S

Reluctance is the ratio of magnetomotive force (MMF) to magnetic flux.
(1. analogous to resistance in an electric circuit)

S = mmf / φ ,
where
S is the reluctance in ampere-turns per weber (At / Wb),
mmf is the MMF in ampere-turns (At), and
φ is the magnetic flux in webers (Wb)

The unit for magnetic reluctance is ampere-turns / Weber or inverse Henry (H -1) 2.

List of formulas,

1. Same Flux flows through limbs in series.

(1) $mmf = S\varphi = NI = Hl$

(2) $S = \frac{l}{\mu A}$ [Ohm's law]

(3) $B = \mu H = \frac{\varphi}{A}$

(4) $V = N\frac{d\varphi}{dt}$ [Faraday's law]

(5) $V = L\frac{di}{dt}$

(6) $L_{ttl} = L_1 + L_2 \pm 2M$ ((+) $\rightarrow \rightarrow$, (−) $\rightarrow \rightarrow$)

(7) $K = \frac{\varphi_{AB}}{\varphi_{AA}} = \frac{M}{\sqrt{L_1 L_2}}$

(8) $L_A = N_A \frac{d\varphi_{AA}}{di_A}$

(9) $M_{AB} = M_{BA} = N_B \frac{d\varphi_{AB}}{di_A} = N_A \frac{d\varphi_{BA}}{di_B}$

(10) $\mu = \mu_0 \mu_r$

(11) $S_{series} = S_1 + S_2 + S_3 \cdots$

(12) $mmf_{series} = H_1 l_1 + H_2 l_2 + H_3 l_3 + \cdots = mmf_1 + mmf_2 + mmf_3 + \cdots$

(13) $mmf_{parallel} = \underbrace{\varphi_1}_{center} S_1 + \varphi_2 S_2 = \underbrace{\varphi_1}_{center} S_1 + \varphi_3 S_3$

$V_0 = I_2 \times 10$

Mutual Inductance is created by the neighboring circuit

KVL for loop 1,

$I_1(4+j8) + I_2(j1) = 200\angle 45^\circ$

KVL for loop 2,

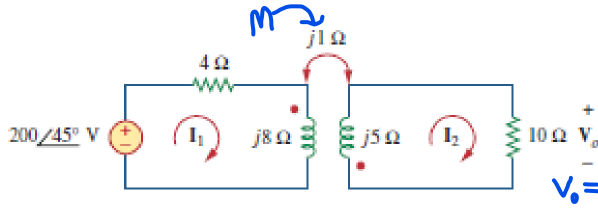
$I_2(10+j5) + I_1(j1) = 0$

ref. book page 562 (pdf page 587)

Question 1

Determine the voltage **V**o in the circuit of given Fig.

M



$V_o = I_2 10$

$I_2 = ?$

Solve by matrix:

* Take advantage of calculator "STO" (store) property

$$\begin{bmatrix} \underset{A}{4+j8} & \underset{B}{j1} \\ \underset{C}{j1} & \underset{D}{10+j5} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \underset{E}{200\angle 45^\circ} \\ \underset{F}{0} \end{bmatrix}$$

$I_2 = \frac{\Delta_2}{\Delta} = \frac{AF-EC}{AD-BC} = -1.399 - j1.428$

$= 1.99\angle -134.4$

$\approx 2\angle -134.4$

$\therefore V_o = 20\angle -134.4$

Ans: 20∠135° V.

ref. book page-562 (pdf 587)

ref. book page 566 (pdf page 591)

Question 2

$K = ?$

Consider the circuit in Fig. 13.16. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t=1s if $v = \underbrace{60}_{V_m} \cos(\underbrace{4}_{\omega}t + \underbrace{30^\circ}_{\varphi})$ V.

