CHAPTER 9

HYPOTHESIS TESTS ABOUT THE MEAN AND PROPORTION

Prem Mann, *Introductory Statistics*, *7/E*

Opening Example

When we travel by plane, does seat assignment make any difference? Do we always prefer a specific seat when we fly? We sure do. In a survey of adults, 61% said that they prefer a window seat, while 38% prefer an aisle seat. Only 1% of the respondents indicated a preference for the middle seat. (See Case Study 9-2).

9.1 Hypothesis Tests: An Introduction

- Two Hypotheses
- Rejection and Nonrejection Regions
- Two Types of Errors
- Tails of a Test

Two Hypotheses

Definition

A ***null hypothesis*** is a claim (or statement) about a population parameter that is assumed to be true until it is declared false.

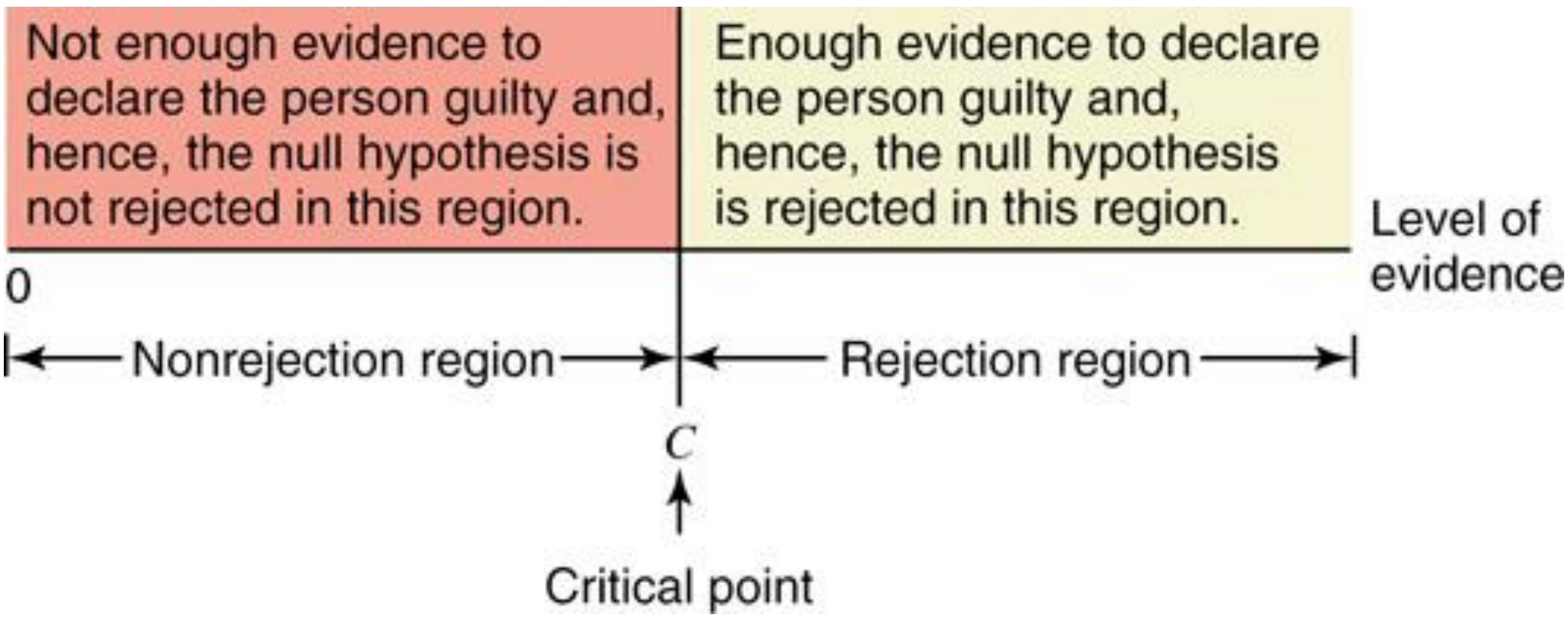
Two Hypotheses

Definition

An ***alternative hypothesis*** is a claim about a population parameter that will be true if the null hypothesis is false.

Rejection and Nonrejection Regions

Figure 9.1 Nonrejection and rejection regions for the court case.



Two Types of Errors

Table 9.1 Four Possible Outcomes for a Court Case

		Actual Situation	
		The Person Is Not Guilty	**The Person Is Guilty**
Court's decision	The person is not guilty	Correct decision	Type II or β error
	The person is guilty	Type I or α error	Correct decision

Two Types of Errors

Definition

A ***Type I error*** occurs when a true null hypothesis is rejected. The value of α represents the probability of committing this type of error; that is,

$$\alpha = P(H_0 \text{ is rejected} \mid H_0 \text{ is true})$$

The value of α represents the ***significance level*** of the test.

Two Types of Errors

Definition

A ***Type II error*** occurs when a false null hypotheses is not rejected. The value of β represents the probability of committing a Type II error; that is,

$$\beta = P\,(H_0 \text{ is not rejected} \mid H_0 \text{ is false})$$

The value of $1 - \beta$ is called the ***power of the test***. It represents the probability of not making a Type II error.

Table 9.2 Four Possible Outcomes for a Test of Hypothesis

		Actual Situation	
		H_0 Is True	**H_0 Is False**
Decision	Do not reject H_0	Correct decision	Type II or β error
	Reject H_0	Type I or α error	Correct decision

Tails of a Test

Definition

A ***two-tailed test*** has rejection regions in both tails, a ***left-tailed test*** has the rejection region in the left tail, and a ***right-tailed test*** has the rejection region in the right tail of the distribution curve.

A Two-Tailed Test

- According to a survey by Consumer Reports magazine conducted in 2008, a sample of sixth graders selected from New York schools showed that their backpacks weighed an average of 18.4 pounds (USA TODAY, August 3, 2009). Another magazine wants to check whether or not this mean has *changed* since that survey. The key word here is *changed*.
- The mean weight of backpacks for sixth-graders in New York has changed if it has either increased or decreased since 2008. This is an example of a two tailed test.

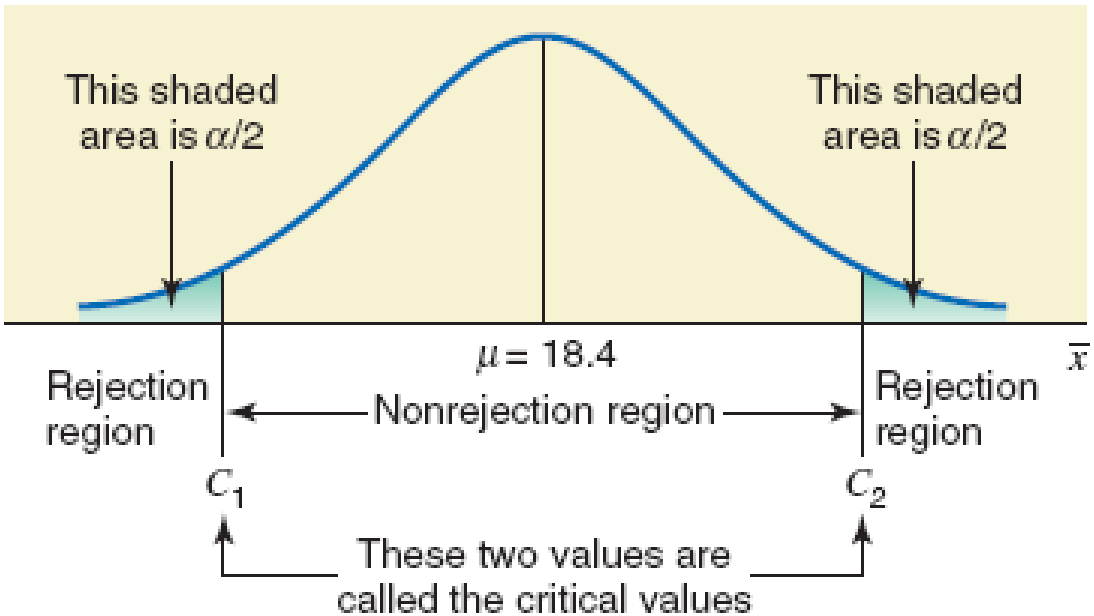
A Two-Tailed Test

- Let μ be the weight of backpacks for the current sixth-graders in New York. The two possible decisions are
  - H_0 : μ = 18.4 pounds (The mean weight of backpacks for sixth-graders in New York has not changed)
  - H_1 : μ ≠ 18.4 pounds (The mean weight of backpacks for sixth-graders in New York has changed)

A Two-Tailed Test

- Whether a test is two–tailed or one–tailed is determined by the sign in the alternative hypothesis.
- If the alternative hypothesis has a *not equal* to (≠) sign, it is a two–tailed test.

Figure 9.2 A two-tailed test.



A Left-Tailed Test

Reconsider the example of the mean amount of soda in all soft-drink cans produced by a company. The company claims that these cans, on average, contain 12 ounces of soda. However, if these cans contain less than the claimed amount of soda, then the company can be accused of cheating. Suppose a consumer agency wants to test whether the mean amount of soda per can is *less than* 12 ounces. Note that the key phrase this time is *less than*, which indicates a left-tailed test.

