Course Code: EEE 4101
Course Title: Electrical Engineering

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Ref. Book: Fundamental of Electrical Circuit: Alexander Sadiku

unit " t " == turn(s)
unit " Wb " == weber
unit " H " == henry
unit " T " == tesla

Magnetic Circuit

The closed path for magnetic flux is called magnetic circuit.

A **magnetic circuit** is made up of magnetic materials having high permeability such as iron, soft steel, etc. **Magnetic circuits** are used in various devices like electric motor, transformers, relays, generators galvanometer, etc.

Unit:
rels
or,
A t / Wb

Analogous to Resistance (R)

Magnetic reluctance which is also known as reluctance, magnetic insulator, or a magnetic resistance is defined as the resistance provide by a magnetic circuit to the flow or production of magnetic flux (magnetic field lines). It is the property of the material that opposes the creation of magnetic flux in a magnetic circuit.

Unit:
H / m
or,
Wb / A m

Analogous to Conductivity (sigma) σ

The **permeability** (μ) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material.

Analogous to Current (I)

φ

The magnetic pressure, which sets up the magnetic flux in a magnetic circuit is called **Magnetomotive Force** (MMF).

Unit:
Wb
or,
T m^2

Unit:
A t

Analogous to Voltage or, EMF (electromotive force) (V)

OHM's Law for Magnetic Circuit and Amperes Circuital Law

Ohm's law for magnetic circuits states that the MMF is directly proportional to the magnetic flux.

mmf

$$F \propto \varphi$$

same as V $\propto$ I

$$=> F = S\varphi$$

$\Rightarrow V = RI$

Where, S= reluctance (the constant of proportionality)

the reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

same as, $R = \frac{L}{\sigma A}$

$$S = \frac{l}{\mu A} \text{ rels}$$

?

l is the length of the magnetic path, and A is the cross-sectional area

Amperes circuital law states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

$$\sum mmf = 0$$

same as, $\Sigma E = 0$ [KVL]

Magnetically Coupled Circuit

The magnetomotive force is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig. 1).

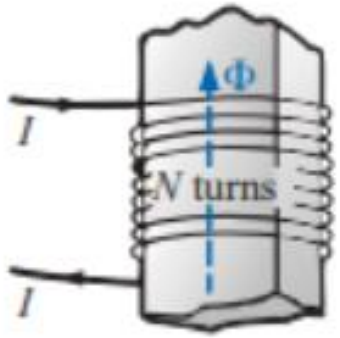


Fig. 1

* $\boxed{\text{m.m.f=NI}}$

The magnetomotive force per unit length is called the magnetizing force (H). In equation form

$$* \boxed{H = \frac{m.m.f}{l}} \left(\frac{At}{m}\right)$$

$$* \boxed{H = \frac{Ni}{l}}$$

The flux density and the magnetizing force are related by the following equation: $B = \mu H$ [unit: Wb m^{-2}]

or,

T (tesla)

Magnetically Coupled Circuit

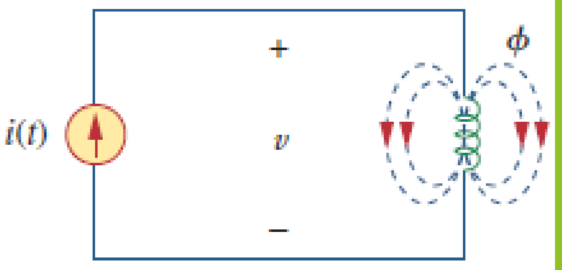
The circuits we have considered so far may be regarded as *conductively coupled*, because one loop affects the neighboring loop through current conduction.

When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled*.

MUTUAL INDUCTANCE

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as *mutual inductance*.

Let us first consider a single inductor, a coil with N turns. When current i flows through the coil, a magnetic flux φ is produced around it. According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of the magnetic flux φ; that is,



$$\boxed{v = N\frac{d\phi}{dt}}$$

Magnetically Coupled Circuit

But the flux is produced by current *i* so that any change in ϕ is caused by a change in the current.

$$v = N\frac{d\phi}{di}\frac{di}{dt}$$

$$\boxed{v = L\frac{di}{dt}}$$

This inductance is commonly called *self-inductance*, because it relates the voltage induced in a coil by a time-varying current in the same coil.

