

CHAPTER 5

DISCRETE RANDOM VARIABLES AND THEIR PROBABILITY DISTRIBUTIONS

5.1 Random Variables

- Discrete Random Variable
- Continuous Random Variable

Table 5.1 Frequency and Relative Frequency Distribution of the Number of Vehicles Owned by Families

Number of Vehicles Owned	Frequency	Relative Frequency
0	30	$30/2000 = .015$
1	470	$470/2000 = .235$
2	850	$850/2000 = .425$
3	490	$490/2000 = .245$
4	160	$160/2000 = .080$
$N = 2000$		Sum = 1.000

RANDOM VARIABLES

Definition

A **random variable** is a variable whose value is determined by the outcome of a random experiment.

Discrete Random Variable

Definition

A *random variable* that assumes countable values is called a **discrete random variable**.

Examples of discrete random variables

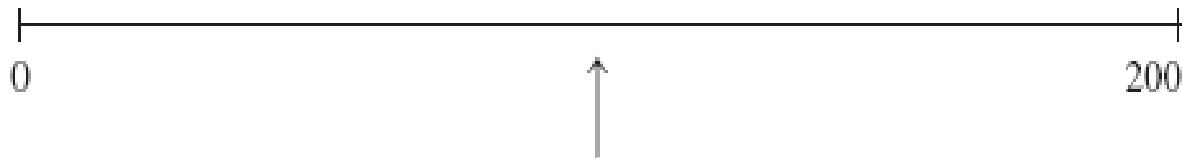
1. The number of cars sold at a dealership during a given month
2. The number of houses in a certain block
3. The number of fish caught on a fishing trip
4. The number of complaints received at the office of an airline on a given day
5. The number of customers who visit a bank during any given hour
6. The number of heads obtained in three tosses of a coin

Continuous Random Variable

Definition

A random variable that can assume any value contained in one or more intervals is called a **continuous random variable**.

Continuous Random Variable



Every point on this line represents a possible value of x that denotes the life of a battery. There is an infinite number of points on this line. The values represented by points on this line are uncountable.

Examples of continuous random variables

1. The height of a person
2. The time taken to commute from home to work
3. The weight of a fish
4. The price of a house

5.2 Probability Distribution of a Discrete Random Variable

Definition

The **probability distribution of a discrete random variable** lists all the possible values that the random variable can assume and their corresponding probabilities.

Example 5-1

Recall the frequency and relative frequency distributions of the number of vehicles owned by families given in Table 5.1. That table is reproduced below as Table 5.2. Let x be the number of vehicles owned by a randomly selected family. Write the probability distribution of x .

Table 5-2 Frequency and Relative Frequency Distributions of the Number of Vehicles Owned by Families

Number of Vehicles Owned	Frequency	Relative Frequency
0	30	.015
1	470	.235
2	850	.425
3	490	.245
4	160	.080
$N = 2000$		Sum = 1.000

Example 5-1: Solution

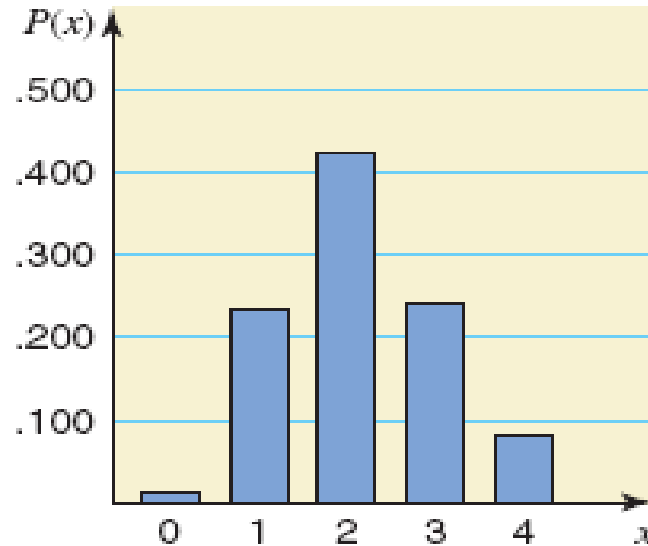
Number of Vehicles Owned x	Probability $P(x)$
0	.015
1	.235
2	.425
3	.245
4	.080
$\Sigma P(x) = 1.000$	

Two Characteristics of a Probability Distribution

The probability distribution of a discrete random variable possesses the following *two characteristics*.

1. $0 \leq P(x) \leq 1$ for each value of x
2. $\sum P(x) = 1$.

Figure 5.1 Graphical presentation of the probability distribution of Table 5.3.



Example 5-2

Each of the following tables lists certain values of x and their probabilities. Determine whether or not each table represents a valid probability distribution.

Example 5-2

(a)

x	$P(x)$
0	.08
1	.11
2	.39
3	.27

(b)

x	$P(x)$
2	.25
3	.34
4	.28
5	.13

(c)

x	$P(x)$
7	.70
8	.50
9	-.20

- a) No, since the sum of all probabilities is not equal to 1.0.
- b) Yes
- c) No since one of the probabilities is negative.

Example 5-3

The following table lists the probability distribution of the number of breakdowns per week for a machine based on past data.

Breakdowns per week	0	1	2	3
Probability	.15	.20	.35	.30

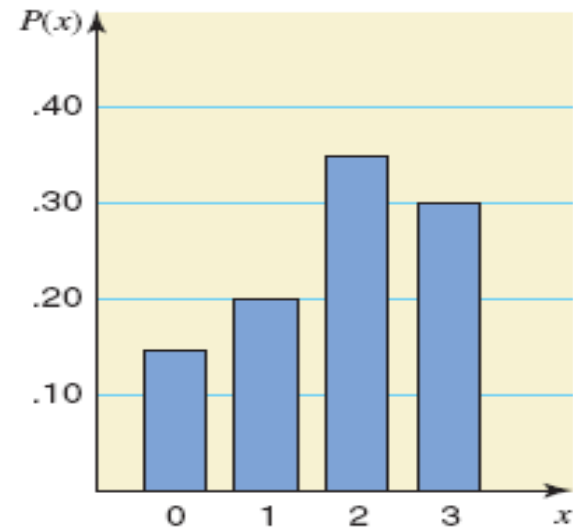
Example 5-3

- a) Present this probability distribution graphically.
- b) Find the probability that the number of breakdowns for this machine during a given week is
 - i. exactly 2 ii. 0 to 2
 - iii. more than 1 iv. at most 1

Example 5-3: Solution

Let x denote the number of breakdowns for this machine during a given week. Table 5.4 lists the probability distribution of x .

x	$P(x)$
0	.15
1	.20
2	.35
3	.30
$\Sigma P(x) = 1.00$	



Example 5-3: Solution

(b) Using Table 5.4,

i. $P(\text{exactly 2 breakdowns}) = P(x = 2) = \mathbf{.35}$

ii. $P(0 \text{ to } 2 \text{ breakdowns}) = P(0 \leq x \leq 2)$
 $= P(x = 0) + P(x = 1) + P(x = 2)$
 $= .15 + .20 + .35 = \mathbf{.70}$

iii. $P(\text{more than 1 breakdown}) = P(x > 1)$
 $= P(x = 2) + P(x = 3)$
 $= .35 + .30 = \mathbf{.65}$

iv. $P(\text{at most one breakdown}) = P(x \leq 1)$
 $= P(x = 0) + P(x = 1)$
 $= .15 + .20 = \mathbf{.35}$

Example 5-4

According to a survey, 60% of all students at a large university suffer from math anxiety. Two students are randomly selected from this university. Let x denote the number of students in this sample who suffer from math anxiety. Develop the probability distribution of x .

