$\int uv = u\int v\,dx - \int \frac{d}{dx}(u)\int (v\,dx)\,dx$

Laplace and Fourier Transformation:

$$\mathcal{L}\{F(t)\} = f(s) = \int_0^{\infty} e^{-st} F(t)\,dt$$

$f(s) \downarrow$ Algebraic Format

$\to$ Parameter

① (i) $F(t) = 1$

$$\mathcal{L}\{F(t)\} = \int_0^{\infty} e^{-st} F(t)\,dt$$

$$\mathcal{L}\{1\} = \int_0^{\infty} e^{-st} \cdot 1\,dt = \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = \frac{1}{-s}[0 - 1] = \frac{1}{s}$$

(ii) $F(t) = t^2$

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{t^2\}$$

$$= \int_0^{\infty} e^{-st} t^2\,dt$$

$$= \left[t^2 \frac{e^{-st}}{-s}\right]_0^{\infty} - \left[\int 2t \cdot \frac{e^{-st}}{-s}\,dt\right]_0^{\infty}$$

$$= \left[\frac{t^2 e^{-st}}{-s}\right]_0^{\infty} + \frac{2}{s}\left[t \cdot \frac{e^{-st}}{-s} - \int 1 \cdot \frac{e^{-st}}{-s}\,dt\right]_0^{\infty}$$

$$= 0 + \frac{2}{s^2}\left[-t e^{-st} + \frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= \cancel{\frac{2}{s^2}\left[0 - \frac{1}{s}(0-1)\right]} = \frac{2}{s^3}$$

$$= \frac{2}{s^2}\left[0 - \frac{1}{s}(0-1)\right] = \frac{2}{s^3}$$

$$\ldots \alpha t - \alpha \cdot \cos \alpha t)\Big]_0^\infty$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{t\} = \frac{1!}{s^2}$$

$$= \frac{1}{s^2}$$

$$② \mathcal{L}\{F(t) = e^{at}\} = \int_0^\infty e^{-st} e^{at} dt$$

$$= \int_0^\infty e^{-t(s-a)} dt \qquad [s > a]$$

$$= \left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty$$

$$= \frac{1}{-(s-a)}(0-1)$$

$$= \frac{1}{s-a}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$e^{-\infty} = 0$

$e^{0} = 1$

$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$

3. $\mathcal{L}\{\sin \alpha t\}$

$$= \int_0^\infty e^{-st} \sin \alpha t \, dt$$

$$= \left[\frac{e^{-st}}{s^2+\alpha^2} (-s.\sin \alpha t - \alpha \cos \alpha t) \right]_0^\infty$$

$$= -\frac{1}{s^2+\alpha^2}(0 - \alpha)$$

$$= \frac{\alpha}{s^2+\alpha^2}$$

4. $\mathcal{L}\{\cos \beta t\} = \int_0^\infty e^{-st} \cos \beta t \, dt$

$$= \left[\frac{e^{-st}}{s^2+\beta^2} (-s \cos \beta t + \beta \sin \beta t) \right]_0^\infty$$

$$= -\frac{1}{s^2+\beta^2}(-s)$$

$$= \frac{s}{s^2+\beta^2}$$

* $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$ * $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

$\int e^{ax}\cos bx \, dx = \dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

$\boxed{Q}$ derivation : [illegible]

$$I = \int e^{\alpha t}\cos\beta t \, dt$$

$$= \cos\beta t \, \frac{e^{\alpha t}}{\alpha} - \int -\sin\beta t \cdot \beta \cdot \frac{e^{\alpha t}}{\alpha} \, dt$$

$$= \frac{1}{\alpha} e^{\alpha t}\cos\beta t + \frac{\beta}{\alpha}\int e^{\alpha t}\sin\beta t \, dt$$

$$= \frac{1}{\alpha} e^{\alpha t}\cos\beta t + \frac{\beta}{\alpha} \cdot \sin\beta t \cdot \frac{e^{\alpha t}}{\alpha} - \frac{\beta}{\alpha}\int \cos\beta t \cdot \beta \cdot \frac{e^{\alpha t}}{\alpha} \, dt$$