A Left-Tailed Test

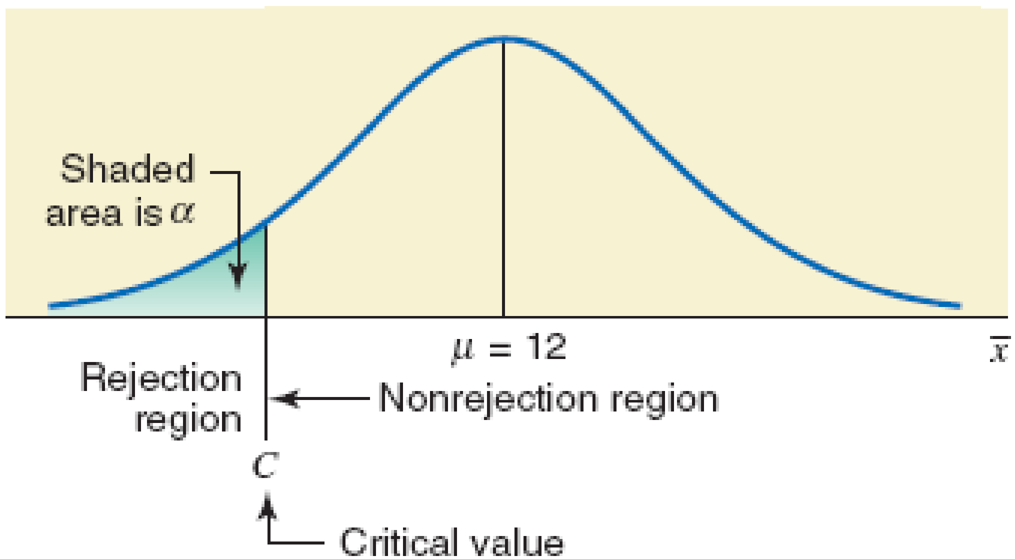
- Let μ be the mean amount of soda in all cans. The two possible decisions are
  - H_0 : μ = 12 ounces (The mean is equal to 12 ounces)
  - H_1 : μ < 12 ounces (The mean is less than 12 ounces)

A Left-Tailed Test

In this case, we can also write the null hypothesis as $H_0 : \mu \geq 12$. This will not affect the result of the test as long as the sign in H_1 is *less than* (<).

When the alternative hypothesis has a *less than* (<) sign, the test is always left–tailed.

Figure 9.3 A left-tailed test.



A Right-Tailed Test

According to www.city-data.com, the average price of homes in West Orange, New Jersey, was $461,216 in 2007. Suppose a real estate researcher wants to check whether the current mean price of homes in this town is *higher than* $461,216. The key phrase in this case is *higher than*, which indicates a right-tailed test.

A Right-Tailed Test

- Let μ be the current mean price of homes in this town. The two possible decisions are
  - H_0 : μ = \$461,216 (The current mean price of homes in this town is not higher than \$461,216)
  - H_1 : μ > \$461,216 (The current mean price of homes in this town is higher than \$461,216)

A Right-Tailed Test

When the alternative hypothesis has a greater than (>) sign, the test is always right–tailed.

Figure 9.4 A right-tailed test.

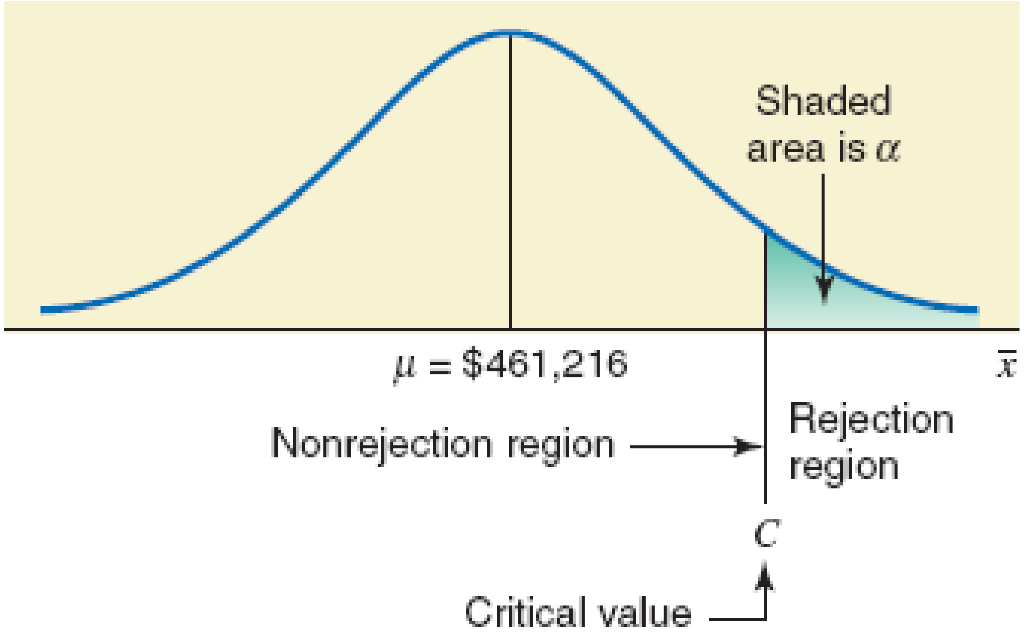


Table 9.3 Signs in H_0 and H_1 and Tails of a Test

	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in the null hypothesis H_0	$=$	$=$ or $\geq$	$=$ or $\leq$
Sign in the alternative hypothesis H_1	$\neq$	$<$	$>$
Rejection region	In both tails	In the left tail	In the right tail

Two Procedures

Two procedures to make tests of hypothesis

1. The p-value approach
2. The critical-value approach

9.2 Hypothesis Tests About μ: σ Known

Three Possible Cases

Case I. If the following three conditions are fulfilled:

1. The population standard deviation σ is known
2. The sample size is small (i.e., $n < 30$)
3. The population from which the sample is selected is normally distributed.

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Three Possible Cases

Case II. If the following two conditions are fulfilled:

1. The population standard deviation σ is known
2. The sample size is large (i.e., $n \geq 30$)

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

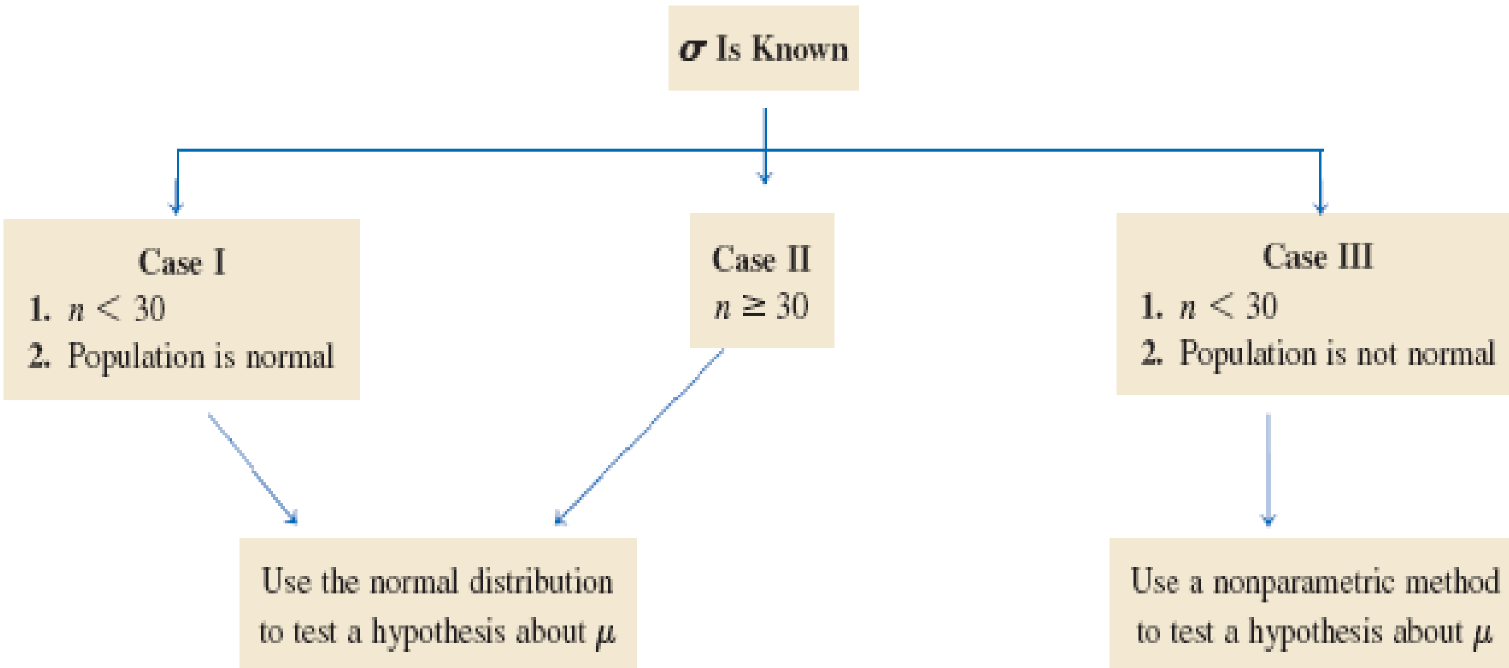
Three Possible Cases

Case III. If the following three conditions are fulfilled:

1. The population standard deviation σ is known
2. The sample size is small (i.e., n < 30)
3. The population from which the sample is selected is not normally distributed (or its distribution is unknown).

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Three Possible Cases



HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Definition

Assuming that the null hypothesis is true, the p-value can be defined as the probability that a sample statistic (such as the sample mean) is at least as far away from the hypothesized value in the direction of the alternative hypothesis as the one obtained from the sample data under consideration. Note that the ***p*–value** is the smallest significance level at which the null hypothesis is rejected.

Figure 9.5 The *p*–value for a right-tailed test.

