CHAPTER 6

CONTINUOUS RANDOM VARIABLES AND THE NORMAL DISTRIBUTION

Prem Mann, *Introductory Statistics*, *7/E*

Opening Example

Have you ever participated in a road race? If you have, where did you stand in comparison to the other runners? Do you think the time taken to finish a road race varies as much among runners as the runners themselves? See Case Study 6-1 for the distribution of times for runners who completed the Manchester (Connecticut) Road Race in 2008.

6.1 Continuous Probability Distribution

Height of a Female Student (inches) x	f	Relative Frequency
60 to less than 61	90	.018
61 to less than 62	170	.034
62 to less than 63	460	.092
63 to less than 64	750	.150
64 to less than 65	970	.194
65 to less than 66	760	.152
66 to less than 67	640	.128
67 to less than 68	440	.088
68 to less than 69	320	.064
69 to less than 70	220	.044
70 to less than 71	180	.036
	$N = 5000$	Sum = 1.0

Figure 6.1 Histogram and polygon for Table 6.1.

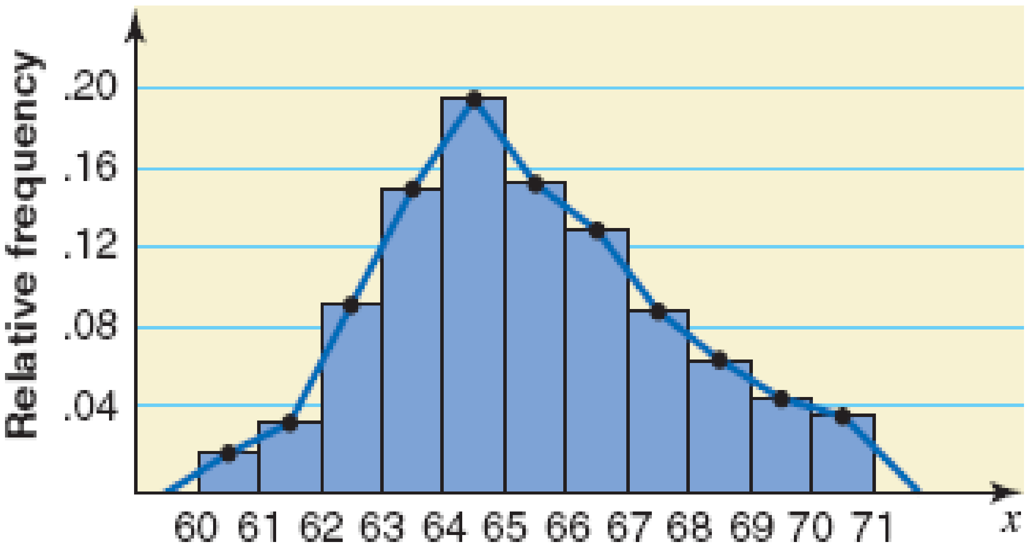
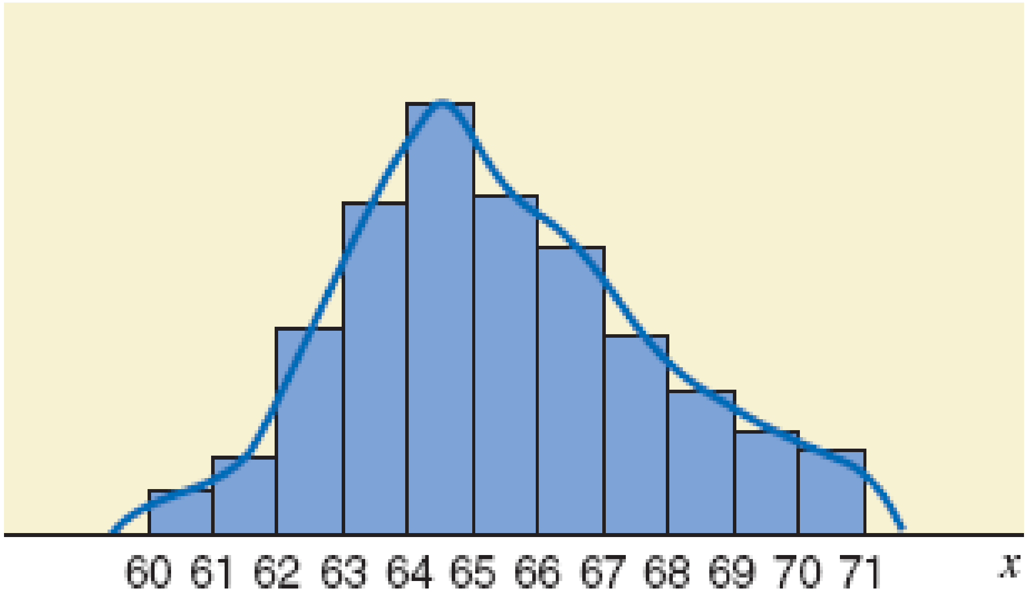


Figure 6.2 Probability distribution curve for heights.



CONTINUOUS PROBABILITY DISTRIBUTION

Two characteristics

1. The probability that x assumes a value in any interval lies in the range 0 to 1
2. The total probability of all the (mutually exclusive) intervals within which x can assume a value of 1.0

Figure 6.3 Area under a curve between two points.

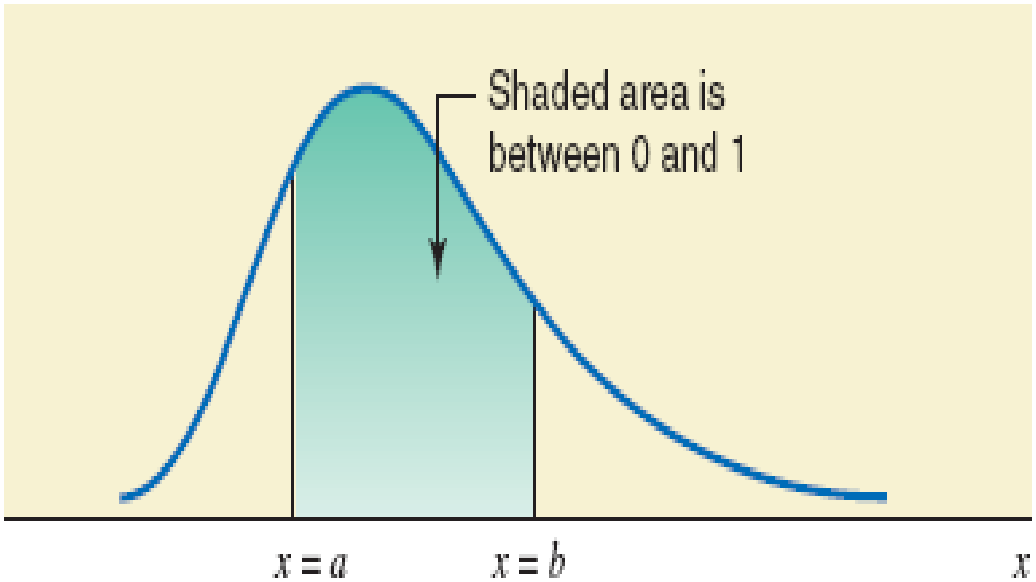


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Figure 6.4 Total area under a probability distribution curve.

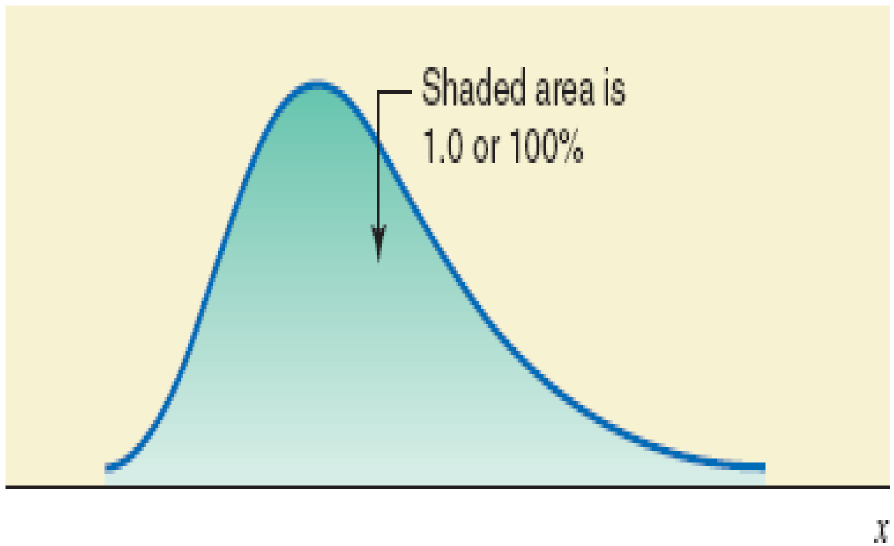

Figure 6.5 Area under the curve as probability.

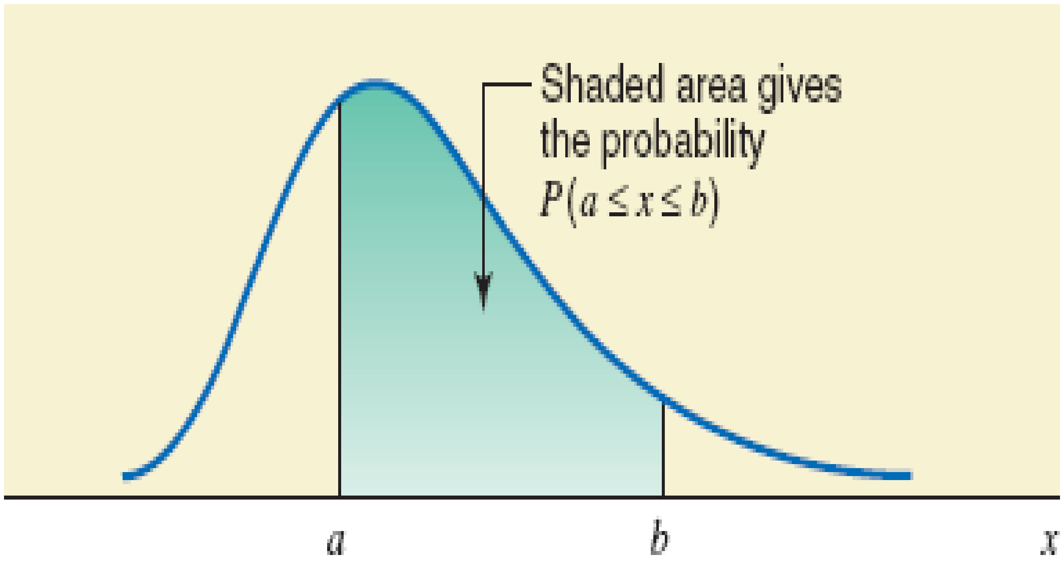

Figure 6.6 Probability that *x* lies in the interval 65 to 68 inches.

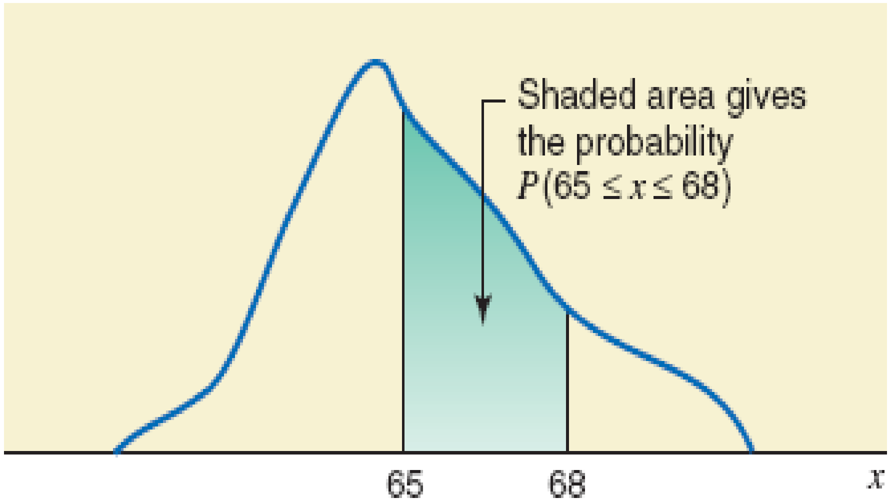

Figure 6.6 Probability that x lies in the interval 65 to 68.

Figure 6.7 The probability of a single value of *x* is zero.

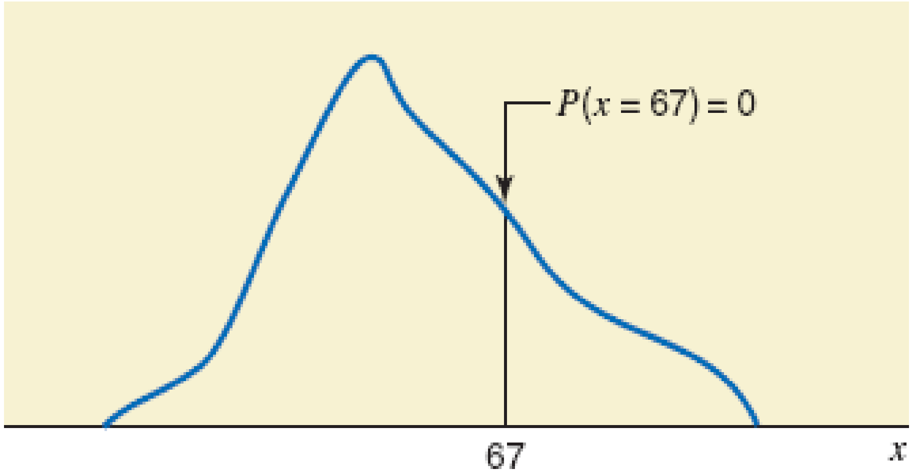

Figure 6.7 The probability of a single value of x is zero.

Figure 6.8 Probability "from 65 to 68" and "between 65 and 68".

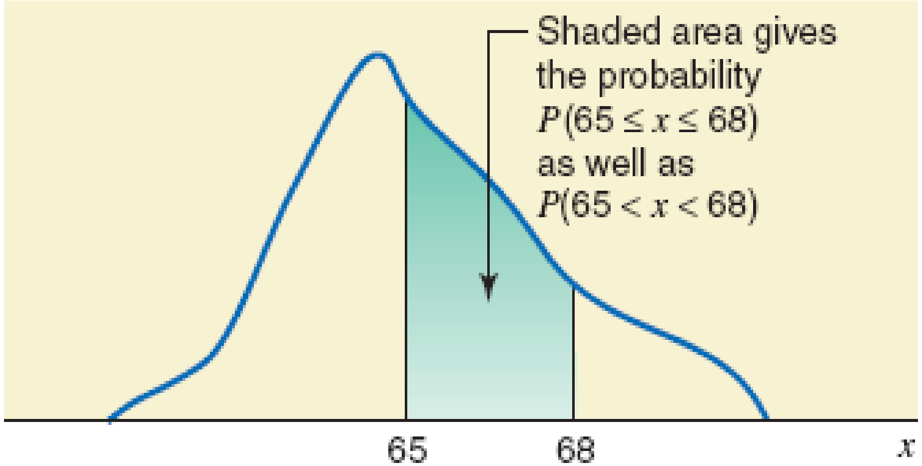

Figure 6.8 Probability "from 65 to 68" and "between 65 and 68."