$$\Rightarrow I = \frac{1}{\alpha} e^{\alpha t}\cos\beta t + \frac{\beta}{\alpha^2} \cdot e^{\alpha t} \cdot \sin\beta t - \frac{\beta}{\alpha} \cdot \frac{\beta}{\alpha}\int e^{\alpha t}\cos\beta t \, dt$$

$$\Rightarrow \left(1 + \frac{\beta^2}{\alpha^2}\right)\left(\int e^{\alpha t}\cos\beta t \, dt\right) = \frac{1}{\alpha} e^{\alpha t}\cos\beta t + \frac{\beta}{\alpha^2} e^{\alpha t}\sin\beta t$$

$$\Rightarrow \int e^{\alpha t}\cos\beta t \, dt = \frac{\alpha^2}{\alpha^2+\beta^2} \cdot \frac{\alpha e^{\alpha t}\cos\beta t + \beta e^{\alpha t}\sin\beta t}{\alpha^2}$$

$$\Rightarrow \int e^{\alpha t}\cos\beta t \, dt = \frac{\alpha e^{\alpha t}\cos\beta t + \beta e^{\alpha t}\sin\beta t}{\alpha^2+\beta^2}$$

$\cosh x = \frac{1}{2}(e^{x} + e^{-x})$

$\sinh x = \frac{1}{2}(e^{x} - e^{-x})$

Q

$$\mathcal{L}\{\cosh(at)\} = \mathcal{L}\left\{\frac{1}{2}\left(e^{at} + e^{-at}\right)\right\}$$

$$= \int_0^\infty e^{-st} \cdot \frac{e^{at} + e^{-at}}{2}\, dt$$

$$= \frac{1}{2}\left(\int_0^\infty e^{-t(s-a)}\, dt + \int_0^\infty e^{-t(s+a)}\, dt\right)$$

$$= \frac{1}{2}\left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty + \frac{1}{2}\left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty$$

$$= \frac{1}{2}\left\{-\frac{1}{-(s-a)}\right\} + \frac{1}{2}\left\{-\frac{1}{-(s-a)}\right\}$$

$$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right)$$

$$= \frac{1}{2}\,\frac{s+a+s-a}{s^2-a^2}$$

$$= \frac{2s}{2(s^2-a^2)} = \frac{s}{s^2-a^2}$$

$\cos \alpha t)\Big]_0^{\infty}$

###

$$\mathcal{L}\{e^{t}\} = \int_0^{\infty} e^{-st} e^{t} \, dt$$

$$= \int_0^{\infty} e^{-t(s-1)} \, dt$$

$$= \left[\frac{e^{-t(s-1)}}{-(s-1)} \right]_0^{\infty}$$

$$= - \frac{1}{-(s-1)}$$

$$= \frac{1}{s-1}$$

$\mathcal{L}\{e^{-t}\} = \int_0^{\infty} e^{-st} e^{-t} \, dt$

$$= \int_0^{\infty} e^{-t(s+1)} \, dt$$

$$= \left[\frac{e^{-t(s+1)}}{-(s+1)} \right]_0^{\infty}$$

$$= \frac{-1}{-(s+1)}$$

$$= \frac{1}{s+1}$$

Properties of laplace Transformation:

Linear property;

$\mathcal{L}\{F_1(t)\} = f_1(s)$

$\mathcal{L}\{F_2(t)\} = f_2(s)$

$\mathcal{L}\{c_1 F_1(t) + c_2 F_2(t)\} = c_1 f_1(s) + c_2 f_2(s)$

Q. $\mathcal{L}\{4t^2 - 3\cos 2t + 5e^{-t}\}$

$= \mathcal{L}\{4t^2\} - \mathcal{L}\{3\cos 2t\} + \mathcal{L}\{5e^{-t}\}$

$= 4 \cdot \frac{2!}{s^3} - 3 \cdot \frac{s}{s^2+2^2} + 5 \frac{1}{s+1}$

$= \frac{8}{s^3} - \frac{3s}{s^2+4} + \frac{5}{s+1}$

First Translation or shiffting property:

if $\mathcal{L}\{F(t)\} = f(s)$,

then, $\mathcal{L}\{e^{at} F(t)\} = f(s-a)$

Proof:

