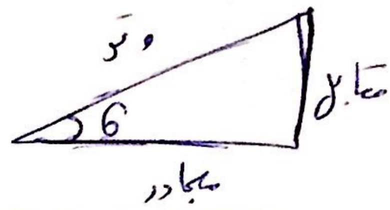
※ Kinematic Analysis ※ 3 D.O.F

$\boxed{1}$

$$\boxed{\sin\theta = \frac{\text{مقابل}}{\text{وتر}}}$$

$$\boxed{\cos\theta = \frac{\text{مجاور}}{\text{وتر}}}$$

$$\boxed{\tan\theta = \frac{\text{مقابل}}{\text{وتر}}}$$



فيثاغورس :- $\boxed{a^2 + b^2 = h^2}$

قانون الجمع الكامل :- $\boxed{(a+b)^2 = a^2 + b^2 + 2ab}$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

∴ متطابقات الزوايا لمجموع زاويتين :-

$$\left\{\begin{array}{l} \sin(A+B) = \sin A \cos B + \cos A \sin B \\ \cos(A+B) = \cos A \cos B - \sin A \sin B \end{array}\right\}$$

※ Forward Kinematic ※

المعطيات

θ_1

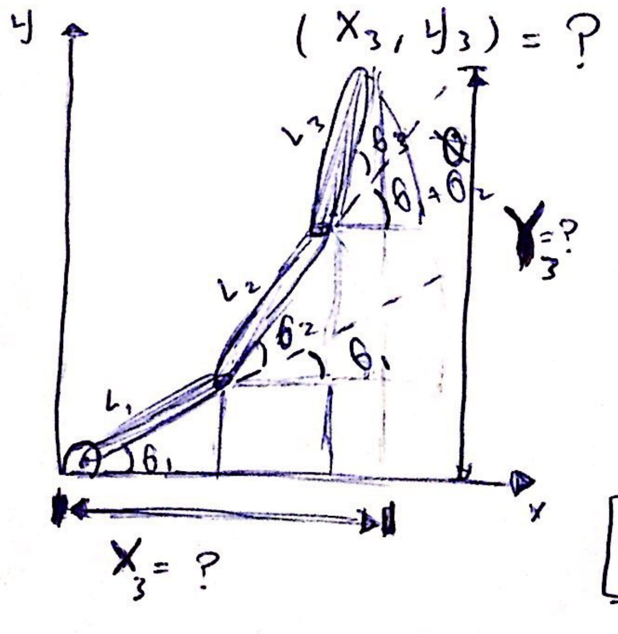
θ_2

θ_3

L_1

L_2

L_3



المجاهيل

X_3

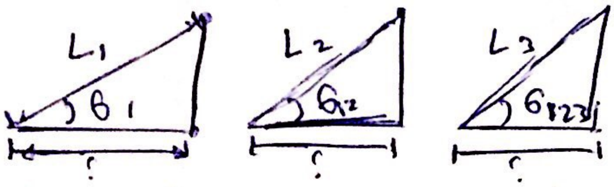
Y_3

$\emptyset$

$$\boxed{\emptyset = \theta_1 + \theta_2 + \theta_3}$$

※ Kinematic Analysis ※ 3 DOF [2]

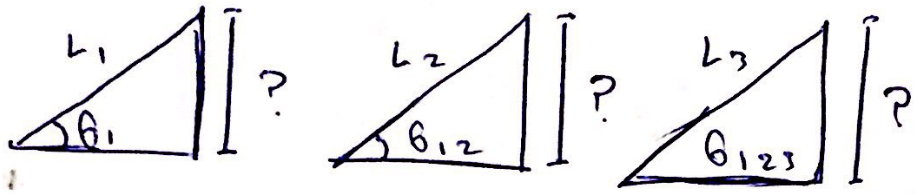
※ [1]



