6.2 The Normal Distribution

Normal Probability Distribution

A ***normal probability distribution*** , when plotted, gives a bell-shaped curve such that:

1. The total area under the curve is 1.0.
2. The curve is symmetric about the mean.
3. The two tails of the curve extend indefinitely.

Figure 6.11 Normal distribution with mean μ and standard deviation σ.

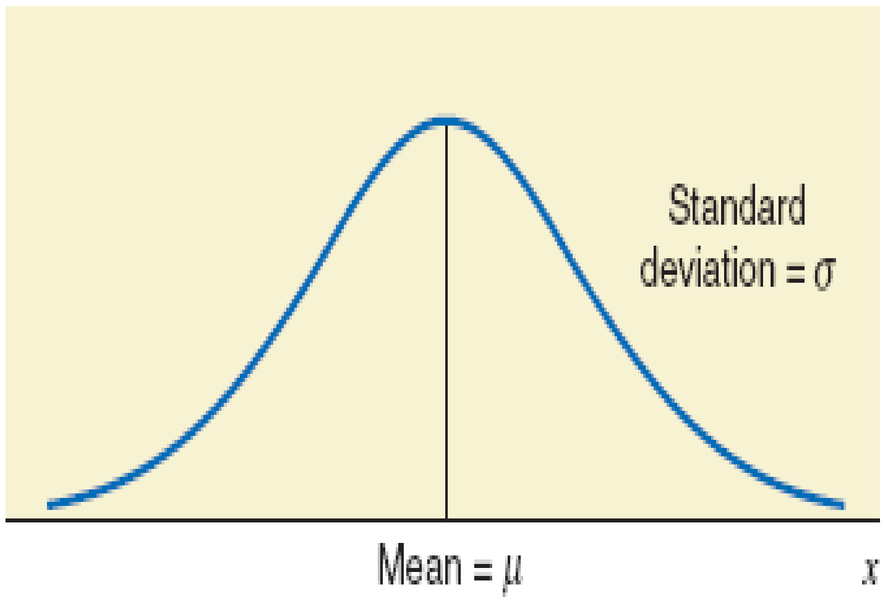

Figure 6.12 Total area under a normal curve.

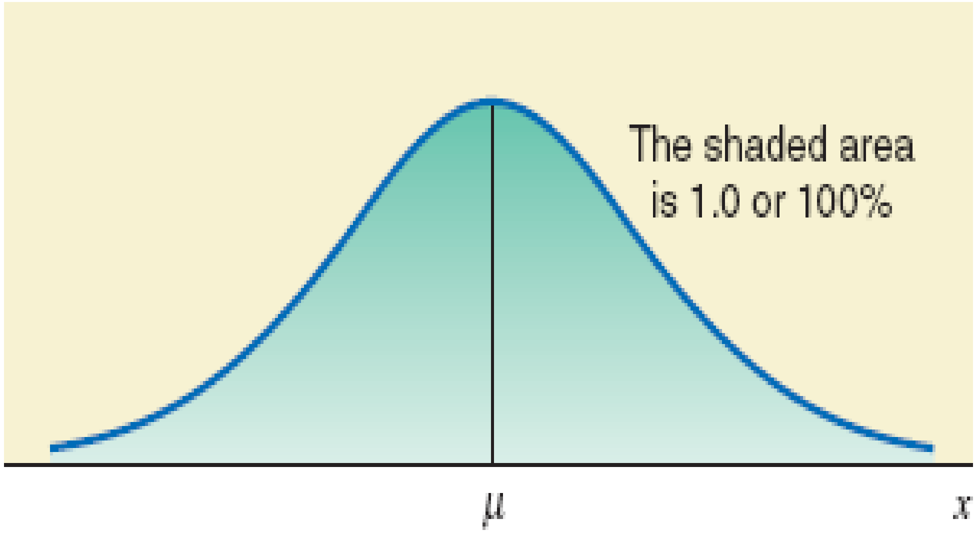

Figure 6.13 A normal curve is symmetric about the mean.

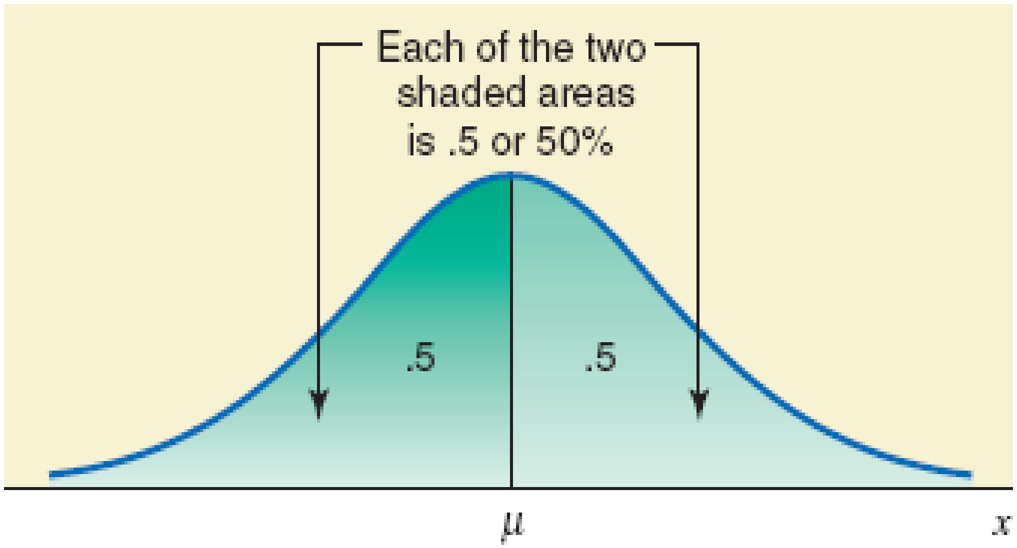

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Figure 6.14 Areas of the normal curve beyond $\mu \pm 3\sigma$.