Example 5-4: Solution

Let us define the following two events:

N = the student selected does not suffer from math anxiety

M = the student selected suffers from math anxiety

$$P(x = 0) = P(NN) = .16$$

$$\begin{aligned} P(x = 1) &= P(NM \text{ or } MN) = P(NM) + P(MN) \\ &= .24 + .24 = .48 \end{aligned}$$

$$P(x = 2) = P(MM) = .36$$

Figure 5.3 Tree diagram.

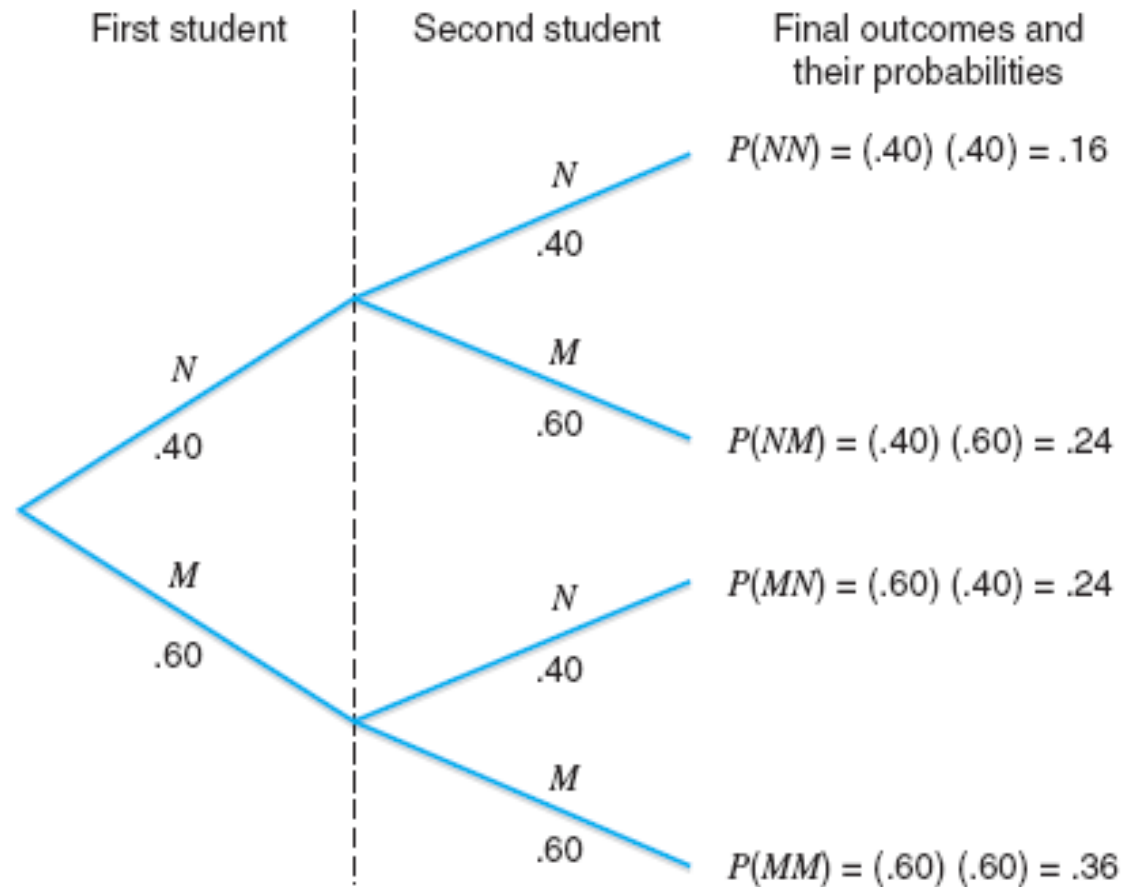


Figure 5.3 Tree diagram.

Table 5.5 Probability Distribution of the Number of Students with Math Anxiety in a Sample of Two Students

x	$P(x)$
0	.16
1	.48
2	.36
$\Sigma P(x) = 1.00$	

5.3 Mean of a Discrete Random Variable

The **mean of a discrete variable** x is the value that is expected to occur per repetition, on average, if an experiment is repeated a large number of times. It is denoted by μ and calculated as

$$\mu = \sum x P(x)$$

The mean of a discrete random variable x is also called its expected value and is denoted by $E(x)$; that is,

$$E(x) = \sum x P(x)$$

Example 5-5

Recall Example 5-3 of Section 5-2. The probability distribution Table 5.4 from that example is reproduced on the next slide. In this table, x represents the number of breakdowns for a machine during a given week, and $P(x)$ is the probability of the corresponding value of x .

Example 5-5

x	$P(x)$
0	.15
1	.20
2	.35
3	.30

Find the mean number of breakdown per week for this machine.

Example 5-5: Solution

Table 5.6 Calculating the Mean for the Probability Distribution of Breakdowns

x	$P(x)$	$xP(x)$
0	.15	$0(.15) = .00$
1	.20	$1(.20) = .20$
2	.35	$2(.35) = .70$
3	.30	$3(.30) = .90$
		$\Sigma xP(x) = 1.80$

The mean is $\mu = \Sigma x P(x) = 1.80$

5.4

Standard Deviation of a Discrete Random Variable

The **standard deviation of a discrete random variable** x measures the spread of its probability distribution and is computed as

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$$

Example 5-6

Baier's Electronics manufactures computer parts that are supplied to many computer companies. Despite the fact that two quality control inspectors at Baier's Electronics check every part for defects before it is shipped to another company, a few defective parts do pass through these inspections undetected. Let x denote the number of defective computer parts in a shipment of 400. The following table gives the probability distribution of x .

Example 5-6

x	0	1	2	3	4	5
$P(x)$.02	.20	.30	.30	.10	.08

Compute the standard deviation of x .

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.02	.00	0	.00
1	.20	.20	1	.20
2	.30	.60	4	1.20
3	.30	.90	9	2.70
4	.10	.40	16	1.60
5	.08	.40	25	2.00
$\Sigma xP(x) = 2.50$			$\Sigma x^2P(x) = 7.70$	

Example 5-6: Solution

$$\mu = \sum xP(x) = 2.50 \quad \text{defective computer parts in 400}$$

$$\sum x^2 P(x) = 7.70$$

$$\begin{aligned}\sigma &= \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{7.70 - (2.50)^2} = \sqrt{1.45} \\ &= 1.204 \quad \text{defective computer parts}\end{aligned}$$

Example 5-7

Loraine Corporation is planning to market a new makeup product. According to the analysis made by the financial department of the company, it will earn an annual profit of \$4.5 million if this product has high sales and an annual profit of \$ 1.2 million if the sales are mediocre, and it will lose \$2.3 million a year if the sales are low. The probabilities of these three scenarios are .32, .51 and .17 respectively.

