**QUIZ-01 (Time: 20 minutes): Marks-20**

**Math-217 (Advanced Mathematics)**

**IOT AND ROBOTICS ENGINEERING DEPARTMENT**

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1) **Find the eigenvalues and eigenvectors of $A \; and \; A^2 \; and \; A^{-1} \; and \; A + 4I$:**

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

**Check the trace $\lambda_1 + \lambda_2 = 4$ and the determinant $\lambda_1 \lambda_2 = 3$. For each case show the eigenvalues and eigenvectors in one graphical representation.**

(1)

(i) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

(ii) $A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix}$

$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

(iii) $A^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

(iv) $A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

Now,
for A, eigen values come from $\det(A - \lambda I) = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)^2 - 1 = 0$

$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$

$\Rightarrow \therefore \lambda = 1, 3$

$\therefore$ The eigen values of A are $\lambda_1 = 1$ & $\lambda_2 = 3$

The trace, $\lambda_1 + \lambda_2 = 4 \Rightarrow 1 + 3 = 4 \Rightarrow 4 = 4$ [shown]

& the determinant, $\lambda_1 \lambda_2 = 3 \Rightarrow 1 \cdot 3 = 3 \Rightarrow 3 = 3$ [shown]

(P.T.O)

The eigen vectors come from $(A-\lambda I)x=0$

for, $\lambda=1$;
$$\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1&-1\\0&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\quad\Big|\; r_2'=r_1+r_2$$

$\therefore$ The system is $x-y=0$

let free variable, $x=t$

$\therefore y=t$

$$\therefore\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}t\\t\end{bmatrix}=t\begin{bmatrix}1\\1\end{bmatrix}$$

$\therefore\begin{bmatrix}1\\1\end{bmatrix}$ is a basis for the eigen space corresponding to solution.

for, $\lambda=3$;
$$\begin{bmatrix}-1&-1\\-1&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1&1\\0&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\quad\Big|\; \begin{matrix}r_1'=r_1\times(-1)\\r_2'=r_1'+r_2\end{matrix}$$

The system is $x+y=0$

let free variable, $x=t$

$\therefore y=-t$

$$\therefore\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}t\\-t\end{bmatrix}=t\begin{bmatrix}1\\-1\end{bmatrix}$$

$\therefore\begin{bmatrix}1\\-1\end{bmatrix}$ is a basis for the eigen space corresponding to solution.

$\therefore$ The eigen vectors, $x_1=\begin{bmatrix}1\\1\end{bmatrix}$, $x_2=\begin{bmatrix}1\\-1\end{bmatrix}$

$A^2$

As the theorem suggests,

If a matrix A has the eigen values and eigen-vectors are $\lambda_1, \lambda_2, \dots \lambda_k$ and $V_1, V_2, \dots V_k$ respectively then the eigen values and eigen vectors of the matrix $A^m$ are $\lambda_1^m, \lambda_2^m, \lambda_3^m \dots \lambda_k^m$ and $V_1, V_2, V_3, \dots V_k$ respectively.

$\therefore$ The eigen vectors for $A^2, A^{-1}$ ~~keep~~ is the same on A which is $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

& their eigen values are,

for $A^2$; $\lambda_1^2 = 1$ & $\lambda_2^2 = 9$

for $A^{-1}$; $\lambda_1^{-1} = 1$ & $\lambda_2^{-1} = \frac{1}{3}$

Now, for, $A+4I$

eigen value comes from ~~$\det(A-\lambda I)=0$~~ $\det(A+4I-\lambda I)=0$

$$\Rightarrow \begin{vmatrix} 2+4-\lambda & -1 \\ -1 & 2+4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (6-\lambda)^2 - 1 = 0$$

$$\Rightarrow \lambda^2 - 12\lambda + 35 = 0$$

$$\therefore \lambda = \cancel{7, 5}\ 5, 7$$

~~$\therefore$ trace, $\lambda_1 + \lambda_2$~~

(P.T.O)

The eigen vectors come from, $(A+4I-\lambda I)\,x = 0$

for, $\lambda_1' = 5$; $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ gives eigen vector $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for, $\lambda_2' = 7$; $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ gives eigen vector, $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Now for each $A^2$, $A^{-1}$ & $(A+4I)$,

for, $A^2$, trace $= 5+5 = 10 = 1+9 = \lambda_1^2 + \lambda_2^2$

determinant $= 9 = 1\cdot 9 = \lambda_1^2 \cdot \lambda_2^2$

for, $A^{-1}$, trace $= \frac{4}{3} = 1 + \frac{1}{3} = \lambda_1^{-1} + \lambda_2^{-1}$

det. $= \frac{1}{3} = 1 \cdot \frac{1}{3} = \lambda_1^{-1} \cdot \lambda_2^{-1}$

for, $(A+4I)$, trace $= 12 = 5+7 = \lambda_1' + \lambda_2'$

det. $= 35 = 5\cdot 7 = \lambda_1' + \lambda_2'$

[shown]

Graphical representation;