$\mathcal{L}\{e^{at} F(t)\} = \int_0^{\infty} e^{-st} e^{at} F(t)\, dt$

$= \int_0^{\infty} e^{-(s-a)t} F(t)\, dt$

$= f(s-a)$ (proved)

$\cos \alpha t)\Big]^{\infty}$

Q $\mathcal{L}\{e^{-t} \cos 2t\}$

Let,

$F(t) = \cos 2t$

$\mathcal{L}\{F(t)\} = \mathcal{L}\{\cos 2t\}$

$\boxed{\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}}$

$$= \frac{s}{s^2+(2)^2}$$

$$= \frac{s}{s^2+4}$$

$\mathcal{L}\{e^{-t} \cos 2t\} = f(s-a)$

$a = -1$

$$= f(s+1)$$

$$= \frac{s+1}{(s+1)^2+2^2}$$

$$= \frac{s+1}{(s+1)^2+4}$$

$$= \frac{s+1}{s^2+2s+5}$$

2nd Translation on shifting :

If $\mathcal{L}\{F(t)\} = f(s)$

and $G(t) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$,

then $\mathcal{L}\{G(t)\} = e^{-as} f(s)$

Q Find the laplace transformation of

$$G(t) = \begin{cases} (t-2)^3 & t > 2 \\ 0 & t < 2 \end{cases}$$

$\Rightarrow$

let,

$$F(t) = t^3$$

$$\mathcal{L}\{F(t)\} = \frac{3!}{s^{3+1}}$$

$$= \frac{6}{s^4}$$

| $a = 2$
| $f(s) = \mathcal{L}\{F(t)\}$

$$\mathcal{L}\{G(t)\} = e^{-2s} \cdot \frac{6}{s^4}$$

$$= \frac{6e^{-2s}}{s^4}$$

Laplace *Transformation of derivative

If $\mathcal{L}\{F(t)\} = f(s)$,

then, $\mathcal{L}\{F'(t)\} = sf(s) - F(0)$

Proof :

$$\mathcal{L}\{F(t)\} = f(s) = \int_0^\infty e^{-st} F(t)\, dt$$

$$\mathcal{L}\{F'(t)\} = \int_0^\infty e^{-st} F'(t)\, dt$$

$\int \frac{d}{dx}(u)\,dx$
$= u$

$= e^{-st}\int_0^{\infty} F'(t)\,dt - \int_0^{\infty}\left\{\frac{d}{dt}(e^{-st})\int_0^{\infty} F'(t)\,dt\right\}dt$

$= \left[e^{-st}F(t)\right]_0^{\infty} - \int_0^{\infty} e^{-st}\cdot(-s)\,F(t)\,dt$

$= 0 - F(0) + s\int_0^{\infty} e^{-st}\,F(t)\,dt$

$= -F(0) + sf(s)$

$= sf(s) - F(0)$

[proved]

$\boxed{Q}$

If $F(t) = \cos 3t$, then Find $\mathcal{L}\{F'(t)\}$

$\mathcal{L}\{F'(t)\} = s\,f(s) - F(0)$

Here,

$F(t) = \cos 3t$

$\mathcal{L}\{F(t)\} = \mathcal{L}\{\cos 3t\}$

$= \frac{s}{s^2 + 3^2}$

$= \frac{s}{s^2+9} = f(s)$

$F(0) = \cos 0 = 1$

$$\mathcal{L}\{F'(t)\} = s \cdot \frac{s}{s^2+9} - 1$$

$$= \frac{s^2 - (s^2+9)}{s^2+9}$$

$$= \frac{-9}{s^2+9}$$

$$\# \mathcal{L}\{F''(t)\} = s^2 f(s) - sF(0) - F'(0)$$

proof :

