

Part 1:

```

1- w=[0:0.1:500]; %Define range
2- H=1./(1+i*0.01*w); %Dot before division allows us to apply a vector.
3- H_mag=abs(H);
4- H_angle=angle(H);
5-
6- figure(1)
7- plot(w,H_mag)
8- grid
9- title('Magnitude')
10- xlabel('w')
11- ylabel('|H(jw)|')
12-
13- figure(2)
14- plot(w,H_angle*180/pi) %Convert to degrees when plotting.
15- grid
16- title('Angle')
17- xlabel('w')
18- ylabel('<H(jw)')

```

$$1. \quad V_R + V_C = x(t) \quad Ri(t) + \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = x(t) \quad Ri(t) + \frac{1}{CD} i(t) = x(t)$$

$$RD i(t) + \frac{i(t)}{C} = Dx(t) \quad \left(RD + \frac{1}{C}\right) i(t) = Dx(t) \quad i(t) = CDy(t)$$

$$\left(RD + \frac{1}{C}\right) (CDy(t)) = Dx(t) \quad (RCD^2 + D)y(t) = Dx(t) \quad (RCD + 1)y(t) = x(t)$$

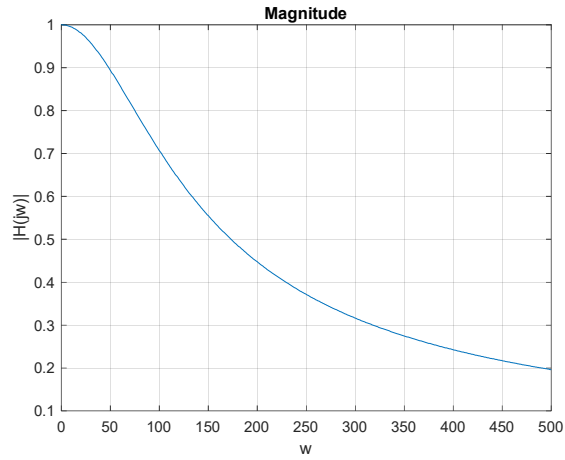
$$RC \frac{dy(t)}{dt} + y(t) = x(t) \quad RC(sY(s) - y(0)) + Y(s) = X(s) \quad \text{Let } y(0) \text{ be } 0.$$

$$RCsY(s) + Y(s) = X(s) \quad (RCs + 1)Y(s) = X(s) \quad H(s) = \frac{Y(s)}{X(s)} = \frac{1}{RCs + 1}$$

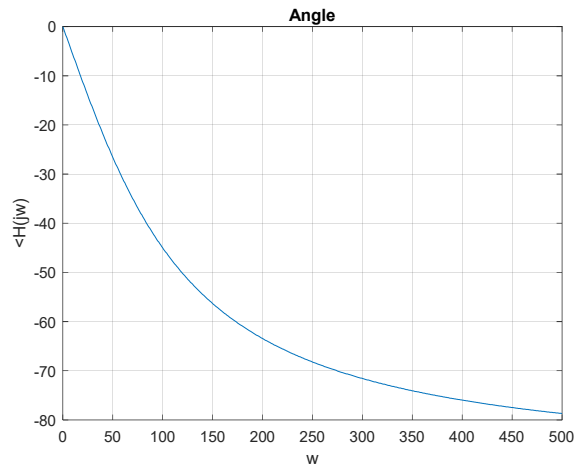
$$H(j\omega) = \frac{1}{jRC\omega + 1} = \frac{1}{j(10000)(10^{-6})\omega + 1} = \frac{1}{j0.01\omega + 1} \quad |H(j\omega)| = \frac{100}{\sqrt{\omega^2 + 10000}}$$

$$\angle H(j\omega) = -\tan^{-1}(0.01\omega)$$

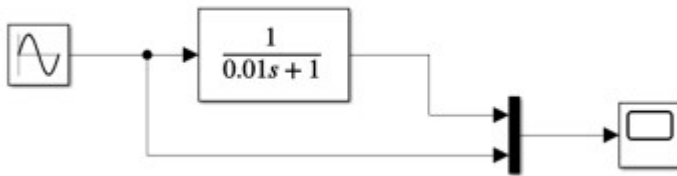
Magnitude:



Angle:



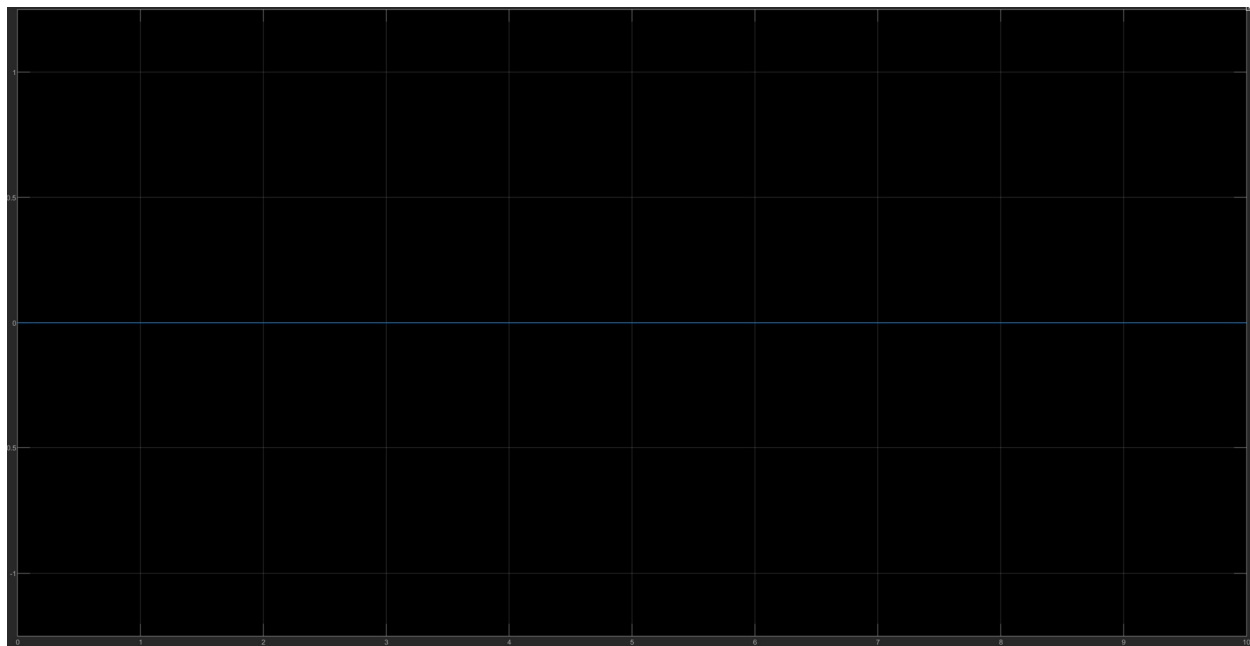
2.



