Find the bases for the eigen spaces of the mat $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Soln i) find eigen values $\det(A-\lambda I)=0$

" " vector $(A-\lambda I)x=0$.

let the eigen value is λ

Characterised eqn is $\det(A-\lambda I)=0$.

Here, $A-\lambda I = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{bmatrix}$

$\det(A-\lambda I) = -\lambda((2-\lambda)(3-\lambda)) - 0 - 2(0-(2-\lambda))$

$= -\lambda(6-2\lambda-3\lambda+\lambda^2) - 2(-2+\lambda)$

$= -6\lambda+2\lambda^2+3\lambda^2-\lambda^3+4-2\lambda$

$= -\lambda^3+5\lambda^2-8\lambda+4$

$\det(A-\lambda I)=0$

$\Rightarrow -\lambda^3+5\lambda^2-8\lambda+4=0$

$\Rightarrow \lambda^3-5\lambda^2+8\lambda-4=0$

$\Rightarrow \therefore \lambda = 1, 2, 2$

for eigen vector,

$(A - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

for $\lambda = 1$,

$$\begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$r_1' = r_1 * (-1)$
$r_2' = r_2 - r_3$
$r_3' = r_1' - r_3$

$\therefore$ the system is, $x_1 + 2x_3 = 0$
$x_2 - x_3 = 0$

let free variable $x_3 = t$

$\therefore x_1 = -2t, \quad x_2 = t.$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t \\ t \\ t \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

so $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ is a basis for the eigen-space corresponding to sol^n.

Now, for, $\lambda = 2$,

$$\begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \left| \begin{array}{l} r_1' = r_1 \div (-2) \\ r_2' = r_1' - r_2 \\ r_3' = r_1' - r_2 \end{array} \right.$$

system, $x_1 + x_3 = 0$

let, free variable $x_2 = t_1$, $x_3 = t_2$

$\therefore x_1 = -t_2$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

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Now, $A^3 - 5A^2 + 8A - 4I = 0$

$$= \begin{bmatrix} -6 & 0 & -14 \\ 7 & 8 & 7 \\ 7 & 0 & 15 \end{bmatrix} - \begin{bmatrix} -10 & 0 & -30 \\ 15 & 20 & 15 \\ 15 & 0 & 35 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & -16 \\ 8 & 16 & 8 \\ 8 & 0 & 24 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6+10-4 & 0 & -14+30-16 \\ 7-15+8 & 8-20+16-4 & 7-15+8 \\ 7-15+8 & 0 & 15-35+24-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{(pr)}$$

Now, $A^3 \cdot A^{-1} - 5A^2 \cdot A^{-1} + 8A \cdot A^{-1} - 4IA^{-1} = 0$

$\Rightarrow A^2 - 5A + 8I - 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = A^2 - 5A + 8I$

$\Rightarrow A^{-1} = \frac{1}{4}\left[A^2 - 5A + 8I\right]$

Now,

$$A^2-5A+8I=\begin{bmatrix}-2 & 0 & -6\\ 3 & 4 & 3\\ 3 & 0 & 7\end{bmatrix}-\begin{bmatrix}0 & 0 & -10\\ 5 & 10 & 5\\ 5 & 0 & 15\end{bmatrix}+\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$$

$$=\begin{bmatrix}-2+8 & 0 & -6+10\\ 3-5 & 4-10+8 & 3-5\\ 3-5 & 0 & 7-15+8\end{bmatrix}$$

$$=\begin{bmatrix}6 & 0 & 4\\ -2 & 2 & -2\\ -2 & 0 & 0\end{bmatrix}$$

$$\therefore\ A^{-1}=\begin{bmatrix}6/4 & 0 & 4/4\\ -2/4 & 2/4 & -2/4\\ -2/4 & 0 & 0\end{bmatrix}=\begin{bmatrix}\frac{3}{2} & 0 & 1\\ -1/2 & 1/2 & -1/2\\ -1/2 & 0 & 0\end{bmatrix}$$

Now, $$A\cdot A^{-1}=\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}3/2 & 0 & 1\\ -1/2 & 1/2 & -1/2\\ -1/2 & 0 & 0\end{bmatrix}$$

$$\frac{3}{2}-\frac{1}{2}-\frac{1}{2}=\frac{3-1-1}{2}=\frac{1}{2}$$

$$=\begin{bmatrix}0+0+1 & 0+0+0 & 0+0+0\\ \frac{3}{2}-1-\frac{1}{2} & 0+1+0 & 1-1+0\\ \frac{3}{2}-\frac{3}{2} & 0+0+0 & 1+0+0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$ Pr.

