, air conditioners, and refrigerators) and industrial loads (such as induction motors) are inductive and operate at a low lagging power factor.
- Although the inductive nature of the load cannot be changed, we can increase its power factor.

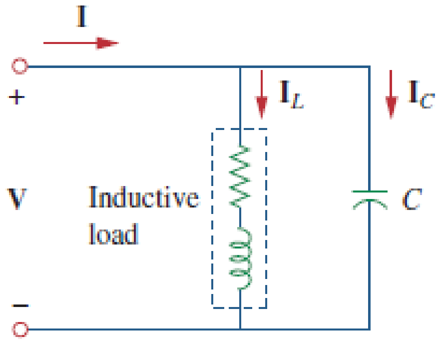
The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

Power Factor Correction

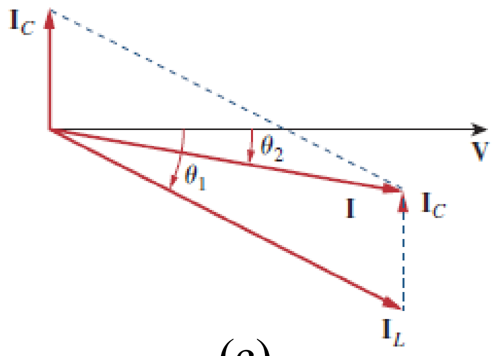
Since most loads are inductive, as shown in Fig. (a), a load's power factor is improved or corrected by deliberately installing a capacitor in parallel with the load, as shown in Fig. (b). The effect of adding the capacitor can be illustrated using either the power triangle or the phasor diagram of the currents involved. Figure (c) shows the latter, where it is assumed that the circuit in Fig. (a) has a power factor of $\cos(\theta_1)$ while the one in Fig. (b) has a power factor of $\cos(\theta_2)$. It is evident from Fig. (c) that adding the capacitor has caused the phase angle between the supplied voltage and current θ_1 to θ_2 reduce from to thereby increasing the power factor. We also notice from the magnitudes of the vectors in Fig. (c) that with the same supplied voltage, the circuit in Fig. (a) draws larger current I_L than the current I drawn by the circuit in Fig. (b). Power companies charge more for larger currents, because they result in increased power losses (by a squared factor, since $P = {I_L}^2 R$). Therefore, it is beneficial to both the power company and the consumer that every effort is made to minimize current level or keep the power factor as close to unity as possible. By choosing a suitable size for the capacitor, the current can be made to be completely in phase with the voltage, implying unity power factor.



(a)



(b)



(c)

Power Factor Correction

If the original inductive load has apparent power S_1, then

$$P = S_1 \cos\theta_1, \qquad Q_1 = S_1 \sin\theta_1 = P \tan\theta_1$$

If we desire to increase the power factor from $\cos(\theta_1)$ to $\cos(\theta_2)$ without altering the real power (i.e, $P_2 = S_2\cos(\theta_2)$) then the new reactive power is

$$Q_2 = P \tan\theta_2$$

The reduction in the reactive power is caused by the shunt capacitor; that is,

$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2)$$

$$Q_C = \frac{{V_{rms}}^2}{X_c} = \omega C {V_{rms}}^2$$

$$C = \frac{Q_c}{\omega {V_{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\mathrm{rms}}^2}$$

Power Factor Correction

V_{rms} $\omega = 2\pi f$ P

Q. When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

pf C $\cos\theta_2 = 0.95$

If the pf = 0.8, then

$$\cos\theta_1 = 0.8 \quad \Rightarrow \quad \theta_1 = 36.87^\circ$$

where θ_1 is the phase difference between voltage and current. We obtain the apparent power from the real power and the pf as

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

The reactive power is

$$Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

When the pf is raised to 0.95,

$$\cos\theta_2 = 0.95 \quad \Rightarrow \quad \theta_2 = 18.19^\circ$$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The new reactive power is

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

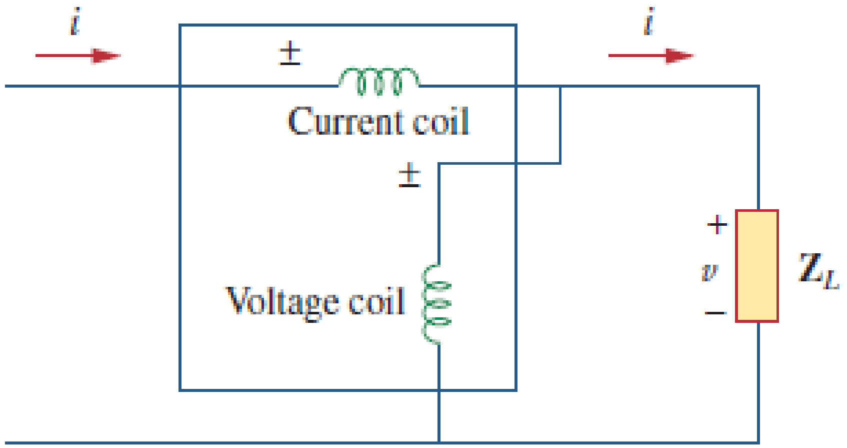
$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

and

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

Power Measurement

The average power absorbed by a load is measured by an instrument called the *wattmeter*.

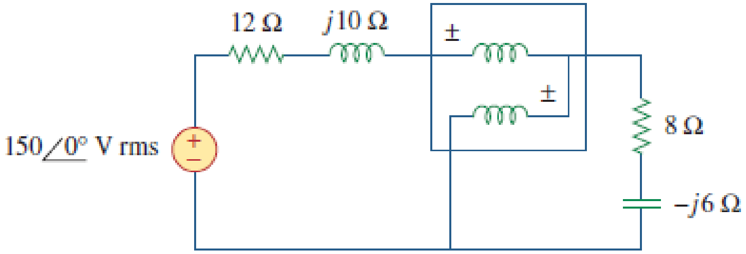


Q. When an wattmeter shows no deflection or down scale deflection?

Each wattmeter coil has two terminals with one marked ±. To ensure upscale deflection, ± the terminal of the current coil is toward the source, while the ± terminal of the voltage coil is connected to the same line as the current coil. Reversing both coil connections still results in upscale deflection. However, reversing one coil and not the other results in downscale deflection and no wattmeter reading.

Power Measurement

Q. Find the wattmeter reading of the circuit in Fig. below.



$$\mathbf{I}_{rms} = \frac{150\angle 0^\circ}{(12 + j10) + (8 - j6)} = \frac{150}{20 + j4} \text{ A}$$

The voltage across the $(8 - j6)\ \Omega$ impedance is

$$\mathbf{V}_{rms} = \mathbf{I}_{rms}(8 - j6) = \frac{150(8 - j6)}{20 + j4} \text{ V}$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = \frac{150(8 - j6)}{20 + j4} \cdot \frac{150}{20 - j4} = \frac{150^2(8 - j6)}{20^2 + 4^2}$$
$$= 423.7 - j324.6 \text{ VA}$$

The wattmeter reads

$$P = \text{Re}(\mathbf{S}) = \mathbf{432.7\ W}$$

Thank You