Part 1:

MATLAB Program:

```
num=[1]; den=[0.01, 1];
X=tf(num,den);
step(X)
grid
title('System Response To Unit Step')
xlabel('t Sec')
ylabel('y(t) V')
```

1. $V_R + V_C = x(t)$ $\quad Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = x(t)$ $\quad Ri(t) + \frac{1}{CD}i(t) = x(t)$

$RDi(t) + \frac{i(t)}{C} = Dx(t)$ $\quad \left(RD + \frac{1}{C}\right)i(t) = Dx(t)$ $\quad i(t) = CDy(t)$

$\left(RD + \frac{1}{C}\right)\left(CDy(t)\right) = Dx(t)$ $\quad (RCD^2 + D)y(t) = Dx(t)$ $\quad (RCD + 1)y(t) = x(t)$

2. $RC\frac{dy(t)}{dt} + y(t) = x(t)$ $\quad RC\left(sY(s) - y(0)\right) + Y(s) = X(s)$ $\quad$ Let $y(0)$ be 0.

$RCsY(s) + Y(s) = X(s)$ $\quad (RCs + 1)Y(s) = X(s)$ $\quad H(s) = \frac{Y(s)}{X(s)} = \frac{1}{RCs+1}$

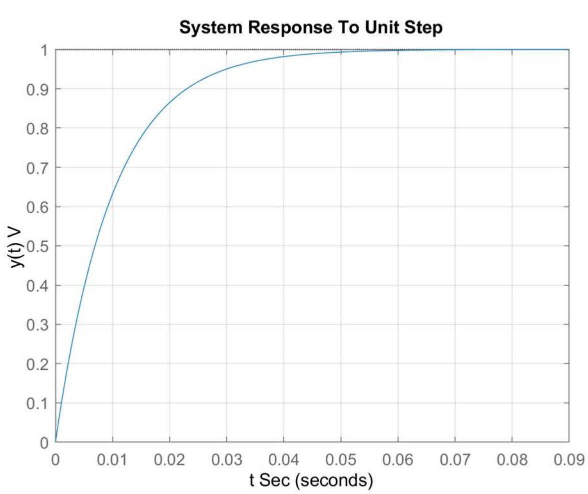
3. $\left((10000)(1 * 10^{-6})s + 1\right)Y(s) = \frac{1}{s}$ $\quad (0.01s + 1)Y(s) = \frac{1}{s}$

$Y(s) = \frac{1}{s(0.01s+1)} = \frac{A}{s} + \frac{B}{0.01s+1}$ $\quad A = \frac{1}{s(0.01s+1)}|_{s=0} = \frac{1}{0+1} = 1$

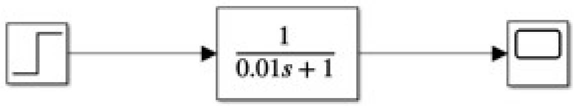
$$B = \frac{1}{s(0.01s+1)}|_{s=-100} = -\frac{1}{100} = -0.01$$

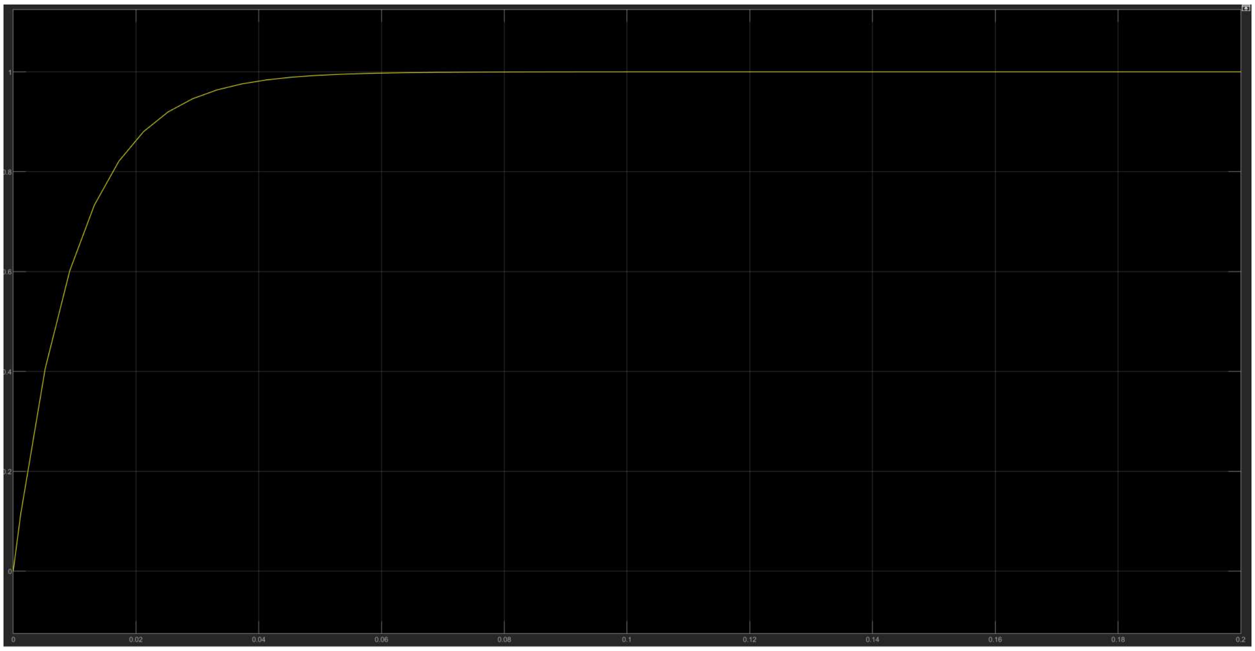
$$Y(s) = \frac{1}{s} - \frac{0.01}{0.01s+1} = \frac{1}{s} - \frac{1}{s+100}$$