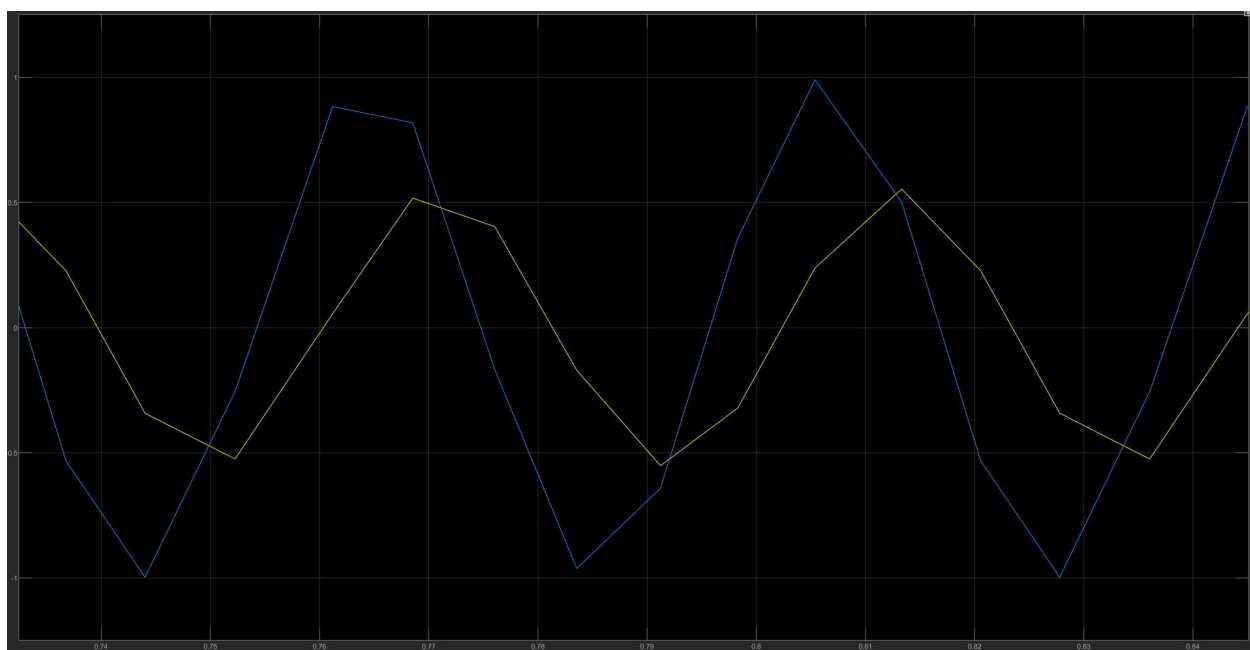
Input Frequency (ω rad/s)	$ H(j\omega) $ (V)	$\angle H(j\omega)$ (Degrees)
0	0	0
150	0.5518	68.7549
300	0.3042	85.9437
400	0.2357	91.6732
500	0.1907	114.592

3.

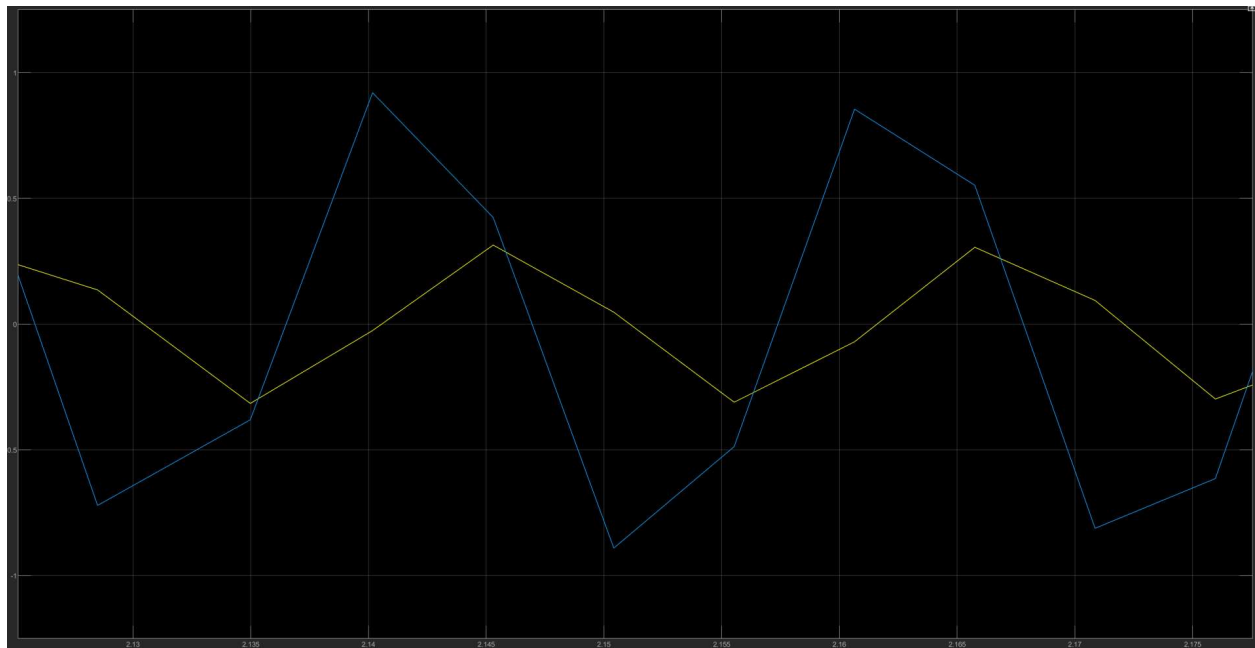
$\omega = 0$:



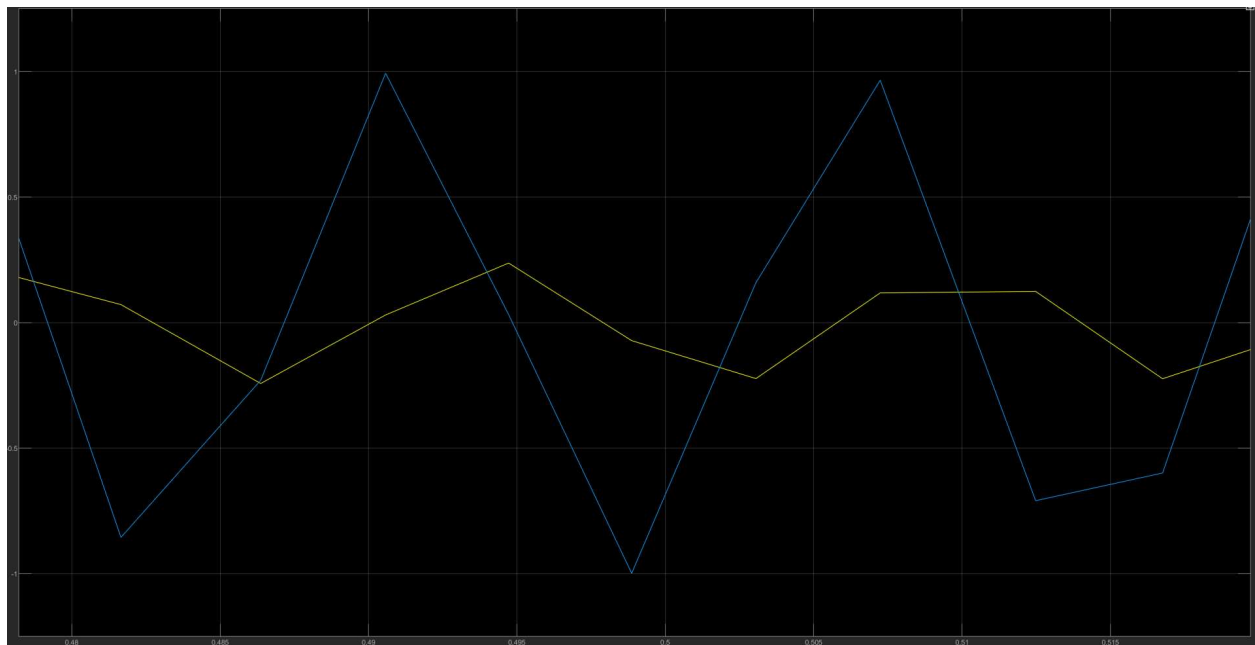
$\omega = 150$:



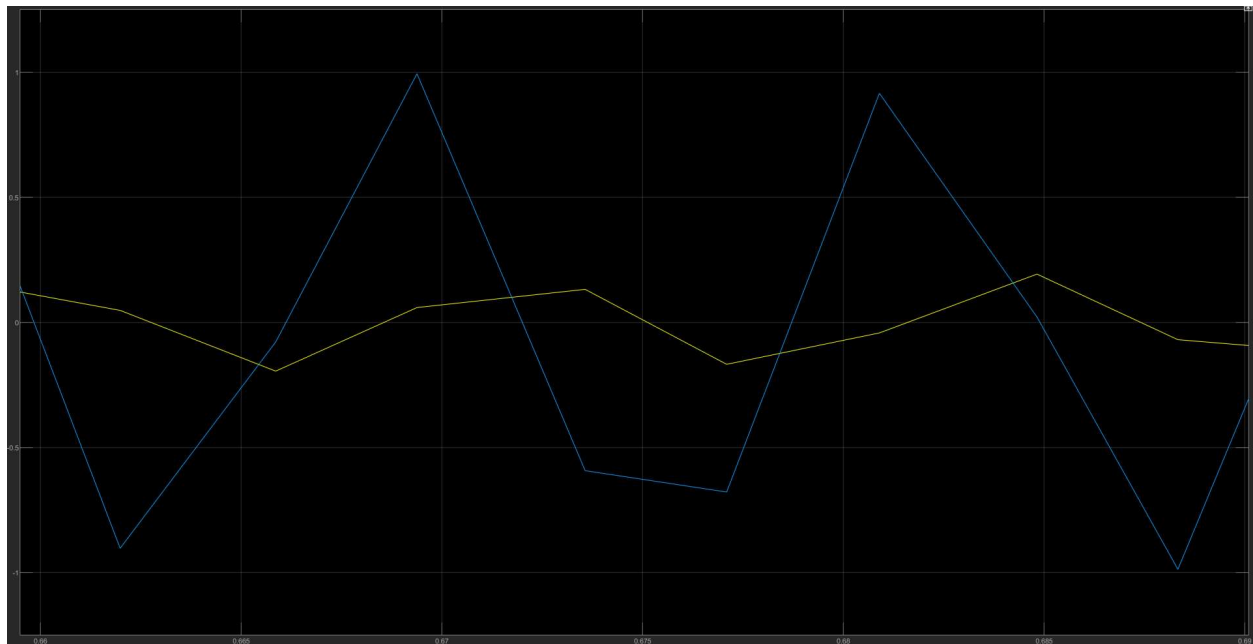
$\omega = 300$:



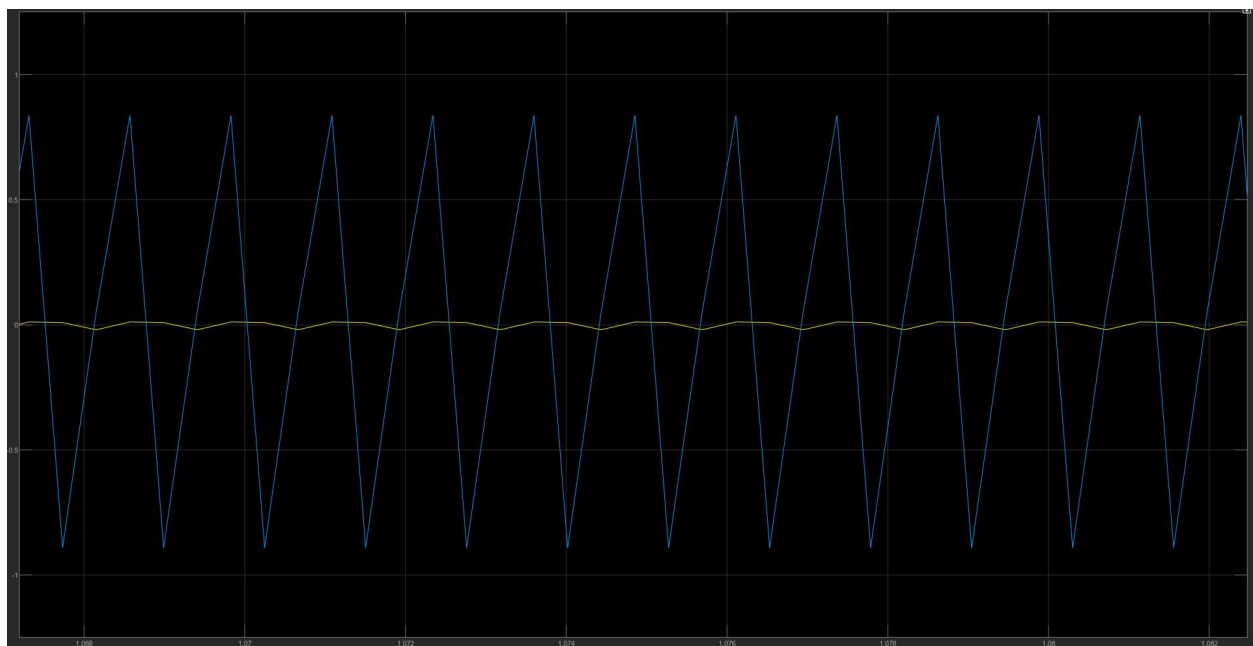
$\omega = 400$:



$\omega = 500$:



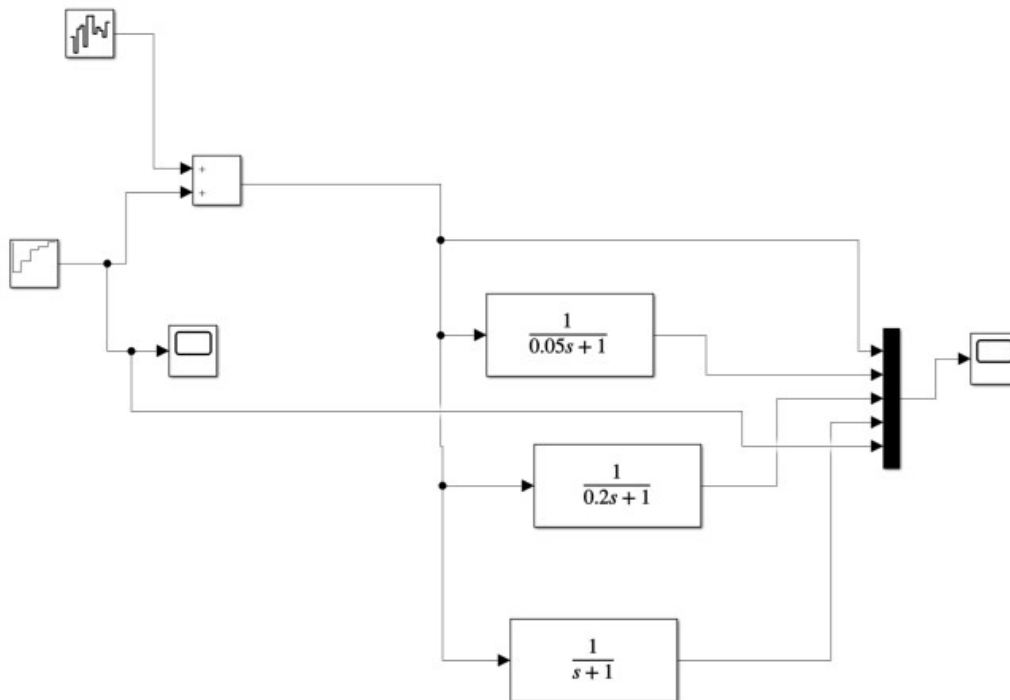
4.



At high frequency, the input signal is much larger in magnitude than the output signal. As such, the system appears to block such input, thus being a low-pass filter due to only having measurable output with lower input frequencies.

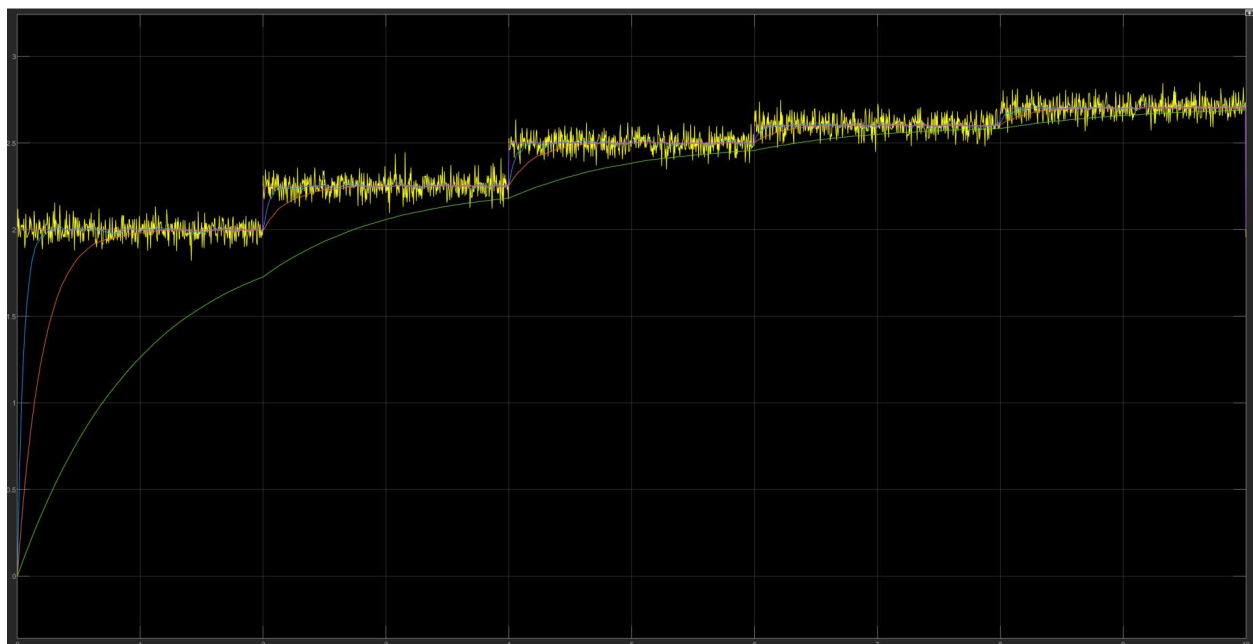
Part 2:

a.

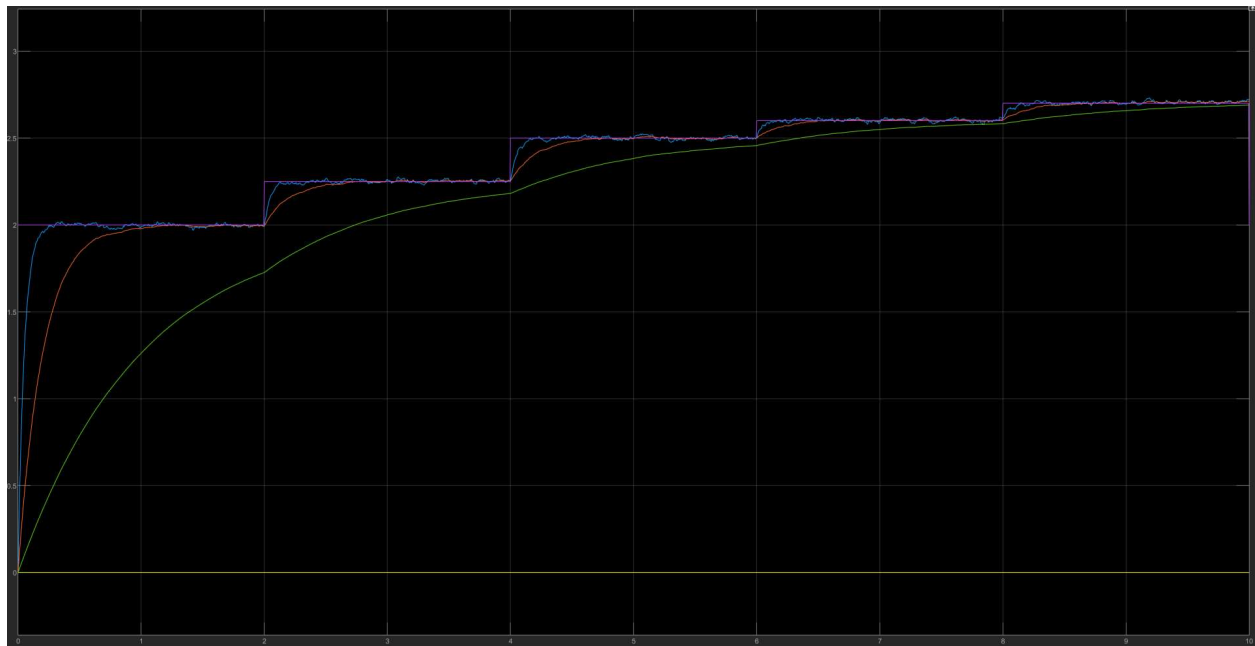


b.

With $n(t)$:



Without $n(t)$:



c. Despite the residual noise, the model with $\tau = 0.05$ (Blue line) captures the actual signal $x(t)$ the most accurately. This is because the other two models transition values too late to accurately resemble the intended value of $x(t)$ each time it changes values.