Lab 12

Gradient descent

Gradient descent is a first-order iterative optimization algorithm for finding a local minimum of a differentiable function.

$$x_{n+1} = x_n - \gamma \nabla F(x_n),\ where\, \gamma\, is\, step\, size$$

Given a function $F(x) = x_1^2 + x_2^2$,

- Assume we start from position (1,1), and trying to find a local minimum, which direction should we go?
- Why step size affect the result?

See demo here

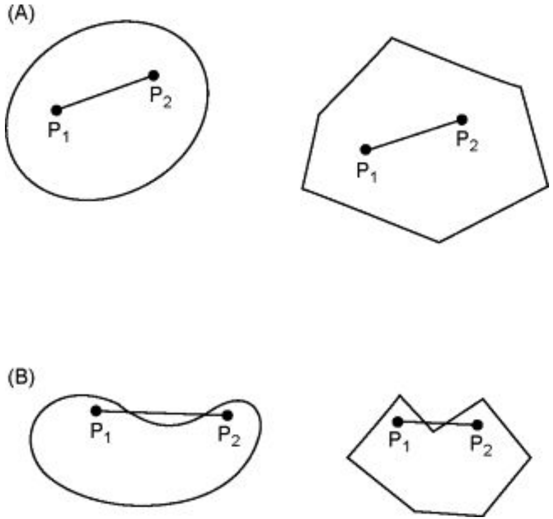
Convex set

A set C is convex if the line segment between any two points in C lies in C.

$$\forall x_1, x_2 \in C, \forall \theta \in [0,1],\ \theta x_1 + (1-\theta)x_2 \in C$$

Some extreme examples:

- The empty set.
- The singleton set.
- The complete set



Prove the convexity of the following sets

1) The unit ball $\{x : ||x|| \leq 1\}$
2) Let A be an m by m PSD matrix, for any a>=0, the set $\left\{x \in \mathbb{R}^m : x^T A x \leq a\right\}$

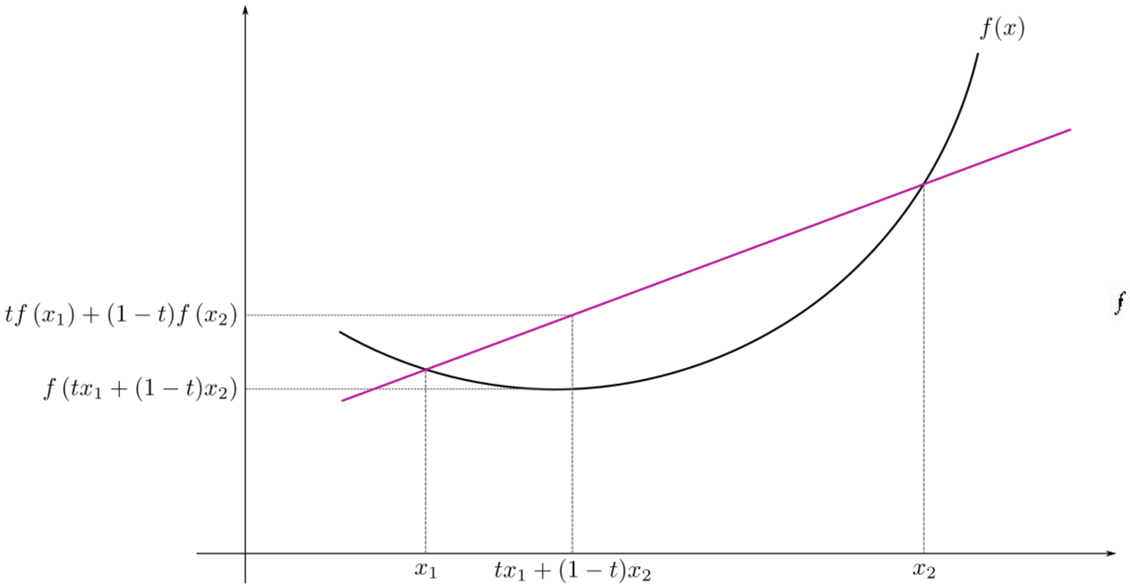
   Hint: by PSD, we know for any non zero vectors x and y,

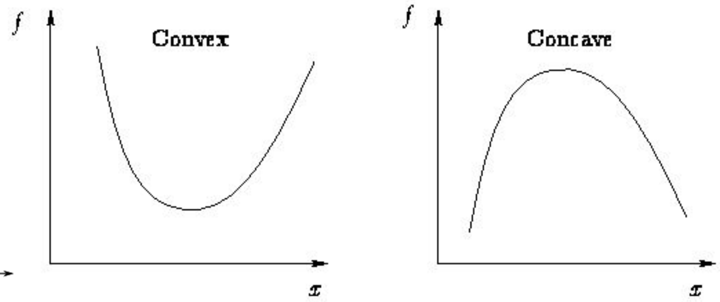
   $$(x - y)^T A(x - y) \geq 0$$

Convex function

A function is convex if its domain is a convex set and $\forall x, y \in dom(f), \forall \theta \in [0,1]$

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y)$$





Convex functions $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y)$

1. All linear functions are convex and concave.
2. For any real value a, e^{ax} is convex
3. x^a Is convex when a >1 or a<=0, and concave when 0<=a<=1.

Prove $x \log x$ is convex on positive domain.