$y(t) = u(t) - e^{-100s}$



4.





Part 2:

MATLAB Program:

```
L=10e-3; C=1e-6; R=[50 10 1 0.5 0];

for i=1:5
    den=[L*C, R(i)*C, 1];
    roots(den)
    r=roots(den);
    figure(1)
    plot(r, 'o'); hold on;
end
xlim([-3000, 500])
ylim([-12000, 12000])
xlabel('Real')
ylabel('Imaginary')
grid

figure(2)
R=50;   %Toggle between all R values
num=[1]; den=[L*C R*C 1];

G=tf(num,den);
step(G);
grid
```

1. $V_L + V_R + V_C = x(t)$ $\quad L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = x(t)$ $\quad LDi(t) + Ri(t) + \frac{1}{CD}i(t) = x(t)$

$LD^2i(t) + RDi(t) + \frac{i(t)}{C} = Dx(t)$ $\quad \left(LD^2 + RD + \frac{1}{C}\right)i(t) = Dx(t)$ $\quad i(t) = CDy(t)$

$\left(LD^2 + RD + \frac{1}{C}\right)\left(CDy(t)\right) = Dx(t)$ $\quad (LCD^3 + RCD^2 + D)y(t) = Dx(t)$

$(LCD^2 + RCD + 1)y(t) = x(t)$ $\quad LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = x(t)$

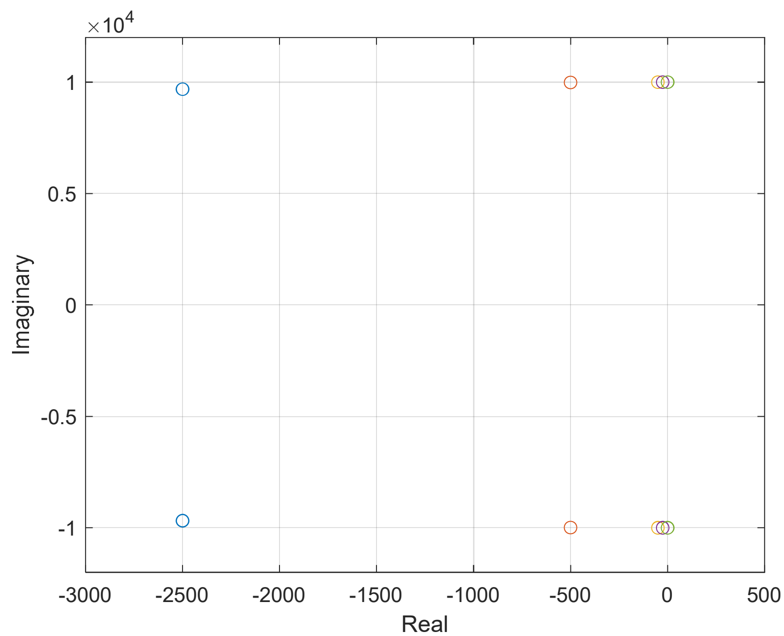
$LC\left(s^2Y(s) - sy(0) - y'(0)\right) + RC\left(sY(s) - y(0)\right) + Y(s) = X(s)$ Let $y(0)$ and $y'(0)$ be 0.

$LCs^2Y(S) + RCsY(s) + Y(s) = X(s)$ $\quad (LCs^2 + RCs + 1)Y(s) = X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{LC\ ^2+RCs+} = \frac{1}{((10*10^{-3})(10^{-6}))+(10^{-6})Rs+1} = \frac{1}{10^{-8}s^2+R*10^{-6}s+1}$

2.