Magnetically Coupled Circuit

Now consider two coils with self-inductances L_1 and L_2 that are in close proximity with each other (Fig.) Coil 1 has N_1 turns, while coil 2 has N_2 turns. For the sake of simplicity, assume that the second inductor carries no current. The magnetic flux ϕ_1 emanating from coil 1 has two components: One component ϕ_{11} links only coil 1, and another component ϕ_{12} links both coils. Hence,

$$\phi_1 = \phi_{11} + \phi_{12} \qquad 1$$

Although the two coils are physically separated, they are said to be *magnetically coupled*. Since the entire flux ϕ_1 links coil 1, the voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_1}{dt} \qquad 2$$

Only flux ϕ_{12} links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_{12}}{dt} \qquad 3$$

Again, as the fluxes are caused by the current i_1 flowing in coil 1, Eq. (2) can be written as

$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

where $L_1 = N_1\, d\phi_1/di_1$ is the self-inductance of coil 1. Similarly, Eq. (3) can be written as

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

where

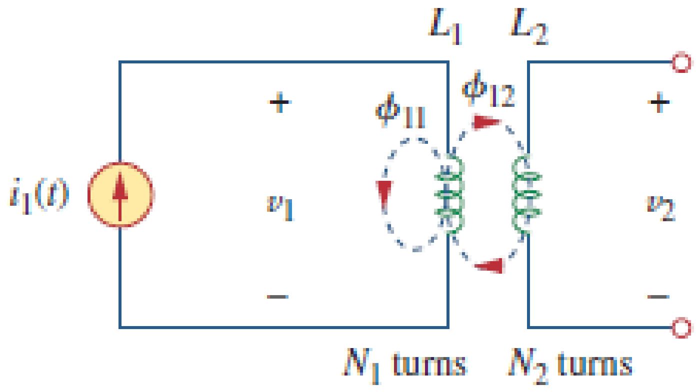
$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

Φ_{AB}

flux induced circuit

flux producing circuit

Magnetic Flux of circuit/coil A links circuit/coil B



M_{AB}

inductance producing circuit

inductance induced circuit

Mutual Inductance of circuit/coil A caused by circuit/coil B

Magnetically Coupled Circuit

M_{21} is known as the *mutual inductance* of coil 2 with respect to coil 1. Subscript 21 indicates that the inductance M_{21} relates the voltage induced in coil 2 to the current in coil 1. Thus, the open-circuit *mutual voltage* (or induced voltage) across coil 2 is

$$\boxed{v_2 = M_{21}\frac{di_1}{dt}}$$

Suppose we now let current i_2 flow in coil 2, while coil 1 carries no current (Fig. 13.3). The magnetic flux ϕ_2 emanating from coil 2 comprises flux ϕ_{22} that links only coil 2 and flux ϕ_{21} that links both coils. Hence,

$$\phi_2 = \phi_{21} + \phi_{22}$$

The entire flux ϕ_2 links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2\frac{d\phi_2}{dt} = N_2\frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2\frac{di_2}{dt}$$

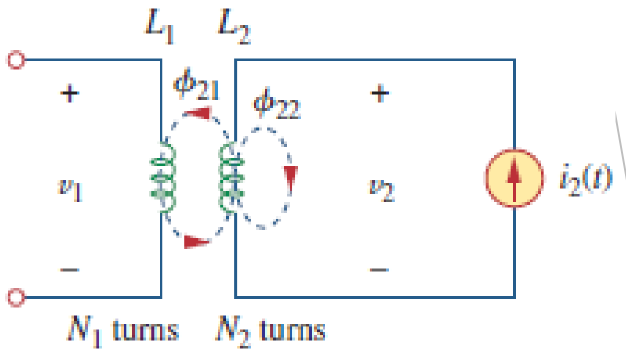
where $L_2 = N_2\, d\phi_2/di_2$ is the self-inductance of coil 2. Since only flux ϕ_{21} links coil 1, the voltage induced in coil 1 is

$$v_1 = N_1\frac{d\phi_{21}}{dt} = N_1\frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt}$$

where

$$M_{12} = N_1\frac{d\phi_{21}}{di_2}$$

which is the *mutual inductance* of coil 1 with respect to coil 2.