Case Study 6-1 Distribution of Time Taken to Run a Road Race

Class	Frequency	Relative Frequency
20 to less than 25	30	.0029
25 to less than 30	208	.0199
30 to less than 35	771	.0739
35 to less than 40	1099	.1054
40 to less than 45	1137	.1090
45 to less than 50	1660	.1591
50 to less than 55	1751	.1679
55 to less than 60	1346	.1290
60 to less than 65	800	.0767
65 to less than 70	419	.0402
70 to less than 75	313	.0300
75 to less than 80	238	.0228
80 to less than 85	178	.0171
85 to less than 90	178	.0171
90 to less than 95	149	.0143
95 to less than 100	107	.0103
100 to less than 105	23	.0022
105 to less than 110	16	.0015
110 to less than 115	8	.0008
	Σf = 10,431	Sum = 1.0001

Case Study 6-1 Distribution of Time Taken to Run a Road Race

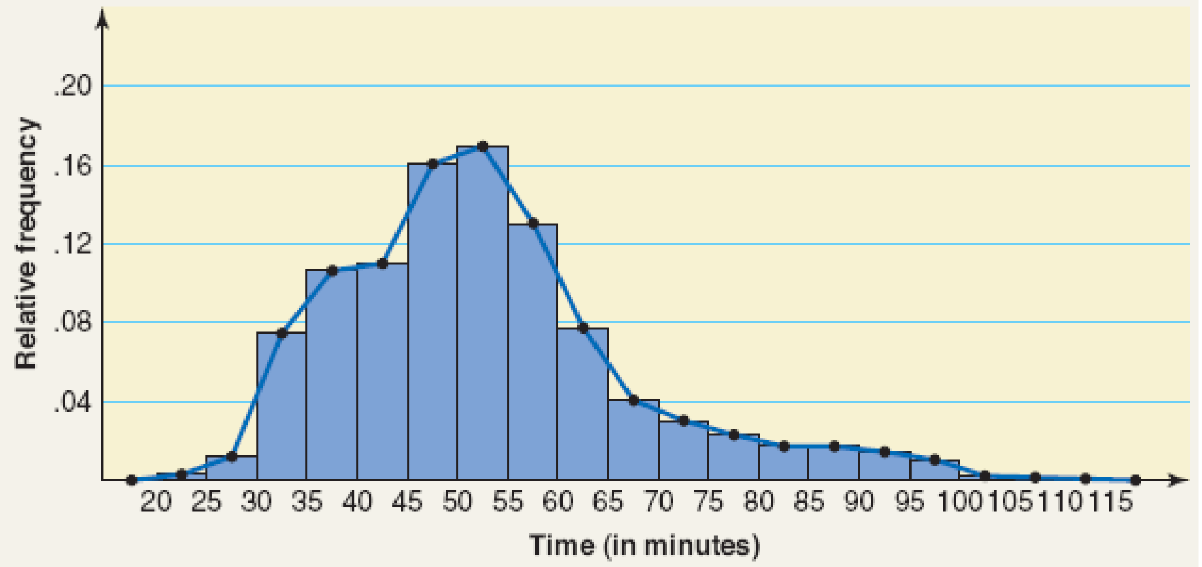

Figure 6.9 Histogram and polygon for the Road Race data.

Case Study 6-1 Distribution of Time Taken to Run a Road Race

Class	Relative Frequency Density
20 to less than 25	.00058
25 to less than 30	.00398
30 to less than 35	.01478
35 to less than 40	.02108
40 to less than 45	.02180
45 to less than 50	.03182
50 to less than 55	.03358
55 to less than 60	.02580
60 to less than 65	.01534
65 to less than 70	.00804
70 to less than 75	.00600
75 to less than 80	.00456
80 to less than 85	.00342
85 to less than 90	.00342
90 to less than 95	.00286
95 to less than 100	.00206
100 to less than 105	.00044
105 to less than 110	.00030
110 to less than 115	.00016

Case Study 6-1 Distribution of Time Taken to Run a Road Race

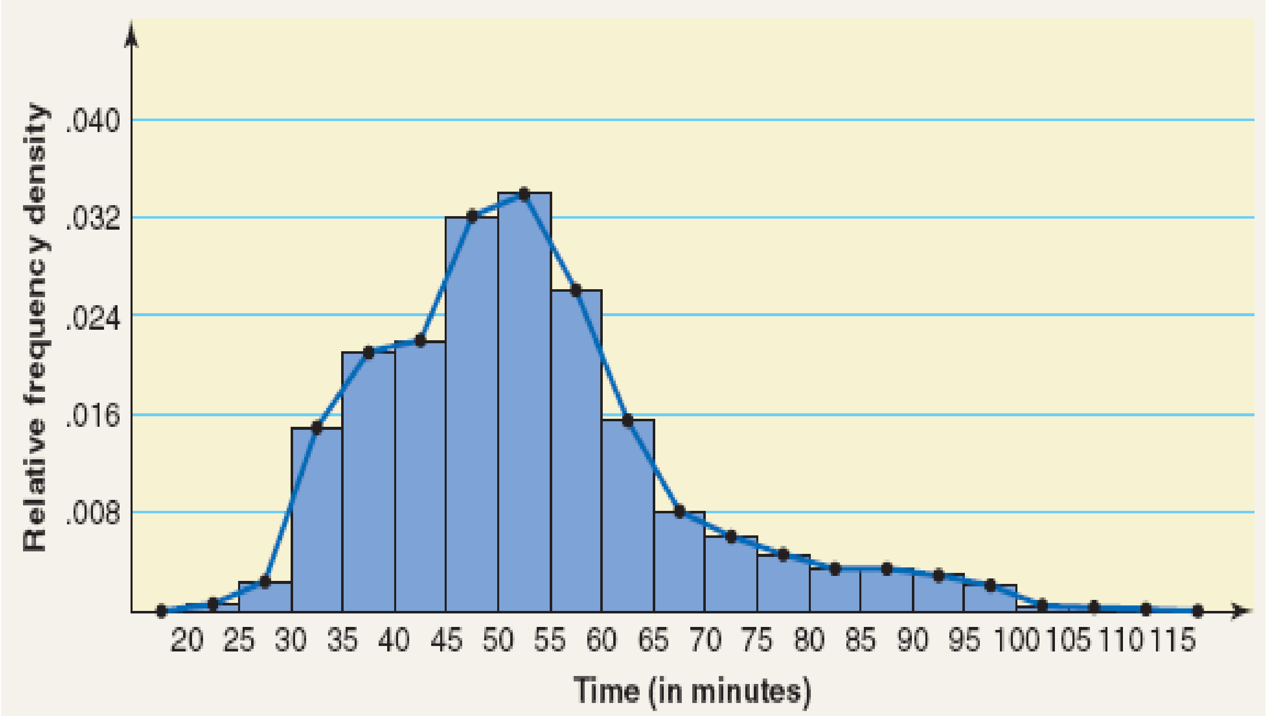

Figure 6.10 Probability distribution curve for the Road Race data.

6.2 The Normal Distribution

Normal Probability Distribution

A ***normal probability distribution*** , when plotted, gives a bell-shaped curve such that:

1. The total area under the curve is 1.0.
2. The curve is symmetric about the mean.
3. The two tails of the curve extend indefinitely.

Figure 6.11 Normal distribution with mean μ and standard deviation σ.

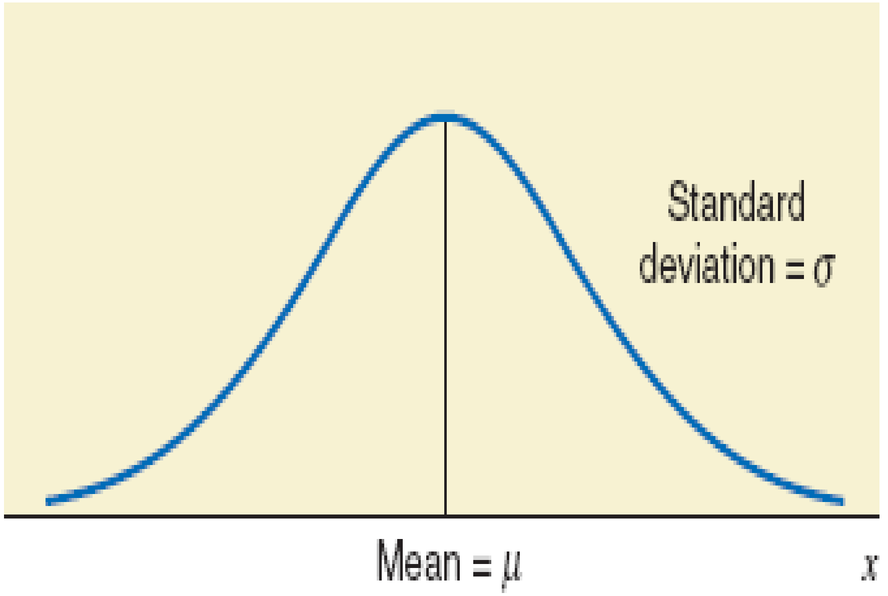

Figure 6.12 Total area under a normal curve.

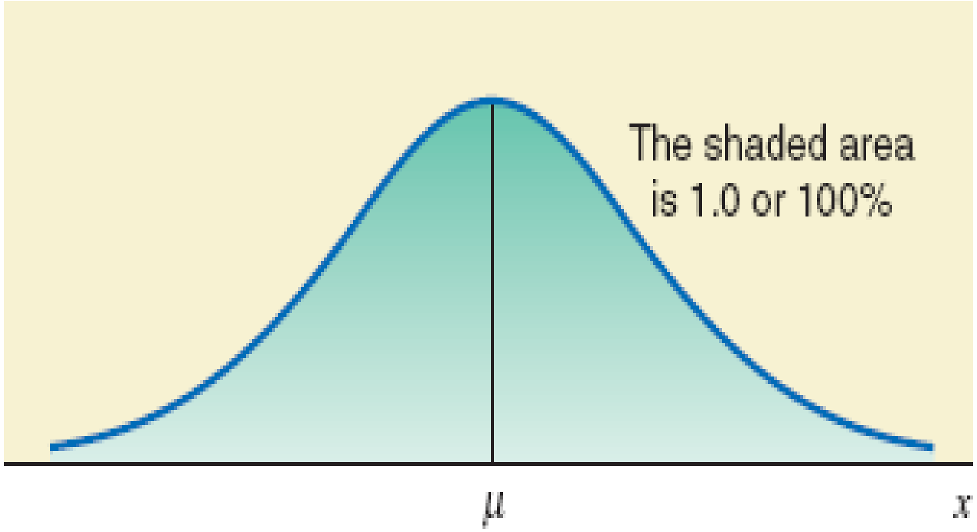

Figure 6.13 A normal curve is symmetric about the mean.

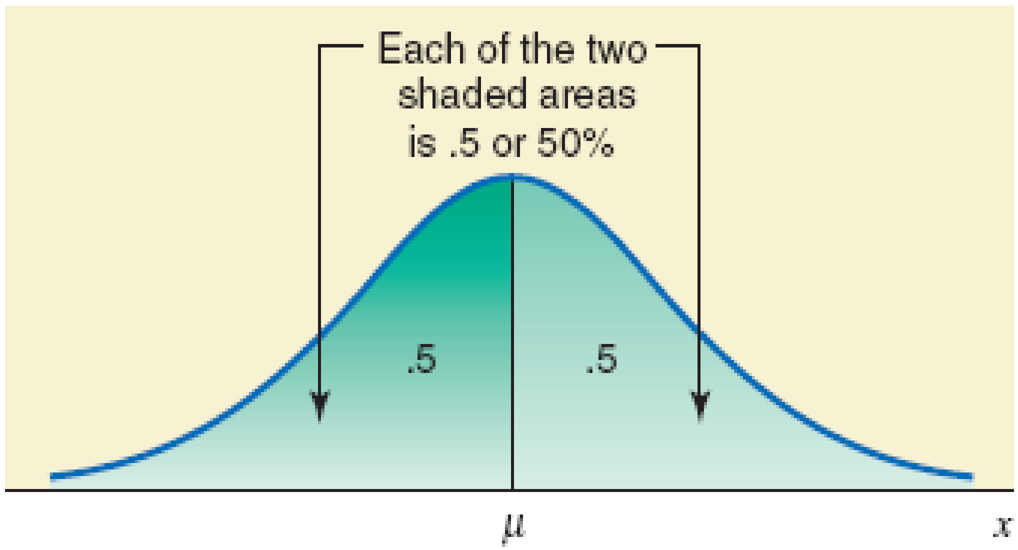

Figure 6.14 Areas of the normal curve beyond $\mu \pm 3\sigma$.

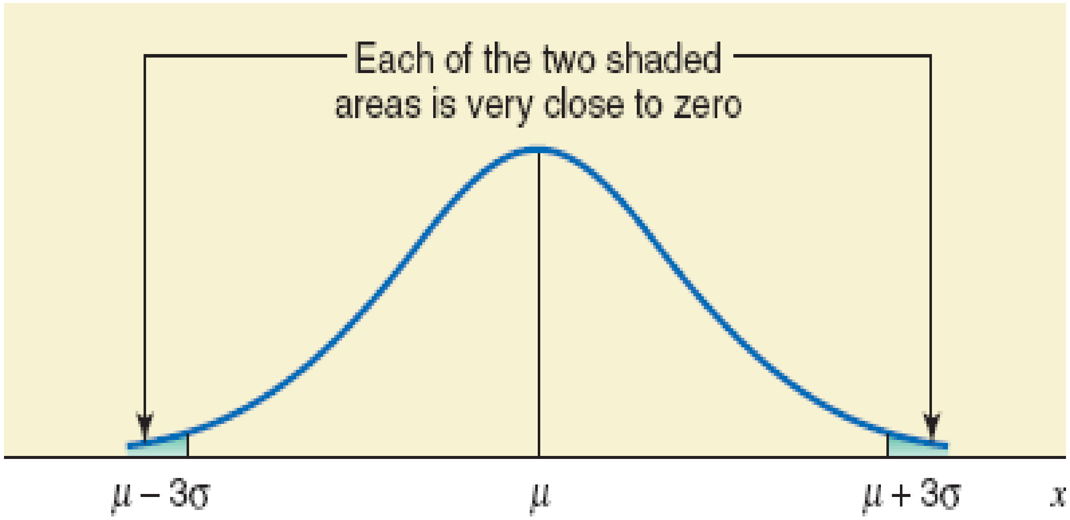

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Figure 6.15 Three normal distribution curves with the same mean but different standard deviations.

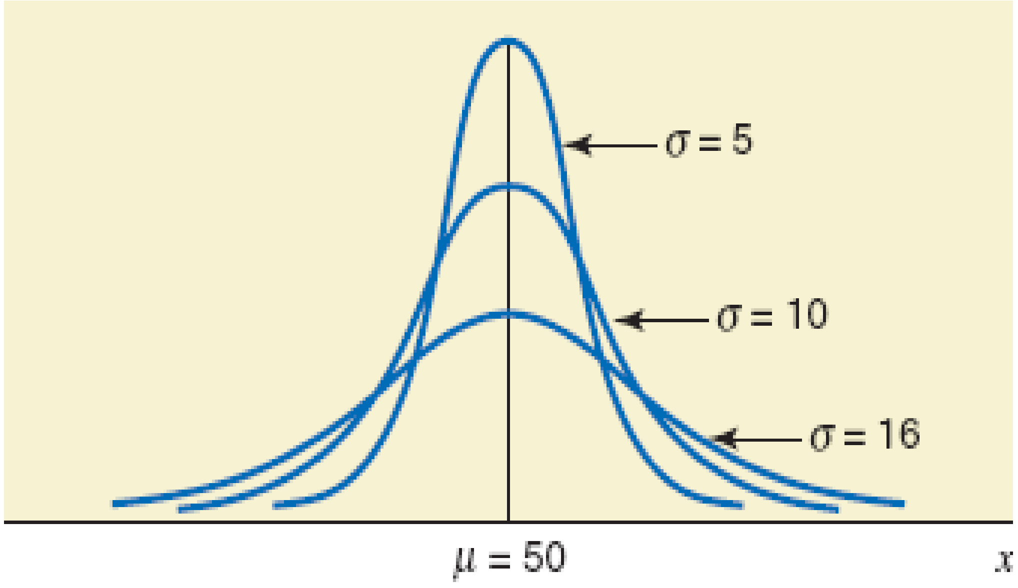
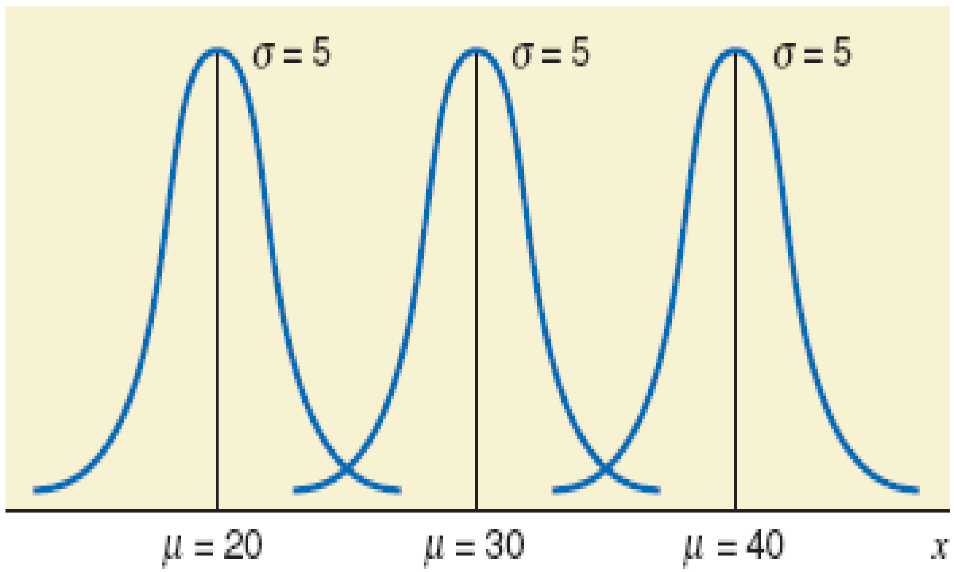


Figure 6.16 Three normal distribution curves with different means but the same standard deviation.

