MATH 217	Advanced Mathematics	3.00

Prerequisite: MATH 209: Engineering Mathematics

Contact Hour: 2 hours Lectures and 2 hours tutorial/week

Objective:

This course is designed with advanced techniques for solving mathematical problems that are not easily solvable by analytical methods. This course is designed with advanced mathematical techniques including Laplace and Fourier concepts.

Outcome: At the end of this course, learners will be able to

- understand the basic concepts of matrix transformation, properties, and solution techniques.
- understand the intuition of eigen values and eigen vectors and matrix factorization or decomposition.
- depicts the concept of a complex variable, its properties, and its application to solving analytical functions.
- apply the Laplace and Fourier transform and use them for solving the initial and boundary-value problems which are found basically in physics or engineering.

Lecture-01

Linear algebra: Solution of a system of linear equations by using Gaussian elimination, Gauss-Jordan elimination, Inverse of a matrix and LU factorization methods. Some Applications of Linear System.

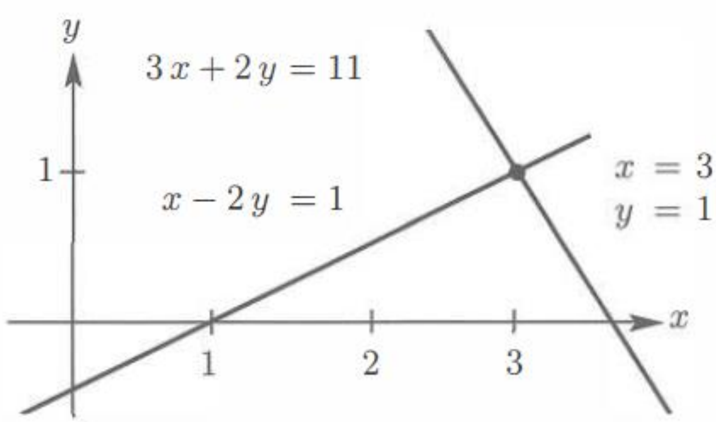

Figure 2.1: *Row picture*: The point $(3, 1)$ where the lines meet solves both equations.

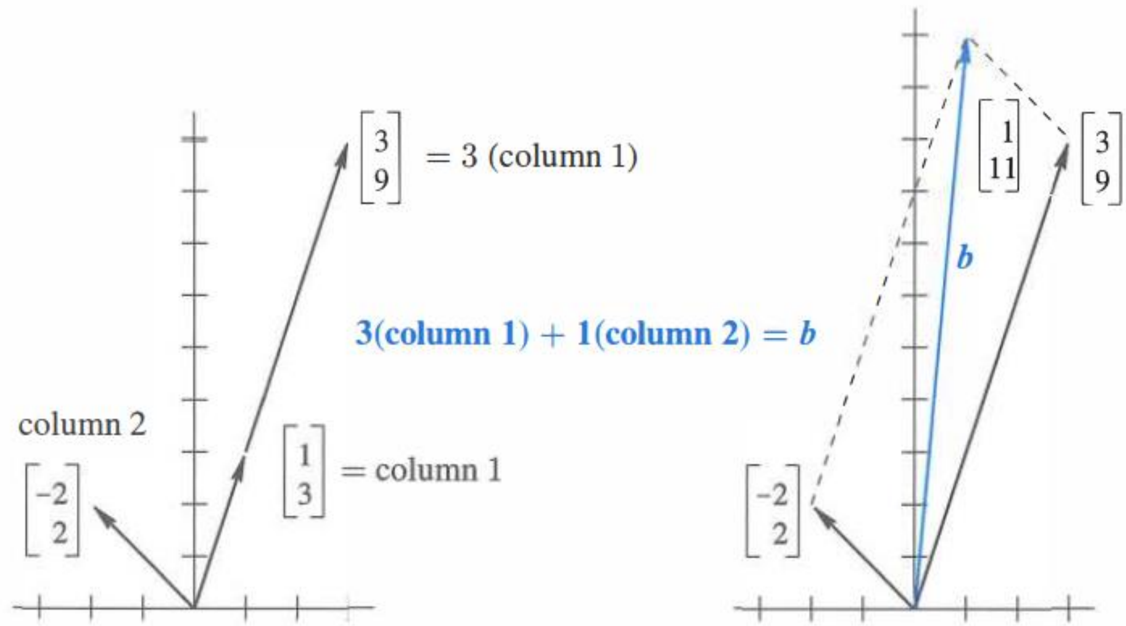

Figure 2.2: *Column picture*: A combination of columns produces the right side (1, 11).