$$\mathcal{L}\{F''(t)\} = \int_0^\infty e^{-st} F''(t)\,dt$$

$$= e^{-st} \int_0^\infty F''(t)\,dt - \int_0^\infty \left[\frac{d}{dt}(e^{-st}) \int_0^\infty F''(t)\,dt\right]$$

$$= \left[e^{-st} F'(t)\right]_0^\infty - \int_0^\infty e^{-st}(-s) F'(t)\,dt$$

$$= 0 - F'(0) + s \int_0^\infty e^{-st} F'(t)\,dt$$

$$= -F'(0) + s\left[e^{-st} \int_0^\infty F'(t)\,dt - \int_0^\infty \left\{\frac{d}{dt}(e^{-st}) \int_0^\infty F'(t)\,dt\right\}\right]$$

$$= -F'(0) + s\left[e^{-st} F(t)\right]_0^\infty - \int_0^\infty e^{-st} \cdot (-s) \cdot F(t)\,dt$$

$$= -F'(0) + s\{0 - F(0)\} + s \cdot s \int_0^\infty e^{-st} F(t)\,dt$$

$$= -F'(0) - sF(0) + s^2 f(s)$$

$$= s^2 f(s) - sF(0) - F'(0)$$

(Proved)

Q Find $\mathcal{L}\{F''(t)\}$.

$$\mathcal{L}\{F''(t)\} = s^2 \cdot \frac{s}{s^2+9} - s \cdot 1 - (-\sin 0)$$

$$= +\frac{s^3}{s^2+9} - s$$

$$= \frac{s^3 - s^3 - 9s}{s^2+9}$$

#Q $\mathcal{L}\{\sinh(at)\} = \mathcal{L}\{\frac{1}{2}(e^{a} - e^{-a})\}$

$$\boxed{\sinh x = \frac{1}{2}(e^{x} - e^{-x})}$$

$$\mathcal{L}\{\sinh(at)\} = \frac{1}{2}\mathcal{L}(e^{a} - e^{-a})$$

$$= \frac{1}{2}\int_0^\infty e^{-st}(e^{a} - e^{-a})\,dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-a)}\,dt - \frac{1}{2}\int_0^\infty e^{-t(s+a)}\,dt$$

$$= \frac{1}{2}\left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty - \frac{1}{2}\left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty$$

$$= \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right)$$

$$= \frac{1}{2}\left(\frac{s+a-s+a}{(s+a)(s-a)}\right)$$

$$= \frac{1}{2} \cdot \frac{2a}{s^2-a^2}$$

$$= \frac{a}{s^2-a^2}$$

Multiplication by the power of t (t^n):

If $\mathcal{L}\{F(t)\} = f(s)$,

then $\mathcal{L}\{t^n F(t)\} = (-1)^n \dfrac{d^n}{ds^n} f(s)$

$$= (-1)^n f^n(s)$$

$\boxed{Q}$ $\mathcal{L}\{te^{2t}\} = ?$

Let,

$F(t) = e^{2t}$

$$\mathcal{L}\{e^{2t}\} = \frac{1}{s-2} = f(s)$$

$$\mathcal{L}\{t e^{2t}\} = (-1)^1 \cdot \frac{d}{ds}\left(\frac{1}{s-2}\right) = -\frac{-1}{(s-2)^2} = \frac{1}{(s-2)^2}$$

$$\mathcal{L}\{t^2 e^{2t}\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right)$$

$$= (-1)^2 \cdot \frac{d}{ds}\left\{\frac{d}{ds}\left(\frac{1}{s-2}\right)\right\}$$

$$= \frac{d}{ds}\left\{\frac{-1}{(s-2)^2}\right\}$$

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$$= -(-2) \cdot (s-2)^{-2-1}$$

$$= 2 \cdot \frac{1}{(s-2)^3}$$

$$= \frac{2}{(s-2)^3}$$

$$\mathcal{L}\{t^3 e^{2t}\} = (-1)^3 \frac{d^3}{ds^3}\left(\frac{1}{s-2}\right)$$