Example 5-7

- a) Let x be the profits (in millions of dollars) earned per annum from this product by the company. Write the probability distribution of x .
- b) Calculate the mean and the standard deviation of x .

Example 5-7: Solution

- a) The table below lists the probability distribution of x . Note that because x denotes profits earned by the company, the loss is written as a negative profit in the table.

x	$P(x)$
4.5	.32
1.2	.51
-2.3	.17

Example 5-7: Solution

- b) Table 4.8 shows all the calculations needed for the computation of the mean and standard deviations of x .

$$\mu = \sum xP(x) = \$1.661 \text{ million}$$

$$\begin{aligned}\sigma &= \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{8.1137 - (1.661)^2} \\ &= \$2.314 \text{ million}\end{aligned}$$

Table 5.8 Computations to Find the Mean and Standard Deviation

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
4.5	.32	1.440	20.25	6.4800
1.2	.51	.612	1.44	.7344
-2.3	.17	-.391	5.29	.8993
$\Sigma xP(x) = 1.661$			$\Sigma x^2P(x) = 8.1137$	

Interpretation of the Standard Deviation

The standard deviation of a discrete random variable can be interpreted or used the same way as the standard deviation of a data set in Section 3.4 of Chapter 3.

5.5 Factorials, Combinations, and Permutations

Factorials

Definition

The symbol $n!$, read as " n factorial," represents the product of all the integers from n to 1. In other words,

$$n! = n(n - 1)(n - 2)(n - 3) \cdot \cdot \cdot 3 \cdot 2 \cdot 1$$

By definition,

$$0! = 1$$

Examples

Evaluate 7!

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{5040}$$

Evaluate 10!

$$\begin{aligned} 10! &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= \mathbf{3,628,800} \end{aligned}$$

Evaluate (12-4)!

$$\begin{aligned} (12-4)! &= 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= \mathbf{40,320} \end{aligned}$$

Example 5-11

Evaluate $(5-5)!$

$$(5-5)! = 0! = \mathbf{1}$$

Note that $0!$ is always equal to 1.

FACTORIALS, COMBINATIONS, AND PERMUTATIONS

Combinations

Definition

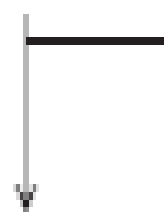
Combinations give the number of ways x elements can be selected from n elements. The notation used to denote the total number of combinations is

$${}_nC_x$$

which is read as “the number of combinations of n elements selected x at a time.”

Combinations

n denotes the total number of elements



${}_nC_x$ = the number of combinations of n elements selected x at a time



x denotes the number of elements selected per selection

Combinations

Number of Combinations

The **number of combinations** for selecting x from n distinct elements is given by the formula

$${}_nC_x = \frac{n!}{x!(n-x)!}$$

where **$n!$** , **$x!$** , and **$(n-x)!$** are read as "*n factorial*," "*x factorial*," "*n minus x factorial*," respectively.

Example 5-12

An ice cream parlor has six flavors of ice cream. Sadia wants to buy two flavors of ice cream. If she randomly selects two flavors out of six, how many combinations are there?

Example 5-12: Solution

n = total number of ice cream flavors = 6

x = # of ice cream flavors to be selected = 2

$${}_6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 15$$

Thus, there are 15 ways for to select two ice cream flavors out of six.

Example 5-13

Three members of a jury will be randomly selected from five people. How many different combinations are possible?

$n = 5$ and $x = 3$

$${}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{120}{6 \cdot 2} = 10$$

Example 5-13: Solution

Example 5-14

Marv & Sons advertised to hire a financial analyst. The company has received applications from 10 candidates who seem to be equally qualified. The company manager has decided to call only 3 of these candidates for an interview. If she randomly selects 3 candidates from the 10, how many total selections are possible?

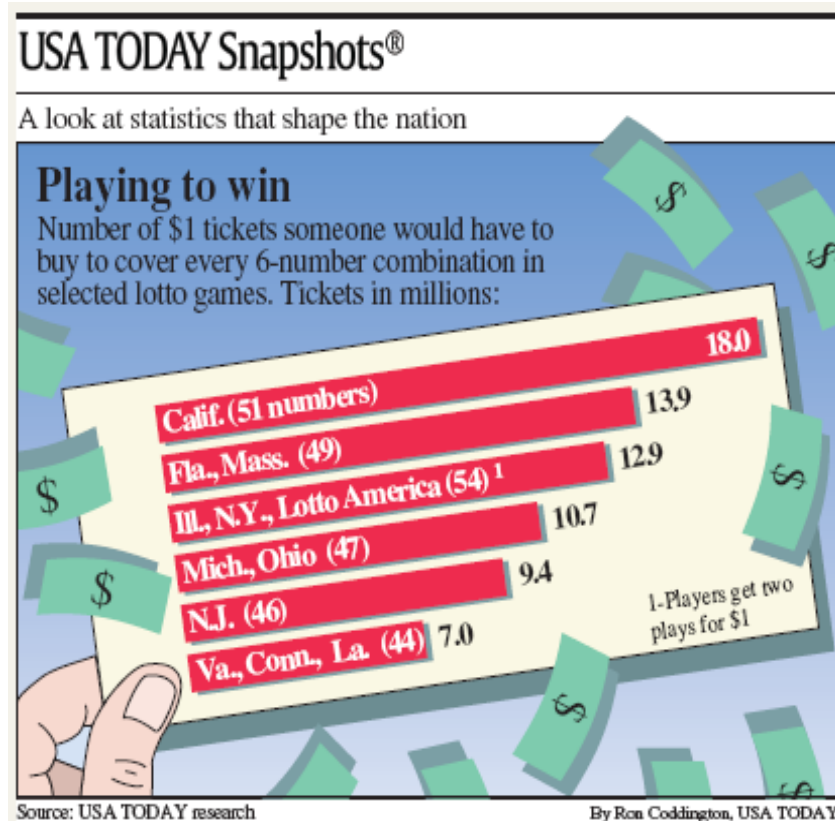
Example 5-14: Solution

$n = 10$ and $x = 3$

$${}_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!} = \frac{3,628,800}{(6)(5040)} = 120$$

Thus, the company manager can select 3 applicants from 10 in 120 ways.

Case Study 5-2 Playing Lotto



Permutations

Permutations Notation

Permutations give the total selections of x element from n (different) elements in such a way that the order of selections is important. The notation used to denote the permutations is

$${}_nP_x$$

which is read as "*the number of permutations of selecting x elements from n elements.*"

Permutations are also called arrangement.

Permutations

Permutations Formula

The following formula is used to find the number of permutations or arrangement of selecting x items out of n items. Note that here, the n items should all be different.

$${}_n P_x = \frac{n!}{(n-x)!}$$

Example 5-15

A club has 20 members. They are to select three office holders – president, secretary, and treasurer – for next year. They always select these office holders by drawing 3 names randomly from the names of all members. The first person selected becomes the president, the second is the secretary, and the third one takes over as treasurer. Thus, the order in which 3 names are selected from the 20 names is important. Find the total arrangements of 3 names from these 20.

