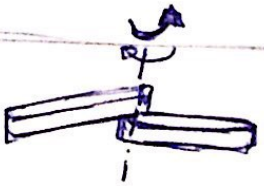
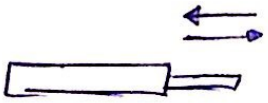
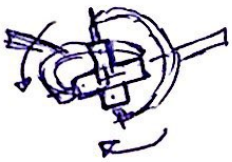
* Kinematic Analysis * 2 D.O.F [1]

[1] How to calculate degrees of freedom?

[Joints]		#of
[A] Revolute		1
[B] Prismatic		1
[C] Universal		2
[D] Spherical		3

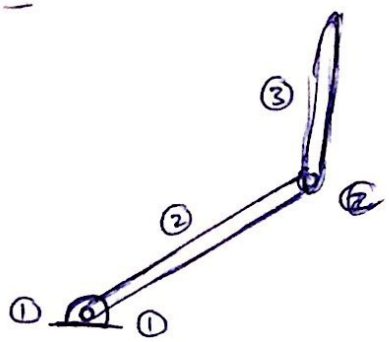
$$d.o.f = m(N-1-J) + J$$

$$\boxed{d.o.f = m(L-1) - 2J} \qquad \boxed{+ H_p}$$

[Dimensions] [Links] [Joints]

✱ Kinematic Analysis ✱ 2 DOF [2]

Example :-

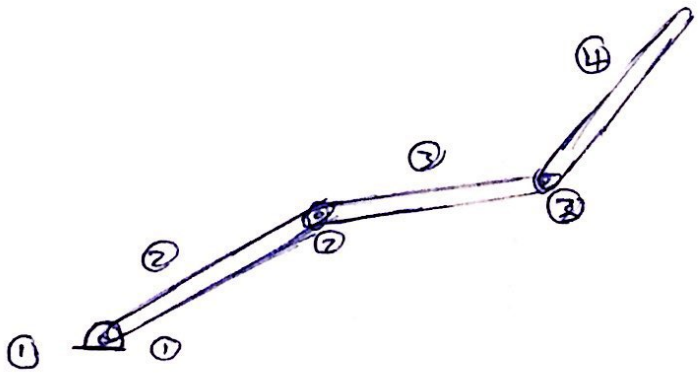


Formula D.O.F = m (L - 1) - 2J

m = 3 , L = 3 , J = 2

D. O. F = 3 (3 - 1) - 2 · (2) = **2 D. O. F**

Example :-



m = 3 , L = 4 , J = 3

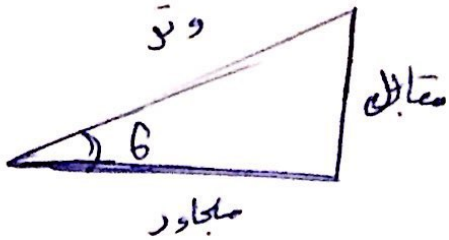
D. O. F = 3 (4 - 1) - 2 (3) = **3 D. O. F**