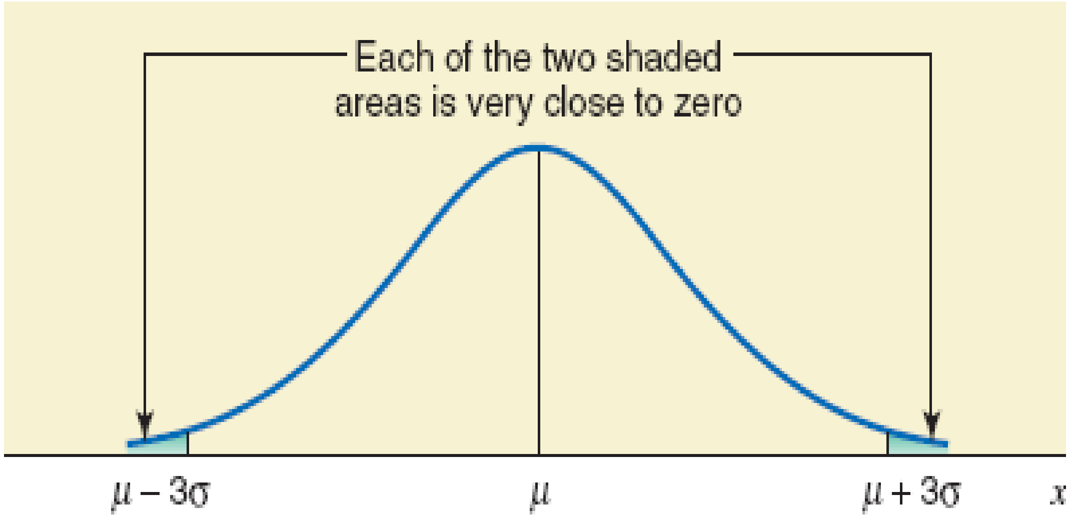


Figure 6.15 Three normal distribution curves with the same mean but different standard deviations.

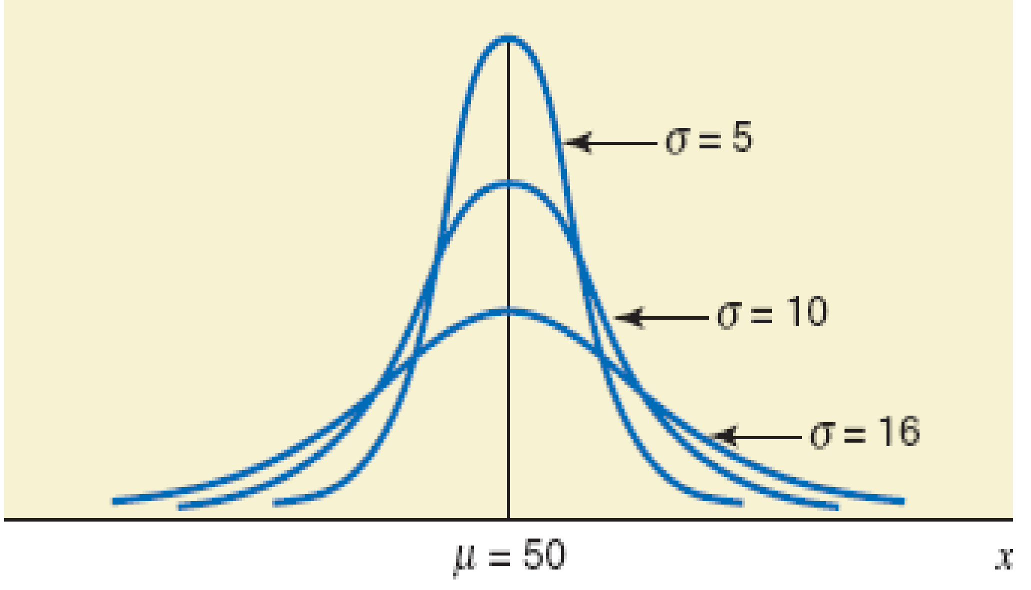
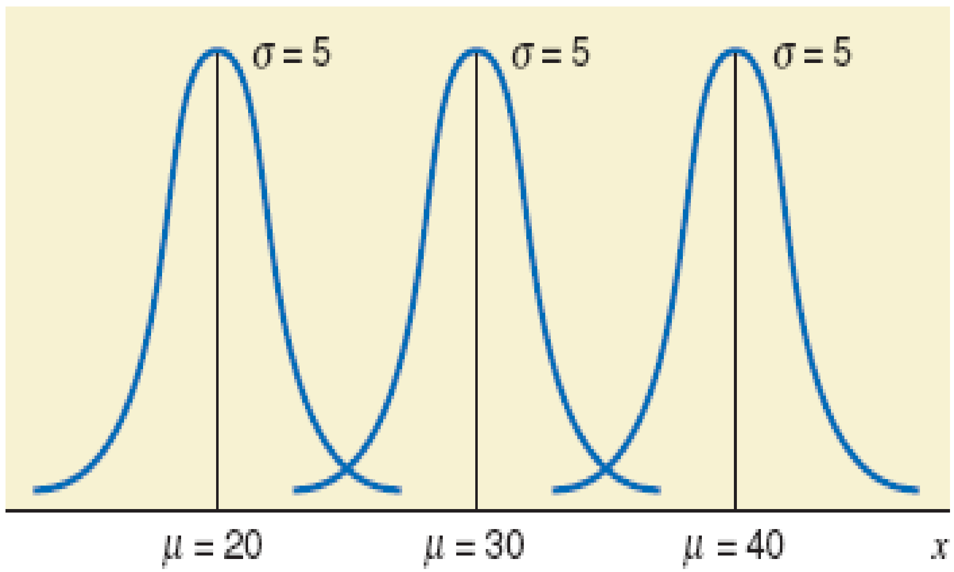


Figure 6.16 Three normal distribution curves with different means but the same standard deviation.

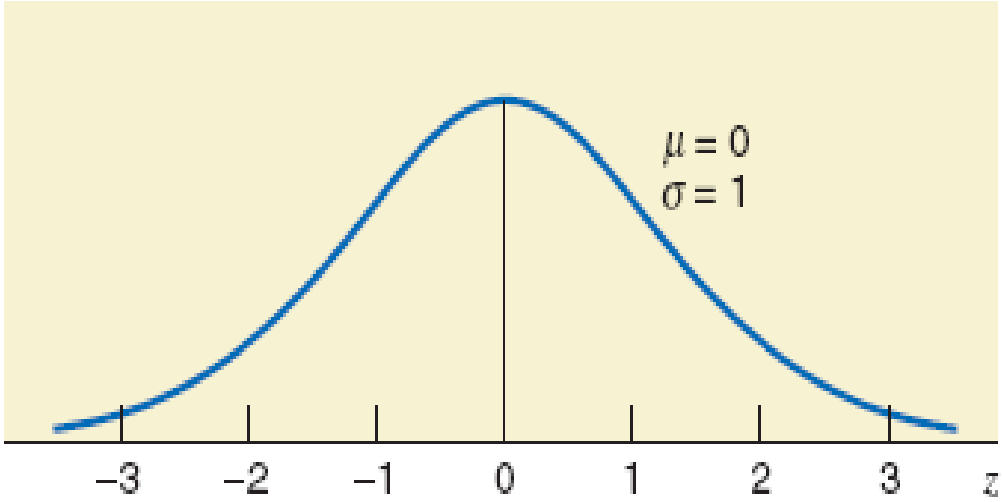


6.3 The Standard Normal Distribution

Definition

The normal distribution with $\mu = 0$ and $\sigma = 1$ is called the ***standard normal distribution***.