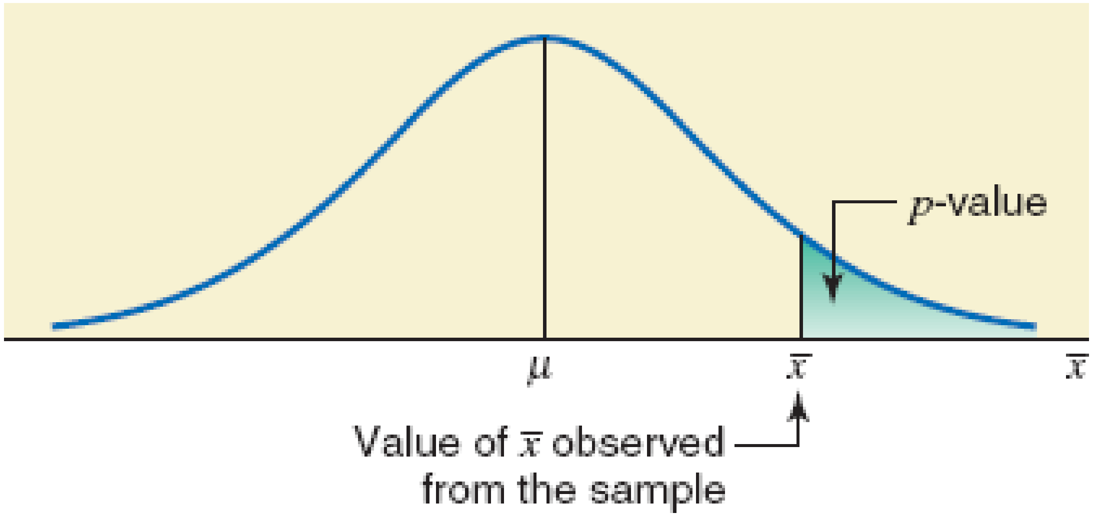
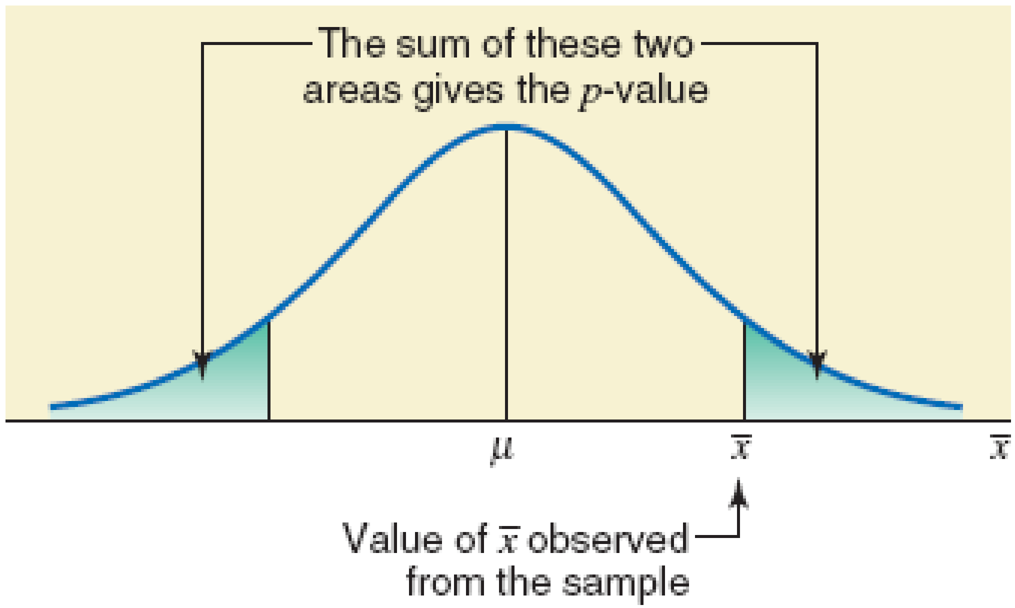


Figure 9.6 The *p*–value for a two-tailed test.



Calculating the z Value for $\bar{x}$

When using the normal distribution, the ***value of z*** for $\bar{x}$ for a test of hypothesis about μ is computed as follows:

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \textbf{where} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

The value of z calculated for $\bar{x}$ using this formula is also called the ***observed value of z***.

Steps to Perform a Test of Hypothesis Using the *p*–Value Approach

1. State the null and alternative hypothesis.
2. Select the distribution to use.
3. Calculate the p–value.
4. Make a decision.

Example 9-1

At Canon Food Corporation, it used to take an average of 90 minutes for new workers to learn a food processing job. Recently the company installed a new food processing machine. The supervisor at the company wants to find if the mean time taken by new workers to learn the food processing procedure on this new machine is different from 90 minutes. A sample of 20 workers showed that it took, on average, 85 minutes for them to learn the food processing procedure on the new machine. It is known that the learning times for all new workers are normally distributed with a population standard deviation of 7 minutes. Find the p–value for the test that the mean learning time for the food processing procedure on the new machine is different from 90 minutes. What will your conclusion be if $\alpha = .01$?

Example 9-1: Solution

- Step 1: H_0: $\mu = 90$ H_1: $\mu \neq 90$
- Step 2: The population standard deviation σ is known, the sample size is small ($n < 30$), but the population distribution is normal. We will use the normal distribution to find the p–value and make the test.

Example 9-1: Solution

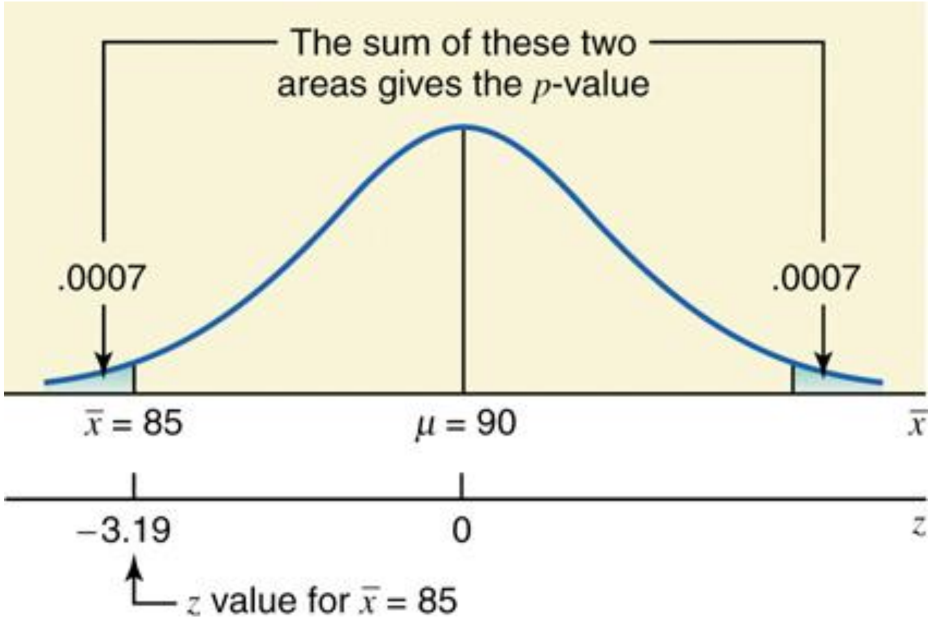
- Step 3:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{20}} = \mathbf{1.56524758\ \ min\textit{utes}}$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{85 - 90}{1.56524758} = -3.19$$

p-value = 2(.0007) = **.0014**

Figure 9-7 The p-value for a two-tailed test.



Example 9-1: Solution

- Step 4: Because α = .01 is greater than the p-value of .0014, we reject the null hypothesis at this significance level.

  Therefore, we conclude that the mean time for learning the food processing procedure on the new machine is different from 90 minutes.

Example 9-2

The management of Priority Health Club claims that its members lose an average of 10 pounds or more within the first month after joining the club. A consumer agency that wanted to check this claim took a random sample of 36 members of this health club and found that they lost an average of 9.2 pounds within the first month of membership with a population standard deviation of 2.4 pounds. Find the p–value for this test. What will you decision be if α = .01? What if α = .05?

Example 9-2: Solution

- Step 1: H_0: $\mu \geq 10$ H_1: $\mu < 10$
- Step 2: The population standard deviation σ is known, the sample size is large (n > 30). Due to the Central Limit Theorem, we will use the normal distribution to find the p–value and perform the test.

Example 9-2: Solution

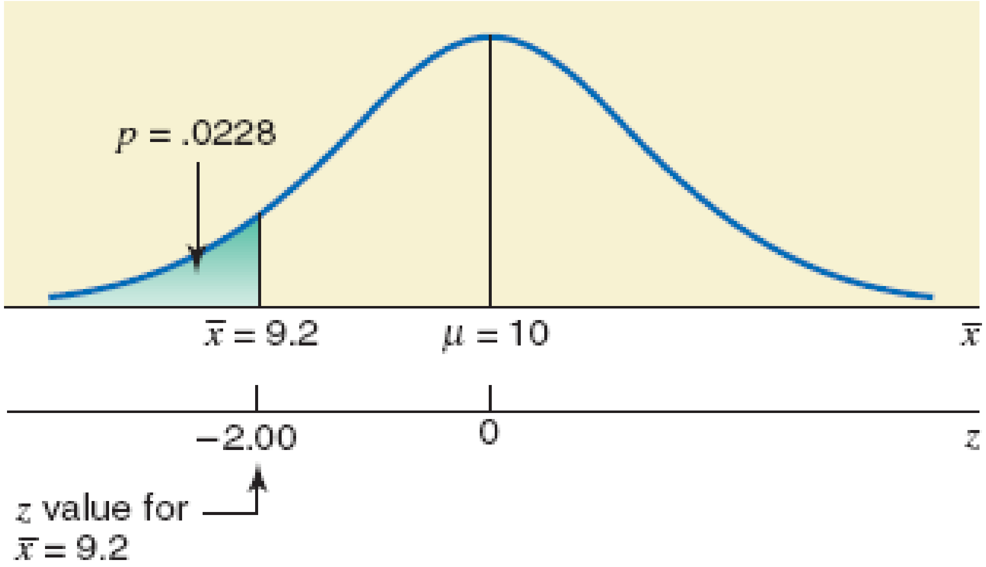
- Step 3:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.4}{\sqrt{36}} = .40$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{9.2 - 10}{.40} = -2.00$$

p-value = **.0228**

Figure 9-8 The p-value for a left-tailed test.



Example 9-2: Solution

- Step 4:
- Since $\alpha = .01$ is less than the p-value of .0228, we do not reject the null hypothesis at this significance level. Consequently, we conclude that the mean weight lost within the first month of membership by the members of this club is 10 pounds or more.

- Because $\alpha = .05$ is greater than the p-value of .0228, we reject the null hypothesis at this significance level. Therefore, we conclude that the mean weight lost within the first month of membership by the members of this club is less than 10 pounds.

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Test Statistic

In tests of hypotheses about μ using the normal distribution, the random variable

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \text{where} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

is called the ***test statistic***. The test statistic can be defined as a rule or criterion that is used to make the decision whether or not to reject the null hypothesis.

HYPOTHESIS TESTS ABOUT μ: σ KNOWN

Steps to Perform a Test of Hypothesis with the Critical-Value Approach

1. State the null and alternative hypotheses.
2. Select the distribution to use.
3. Determine the rejection and nonrejection regions.
4. Calculate the value of the test statistic.
5. Make a decision.

Example 9-3

The TIV Telephone Company provides long-distance telephone service in an area. According to the company's records, the average length of all long-distance calls placed through this company in 2009 was 12.44 minutes. The company's management wanted to check if the mean length of the current long-distance calls is different from 12.44 minutes. A sample of 150 such calls placed through this company produced a mean length of 13.71 minutes with a standard deviation of 2.65 minutes. Using the 2% significance level, can you conclude that the mean length of all current long-distance calls is different from 12.44 minutes?