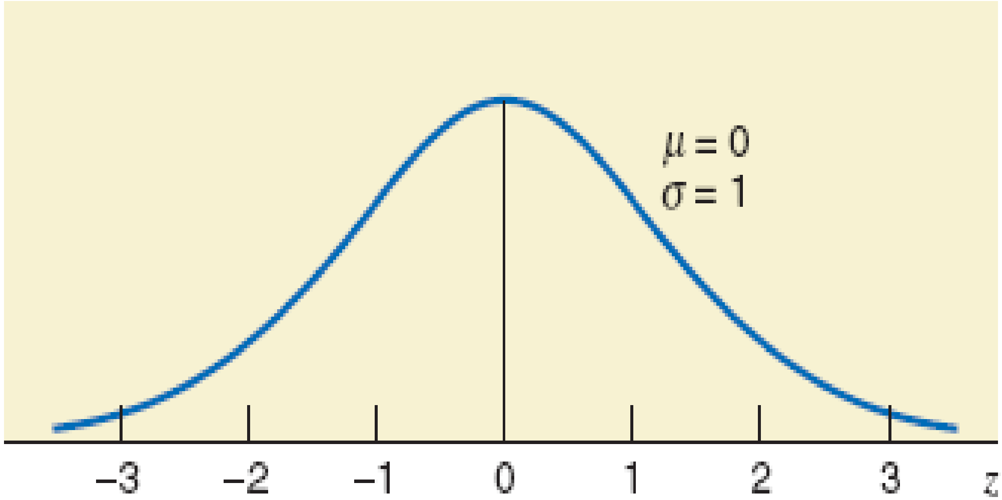


6.3 The Standard Normal Distribution

Definition

The normal distribution with $\mu = 0$ and $\sigma = 1$ is called the ***standard normal distribution***.

Figure 6.17 The standard normal distribution curve.



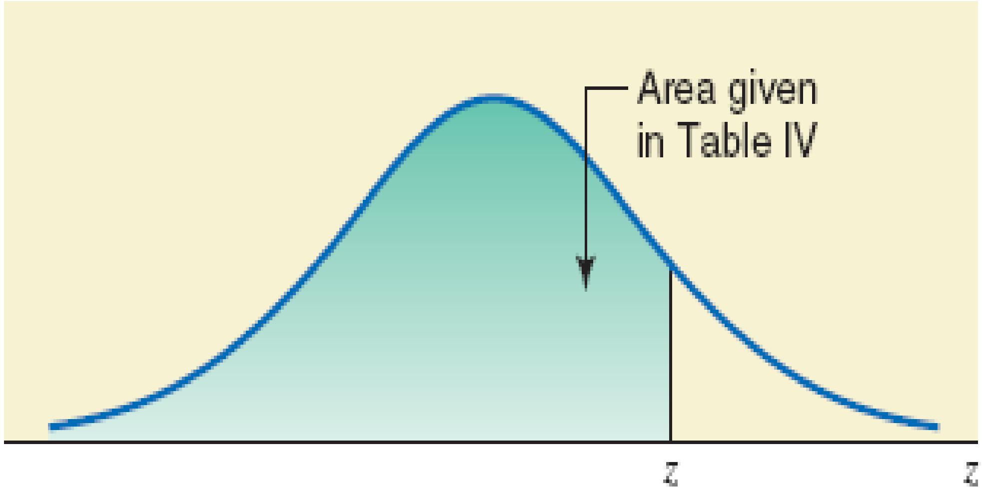
THE STANDARD NORMAL DISTRIBTUION

z Values or *z* Scores

Definition

The units marked on the horizontal axis of the standard normal curve are denoted by ***z*** and are called the ***z values or z scores***. A specific value of z gives the distance between the mean and the point represented by z in terms of the standard deviation.

Figure 6.18 Area under the standard normal curve.



Example 6-1

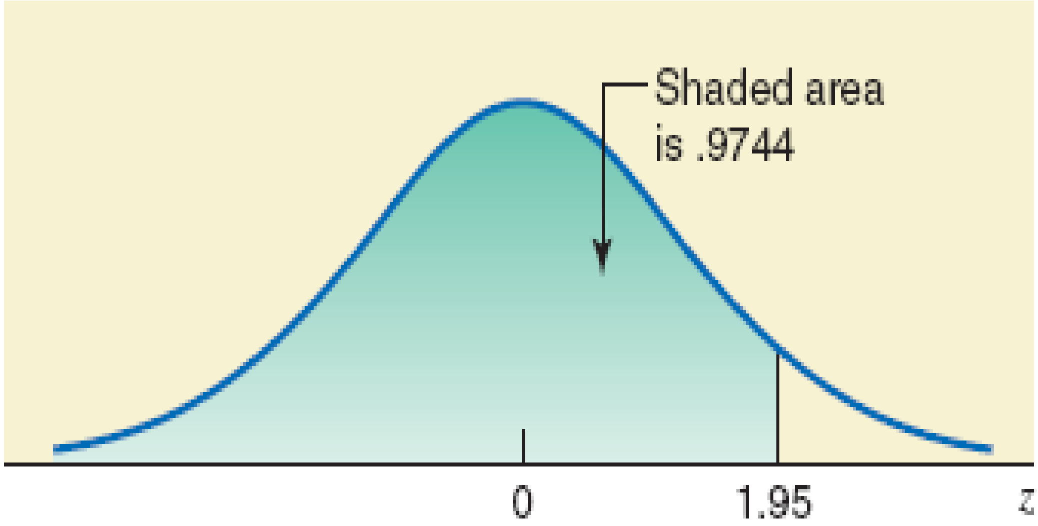
Find the area under the standard normal curve between $z = 0$ and $z = 1.95$.

Table 6.2 Area Under the Standard Normal Curve to the Left of $z = 1.95$

z	.00	.01	...	$\boxed{.05}$	...	.09
−3.4	.0003	.0003	...	.0003	...	.0002
−3.3	.0005	.0005	...	.0004	...	.0003
−3.2	.0007	.0007	...	.0006	...	.0005
.	.	.	...	.	...	.
.	.	.	...	.	...	.
.	.	.	...	.	...	.
$\boxed{1.9}$	.9713	.9719	...	$\boxed{.9744}$	...	.9767
.	.	.	...	.	...	.
.	.	.	...	.	...	.
.	.	.	...	.	...	.
3.4	.9997	.9997	...	.9997	...	.9998

Required area

Figure 6.19 Area to the left of $z = 1.95$.



Example 6-2

Find the area under the standard normal curve from z = -2.17 to z = 0.

Example 6-2: Solution

- To find the area from z=-2.17 to z =0 , first we find the areas to the left of z=0 and to the left of z=-2.17 in Table IV. As shown in Table 6.3, these two areas are .5 and .0150, respectively. Next we subtract .0150 from .5 to find the required area.
- Area from -2.17 to 0 = $P(-2.17 \le z \le 0)$

  = .5000 - .0150 = **.4850**

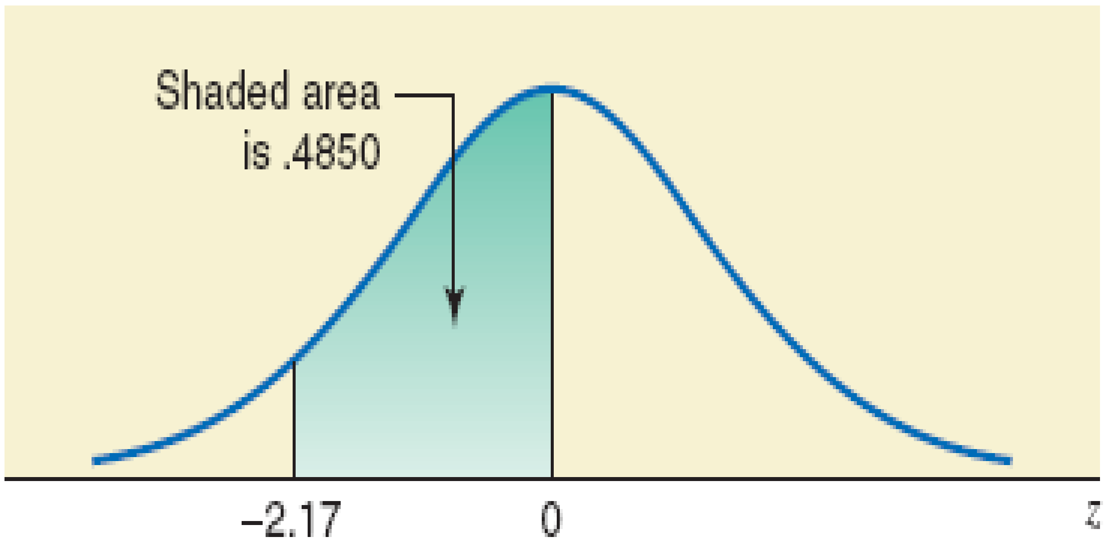
Table 6.3 Area Under the Standard Normal Curve

z	00	.01	...	.07	...	.09
−3.4	.0003	.0003	...	.0003	...	.0002
−3.3	.0005	.0005	...	.0004	...	.0003
−3.2	.0007	.0007	...	.0005	...	.0005
.	.	.	...	.	...	.
.	.	.	...	.	...	.
−2.1	.0179	.0174	...	.0150	...	.0143
.	.	.	...	.	...	.
.	.	.	...	.	...	.
0.0	.5000	.5040	...	.5279	...	.5359
.	.	.	...	.	...	.
.	.	.	...	.	...	.
3.4	.9997	.9997	...	.9997	...	.9998

Area to the left of $z = 0$

Area to the left of $z = -2.17$

Figure 6.20 Area from z = -2.17 to z = 0.



Example 6-3

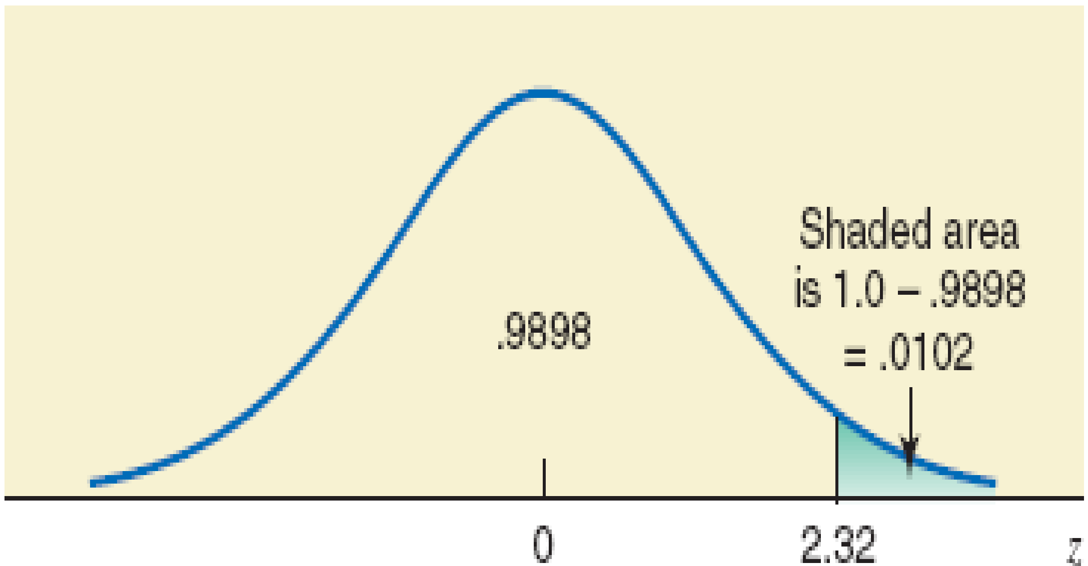
Find the following areas under the standard normal curve.

a) Area to the right of $z = 2.32$
b) Area to the left of $z = -1.54$

Example 6-3: Solution

a) To find the area to the right of z=2.32, first we find the area to the left of z=2.32. Then we subtract this area from 1.0, which is the total area under the curve. The required area is 1.0 - .9898 = .0102.

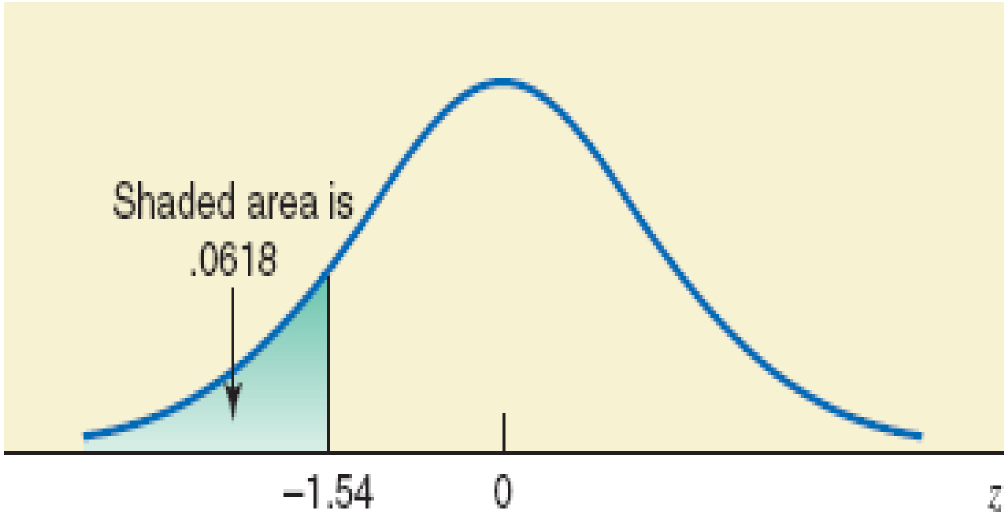
Figure 6.21 Area to the right of $z = 2.32$.



Example 6-3: Solution

b) To find the area under the standard normal curve to the left of z=-1.54, we find the area in Table IV that corresponds to -1.5 in the z column and .04 in the top row. This area is .0618.
Area to the left of -1.54
$= P(z < -1.54) = .0618$

Figure 6.22 Area to the left of *z* = -1.54.



