Linear Programming Solver

Names and Ids

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the LP-Solver provides 4 methods :

- Simplex Method
- Big-M Method
- Two-Phase Method
- Goal Programming Method

Simplex Method

[A] Step By Step explanation

parameters send to it :

- tableau : a matrix containing (the objective function in the first row and the rest rows for the constraints)
- vararr : a list of all variables basic and non-basic
- basicarr : a list of the basic variables (initially the slack variables)
- ismaxi : to determine maximization or minimization of the objective

the Basic Iteration :

first : Select the Entering Variable (Pivot Column)

- maximization : the most negative coefficient in the objective row
- minimization : the most positive coefficient in the objective row

second : Select the Leaving Variable (Pivot Row)

- The leaving variable is determined by minimum positive ratio test :

$$\frac{\text{Solution Column (RHS)}}{\text{Pivot Column Coefficient}}$$

- if no positive ratio is found then the solution is infeasible

third : Update the Tableau

- the new basic variable replaces the leaving variable
- the pivot row is divided by the pivot element to make it 1
- all other rows are updated using row operations to make the pivot column values zero

fourth : Check for Optimality again as we checked it first before entering the main iterations

- maximization case: If any coefficient in the objective row (excluding the last column) is negative, the solution is not optimal (continue iterating)
- minimization case: If any coefficient in the objective row is positive, the solution is not optimal
- if the optimality hasn't reached then the iterations will continue finding the pivot row/column

[B] Handled Condition in it :

The code handles if any unrestricted variables are in the problem and reformulate the tableau at the parameter before entering the function

Now if in our Constraints we encountered (≥) then Slack Variables will not be Sufficient and we will need Extra Variables which are the (Artificial Variables) and then we will use Automatically either

→ Big-M method or Two-Phase Method

Big-M Method

[A] Step By Step explanation

first : parameters send to it :

- tableau : a matrix containing (the objective function in the first row and the rest rows for the constraints)
- vararr : a list of all variables basic and non-basic
- basicarr : a list of the basic variables (including the artificial variables)
- ismaxi : to determine maximization or minimization of the objective

second : Assigning the Big-M penalty

A large value M is chosen based on the optimization type :

- If maximization, artificial variables should be removed quickly, so M = 100
- If minimization, the opposite approach is taken, so M = -100

third : the objective function row is multiplied by -1 to convert a maximization problem into the simplex

fourth : Inserting M in the Objective Function for the Artificial Variables

fifth : do row operations to remove the M in the Objective to make the Objective Consistent

sixth : Call the Simplex Method to Handle the Formulated tableau for the Big-M

[B] Handled Condition in it :

the Big-M also handles the Unrestricted Variables sent to it ready from the Construction helping method

Two-Phase

[A] Step By Step explanation

The process consists of two phases:

1. **Phase 1:** minimize the sum of artificial variables to determine if a feasible solution exists
2. **Phase 2:** remove artificial variables and proceed with the original objective function

first : parameters send to it :

- same as the parameters sent to the Big-M but increasingly we send the Original Objective Row

(arr parameter) to be able to use it in Phase-2

second : minimize sum of artificial variables to find a feasible solution

- the original objective function is temporarily ignored
- we introduce a **new objective function**, which minimizes the sum of artificial variables

third : use Simplex Iterations to check if the artificial variables will become less than or equal to 0 at the end well find feasible solution or not

fourth : if found remove the artificial variables from the latest simplex tableau and restore the

Original Objective Function at the first row , then continue Simplex Solution for the rest of Basic Variables till finding the Optimal Solution

[B] Handled Condition in it :

the Two-Phase also handles the Unrestricted Variables sent to it ready from the Construction helping method

Preemptive Goal Programming

[A] Step By Step explanation

this function implements multiple goals and multiple constraints Problem

first : parameters send to it :

- goal_arr : the multiple goals array
- constr_arr : the multiple constraints array
- num_goals : number of goals the user entered
- num_constraints : number of constraints the user entered
- maxi : if maximizing or minimizing the goals

second : the function generate names for all variables :

- the desicion variables
- slack variables for each constraint
- deviation variables (s- , s+) for each goal

third : populating the tableau :

- for each goal row set -1 for s- (underachievement)
- for each goal row set 1 for s+ (overachievement)
- each constraint row contains its coefficients and a 1 in the column of its slack variable
- removing the inconsistency of the z rows by adding to their goal rows

fourth : apply the simplex method on the updated tableau (minimizing the first z rows)

- put into consideration the stopping conditions for optimality or feasible solutions
- finding the minimum value in the z row and apply the simplex on all the rows

- if positive and can not apply further then the solution is infeasible
- if a lower priority row encounters a negative value above it achievable z row then we stop simplex that z row and it will not be achievable goal

fifth : interpreting results based on goal achievements

Helping Method

The construct_tableau method

- this method is sent to it the Matrix of Objective Function and Constraints in the first three methods so that to reformulate them to have added the artificial or/and slack variables constructed in the tableau
- also putting into consideration if there are unrestricted variables or not