* Kinematic Analysis * 2 D.O.F

$$\sin\theta = \frac{\text{مقابل}}{\text{وتر}}$$

$$\cos\theta = \frac{\text{مجاور}}{\text{وتر}}$$

$$\tan\theta = \frac{\text{مقابل}}{\text{مجاور}}$$



$$a^2 + b^2 = h^2$$ * فيثاغورس

$$(a \pm b)^2 = a^2 + b^2 + 2ab$$ قانون الحوبع الكامل :-

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* Forward Kinematic *

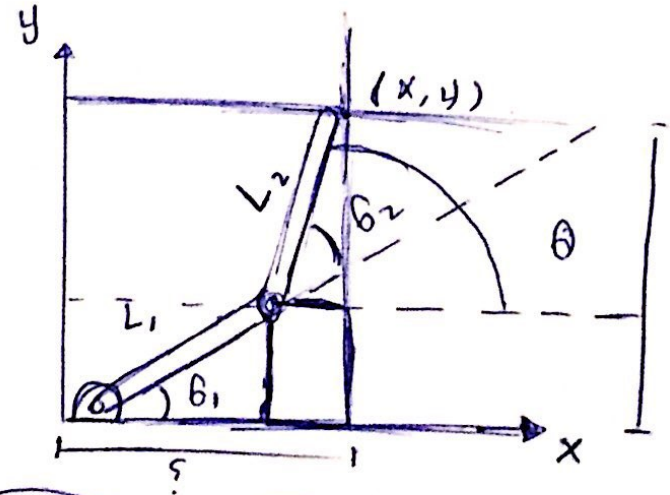
المعطيات

$\emptyset_1$

$\emptyset_2$

L_1

L_2



المطلوب

X

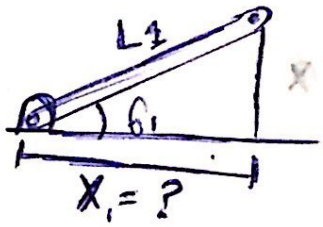
Y

θ

$$\theta = \theta_1 + \theta_2$$

※ Kinematic Analysis ※ 2 D.O.F 4

※ 1

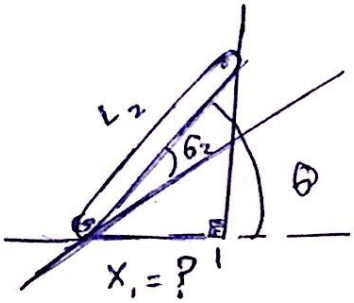


$$\cos\theta = \frac{\text{مجاور}}{\text{وتر}}$$

$$\cos\theta_1 = \frac{X}{L_1}$$

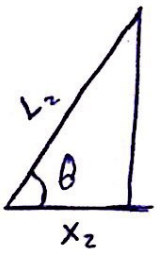
$$X_1 = L_1 \cos\theta_1$$

※ 2



$$\theta = \theta_1 + \theta_2$$

$$\cos\theta = \frac{\text{مجاور}}{\text{وتر}}$$



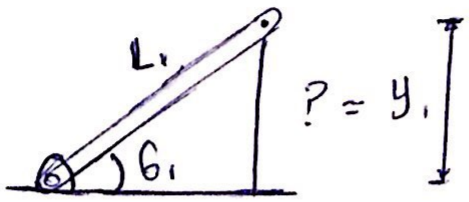
$$\cos\theta = \frac{X_2}{L_2}$$

$$X_2 = L_2 \cos\theta = L_2 \cos(\theta_1 + \theta_2)$$

$$X = X_1 + X_2 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

※ Kinematic Analysis ※ 2 D.O.F [5]

※ [3]

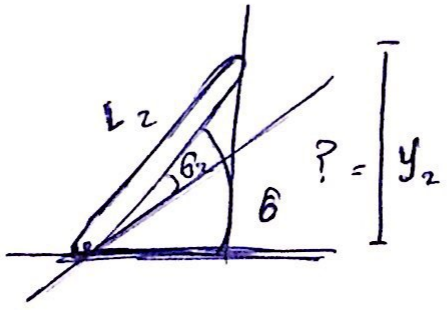


$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin\theta_1 = \frac{y_1}{L_1}$$

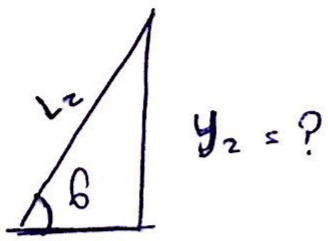
$$y_1 = L_1 \sin\theta_1$$

※ [4]



$$\theta = \theta_1 + \theta_2$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$



$$\sin\theta = \frac{y_2}{L_2}$$

$$y_2 = L_2 \sin\theta = L_2 \sin(\theta_1 + \theta_2)$$

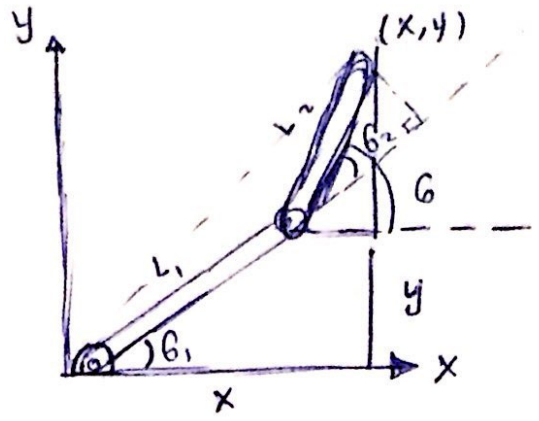
$$Y = y_1 + y_2 = L_2 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