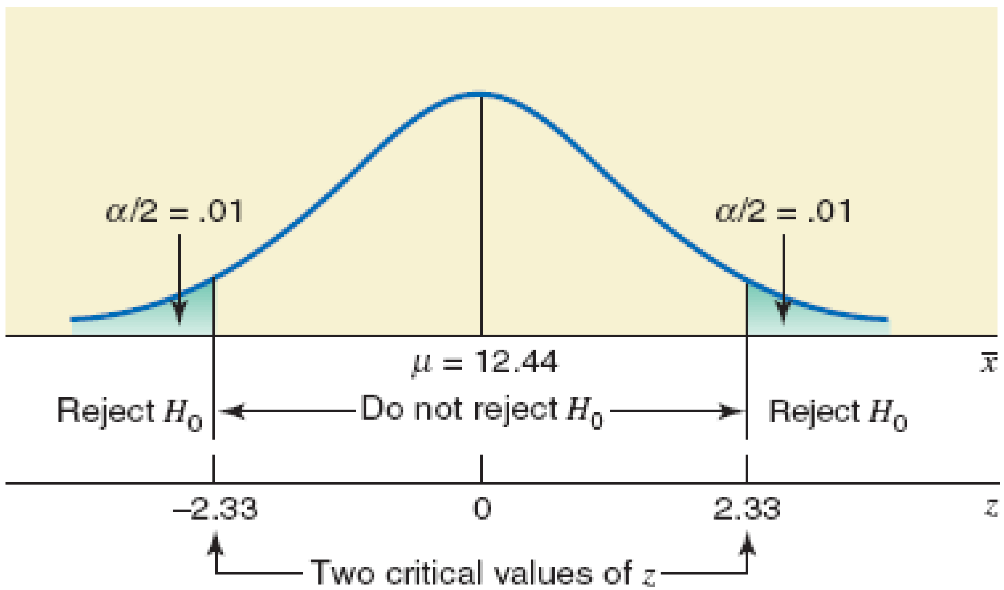
Example 9-3: Solution

- Step 1: $H_0 : \mu = 12.44 \quad H_1 : \mu \neq 12.44$
- Step 2: The population standard deviation σ is known, and the sample size is large ($n > 30$). Due to the Central Limit Theorem, we will use the normal distribution to perform the test.

Example 9-3: Solution

- Step 3: α = .02
- The ≠ sign in the alternative hypothesis indicates that the test is two-tailed
- Area in each tail = α / 2= .02 / 2 = .01
- The z values for the two critical points are -2.33 and 2.33

Figure 9.9



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Calculating the Value of the Test Statistic

When using the normal distribution, the ***value of the test statistic z*** for $\bar{x}$ for a test of hypothesis about μ is computed as follows:

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \textbf{where} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

The value of z for $\bar{x}$ is also called the ***observed value of z***.

Example 9-3: Solution

- Step 4:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.65}{\sqrt{150}} = .21637159$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{13.71 - 12.44}{.21637159} = 5.87$$

Example 9-3: Solution

- Step 5: This value of z = 5.87 is greater than the critical value of z = 2.33, and it falls in the rejection region in the right tail in Figure 9.9. Hence, we reject H_0 and conclude that based on the sample information, it appears that the mean length of all such calls is not equal to 12.44 minutes.

Example 9-4

The mayor of a large city claims that the average net worth of families living in this city is at least \$300,000. A random sample of 25 families selected from this city produced a mean net worth of \$288,000. Assume that the net worths of all families in this city have a normal distribution with the population standard deviation of \$80,000. Using the 2.5% significance level, can you conclude that the mayor's claim is false?

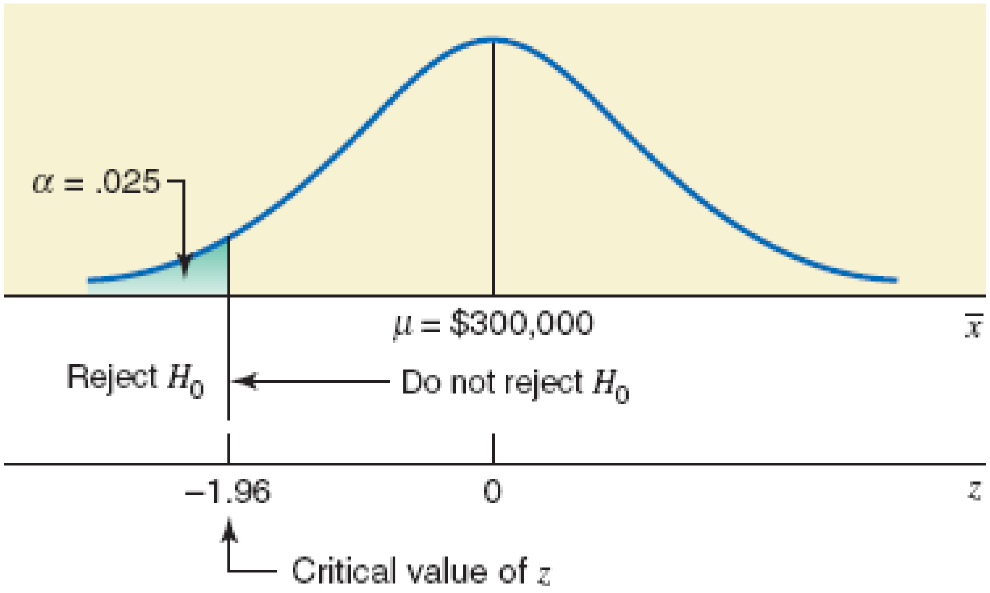
Example 9-4: Solution

- Step 1: H_0 : $\mu \geq \$300,000$
  H_1 : $\mu < \$300,000$
- Step 2: The population standard deviation σ is known, the sample size is small ($n < 30$), but the population distribution is normal. Consequently, we will use the normal distribution to perform the test.

Example 9-4: Solution

- Step 3: $\alpha = .025$
- The < sign in the alternative hypothesis indicates that the test is left-tailed
- Area in the left tail $= \alpha = .025$
- The critical value of z is -1.96

Figure 9.10



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Example 9-4: Solution

- Step 4:

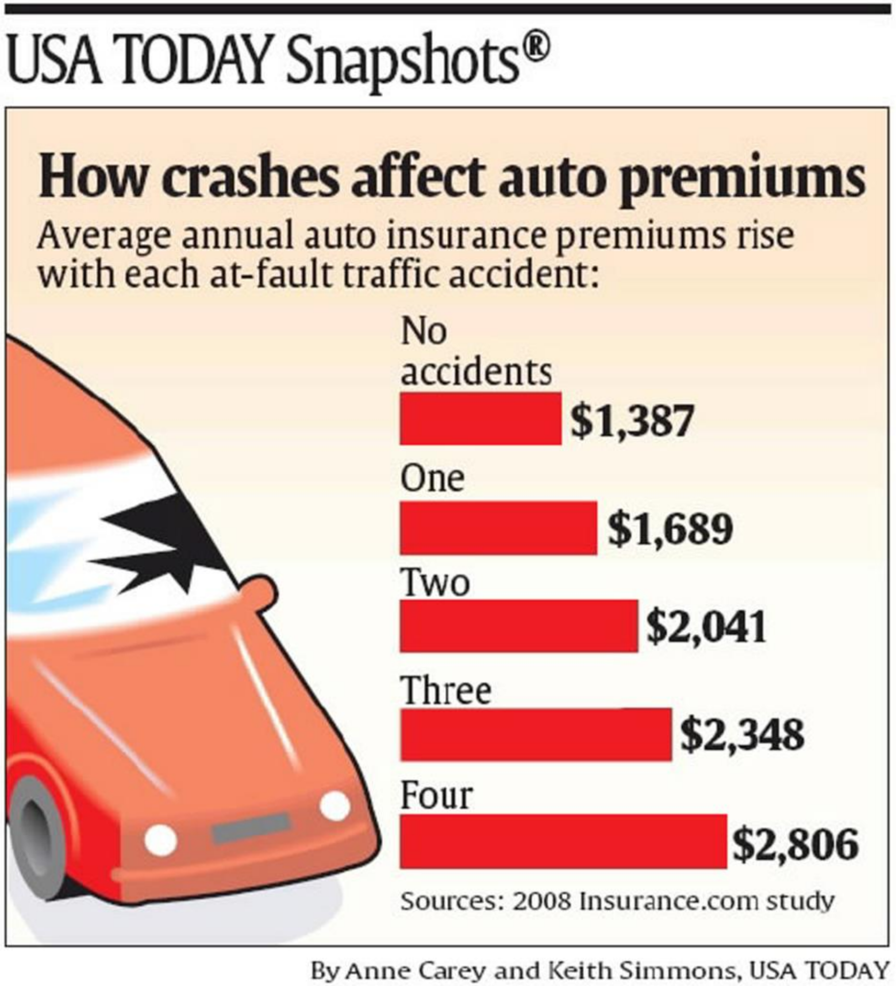
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{80,000}{\sqrt{25}} = \$16,000$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{288,000 - 300,000}{16,000} = -.75$$

Example 9-4: Solution

- Step 5: This value of z = -.75 is greater than the critical value of z = -1.96, and it falls in the nonrejection region. As a result, we fail to reject H_0. Therefore, we can state that based on the sample information, it appears that the mean net worth of families in this city is not less than $300,000.

Case Study 9-1 How Crashes Affect Auto Premiums



9.3 Hypothesis Tests About μ: σ Not Known

Three Possible Cases

Case I. If the following three conditions are fulfilled:

1. The population standard deviation σ is not known
2. The sample size is small (i.e., $n < 30$)
3. The population from which the sample is selected is normally distributed.