Figure 6.17 The standard normal distribution curve.



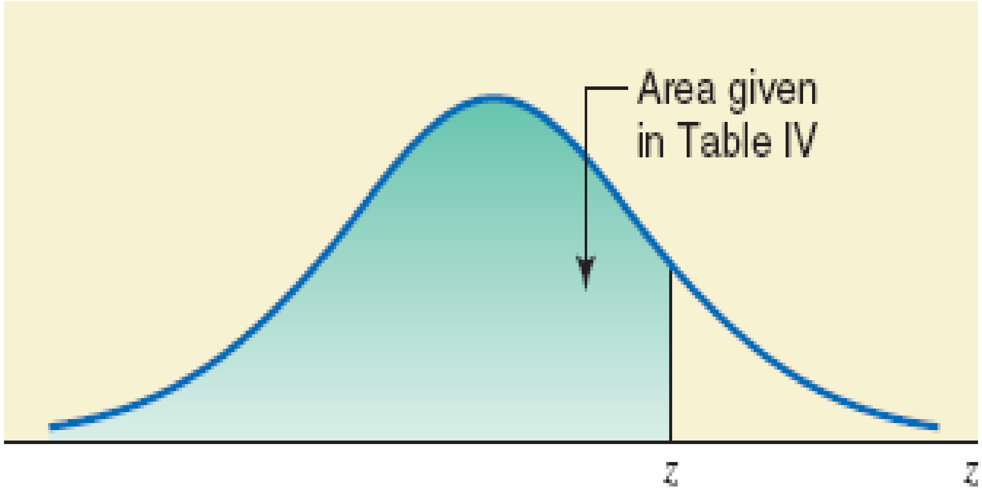
THE STANDARD NORMAL DISTRIBTUION

z Values or *z* Scores

Definition

The units marked on the horizontal axis of the standard normal curve are denoted by ***z*** and are called the ***z* values or *z* scores**. A specific value of z gives the distance between the mean and the point represented by z in terms of the standard deviation.

Figure 6.18 Area under the standard normal curve.



Example 6-1

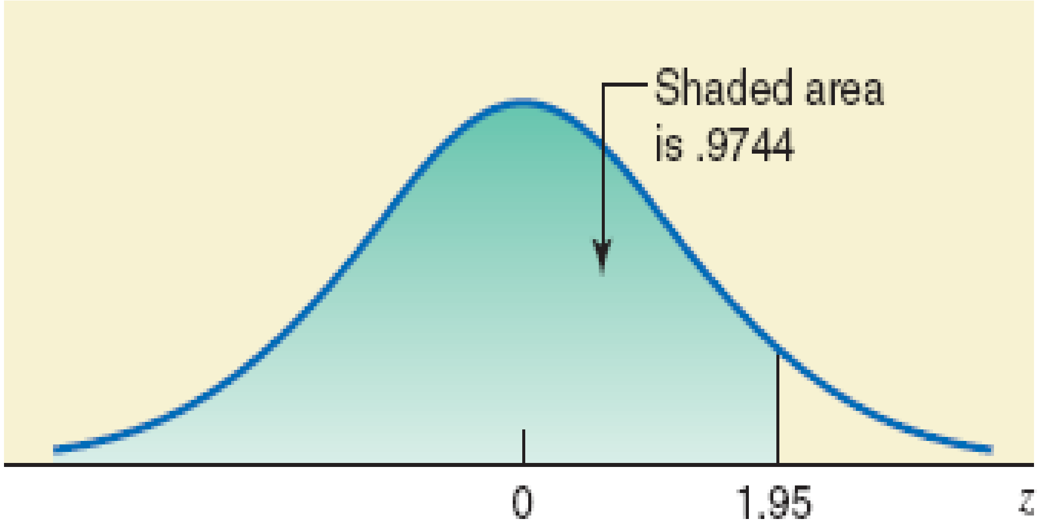
Find the area under the standard normal curve between z = 0 and z = 1.95.

Table 6.2 Area Under the Standard Normal Curve to the Left of $z = 1.95$

z	.00	.01	...	.05	...	.09
−3.4	.0003	.0003	...	.0003	...	.0002
−3.3	.0005	.0005	...	.0004	...	.0003
−3.2	.0007	.0007	...	.0006	...	.0005
.	.	.	...	.	...	.
.	.	.	...	.	...	.
.	.	.	...	.	...	.
1.9	.9713	.9719	...	.9744	...	.9767
.	.	.	...	.	...	.
.	.	.	...	.	...	.
.	.	.	...	.	...	.
3.4	.9997	.9997	...	.9997	...	.9998

Required area

Figure 6.19 Area to the left of $z = 1.95$.



Example 6-2

Find the area under the standard normal curve from z = -2.17 to z = 0.

Example 6-2: Solution

- To find the area from z=-2.17 to z =0 , first we find the areas to the left of z=0 and to the left of z=-2.17 in Table IV. As shown in Table 6.3, these two areas are .5 and .0150, respectively. Next we subtract .0150 from .5 to find the required area.
- Area from -2.17 to 0 = $P(-2.17 \le z \le 0)$