Magnetically Coupled Circuit

Thus, the open-circuit *mutual voltage* across coil 1 is

$$v_1 = M_{12}\frac{di_2}{dt}$$

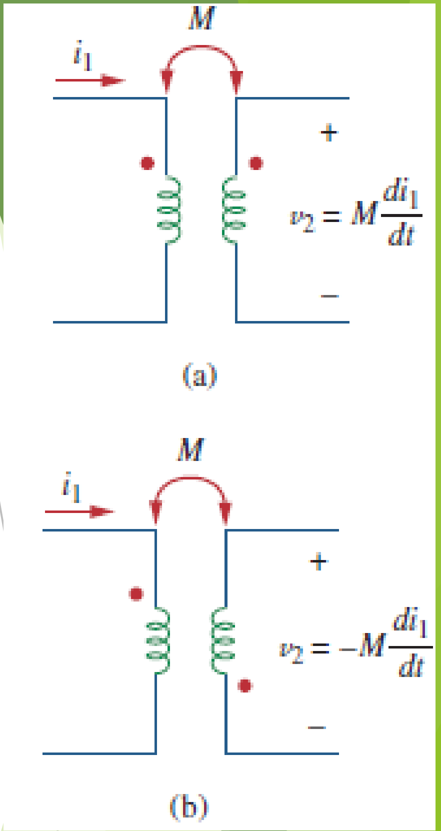
Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

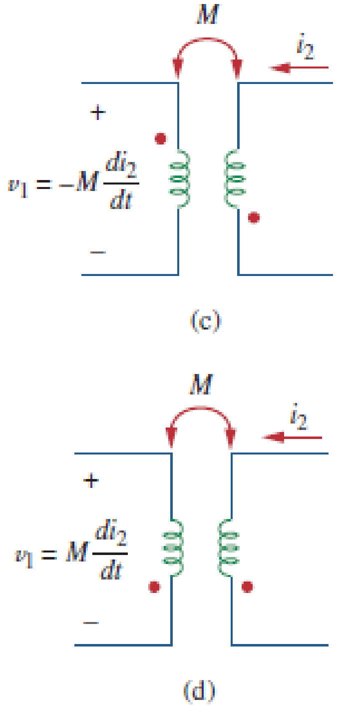
Dot Convention

The *dot convention* is a technique which gives the details about voltage polarity at the dotted terminal. This information is important for getting KVL equation for circuit analysis

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

Alternatively, If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.





Dot Convention

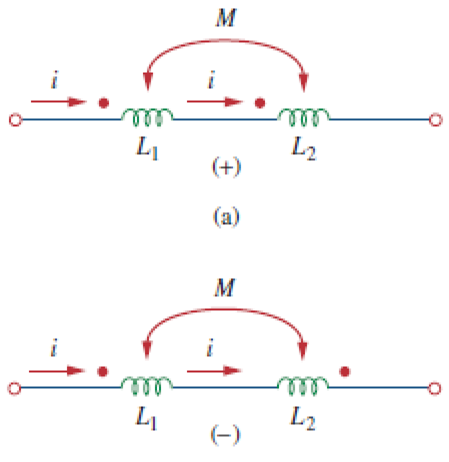


Figure
Dot convention for coils in series; the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

Figure shows the dot convention for coupled coils in series. For the coils in Fig. (a), the total inductance is

$$\boxed{L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}} \tag{13.18}$$

For the coils in Fig. (b),

$$\boxed{L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}} \tag{13.19}$$

Now that we know how to determine the polarity of the mutual voltage, we are prepared to analyze circuits involving mutual inductance. As the first example, consider the circuit in Fig. (a). Applying KVL to coil 1 gives

$$v_1 = i_1R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

For coil 2, KVL gives

$$v_2 = i_2R_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

We can write

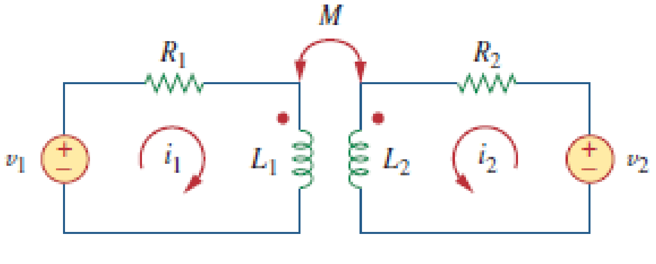
$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M\mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2$$

As a second example, consider the circuit in Fig. 13.7(b). We analyze this in the frequency domain. Applying KVL to coil 1, we get