The GUI

A Friendly User-Interface So that the user enters

In First Three Methods

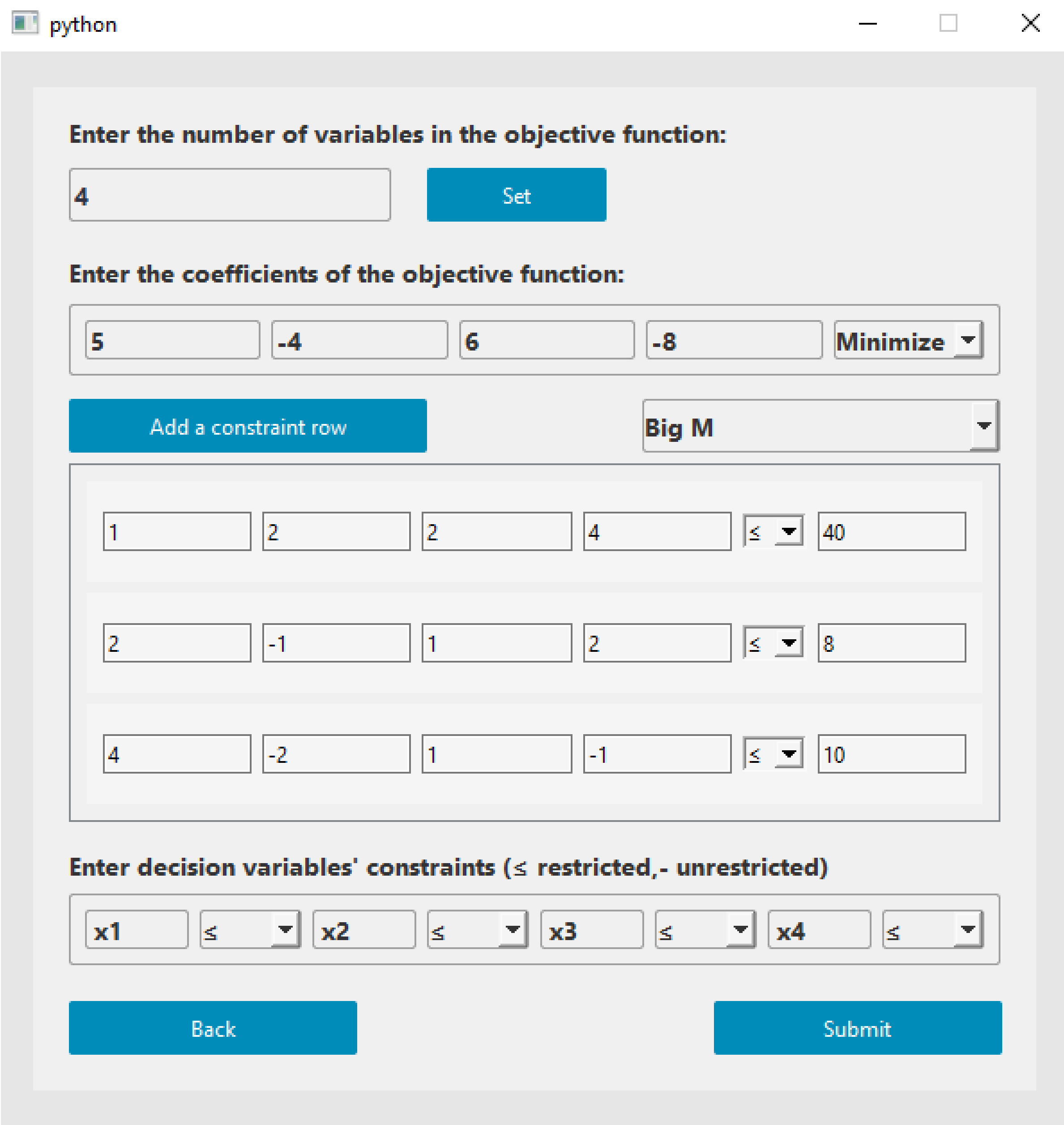
- the number of Decision Variables
- Coefficients of Decision Variables
- Coefficients , Sign , RHS Of Constraint functions
- Wether the Decision variables are restricted or not
- if we encounter ≥ sign in any of the Constraint functions : the user is directed to Big-M by default and Two-Phase if he chose it
- other wise he will be directed To Simplex

In Goal Programming

- the number of Decision Variables
- the Goal functions
- the Constraint functions