Example 6-4

Find the following probabilities for the standard normal curve.

a) $P(1.19 < z < 2.12)$
b) $P(-1.56 < z < 2.31)$
c) $P(z > -.75)$

Example 6-4: Solution

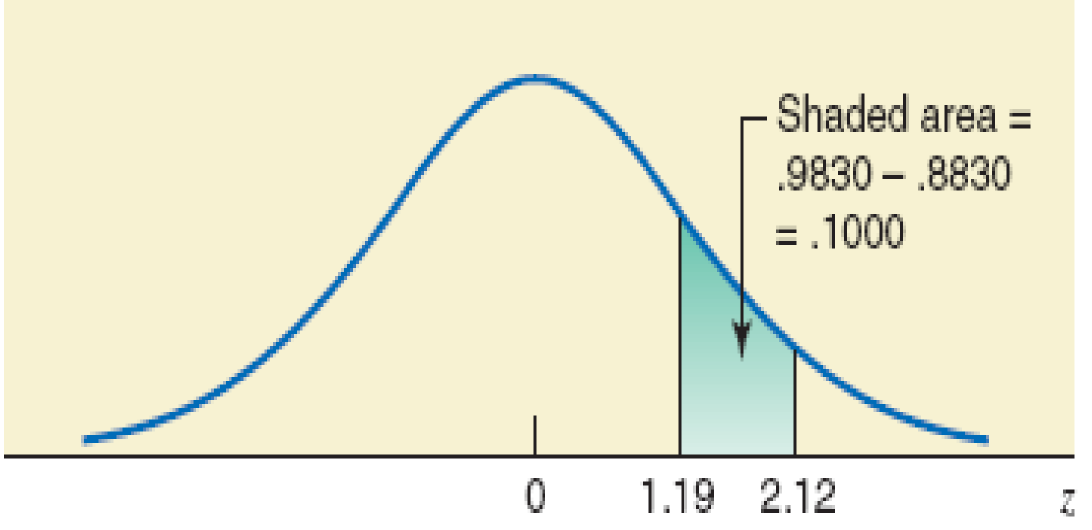
a) $P(1.19 < z < 2.12)$

$= \text{Area between 1.19 and 2.12}$

$= .9830 - .8830$

$= .1000$

Figure 6.23 Finding $P(1.19 < z < 2.12)$.



Example 6-4: Solution

b) $P(-1.56 < z < 2.31)$

$= \text{Area between } -1.56 \text{ and } 2.31$

$= .9896 - .0594$

$= .9302$

Figure 6.24 Finding *P* (-1.56 < *z* < 2.31).

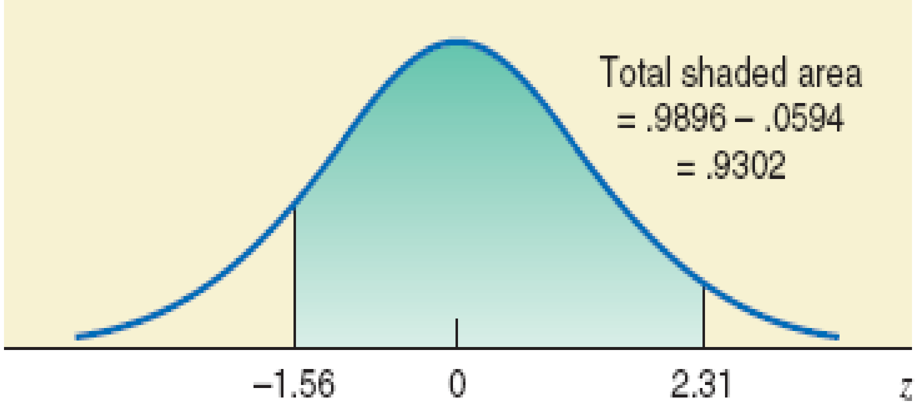


Figure 6.24 Finding $P(-1.56 < z < 2.31)$.

Example 6-4: Solution

c) $P(z > -.75)$

$= \text{Area to the right of } -.75$

$= 1.0 - .2266$

$= .7734$

Figure 6.25 Finding P (z > -.75).

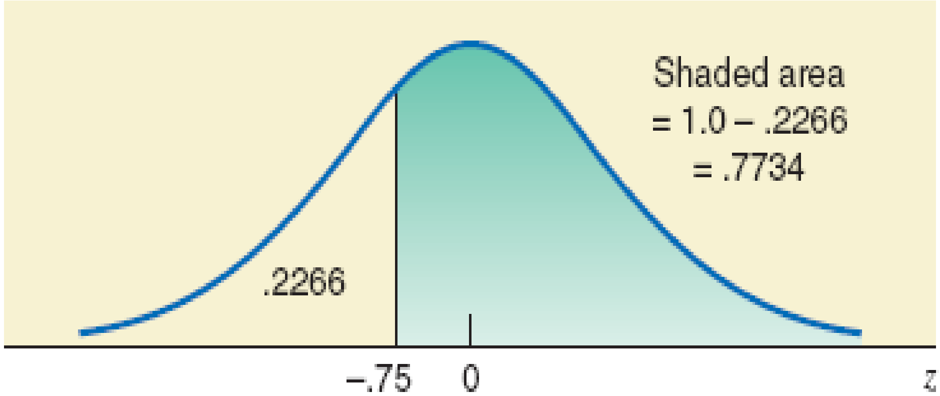


Figure 6.25 Finding $P(z > -.75)$.

Figure 6.26 Area within one standard deviation of the mean.

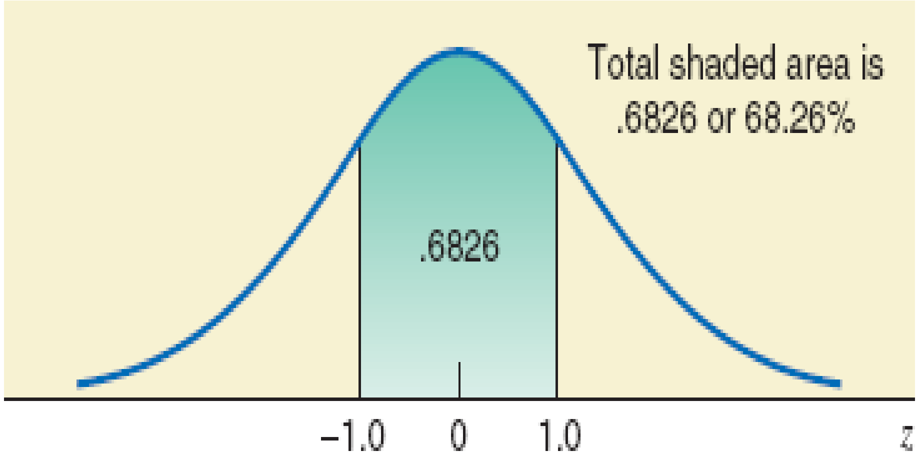


Figure 6.27 Area within two standard deviations of the mean.

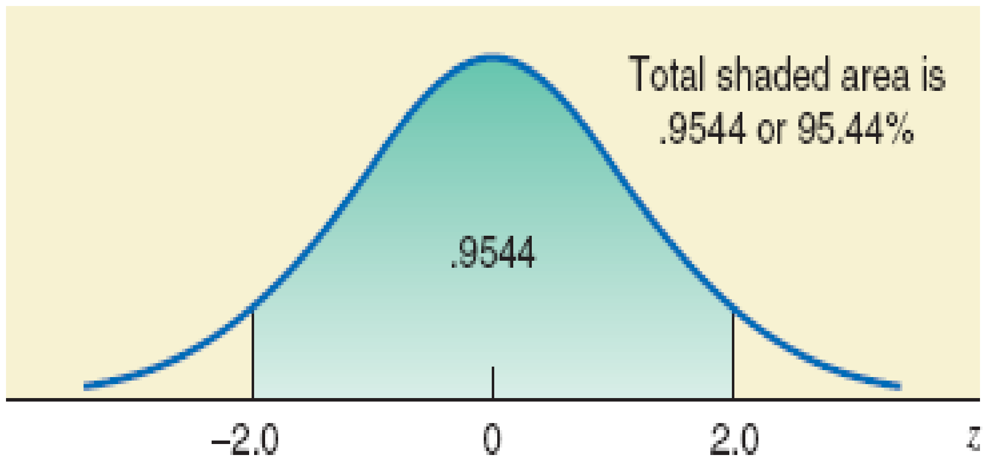
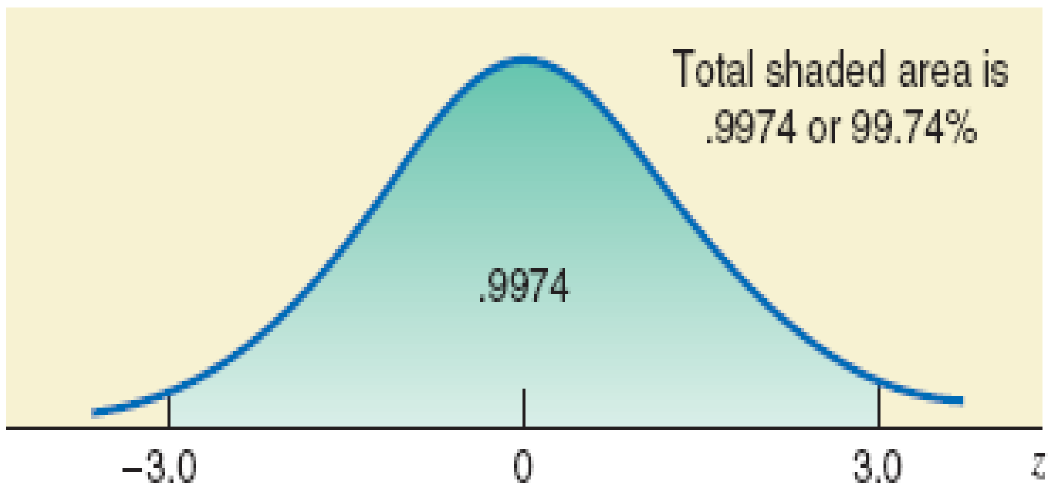


Figure 6.28 Area within three standard deviations of the mean.



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Example 6-5

Find the following probabilities for the standard normal curve.

a) $P\ (0 < z < 5.67)$

b) $P\ (z < -5.35)$

Example 6-5: Solution

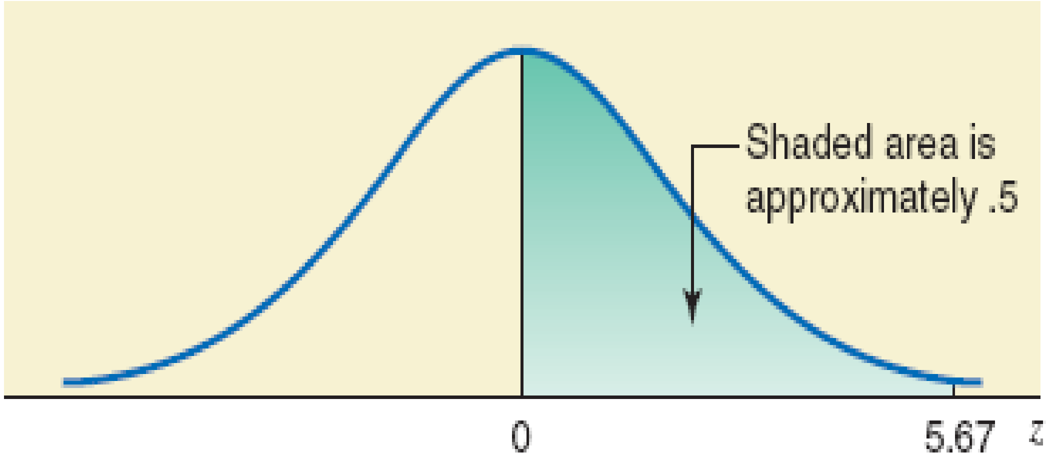
a) $P\,(0 < z < 5.67)$

$= \text{Area between 0 and 5.67}$

$= 1.0 - .5$

$=$ **.5** approximately

Figure 6.29 Area between $z = 0$ and $z = 5.67$.



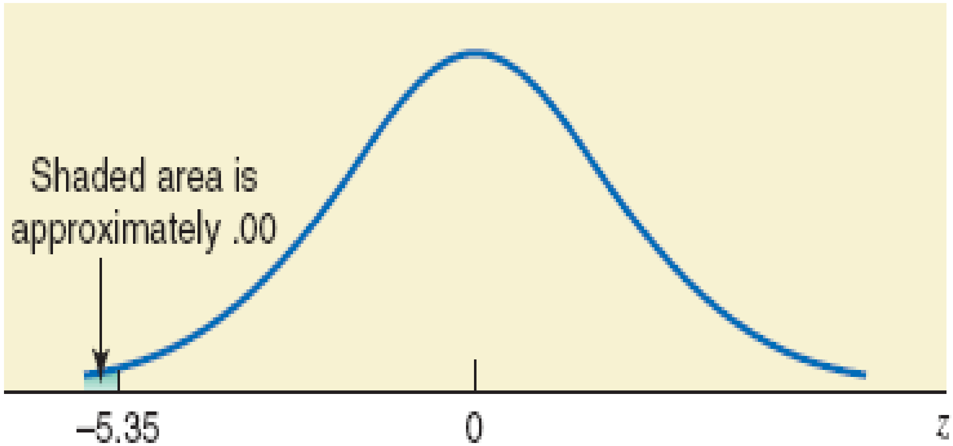
Example 6-5: Solution

b) $P(z < -5.35)$

= Area to the left of -5.35

= .00 approximately

Figure 6.30 Area to the left of z = -5.35.



6.4 Standardizing a Normal Distribution

Converting an x Value to a z Value

For a normal random variable x, a particular value of x can be converted to its corresponding z value by using the formula

$$z = \frac{x - \mu}{\sigma}$$

where μ and σ are the mean and standard deviation of the normal distribution of x, respectively.

Example 6-6

Let x be a continuous random variable that has a normal distribution with a mean of 50 and a standard deviation of 10. Convert the following x values to z values and find the probability to the left of these points.

a) x = 55

b) x = 35

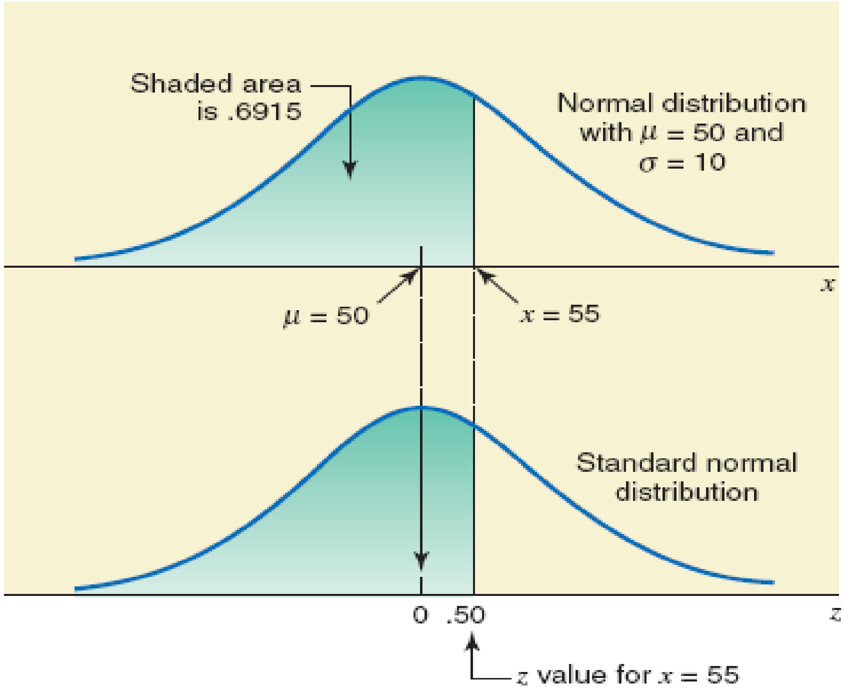
Example 6-6: Solution

a) x = 55

$$z = \frac{x-\mu}{\sigma} = \frac{55-50}{10} = .50$$

P(x < 55) = P(z < .50) = .6915

Figure 6.31 z value for $x = 55$.