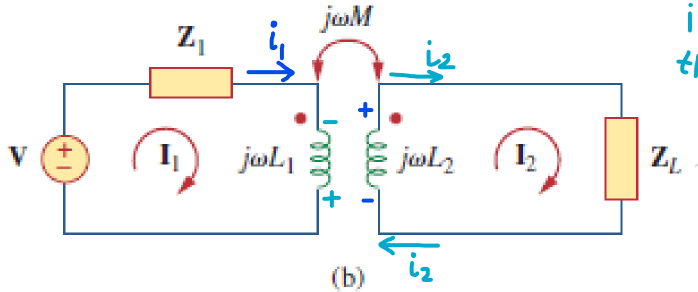
$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2 \tag{13.22a}$$

For coil 2, KVL yields

$$0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2 \tag{13.22b}$$



(a)



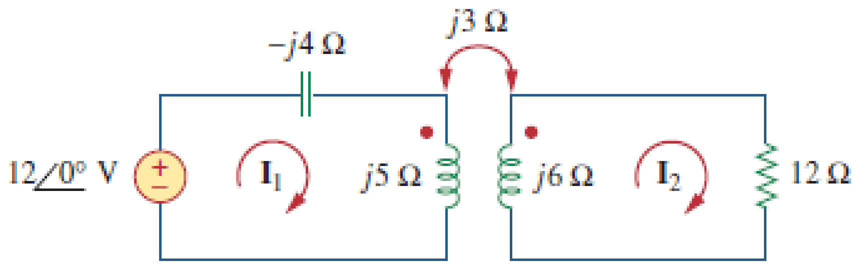
(b)

if i flows to the (-) of a circuit, then the other circuit's (-) will be (+), otherwise (-)

If current enters through both of the dots, M (+)
if differs, M (-)

Dot Convention

Q. Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig.



For loop 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

or

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12 \quad \text{①}$$

$$\begin{bmatrix} j & -j3 \\ -j3 & 12+j6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

For loop 2, KVL gives

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0 \quad \text{②}$$

or

$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2$$

Substituting this in Eq. (13.1.1), we get

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

or

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\angle 14.04° \text{ A}$$

From Eqs. (13.1.2) and (13.1.3),

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\angle -63.43°)(2.91\angle 14.04°)$$
$$= 13.01\angle -49.39° \text{ A}$$

Coefficient of Coupling

★★★ Proof!

It is defined as the ratio of mutual flux to the self flux. It is denoted by symbol “k”

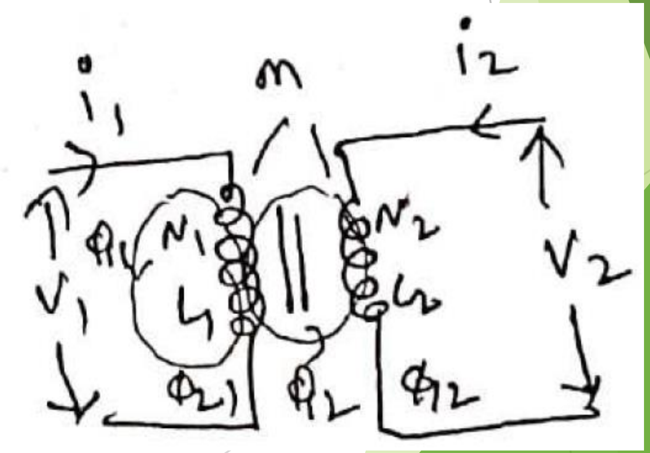
$$k = \frac{\varphi_{12}}{\varphi_{11}} \, or \, \frac{\varphi_{21}}{\varphi_{22}}$$

Properties:

1. It is unitless.
2. If k=0, there is no coupling between two coils.
3. If k=1, ideal coupling.
4. The range of k lies between 0 to 1.
5. If k decreases then distance between two coils increases.
6. k<0.5 => loosely coupled & k>0.5 => tightly coupled

Derivation:

Consider two coils which are magnetically coupled as shown in figure



For coil 1

Self inductance $L_1 = \frac{N_1 d\Phi_{11}}{di_1} = \frac{N_1 \Phi_{11}}{i_1}$

mutual inductance $m = \frac{N_2 d\Phi_{12}}{di_1} = \frac{N_2 \Phi_{12}}{i_1}$

For coil 2

self inductance $L_2 = \frac{N_2 d\Phi_{22}}{di_2} = \frac{N_2 \Phi_{22}}{i_2}$

mutual inductance $m = \frac{N_1 d\Phi_{21}}{di_2} = \frac{N_1 \Phi_{21}}{i_2}$

Coefficient of Coupling

mutual inductance of both coils

$$m \cdot m = \frac{N_2 \phi_{12}}{i_1} \times \frac{N_1 \phi_{21}}{i_2} \quad -(1)$$

but we know $K = \frac{\phi_{12}}{\phi_{11}}$, $K = \frac{\phi_{21}}{\phi_{22}}$ $-(2)$

substitute eq (2) in eq (1)

$$m^2 = \frac{N_1 N_2 (K\phi_{11})(K\phi_{22})}{i_1 \cdot i_2}$$

$$= K^2 \left(\frac{N_1 \phi_{11}}{i_1}\right)\left(\frac{N_2 \phi_{22}}{i_2}\right)$$

$$m^2 = K^2 L_1 L_2$$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}} \quad \text{or} \quad \boxed{m = K \cdot \sqrt{L_1 L_2}}$$

Coefficient of Coupling

→ Two inductive coupled coils have self inductance $L_1 = 50\,mH$ $L_2 = 200\,mH$. with the coefficient of coupling is 0.5

(i) Find mutual inductance.

(ii) what is the maximum possible value of m.

$L_1 = 50\,mH$, $L_2 = 200\,mH$, $K = 0.5$

(i) we know

$$K = \frac{m}{\sqrt{L_1 L_2}}$$

$$m = K \cdot \sqrt{L_1 L_2} = 0.5 \sqrt{50 \times 200 \times 10^{-6}}$$

$$= 0.5 \times 0.1$$

$$= 0.05\,H$$

$$\boxed{m = 50\,mH}$$

(ii) To obtain maximum possible value of m, (put K=1)

$$m = K \sqrt{L_1 L_2}$$

$$= 1 \times \sqrt{50 \times 10^{-3} \times 200 \times 10^{-3}}$$

$$= 0.1$$

$$\boxed{m = 100\,mH}$$

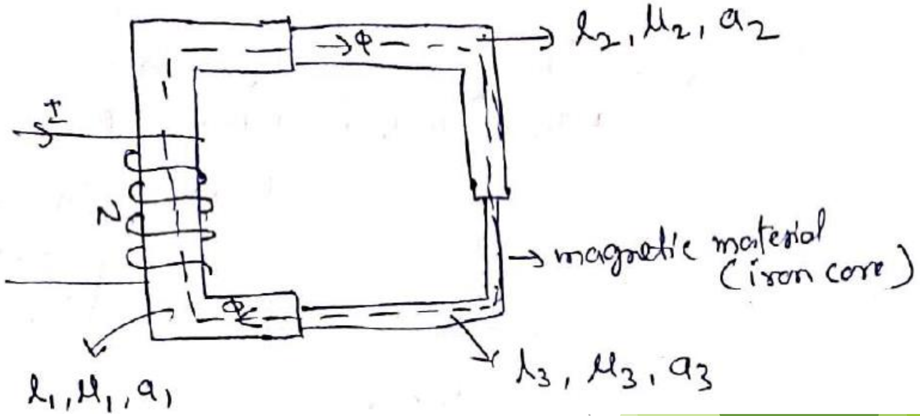
Series Magnetic Circuit

Def :

magnetic circuit having a number of different dimensions of magnetic materials (iron core) and are in series the materials carrying the same magnetic field is called series magnetic ~~field~~ circuit.

Explanation

consider a three different dimentions of magneti materials are connected in series which is shown in fig. current I is passed through one limb of magnetic ckt, ϕ is the flux setup in the material. In this ckt, l_1, l_2, l_3 are the length of the magnetic material. a_1, a_2, a_3 are the areas of three magnetic materials and μ_1, μ_2, μ_3 are the relative permeability of the three materials. l_g, a_g are length & area of air gap



Series Magnetic Circuit

Now, the total reluctance (S) of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g$$

$$S = \frac{l_1}{\mu_0 \mu_{r_1} a_1} + \frac{l_2}{\mu_0 \mu_{r_2} a_2} + \frac{l_3}{\mu_0 \mu_{r_3} a_3} + \frac{l_g}{\mu_0 a_g} \quad \text{—(1)}$$

[$S = \frac{l}{\mu a}$

$= \frac{l}{\mu_0 \mu_r a}$

μ_0 → Permeability of free space.]

$\mu_r = 1$ for air gap.

