Chapter - 1
Configuration Space

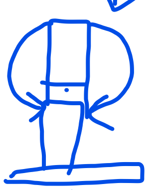
1. Introduction to Configuration Space

1.1 What is Configuration Space (C-Space)?

In robotics, the **Configuration Space (C-Space)** is a mathematical representation of all possible positions and orientations a robot can take. It abstracts away the physical dimensions of the robot and represents its state using a set of **configuration variables**.
=(degree of freedom" or "DOF"

- **Definition**:
  =(degree of freedom" or "DOF"
  C-Space is an **n-dimensional space** where each point represents a possible configuration (pose) of the robot.
- **Mathematical Representation**:
  If a robot has n **degrees of freedom (DOF)**, then its configuration can be expressed as a vector:

R^n



qi=joint parameter

$$q = (q_1, q_2, \ldots, q_n) \in \mathbb{R}^n$$

configuration space

where each qi represents a joint parameter (e.g., an angle or displacement). R^n represents the configuration space.

DoF : 1
#pos : ∞(0°–360°)
'Revolt joint'
q(Angle)

1.2 Configuration Space vs. Workspace

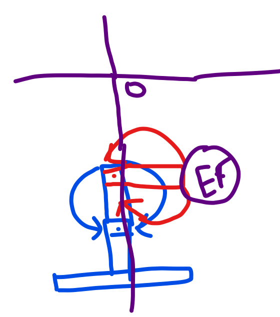
- fixed dimension
  Workspace (Task Space): The physical space in which the robot operates. It is represented in **Cartesian coordinates** (x, y, z).
- **Configuration Space**: The space of all possible robot configurations, represented using **joint parameters**.
  flexible dimension

Example: A Simple Robotic Arm

Consider a **2-link planar manipulator** with two **revolute joints**:

- **Workspace Representation**:
  The end-effector moves in a 2D plane, so its position is given by **(x, y)**.
- **Configuration Space Representation**:
  The two joint angles, q1 and q2, define the arm's posture:

q=(q1,q2)

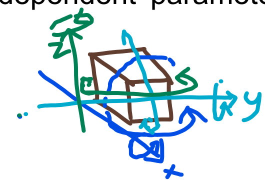


This means the C-Space is **2D**, where each axis represents a joint angle.

DOF = number of independent ways a system can move. MAX 6

1.3 Degrees of Freedom (DOF) and Configuration Variables

The **Degrees of Freedom (DOF)** define the number of independent parameters needed to specify the robot's configuration.



- **A free-moving object in 3D space has 6 DOF**:
  - 3 for position (**x, y, z**)
  - 3 for orientation (**roll, pitch, yaw**)
- **Examples**:
  - A **prismatic joint** (sliding) contributes **1 DOF** (linear displacement).
  - A **revolute joint** (rotating) contributes **1 DOF** (rotation angle).

Example: DOF of Different Robots

Robot Type	DOF	Configuration Variables
1-Link Arm	1	q1 (angle)
2-Link Arm	2	(q1, q2)
3D Drone	6	(x, y, z, θx, θy, θz)
also a 2 link arm with only revolute joint on the same direction	1	q1(angle), q2(angle)

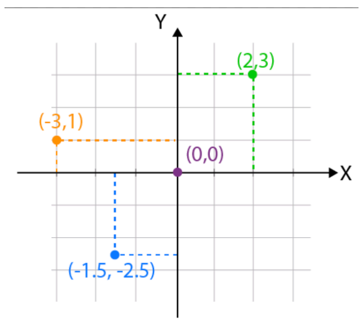
1.4 Representation of Configuration Space

1. **Discrete vs. Continuous C-Space**:
   - Discrete C-Space: A grid-based representation.
   - Continuous C-Space: Uses real numbers for smooth motion.
2. **Dimensionality of C-Space**:
   - A **single-jointed robot** has a **1D** C-Space.
   - A **2-joint planar robot** has a **2D** C-Space.
   - A **6-DOF robot arm** has a **6D** C-Space.