Diagonozation :-

$P^{-}AP$ is diagoral

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

for $\lambda = 2$, $P_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 1$, $P_3 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$P = [P_1 \;\; P_2 \;\; P_3]$

$$\therefore P = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\therefore P^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\therefore P^{-1}AP = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & 0+0+0 & -2+0+6 \\ 0+1+1 & 0+2+0 & -2+1+3 \\ 0+0-1 & 0+0+0 & 2+0-3 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 2 & 0 & 4 \\ 2 & 2 & 2 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+4 & 0 & -4+0+4 \\ -2+0+2 & 0+2+0 & -4+2+2 \\ 1+0-1 & 0+0+0 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

L-6

Solve the following system by QR decomposition:

$x_1 + 2x_2 + 3x_3 = 5$

$2x_1 + 5x_2 + 3x_3 = 3$

$x_1 \quad + 8x_3 = 17$

Soln $A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{vmatrix}$

$\therefore |A| = 1(40-0) - 2(16-3) + 3(0-5)$

$= 40 - 26 - 15$

$= -1$

Now, $u_1 = (1,2,1)$, $u_2 = (2,5,0)$, $u_3 = (3,3,8)$

$u_1 = v_1 = (1,2,1)$

$v_2 = u_2 - \frac{\langle u_2 v_1 \rangle}{\|v_1\|^2} v_1$

$= (2,5,0) - \frac{(2,5,0)\cdot(1,2,1)}{\left(\sqrt{1^2+2^2+1^2}\right)^2}(1,2,1)$

$= (2,5,0) - \frac{2+10+0}{6}(1,2,1)$

$= (2,5,0) - (2,4,2)$

$= (0,1,-2)$

$3-\frac{241}{30}$ $=\frac{240-241}{30}$

$-\frac{26}{5}-\frac{17}{6}$ $=\frac{-156-}{30}$

$3-\frac{46}{15}$ $=\frac{45-46}{15}$

$\frac{13}{5}-\frac{17}{3}$ $=\frac{39-85}{15}$

$\frac{13}{39}$

$3+\frac{17}{6}$ $=\frac{18+17}{6}=\frac{1}{6}$

$$v_3 = u_3 - \frac{\langle u_3\, v_2\rangle}{\|v_2\|^2} v_2 - \frac{\langle u_3\, v_1\rangle}{\|v_1\|^2} v_1$$

$$= u_3 - \frac{(3,3,8)(0,1,-2)}{\left(\sqrt{0+1^2+(-2)^2}\right)^2} v_2 - \frac{(3,3,8)(1,2,1)}{6}(1,2,1)$$

$$= u_3 - \frac{0+3-16}{5}(0,1,-2) - \frac{3+6+8}{6}(1,2,1)$$

$$= u_3 + \frac{13}{5}(0,1,-2) - \frac{17}{6}(1,2,1)$$

$$= (3,3,8) + \left(0, \frac{13}{5}, -\frac{26}{5}\right) - \left(\frac{17}{6}, \frac{17}{3}, \frac{17}{6}\right)$$

$$= (3,3,8) + \left(-\frac{17}{6}, -\frac{46}{15}, -\frac{241}{30}\right)$$

$$= \left(\frac{1}{6}, -\frac{1}{15}, -\frac{1}{30}\right)$$

$$q_1 = \frac{u_1}{\|u_1\|} = \frac{(1,2,1)}{\sqrt{6}} = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$q_2 = \frac{v_2}{\|v_2\|} = \frac{(0,1,-2)}{\sqrt{5}} = \left(0, \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$$

$\frac{8}{3}$ 13 $\frac{39-40}{}$ $=\frac{}{13}$

$$q_3 = \frac{v_3}{\|v_3\|} = \frac{\left(\frac{1}{6}, -\frac{1}{15}, -\frac{1}{30}\right)}{\sqrt{\left(\frac{1}{6}\right)^2+\left(-\frac{1}{15}\right)^2+\left(-\frac{1}{30}\right)^2}}$$

$$= \frac{\left(\frac{1}{6}, -\frac{1}{15}, -\frac{1}{30}\right)}{\frac{1}{\sqrt{30}}}$$

$$= \left(\frac{\sqrt{30}}{6}, -\frac{\sqrt{30}}{15}, -\frac{\sqrt{30}}{30}\right)$$

$$= \left(\frac{\sqrt{30}}{6}, -\frac{\sqrt{30}}{13}, -\frac{1}{\sqrt{30}}\right)$$