The three unknowns are x, y, z. We have three linear equations:

$$A\boldsymbol{x} = \boldsymbol{b} \qquad \begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 6 \\ 2x & + & 5y & + & 2z & = & 4 \\ 6x & - & 3y & + & z & = & 2 \end{array}$$

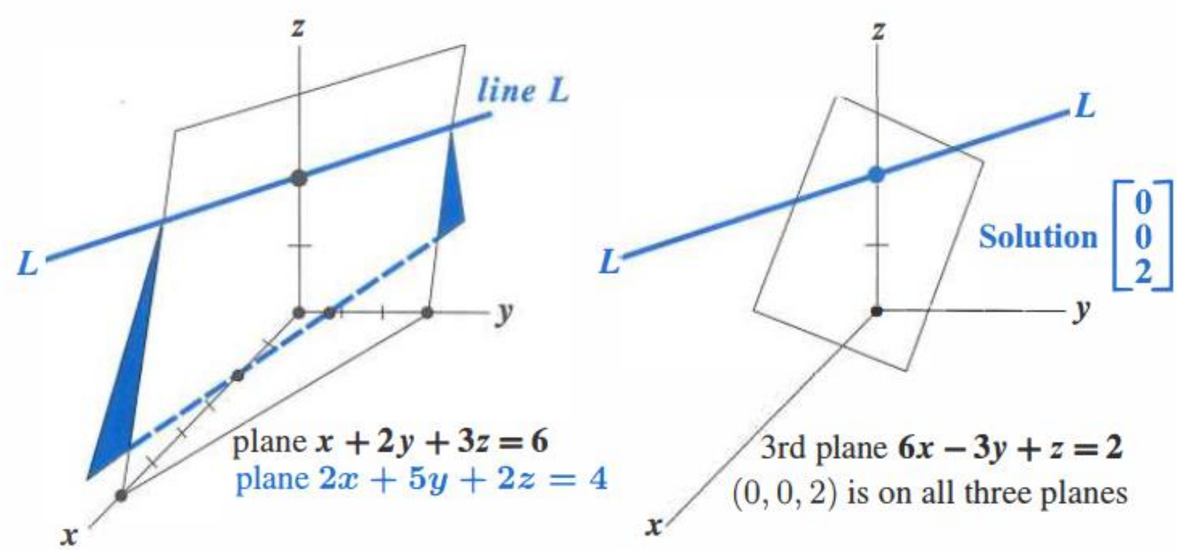

Figure 2.3: *Row picture*: Two planes meet at a line L. Three planes meet at a point.

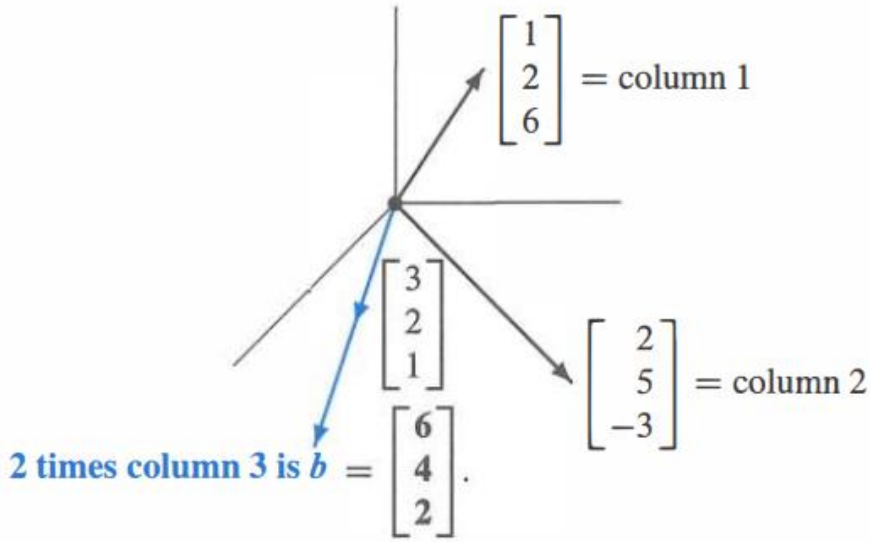

Figure 2.4: *Column picture: Combine the columns with weights* $(x, y, z) = (0, 0, 2)$.