$\cos\theta = \dfrac{\text{مجاور}}{\text{وتر}}$

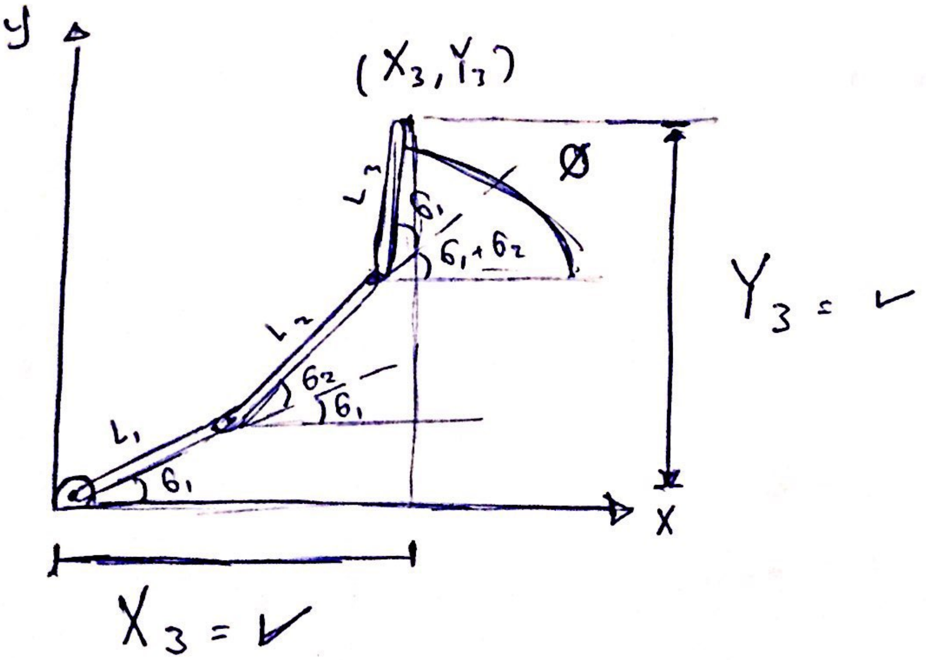
$$\left[X_3 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3) \right]$$

※ [2]



$\sin\theta = \dfrac{\text{مقابل}}{\text{وتر}}$

$$\left[Y_3 = L_1 \sin\theta_1 + L_1 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3) \right]$$



$$\therefore \boxed{\phi = \theta_1 + \theta_2 + \theta_3}$$

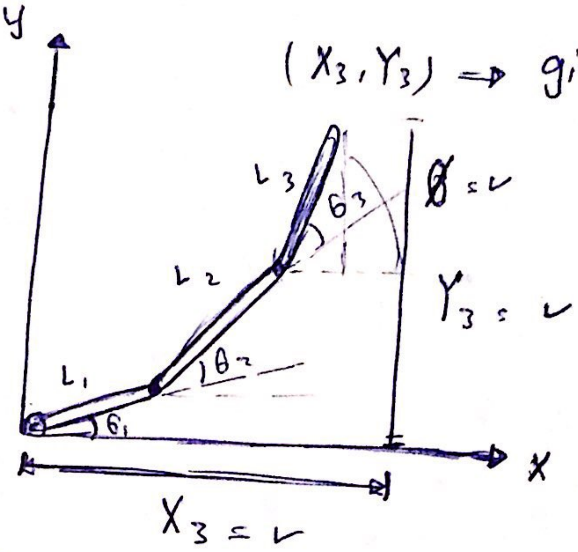
※ Kinematic Analysis ※ 3 DOF [3]

※ Inverse Kinematic ※

المعطى

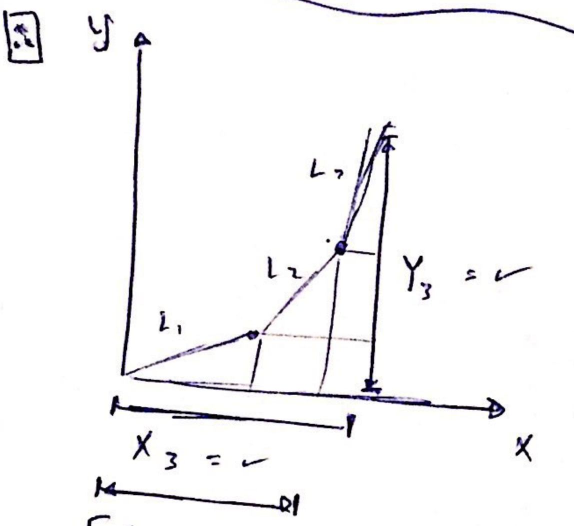
X_3
Y_3
ϕ
L_1
L_2
L_3

$(X_3, Y_3) \Rightarrow$ given



المجاهيل

θ_1
θ_2
θ_3



$$[\,y_2 = Y_3 - L_3 \sin(\theta_1 + \theta_2 + \theta_3)\,]$$

$$[\,X_2 = X_3 - L_3 \cos(\theta_1 + \theta_2 + \theta_3)\,]$$

∴ بعد أن عرفنا قيمة $[y_2, X_2]$ سوف نهمل $[L_3]$ ونوجد $[\theta_2]$
مثل السابق في 2DOF .