HYPOTHESIS TESTS ABOUT μ: σ NOT KNOWN

Three Possible Cases

Case II. If the following two conditions are fulfilled:

1. The population standard deviation σ is not known
2. The sample size is large (i.e., $n \geq 30$)

HYPOTHESIS TESTS ABOUT μ: σ NOT KNOWN

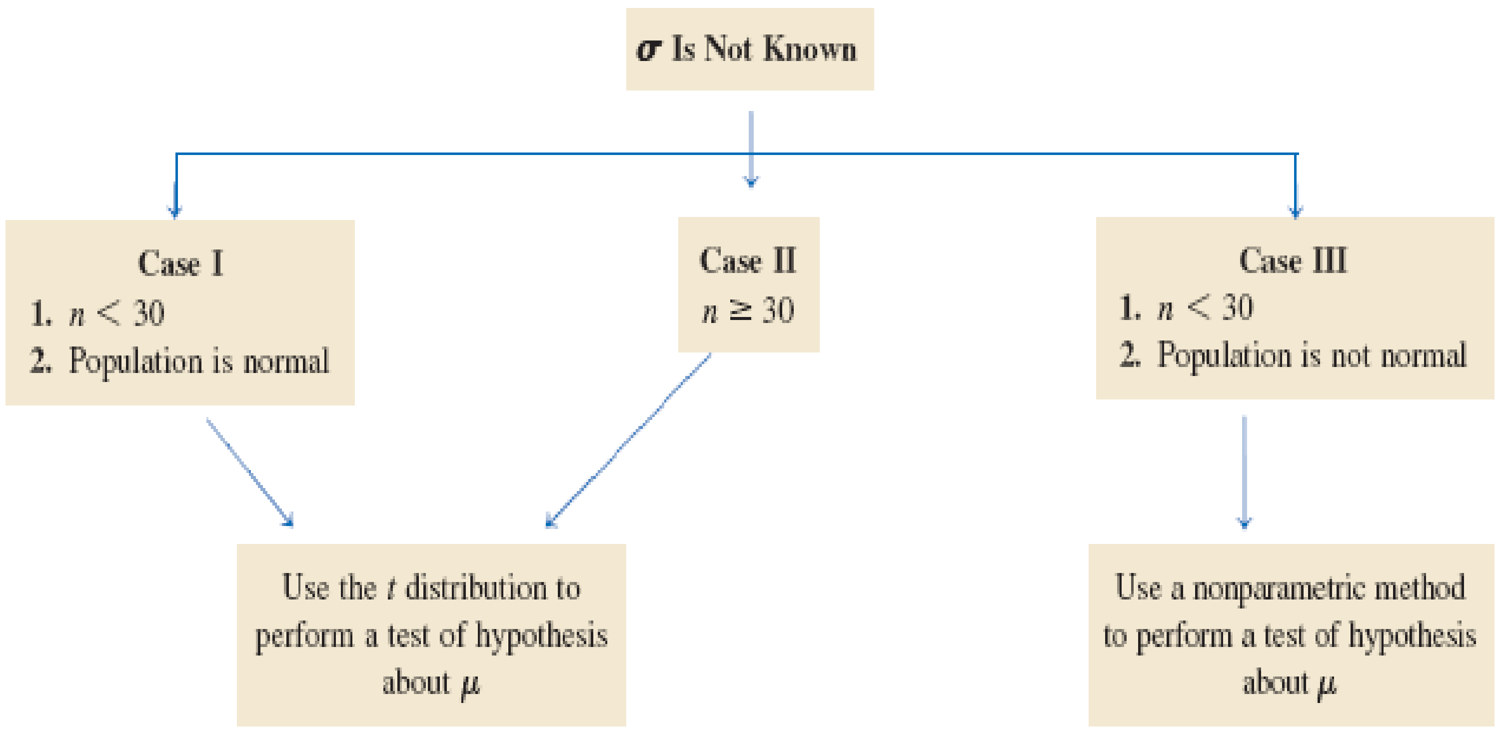
Three Possible Cases

Case III. If the following three conditions are fulfilled:

1. The population standard deviation σ is not known
2. The sample size is small (i.e., $n < 30$)
3. The population from which the sample is selected is not normally distributed (or its distribution is unknown).

HYPOTHESIS TESTS ABOUT μ: σ NOT KNOWN

Three Possible Cases



HYPOTHESIS TESTS ABOUT μ: σ NOT KNOWN

Test Statistic

The value of the ***test statistic*** t for the sample mean $\bar{x}$ is computed as

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{where} \quad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

The value of t calculated for $\bar{x}$ by using this formula is also called the ***observed value of*** t.

Example 9-5

A psychologist claims that the mean age at which children start walking is 12.5 months. Carol wanted to check if this claim is true. She took a random sample of 18 children and found that the mean age at which these children started walking was 12.9 months with a standard deviation of .80 month. It is known that the ages at which all children start walking are approximately normal distributed. Find the p-value for the test that the mean age at which all children start walking is different from 12.5 months. What will your conclusion be if the significance level is 1%?

Example 9-5: Solution

- Step 1: $H_0 : \mu = 12.5$

  $H_1 : \mu \neq 12.5$

- Step 2: The population standard deviation σ is not known, the sample size is small ($n < 30$), and the population is normally distributed. Consequently, we will use the t distribution to find the p-value for the test.

Example 9-5: Solution

- Step 3: The ≠ sign in the alternative hypothesis indicates that the test is two-tailed

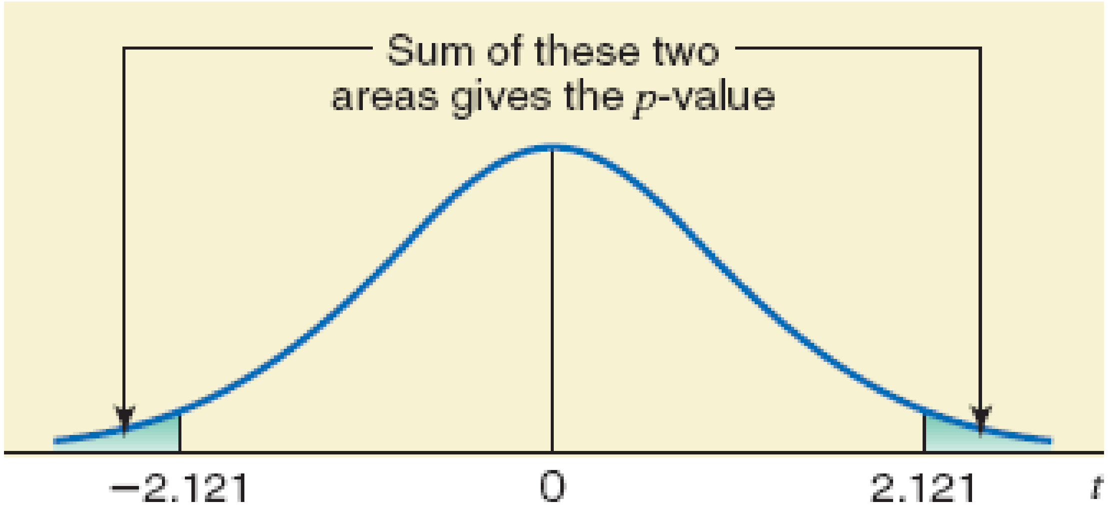
$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{.80}{\sqrt{18}} = .18856181$$

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{12.9 - 12.5}{.18856181} = 2.121$$

and $df = n - 1 = 18 - 1 = 17$

$.02 < \text{p-value} < .05$

Figure 9.11 The required *p*-value



Example 9-5: Solution

- Step 4: For any α greater than .05, we will reject the null hypothesis. For any α less than .02, we will not reject the null hypothesis. For our example, α = .01, which is less than the lower limit of the p-value ranges of .02. As a result, we fail to reject H_0 and conclude that the mean age at which all children start walking is not different from 12.5 months.

Example 9-6

Grand Auto Corporation produces auto batteries. The company claims that its top-of-the-line Never Die batteries are good, on average, for at least 65 months. A consumer protection agency tested 45 such batteries to check this claim. It found the mean life of these 45 batteries to be 63.4 months with a standard deviation of 3 months. Find the p-value for the test that mean life of all such batteries is less than 65 months. What will your conclusion be if the significance level is 2.5%?

Example 9-6: Solution

- Step 1: $H_0 : \mu \geq 65$
  $H_1 : \mu < 65$
- Step 2: The population standard deviation σ is not known and the sample size is large (n > 30). Consequently, we will use the t distribution to find the *p*-value for the test.

Example 9-6: Solution

- Step 3: The < sign in the alternative hypothesis indicates that the test is left-tailed

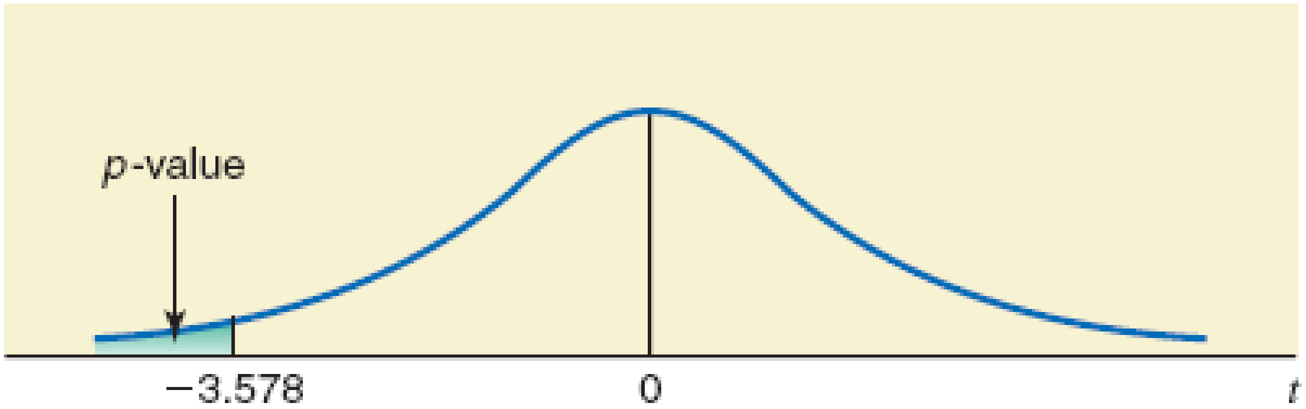
$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3}{\sqrt{45}} = .44721360$$

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{63.4 - 65}{.44721360} = -3.578$$

and $df = n - 1 = 45 - 1 = 44$

p-value < .001

Figure 9.12 The required *p*-value



Example 9-7

Refer to Example 9-5. A psychologist claims that the mean age at which children start walking is 12.5 months. Carol wanted to check if this claim is true. She took a random sample of 18 children and found that the mean age at which these children started walking was 12.9 months with a standard deviation of .80 month. Using the 1% significance level, can you conclude that the mean age at which all children start walking is different from 12.5 months? Assume that the ages at which all children start walking have an approximately normal distribution.