$\frac{\frac{1}{2}+\frac{1}{5}}{\frac{2}{5}}$

$$q_1 = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \qquad q_2 = \begin{bmatrix} 0 \\ 1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix} \qquad q_3 = \begin{bmatrix} -\frac{\sqrt{30}}{6} \\ -\frac{\sqrt{30}}{15} \\ -1/\sqrt{30} \end{bmatrix}$$

Now,

$$\langle u_1 q_1 \rangle = (1,2,1)\left(1/\sqrt{6},\ 2/\sqrt{6},\ \frac{1}{\sqrt{6}}\right)$$

$$= \frac{1}{\sqrt{6}} + 4/\sqrt{6} + 1/\sqrt{6} = \frac{1+4+1}{\sqrt{6}} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

$$\langle u_2 q_1 \rangle = (2,5,0)\ \left(1/\sqrt{6},\ 2/\sqrt{6},\ 1/\sqrt{6}\right)$$

$$= \left(2/\sqrt{6} + \frac{10}{\sqrt{6}} + 0\right) = \frac{2+10}{\sqrt{6}} = \frac{2\times 6}{\sqrt{6}} = 2\sqrt{6}$$

$$\langle u_3 q_1 \rangle = (3,3,8)\ \left(1/\sqrt{6},\ 2/\sqrt{6},\ 1/\sqrt{6}\right)$$

$$= 3/\sqrt{6} + 6/\sqrt{6} + 8/\sqrt{6} = \frac{17}{\sqrt{6}}$$

$$\langle u_2 q_2 \rangle = (2,5,0)\left(0,\ 1/\sqrt{5},\ -2/\sqrt{5}\right)$$

$$= 0 + 5/\sqrt{5} + 0 = \sqrt{5}$$

$$\langle u_3 q_2 \rangle = (3,3,8)\ \left(0,\ 1/\sqrt{5},\ -2/\sqrt{5}\right)$$

$$= 0 + 3/\sqrt{5} - \frac{16}{\sqrt{5}} = \frac{-13}{\sqrt{5}}$$

$$\langle u_3 q_3 \rangle = (3,3,8)\ \left(\frac{\sqrt{30}}{6},\ -\frac{\sqrt{30}}{15},\ -\frac{1}{\sqrt{30}}\right)$$

$$= \frac{3\sqrt{30}}{6} + \frac{3\sqrt{30}}{15} - \frac{8}{\sqrt{30}}$$

$$= \frac{\sqrt{30}}{2} - \frac{\sqrt{30}}{5} - \frac{8}{\sqrt{30}}$$

$$= \frac{5\sqrt{30} - 2\sqrt{30}}{10} - 8/\sqrt{30}$$

$$= \frac{3\sqrt{30}}{10} - \frac{8}{\sqrt{30}} = \cancel{0.024}.\ 0.1825.$$

3×3 3×1

3×1 3×3

$$\therefore R = \begin{bmatrix} \sqrt{6} & 2\sqrt{6} & 17/\sqrt{6} \\ 0 & \sqrt{5} & -13/\sqrt{5} \\ 0 & 0 & 0.1825 \end{bmatrix}$$

$Ax = b$

$\Rightarrow QRx = b$

$\Rightarrow QQ^TRx = Q^Tb$

$\Rightarrow I \quad Rx = Q^Tb$

$\Rightarrow Rx = Q^Tb$

$$R^{-1} = \begin{bmatrix} 0.408 & -0.894 & -33.47 \\ 0 & 0.4472 & 10.833 \\ 0 & 0 & 4.167 \end{bmatrix}$$

$$\Rightarrow R\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} & 0 & \sqrt{30}/6 \\ 2/\sqrt{6} & 1/\sqrt{5} & -\sqrt{30}/15 \\ 1/\sqrt{6} & -2/\sqrt{5} & 1/\sqrt{30} \end{bmatrix}^T \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} \\ \sqrt{30}/6 & -\sqrt{30}/15 & -1/\sqrt{30} \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

$$R\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11.43 \\ -13.86 \\ 0.3651 \end{bmatrix} * R^{-1} =$$

$$\therefore \sqrt{6}\,x_1 + 2\sqrt{6}\,x_2 + \frac{17}{\sqrt{6}}\,x_3 = 11.43$$

$$\sqrt{5}\,x_2 - {}^{13}/_{\sqrt{5}}\,x_3 = -13.86$$

$$0.1825\,x_3 = \cancel{6.37}\,0.3651$$

$$\therefore x_3 = 2.0005.$$

$$\boxed{\begin{aligned} x_1 &= 0.9719 \\ x_2 &= -0.9969 \\ x_3 &= 2.0005. \end{aligned}}$$