$$= (-1)\frac{d^2}{ds^2}\left[\frac{d}{ds}\left(\frac{1}{s-2}\right)\right]$$

$$= -1 \frac{d}{ds}\left[\frac{d}{ds} \cdot \frac{1}{(s-2)^2}\right]$$

$$= -1 \frac{d}{ds}\left[\frac{-2}{(s-2)^3}\right]$$

$$= 2 \frac{d}{ds} \cdot \frac{1}{(s-2)^3}$$

$$= 2 \cdot \frac{-3}{(s-2)^4}$$

$$= \frac{6}{(s-2)^4}$$

$$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$$

$\boxed{Q}$ Find $\mathcal{L}\{t^2 \cos at\}$

By the def^n:

$$\mathcal{L}\{t^2 \cos at\} = (-1)^n \underbrace{f^2(s)}_{\hookrightarrow \frac{d^2}{ds^2}(\mathcal{L}\{F(t)\})}$$

Let, $F(t) = \cos at$

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{\cos at\}$$

$$= \frac{s}{s^2+a^2} = f(s)$$

$$\frac{d}{ds}[\mathcal{L}\{F(t)\}] = \frac{(s^2+a^2)\cdot 1 - s(2s)}{(s^2+a^2)^2}$$

$$= \frac{s^2+a^2-2s^2}{(s^2+a^2)^2}$$

$$= \frac{a^2-s^2}{(s^2+a^2)^2}$$

$$\frac{d^2}{ds^2}[\mathcal{L}\{F(t)\}] = \frac{(s^2+a^2)^2(-2s) - (a^2-s^2)\cdot 2(s^2+a^2)\cdot 2s}{\{(s^2+a^2)^2\}^2}$$

$$= \frac{-2s(s^2+a^2)^2 - 4s(s^2+a^2)(a^2-s^2)}{(s^2+a^2)^4}$$

$$= \frac{(s^2+a^2)(-2s^3 - 2sa^2 - 4sa^2 + 4s^3)}{(s^2+a^2)^4}$$

$$= \frac{2s^3 - 6sa^2}{(s^2+a^2)^3}$$

$$= \frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$$

$$\mathcal{L}\{t^2 \cos at\} = (-1)^2 \cdot \frac{2s(s^2-3a^2)}{(s^2+a^2)}$$

$$= \frac{-2s(3a^2-s^2)}{(s^2+a^2)}$$

Division by t:

If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(u)\,du$$

$\boxed{Q}$ Show that, $\int_0^\infty \frac{\sin t}{t}\,dt = \pi/2$

Let, $F(t) = \sin t$

$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

$\mathcal{L}\{\underbrace{\sin t}_{F(t)}\} = \frac{1}{1+s^2} \rightarrow f(s)$

$$\mathcal{L}\{F(t)\} = \frac{1}{1+s^2} = f(s)$$

$$f(u) = \frac{1}{1+u^2}$$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x$

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$$\int_0^\infty \frac{\sin t}{t}\,dt = \int_0^\infty \frac{1}{1+u^2}\,du$$

$$= \left[\tan^{-1}u\right]_0^\infty$$

$$= \pi/2 - 0$$

$$= \pi/2$$

Inverse Laplace Transformation:

$F(t) = L^{-1}\{F(s)\}$ $\quad L\{F(t)\} = f(s)$

1	$1/s$	$s>0$
t	$1/s^2$	$s>0$
t^n	$\frac{n!}{s^{n+1}}$	$s>0$
e^{at}	$\frac{1}{s-a}$	$s>0$
$\sin at$	$\frac{a}{s^2+a^2}$	$s>0$
$\cos at$	$\frac{s}{s^2+a^2}$	$s>0$
$\sinh(at)$	$\frac{a}{s^2-a^2}$	$s>\lvert a\rvert$
$\cosh(at)$	$\frac{s}{s^2-a^2}$	$s>\lvert a\rvert$

