শিক্ষা নিয়ে গড়ব দেশ

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Faculty of Cyber Physical System

Department of IoT and Robotics Engineering (IRE)

OBE Description of "Probability and Statistics"

For the

Undergraduate Programs of BDU

Assignment-2

Question-1.

A commuter train arrives punctually at a station every 25 minutes. Each morning, a commuter leaves his house and casually walks to the train station. Let X denote the amount of time, in minutes, that commuter waits for the train from the time he reaches the train station. It is known that the probability density function of X is

$$f(x)=\begin{cases}\frac{1}{25}, & for\ 0<x<25\\ 0, & otherwise.\end{cases}$$

Obtain and interpret the expected value of the random variable X.

Question-2.

Six men and five women apply for an executive position in a small company. Two of the applicants are selected for an interview. Let X denote the number of women in the interview pool. We have found the probability mass function of X.

$X = x$	0	1	2
$P(x)$	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{4}{11}$

How many women do you expect in the interview pool?

Question-3.

In a population of people in a city, the distribution of heights of individuals (crudely measured using a scaled with least count 2 cm) is given as per the following table:

Height (in cm)	160	162	164	166	168	170	172	174	176	178	180
People (in 1000's)	5	10	25	50	100	120	140	100	50	10	5

X denotes the random variable "the height of a randomly chosen person from the city where each person is equally likely to be chosen". Calculate the following:

1. The mathematical expectation $E(X)$ (also called $\mu(X)$).
2. The most likely height.
3. The smallest number h so that at least 50% of the population has neight less than or equal to h.
4. The variance $\sigma^2(X)$ and the standard deviation $\sigma(X)$.

Use a calculator if necessary. Also, try to understand the way in which X is defined.

Question-4.

The number of miles that Anita's motorbike will travel on one gallon of petrol may be modelled by a normal distribution with mean 135 and standard deviation 12.

(a) Given that Anita starts a journey with one gallon of petrol in her motorbike's tank, find the probability that, without refuelling, she can travel:

(i) more than 111 miles;

(ii) between 141 and 150 miles.

Question-05.

Grand Auto Corporation produces auto batteries. The company claims that its top-of-the-line Never Die batteries are good, on average, for at least 65 months. A consumer protection agency tested 45 such batteries to check this claim. It found the mean life of these 45 batteries to be 63.4 months with a standard deviation of 3 months. Find the p-value for the test that mean life of all such batteries is less than 65 months. What will your conclusion be if the significance level is 2.5%?

Question-06.

Pop-Donuts Company makes two products, donuts and cakes. It has bottlenecks in its capital used to purchase flour that is required in making both donuts and cakes, and also in labor hours used to make these two products. The goal of Pop-Donuts is to make as much profits as possible within its capacities. Currently contribution margins for a dozen donuts and a cake are $3 and $9, respectively. Pop-Donuts will adjust prices to keep same contribution margins when variable costs change. Required direct labor hours are 0.25 h per a dozen donuts and 1 h per cake. Pop-Donuts has business uncertainties and several possible scenarios in its business operations are summarized in the following:

Scenario 1: Pop-Donuts has a tight capital budget for only $15,000 of flour costs and 10,000 labor hours annually. The flour for donuts costs $0.5 per a dozen donuts and $1.2 per cake.

Scenario 2: Pop-Donuts has a modest capital budget for $17,000 of flour costs and 10,000 labor hours annually. The flour for donuts costs $0.6 per a dozen donuts and $1.3 per cake.

Scenario 3: Pop-Donuts has a very good capital budget for $20,000 of flour costs and 10,000 labor hours annually. The flour for donuts costs $0.7 per a dozen donuts and $1.4 per cake.

Question-7.

The Acme Machine Shop has a tool crib to store tools required by the shop mechanics. Two clerks run the tool crib. The clerks hand out the tools as the mechanics arrive and request them. The tools then are returned to the clerks when they are no longer needed. There have been complaints from supervisors that their mechanics have had to waste too much time waiting to be served at the tool crib, so it appears as if there should be more clerks. On the other hand, management is exerting pressure to reduce overhead in the plant, and this reduction would lead to fewer clerks. To resolve these conflicting pressures, an OR study is being conducted to determine just how many clerks the tool crib should have.

Question-8

A multiple choice paper has 10 questions. Each question has 4 choices. The correct answer gets 3 marks and a wrong answer gets -1 mark. A student throws a 4-sided unbiased die to answer each question (the different throws are independent). Let X be the random variable that denotes the score of the student in the examination.

1. Calculate the expected score $E(X)$.
2. Calculate the value of X for which the probability is the highest.
3. What is the smallest s so that $P(X \leq s) \geq 1/2$?
4. Calculate the variance $\sigma^2(X)$.

Use a calculator if necessary.

N.B: Submission of the last date is 25-05-2024.

Best of Luck