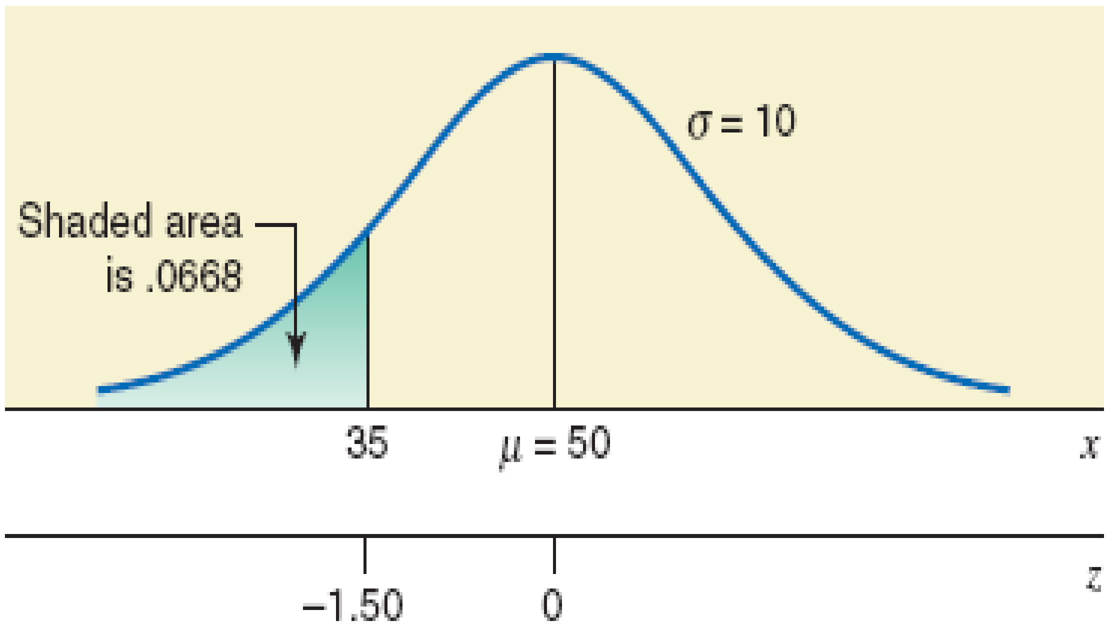
Example 6-6: Solution

b) x = 35

$$z = \frac{x - \mu}{\sigma} = \frac{35 - 50}{10} = -1.50$$

P(x < 35) = P(z < -1.50) = .0668

Figure 6.32 *z* value for $x = 35$.



Example 6-7

Let x be a continuous random variable that is normally distributed with a mean of 25 and a standard deviation of 4. Find the area

a) between $x = 25$ and $x = 32$
b) between $x = 18$ and $x = 34$

Example 6-7: Solution

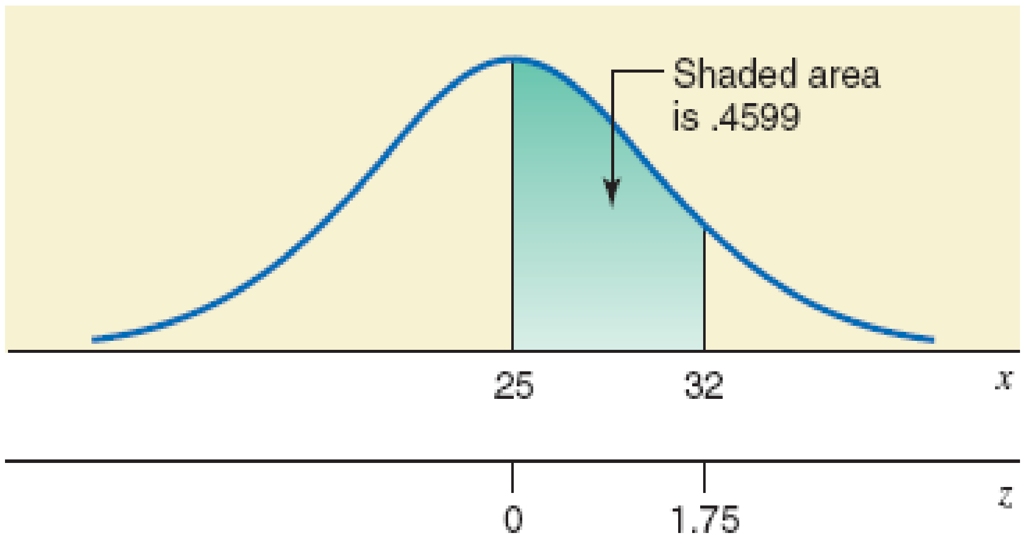
a)

- The z value for x = 25 is 0
- The z value for x = 32 is

$$z = \frac{x-\mu}{\sigma} = \frac{32-25}{4} = 1.75$$

- $P\,(25 < x < 32) = P(0 < z < 1.75)$
  $= .4599$

Figure 6.33 Area between x = 25 and x = 32.

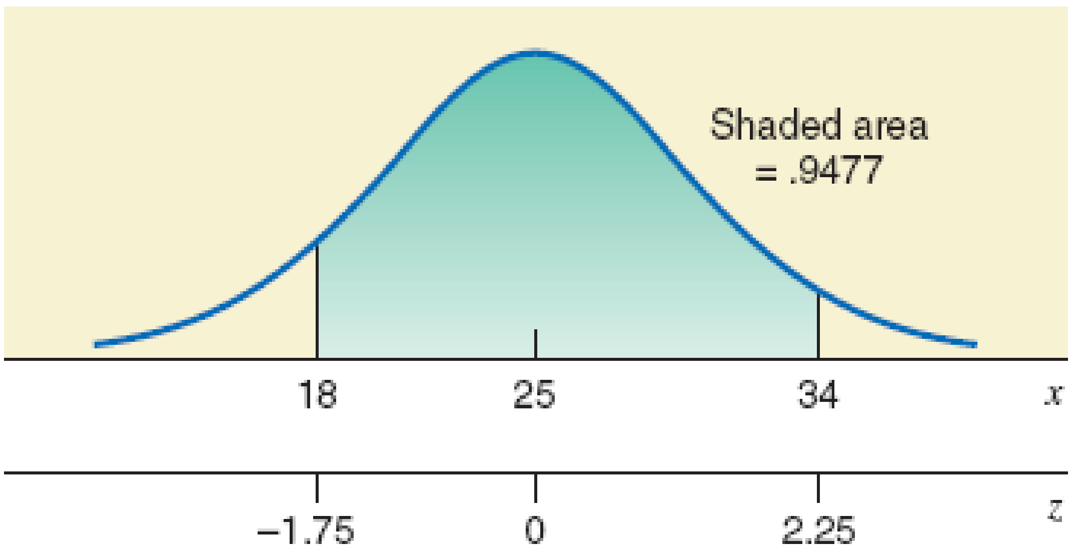


Example 6-7: Solution

b)

- For x = 18: $$z = \frac{18-25}{4} = -1.75$$
- For x = 34: $$z = \frac{34-25}{4} = 2.25$$
- P (18 < x < 34) = P (-1.75 < z < 2.25) = .9878 - .0401 = .9477

Figure 6.34 Area between $x = 18$ and $x = 34$.



Example 6-8

Let x be a normal random variable with its mean equal to 40 and standard deviation equal to 5. Find the following probabilities for this normal distribution

a) $P(x > 55)$

b) $P(x < 49)$

Example 6-8: Solution

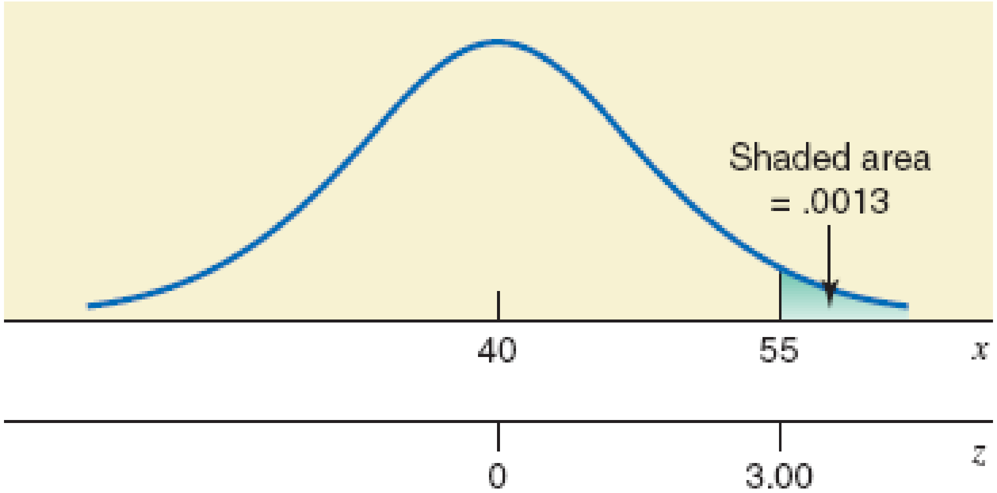
a)

- For x = 55:

$$z = \frac{55 - 40}{5} = 3.00$$

- P (x > 55) = P (z > 3.00) = 1.0 - .9987
  = .0013

Figure 6.35 Finding $P(x > 55)$.



Example 6-8: Solution

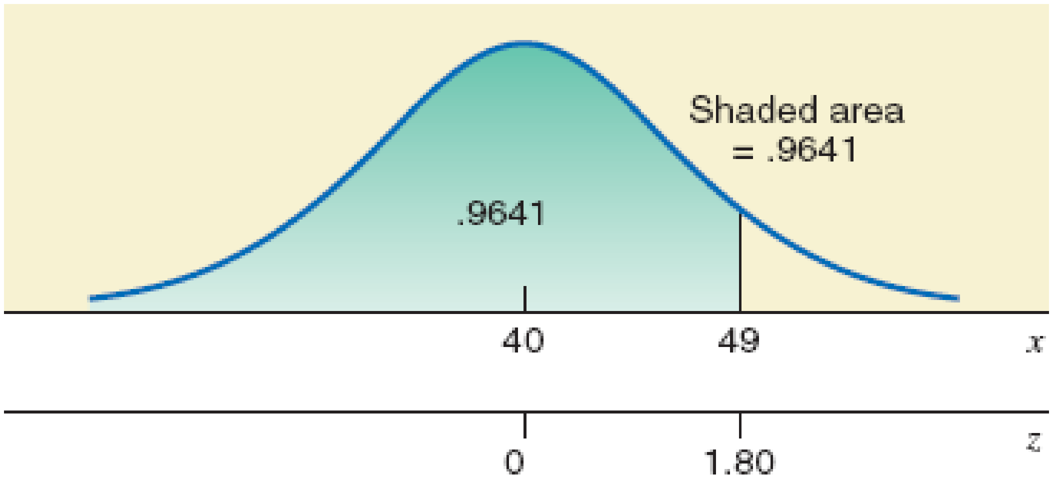
b)

- For x = 49:

$$z = \frac{49-40}{5} = 1.80$$

- $P\ (x < 49) = P\ (z < 1.80) = .9461$

Figure 6.36 Finding $P(x < 49)$.



Example 6-9

Let x be a continuous random variable that has a normal distribution with $\mu = 50$ and $\sigma = 8$. Find the probability $P(30 \le x \le 39)$.

Example 6-9: Solution

- For x = 30:

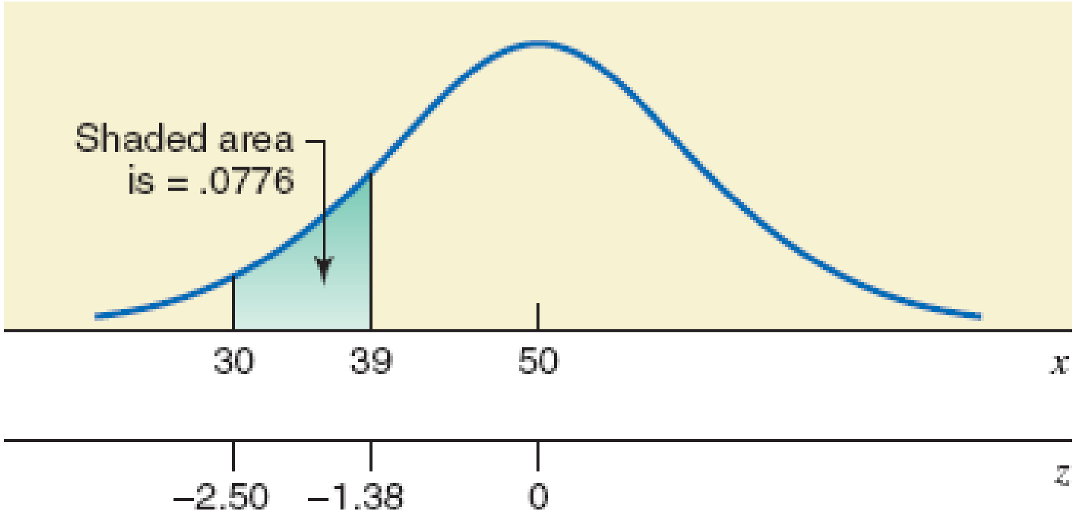
$$z = \frac{30 - 50}{8} = -2.50$$

- For x = 39:

$$z = \frac{39 - 50}{8} = -1.38$$

- P (30 ≤ x ≤ 39) = P (-2.50 ≤ z ≤ -1.38) = .0838 - .0062 = .0776

Figure 6.37 Finding $P(30 \leq x \leq 39)$.



Example 6-10

Let x be a continuous random variable that has a normal distribution with a mean of 80 and a standard deviation of 12. Find the area under the normal distribution curve

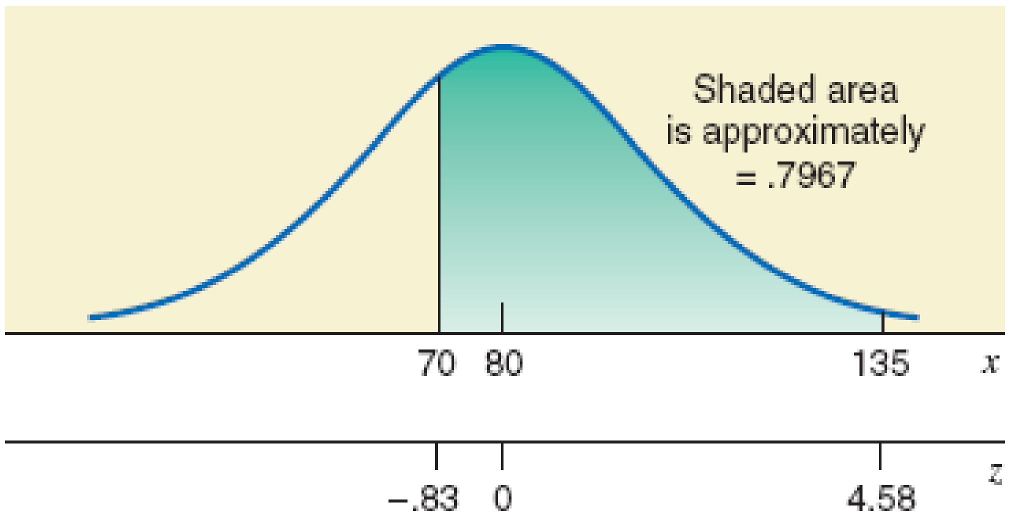
a) from x = 70 to x = 135
b) to the left of 27

Example 6-10: Solution

a)

- For x = 70: $$z = \frac{70-80}{12} = -.83$$
- For x = 135: $$z = \frac{135-80}{12} = 4.58$$
- P (70 ≤ x ≤ 135) = P (-.83 ≤ z ≤ 4.58)
  = 1 - .2033
  = .7967 approximately

Figure 6.38 Area between x = 70 and x = 135.