$K = \frac{M}{\sqrt{L_1 L_2}}$

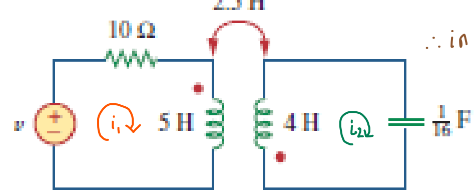
$= 0.559$ [No unit]

Mutual inductance, M = 2.5 H

Inductance of the coupled inductors,

$L_1 = 5$ H

$L_2 = 4$ H



$\therefore$ in phasor, $V = V_m\angle\varphi$

$= 60\angle 30^\circ$

Total energy stored in the coupled inductors, $t=1$

$W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$

$= 20.63$ J

$i_1 = -3.38$ A

$i_2 = 2.82$ A

here, inductance, $L_1 = 5H$, $L_2 = 4H$, C

$\therefore$ impedence, $Z_1 = j\omega L_1$, $Z_2 = j\omega L_2$, $Z_3 = \frac{1}{j\omega C}$

$= j\cdot 4\cdot 5$ $= j16\ \Omega$ $= \frac{1}{j\cdot 4\cdot \frac{1}{16}} = -j4$

$= j20\ \Omega$

also writing the mutual inductance in frequency domain or, phasor

$j\times 4\times 2.5 = j10$

$\therefore$ KVL for loop 1,

$i_1(10+j20) + i_2(j10) = 60\angle 30^\circ \cdots$ (i)

KVL for loop 2,

$i_2(j16-j4) + i_1(j10) = 0$

$\Rightarrow i_2 j12 + i_2 j10 = 0 \cdots$ (ii)

$\therefore$ Solving (i) & (ii), $i_1 = 3.9\angle -19.3^\circ$, $i_2 = 3.25\angle 160.6^\circ$

Now bring them into TIME DOMAIN, to input them into the equation

$\therefore i_1 = i_m \times \cos(\omega t + \varphi)$

$= 3.9 \times \cos(4t - 19.3^\circ)$

$\therefore i_2 = 3.25\cos(4t + 160.6^\circ)$

Now if $t=1$,

$\omega = 4t = 4$ rad

$= (4 \times \frac{180}{\pi})^\circ = 229.18^\circ$

$\therefore i_1 = -3.38$ A, $i_2 = 2.82$ A

Question 3

A ring composed of three sections. The cross-sectional area is 0.001 m 2 for each section. The mean arc lengths are la= 0.3 m, l_b= 0.2 m, lc = 0.1 m. An air-gap length of 0.1 mm is cut in the ring. μ_r for sections a, b, c are 5000, 1000, and 10,000 respectively. Flux in the air gap is 7.5×10[4] Wb. Find (i) mmf, (ii) exciting current if the coil has 100 turns, (iii) reluctance of the sections.

$S = \frac{l}{\mu A}$ rels, mmf=Sφ, m.m.f=NI

Question 4

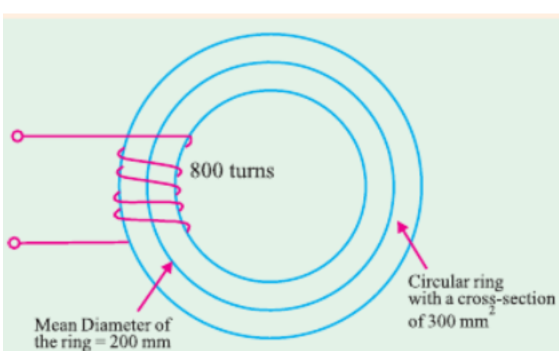
A wooden ring has a circular cross-section of 300 sq. mm and a mean diameter of the ring is 200 mm. It is uniformly wound with 800 turns.

Calculate:

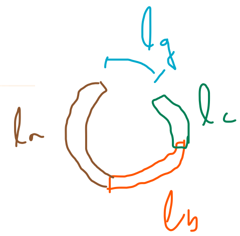
(i) the field strength produced in the coil by a current of 2 amperes :(assume = 1) $H = \frac{m.m.f}{l}$

(ii) the magnetic flux density produced by this current and $B = \mu H$

(iii) the current required to produce a flux density of 0.02 wb/m^2. $\frac{B_1}{I_1} = \frac{B_2}{I_2}$



3.