$F(t)$ $\quad f(s)$

$$\frac{t^n}{n!} \rightarrow \frac{1}{s^{n+1}}$$

$$\frac{\sin at}{a} \rightarrow \frac{1}{s^2+a^2}$$

$$\frac{\sinh at}{a} \rightarrow \frac{a}{s^2-a^2}$$

1. Linear Property:

$$\mathcal{L}^{-1}\{c_1 f_1(s) + c_2 f_2(s)\} = c_1 f_1(t) + c_2 F_2(t)$$

2. First Translation:

If $\mathcal{L}^{-1}\{f(s)\} = F(t)$

then $\mathcal{L}^{-1}\{f(s-a)\} = e^{at} F(t)$

Ex:

$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

$\mathcal{L}^{-1} \frac{a}{s^2+a^2} = \sin at$

(i) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{2} \cdot \frac{2}{s^2+2^2}\right\}$

$= \frac{1}{2} \cdot \sin 2t$

ii) $\mathcal{L}^{-1}\left\{\frac{1}{s^2-2s+5}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s^2-2s+1+4}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+2^2}\right\}$

$= \frac{1}{2} \mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+2^2}\right\}$

we know

$$\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at = F(t).$$

$$\& \ \mathcal{L}^{-1}\{f(s-a)\} = e^{at} F(t)$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{1}{s^2-2s+5}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+2^2}\right\}$$

$$= \frac{1}{2}\cdot e^{t}\sin 2t$$

(Ans)

Second Translation:

If $\mathcal{L}^{-1}\{f(s)\} = F(t)$,

then $\mathcal{L}^{-1}\{e^{-as}f(s)\} = \begin{cases} F(t-a) & t>a \\ 0 & t<a \end{cases} = G(t)$

~~Proof~~:

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s/3}}{s^2+1}\right\} = ?$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{e^{-\pi s/3}}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{e^{-\frac{\pi}{3}\cdot s}\cdot\frac{1}{s^2+1}\right\}$$

$a = \pi/3$

$f(s) = \frac{1}{s^2+1} = \mathcal{L}\{\sin t\}$

$$= \begin{cases} F(t-\pi/3) & t > \pi/3 \\ 0 & t < \pi/3 \end{cases}$$

Invense laplace Theonem of denivatives

If $\mathcal{L}^{-1}\{f(s)\} = F(t)$

then $\mathcal{L}^{-1}\{f^n(s)\} = (-1)^n t^n F(t)$

Q. Invense laplace transfonmation of $\frac{s}{(s^2+1)^2}$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$$

We know, $\mathcal{L}^{-1}\left\{f(s) = \frac{1}{s^2+1}\right\} = \sin t = F(t)$

$$\frac{d}{ds}\left(\frac{1}{s^2+1}\right) = \frac{(s^2+1)\cdot 0 - 1\cdot(2s+0)}{(s^2+1)^2}$$

$$= \frac{-2s}{(s^2+1)^2}$$

Now, $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{-2s}{(s^2+1)^2}\right\}\left(-\frac{1}{2}\right)$$

$\mathcal{L}\{\sin t\}$

$$= t\,\frac{1}{2}\left\{\mathcal{L}^{-1}\frac{d}{ds}\left(\frac{\cancel{s}\,1}{(s^2+1)^4}\right)\right\} = t\,\frac{1}{2}\left\{\mathcal{L}^{-1}\left\{(-1)^1\frac{d}{ds}\frac{1}{s^2+1}\right\}\right.$$

$$= \frac{1}{2}\left\{\mathcal{L}^{-1}\left\{\mathcal{L}\left\{t^1 \cdot \sin t\right\}\right.\right.$$

$$= \frac{t\sin t}{2}$$

$$= -\frac{1}{2}\mathcal{L}^{-1}\{f(s)\}$$

$$= -\frac{1}{2}(-1)^1 t^1 \sin t$$

$$= \frac{1}{2}t\sin t$$

Inverse laplace transformation of Integrals:

if $\mathcal{L}^{-1}\{f(s)\} = F(t)$

the $\mathcal{L}^{-1}\{\int_0^\infty f(u)\,du = \frac{f(t)}{t}$

$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$= t - e^{-t}$$