$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx.$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx.$$

(a) $f(x)$ is defined in the interval $-l<x<l$.

(b) $f(x)$ and $f'(x)$ are sectional continuous in $-l<x<l$.

(c) $f(x+2l) = f(x)$; $f(x)$ is periodic with period $2l$.

Complex form of Fourier series:

$$f(x) = \sum_{n=-\infty}^{\infty} \boxed{c_n} e^{\left(\frac{in\pi x}{l}\right)} \quad \text{where, } c_n = \frac{1}{2l}\int_{-l}^{l} f(x)\, e^{\frac{-in\pi x}{l}}\, dx.$$

$$e^{ix} = \boxed{\cos x + i\sin x}$$

$$= \frac{e^x \sin nx}{n} - \frac{1}{n}\left[-e^x \frac{\cos nx}{n} + \int e^x \frac{\cos nx}{n} dx\right]$$

$$I = \frac{e^x \sin nx}{n} + \frac{1}{n^2} e^x \cos nx - \frac{1}{n^2} I$$

$$\Rightarrow I\left(\frac{1+n^2}{n^2}\right) = \frac{1}{n^2}\left[n e^x \sin nx + e^x \cos nx\right]$$

$$\Rightarrow I = \frac{e^x(\cos nx + n \sin nx)}{(1+n^2)}$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-\pi}(\cos n\pi + 0)\right]$$

$$= \frac{1}{\pi(1+n^2)}\left[(e^{\pi} - e^{-\pi})(-1)^n\right]$$

$$= \frac{(-1)^n(e^{\pi} - e^{-\pi})}{\pi(1+n^2)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \, dx = \frac{1}{\pi}\left[e^x\right]_{-\pi}^{\pi} = \left(\frac{e^{\pi} - e^{-\pi}}{\pi}\right)$$

Now; $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin \frac{n\pi x}{\pi} dx$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin nx \, dx$$

$$b_n = \frac{(-1)^{n+1}}{\pi(1+n^2)} n\left(e^{\pi} - e^{-\pi}\right)$$

$$\therefore f(x) = \frac{e^{\pi} - e^{-\pi}}{2\pi} + \sum_{n=1}^{\infty} \frac{(-1)^n}{\pi(1+n^2)}\left(e^{\pi} - e^{-\pi}\right) \cos nx + \frac{(-1)^{n+1}}{\pi(1+n^2)} n\left(e^{\pi} - e^{-\pi}\right) \sin nx$$

Ans

$F(-x) = -F(x) \rightarrow$ odd. $\rightarrow x, \ldots$

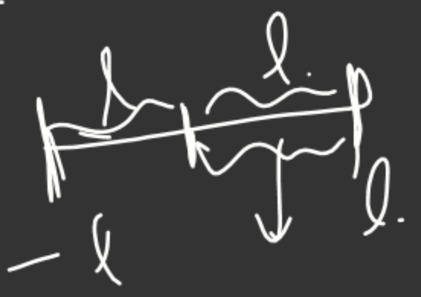
$\cos(-18°)$

$$\underbrace{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{l}\right.}\left.+ b_n \sin\frac{n\pi x}{l}\right]$$

$2l$

<u>even:</u> $b_n = 0$; $\quad F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$

<u>odd:</u> $\quad a_n = 0$; $\quad F(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$



$0 \le x \le l$

$l \geq 0$

Half Range Sine and cosine Series:

<u>Half Range Sine:</u> $\quad a_n = a_0 = 0 \quad ; \quad b_n = \frac{1 \cdot 2}{l}\int_0^l F(x)\sin\frac{n\pi x}{l}\,dx$

$$= \frac{2}{l}\int_0^l F(x)\sin\frac{n\pi x}{l}\,dx$$

<u>Half Range cos:</u> $\quad b_n = 0$

$$a_n = \frac{2}{l}\int_0^l F(x)\cos\frac{n\pi x}{l}\,dx$$

Inverse ... $n=1$

- - - - cosine - - $f_c(n) = \int_0^l F(x) \cos \frac{n\pi x}{l} \, dx$.

Inverse - - - $F(x) = \frac{1}{l} f_c(0) + \frac{2}{l} \sum_{n=1}^{\infty} f_c(n) \cos \frac{n\pi x}{l}$.

Infinite F. Sine: $f_s(n) = \int_0^\infty F(x) \sin nx \, dx$. $0 < x < \infty$.

Inverse: $F(x) = \frac{2}{\pi} \int_0^\infty f_s(n) \sin nx \, dn$

Infinite F. cos: $f_c(n) = \int_0^\infty F(x) \cos nx \, dx$.

Inverse: - - - $F(x) = \frac{2}{\pi} \int_0^\infty f_c(n) \cos nx \, dx$.

Find the Fourier Cosine Transform of $\sin kx$ $0 < x < \pi$;

$$f_c(n) = \int_0^{\pi} \sin kx \cos \frac{n\pi x}{\pi} dx = \int_0^{\pi} \sin kx \cos nx \, dx.$$

Ans: $\frac{k}{n^2 - k^2}\left[(-1)^n \cos k\pi - 1\right]$