Now,
we know
mmF = magneto motive force

$= \phi \times S$

$$\text{mmF} = \frac{\phi l_1}{\mu_0 \mu_{r} a_1} + \frac{\phi l_2}{\mu_0 \mu_{r_2} a_2} + \frac{\phi l_3}{\mu_0 \mu_{r_3} a_3} + \frac{\phi l_g}{\mu_0 a_g} \quad \text{—(2)}$$

Also ~~we~~ we know

magnetic Flux Density $B = \frac{\phi}{a}$ wb/m² or Tesla. —(3)

A (area)

Now, eq (2) becomes

$$\text{mmF} = \frac{B_1 l_1}{\mu_0 \mu_{r_1}} + \frac{B_2 l_2}{\mu_0 \mu_{r_2}} + \frac{B_3 l_3}{\mu_0 \mu_{r_3}} + \frac{B_g l_g}{\mu_0} \quad \text{—(4)}$$

Also

$B = \mu H$, $H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r}$ —(5)

sub eq(5) in eq (4), then eq (4) becomes

$$\boxed{\text{mmF} = H_1 \times l_1 + H_2 \times l_2 + H_3 \times l_3 + H_g \times l_g} \quad \text{—(6)}$$

magnetic field intensity (H) → AT/m

Series Magnetic Circuit

Problem on series magnetic ckt

① An iron ring has a cross section area $2\,cm^2$ and a mean diameter of 20cm. An airgap of 0.4 mm has been cut across the section of the ring. The ring is wound with a coil of 300 turn. The total magnetic flux is 0.20 mwb. The relative permeability of iron is 1000. Find the value of current passed in turn.

Given

$a = 2\,cm^2 = 2\times10^{-4}\,m^2$

$D_m = 20\,cm = 20\times10^{-2}\,m$

$l_T = 2\pi r = \pi D_m = \pi\times20\times10^{-2}$

$l_T = 0.628\,m$, $l_g = 0.4\,mm = 0.4\times10^{-3}\,m$

$N = 300$

$\Phi = 0.2\,mwb = 0.2\times10^{-3}\,wb$.

$\mu_{ri} = 1000$, $I = ?$

$(mmF)_T = (mmF)_i + (mmF)_g$

$N\cdot I = H_i\times l_i + H_g\times l_g$ —①

we know, $B = \mu H$, $H = \frac{B}{\mu} = \frac{B}{\mu_0\mu_r}$

for gap or air $\mu_r = 1$

$$N\cdot I = \frac{B}{\mu_0\mu_{ri}}(l_T - l_g) + \frac{B}{\mu_0}l_g \quad \left(\begin{array}{c}\text{for gap}\\ \mu_r = 1\end{array}\right)$$

also we know, $B = \frac{\Phi}{a}$

$$N\cdot I = \frac{\Phi}{\mu_0\mu_{ri}a}(l_T - l_g) + \frac{\Phi}{\mu_0 a}l_g$$

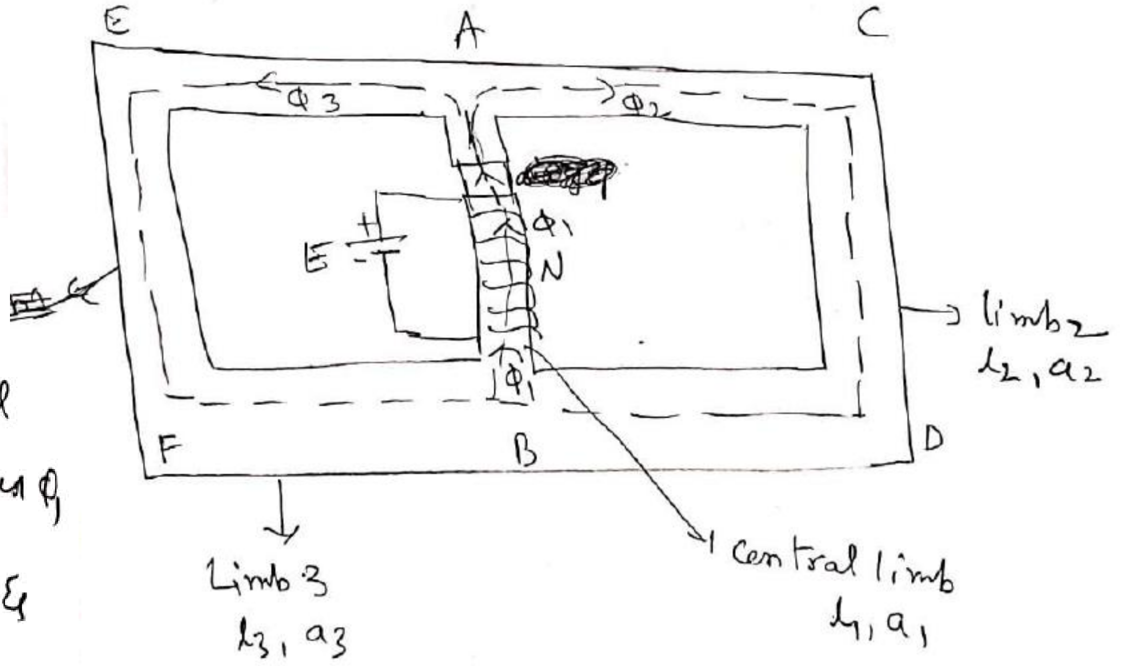
$$300\times I = \frac{0.2\times10^{-3}(0.628 - 0.4\times10^{-3})}{4\pi\times10^{-7}\times1000\times2\times10^{-4}} + \frac{0.2\times10^{-3}}{4\pi\times10^{-7}\times2\times10^{-4}}$$