Screen Shots Of the Program

Simplex



python

Simplex Method Steps

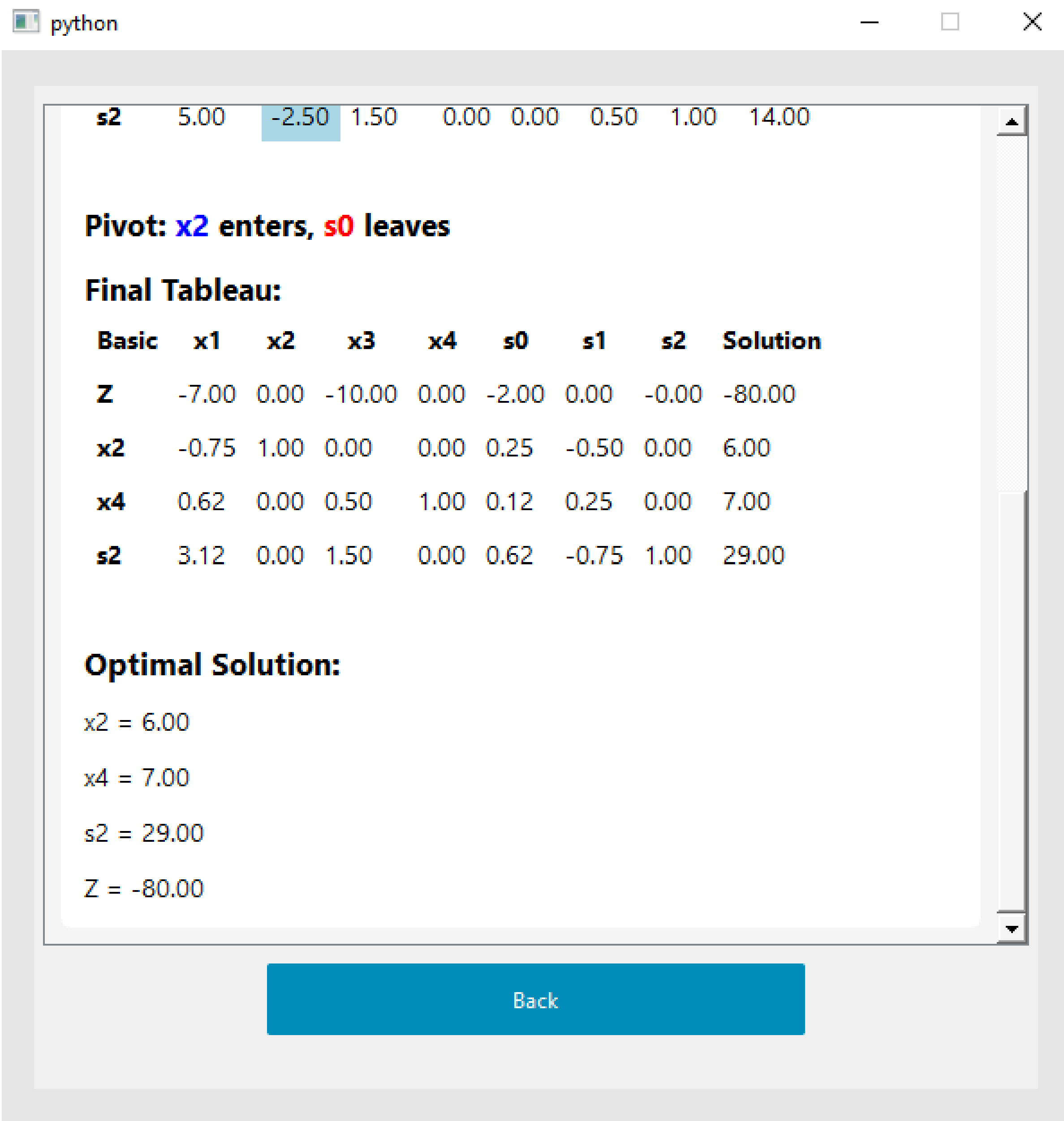
Basic	x1	x2	x3	x4	s0	s1	s2	Solution
Z	-5.00	4.00	-6.00	8.00	-0.00	-0.00	-0.00	-0.00
s0	1.00	2.00	2.00	4.00	1.00	0.00	0.00	40.00
s1	2.00	-1.00	1.00	**2.00**	0.00	1.00	0.00	8.00
s2	4.00	-2.00	1.00	-1.00	0.00	0.00	1.00	10.00

Pivot: x4 enters, s1 leaves

Basic	x1	x2	x3	x4	s0	s1	s2	Solution
Z	-13.00	8.00	-10.00	0.00	-0.00	-4.00	-0.00	-32.00
s0	-3.00	**4.00**	0.00	0.00	1.00	-2.00	0.00	24.00
x4	1.00	-0.50	0.50	1.00	0.00	0.50	0.00	4.00
s2	5.00	-2.50	1.50	0.00	0.00	0.50	1.00	14.00

Pivot: x2 enters, s0 leaves

Back



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s2	5.00	-2.50	1.50	0.00	0.00	0.50	1.00	14.00

Pivot: x2 enters, s0 leaves

Final Tableau:

Basic	x1	x2	x3	x4	s0	s1	s2	Solution
Z	-7.00	0.00	-10.00	0.00	-2.00	0.00	-0.00	-80.00
x2	-0.75	1.00	0.00	0.00	0.25	-0.50	0.00	6.00
x4	0.62	0.00	0.50	1.00	0.12	0.25	0.00	7.00
s2	3.12	0.00	1.50	0.00	0.62	-0.75	1.00	29.00

Optimal Solution:

x2 = 6.00

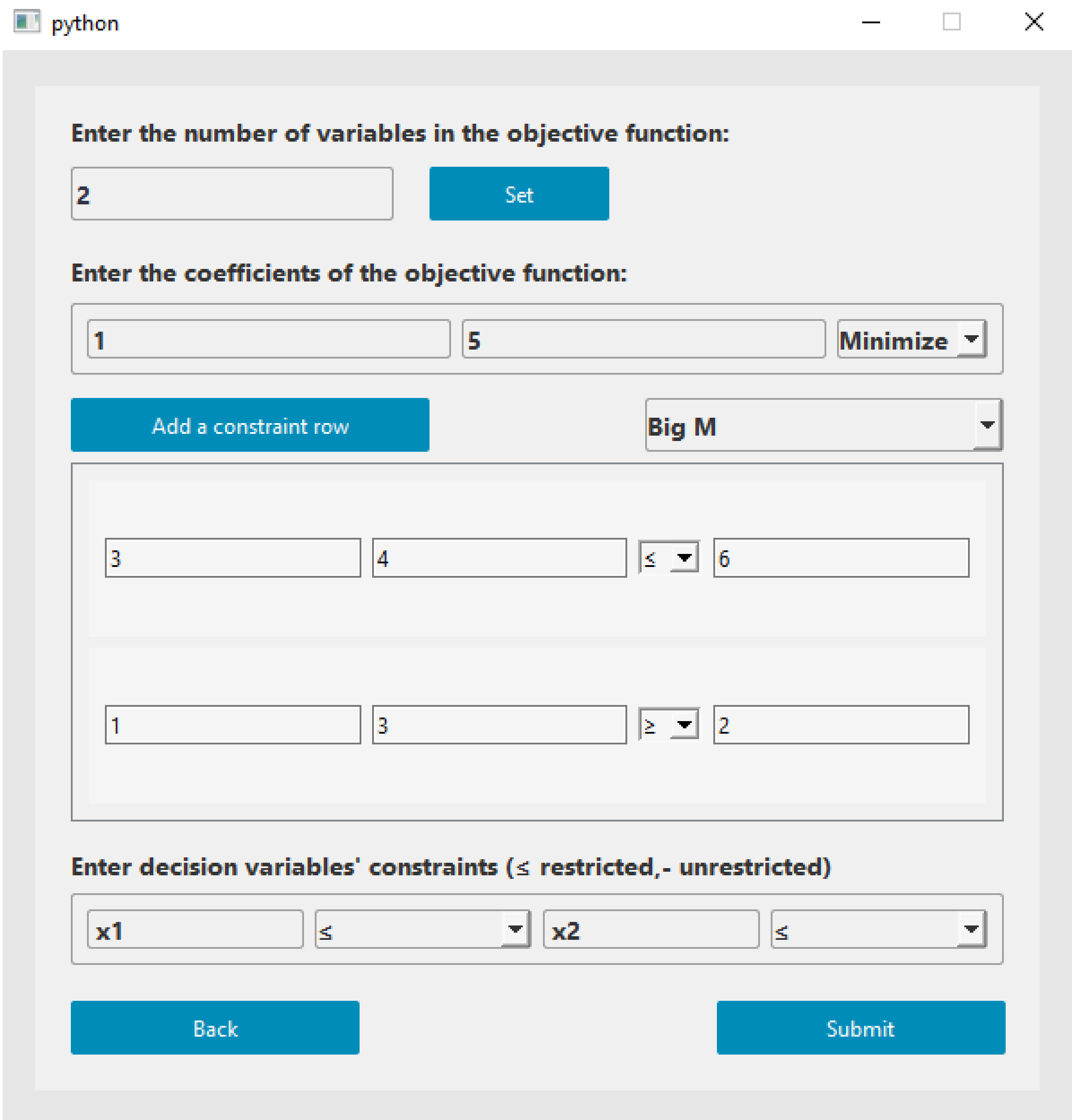
x4 = 7.00

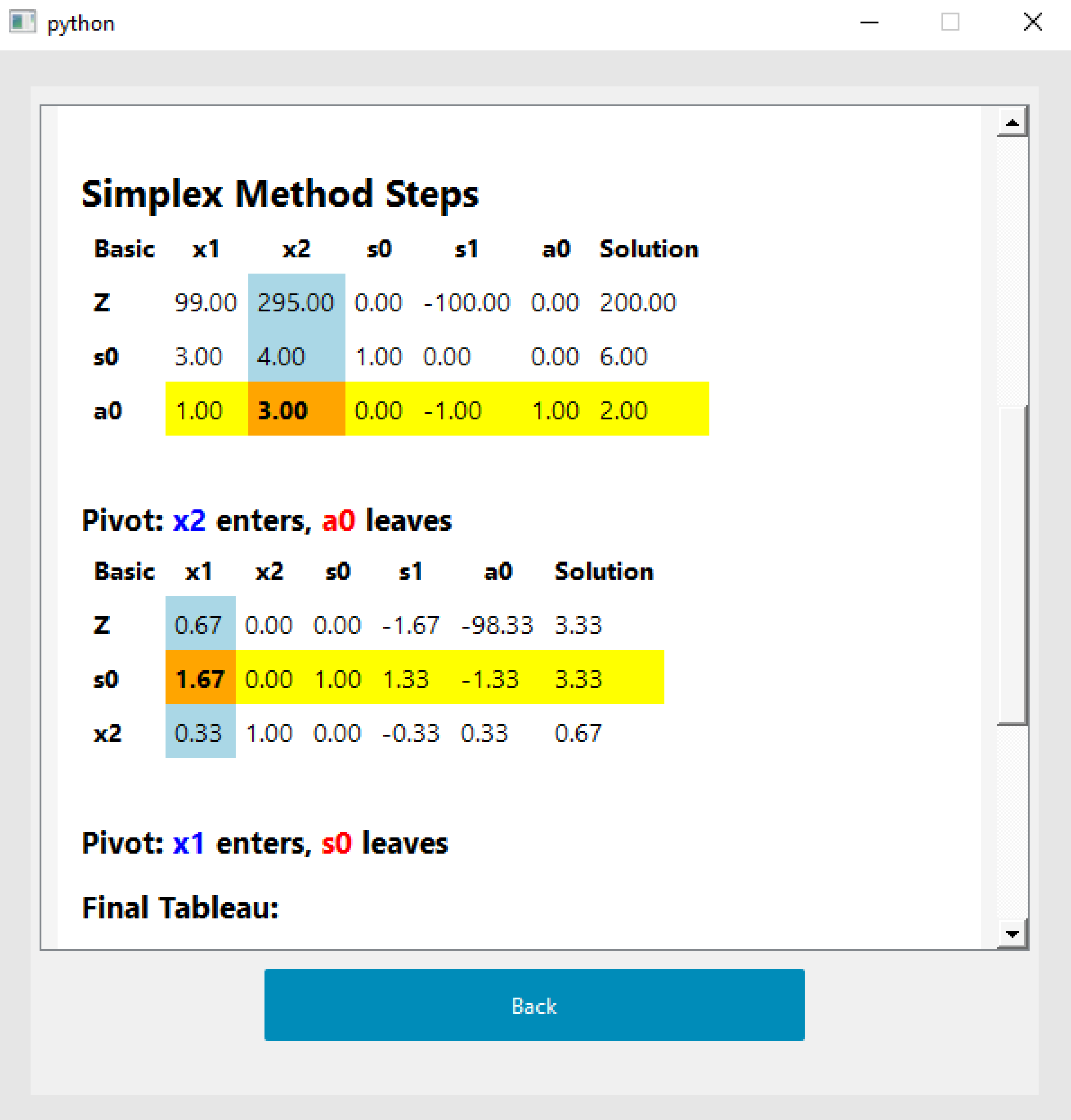
s2 = 29.00

Z = -80.00

Back

Big-M


python
Enter the number of variables in the objective function:
2
Set
Enter the coefficients of the objective function:
1
5
Minimize
Add a constraint row
Big M
3
4
≤
6
1
3
≥
2
Enter decision variables' constraints (≤ restricted,- unrestricted)
x1
≤
x2
≤