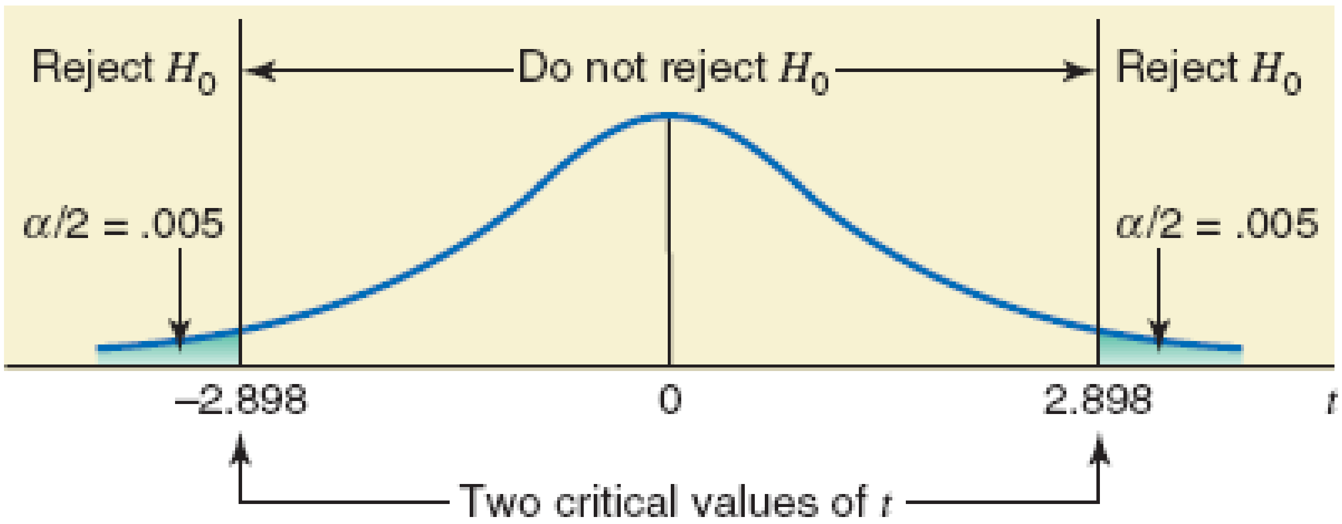
Example 9-7: Solution

- Step 1: $H_0 : \mu = 12.5$
  $H_1 : \mu \neq 12.5$
- Step 2: The population standard deviation σ is not known, the sample size is small ($n < 30$), and the population is normally distributed. Consequently, we will use the t distribution to perform the test.

Example 9-7: Solution

- Step 3: Significance level = .01. The ≠ sign in the alternative hypothesis indicates that the test is two-tailed and the rejection region lies in both tails.
- Area in each tail = $\alpha/2$ = .01/2 = .005
- $df = n - 1 = 18 - 1 = 17$
- The critical values for t for 17 df and .005 area in each tail are -2.898 and 2.898.

Figure 9.13 The required *p*-value



Example 9-7: Solution

- Step 4: $$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{.80}{\sqrt{18}} = .18856181$$

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{12.9 - 12.5}{.18856181} = 2.121$$

The value of the test statistic t = 2.121 falls between the two critical points, -2.898 and 2.898, which is the nonrejection region. Consequently, we fail to reject H0. As a result, we can state the difference between the hypothesized population mean and the sample mean is so small that it may have occurred because of sampling error.

Example 9-8

The management at Massachusetts Savings Bank is always concerned about the quality of service provided to its customers. With the old computer system, a teller at this bank could serve, on average, 22 customers per hour. The management noticed that with this service rate, the waiting time for customers was too long. Recently the management of the bank installed a new computer system in the bank, expecting that it would increase the service rate and consequently make the customers happier by reducing the waiting time.

Example 9-8

To check if the new computer system is more efficient than the old system, the management of the bank took a random sample of 70 hours and found that during these hours the mean number of customers served by tellers was 27 per hour with a standard deviation of 2.5. Testing at the 1% significance level, would you conclude that the new computer system is more efficient than the old computer system?

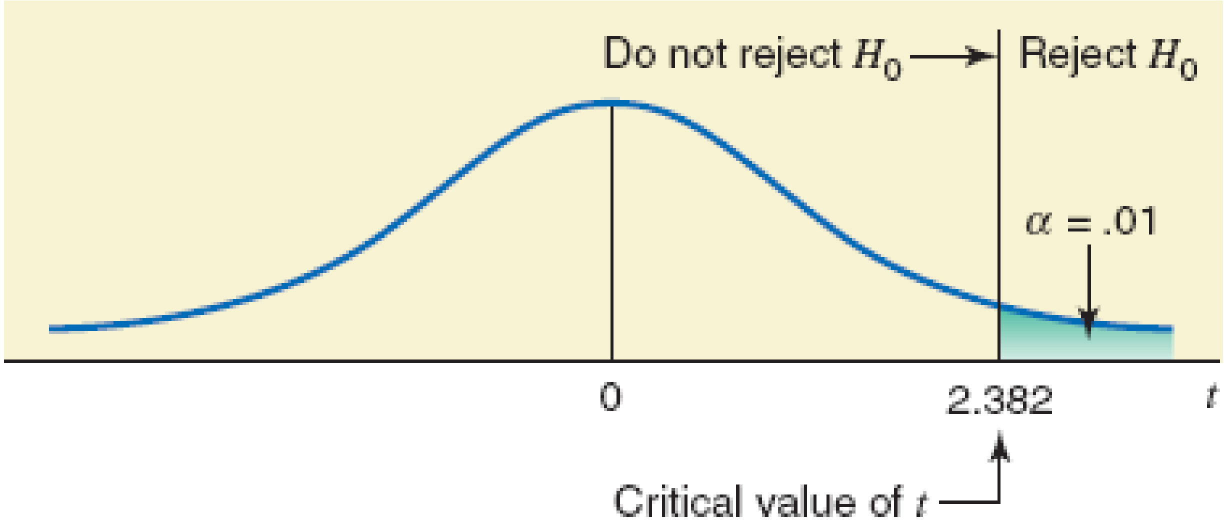
Example 9-8: Solution

- Step 1: $H_0 : \mu = 22$
  $H_1 : \mu > 22$
- Step 2: The population standard deviation σ is not known and the sample size is large (n > 30). Consequently, we will use the t distribution to perform the test.

Example 9-8: Solution

- Step 3: Significance level = .01. The > sign in the alternative hypothesis indicates that the test is right-tailed and the rejection region lies in the right tail.
- Area in the right tail = α = .01
- $df = n - 1 = 70 - 1 = 69$
- The critical value for t for 69 *df* and .01 area in the right tail is 2.382.

Figure 9.14


Do not reject H_0
Reject H_0
$\alpha = .01$
0
2.382
t
Critical value of t

Example 9-8: Solution

- Step 4: $$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.5}{\sqrt{70}} = .29880715$$

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{27 - 22}{.29880715} = 16.733$$

The value of the test statistic t = 16.733 is greater than the critical value of t = 2.382, and it falls in the rejection region. Consequently, we reject H0. As a result, we conclude that the value of the sample mean is too large compared to the hypothesized value of the population mean, and the difference between the two may not be attributed to chance alone.

What If the Sample Size Is Too Large?

1. Use the *t* value from the last row (the row of ∞) in Table V of Appendix C.
2. Use the normal distribution as an approximation to the *t* distribution.

9.4 Hypothesis Tests About a Population Proportion: Large Samples

Test Statistic

The value of the ***test statistic z*** for the sample proportion, $\hat{p}$, is computes as

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \quad \text{where} \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

The value of p that is used in this formula is the one from the null hypothesis. The value of q is equal to 1-p. The value of z calculated for $\hat{p}$ using the above formula is also called the observed value of z.

Example 9-9

According to a Nationwide Mutual Insurance Company *Driving While Distracted* Survey conducted in 2008, 81% of the drivers interviewed said that they have talked on their cell phones while driving (*The New York Times*, July 19, 2009). The survey included drivers aged 16 to 61 years selected from 48 states. Assume that this result holds true for the 2008 population of all such drivers in the United States. In a recent random sample of 1600 drivers aged 16 to 61 years selected from the United States, 83% said that they have talked on their cell phones while driving.

Example 9-9

Find the p-value to test the hypothesis that the current percentage of such drivers who have talked on their cell phones while driving is different from 81%. What is your conclusion if the significance level is 5%?

Example 9-9: Solution

- Step 1: $H_0 : p = .81$

  $H_1 : p \neq .81$
- Step 2: To check whether the sample is large, we calculate the values of np and nq:

  $np = 1600(.81) = 1296 > 5$

  $nq = 1600(.19) = 304 > 5$

  Consequently, we will use the normal distribution to find the p-value for this test.

Example 9-9: Solution

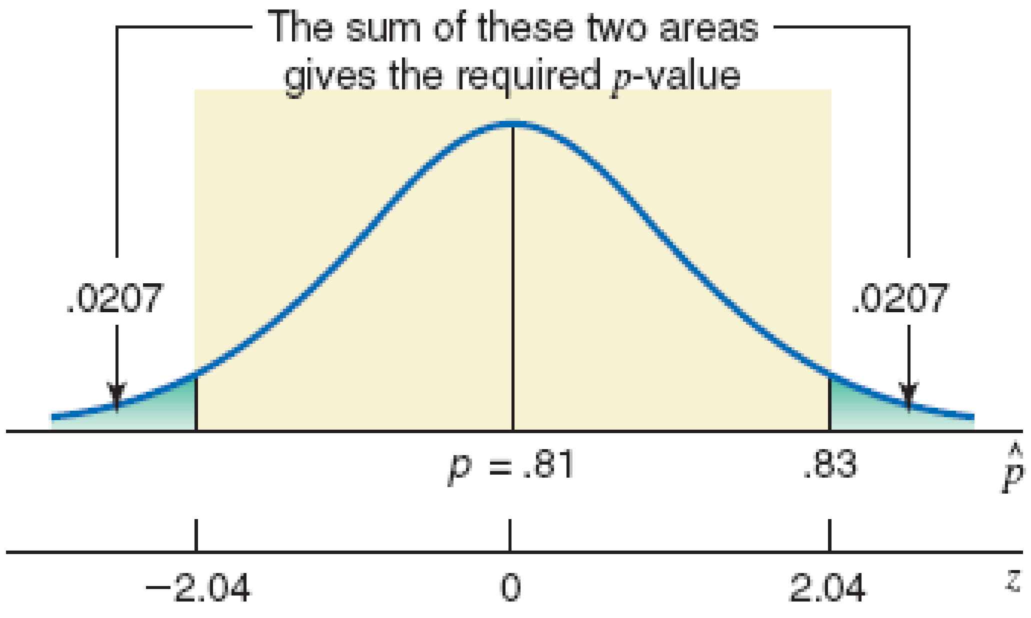
- Step 3: The ≠ sign in the alternative hypothesis indicates that the test is two-tailed.