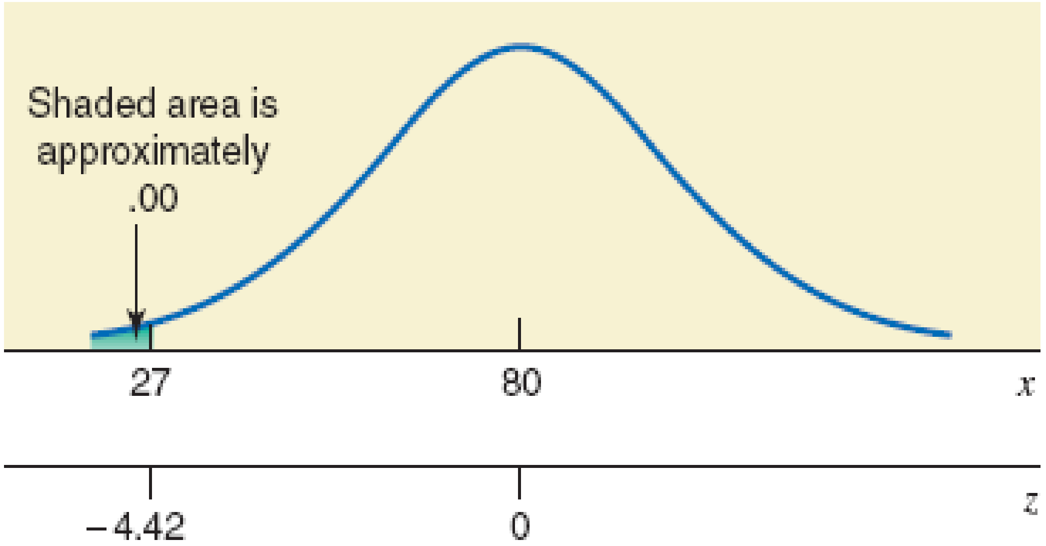


Example 6-10: Solution

b)

- For x = 27: $$z = \frac{27 - 80}{12} = -4.42$$
- P (x < 27) = P (z < -4.42)
  =.00 approximately

Figure 6.39 Area to the left of $x = 27$.



6.5 Applications of the Normal Distribution

Section 6.2 through 6.4 discussed the normal distribution, how to convert a normal distribution to the standard normal distribution, and how to find areas under a normal distribution curve. This section presents examples that illustrate the applications of the normal distribution.

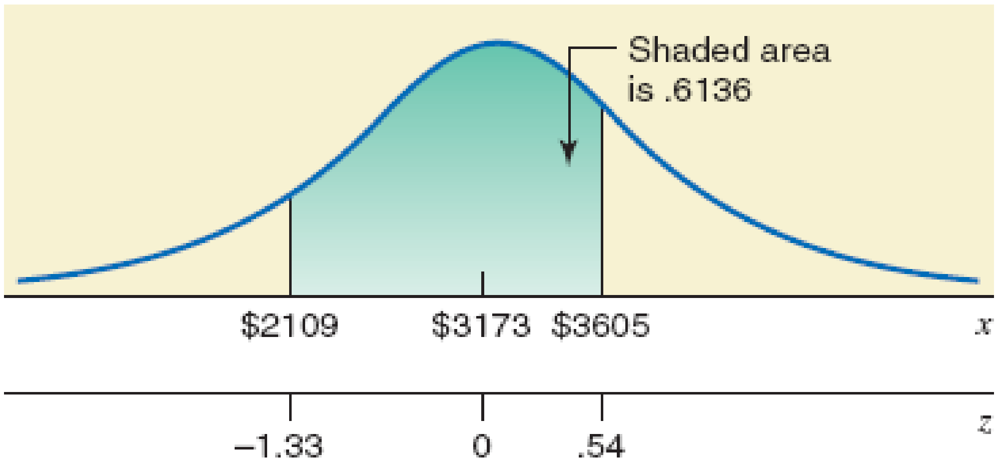
Example 6-11

According to a Sallie Mae and credit bureau data, in 2008, college students carried an average of $3173 debt on their credit cards (*USA TODAY*, April 13, 2009). Suppose that current credit card debts for all college students have a normal distribution with a mean of $3173 and a standard deviation of $800. Find the probability that credit card debt for a randomly selected college student is between $2109 and $3605.

Example 6-11: Solution

- For x = \$2109: $$z = \frac{2109 - 3173}{800} = -1.33$$
- For x = \$3605: $$z = \frac{3605 - 3173}{800} = .54$$
- P (\$2109 < x < \$3605)
  = P (-1.33 < z < .54)
  = .7054 - .0918
  = .6136 = 61.36%

Figure 6.40 Area between x = \$2109 and x = \$3605.



Example 6-12

A racing car is one of the many toys manufactured by Mack Corporation. The assembly times for this toy follow a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5 p.m. every day. If one worker starts to assemble a racing car at 4 p.m., what is the probability that she will finish this job before the company closes for the day?

Example 6-12: Solution

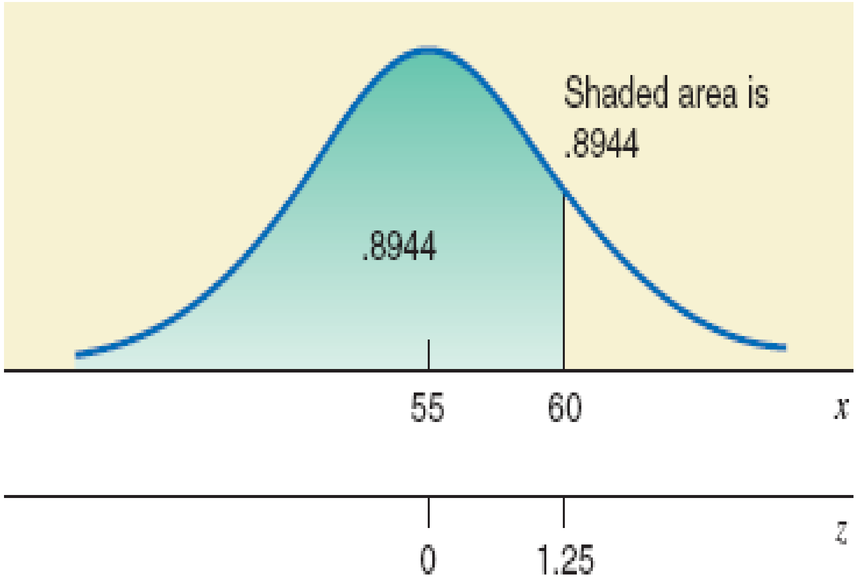
- For x = 60:

$$z = \frac{60 - 55}{4} = -1.25$$

- $P(x \leq 60) = P(z \leq 1.25) = .8944$

- Thus, the probability is .8944 that this worker will finish assembling this racing car before the company closes for the day.

Figure 6.41 Area to the left of $x = 60$.



Example 6-13

Hupper Corporation produces many types of soft drinks, including Orange Cola. The filling machines are adjusted to pour 12 ounces of soda into each 12-ounce can of Orange Cola. However, the actual amount of soda poured into each can is not exactly 12 ounces; it varies from can to can. It has been observed that the net amount of soda in such a can has a normal distribution with a mean of 12 ounces and a standard deviation of .015 ounce.

Example 6-13

a) What is the probability that a randomly selected can of Orange Cola contains 11.97 to 11.99 ounces of soda?

b) What percentage of the Orange Cola cans contain 12.02 to 12.07 ounces of soda?

Example 6-13: Solution

a)

- For x = 11.97: $$z = \frac{11.97 - 12}{.015} = -2.00$$
- For x = 11.99: $$z = \frac{11.99 - 12}{.015} = -.67$$
- P (11.97 ≤ x ≤ 11.99)
  = P (-2.00 ≤ z ≤ -.67) = .2514 - .0228
  = .2286

Figure 6.42 Area between x = 11.97 and x = 11.99.

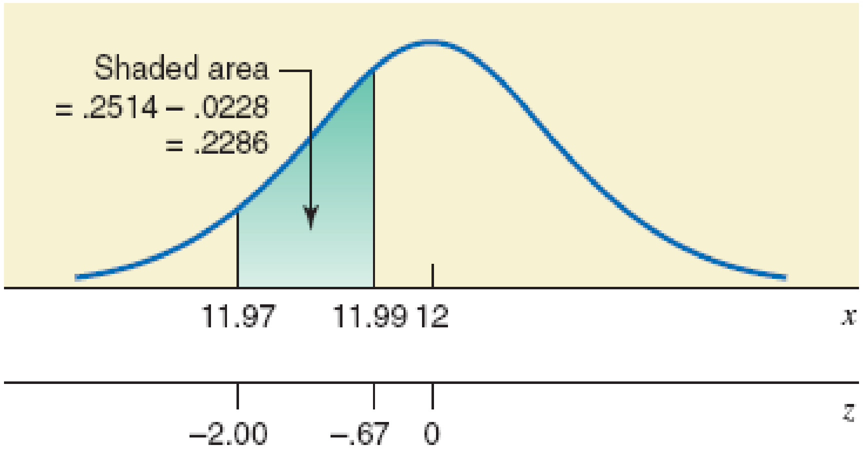


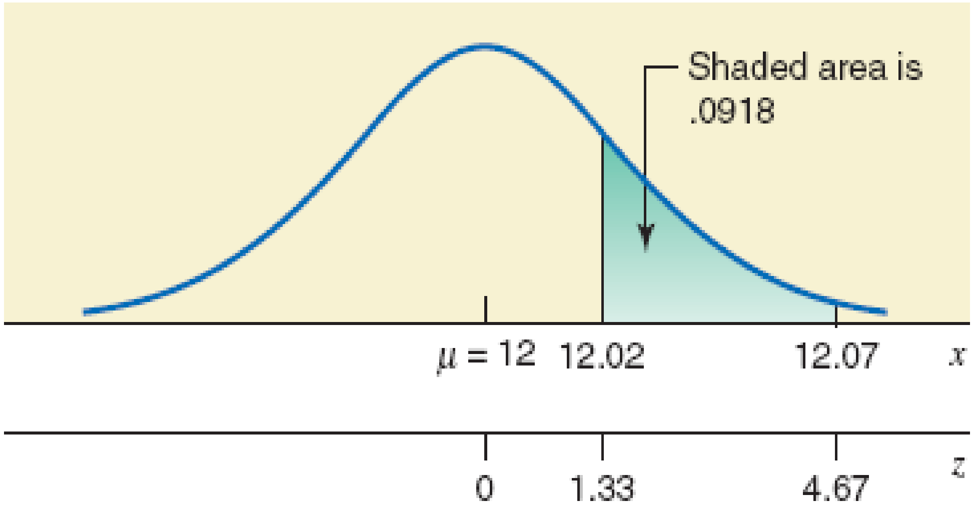
Figure 6.42 Area between x = 11.97 and x = 11.99.

Example 6-13: Solution

b)

- For x = 12.02: $$z = \frac{12.02 - 12}{.015} = 1.33$$
- For x = 12.07: $$z = \frac{12.07 - 12}{.015} = 4.67$$
- $P(12.02 \le x \le 12.07)$
  $= P(1.33 \le z \le 4.67) = 1 - .9082$
  $= .0918$

Figure 6.43 Area from $x = 12.02$ to $x = 12.07$.



Example 6-14

The life span of a calculator manufactured by Texas Instruments has a normal distribution with a mean of 54 months and a standard deviation of 8 months. The company guarantees that any calculator that starts malfunctioning within 36 months of the purchase will be replaced by a new one. About what percentage of calculators made by this company are expected to be replaced?

Example 6-14: Solution

- For x = 36:

$$z = \frac{36 - 54}{8} = -2.25$$

- $P(x < 36) = P\ (z < -2.25)$
  $= .0122$

- Hence, 1.22% of the calculators are expected to be replaced.

Figure 6.44 Area to the left of $x = 36$.

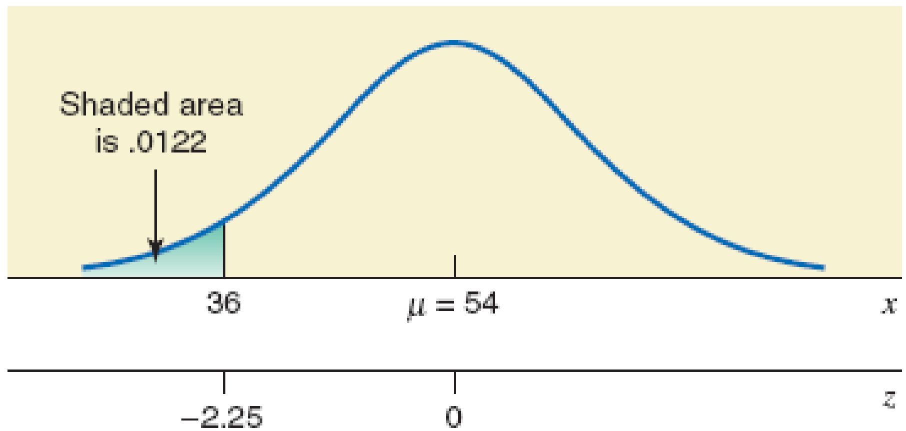


Figure 6.44 Area to the left of $x = 36$.

6.6 Determining the z and x Values When an Area Under the Normal Distribution Curve Is Known

Now we learn how to find the corresponding value of z or x when an area under a normal distribution curve is known.

Example 6-15

Find a point z such that the area under the standard normal curve to the left of z is .9251.

Figure 6.45 Finding the *z* value.

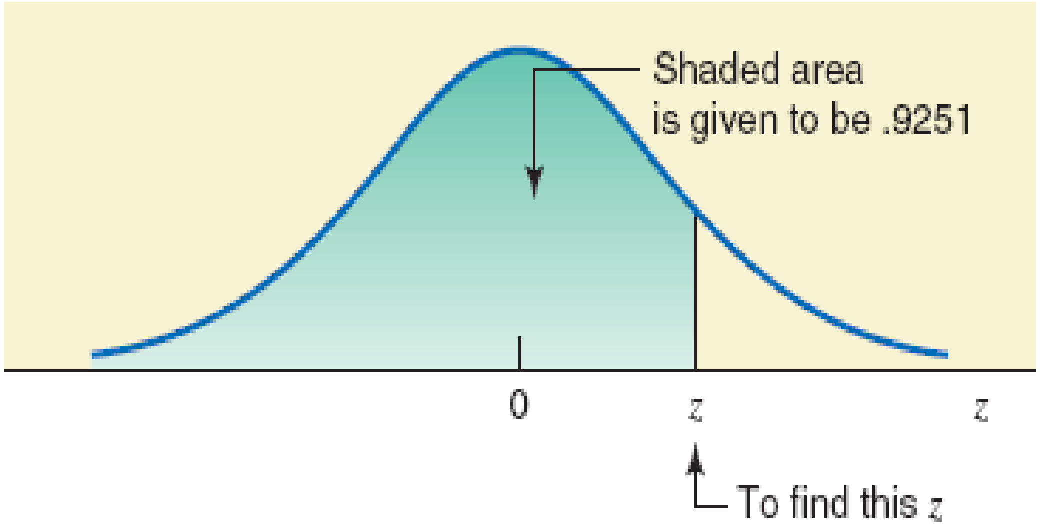


Table 6.4 Finding the *z* Value When Area Is Known.

z	**.00**	**.01**	⋯	$\boxed{\textbf{.04}}$	⋯	**.09**
−3.4	.0003	.0003	⋯		⋯	.0002
−3.3	.0005	.0005	⋯		⋯	.0003
−3.2	.0007	.0007	⋯		⋯	.0005
.	.	.	⋯		⋯	.
.	.	.	⋯		⋯	.
.	.	.	⋯		⋯	.
$\boxed{1.4}$				$\boxed{.9251}$	⋯	⋯
.	.	.	⋯	.	⋯	.
.	.	.	⋯	.	⋯	.
.	.	.	⋯	.	⋯	.
3.4	.9997	.9997	⋯	.9997	⋯	.9998

We locate this value in Table IV of Appendix C

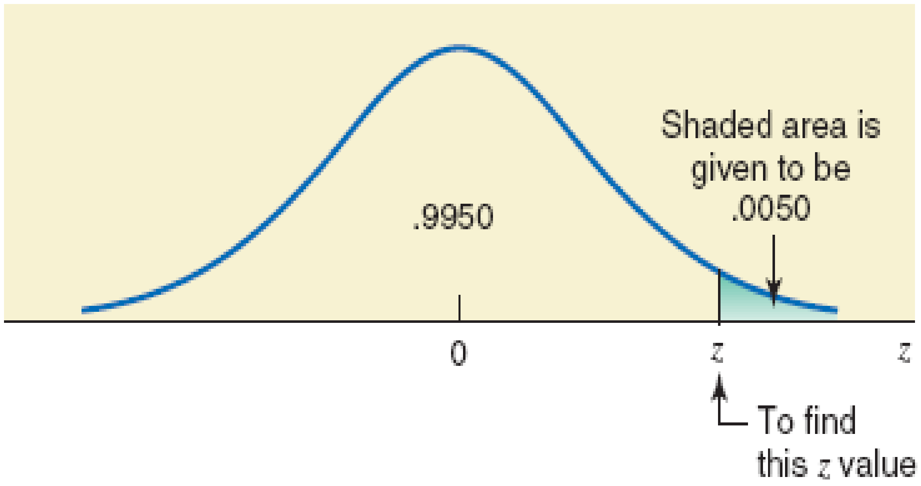
Example 6-16

Find the value of z such that the area under the standard normal curve in the right tail is .0050.