Back
Submit



python

Simplex Method Steps

Basic	x1	x2	s0	s1	a0	Solution
Z	99.00	295.00	0.00	-100.00	0.00	200.00
s0	3.00	4.00	1.00	0.00	0.00	6.00
a0	1.00	**3.00**	0.00	-1.00	1.00	2.00

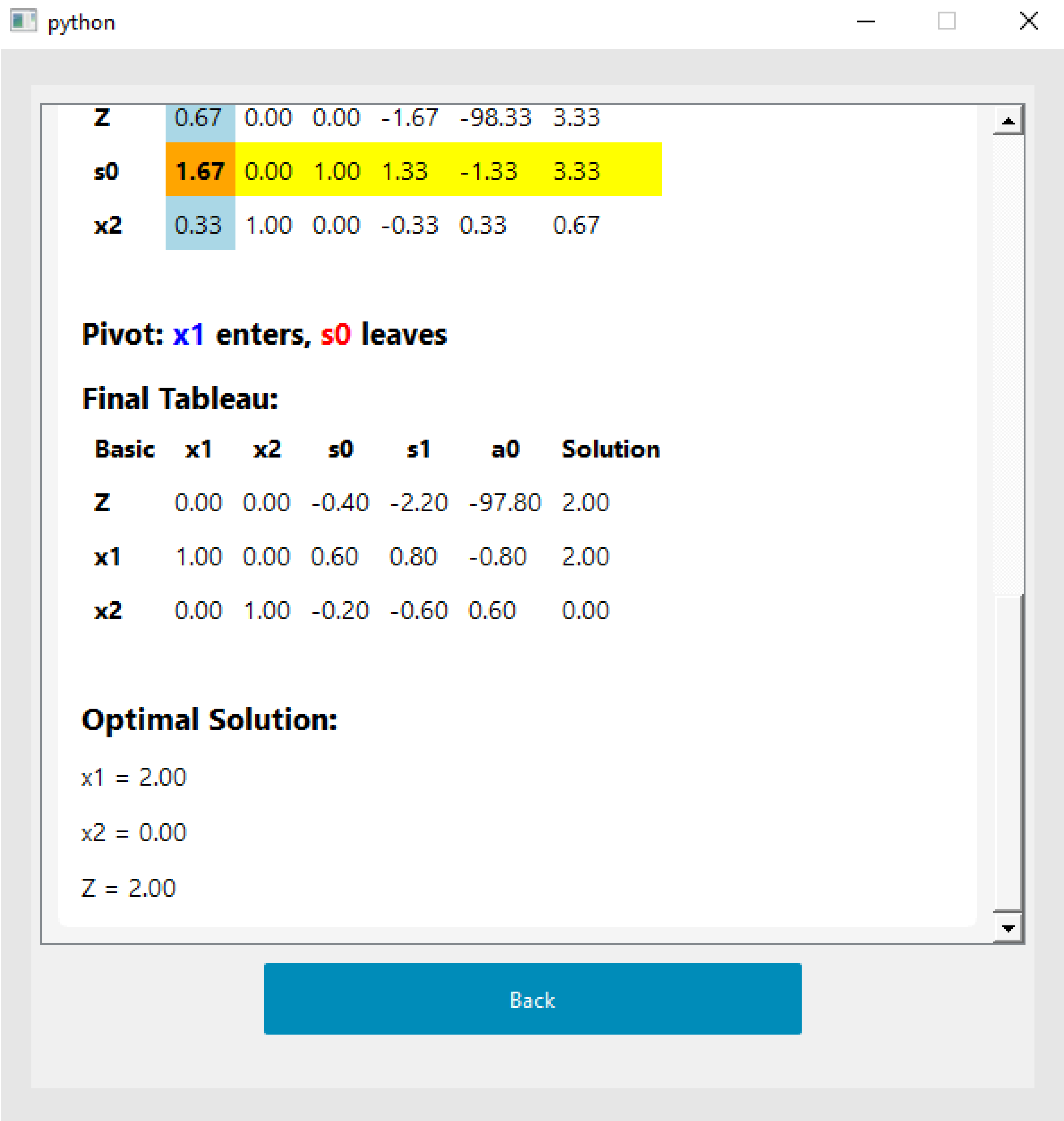
Pivot: x2 enters, a0 leaves

Basic	x1	x2	s0	s1	a0	Solution
Z	0.67	0.00	0.00	-1.67	-98.33	3.33
s0	**1.67**	0.00	1.00	1.33	-1.33	3.33
x2	0.33	1.00	0.00	-0.33	0.33	0.67