∴ After we know the value of $[X_2, y_2]$ we will neglect $[L_3]$ and Find $[\theta_2]$ like the previous in 2 DOF.

※ Kinematic Analysis ※ 3 DOF

$$\cos\theta_2 = \frac{X_2^2 + Y_2^2 - L_1^2 - L_2^2}{2 L_1 L_2} \quad , \quad \sin\theta_2 = \pm\sqrt{1 - \cos^2\theta_2}$$

$$\theta_2 = \cos^{-1}\left(\frac{X_2^2 + Y_2^2 - L_1^2 - L_2^2}{2 L_1 L_2}\right) \quad \left[\begin{array}{c}\text{two}\\ \text{solotions}\end{array}\right]$$

$$\theta_2 = \sin^{-1}(\theta) = \checkmark$$

To Find θ_1 = ?

$$X_2 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$Y_2 = L_1 \sin\theta_1 + L_2 \sin\theta(\theta_1 + \theta_2)$$

∴ بإستخدام متطابقات الزاوية لمجموع زاويتين :-

$$X_2 = L_1 \cos\theta_1 + L_2(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)$$

$$Y_2 = L_1 \sin\theta_1 + L_2(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2)$$

∴ عامل مشترك :-

$$\left\{\begin{array}{l} X_2 = \cos\theta_1 (L_1 + L_2\cos\theta_2) - (L_2 \sin\theta_2)\sin\theta_1 \\ Y_2 = \sin\theta_1 (L_1 + L_2\cos\theta_2) + (L_2 \sin\theta_2)\cos\theta_1 \end{array}\right\}$$

∴ معادلتين بمجهولين

※ kinematic Analysis ※ 3 DOF

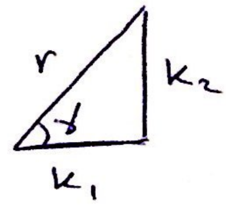
Let say :-

$$k_1 = L_1 + L_2 \cos\theta_2 \quad \text{---- ①}$$

$$k_2 = L_2 \sin\theta_2 \quad \text{---- ②}$$

$$x_2 = \cos\theta_1 (k_1) - (k_2) \sin\theta_1$$

$$y_2 = \sin (k_1) + (k_2) \cos\theta_1$$



$\cos\gamma = \frac{\text{مجاور}}{\text{وتر}}$

$\sin\gamma = \frac{\text{مقابل}}{\text{وتر}}$

$\gamma = \sqrt{k_1 + k_2}$

بالتعويض

$$\frac{X}{r} = \cancel{\frac{r}{r}} \cos\gamma \cos\theta_1 - \cancel{\frac{r}{r}} \sin\gamma \sin\theta_1$$

$$\frac{y}{r} = \cancel{\frac{r}{r}} \cos\gamma \sin\theta_1 + \cancel{\frac{r}{r}} \sin\gamma \cos\theta_1$$

∴ بالقسمة على r

∴

$$\frac{X}{r} = \cos\gamma \cos\theta_1 - \sin\gamma \sin\theta_1$$

$$\frac{y}{r} = \cos\gamma \sin\theta_1 + \sin\gamma \cos\theta_1$$

∴ باستخدام متطابقات الزاوية لمجموع زاويتين :-

$$\frac{X}{r} = \cos(\gamma + \theta_1)$$

$$\frac{y}{r} = \sin(\gamma + \theta_1)$$

$$\left[\tan = \frac{\sin\theta}{\cos\theta} \right]$$

$$\tan(\gamma + \theta_1) = \frac{\frac{y}{\cancel{r}}}{\frac{X}{\cancel{r}}}$$

then :-

$$\gamma + \theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \gamma$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{k_2}{k_1}\right) = \checkmark$$

$\theta_1 = \checkmark \qquad \theta_2 = \checkmark$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

$$\theta_3 = \phi - \theta_1 + \theta_2 = \checkmark$$

∴ هناك حلين لـ θ_2 تعتمد لأخذك قيمة $\pm \sin\theta_2$

$$\theta_2 \begin{cases} + \sin\theta_2 \rightarrow k_2\ (+ve) \rightarrow \gamma' \rightarrow \theta_1' \\ - \sin\theta_2 \rightarrow k_2\ (-ve) \rightarrow \gamma'' \rightarrow \theta_1'' \end{cases}$$