$$A=\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

$$A=\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 0 & -1 & 2 & 12 & 16 & -5 \\ 0 & -1 & 2 & 12 & 16 & -5 \\ 0 & -1 & 2 & 12 & 16 & -5 \end{bmatrix} \quad \begin{bmatrix} r_2' = r_1\times 3 + r_2 \\ r_3' = r_1 \times 2 + r_3 \\ r_4' = r_1 \times 4 + r_4 \end{bmatrix}$$

$$A=\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 0 & -1 & 2 & 12 & 16 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} r_3' = r_2 - r_3 \\ r_4' = r_2 - r_4 \end{bmatrix}$$

$$A=\begin{bmatrix} 1 & -2 & 0 & -4 & -5 & 3 \\ 0 & 1 & -2 & -12 & -16 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} r_1' = r_1(-1) \\ r_2' = r_2(-1) \end{bmatrix}$$

$$A=\begin{bmatrix} \boxed{1} & 0 & -4 & -28 & -37 & 13 \\ 0 & \boxed{1} & -2 & -12 & -16 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} r_1' = \\ r_1 + r_2 \times 2 \end{bmatrix}$$

Since this matrix has two leading 1's, so it has rank 2.

In order to find the nullity we have to find the solution space of the system:

$$AX = 0$$

the system is:

$$\boxed{x_1} - 4x_3 - 28x_4 - 37x_5 + 13x_6 = 0$$

$$\boxed{x_2} - 2x_3 - 12x_4 - 16x_5 + 5x_6 = 0$$

leading variable. x_1, x_2

free variable x_3, x_4, x_5, x_6

let,

$x_3 = t_1$

$x_4 = t_2$

$x_5 = t_3$

$x_6 = t_4$

$x_1 = 4x_3 + 28x_4 + 37x_5 - 13x_6$

$= 4t_1 + 28t_2 + 37t_3 - 13t_4$

$x_2 = 2t_1 + 12t_2 + 16t_3 - 5t_4$

sol^n:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4t_1 + 28t_2 + 37t_3 - 13t_4 \\ 2t_1 + 12t_2 + 16t_3 - 5t_4 \\ t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} = t_1 \underbrace{\begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{v_1} + t_2 \underbrace{\begin{bmatrix} 28 \\ 12 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{v_2} + t_3 \underbrace{\begin{bmatrix} 37 \\ 16 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{v_3} + t_4 \underbrace{\begin{bmatrix} -13 \\ -5 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{v_4}$$

$\{v_1, v_2, v_3, v_4\} \to R^6$

Here, the vectors v_1, v_2, v_3 & v_4 form the basis for the null space. so it has nullity 4.

$$k_1 v_1 + k_2 v_2 + k_3 v_3 + \cdots + k_n v_n = 0$$

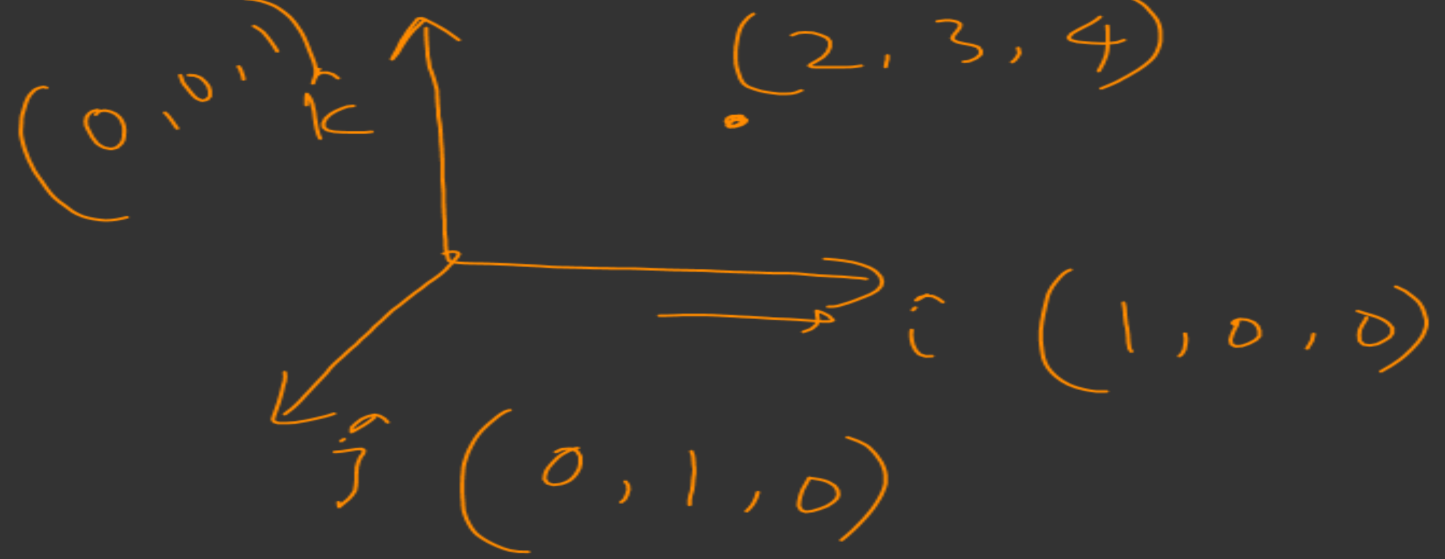
R^4

R^2

$$\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right.$$

$2\hat{i} + 3\hat{j} + 4\hat{k}$

$(2, 3, 4)$

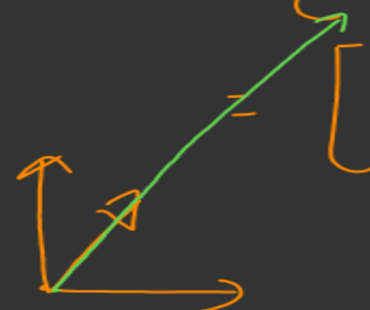


Eigenvalues and eigenvectors.

$$\overbrace{\underbrace{A}_{\text{Matrix}}\underbrace{x}_{\text{vector}}}^{\text{vector}} = \overbrace{\underbrace{\lambda}_{\text{scalar}}\underbrace{x}_{\text{vector}}}^{\text{vector}}$$

$$V = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$



$$AV = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 \\ 8-2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$AV = \underbrace{3}_{\text{eigen-value}}V$$

$Ax = \lambda x$

$\Rightarrow Ax = \lambda I x$

$\Rightarrow (A - \lambda I)x = 0$

solvable iff $\boxed{\det(A - \lambda I) = 0}$

theorem: If A is an $n \times n$ matrix, then λ is an eigen-value of A if and only if it satisfies the eqn:

$$\det(\lambda I - A) = \det(A - \lambda I) = 0$$

this eqn is also called the characteristic eqn of A.

and $P(\lambda) = \det(A - \lambda I)$ is the characteristic polynomial.

Find the eigenvalues of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

Sol: Let, the eigen value is λ.

characteristic eqn is: $\det(A - \lambda I) = 0$ —— ①

Here, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

$\lambda I = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$\Rightarrow ① \Rightarrow \det(A - \lambda I) = 0$

$\Rightarrow \det \begin{pmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{pmatrix} = 0$

$\Rightarrow -\lambda(-8\lambda + \lambda^2 + 17) - 1(-4) = 0$

$\Rightarrow 8\lambda^2 - \lambda^3 - 17\lambda + 4 = 0$

$\Rightarrow \lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$

Solving this we get,

$\lambda_1 = 4$

$\lambda_2 = 2 + \sqrt{3}$, $\lambda_3 = 2 - \sqrt{3}$