Course Code: EEE 4101
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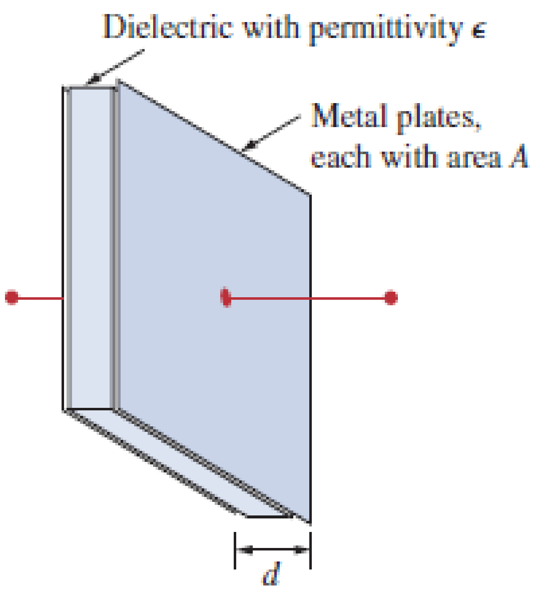
Ref. Book: Fundamental of Electrical Circuit: Alexander Sadiku

Capacitor and Inductor

Another two new and important passive linear circuit elements: the capacitor and the inductor

Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).



In DC:

Cpacitors are replaced with Open circuit

Inductors are replaced with Short circuit

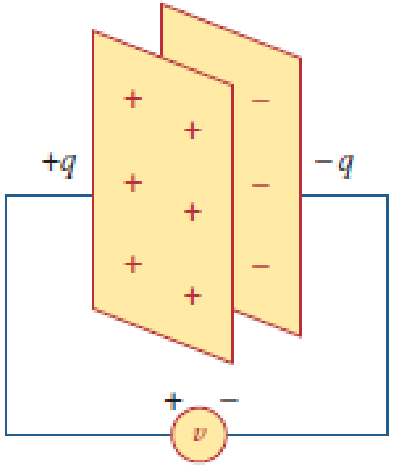
Capacitor

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica. Al

When a voltage source is connected to the capacitor, as in given figure, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that

$$q = Cv$$

where C, the constant of proportionality, is known as the *capacitance* of the capacitor. The unit of capacitance is the farad (F), in honor of the English physicist Michael Faraday (1791–1867).

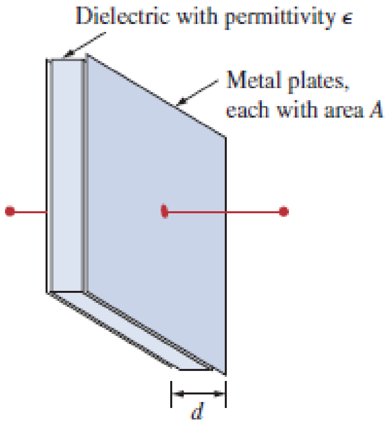


Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Capacitor (Contd.)

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in the figure, the capacitance is given by

$$C = \frac{\mathcal{E}A}{d}$$



Where, A= is the surface area of each plate,
d = distance between the plates, and
$\mathcal{E}$ =permittivity of the dielectric material between the plates.

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

Capacitor (Contd.)

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of the following Eq. Since

$$q = Cv$$

$$\Rightarrow \frac{dq}{dt} = C\frac{dv}{dt}$$

$$\Rightarrow i = C\frac{dv}{dt}$$

The instantaneous power delivered to the capacitor is

$$P = vi = Cv\frac{dv}{dt}$$

Energy stored in capacitor, $w = \frac{1}{2}Cv^2$

$$W = \int_{-\infty}^{t} p = \int_{-\infty}^{t} Cv\frac{dv}{dt}\,dt = \int_{v(-\infty)}^{v(t)} Cv\,dv = C\left.\frac{v^2}{2}\right|_{v(-\infty)}^{v(t)} = \frac{1}{2}Cv^2$$

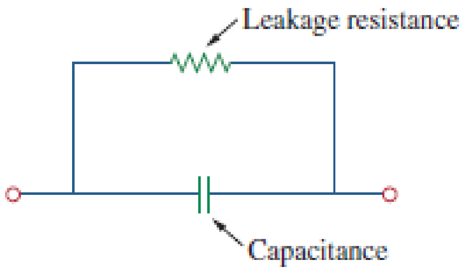
[$v(-\infty) = 0$; as the capacitor was uncharged at $t = -\infty$]

Capacitor (Contd.)