$$\boxed{I = \frac{817.73}{300} = 2.72\,A}$$

Parallel Magnetic Circuit

Def: A magnetic circuit having two or more than two paths for the magnetic flux is called parallel magnetic circuit.

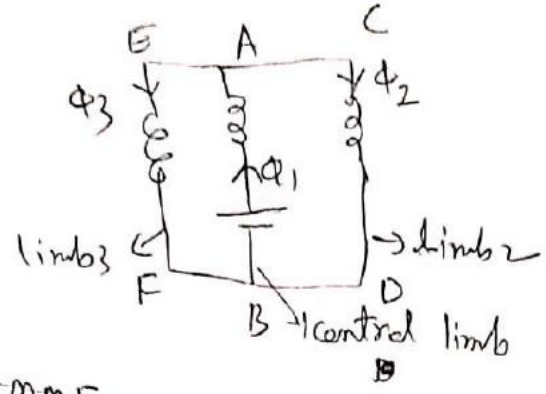
— The parallel magnetic circuit contains different dimensional areas and materials having various number of paths.



In the above ckt, the current carrying coil is wound on the central limb. This coil sets up the magnetic flux ϕ_1 in the central limb.

This flux ϕ_1 is devided into two fluxes i.e ϕ_2 & ϕ_3. $\therefore \phi_1 = \phi_2 + \phi_3$.

Parallel Magnetic Circuit



~~Total mmF = mmF of~~

Reluctance of $\boxed{\text{AB path } (S_{AB}) = \dfrac{l_1}{\mu_0 \mu_{r_1} a_1}}$

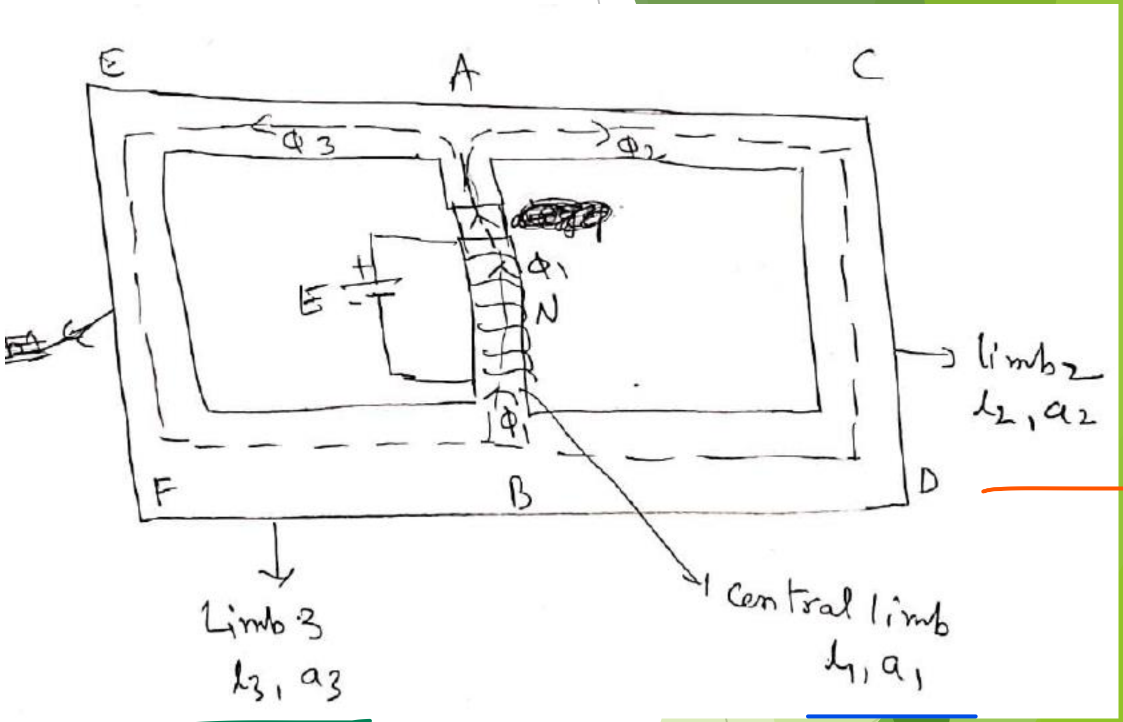
" $\boxed{\text{ABCD } (S_{ABCD}) = \dfrac{l_2}{\mu_0 \mu_{r_2} a_2}}$