* Kinematic Analysis * 2 D.O.F 6

* Inverse kinematic *

المعلوم

X
Y
θ
L_1
L_2



المجاهيل

$\emptyset_1$
$\emptyset_2$

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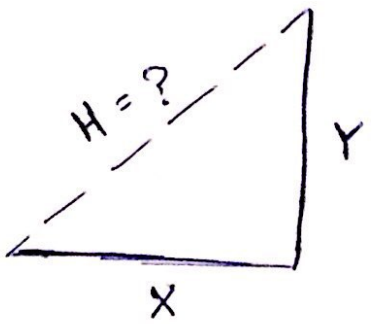
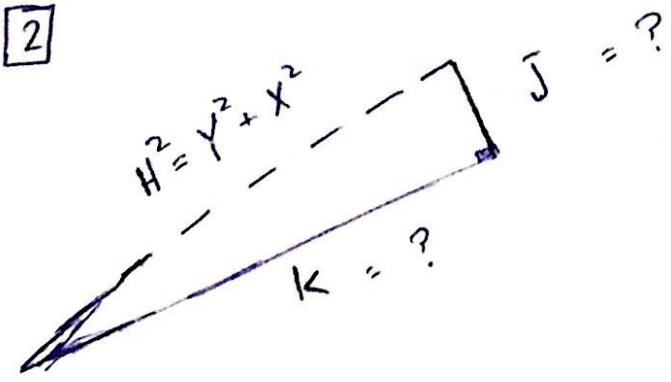
* 1



نظرية فيثاغورس :-

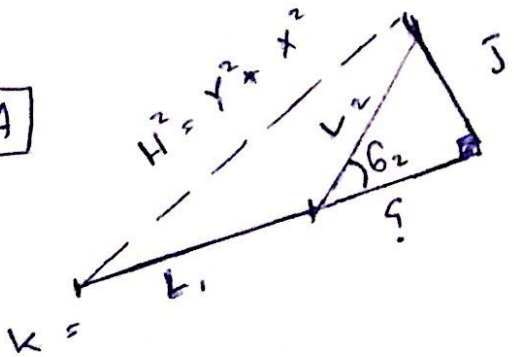
$$H^2 = X^2 + Y^2$$

* 2



$$\sin\theta = \frac{\text{المقابل}}{\text{الوتر}}$$

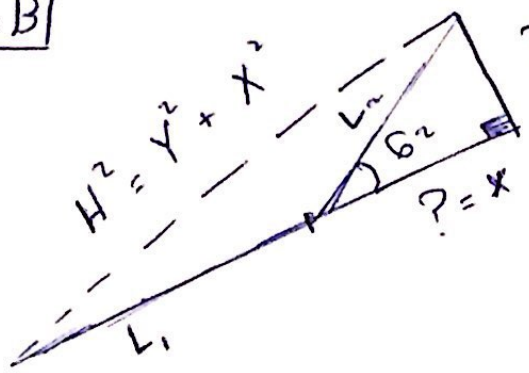
2. A



$$\sin\theta_2 = \frac{\bar{J}}{L_2}$$

* $\bar{J} = L_2 \sin\theta_2$

2.B



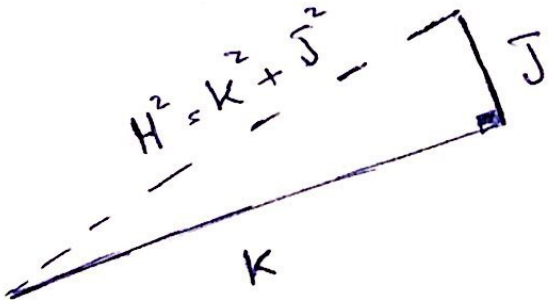
$$\cos\theta = \frac{\text{مجاور}}{\text{وتر}}$$