Example 5-15: Solution

n = total members of the club = 20

x = number of names to be selected = 3

$${}_nP_x = \frac{n!}{(n-x)!} = \frac{20!}{(20-3)!} = \frac{20!}{17!} = 6840$$

Thus, there are 6840 permutations or arrangement for selecting 3 names out of 20.

5.6 The Binomial Probability Distribution

- The Binomial Experiment
- The Binomial Probability Distribution and Binomial Formula
- Using the Table of Binomial Probabilities
- Probability of Success and the Shape of the Binomial Distribution
- Mean and Standard Deviation of the Binomial Distribution

The Binomial Experiment

Conditions of a Binomial Experiment

A binomial experiment must satisfy the following four conditions.

1. There are n identical trials.
2. Each trial has only two possible outcomes.
3. The probabilities of the two outcomes remain constant.
4. The trials are independent.

Example 5-16

Consider the experiment consisting of 10 tosses of a coin. Determine whether or not it is a binomial experiment.

Example 5-16: Solution

1. There are a total of 10 trials (tosses), and they are all identical. Here, $n=10$.
2. Each trial (toss) has only two possible outcomes: a head and a tail.
3. The probability of obtaining a head (success) is $\frac{1}{2}$ and that of a tail (a failure) is $\frac{1}{2}$ for any toss.

That is,

$$p = P(H) = \frac{1}{2} \quad \text{and} \quad q = P(T) = \frac{1}{2}$$

4. The trials (tosses) are independent.

Consequently, the experiment consisting of 10 tosses is a binomial experiment.

Example 5-17

Five percent of all DVD players manufactured by a large electronics company are defective. Three DVD players are randomly selected from the production line of this company. The selected DVD players are inspected to determine whether each of them is defective or good. Is this experiment a binomial experiment?

Example 5-17: Solution

1. This example consists of three identical trials.
2. Each trial has two outcomes: defective or good.
3. The probability p that a DVD player is defective is .05. The probability q that a DVD player is good is .95.
4. Each trial (DVD player) is independent.

Because all four conditions of a binomial experiment are satisfied, this is an example of a binomial experiment.

The Binomial Probability Distribution and Binomial Formula

For a binomial experiment, the probability of exactly x successes in n trials is given by the binomial formula

$$P(x) = {}_n C_x p^x q^{n-x}$$

where

n = total number of trials

p = probability of success

$q = 1 - p$ = probability of failure

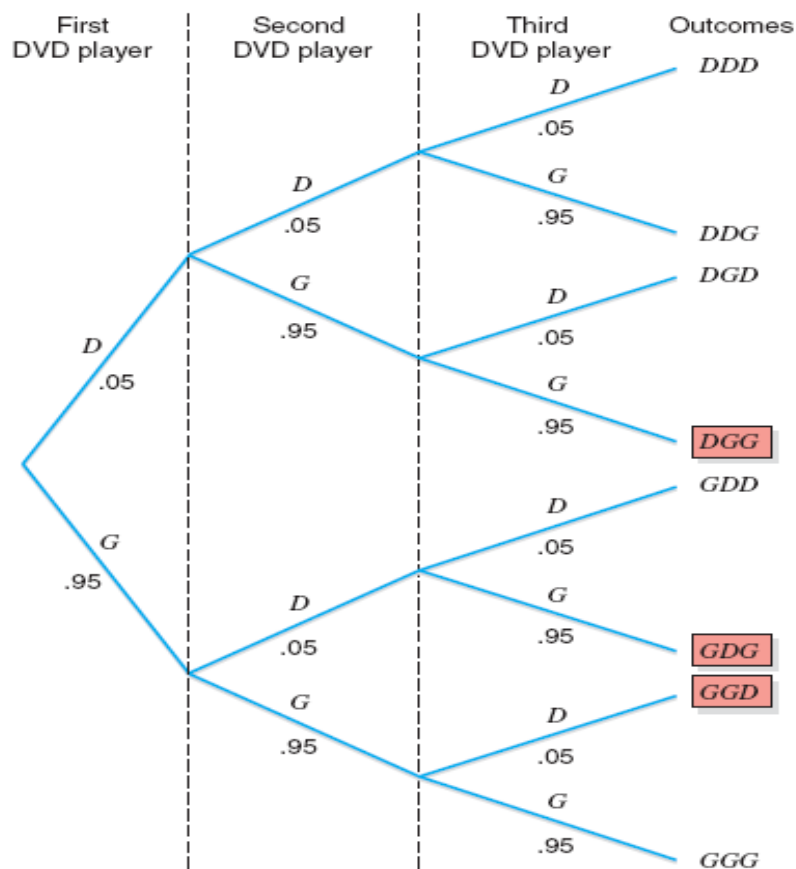
x = number of successes in n trials

$n - x$ = number of failures in n trials

Example 5-18

Five percent of all DVD players manufactured by a large electronics company are defective. A quality control inspector randomly selects three DVD player from the production line. What is the probability that exactly one of these three DVD players is defective?

Figure 5.4 Tree diagram for selecting three DVD Players.



Example 5-18: Solution

Let

D = a selected DVD player is defective $P(D) = .05$

G = a selected DVD player is good $P(G) = .95$

$$\begin{aligned} P(DGG) &= P(D) P(G) P(G) \\ &= (.05)(.95)(.95) = .0451 \end{aligned}$$

$$\begin{aligned} P(GDG) &= P(G) P(D) P(G) \\ &= (.95)(.05)(.95) = .0451 \end{aligned}$$

$$\begin{aligned} P(GGD) &= P(G) P(G) P(D) \\ &= (.95)(.95)(.05) = .0451 \end{aligned}$$

Example 5-18: Solution

$$\begin{aligned} &P(\text{1 DVD player in 3 is defective}) \\ &= P(DGG \text{ or } GDG \text{ or } GGD) \\ &= P(DGG) + P(GDG) + P(GGD) \\ &= .0451 + .0451 + .0451 \\ &= \mathbf{.1353} \end{aligned}$$

Example 5-18: Solution

n = total number of trials = 3 DVD players

x = number of successes = number of defective DVD players = 1

$n - x$ = number of failures = number of good DVD players = $3 - 1 = 2$

$p = P(\text{success}) = .05$

$q = P(\text{failure}) = 1 - p = .95$

Example 5-18: Solution

The probability of selecting exactly one defective DVD player is

Number of ways to obtain 1 success in 3 trials Number of successes Number of failures

Probability of success Probability of failure

$$P(x = 1) = {}_3C_1(.05)^1(.95)^2 = (3)(.05)(.9025) = .1354$$

Example 5-19

At the Express House Delivery Service, providing high-quality service to customers is the top priority of the management. The company guarantees a refund of all charges if a package it is delivering does not arrive at its destination by the specified time. It is known from past data that despite all efforts, 2% of the packages mailed through this company do not arrive at their destinations within the specified time. Suppose a corporation mails 10 packages through Express House Delivery Service on a certain day.

Example 5-19

- a) Find the probability that exactly one of these 10 packages will not arrive at its destination within the specified time.
- b) Find the probability that at most one of these 10 packages will not arrive at its destination within the specified time.

