# Electric Circuits

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# 11.1 Electric Current

## Electric Current

Electric current is the movement of charge carries, such as electrons, through a material. It measures the number of coulombs of charge that move through an area in a given time. The unit of electric current is the ampere, or coulombs per second.

$$I = \frac{Q}{\Delta t} = \frac{dq}{dt}$$

In a wire, each of the charge carriers move with many different velocities, with the net movement of charge carriers being represented by the drift velocity, $v_d$. We can also calculate electric current through the formula

$$I = nqv_dA$$

Where:

- $n$ is the number of charged particles per cubic meter of volume
- $q$ is the quantity of of the charge
- $A$ is the cross-sectional area of the wire

To create a movement of a fluid in a pipe, there must be a pressure difference between the ends of a pipe. Likewise, to create an electric current, there must be a potential difference between the ends of the wire. This electric potential difference is also referred to as an electromotive force, or $\epsilon$. We can create this force through a battery or generator.

## Electric Current Density

Recall that the electric current directly depends on the cross sectional area of the wire. If we were to measure a smaller cross sectional area within the wire, we would get a smaller value. The quantity that stays the same for both areas is the electric current density, which is a vector quantity. This can be intuitively derived:

$$J = \frac{I}{A} = nqv_d$$

Current is the dot product of current density and area.

$$I = \vec{J} \cdot \vec{A} = ||\vec{J}||\,||\vec{A}|| \cos\theta$$

And if electric current density is not constant throughtout the cross section of a wire:

$$I = \int \vec{J} \cdot d\vec{A}$$

The idea of charge flow as current implies that electric current has a direction, which contradicts the idea of current being a dot product. Despite this, current does have a direction, but not like a traditional vector quantity. Electric current is relative to the charge carriers and does not obey the laws of vector addition. By convention, the direction of current in a wire is based on the movement of positive charges, however, the charge carriers in a wire are electrons, which have a negative charge. Therefore, electrons in a wire move in the opposite direction as conventional current.

**Problem**

Suppose a wire with a radius of 2mm has an electric current density that varies with radial distance $r$ as $J = ar^2$, where $a = 1.5 \times 10^{10}$ and $r$ is in meters. What is the current flowing through the outer portion of the wire between $\frac{R}{2}$ and $R$?

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*Solution.* We can sum together concentric rings with an infinitely small thickness $dr$ to find $dA$. Find the area of a infinitely thin ring at radius $r$ with an area $dA$ by imagining bending the ring into an infinitely thin rectangle with width $dr$. Since we bent the rectangle from a circle, the length of the rectangle is the circumfrence of the circle. The area of the rectangle is thus

$$dA = 2\pi r dr$$

With the limits of integration being $\frac{R}{2}$ and $R$, we can replace $J$ with the given equation and $dA$. Since $J$ and $A$ are in the same direction, there is no need for additional consideration.

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\begin{aligned}
I &= \int J dA \\
&= \int_{\frac{R}{2}}^{R} ar^2 \cdot 2\pi r dr \\
&= \frac{15}{32}\pi a R^4 \\
&= \frac{15}{32}\pi (1.5 \times 10^{10})(0.002)^4 = 0.35A
\end{aligned}
$$

### Current and Electric Field

Recall that an electric field causes charge carriers to move through a wire. An increase in electric field strength causes an increase in electromotive force, which increases the current density. Current density and electric field are proportionally related to each other.

## 10.2 Simple Electric Circuits

An electric circuit is a complete loop through which current can flow. A simple electric circuit is composed of electrical loops, which can include the following individually or in combinations.

- Wires
- Batteries
- Resistors
- Lightbulbs
- Capacitors
- Inductors
- Switches
- Ammeters
- Voltmeters