$$\cos\theta_2 = \frac{X}{L_2}$$

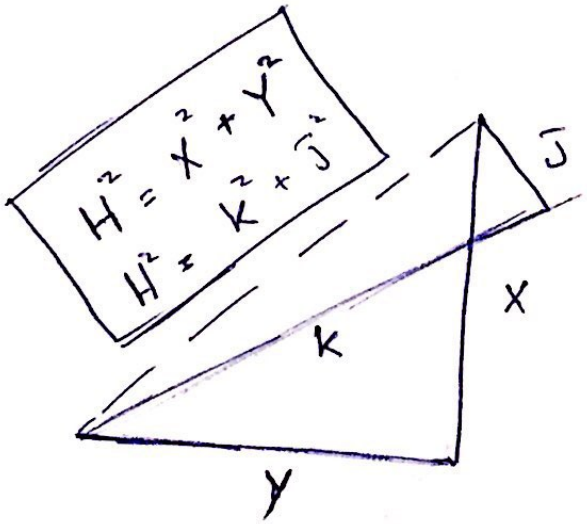
$$X = L_2 \cos\theta_2$$

K =

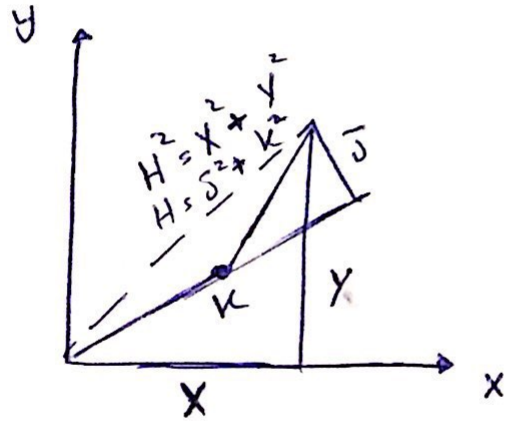
$$K = L_1 + X = \boxed{L_1 + L_2 \cos\theta_2}$$



∴



∴ عندنا مثلثين لوتر واحد :-



* $\bar{J} = L_2 \sin\theta_2$

* $K = L_1 + L_2 \cos\theta_2$

قانون المربع الكامل :-

$$[(a \pm b)^2 = a^2 + b^2 + 2ab]$$

$$H^2 = H^2$$

$$X^2 + Y^2 = \bar{J}^2 + K^2$$

$$X^2 + Y^2 = [L_2 \sin\theta_2]^2 + [L_1 + L_2 \cos\theta_2]^2$$

$$X^2 + Y^2 = [L_2^2 \sin\theta_2^2] + [L_1^2 + L_2^2 \cos\theta_2^2 + 2 L_1 L_2 \cos\theta_2]$$

$$X^2 + Y^2 = [L_2^2 \sin\theta_2^2] + [L_1^2 + L_2^2 \cos\theta_2^2 + 2 L_1 L_2 \cos\theta_2]$$

$$\sin^2\theta + \cos\theta^2 = 1 \quad = \text{المثلثي}$$

$$X^2 + Y^2 = L_1^2 + L_2^2 + 2 L_1 L_2 \cos\theta_2$$

$$\cos\theta_2 = \frac{X^2 + Y^2 - L_1^2 + L_2^2}{2 L_1 L_2}$$

Final Answer :~

Forward kinematic :-

$$X = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$Y = L_1 \sin\theta + L_2 \sin(\theta_1 + \theta_2)$$

Inverse kinematic :~

$$\cos\theta_2 = \frac{X^2 + Y^2 - L_1^2 + L_2^2}{2 L_1 L_2}$$

$$\theta_2 = \cos^{-1}\left(\frac{X^2 + Y^2 - L_1^2 + L_2^2}{2 L_1 L_2}\right)$$

$$\boxed{\theta_1 = \theta - \theta_2}$$

Done by
Mohammed Gamal