Example 5-19: Solution

n = total number of packages mailed = 10

p = $P(\text{success})$ = .02

q = $P(\text{failure})$ = $1 - .02 = .98$

Example 5-19: Solution

x = number of successes = 1

a) $n - x$ = number of failures = $10 - 1 = 9$

$$\begin{aligned} P(x = 1) &= {}_{10}C_1 (.02)^1 (.98)^9 = \frac{10!}{1!(10-1)!} (.02)^1 (.98)^9 \\ &= (10)(.02)(.83374776) = .1667 \end{aligned}$$

Thus, there is a .1667 probability that exactly one of the 10 packages mailed will not arrive at its destination within the specified time.

Example 5-19: Solution

b) At most one $\Rightarrow x = 0$ and $x = 1$

$$\begin{aligned}P(x \leq 1) &= P(x = 0) + P(x = 1) \\&= {}_{10}C_0 (.02)^0 (.98)^{10} + {}_{10}C_1 (.02)^1 (.98)^9 \\&= (1)(1)(.81707281) + (10)(.02)(.83374776) \\&= .8171 + .1667 = .9838\end{aligned}$$

Thus, the probability that at most one of the 10 packages mailed will not arrive at its destination within the specified time is .9838.

Example 5-20

In a Robert Half International survey of senior executives, 35% of the executives said that good employees leave companies because they are unhappy with the management (*USA TODAY*, February 10, 2009). Assume that this result holds true for the current population of senior executives. Let x denote the number in a random sample of three senior executives who hold this opinion. Write the probability distribution of x and draw a bar graph for this probability distribution.

Example 5-20: Solution

- n = total senior executives in the sample = 3
- p = $P(\text{a senior executive holds the said opinion}) = .35$
- q = $P(\text{a senior executive does not hold the said opinion}) = 1 - .35 = .65$

Example 5-20: Solution

$$P(x = 0) = {}_3C_0 (.56)^0 (.44)^3 = (1)(1)(.085184) = .0852$$

$$P(x = 1) = {}_3C_1 (.56)^1 (.44)^2 = (3)(.56)(.1936) = .3252$$

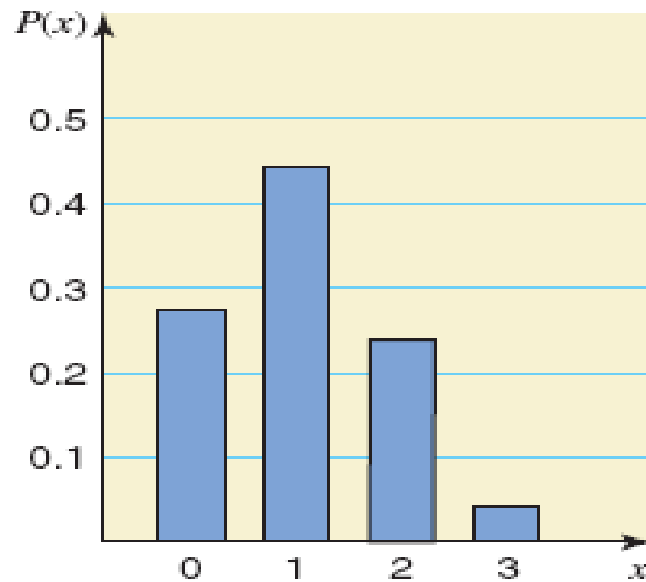
$$P(x = 2) = {}_3C_2 (.56)^2 (.44)^1 = (3)(.3136)(.44) = .4140$$

$$P(x = 3) = {}_3C_3 (.56)^3 (.44)^0 = (1)(.175616)(1) = .1756$$

Table 5.9 Probability Distribution of x

x	$P(x)$
0	.2746
1	.4436
2	.2389
3	.0429

Figure 5.5 Bar graph of the probability distribution of x .



Using the Table of Binomial Probabilities

Table I in Appendix C, the table of binomial probabilities.

List the probabilities of x for $n = 1$ to $n = 25$.

List the probabilities of x for selected values of p .

Example 5-21

In an Accountemps survey of senior executives, 30% of the senior executives said that it is appropriate for job candidates to ask about compensation and benefits during the first interview (*USA TODAY*, April 13, 2009). Suppose that this result holds true for the current population of senior executives in the United States. A random sample of six senior executives is selected. Using Table I of Appendix C, answer the following.

Example 5-21

- a) Find the probability that exactly three senior executives in this sample hold the said opinion.
- b) Find the probability that at most two senior executives in this sample hold the said opinion.
- c) Find the probability that at least three senior executives in this sample hold the said opinion.
- d) Find the probability that one to three senior executives in this sample hold the said opinion.
- e) Let x be the number of senior executives in this sample hold the said opinion. Write the probability distribution of x , and draw a bar graph for this probability distribution.

Table 5.10 Determining $P(x = 3)$ for $n = 6$ and $p = .30$

		p					
		.05	.10	.20	.3095
$n = 6 \rightarrow$	6	.7351	.5314	.2621	.11760000
	1	.2321	.3543	.3932	.30250000
	2	.0305	.0984	.2458	.32410001
$x = 3 \rightarrow$	3	.0021	.0146	.0819	.18520021
	4	.0001	.0012	.0154	.05950305
	5	.0000	.0001	.0015	.01022321
	6	.0000	.0000	.0001	.00077351

$P(x = 3) = .1852$

Table 5.11 Portion of Table I for $n = 6$ and $p = .30$

n	x	p
		.30
6	0	.1176
	1	.3025
	2	.3241
	3	.1852
	4	.0595
	5	.0102
	6	.0007

Example 5-21: Solution

(a) $P(x = 3) = \mathbf{.1852}$

(b) $P(\text{at most } 2) = P(0 \text{ or } 1 \text{ or } 2)$
 $= P(x = 0) + P(x = 1) + P(x = 2)$
 $= .1176 + .3025 + .3241 = \mathbf{.7442}$

(c) $P(\text{at least } 3) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6)$
 $= P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$
 $= .1852 + .0595 + .0102 + .0007$
 $= \mathbf{.2556}$

(d) $P(1 \text{ to } 3) = P(x = 1) + P(x = 2) + P(x = 3)$
 $= .3025 + .3241 + .1852 = \mathbf{.8118}$

Table 5.12 Probability Distribution of x for $n = 6$ and $p = .30$

x	$P(x)$
0	.1176
1	.3025
2	.3241
3	.1852
4	.0595
5	.0102
6	.0007

Figure 5.6 Bar graph for the probability distribution of x .