= .5000 - .0150 = **.4850**

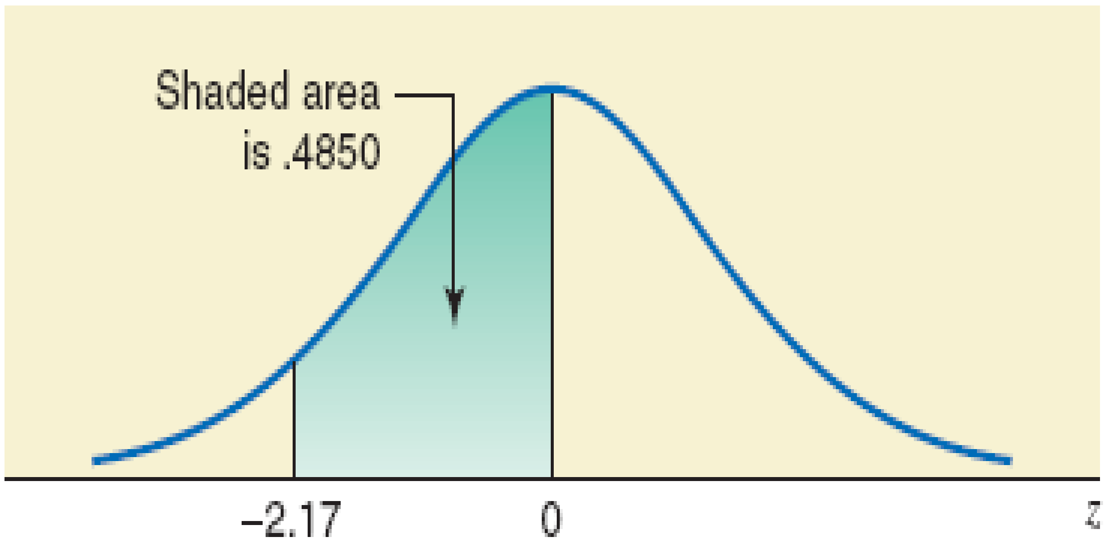
Table 6.3 Area Under the Standard Normal Curve

z	00	.01	. . .	.07	. . .	.09
−3.4	.0003	.0003	. . .	.0003	. . .	.0002
−3.3	.0005	.0005	. . .	.0004	. . .	.0003
−3.2	.0007	.0007	. . .	.0005	. . .	.0005
.	.	.	. . .	.	. . .	.
.	.	.	. . .	.	. . .	.
−2.1	.0179	.0174	. . .	.0150	. . .	.0143
.	.	.	. . .	.	. . .	.
.	.	.	. . .	.	. . .	.
0.0	.5000	.5040	. . .	.5279	. . .	.5359
.	.	.	. . .	.	. . .	.
.	.	.	. . .	.	. . .	.
3.4	.9997	.9997	. . .	.9997	. . .	.9998

Area to the left of $z = 0$

Area to the left of $z = -2.17$

Figure 6.20 Area from z = -2.17 to z = 0.



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Example 6-3

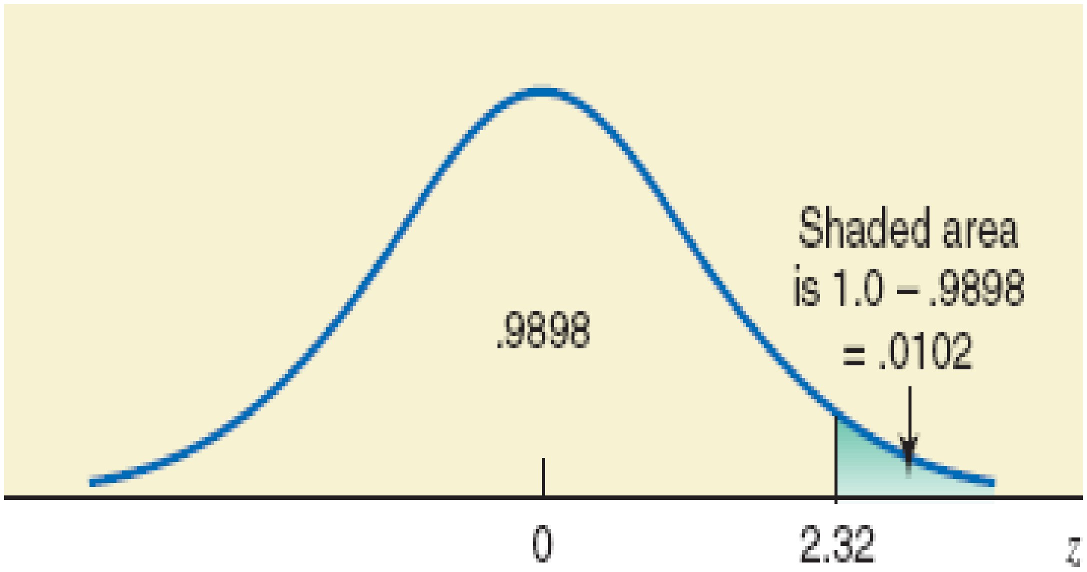
Find the following areas under the standard normal curve.

a) Area to the right of $z = 2.32$
b) Area to the left of $z = -1.54$

Example 6-3: Solution

a) To find the area to the right of $z=2.32$, first we find the area to the left of $z=2.32$. Then we subtract this area from 1.0, which is the total area under the curve. The required area is 1.0 - .9898 = .0102.

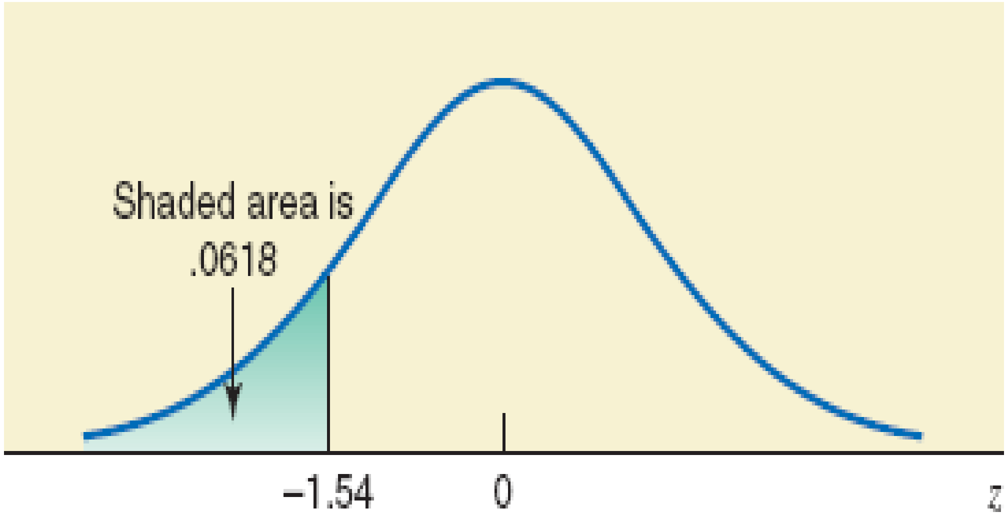
Figure 6.21 Area to the right of $z = 2.32$.