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.81)(.19)}{1600}} = .00980752$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.83 - .81}{.00980752} = 2.04$$

p-value = 2(.0207) = .0414

Figure 9.15 The required p-value



Example 9-9: Solution

- Step 4: We can state that for any α greater than .0414 we will reject the null hypothesis. For our example, α = .05, which is greater than the p-value of .0414. As a result, we reject H_0 and conclude that the current percentage of all U.S. drivers aged 16 to 61 years who have talked on their cell phones while driving is different from .81.

Example 9-10

When working properly, a machine that is used to make chips for calculators does not produce more than 4% defective chips. Whenever the machine produces more than 4% defective chips, it needs an adjustment. To check if the machine is working properly, the quality control department at the company often takes samples of chips and inspects them to determine if they are good or defective.

Example 9-10

One such random sample of 200 chips taken recently from the production line contained 12 defective chips. Find the p-value to test the hypothesis whether or not the machine needs an adjustment. What would your conclusion be if the significance level is 2.5%?

Example 9-10: Solution

- Step 1: H_0: $p \leq .04$

  H_1: $p > .04$

- Step 2: To check whether the sample is large, we calculate the values of np and nq:

  $np = 200(.04) = 8 > 5$

  $nq = 200(.96) = 192 > 5$

  Consequently, we will use the normal distribution to find the p-value for this test.

Example 9-10: Solution

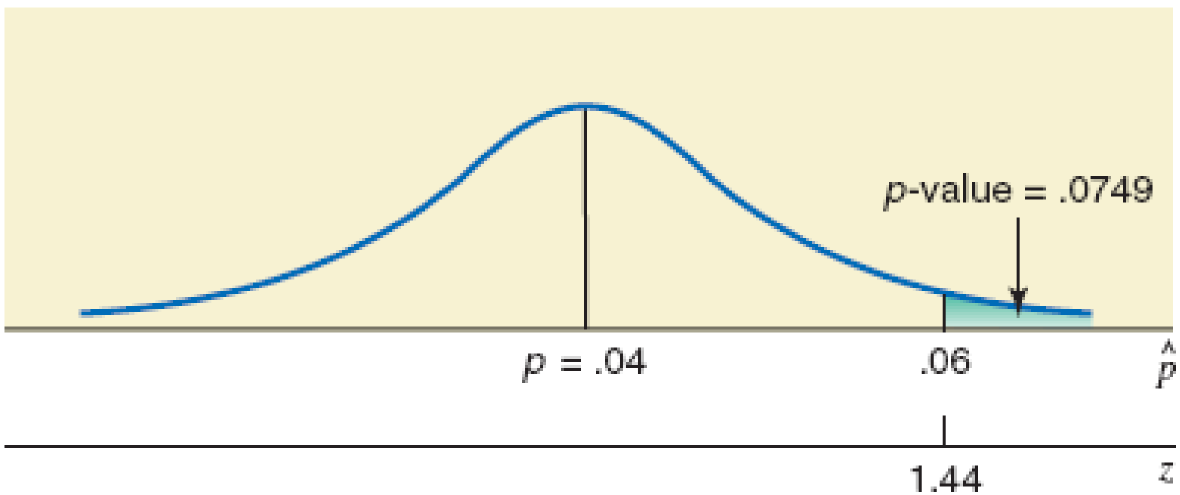
- Step 3: The > sign in the alternative hypothesis indicates that the test is right-tailed.

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.04)(.96)}{200}} = .01385641$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.06 - .04}{.01385641} = 1.44$$

p-value = .0749

Figure 9.16 The required p-value



Example 9-10: Solution

- Step 4: We can state that for any α greater than .0749 we will reject the null hypothesis, and for any α less than or equal to .0749 we will not reject the null hypothesis. For our example, α = .025, which is less than the p-value of .0749. As a result, we fail to reject H_0 and conclude that the machine does not need an adjustment.

Example 9-11

Refer to Example 9-9. According to a Nationwide Mutual Insurance Company *Driving While Distracted* Survey conducted in 2008, 81% of the drivers interviewed said that they have talked on their cell phones while driving (*The New York Times*, July 19, 2009). The survey included drivers aged 16 to 61 years selected from 48 states. Assume that this result holds true for the 2008 population of all such drivers in the United States. In a recent random sample of 1600 drivers aged 16 to 61 years selected from the United States, 83% said that they have talked on their cell phones while driving.

Example 9-11

Using the 5% significance level, can you conclude that the current percentage of such drivers who have talked on their cell phones while driving is different from 81%.

Example 9-11: Solution

- Step 1: $H_0 : p = .81$

  $H_1 : p \neq .81$
- Step 2: To check whether the sample is large, we calculate the values of np and nq:

  $np = 1600(.81) = 1296 > 5$

  $nq = 1600(.19) = 304 > 5$

  Consequently, we will use the normal distribution to make the test.

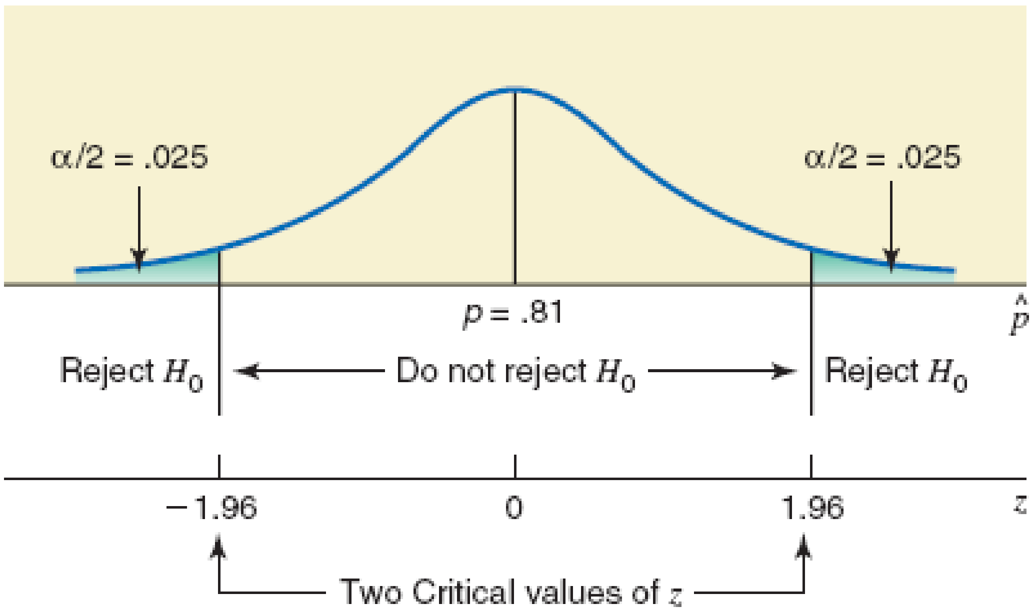
Example 9-11: Solution

- Step 3: The ≠ sign in the alternative hypothesis indicates that the test is two-tailed. The significance level is .05. Therefore, the total area of the two rejection regions is .05.

Area in each tail = α / 2 = .05 / 2 = .025

The critical values of z are -1.96 and 1.96.

Figure 9.17 The critical values of z



Example 9-11: Solution

- Step 4:

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.81)(.19)}{1600}} = .00980752$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.83 - .81}{.00980752} = 2.04$$

Example 9-11: Solution

- Step 5: the value of test statistic z = 2.04 falls in the rejection region. As a result, we reject H0 and conclude that the current percentage of all U.S. drivers aged 16 to 61 years who have talked on their cell phones while driving is different from .81.

Example 9-12

Direct Mailing Company sells computers and computer parts by mail. The company claims that at least 90% of all orders are mailed within 72 hours after they are received. The quality control department at the company often takes samples to check if this claim is valid. A recently taken sample of 150 orders showed that 129 of them were mailed within 72 hours. Do you think the company's claim is true? Use a 2.5% significance level.

Example 9-12: Solution

- Step 1: $H_0 : p \geq .90$
  $H_1 : p < .90$
- Step 2: To check whether the sample is large, we calculate the values of np and nq:

  $np = 150(.90) = 135 > 5$

  $nq = 150(.10) = 15 > 5$

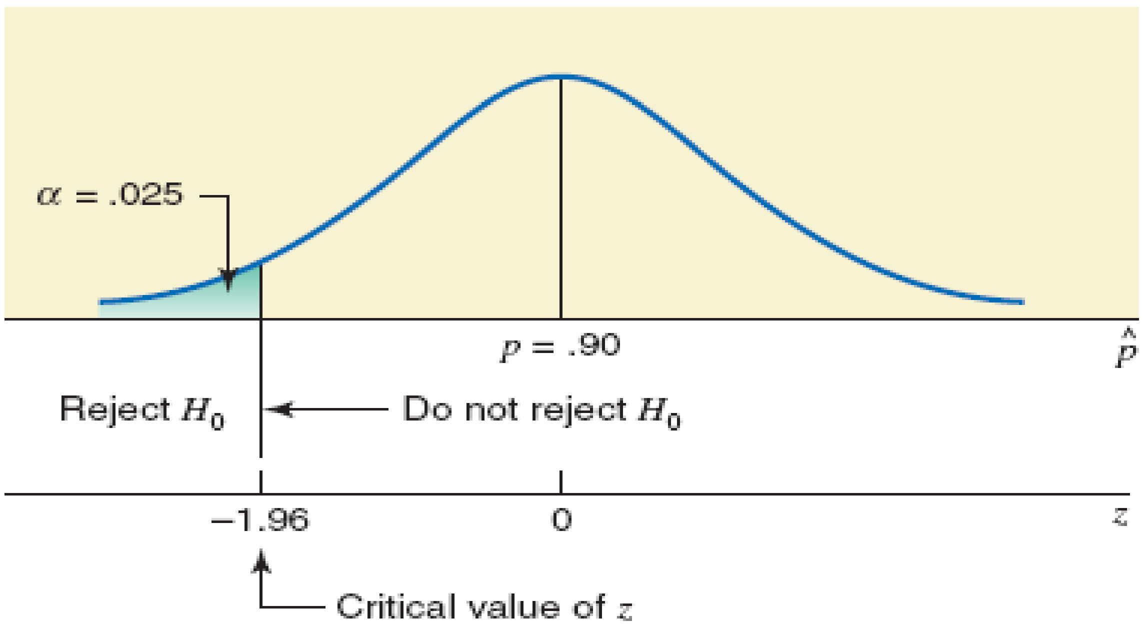
  Consequently, we will use the normal distribution to make the test.