Example 6-16: Solution

- Area to the left of z = 1.0 - .0050 = .9950
- Look for .9950 in the body of the normal distribution table. Table VII does not contain .9950.
- Find the value closest to .9950, which is either .9949 or .9951.
- If we choose .9951, the z = **2.58**.
- If we choose .0049, the z = **2.57**.

Figure 6.46 Finding the *z* value.



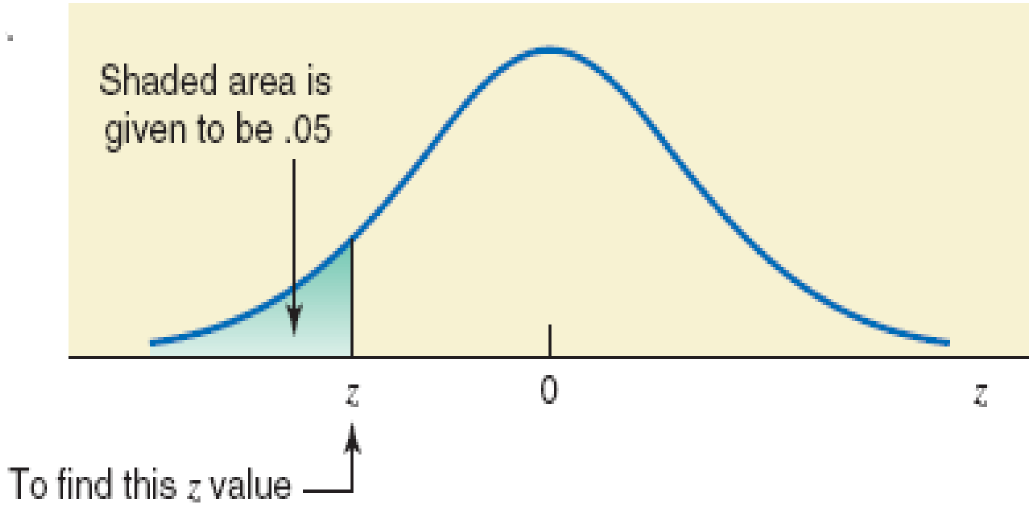
Example 6-17

Find the value of z such that the area under the standard normal curve in the left tail is .05.

Example 6-17: Solution

- Because .05 is less than .5 and it is the area in the left tail, the value of z is negative.
- Look for .0500 in the body of the normal distribution table. The value closes to .0500 in Table IV is either .0505 or .0495.
- If we choose .0495, the $z = -1.65$.

Figure 6.47 Finding the z value.



Finding an *x* Value for a Normal Distribution

For a normal curve, with known values of μ and σ and for a given area under the curve to the left of x, the x value is calculated as

$$x = \mu + z\sigma$$

Example 6-18

Recall Example 6-14. It is known that the life of a calculator manufactured by Texas Instruments has a normal distribution with a mean of 54 months and a standard deviation of 8 months. What should the warranty period be to replace a malfunctioning calculator if the company does not want to replace more than 1% of all the calculators sold?

Example 6-18: Solution

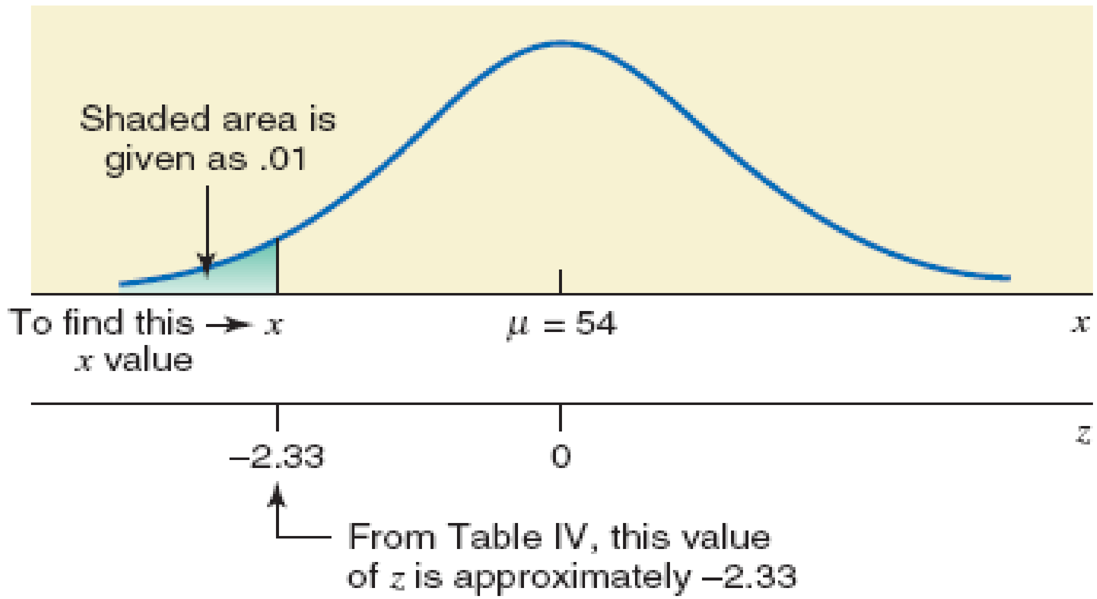
- Area to the left of x = .01 or 1%
- Find the z value from the normal distribution table for .0100. Table IV does not contain a value that is exactly .0100.
- The value closest to .0100 in the table is .0099. The z = -2.33.
- $x = \mu + z\sigma = 54 + (-2.33)(8)$

  $= 54 - 18.64 = 35.36$

Example 6-18: Solution

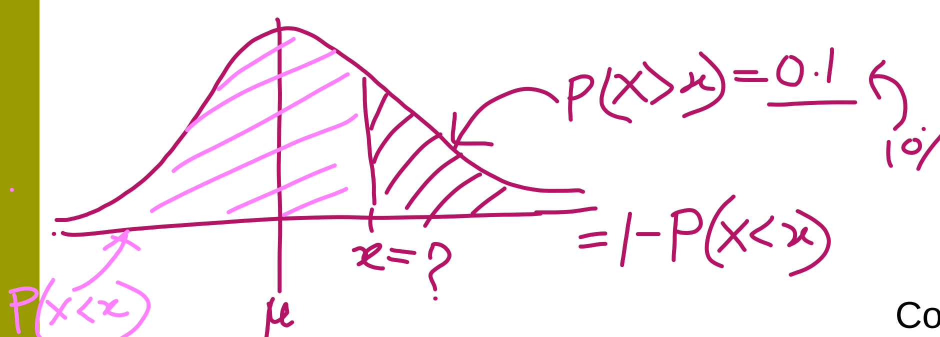
Thus, the company should replace all calculators that start to malfunction within 35.36 months (which can be rounded to 35 months) of the date of purchase so that they will not have to replace more than 1% of the calculators.

Figure 6.48 Finding an *x* value.



Example 6-19

Almost all high school students who intend to go to college take the SAT test. In a recent test, the mean SAT score (in verbal and mathematics) of all students was 1020. μ Debbie is planning to take this test soon. Suppose the SAT scores of all students who take this test with Debbie will have a normal distribution with a mean of 1020 and a standard deviation of 153. What should her score x be on this test so that only 10% of all examinees score higher than she does?



Prem Mann, *Introductory Statistics*, 7/E

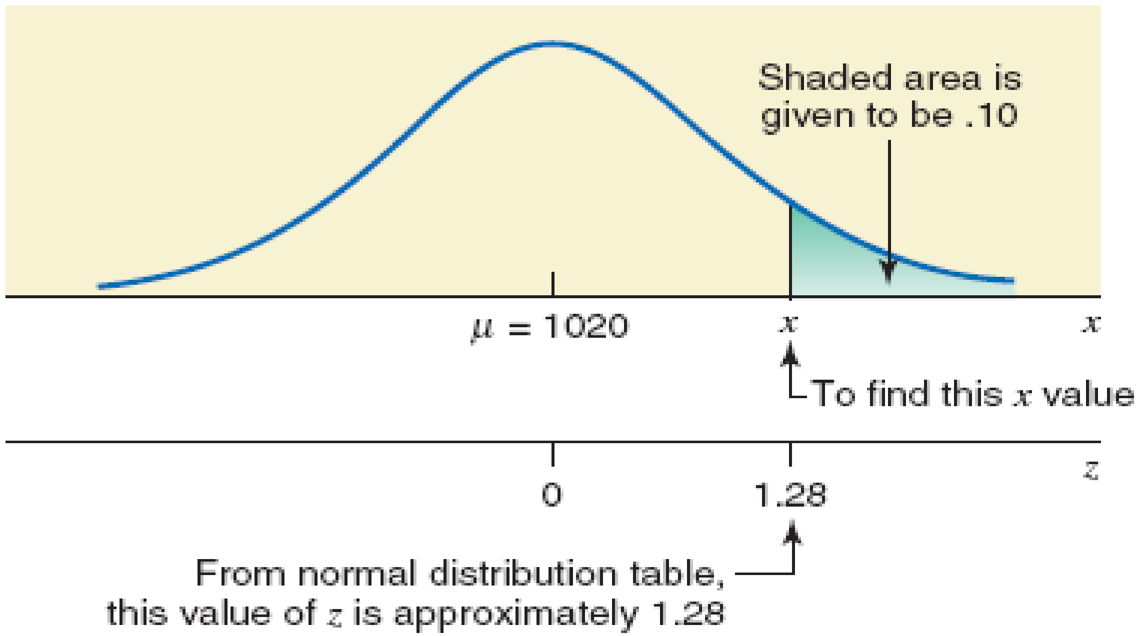
Example 6-19: Solution

- Area to the left of the x value

  $= 1.0 - .10 = .9000$
- Look for .9000 in the body of the normal distribution table. The value closest to .9000 in Table IV is .8997, and the z value is 1.28.
- $x = \mu + z\sigma = 1020 + 1.28(153)$

  $= 1020 + 195.84 = 1215.84 \approx 1216$
- Thus, if Debbie scores 1216 on the SAT, only about 10% of the examinees are expected to score higher than she does.

Figure 6.49 Finding an x value.



6.7 The Normal Approximation to the Binomial Distribution

1. The binomial distribution is applied to a discrete random variable.
2. Each repetition, called a trial, of a binomial experiment results in one of two possible outcomes, either a success or a failure.
3. The probabilities of the two (possible) outcomes remain the same for each repetition of the experiment.
4. The trials are independent.

THE NORMAL APPROXIMATION OF THE BINOMIAL DISTRIBUTION

- The binomial formula, which gives the probability of x successes in n trials, is

$$P(x) = {}_n C_x \, p^x q^{n-x}$$

THE NORMAL APPROXIMATION OF THE BINOMIAL DISTRIBUTION

Normal Distribution as an Approximation to Binomial Distribution

Usually, the normal distribution is used as an approximation to the binomial distribution when np and nq are both greater than 5 -- that is, when

$$np > 5 \quad \text{and} \quad nq > 5$$

Table 6.5 The Binomial Probability Distribution for $n = 12$ and $p = .50$

x	$P(x)$
0	.0002
1	.0029
2	.0161
3	.0537
4	.1208
5	.1934
6	.2256
7	.1934
8	.1208
9	.0537
10	.0161
11	.0029
12	.0002

Figure 6.50 Histogram for the probability distribution of Table 6.5.

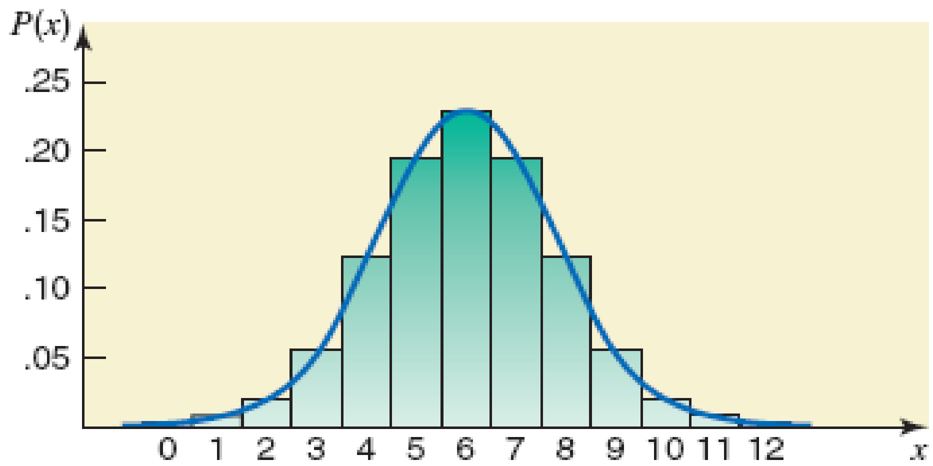


Figure 6.50 Histogram for the probability distribution of Table 6.5.

Example 6-20

$p = q$

According to an estimate, 50% of the people in the United States have at least one credit card. If a random sample of 30 n persons is selected, what is the probability that 19 x of them will have at least one credit card?

Example 6-20: Solution

- $n = 30$, $p = .50$, $q = 1 - p = .50$
- $x = 19$, $n - x = 30 - 19 = 11$
- From the binomial formula,

$$P(19) = {}_{30}C_{19}(.5)^{19}(.5)^{11} = .0509$$

Example 6-20: Solution

- Let's solve this problem using the normal distribution as an approximation to the binomial distribution.
- np = 30(.50) = 15 **> 5** and

  nq = 30(.50) = 15 **> 5**.
- Can use the normal distribution as an approximation to solve this binomial problem.

Example 6-20: Solution

- Step 1. Compute μ and σ for the binomial distribution.

$$\mu = np = 30(.50) = 15$$

$$\sigma = \sqrt{npq} = \sqrt{30(.50)(.50)} = 2.73861279$$

- Step 2. Convert the discrete random variable into a continuous random variable (by making the **correction for continuity**).