2. Types of Coordinate Systems in Robotics

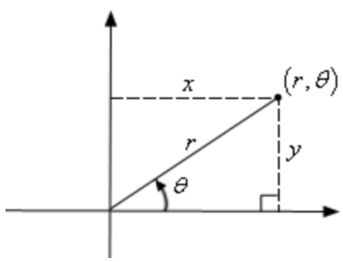
A robot's position and orientation can be represented in different **coordinate systems**. The three most common ones are:

2.1 Cartesian (Rectangular) Coordinate System



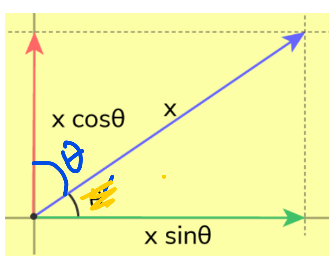
- Uses **(x, y, z)** to define a position.
- Represents position in **Euclidean space**.
- Used in **industrial robots, CNC machines, and Cartesian manipulators**.

2.2 Polar Coordinate System



- Uses **(r, θ)** to represent a point in 2D.
- rrr = Distance from the origin.
- θ\thetaθ = Angle with the reference axis.

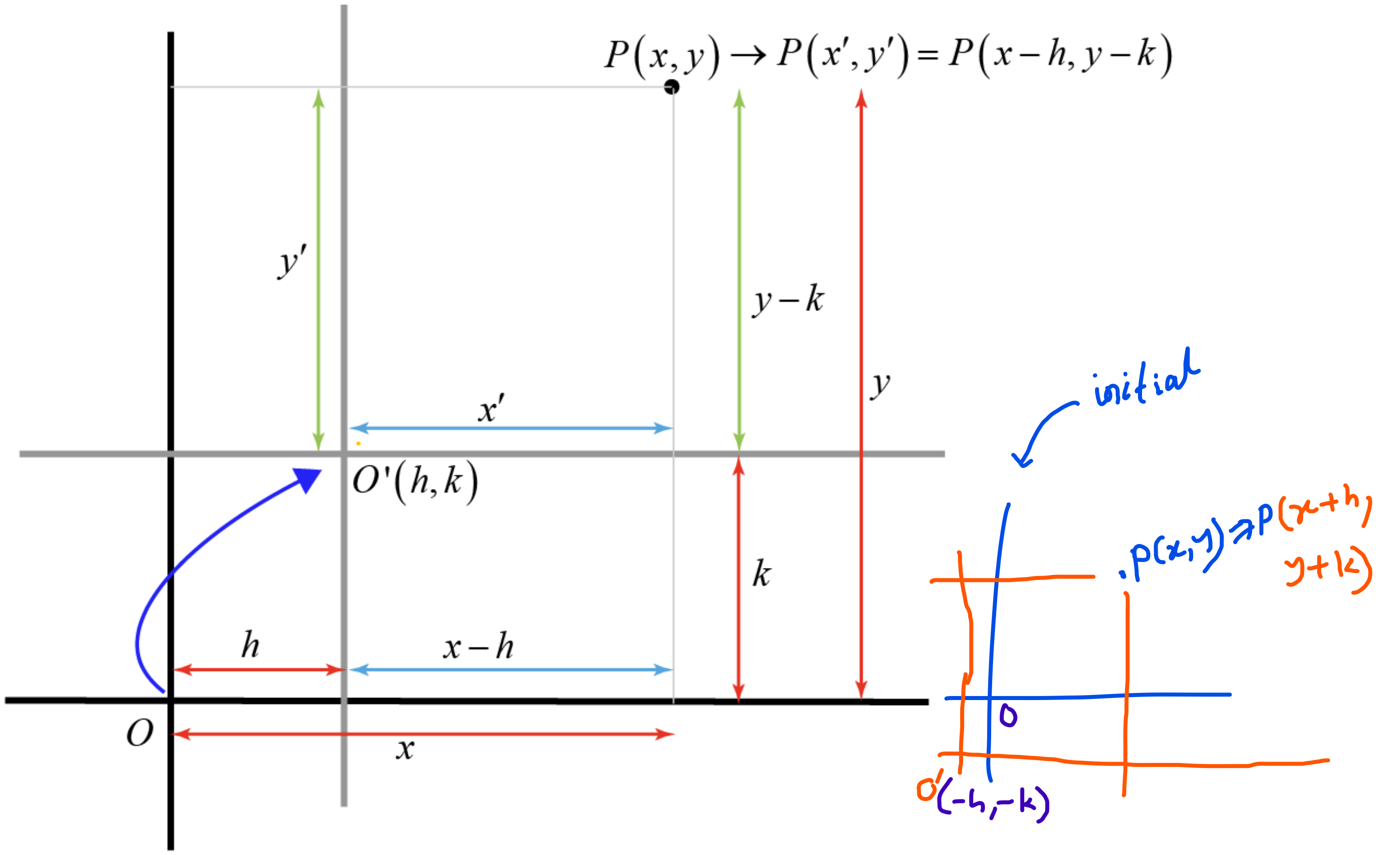
2.3 Vector Space Representation



- Represents robot configurations as **vectors in an n-dimensional space**.
- Each joint or coordinate contributes to a dimension.

2.4 Transformations of Axes

2.4.1 Translation



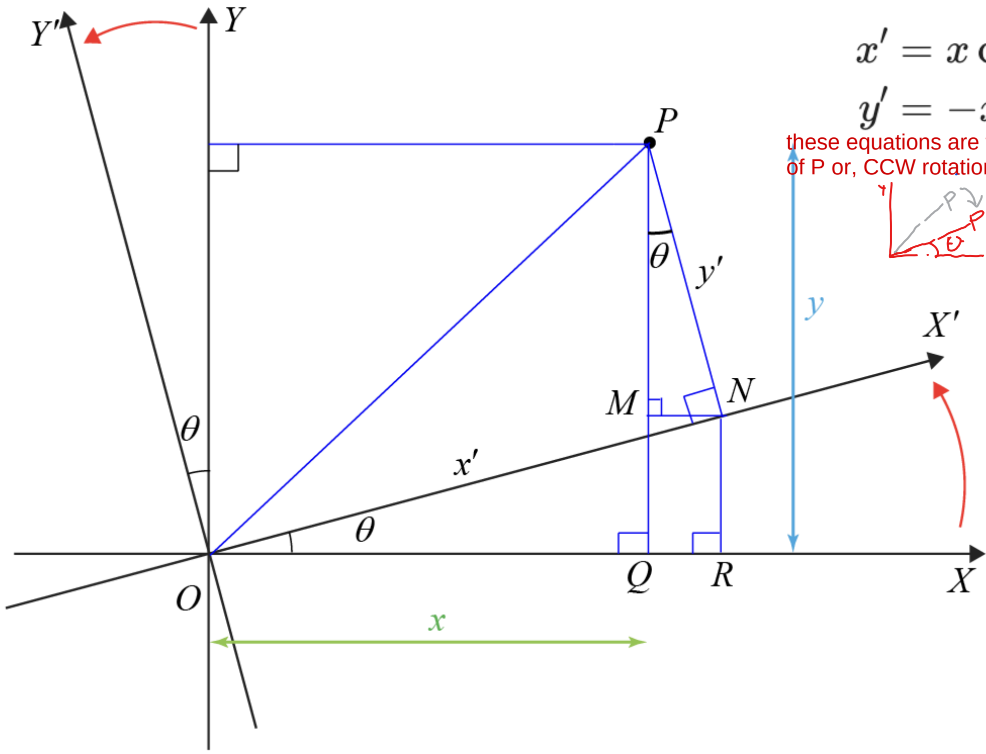
Consider a Cartesian coordinate system with its origin at O. Let P be a point within this system, having coordinates (x,y). Now, if we shift the origin to a new point O′(h,k) without changing the orientation in the original system, we establish a new coordinate system. In this new system, the coordinates of O′ become (0,0), indicating its position as the new origin. As a result of this shift, the coordinates of point P will change to accommodate the new frame of reference. Suppose the new coordinates of P in this shifted system are (x′,y′). Given the translation of the origin and maintaining the orientation of the axes, it follows that the relationship between the old and new coordinates of P can be expressed as x′=x−h and y′=y−k. This transformation effectively recalibrates the coordinate system, considering O′ as the new reference point, thus altering the perceived position of P relative to this new origin.