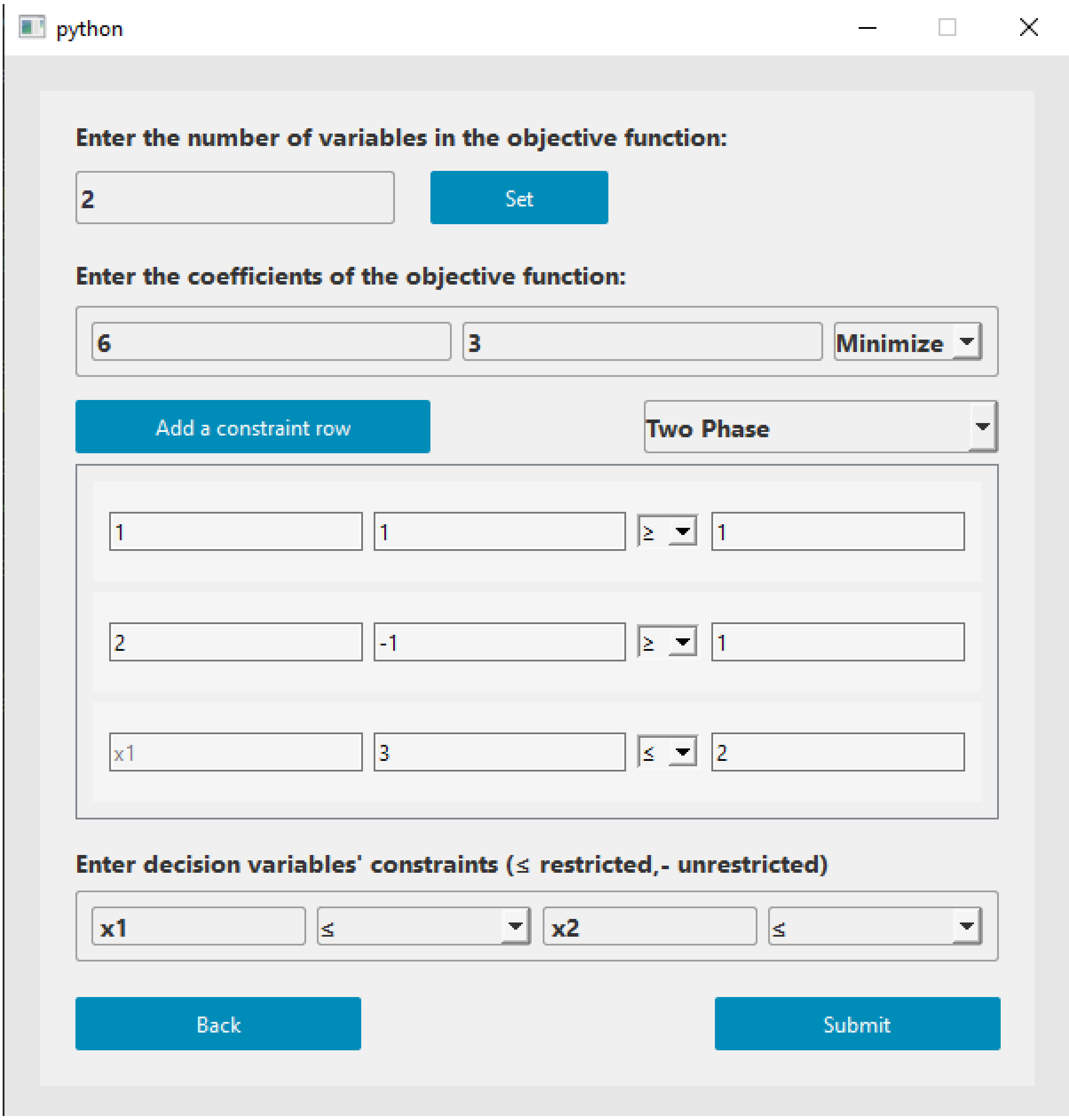
Pivot: x1 enters, s0 leaves

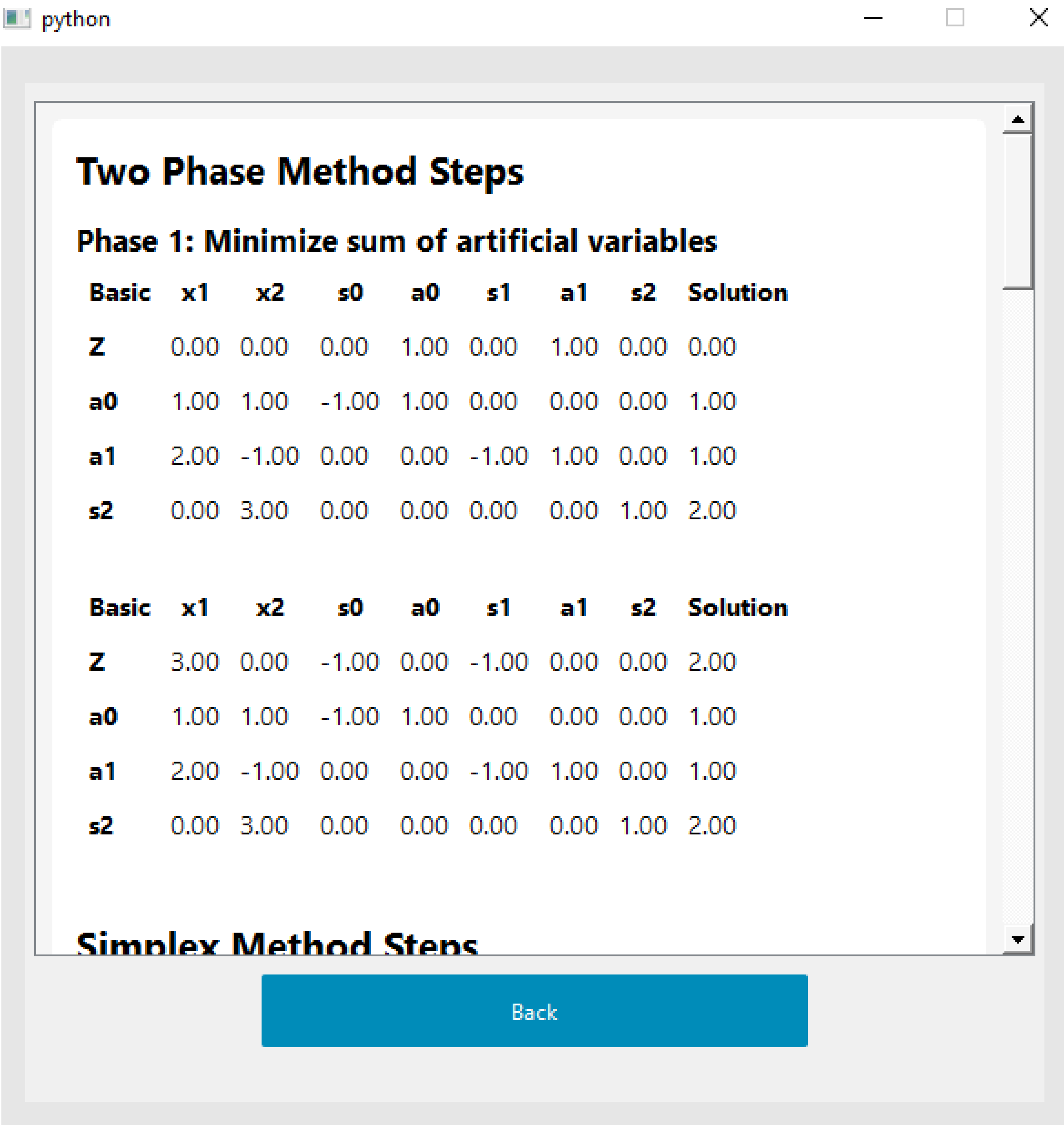
Final Tableau:

Back


python
Z 0.67 0.00 0.00 -1.67 -98.33 3.33
s0 1.67 0.00 1.00 1.33 -1.33 3.33
x2 0.33 1.00 0.00 -0.33 0.33 0.67
Pivot: x1 enters, s0 leaves
Final Tableau:
Basic x1 x2 s0 s1 a0 Solution
Z 0.00 0.00 -0.40 -2.20 -97.80 2.00
x1 1.00 0.00 0.60 0.80 -0.80 2.00
x2 0.00 1.00 -0.20 -0.60 0.60 0.00
Optimal Solution:
x1 = 2.00
x2 = 0.00
Z = 2.00
Back

Two-Phase


python
Enter the number of variables in the objective function:
2
Set
Enter the coefficients of the objective function:
6
3
Minimize
Add a constraint row
Two Phase
1
1
≥
1
2
-1
≥
1
x1
3
≤
2
Enter decision variables' constraints (≤ restricted,- unrestricted)
x1
≤
x2
≤
Back
Submit



python

Two Phase Method Steps

Phase 1: Minimize sum of artificial variables

Basic	x1	x2	s0	a0	s1	a1	s2	Solution