Example 6-3: Solution

b) To find the area under the standard normal curve to the left of z=-1.54, we find the area in Table IV that corresponds to -1.5 in the z column and .04 in the top row. This area is .0618.
Area to the left of -1.54
$= P(z < -1.54) = .0618$

Figure 6.22 Area to the left of *z* = -1.54.



Example 6-4

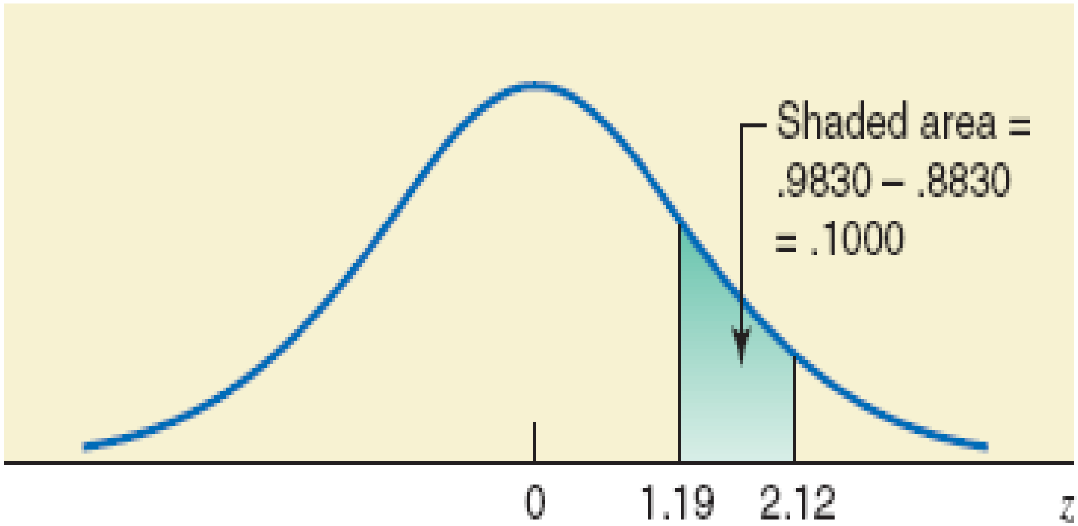
Find the following probabilities for the standard normal curve.

a) $P\ (1.19 < z < 2.12)$
b) $P\ (-1.56 < z < 2.31)$
c) $P\ (z > -.75)$

Example 6-4: Solution

a) $P\,(1.19 < z < 2.12)$
$= \text{Area between } 1.19 \text{ and } 2.12$
$= .9830 - .8830$
$= .1000$

Figure 6.23 Finding $P(1.19 < z < 2.12)$.



Example 6-4: Solution

b) $P(-1.56 < z < 2.31)$

$= \text{Area between } -1.56 \text{ and } 2.31$

$= .9896 - .0594$

$= .9302$

Figure 6.24 Finding P (-1.56 < z < 2.31).

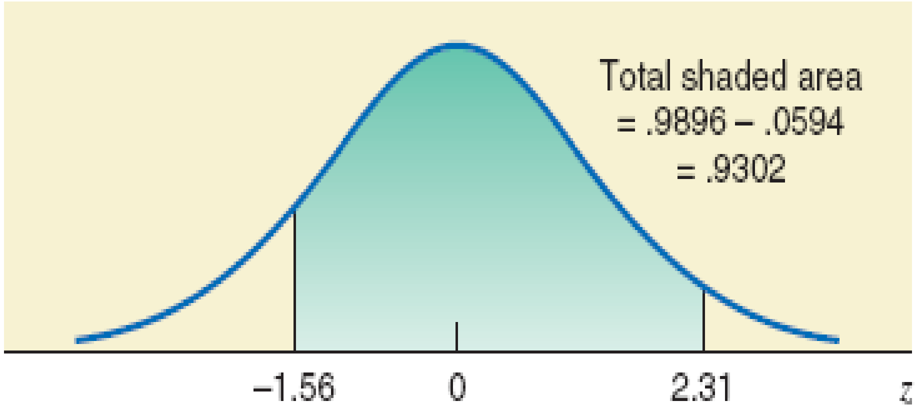


Figure 6.24 Finding $P(-1.56 < z < 2.31)$.

Example 6-4: Solution

c) $P\,(z > -.75)$

$= \text{Area to the right of } -.75$

$= 1.0 - .2266$

$= .7734$

Figure 6.25 Finding $P(z > -.75)$.

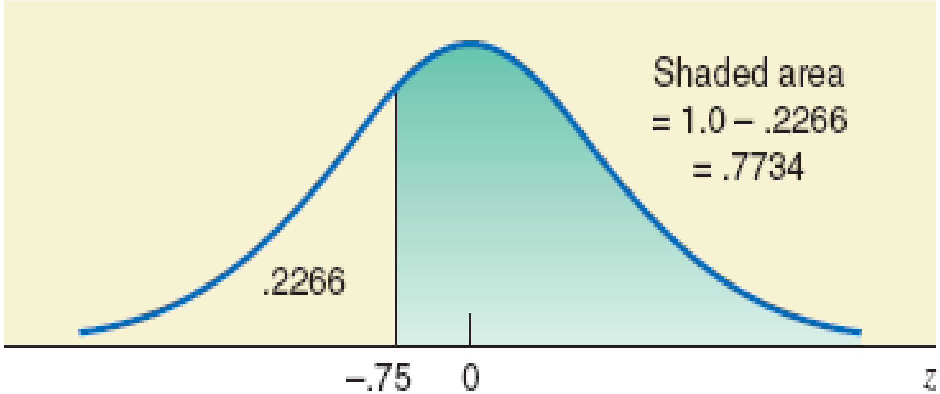


Figure 6.25 Finding $P(z > -.75)$.

Figure 6.26 Area within one standard deviation of the mean.