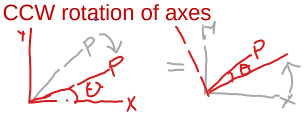
rotate the point (P)
counterclockwise(CCW) by "theta",
x' = xcosA -ysinA
y' = xsinA + ycosA

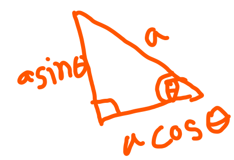
2.4.2 Rotation

$$x' = x\cos(\theta) + y\sin(\theta)$$
$$y' = -x\sin(\theta) + y\cos(\theta)$$

these equations are for clockwise(CW) rotation by "theta"
of P or, CCW rotation of axes







From the construction, line PM is parallel to the OY axis and line PN is parallel to the OY' axis, which implies $\angle MPN = \theta$ since it corresponds to the angle between OY and OY'.

In $\triangle PMN$, the length MN can be determined using y' and θ:

$$MN = y'\sin(\theta)$$

In $\triangle ONR$, which is right-angled at N, OR represents x' projected onto the OX axis:

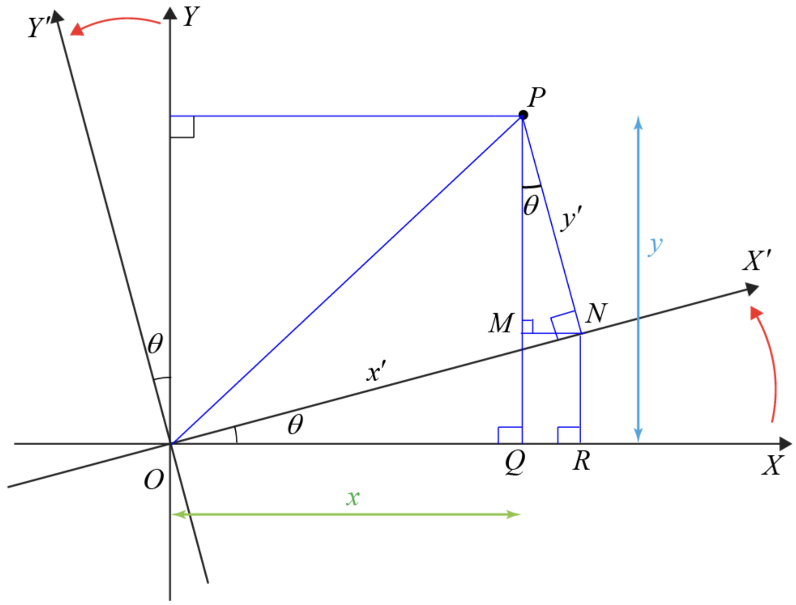
$$OR = x'\cos(\theta)$$

The horizontal distance OQ can be found by subtracting MN from OR:

$$OQ = OR - MN = x'\cos(\theta) - y'\sin(\theta)$$

The vertical component NR in $\triangle ONR$ is the projection of x' on the OY axis:

$$NR = x'\sin(\theta)$$



For PQ, which is the vertical distance from the original OX axis to point P, we consider the vertical distances PM and MQ:

$$PM = y' \cos(\theta)$$

$$MQ = NR = x' \sin(\theta)$$

Summing these gives the y coordinate in the original system:

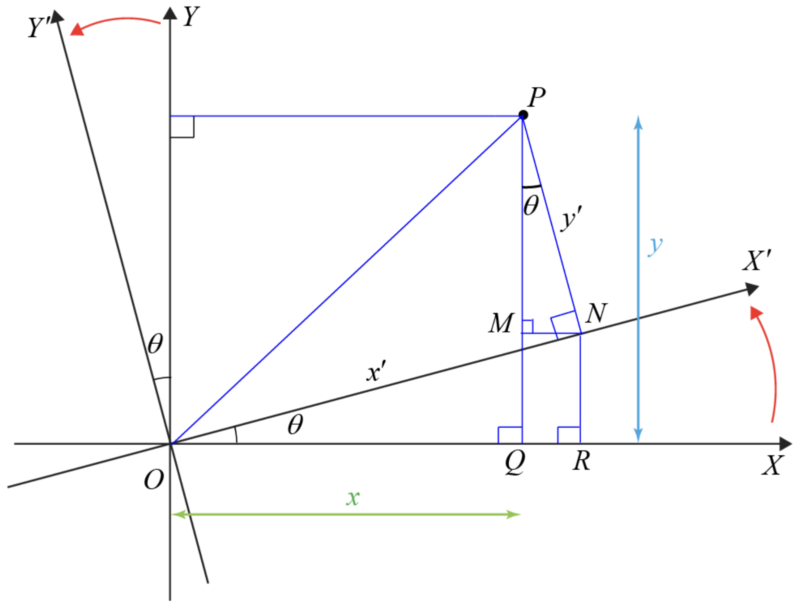
$$PQ = PM + MQ = y' \cos(\theta) + x' \sin(\theta)$$