Z	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00
a0	1.00	1.00	-1.00	1.00	0.00	0.00	0.00	1.00
a1	2.00	-1.00	0.00	0.00	-1.00	1.00	0.00	1.00
s2	0.00	3.00	0.00	0.00	0.00	0.00	1.00	2.00

Basic	x1	x2	s0	a0	s1	a1	s2	Solution
Z	3.00	0.00	-1.00	0.00	-1.00	0.00	0.00	2.00
a0	1.00	1.00	-1.00	1.00	0.00	0.00	0.00	1.00
a1	2.00	-1.00	0.00	0.00	-1.00	1.00	0.00	1.00
s2	0.00	3.00	0.00	0.00	0.00	0.00	1.00	2.00

Simplex Method Steps

Back

python

Simplex Method Steps

Basic	x1	x2	s0	a0	s1	a1	s2	Solution
Z	3.00	0.00	-1.00	0.00	-1.00	0.00	0.00	2.00
a0	1.00	1.00	-1.00	1.00	0.00	0.00	0.00	1.00
a1	**2.00**	-1.00	0.00	0.00	-1.00	1.00	0.00	1.00
s2	0.00	3.00	0.00	0.00	0.00	0.00	1.00	2.00

Pivot: x1 enters, a1 leaves

Basic	x1	x2	s0	a0	s1	a1	s2	Solution
Z	0.00	1.50	-1.00	0.00	0.50	-1.50	0.00	0.50
a0	0.00	**1.50**	-1.00	1.00	0.50	-0.50	0.00	0.50
x1	1.00	-0.50	0.00	0.00	-0.50	0.50	0.00	0.50
s2	0.00	3.00	0.00	0.00	0.00	0.00	1.00	2.00