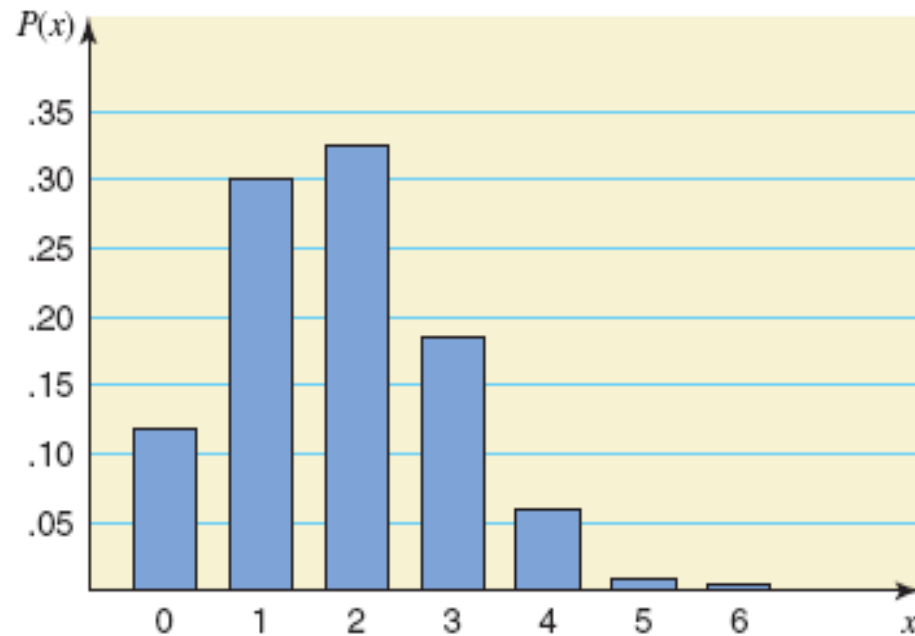


Figure 5.6 Bar graph for the probability distribution of x .

Probability of Success and the Shape of the Binomial Distribution

1. The binomial probability distribution is symmetric if $p = .50$.
2. The binomial probability distribution is skewed to the right if p is less than .50.
3. The binomial probability distribution is skewed to the left if p is greater than .50.

Table 5.13 Probability Distribution of x for $n = 4$ and $p = .50$

x	$P(x)$
0	.0625
1	.2500
2	.3750
3	.2500
4	.0625

Figure 5.7 Bar graph from the probability distribution of Table 5.13.

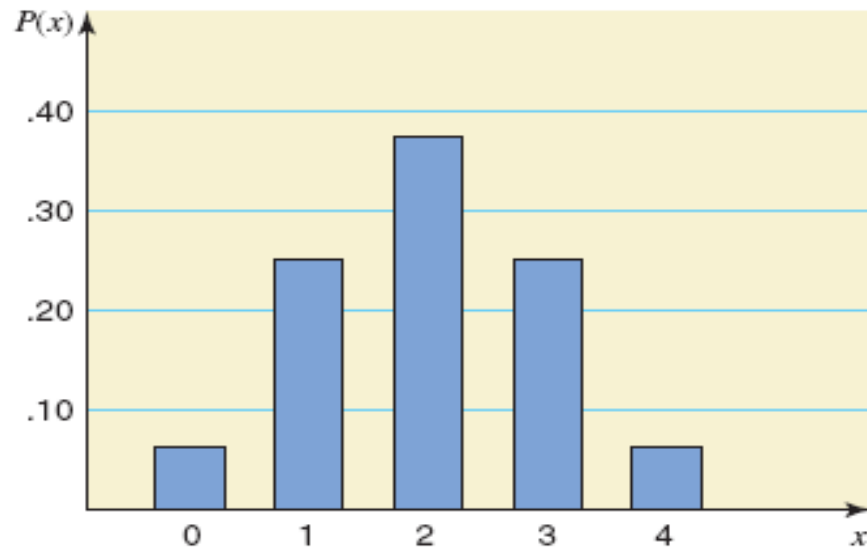


Figure 5.7 Bar graph for the probability distribution of Table 5.13.

Table 5.14 Probability Distribution of x for $n = 4$ and $p = .30$

x	$P(x)$
0	.2401
1	.4116
2	.2646
3	.0756
4	.0081

Figure 5.8 Bar graph from the probability distribution of Table 5.14.

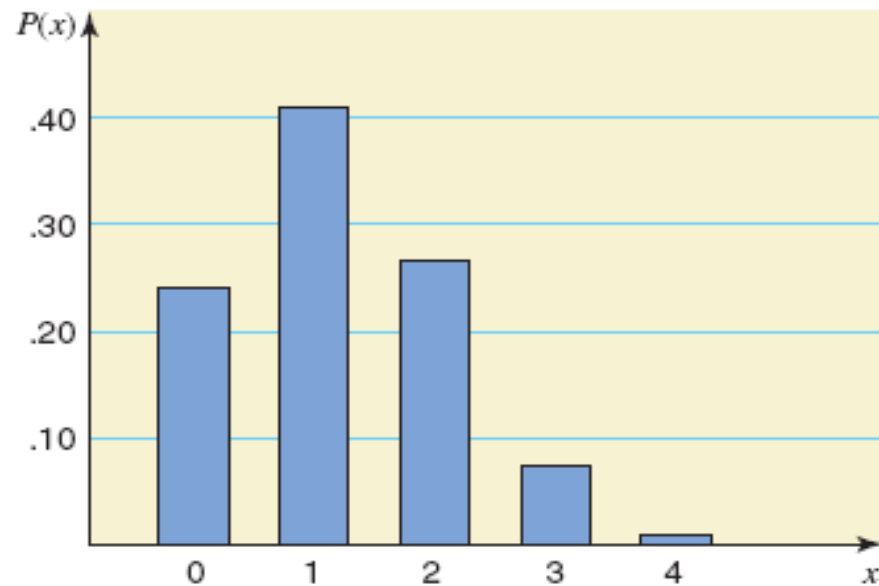


Figure 5.8 Bar graph for the probability distribution of Table 5.14.

Table 5.15 Probability Distribution of x for $n = 4$ and $p = .80$

x	$P(x)$
0	.0016
1	.0256
2	.1536
3	.4096
4	.4096

Figure 5.9 Bar graph from the probability distribution of Table 5.15.

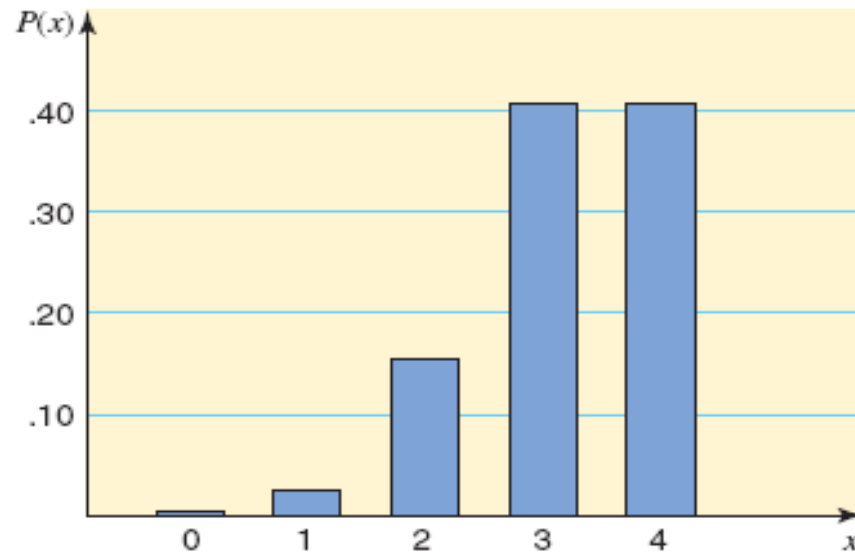


Figure 5.9 Bar graph for the probability distribution of Table 5.15.