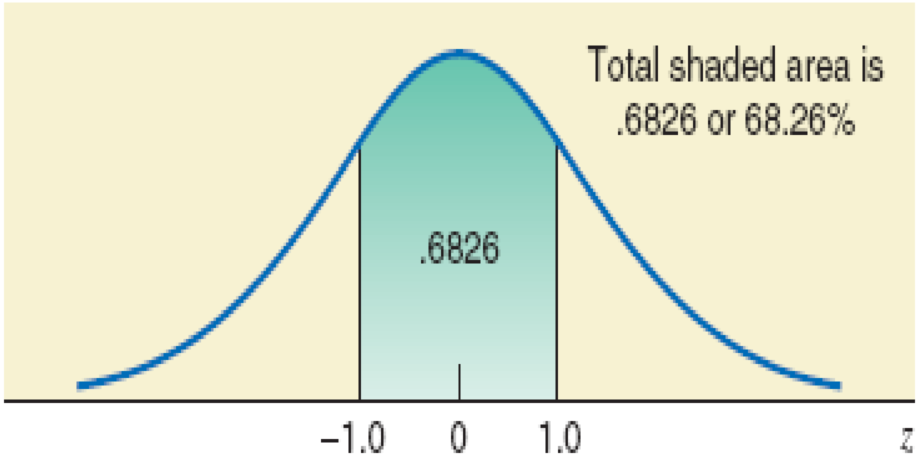


Figure 6.27 Area within two standard deviations of the mean.

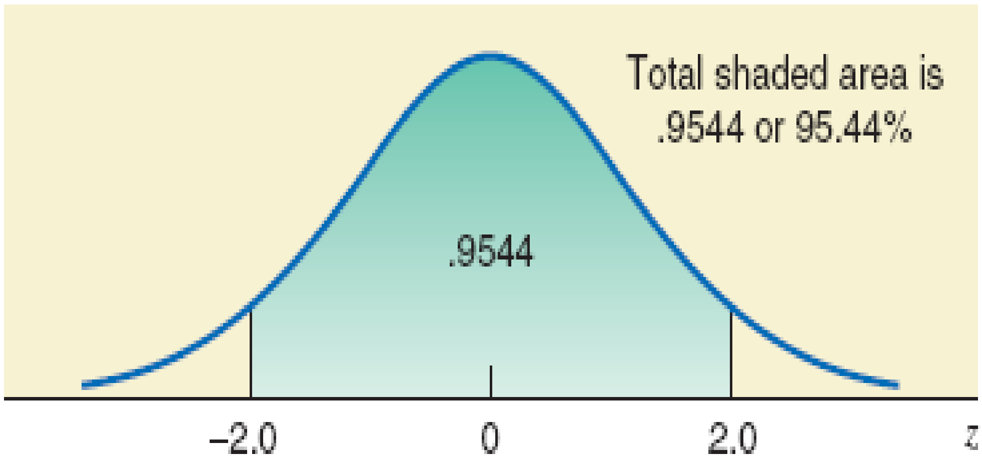
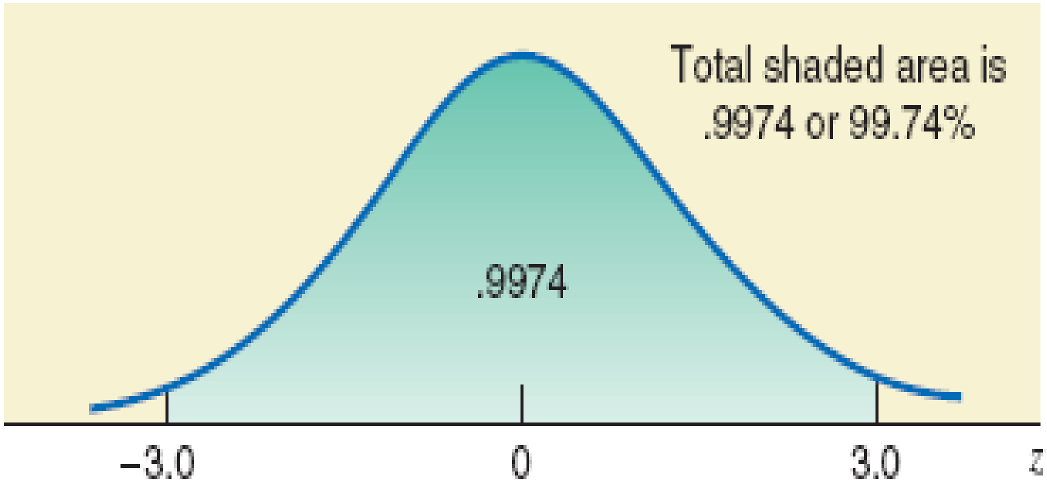


Figure 6.28 Area within three standard deviations of the mean.



Example 6-5

Find the following probabilities for the standard normal curve.

a) $P\ (0 < z < 5.67)$

b) $P\ (z < -5.35)$

6.4 Standardizing a Normal Distribution

Converting an x Value to a z Value

For a normal random variable x, a particular value of x can be converted to its corresponding z value by using the formula

$$z = \frac{x - \mu}{\sigma}$$

where μ and σ are the mean and standard deviation of the normal distribution of x, respectively.

Example 6-6

Let x be a continuous random variable that has a normal distribution with a mean of 50 and a standard deviation of 10. Convert the following x values to z values and find the probability to the left of these points.

a) x = 55

b) x = 35

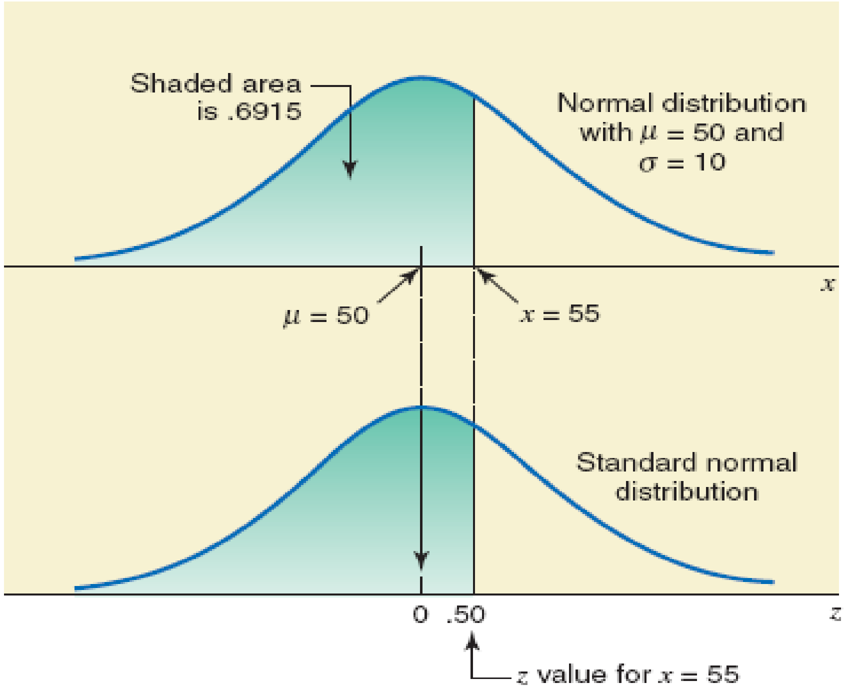
Example 6-6: Solution

a) x = 55

$$z = \frac{x - \mu}{\sigma} = \frac{55 - 50}{10} = .50$$

P(x < 55) = P(z < .50) = .6915

Figure 6.31 z value for $x = 55$.