$\frac{dv}{dt} = 0 \Rightarrow i = C\frac{dv}{dt} = 0$

Important properties of a capacitor:

1. when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus, A capacitor is an open circuit in DC.
2. The voltage on the capacitor must be continuous.
3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.
4. A real, nonideal capacitor has a parallel-model leakage resistance, as shown in the given figure. The leakage resistance may be as high as 100MΩ and can be neglected for most practical applications. For this reason, we will assume ideal capacitors in this book



Capacitor (Contd.)

Q. The voltage across a capacitor is 5 μF is $v = 10\ Cos6000t\ V$. Calculate the current through it.

$$\Rightarrow i = C\frac{dv}{dt} = 5\times10^{-6}\times10\times6000\times(-\sin6000t) = -0.3\sin6000t$$

Q. Determine the current through a 200 μF capacitor whose voltage is shown in below figure.

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$$\frac{x-x_2}{x_1-x_2} = \frac{y-y_2}{y_1-y_2}$$

$$i = C\frac{dv}{dt}$$

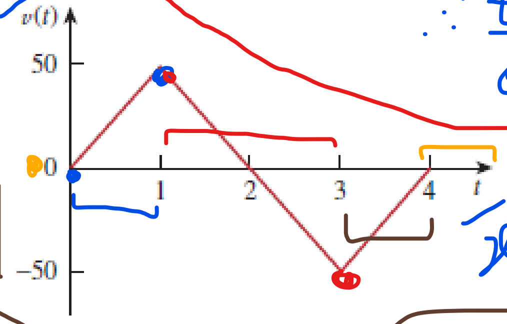
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t\ \text{V} & 0 < t < 1 \\ 100 - 50t\ \text{V} & 1 < t < 3 \\ -200 + 50t\ \text{V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore \frac{t-1}{0-1} = \frac{v(t)-50}{0-50}$$

$$\therefore \frac{t-3}{1-3} = \frac{v(t)+50}{50+50}$$

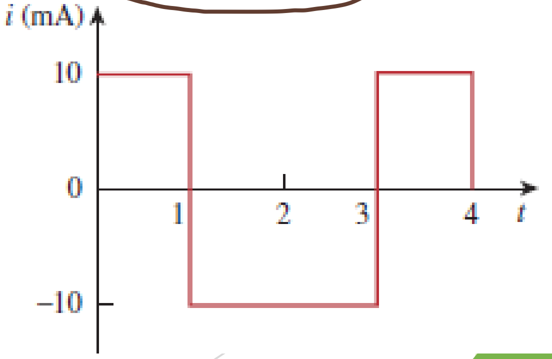
$$\therefore \frac{t-4}{3-4} = \frac{v(t)-0}{-50-0}$$



Since $i = C\,dv/dt$ and $C = 200\ \mu\text{F}$, we take the derivative of v to obtain

$$i(t) = 200\times10^{-6}\times\begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10\ \text{mA} & 0 < t < 1 \\ -10\ \text{mA} & 1 < t < 3 \\ 10\ \text{mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

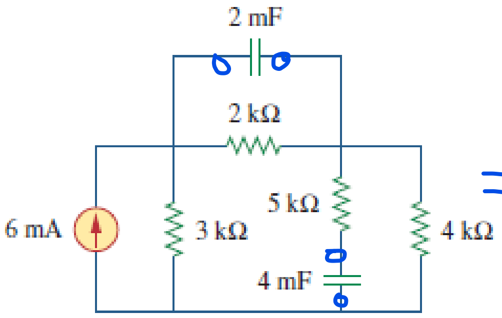


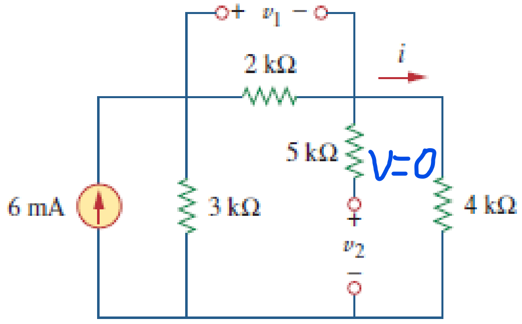
Thus the current waveform is as shown in the second picture.

