Solution

1.

[**A**nswer:

(a) No. $F1 : A^* = AC, B^* = ABC, C^* = C$; $F2 : A^* = A, B^* = ABC, C^* = C$
Hence, $F1 \Rightarrow F2$, but $F2 \not\Rightarrow F1$ since $A \to C$ does not hold based on $A^* = A$ in $F2$.

(b) No. $F1 : A^* = AC, B^* = ABC, C^* = C$; $F3 : A^* = A, B^* = ABC, C^* = C$.
Hence, $F1 \Rightarrow F3$, but $F3 \not\Rightarrow F1$ since $A \to C$ does not hold based on $A^* = A$ in $F3$.

(c) Yes. $F2 : A^* = A, B^* = ABC, C^* = C$; $F3 : A^* = A, B^* = ABC, C^* = C$. Hence, $F2 \Rightarrow F3$, and $F3 \Rightarrow F2$.

2.

Consider a relation R(A, B, C), satisfying some functional dependency. Two instances of R are given as below:

A	B	C
2	3	1
2	2	4

A	B	C
2	2	1
3	3	2
4	2	1

Based on R's schema, enumerate all possible completely nontrivial functional dependencies (FDs) with only a single attribute on the right-hand side. Then, based on the instances above, for each FD you listed, label whether it:
H: Definitely holds in R.
NH: Definitely does not hold in R.
CD: Cannot be determined from the information given whether or not it holds in R.

- $B \to A$: NH in R.
- $C \to A$: NH in R.
- $BC \to A$: NH in R.
- $A \to B$: NH in R.
- $C \to B$: CD in R.
- $AC \to B$: CD in R.
- $A \to C$: NH in R.

3.

Consider the following relational schema for a chain store:
Sale(clerk, store, city, date, dish, size)
// a clerk sold a dish on a particular day at a given store in a city
Menu(dish, size, price)
// prices and available size for the dish

Make the following assumptions:

- Each clerk works in one store.
- Each store is in one city.
- The price of a dish is different for different sizes. The store has standardized prices: the same sized dish cannot be sold to two persons at two different prices.

1. Specify a set of completely nontrivial functional dependencies for relations Sale and Menu that encodes the assumptions described above and no additional assumptions.
   $clerk \rightarrow store$
   $store \rightarrow city$
   $dish, size \rightarrow price$

2. Based on your functional dependencies in part (1), specify all minimal keys for relations Sale and Menu.
   Sale: {clerk, date, dish, size}
   Menu: {dish, size}

3. Are the schema of Sale and Menu in Boyce-Codd Normal Form (BCNF) according to your answers to (1) and (2)? If not, give a decomposition into BCNF. If yes, justify your answer.
   For Menu, it is in Boyce-Codd Normal Form (BCNF). However, Sale is not in BCNF.
   Menu: The closure for the minimal key in previous question is {dish, size, price}. It contains all the attributes in Menu schema.
   Sale:
   - {clerk, store}
   - {store, city}
   - {clerk, date, dish, size}

4. Now add the following assumption:
   - Each city has at most one store and each store has only one clerk.

Specify additional functional dependencies to take these new assumptions into account.
Answer: $city \rightarrow store$, $store \rightarrow clerk$

5. Based on your functional dependencies for parts (1) and (4) together, specify all minimal keys for relation Sale.
   Sale:
   - {clerk, date, dish, size }
   - {store, date, dish, size }
   - {city, date, dish, size }

6. Are the schema of Sale and Menu in 3NF according to your answers to (1), (4) and (5)? If not, give a decomposition into 3NF. If yes, justify your answer.
   Answer: Sale is not 3NF, Menu is 3NF.
   Sale: (clerk, store), (store, city), (clerk, date, dish, size)
   Menu: $dish, size \rightarrow price$, so (dish, size, price) is the minimal-key, the result from step 2. We do not need decompose this.