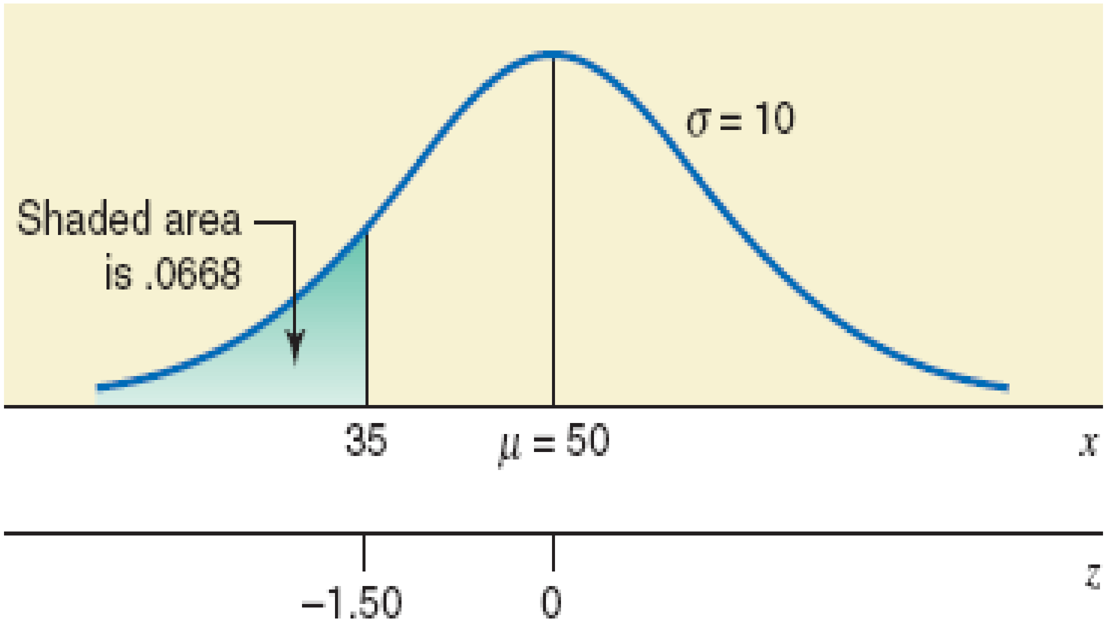
Example 6-6: Solution

b) x = 35

$$z = \frac{x - \mu}{\sigma} = \frac{35 - 50}{10} = -1.50$$

P(x < 35) = P(*z* < -1.50) = .0668

Figure 6.32 z value for $x = 35$.



Example 6-7

Let x be a continuous random variable that is normally distributed with a mean of 25 and a standard deviation of 4. Find the area

a) between x = 25 and x = 32

b) between x = 18 and x = 34

Example 6-7: Solution

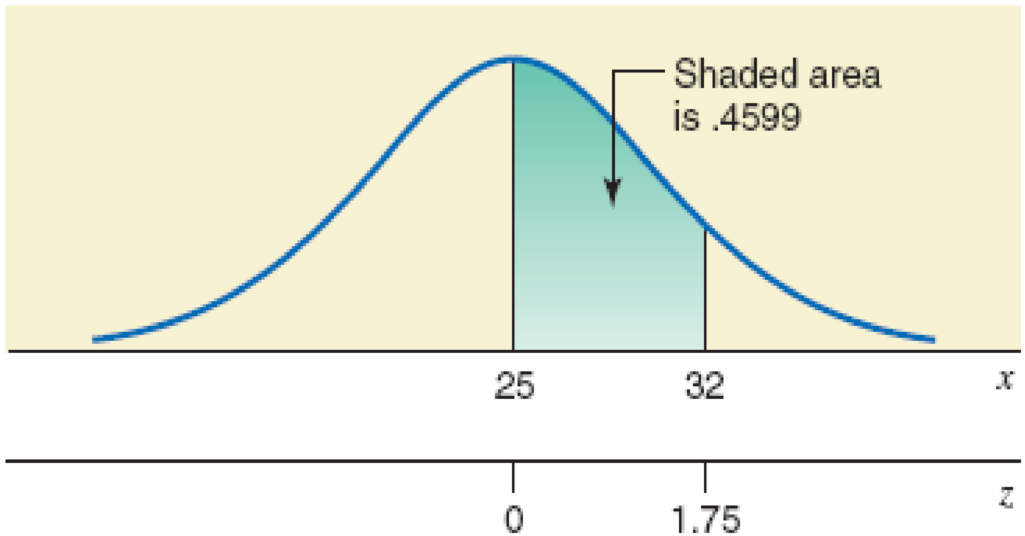
a)

- The z value for x = 25 is 0
- The z value for x = 32 is

$$z = \frac{x - \mu}{\sigma} = \frac{32 - 25}{4} = 1.75$$

- $P(25 < x < 32) = P(0 < z < 1.75)$
  $= .4599$

Figure 6.33 Area between $x = 25$ and $x = 32$.



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6.5 Applications of the Normal Distribution

Section 6.2 through 6.4 discussed the normal distribution, how to convert a normal distribution to the standard normal distribution, and how to find areas under a normal distribution curve. This section presents examples that illustrate the applications of the normal distribution.