Capacitor (Contd.)

$W = \frac{1}{2} C V^2$

Q. Obtain the energy stored in each capacitor in given Fig. under dc conditi





$i = 6 \times \frac{3}{3+(2+4)} = 2\,mA$

$V_1 = 2i = 4\,V$

$V_2 = 4i = 8V$

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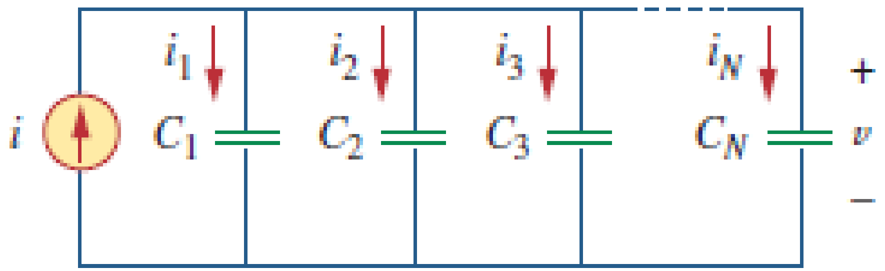
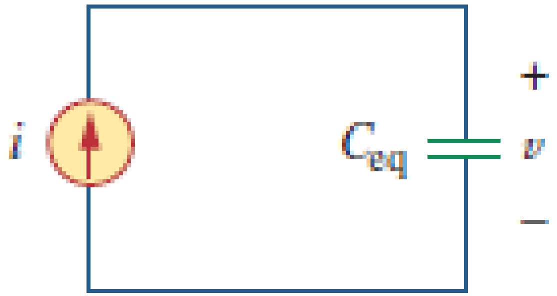
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

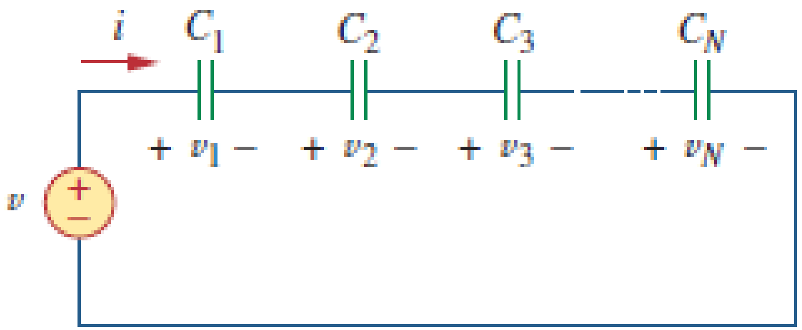
$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

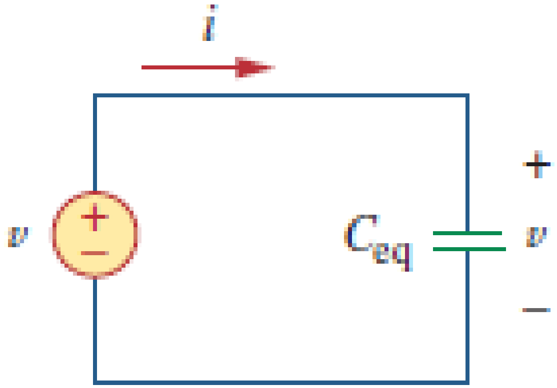
Series and Parallel Capacitor





$C_1 + C_2 + \cdots$

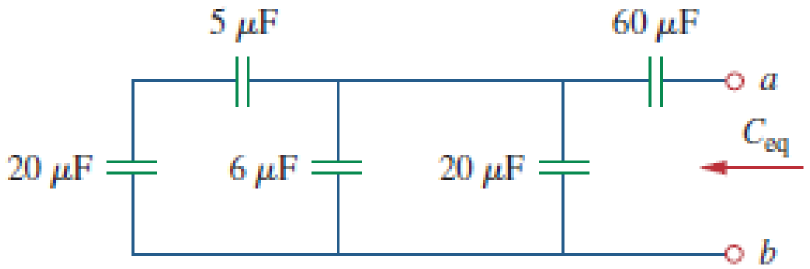




$\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right)^{-1}$

Capacitor (Contd.)