" $\boxed{\text{ABFE } (S_{ABFE}) = \dfrac{l_3}{\mu_0 \mu_{r_3} a_3}}$

Total mmF required = mmF path AB + (mmF path ABDC) or (mmF path ABFE)

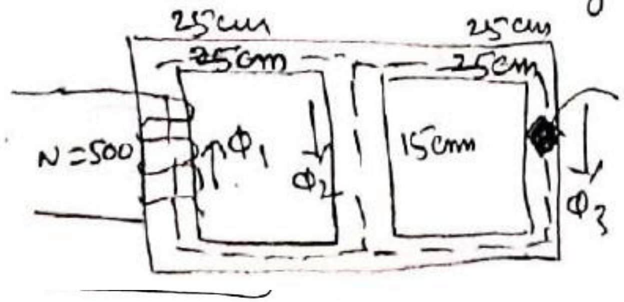
$$\boxed{mmF_{Total} = \Phi_1 S_{AB} + (\Phi_2 S_{ABCD} \text{ or } \Phi_3 S_{ABFE})}$$

where $\Phi_2 S_{ABCD} = \Phi_3 S_{ABFE}$

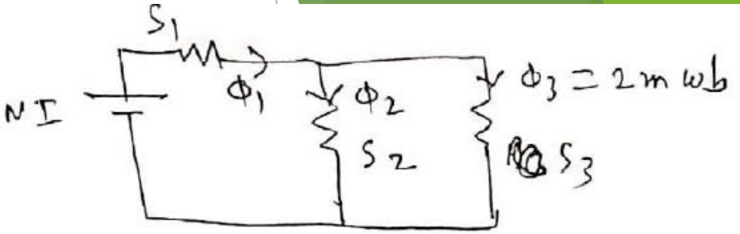


Parallel Magnetic Circuit

① A cast steel magnetic structure made for a bar of Section 2cm x 2cm. Determine the current that a 500 turn coil on the left limb should carry so that a flux of 2 mwb is produced in the right limb. Take $\mu_r = 600$. Neglect leakage.



$A = 2 \times 2 cm$
$= 4 cm^2$
$= 4 \times 10^{-4} m^2$



$MMF = NI = S\phi$
$= \frac{l}{\mu A}\phi = \frac{l}{\mu_0 \mu_r A}\phi$

$\frac{S_2}{S_3} = \frac{15}{25}$

$\phi_3 = 2 mwb$

$\phi_2 = 2 \times \frac{5}{3} = \frac{10}{3} = 3.33 mwb$

$\phi_1 = \phi_2 + \phi_3$
$= 2 + 3.33$
$\boxed{\phi_1 = 5.33 \ mwb}$

$MMF_{left} = AT_{left} = \phi_1 S_1$
$= \frac{5.33 \times 10^{-3} \times 25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$
$= 4420 AT$

$MMF_{right} = AT_{Right} = \phi_3 S_3$
$= \frac{2 \times 10^{-3} \times 25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$
$= 1658 AT$

$AT_{total} = 4420 + 1658$
$NI_{total} = 6078 AT$.

If $N = 500$
$I = \frac{6078}{500} = 12.15 Amp$

Thank You