Mean and Standard Deviation of the Binomial Distribution

The mean and standard deviation of a binomial distribution are, respectively,

$$\mu = np \quad \text{and} \quad \sigma = \sqrt{npq}$$

where n is the total number of trials, p is the probability of success, and q is the probability of failure.

Example 5-22

According to a Harris Interactive survey conducted for World Vision and released in February 2009, 56% of teens in the United States volunteer time for charitable causes. Assume that this result is true for the current population of U.S. teens. A sample of 60 teens is selected. Let x be the number of teens in this sample who volunteer time for charitable causes. Find the mean and standard deviation of the probability distribution of x .

Example 5-22: Solution

$$n = 60 \quad p = .56, \quad \text{and} \quad q = .44$$

Using the formulas for the mean and standard deviation of the binomial distribution,

$$\mu = np = 40(.45) = 18$$

$$\sigma = \sqrt{npq} = \sqrt{(40)(.45)(.55)} = 3.146$$

5.7 The Hypergeometric Probability Distribution

Let

N = total number of elements in the population

r = number of successes in the population

$N - r$ = number of failures in the population

n = number of trials (sample size)

x = number of successes in n trials

$n - x$ = number of failures in n trials

THE HYPERGEOMETRIC PROBABILITY DISTRIBUTION

The probability of x successes in n trials is given by

$$P(x) = \frac{{}_r C_x {}_{N-r} C_{n-x}}{{}_N C_n}$$

Example 5-23

Brown Manufacturing makes auto parts that are sold to auto dealers. Last week the company shipped 25 auto parts to a dealer. Later on, it found out that 5 of those parts were defective. By the time the company manager contacted the dealer, 4 auto parts from that shipment have already been sold. What is the probability that 3 of those 4 parts were good parts and 1 was defective?

Example 5-23: Solution

$$N = 25, r = 20, N - r = 5, n = 4, x = 3, n - x = 1$$

$$\begin{aligned} P(x = 3) &= \frac{{}_r C_x {}_{N-r} C_{n-x}}{{}_N C_n} = \frac{{}_{20} C_3 {}_5 C_1}{{}_{25} C_4} = \frac{\frac{20!}{3!(20-3)!} \cdot \frac{5!}{1!(5-1)!}}{\frac{25!}{4!(25-4)!}} \\ &= \frac{(1140)(5)}{12,650} = .4506 \end{aligned}$$

Thus, the probability that three of the four parts sold are good and one is defective is .4506.

Example 5-24

Dawn Corporation has 12 employees who hold managerial positions. Of them, 7 are female and 5 are male. The company is planning to send 3 of these 12 managers to a conference. If 3 managers are randomly selected out of 12,

- a) Find the probability that all 3 of them are female
- b) Find the probability that at most 1 of them is a female

Example 5-24: Solution

$$(a) N = 12, r = 7, N - r = 5, n = 3, x = 3, n - x = 0$$

$$P(x = 3) = \frac{{}_r C_x {}_{N-r} C_{n-x}}{{}_N C_n} = \frac{{}_7 C_3 {}_5 C_0}{{}_{12} C_3} = \frac{(35)(1)}{220} = .1591$$

Thus, the probability that all 3 of managers selected are female is .1591.

Example 5-24: Solution

(b) $N = 12, r = 7, N - r = 5, n = 3, x = 0$ and $1, n - x = 3$

$$P(x = 0) = \frac{{}_r C_x {}_{N-r} C_{n-x}}{{}_N C_n} = \frac{{}_7 C_0 {}_5 C_3}{{}_{12} C_3} = \frac{(1)(10)}{220} = .0455$$

$$P(x = 1) = \frac{{}_r C_x {}_{N-r} C_{n-x}}{{}_N C_n} = \frac{{}_7 C_1 {}_5 C_2}{{}_{12} C_3} = \frac{(7)(10)}{220} = .3182$$

$$P(x \leq 1) = P(x = 0) + P(x = 1) = .0455 + .3182 = .3637$$

Thus, the probability that at most 1 of 3 managers selected is female is .3637.

5.8 The Poisson Probability Distribution

- Using the Table of Poisson probabilities
- Mean and Standard Deviation of the Poisson Probability Distribution

THE POISSON PROBABILITY DISTRIBUTION

Conditions to Apply the Poisson Probability Distribution

The following three conditions must be satisfied to apply the Poisson probability distribution.

1. x is a discrete random variable.
2. The occurrences are random.
3. The occurrences are independent.

Examples of Poisson Probability Distribution

1. The number of accidents that occur on a given highway during a 1-week period.
2. The number of customers entering a grocery store during a 1-hour interval.
3. The number of television sets sold at a department store during a given week.

THE POISSON PROBABILITY DISTRIBUTION

Poisson Probability Distribution Formula

According to the Poisson probability distribution, the probability of x occurrences in an interval is

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where λ (pronounced *lambda*) is the mean number of occurrences in that interval and the value of e is approximately 2.71828.

Example 5-25

On average, a household receives 9.5 telemarketing phone calls per week. Using the Poisson distribution formula, find the probability that a randomly selected household receives exactly 6 telemarketing phone calls during a given week.

Example 5-25: Solution

$$\begin{aligned}P(x = 6) &= \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(9.5)^6 e^{-9.5}}{6!} \\&= \frac{(735,091.8906)(.00007485)}{720} \\&= 0.0764\end{aligned}$$

Example 5-26

A washing machine in a laundromat breaks down an average of three times per month. Using the Poisson probability distribution formula, find the probability that during the next month this machine will have

- a) exactly two breakdowns
- b) at most one breakdown

Example 5-26: Solution

(a) P(exactly two breakdowns)

$$P(x = 2) = \frac{(3)^2 e^{-3}}{2!} = \frac{(9)(.04978707)}{2} = .2240$$

(b) P(at most 1 breakdown) = P(0 or 1 breakdown)

$$\begin{aligned} P(x = 0) + P(x = 1) &= \frac{(3)^0 e^{-3}}{0!} + \frac{(3)^1 e^{-3}}{1!} \\ &= \frac{(1)(.04978707)}{1} + \frac{(3)(.04978707)}{1} \\ &= .0498 + .1494 = .1992 \end{aligned}$$

Example 5-27

Cynthia's Mail Order Company provides free examination of its products for 7 days. If not completely satisfied, a customer can return the product within that period and get a full refund. According to past records of the company, an average of 2 of every 10 products sold by this company are returned for a refund. Using the Poisson probability distribution formula, find the probability that exactly 6 of the 40 products sold by this company on a given day will be returned for a refund.

Example 5-27: Solution

$$\lambda = 8, \quad x = 6$$

$$P(x = 6) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(8)^6 e^{-8}}{6!} = \frac{(262,144)(.00033546)}{720} = .1221$$

Thus, the probability is .1221 that exactly 6 products out of 40 sold on a given day will be returned.

Case Study 5-3 Ask Mr. Statistics

Fortune magazine used to publish a column titled *Ask Mr. Statistics*, which contained questions and answers to statistical problems. The following excerpts are reprinted from one such column.