Q. Find the equivalent capacitance seen between terminals *a* and *b* of the circuit in given Fig.



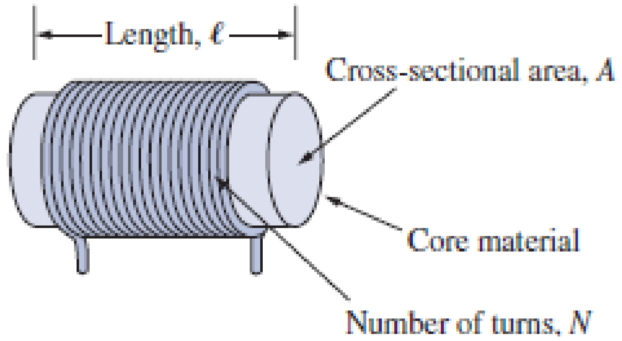
$((20 \sim 5) \,||\, 6 \,||\, 20) \sim 60$

$= (4+6+20) \sim 60$

$= 20\ \mu F$

Inductor

Inductor consists of a coil of conducting wire.



Typical form of an inductor.

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.

$$V = L\frac{dI}{dt}$$

where L is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H)

The inductance of an inductor depends on its physical dimension and construction.

$$L = \frac{N^2 \mu A}{l}$$

where N is the number of turns, is the length, A is the cross-sectional area, and μ is the permeability of the core.

Inductor (Contd.)

The power delivered to the inductor is

$$P = vi$$

$$\Rightarrow P = L\frac{di}{dt} * i$$

Energy stored in the inductor is

$$w = \int Pdt = \int L\frac{di}{dt}idt = L\int idi = \frac{1}{2}Li^2$$

Important properties of a ~~capacitor~~: inductor:

1. The voltage across an inductor is zero when the current is constant. The inductor acts as a short circuit.
2. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.
3. A practical, nonideal inductor has a significant resistive component. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the *winding resistance*, R_w , and it appears in series with the inductance of the inductor. The presence of R_w makes it both an energy storage device and an energy dissipation device. Since is usually very small, it is ignored in most cases.

Inductor (Contd.)

i L

Q. Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at t=5s, Assume i(v)>0

$\int_0^t v\,dt = \int_{i(0)}^{i(t)} L \frac{di}{dt} dt \Rightarrow i = \frac{1}{L}\int_0^t v\,dt$

$\Rightarrow i = \frac{1}{5}\int_0^t 30t^2\,dt$

$= \frac{1}{5} \times \frac{30t^3}{3} = 2t^3$ A

$P = iv$
$= 2t^3 \times 30t^2$
$= 60t^5$ W

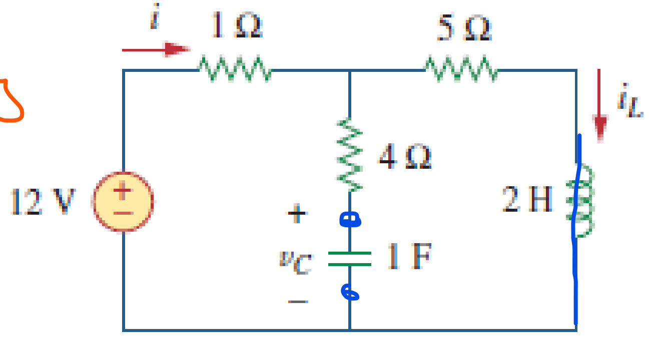
$W = \int_0^5 P\,dt$
$= 10t^6 \Big|_0^5 = 15625$ J

or,

$W\Big|_0^5 = \frac{1}{2} L\, i(5)^2 = 15625$ J

Inductor (Contd.)

