শিক্ষা নিয়ে গড়বো দেশ তথ্য-প্রযুক্তির বাংলাদেশ

Bangabandhu Sheikh Mujibur Rahman Digital University, Bangladesh

ASSIGNMENT NO: 02

COURSE NO.-**DS 107**
COURSE TITLE-**PROBABILITY AND STATISTICS**

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Date of Submission: 25 May, 2024

1.
Given probability density function of x,

$$f(x) = \begin{cases} \frac{1}{25} & , 0 < x < 25 \\ 0 & , otherwise \end{cases}$$

Expected value of x,

$$E(x) = \int_0^{25} x\ f(x)dx$$

$$= \int_0^{25} x.\ \frac{1}{25}dx$$

$$= \frac{1}{25}[\frac{x^2}{2}]_0^{25}$$

$$= \frac{1}{25}[\frac{25^2}{2} - \frac{0^2}{2}]$$

$$= 12.5$$

Therefore, in long run the commuter is expected to wait, at the station for the train to arrive from the time he reached the station, for 12.5 minutes.

2.
Given random variable, x and it's probability mass function P(x)=f(x),

$X = x$	0	1	2
$P(x)$	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{4}{11}$

So, the expected value of x,

$$E(x) = \sum_x x\ f(x)$$

$$= (0 \times \frac{2}{11}) + (1 \times \frac{5}{11}) + (2 \times \frac{4}{11})$$

$$= (0 \times \frac{2}{11}) + (1 \times \frac{5}{11}) + (2 \times \frac{4}{11})$$

$$= \frac{13}{11}$$

$$= 1.182$$

Therefore, the expected number of women in the interview pool is 1.182

3.

(3.1) Given table,

Height (in cm)	160	162	164	166	168	170	172	174	176	178	180
People (in 1000's)	5	10	25	50	100	120	140	100	50	10	5

So, total number of people (in 1000's)
= (5+10+25+50+100+120+140+5+10+50+100) = 615

X	P(X)
160	0.008
162	0.016
164	0.041
166	0.081
168	0.163
170	0.195
172	0.228
174	0.163
176	0.081
178	0.016
180	0.008
	ΣP(X) = 1

So, the mathematical expectation,

$$E(X) = \sum_{x} X\, P(X)$$

$$= (160 \times 0.008) + (162 \times 0.016) + (164 \times 0.041) + (166 \times 0.081) + (168 \times 0.163)$$

$$+(170 \times 0.195) + (172 \times 0.228) + (174 \times 0.163) + (176 \times 0.081) + (178 \times 0.016) + (180 \times 0.008)$$

$$= 170.699$$

∴. E(X) = 170.699

(3.2)
The most likely hight is the one with the highest propability. That is;
MAX(P(X)) = 0.228 with a hight of 172 cm.

So, the most likely height is 172 cm.

(3.3)
Total population = 5 + 10 + 25 + 50 + 100 + 120 + 140 + 100 + 50 + 10 + 5
= 615 Thousands

50% of the population = 615 * 50%
= 307.5 Thousands

$P(X \leq h) \geq 0.5$

We sum the probabilities starting from the smallest height until we reach a cumulative probability of at least 0.5

Height (h)	Frequency (f)	Cumulative frequency (F_C)
160	5	5
162	10	15
164	25	40
166	50	90
168	100	190
170	120	310
172	140	450
174	100	550
176	50	600
178	10	610
180	5	615

A rolling sum shows that the population of people with height less than or equal to 170 cm is 310 thousands which is nearly 50% of the population.