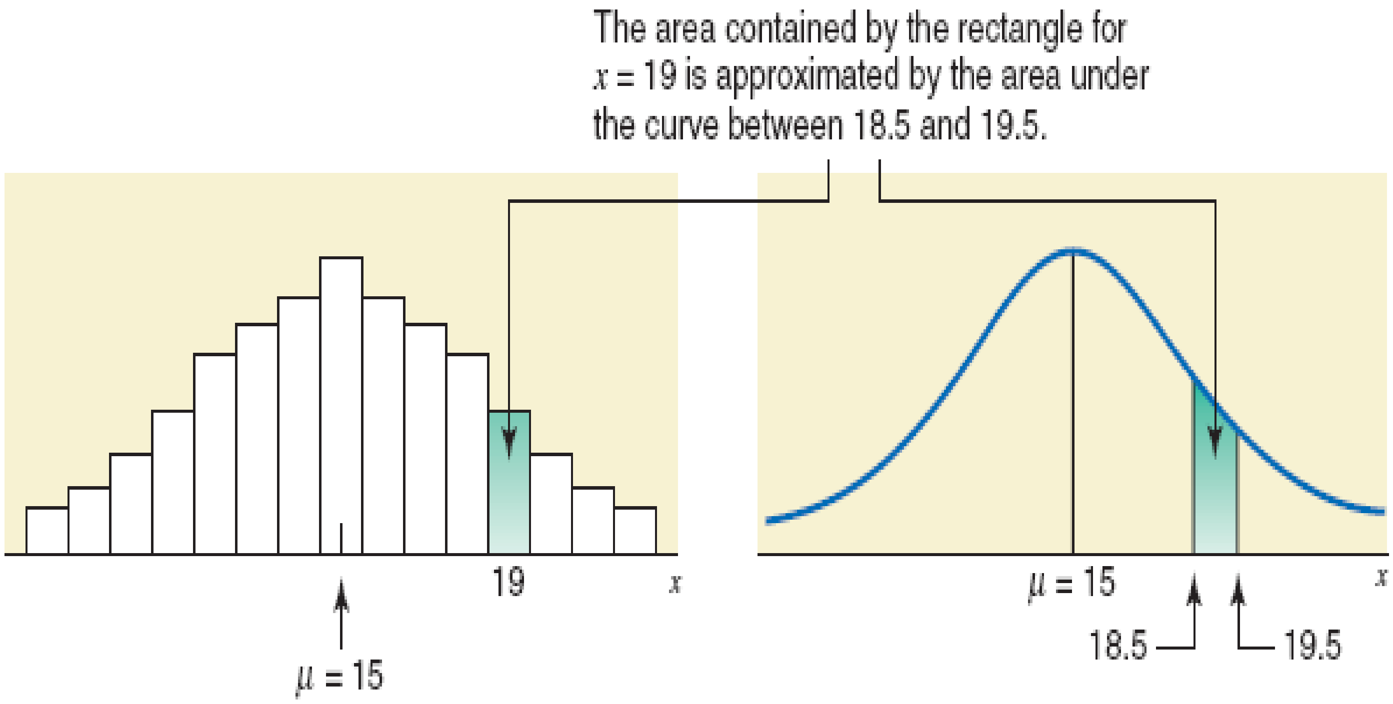
Continuity Correction Factor

Continuity Correction Factor

Definition

The addition of .5 and/or subtraction of .5 from the value(s) of x when the normal distribution is used as an approximation to the binomial distribution, where x is the number of successes in n trials, is called the ***continuity correction factor***.

Figure 6.51



The area contained by the rectangle for
x = 19 is approximated by the area under
the curve between 18.5 and 19.5.
19
x
μ = 15
μ = 15
x
18.5
19.5

Example 6-20: Solution

- Step 3. Compute the required probability using the normal distribution.

- For x = 18.5: $$z = \frac{18.5 - 15}{2.73861279} = 1.28$$

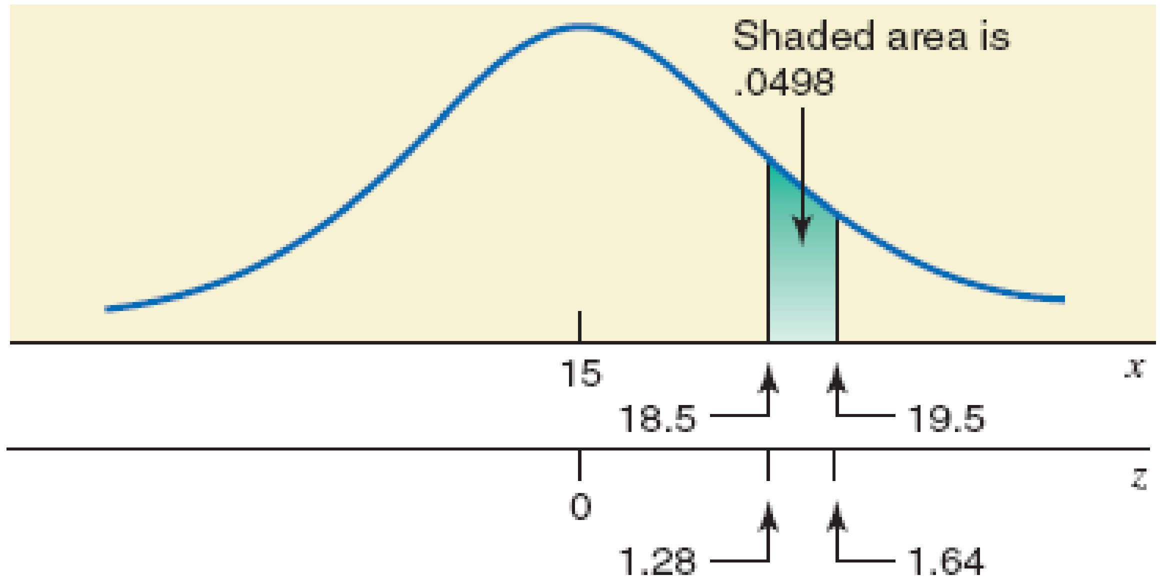
- For x = 19.5: $$z = \frac{19.5 - 15}{2.73861279} = 1.64$$

- $P(18.5 \le x \le 19.5) = P(1.28 \le z \le 1.64)$ $= .9495 - .8997 = .0498$

Example 6-20: Solution

- Thus, based on the normal approximation, the probability that 19 persons in a sample of 30 will have at least one credit card is approximately .0498.
- Using the binomial formula, we obtain the exact probability .0509.
- The error due to using the normal approximation is .0509 - .0498 = .0011.

Figure 6.52 Area between x = 18.5 and x = 19.5.



Example 6-21

According to a joint reader survey by USATODAY.com and TripAdvisor.com, 34% of the people surveyed said that the first thing they do after checking into a hotel is to adjust the thermostat (*USA TODAY*, June 12, 2009). Suppose that this result is true for the current population of all adult Americans who stay in hotels. What is the probability that in a random sample of 400 adult Americans who stay in hotels, 115 to 130 will say that the first thing they do after checking into a hotel is to adjust the thermostat?

Example 6-21: Solution

- n = 400, p = .34, q = 1 – .34 = .66

$$\mu = np = 400(.34) = 136$$

$$\sigma = \sqrt{npq} = \sqrt{400(.34)(.66)} = 9.47417543$$

- For x = 114.5:

$$z = \frac{114.5 - 136}{9.47417543} = -2.27$$

- For x = 130.5

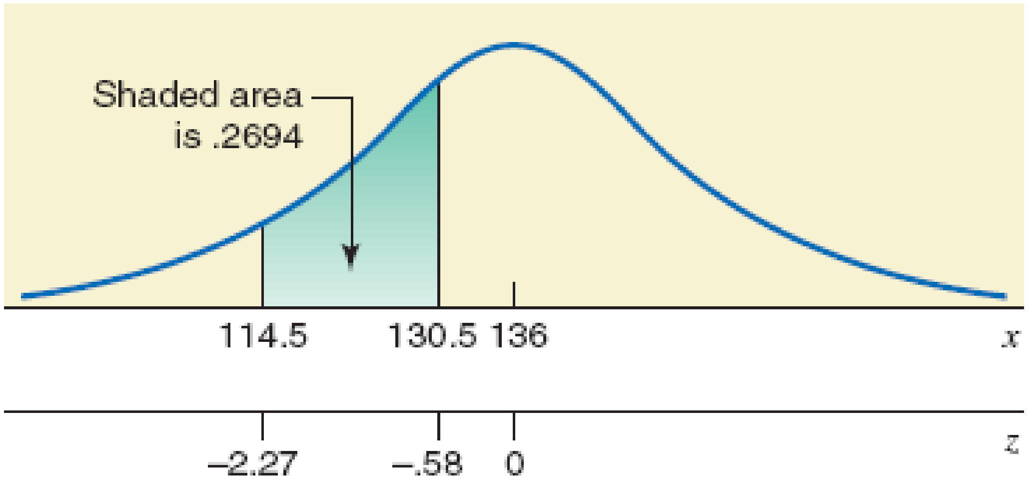
$$z = \frac{130.5 - 136}{9.47417543} = -.58$$

Example 6-21: Solution

- $P(114.5 \le x \le 130.5)$
  $= P(-2.27 \le z \le -.58)$
  $= .2810 - .0116 = .2694$

- Thus, the probability that 115 to 130 adults in a sample of 400 will say that the first thing they do after checking into a hotel is to adjust the thermostat is approximately .2694.

Figure 6.53 Area between x = 114.5 and x = 130.5



Example 6-22

According to an American Laser Centers survey, 32% of adult men said that their stomach is the least favorite part of their body (*USA TODAY*, March 10, 2009). Assume that this percentage is true for the current population of all men. What is the probability that 170 or more adult in a random sample of 500 will say that their stomach is the least favorite part of their body?

Example 6-22: Solution

- n = 500, p = .32, q = 1 – .32 = .68

$$\mu = np = 500(.32) = 160$$

$$\sigma = \sqrt{npq} = \sqrt{500(.32)(.68)} = 10.43072385$$

- For x = 169.5:

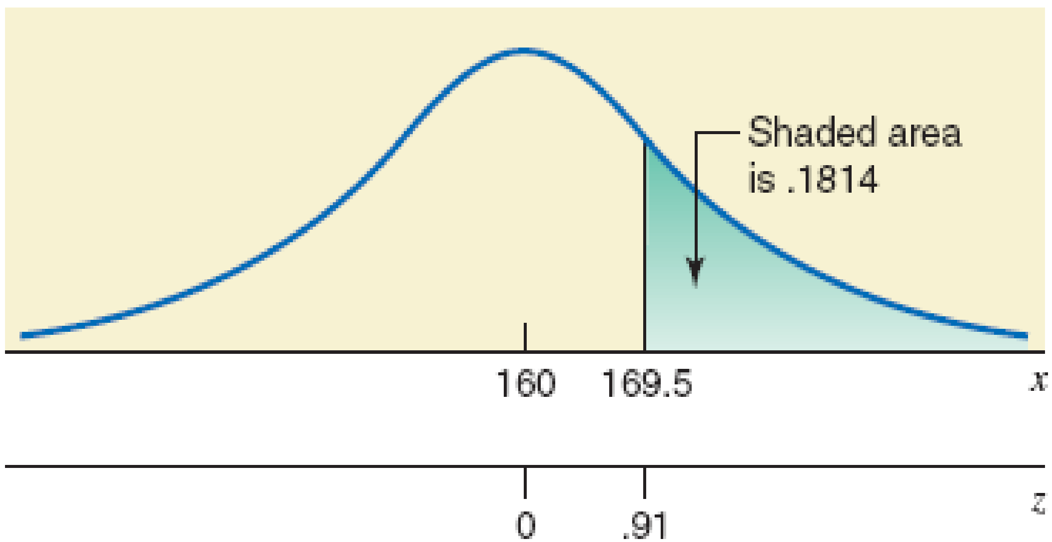
$$z = \frac{169.5 - 160}{10.430772385} = .91$$

Example 6-22: Solution

- $P(x \geq 169.5) = P(z \geq .91)$
  $= 1.0 - .8186 = .1814$

- Thus, the probability that 170 170 or more adult in a random sample of 500 will say that their stomach is the least favorite part of their body is approximately .1814.

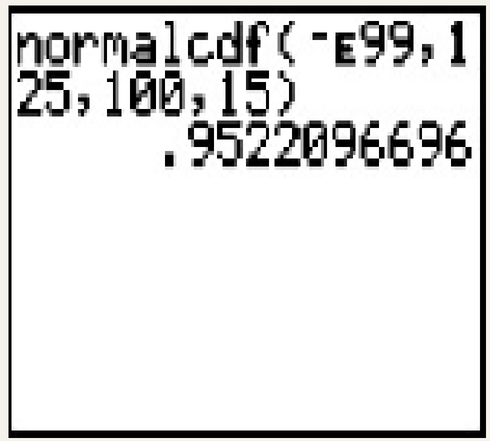
Figure 6.54 Area to the right of $x = 169.5$



TECHNOLOGY INSTRUCTION

Normal and Inverse Normal Probabilities

TI-84

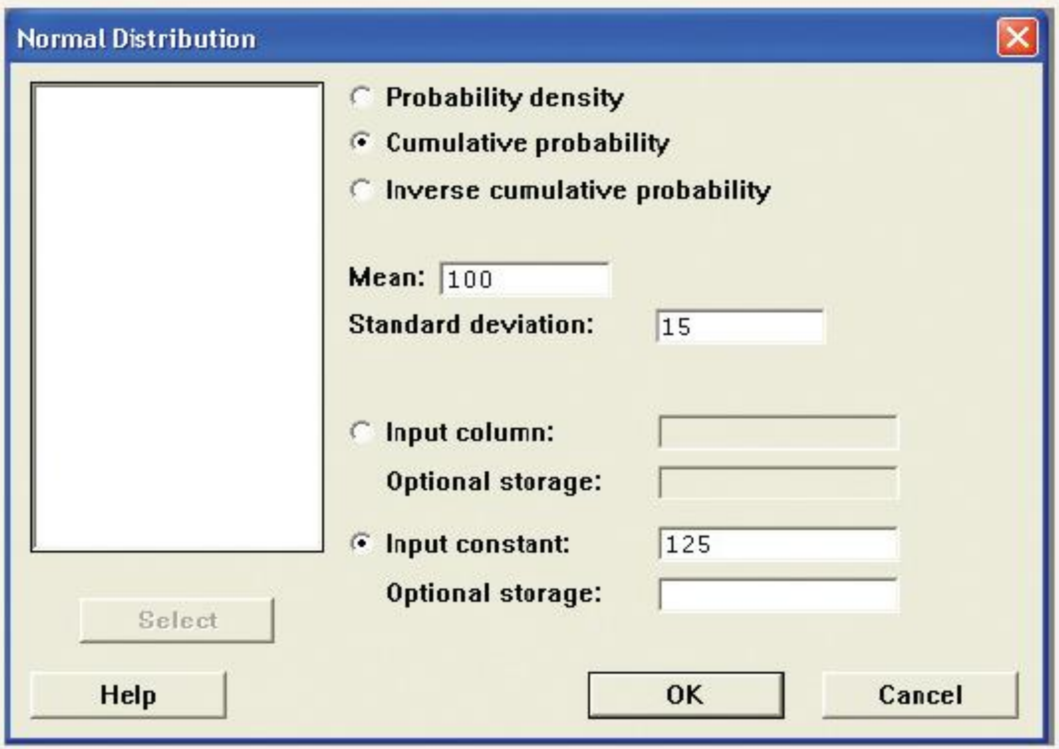


Screen 6.1

TECHNOLOGY INSTRUCTION

Normal and Inverse Normal Probabilities

Minitab



Screen 6.2

TECHNOLOGY INSTRUCTION

Normal and Inverse Normal Probabilities

Minitab

```
Cumulative Distribution Function

Normal with mean = 100 and standard deviation = 15

  x  P( X <= x )
125     0.952210
```

Screen 6.3

TECHNOLOGY INSTRUCTION

Normal and Inverse Normal Probabilities

Excel

	A	B	C	D
1	Mean	100		
2	Std. Dev.	15		
3				
4	P(X<125)	=NORMDIST(125,100,15,1)		

Screen 6.4