Q. Consider the circuit in following Fig. Under dc conditions, find: (a) i, v_c and i_L (b) the energy stored in the capacitor and inductor.



as no current will flow throgh 4 ohm

a) $R_{eq} = 5 + 1 = 6\ \Omega$

$\therefore i = \frac{12}{6} = 2A$

Mesh a...,

$-12 + 1(2) + 4(0) + v_c = 0$

$\therefore V_c = 10V$

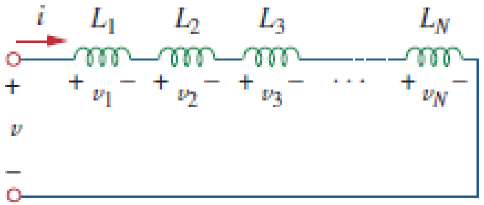
$i_L = i = 2A$

b) $W_c = \frac{1}{2} C v_c^2 = 50J$

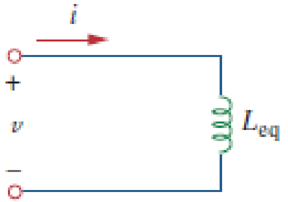
$W_L = \frac{1}{2} L i_L^2 = 4J$

Series Parallel Inductors

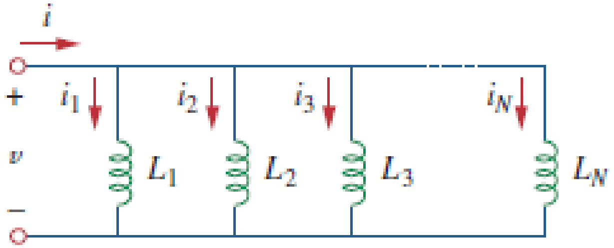
$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$



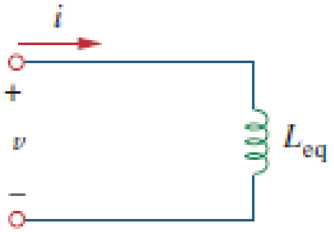
(a)



(b)



(a)

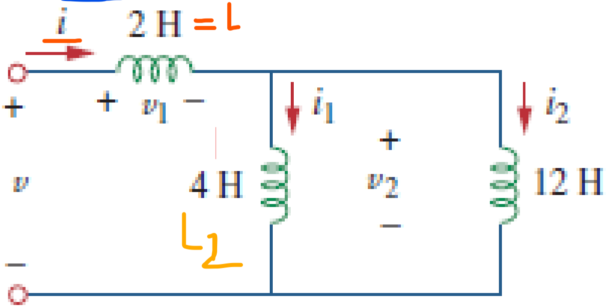


(b)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

Series Parallel Inductors (Contd.)

Q. For the circuit in the given figure $i(t) = 4\left(2 - e^{-10t}\right)mA$. If $i_2(0) = -1\ mA.$ Find (a) $i_1(0)$, (b) v(t), $v_1(t)$, $and\ v_2(t)$, (c) $i_1(t)$ and $i_2(t)$.



a) $i(0) - i_1(0) - i_2(0) = 0$ [KCL]

$\Rightarrow 4(2 - e^{-10\times 0}) - i_1(0) - (-1) = 0$

$\therefore i_1(0) = 5\ mA$

b) $V(t) = L_{eq} \times \frac{d\,i(t)}{dt}$

$= ((4 \| 12) + 2) \times \frac{d}{dt}\left[4(2 - e^{-10t})\right]$

$= 5 \times 4\,(10e^{-10t})$

$= 200\,e^{-10t}$ V

$V_1(t) = L \times \frac{d\,i(t)}{dt}$

$= 2 \times 4 \times 10 \times e^{-10t}$

$= 80\,e^{-10t}$ V

$V_2(t) = V(t) - V_1(t)$ [KVL]

$= 120\,e^{-10t}$ V

(c) $V_1(t) = L_2 \frac{d\,i_1(t)}{dt}$

$\therefore \int_0^t V_1(t)\,dt = L_2 \int_0^t \frac{d\,i_1(t)}{dt}\,dt = L_2\left[i_1(t) - i_1(0)\right]$

$\therefore i_1(t) = \frac{1}{L_2}\int_0^t V_2(t)\,dt + i_1(0)$

$= \frac{1}{4}\int_0^t 120\,e^{-10t}\,dt + 5$

$= \frac{120}{4}\left[\frac{e^{-10t}}{-10}\right]_0^t + 5$

$= 30\left[\frac{e^{-10t}}{-10} - \frac{1}{-10}\right] + 5$

$= -3(e^{-10t} - 1) + 5$

$= 8 - 3e^{-10t}$ A

$i_2(t) = i(t) - i_1(t)$ [KCL]

$= (4(2 - e^{-10t})) - (8 - 3e^{-10t})$

$= -e^{-10t}$ A

Thank You