Example 9-12: Solution

- Step 3: Significance level = .025. The < sign in the alternative hypothesis indicates that the test is left-tailed, and the rejection region leis in the left tail.

  The critical values of z for .0250 area in the left tail is -1.96.

Figure 9.18 The critical values of z



Prem Mann, *Introductory Statistics*, *7/E*

Example 9-12: Solution

- Step 4:

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.90)(.10)}{150}} = .02449490$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.86 - .90}{.02449490} = -1.63$$

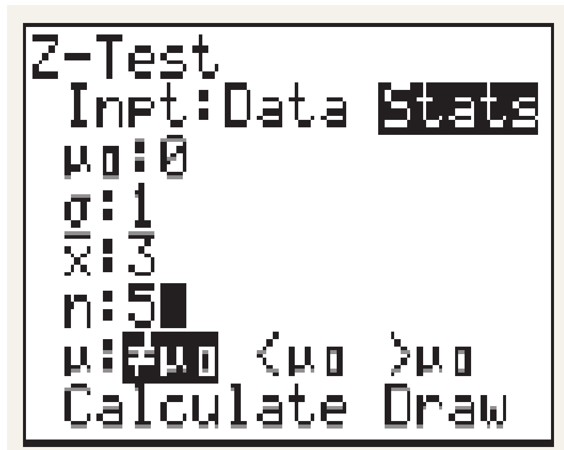
Example 9-12: Solution

- Step 5: The value of test statistic z = -1.63 is greater than the critical value of z = -1.96, and it falls in the nonrejection region. Therefore, we fail to reject H_0. We can state that the difference between the sample proportion and the hypothesized value of the population proportion is small, and this difference may have occurred owing to the chance alone.

TECHNOLOGY INSTRUCTION

Hypothesis Testing

TI-84

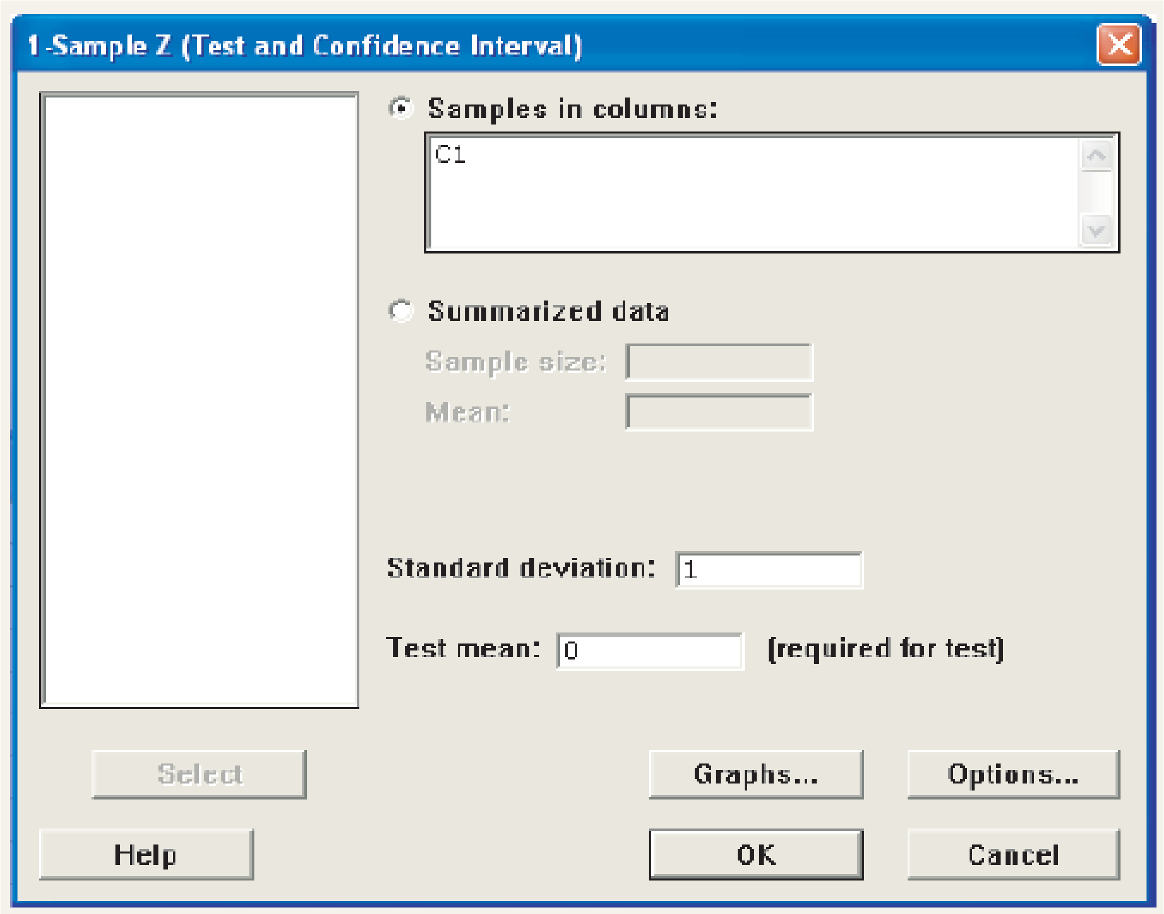


Screen 9.1

Screen 9.1

Hypothesis Testing

Minitab



Screen 9.2

Excel

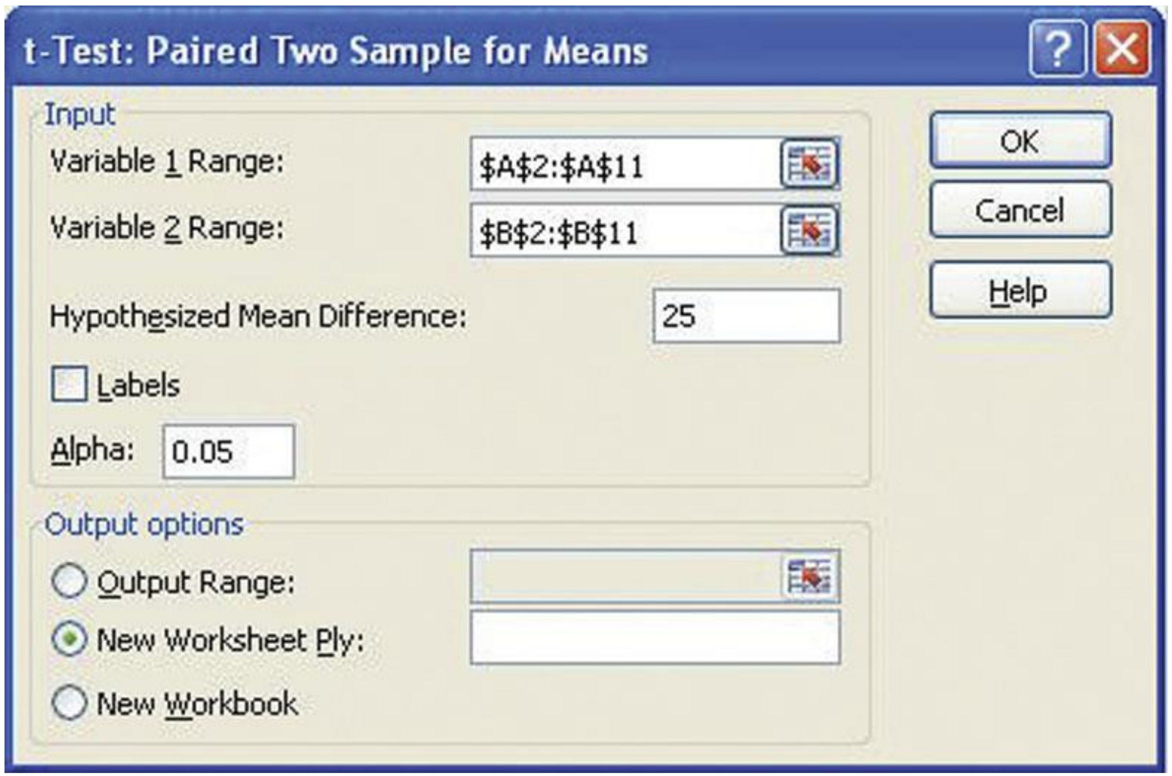
	A	B
1	data	
2	14.3	0
3	25.2	0
4	22.5	0
5	38.3	0
6	16.9	0
7	26.7	0
8	19.5	0
9	23.1	0
10	41	0
11	33.9	0

Screen 9.3

TECHNOLOGY INSTRUCTION

Hypothesis Testing

Excel



Screen 9.4

TECHNOLOGY INSTRUCTION

Hypothesis Testing

Excel

	A	B	C
1	t-Test: Paired Two Sample for Means		
2			
3		*Variable 1*	*Variable 2*
4	Mean	26.14	0
5	Variance	80.24933333	0
6	Observations	10	10
7	Pearson Correlation	#DIV/0!	
8	Hypothesized Mean Difference	25	
9	df	9	
10	t Stat	0.402424242	
11	P(T<=t) one-tail	0.348381148	
12	t Critical one-tail	1.833112923	
13	P(T<=t) two-tail	0.696762296	
14	t Critical two-tail	2.262157158	

Screen 9.5