Dear Oddgiver: I am in the seafood distribution business and find myself endlessly wrangling with supermarkets about appropriate order sizes, especially with high-end tidbit products like our matjes herring in superspiced wine, which we let them have for \$4.25, and still they take only a half-dozen jars, thereby running the risk of getting sold out early in the week and causing the better class of customers to storm out empty-handed. How do I get them to realize that lowballing on inventories is usually bad business, also to at least try a few jars of our pickled crappie balls?

—HEADED FOR A BREAKDOWN

Using the Table of Poisson Probabilities

Table III in appendix C, the table of Poisson probabilities.

Example 5-28

On average, two new accounts are opened per day at an Imperial Saving Bank branch. Using Table III of Appendix C, find the probability that on a given day the number of new accounts opened at this bank will be

a) exactly 6 b) at most 3 c) at least 7

Table 5.16 Portion of Table III for $\lambda = 2.0$

		λ			
x		1.1	1.2	...	2.0
0					.1353
1					.2707
2					.2707
3					.1804
4					.0902
5					.0361
$x = 6 \longrightarrow$	6				.0120
7					.0034
8					.0009
9					.0002

← $\lambda = 2.0$

← $P(x = 6)$

Example 5-28: Solution

- a) $P(x = 6) = .0120$
- b) $P(\text{at most } 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) = .1353 + .2707 + .2707 + .1804 = .8571$
- c) $P(\text{at least } 7) = P(x = 7) + P(x = 8) + P(x = 9) = .0034 + .0009 + .0002 = .0045$

Case Study 5-4 Living and Dying in The Sea



Example 5-29

An auto salesperson sells an average of .9 car per day. Let x be the number of cars sold by this salesperson on any given day. Using the Poisson probability distribution table, write the probability distribution of x . Draw a graph of the probability distribution.

Table 5.17 Probability Distribution of x for $\lambda = .9$

x	$P(x)$
0	.4066
1	.3659
2	.1647
3	.0494
4	.0111
5	.0020
6	.0003

Figure 5.10 Bar graph for the probability distribution of Table 5.17.

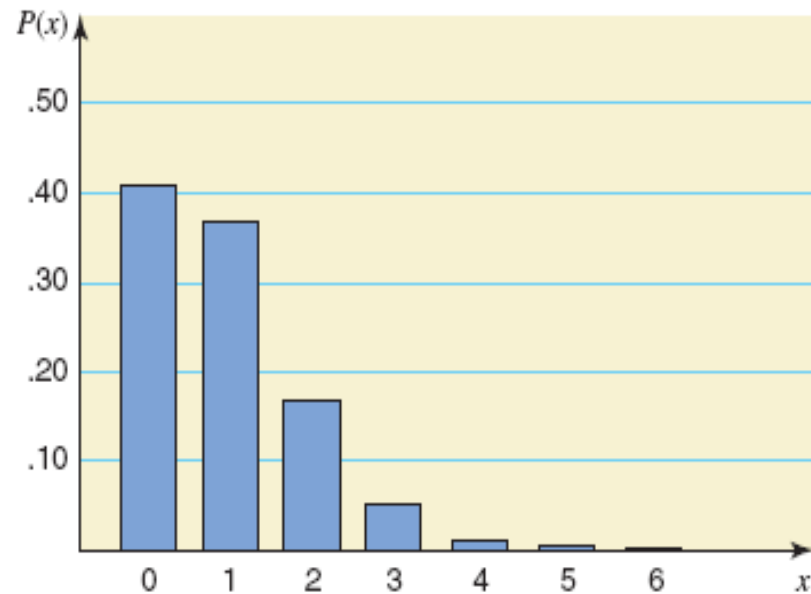


Figure 5.10 Bar graph for the probability distribution of Table 5.17.

Mean and Standard Deviation of the Poisson Probability Distribution

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

$$\sigma = \sqrt{\lambda}$$

Example 5-29

An auto salesperson sells an average of .9 car per day. Let x be the number of cars sold by this salesperson on any given day. Find the mean, variance, and standard deviation.

$$\mu = \lambda = .9 \text{ car}$$

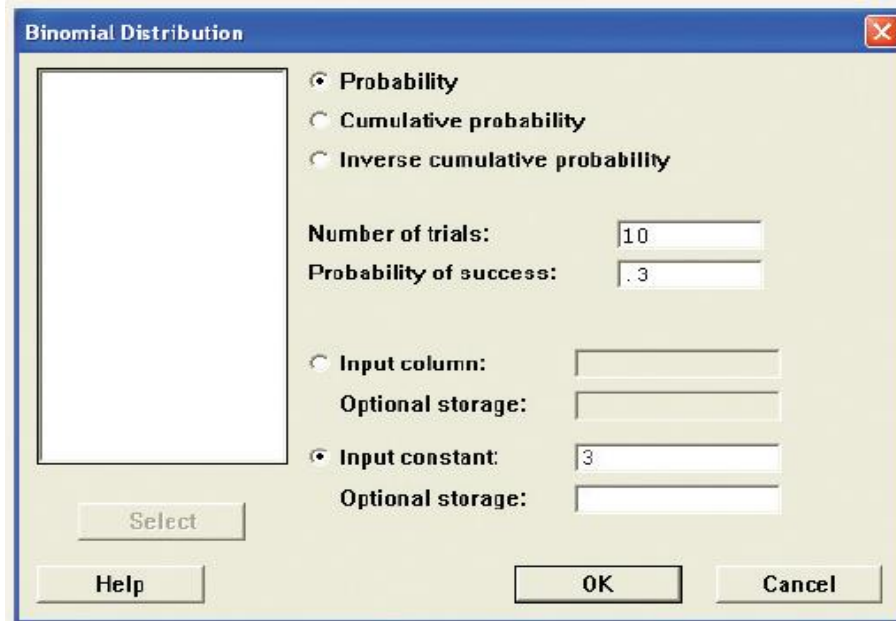
$$\sigma^2 = \lambda = .9$$

$$\sigma = \sqrt{\lambda} = \sqrt{.9} = .949 \text{ car}$$

TI-84

DISTR DRAW
6:χ²pdf()
7:χ²cdf()
8:Fpdf()
9:Fcdf()
0:binompdf()
A:binomcdf()
B:poissonpdf()

Screen 5.1



The image shows the 'Binomial Distribution' dialog box in Minitab. The window has a title bar with the text 'Binomial Distribution' and a close button. On the left is a large empty rectangular box. To its right are three radio buttons: 'Probability' (selected), 'Cumulative probability', and 'Inverse cumulative probability'. Below these are two input fields: 'Number of trials:' with the value '10' and 'Probability of success:' with the value '.3'. Further down are two sets of input fields, each with a radio button: 'Input column:' (unselected) and 'Input constant:' (selected). Each set also has an 'Optional storage:' field. At the bottom left is a 'Select' button, and at the bottom right are 'OK' and 'Cancel' buttons. A 'Help' button is located at the bottom left of the dialog area.

Screen 5.2

	A	B	C	D	E
1	Number of combinations of 3 objects out of 10:				
2					
3	120				

Screen 5.3

Excel

	A	B	C	D
1	Number of combinations of 3 objects out of 10:			
2				
3	=COMBIN(10,3)			
4	COMBIN(number, number_chosen)			

Screen 5.4