that diagonalizes A.

ⓐ $T(x_1, x_2, x_3) = (8x_1 + 3x_2 - 4x_3, -3x_1 + x_2 + 3x_3, 4x_1 + 3x_2)$

ⓑ using Caylay-Hamilton theorem find A^{-1}. also show that the sum of rank and nullity of the matrix A^{-1} is equal to the number of column A^{-1}.

$$\begin{vmatrix} 8-\lambda & 3 & -4 \\ -3 & 1-\lambda & 3 \\ 4 & 3 & -\lambda \end{vmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\Rightarrow (8-\lambda)(-\lambda+\lambda^2-9) - 3(3\lambda-12) - 4(-9-4+4\lambda) = 0$

$\Rightarrow -8\lambda + 8\lambda^2 - 72 + \lambda^2 - \lambda^3 + \cancel{9\lambda} - \cancel{9\lambda} + 36 + 52 - 16\lambda = 0$

$\Rightarrow -\lambda^3 + 9\lambda^2 - 24\lambda + 16 = 0$

$\Rightarrow \lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$

$\Rightarrow \lambda^3 - \lambda^2 - 8\lambda^2 + 8\lambda + 16\lambda - 16 = 0$

$\Rightarrow \lambda^2(\lambda-1) - 8\lambda(\lambda-1) + 16(\lambda-1) = 0$

$\Rightarrow (\lambda-1)(\lambda-4)^2 = 0$

$\Rightarrow \boxed{\lambda = 1, 4, 4}$