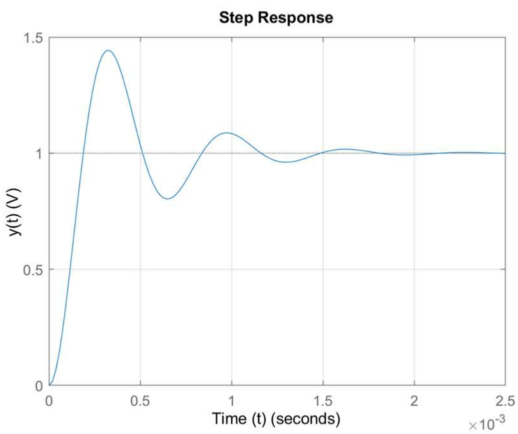
R Value (Ω)	Roots
50	$1000(-2.5 \pm j9.6825)$
10	$1000(-0.5 \pm j9.9875)$
1	$1000(-0.05 \pm j9.9991)$
0.5	$1000(-0.025 \pm j10.0000)$
0	$\pm j10000$



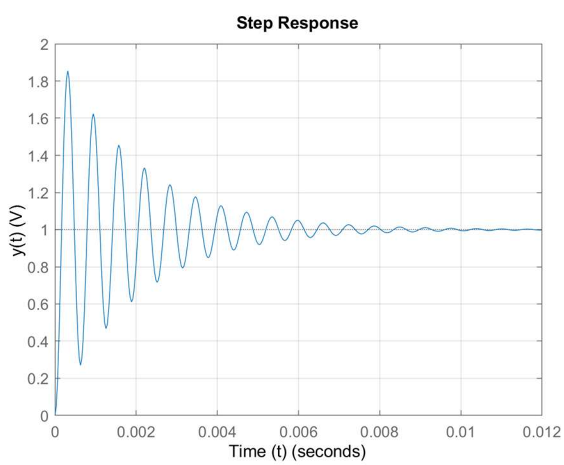
Decreasing the resistance decreases system stability.

3.

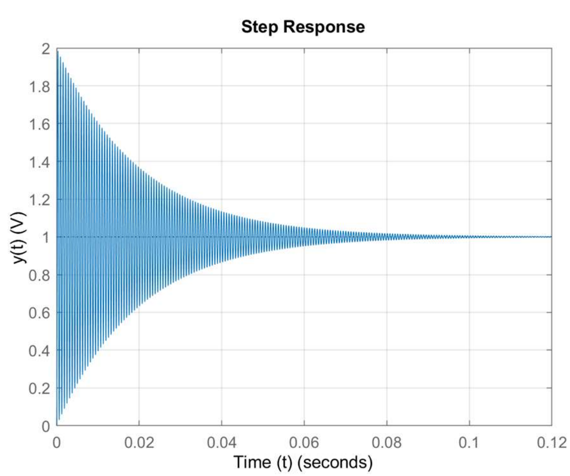
$R = 50\ \Omega$:



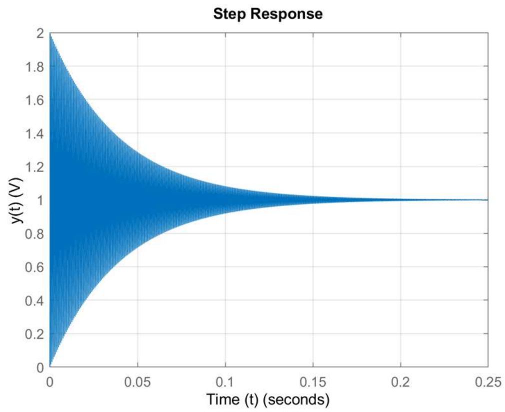
$R = 10\ \Omega$:



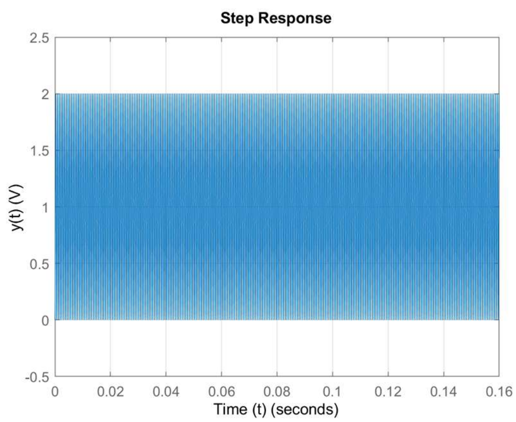
$R = 1\ \Omega$:



$R = 0.5\ \Omega$:



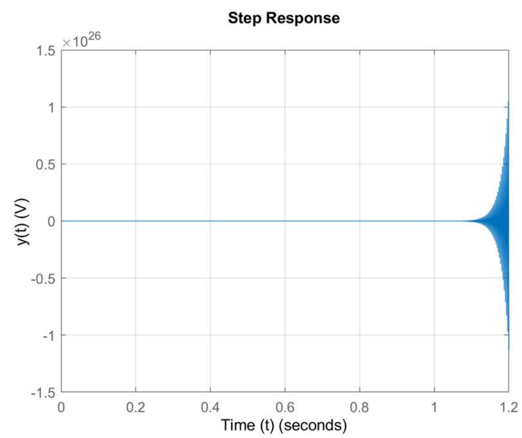
$R = 0\ \Omega$:



4. The further away from the complex axis (imaginary part = 0), the faster the signal dampens. When the poles are on the complex axis, the output oscillates permanently.

5.

Poles: $1000(0.05 \pm 9.9999i)$



The system becomes unstable with negative resistance, with the magnitude of oscillation increasing infinitely.