Thus, the new coordinates (x, y) of point P after the rotation by θ in terms of the rotated coordinates (x', y') are:

$$x = x' \cos(\theta) - y' \sin(\theta)$$
$$y = x' \sin(\theta) + y' \cos(\theta)$$

Starting with the equations for the coordinates (x, y) after an anticlockwise rotation by θ in terms of the new coordinates (x', y'):

$$x = x' \cos(\theta) - y' \sin(\theta) \quad (1)$$
$$y = x' \sin(\theta) + y' \cos(\theta) \quad (2)$$



To express x' in terms of x and y, we multiply equation (1) by $\cos(\theta)$ and equation (2) by $\sin(\theta)$, then add the results:

$$x\cos(\theta) = x'\cos^2(\theta) - y'\sin(\theta)\cos(\theta) \quad (3)$$

$$y\sin(\theta) = x'\sin^2(\theta) + y'\sin(\theta)\cos(\theta) \quad (4)$$

Adding equations (3) and (4):

$$x\cos(\theta) + y\sin(\theta) = x'(\cos^2(\theta) + \sin^2(\theta))$$

$$x' = x\cos(\theta) + y\sin(\theta)$$

For y', we multiply equation (1) by $\sin(\theta)$ and equation (2) by $\cos(\theta)$, then subtract equation (1) from equation (2):

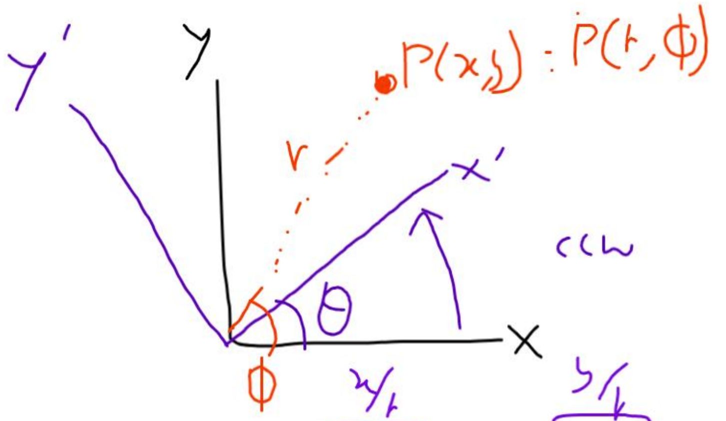
$$x\sin(\theta) = x'\sin(\theta)\cos(\theta) - y'\sin^2(\theta) \quad (5)$$

$$y\cos(\theta) = x'\sin(\theta)\cos(\theta) + y'\cos^2(\theta) \quad (6)$$

Now, subtract equation (5) from equation (6):

$$y\cos(\theta) - x\sin(\theta) = y'(\cos^2(\theta) + \sin^2(\theta))$$

$$y' = (y\cos(\theta) - x\sin(\theta))$$



point P has been rotated clockwise
but the axis has been rotated
counterclockwise

$$x' = r\cos(\phi-\theta) = r(\overbrace{\cos\phi}^{x/r}\cos\theta + \overbrace{\sin\phi}^{y/r}\sin\theta) = x\cos\theta + y\sin\theta$$

$$y' = r\sin(\phi-\theta) = r(\underbrace{\sin\phi}_{y/r}\cos\theta - \underbrace{\cos\phi}_{x/r}\sin\theta) = -x\sin\theta + y\cos\theta$$