(3.4)
We got from (3.1),
$E(X) = \mu(X) = 170.699$

Variance,
$\sigma^2 = E(X - \mu)^2$
$= E(X^2) - \mu^2$

$$= (160^2 \times 0.008) + (162^2 \times 0.016) + (164^2 \times 0.041) + (166^2 \times 0.081) + (168^2 \times 0.163)$$

$$+(170^2 \times 0.195) + (172^2 \times 0.228) + (174^2 \times 0.163) + (176^2 \times 0.081) + (178^2 \times 0.016) + (180^2 \times 0.008)$$

$$-170.699^2$$

$$= 29150.828 - 170.699^2$$

$$= 12.679$$

Standard deviation,
$\sigma(X) = \sqrt{12.679}$
$= 3.56$

.'. The variance, $\sigma^2 (X) = 12.679$
The standard deviation, $\sigma (X) = 3.62$

4.
(4. a)
X = Number of miles that Anita's motorbike will travel on one gallon of petrol

We know that,
For a normal random variable x, a particular value of x can be converted to its corresponding z value by using the following formula,

$$z = \frac{x-\mu}{\sigma}$$

Here,

$$z = \frac{X-135}{12}$$

(5. a.i)

$$P(x > 111) = P(z > \frac{111-135}{12})$$

$$P(x > 111) = P(-2)$$

$$P(x > 111) = 0.9772$$

Probability that, without refueling, Anita can travel more than 111 miles = 0.9772

(4. a.ii)

$$P(141 < x < 150) = P(\frac{141-135}{12} < z < \frac{150-135}{12})$$

$$P(141 < x < 150) = P(0.5 < z < 1.25)$$

$$P(141 < x < 150) = P(0 < z < 1.25) - P(0 < z < 0.5)$$

$$P(141 < x < 150) = 0.3944 - 0.1915$$

$$P(141 < x < 150) = 0.2029$$

Probability that, without refueling, Anita can travel between 141 and 150 miles
= 0.2029

5.
$n = 45$

$\overline{x} = 63.4$

$s = 3$

$H_0 : \mu \geq 65$

$H_1 : \mu < 65$

t - distribution with $df = n - 1$
$= 45 - 1$
$= 44$

Test Statistic
We know that, the value of the test statistic t for the sample mean $\overline{x}$ is computed as

$$t = \frac{\overline{x} - \mu}{s_{\overline{x}}} \quad \text{where} \quad s_{\overline{x}} = \frac{s}{\sqrt{n}}$$

The population standard deviation σ is not known and the sample size is large (n > 30)

Consequently, we will use the t distribution to find the p - value for the test.

$$s_{\overline{x}} = \frac{s}{\sqrt{n}}$$

$$= \frac{3}{\sqrt{45}}$$

$$= 0.447$$

$$t = \frac{\overline{x} - \mu}{s_{\overline{x}}}$$

$$= \frac{63.4 - 65}{0.447}$$

$$= -3.578$$

df = n - 1
= 45 -1
= 44

p - value < 0.001

The < sign in the alternative hypothesis indicates that the test is left-tailed.

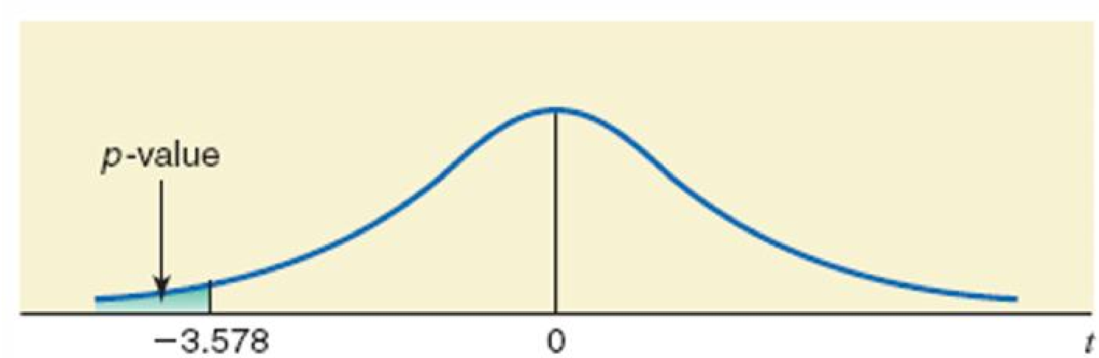

Fig. 1: The required p-value

6.
For Discrete Distribution:
When discrete distribution are used, the problem will have a finite number of scenarios.
The probability of these three scenarios are 1/3 each. Them the problem can be formulated as follows:

Scenario 1:
Max
$Z_1 = 3x_1 + 9x_2$

Subject to
$0.5x_1 + 1.2x_2 \leq 15000$

$2.5x_1 + x_2 \leq 10000$

$x_1 , x_2 \leq 0$

Scenario 2:
Max
$Z_1 = 3x_1 + 9x_2$

Subject to
$0.6x_1 + 1.3x_2 \leq 17000$

$2.5x_1 + x_2 \leq 10000$

$x_1 , x_2 \leq 0$

Scenario 3:
Max
$Z_1 = 3x_1 + 9x_2$

Subject to
$0.7x_1 + 1.4x_2 \leq 20000$

$2.5x_1 + x_2 \leq 10000$

$x_1 , x_2 \leq 0$

7.
The tool crib constitutes a queuing system, with the clerks as its servers and the mechanics as its customers. After gathering some data on inter-arrival times and service times, the OR team has concluded that the queuing system best is the M/M/s model. The estimates of the mean arrival rate λ and the mean service rate (per server) μ are

λ = 120 customers per hour
μ = 80 customers per hour

So, the utilization factor for the two clerks is:

$$\rho = \frac{\lambda}{s\,\mu}$$

$$= \frac{120}{2 \times 80}$$

$$= 0.75$$

The total cost to the company of each tool crib clerk is about ＄20 per hour,

So, C_s = ＄20

While a mechanic is busy, the value to the company of his or her output averages about ＄48 per hour

So, C_w = ＄48

Therefore, the OR team now needs to find the number of servers (tool crib clerks) s that will

Minimize E(TC) = ＄20 s + ＄48 L

It turns out s = 3 yields the minimum total cost.

8.
If there are r correct answers, there are 10 - r wrong answers and the score is

3r - (10 - r) = 3r - 10 + r
= 4r - 10

The probability of a correct answer = 1/4
The probability of a wrong answer = 1 - (1/4)
= 3/4

Here, by the binomial distribution, the probability of r correct answer is:

$$\binom{10}{r} 3^{10-r}/4^{10} = \binom{10}{r} (3/4)^{10} \ (1/3)^r$$

By the Binomial theorem,

$$(1+t)^{10} = \sum_{r=0}^{10} \binom{10}{r} t^r$$

Differentiating both sides with respect to t and then multiplying by t, we have,

$$10(1+t)^9 = \sum_{r=0}^{10} r \binom{10}{r} t^r$$

Doing the same operation once more, we have,

$$10t(1+t)^9 + 90t^2(1+t)^8 = \sum_{r=0}^{10} r^2 \binom{10}{r} t^r$$

The second formula gives us, on putting t =1/3

$$\sum_{r=0}^{10} r \binom{10}{r} (1/3)^r = (10/3)(4/3)^9$$

Or Equivalently,

$$E(X+10) = \sum_{r=0}^{10} 4r \binom{10}{r} (3/4)^{10}(1/3)^r$$

$$= 4(3/4)(10/3)$$
$$= 10$$

In other words, as we might expect we get,

E(X) = 0

Similarly, we can use the third formula with t = 1/3 to get,

$$\sum_{r=0}^{10} r^2 \binom{10}{r} (1/3)^r$$
$$= (10/3)(4/3)^9 + 10(4/3)^8$$

Or Equivalently,

$$E((X+10)^2) = \sum_{r=0}^{10} 16r^2 \binom{10}{r} (3/4)^{10}(1/3)^r$$
$$= 16((3/4)(10/3) + 10(3/4)^2)$$
$$= 130$$

This gives the variance , $\sigma^2(X) = \sigma^2(X+10) = 30$

Hence, the standard deviation, $\sigma(X) = \sqrt{30} = 5.477$

The table of probabilities,

r	0	1	2	3	4	5	6	7	8	9	10
4^{10} * P	59049	196830	295245	26244	153090	61236	17010	3240	405	30	1

We note that to get at least 50% of the population we can take r ≤ 2
Since the score is then (4r - 10) ≤ -2
It means that more than 50% of the time we will get a negative score.

(8.1) E(X) = 0

(8.2) Most probable score : 0

(8.3) Smallest s such that P(x ≤ s) ≥ (1/2) : ‘negative score’

(8.4) Variance, $\sigma^2(X) = 30$