Solution.

$$\text{Area} = 0.001 \text{ sq.m}$$
$$l_a = 0.3 \text{ m}, \; l_b = 0.2 \text{ m}, \; l_c = 0.1 \text{ m}, \; l_g = 0.1 \times 10^{-3} \text{ m}$$
$$\mu_{ra} = 5000, \; \mu_{rb} = 1000, \; \mu_{rc} = 10{,}000 \;\; \mu_o = 4\pi \times 10^{-7}$$
$$\phi = 7.5 \times 10^{-4} \text{ Wb}$$

(*iii*) Calculations of Reluctances of four parts of the magnetic circuit :

(*a*) Reluctance of air gap, $R_{eg} = \frac{1}{\mu_o} \times \frac{0.1 \times 10^{-3}}{0.001} = \frac{1000}{4\pi \times 0.001} = 79618$

(*b*) Reluctance of section '*a*' of ring

$$= R_{ea} = \frac{1}{\mu_o \mu_{ra}} \times \frac{0.3}{0.001} = \frac{10^7 \times 0.3}{4\pi \times 47770 \times 5000 \times 0.001} = 47770$$

(*c*) Reluctance of section '*b*' of the ring

$$= R_{eb} = \frac{1}{\mu_o \mu_{rb}} \times \frac{0.20}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 15923.6$$

(*d*) Reluctance of section '*c*' of the ring

$$= R_{ec} = \frac{1}{\mu_o \mu_{rc}} \times \frac{0.10}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 7961$$

$$\text{Total Reluctance} = R_{eg} + R_{ea} + R_{eb} + R_{ec} = 294585$$

(*i*) $\text{Total mmf required} = \text{Flux} \times \text{Reluctance}$ [Same Flux flows through limbs in series]

$$= 7.5 \times 10^{-4} \times 294585 = 221 \text{ amp-turns}$$

(*ii*) $\text{Current required} = 221/100 = 2.21 \text{ amp}$

2.

The coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

indicating that the inductors are tightly coupled. To find the energy stored, we need to calculate the current. To find the current, we need to obtain the frequency-domain equivalent of the circuit.

$$60 \cos(4t + 30^\circ) \Rightarrow 60\angle 30^\circ, \quad \omega = 4 \text{ rad/s}$$
$$5 \text{ H} \Rightarrow j\omega L_1 = j20\ \Omega$$
$$2.5 \text{ H} \Rightarrow j\omega M = j10\ \Omega$$
$$4 \text{ H} \Rightarrow j\omega L_2 = j16\ \Omega$$
$$\frac{1}{16} \text{F} \Rightarrow \frac{1}{j\omega C} = -j4\ \Omega$$

The frequency-domain equivalent is shown in Fig. 13.17. We now apply mesh analysis. For mesh 1,

$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\angle 30^\circ$$

For mesh 2,

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = -1.2\mathbf{I}_2$$

Substituting this into Eq. (13.3.1) yields

$$\mathbf{I}_2(-12 - j14) = 60\angle 30^\circ \quad \Rightarrow \quad \mathbf{I}_2 = 3.254\angle 160.6^\circ \text{ A}$$

and

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 = 3.905\angle -19.4^\circ \text{ A}$$

In the time-domain,

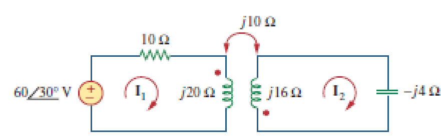
$$i_1 = 3.905 \cos(4t - 19.4^\circ), \qquad i_2 = 3.254 \cos(4t + 160.6^\circ)$$

At time $t = 1$ s, $4t = 4$ rad $= 229.2^\circ$, and

$$i_1 = 3.905 \cos(229.2^\circ - 19.4^\circ) = -3.389 \text{ A}$$
$$i_2 = 3.254 \cos(229.2^\circ + 160.6^\circ) = 2.824 \text{ A}$$

The total energy stored in the coupled inductors is

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$
$$= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$



4.

Solution. The question assumes that the flux-path is through the ring, as shown by the dashed line, in figure, at the mean diameter, in Fig. 6.40.

With a current of 2 amp,

$$\text{Coil m.m.f.} = 800 \times 2 = 1600 \text{ AT}$$

$$\text{Mean length of path} = \pi \times 0.2 = 0.628 \text{ m}$$

(*i*) $$H = \frac{1600}{0.628} = 2548 \text{ amp-turns/meter}$$

(*ii*) $$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1 \times 2548 = 3.20 \times 10^{-3} \text{ Wb/m}^2$$

This Flux density is produced by a coil current of 2-amp

(*iii*) For producing a flux of 0.02 Wb/m^2, the coil current required is

$$2 \times \frac{0.02}{0.0032} = \textbf{12.5 amp}$$