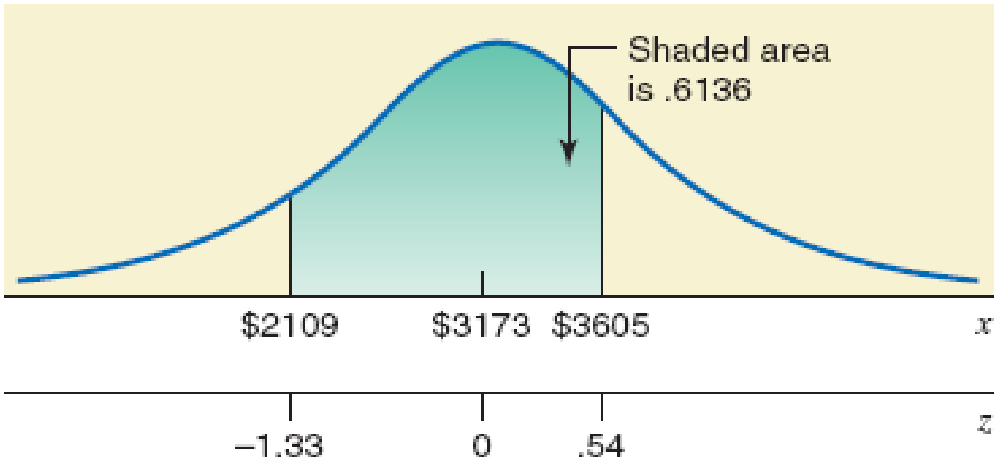
Example 6-11

According to a Sallie Mae and credit bureau data, in 2008, college students carried an average of \$3173 debt on their credit cards (*USA TODAY*, April 13, 2009). Suppose that current credit card debts for all college students have a normal distribution with a mean of \$3173 and a standard deviation of \$800. Find the probability that credit card debt for a randomly selected college student is between \$2109 and \$3605.

Example 6-11: Solution

- For x = \$2109: $$z = \frac{2109 - 3173}{800} = -1.33$$
- For x = \$3605: $$z = \frac{3605 - 3173}{800} = .54$$
- P (\$2109 < x < \$3605)
  = P (-1.33 < z < .54)
  = .7054 - .0918
  = .6136 = 61.36%

Figure 6.40 Area between x = \$2109 and x = \$3605.



Example 6-12

A racing car is one of the many toys manufactured by Mack Corporation. The assembly times for this toy follow a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5 p.m. every day. If one worker starts to assemble a racing car at 4 p.m., what is the probability that she will finish this job before the company closes for the day?

Example 6-12: Solution

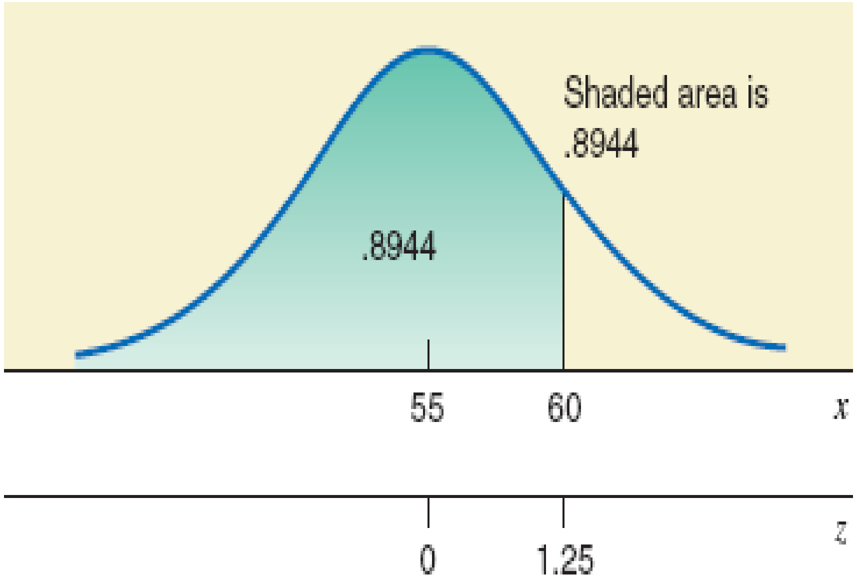
- For x = 60:

$$z = \frac{60 - 55}{4} = -1.25$$

- $P(x \leq 60) = P(z \leq 1.25) = .8944$

- Thus, the probability is .8944 that this worker will finish assembling this racing car before the company closes for the day.

Figure 6.41 Area to the left of $x = 60$.



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Example 6-13

Hupper Corporation produces many types of soft drinks, including Orange Cola. The filling machines are adjusted to pour 12 ounces of soda into each 12-ounce can of Orange Cola. However, the actual amount of soda poured into each can is not exactly 12 ounces; it varies from can to can. It has been observed that the net amount of soda in such a can has a normal distribution with a mean of 12 ounces and a standard deviation of .015 ounce.

Example 6-13

a) What is the probability that a randomly selected can of Orange Cola contains 11.97 to 11.99 ounces of soda?
b) What percentage of the Orange Cola cans contain 12.02 to 12.07 ounces of soda?

Example 6-13: Solution

a)

- For x = 11.97: $$z = \frac{11.97 - 12}{.015} = -2.00$$
- For x = 11.99: $$z = \frac{11.99 - 12}{.015} = -.67$$
- $P(11.97 \le x \le 11.99)$
  $= P(-2.00 \le z \le -.67) = .2514 - .0228$
  $= .2286$

Figure 6.42 Area between $x = 11.97$ and $x = 11.99$.

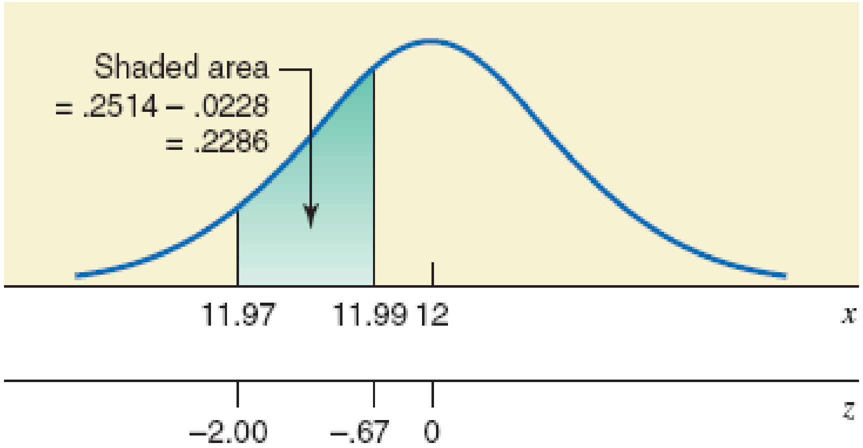

Figure 6.42 Area between $x = 11.97$ and $x = 11.99$.

Example 6-13: Solution

b)

- For x = 12.02: $$z = \frac{12.02 - 12}{.015} = 1.33$$
- For x = 12.07: $$z = \frac{12.07 - 12}{.015} = 4.67$$
- $P(12.02 \leq x \leq 12.07)$
  $= P(1.33 \leq z \leq 4.67) = 1 - .9082$
  $= .0918$

Figure 6.43 Area from $x = 12.02$ to $x = 12.07$.