Row Echelon and Reduced Row Echelon form

Solve by Gauss–Jordan elimination.

$$
\begin{aligned}
x_1 + 3x_2 - 2x_3 \qquad\quad + 2x_5 \qquad\quad &= 0 \\
2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\
5x_3 + 10x_4 \qquad\quad + 15x_6 &= 5 \\
2x_1 + 6x_2 \qquad\quad + 8x_4 + 4x_5 + 18x_6 &= 6
\end{aligned}
$$

Final form

$$
\begin{bmatrix}
1 & 3 & 0 & 4 & 2 & 0 & 0 \\
0 & 0 & 1 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The solution is

$x_1 = -3r - 4s - 2t, \quad x_2 = r, \quad x_3 = -2s, \quad x_4 = s, \quad x_5 = t, \quad x_6 = \frac{1}{3}$

Inverse of a matrix

THEOREM 1.4.5 *The matrix*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible if and only if $ad - bc \neq 0$, *in which case the inverse is given by the formula*

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{2}$$

THEOREM 1.4.6 *If A and B are invertible matrices with the same size, then AB is invertible and*

$$(AB)^{-1} = B^{-1}A^{-1}$$

Inversion form $\quad [\boldsymbol{A}|\boldsymbol{I}] \rightarrow [\boldsymbol{I}|\boldsymbol{A}^{-1}]$

Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Solution: $[\boldsymbol{A}|\boldsymbol{I}]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

Final form: $[\boldsymbol{I}|\boldsymbol{A}^{-1}]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

Thus,

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

T1. Colors in print media, on computer monitors, and on television screens are implemented using what are called "color models". For example, in the RGB model, colors are created by mixing percentages of red (R), green (G), and blue (B), and in the YIQ model (used in TV broadcasting), colors are created by mixing percentages of luminescence (Y) with percentages of a chrominance factor (I) and a chrominance factor (Q). The conversion from the RGB model to the YIQ model is accomplished by the matrix equation

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} .299 & .587 & .114 \\ .596 & -.275 & -.321 \\ .212 & -.523 & .311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

What matrix would you use to convert the YIQ model to the RGB model?

LU Factorization

DEFINITION 1 A factorization of a square matrix A as

$$A = LU \tag{1}$$

where L is lower triangular and U is upper triangular, is called an ***LU-decomposition*** (or ***LU-factorization***) of A.

The Method of LU-Decomposition

Step 1. Rewrite the system $A\mathbf{x} = \mathbf{b}$ as

$$LU\mathbf{x} = \mathbf{b} \tag{2}$$

Step 2. Define a new $n \times 1$ matrix $\mathbf{y}$ by

$$U\mathbf{x} = \mathbf{y} \tag{3}$$

Step 3. Use (3) to rewrite (2) as $L\mathbf{y} = \mathbf{b}$ and solve this system for $\mathbf{y}$.

Step 4. Substitute $\mathbf{y}$ in (3) and solve for $\mathbf{x}$.

$$A = LU = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Solve the following system of equations using *LU* factorization

$$\begin{bmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$

Solution

$x_1 = -1, x_2 = 1, x_3 = 0$

Some Application questions on the Matrix Linear system:

1.

Find a cubic polynomial whose graph passes through the points

$$(1, 3), \quad (2, -2), \quad (3, -5), \quad (4, 0)$$

2.

Table 1

Input Required per Dollar Output

Provider		Manufacturing	Agriculture	Utilities
	Manufacturing	\$ 0.50	\$ 0.10	\$ 0.10
	Agriculture	\$ 0.20	\$ 0.50	\$ 0.30
	Utilities	\$ 0.10	\$ 0.30	\$ 0.40

Composition matrix is

$$C = \begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{bmatrix}$$

$$\underbrace{\mathbf{x}}_{\boxed{\text{Amount produced}}} - \underbrace{C\mathbf{x}}_{\boxed{\text{Intermediate demand}}} = \underbrace{\mathbf{d}}_{\boxed{\text{Outside demand}}}$$

which we will find convenient to rewrite as

$$(I - C)\mathbf{x} = \mathbf{d}$$

Consider the economy described in Table 1. Suppose that the open sector has a demand for \$7900 worth of manufacturing products, \$3950 worth of agricultural products, and \$1975 worth of utilities.

(a) Can the economy meet this demand?

(b) If so, find a production vector $\mathbf{x}$ that will meet it exactly.