Pivot: x2 enters, a0 leaves

Back

python

Pivot: x2 enters, a0 leaves

Final Tableau:

Basic	x1	x2	s0	a0	s1	a1	s2	Solution
Z	0.00	0.00	0.00	-1.00	0.00	-1.00	0.00	0.00
x2	0.00	1.00	-0.67	0.67	0.33	-0.33	0.00	0.33
x1	1.00	0.00	-0.33	0.33	-0.33	0.33	0.00	0.67
s2	0.00	0.00	2.00	-2.00	-1.00	1.00	1.00	1.00

Optimal Solution:

x2 = 0.33

x1 = 0.67

s2 = 1.00

Z = 0.00

Phase 2: Remove artificial variables and restore original objective

Back

python

Phase 2: Remove artificial variables and restore original objective

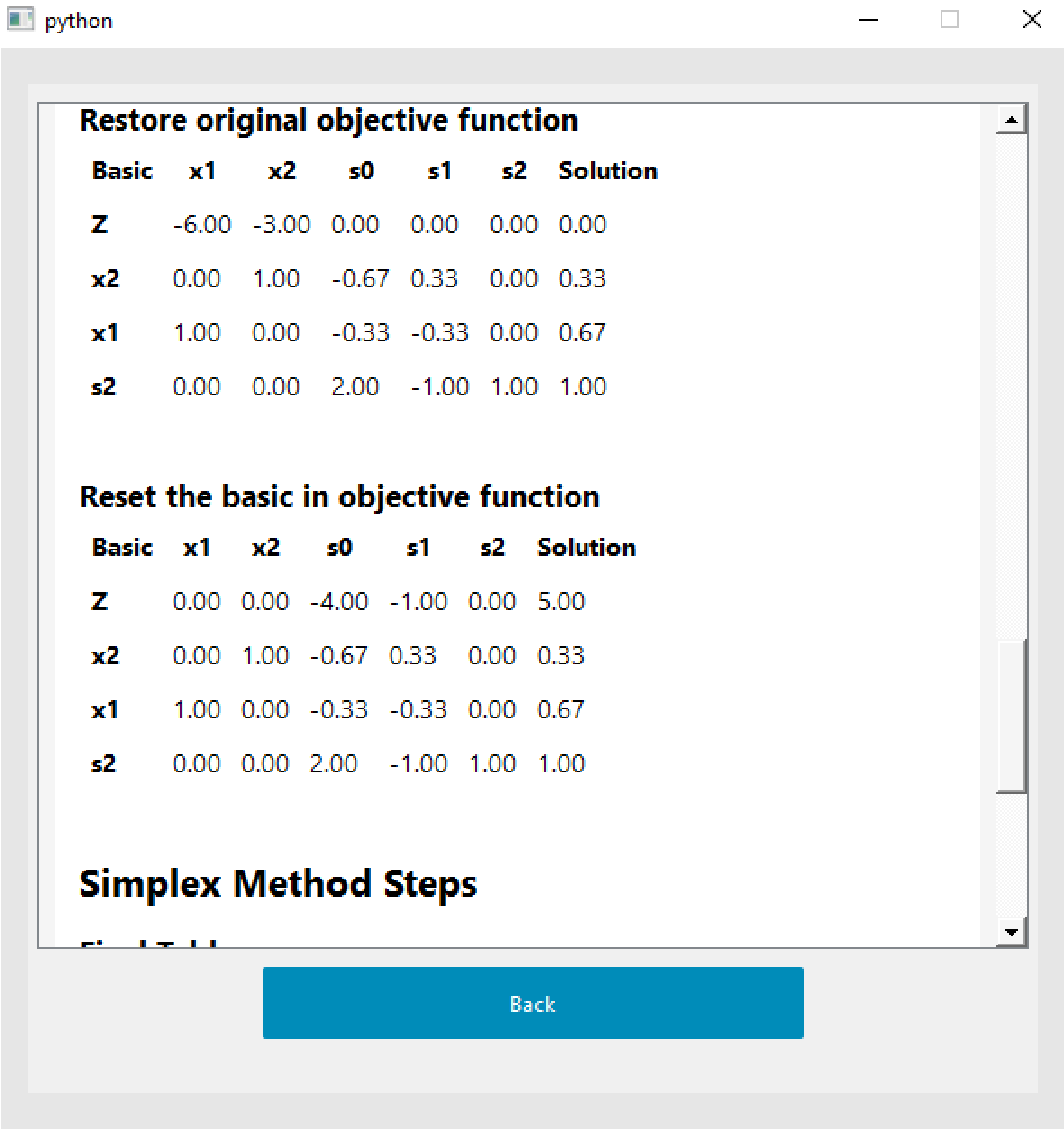
Basic	x1	x2	s0	s1	s2	Solution
Z	0.00	0.00	0.00	0.00	0.00	0.00
x2	0.00	1.00	-0.67	0.33	0.00	0.33
x1	1.00	0.00	-0.33	-0.33	0.00	0.67
s2	0.00	0.00	2.00	-1.00	1.00	1.00

Restore original objective function

Basic	x1	x2	s0	s1	s2	Solution
Z	-6.00	-3.00	0.00	0.00	0.00	0.00
x2	0.00	1.00	-0.67	0.33	0.00	0.33
x1	1.00	0.00	-0.33	-0.33	0.00	0.67
s2	0.00	0.00	2.00	-1.00	1.00	1.00

Reset the basic in objective function

Back



python

Restore original objective function

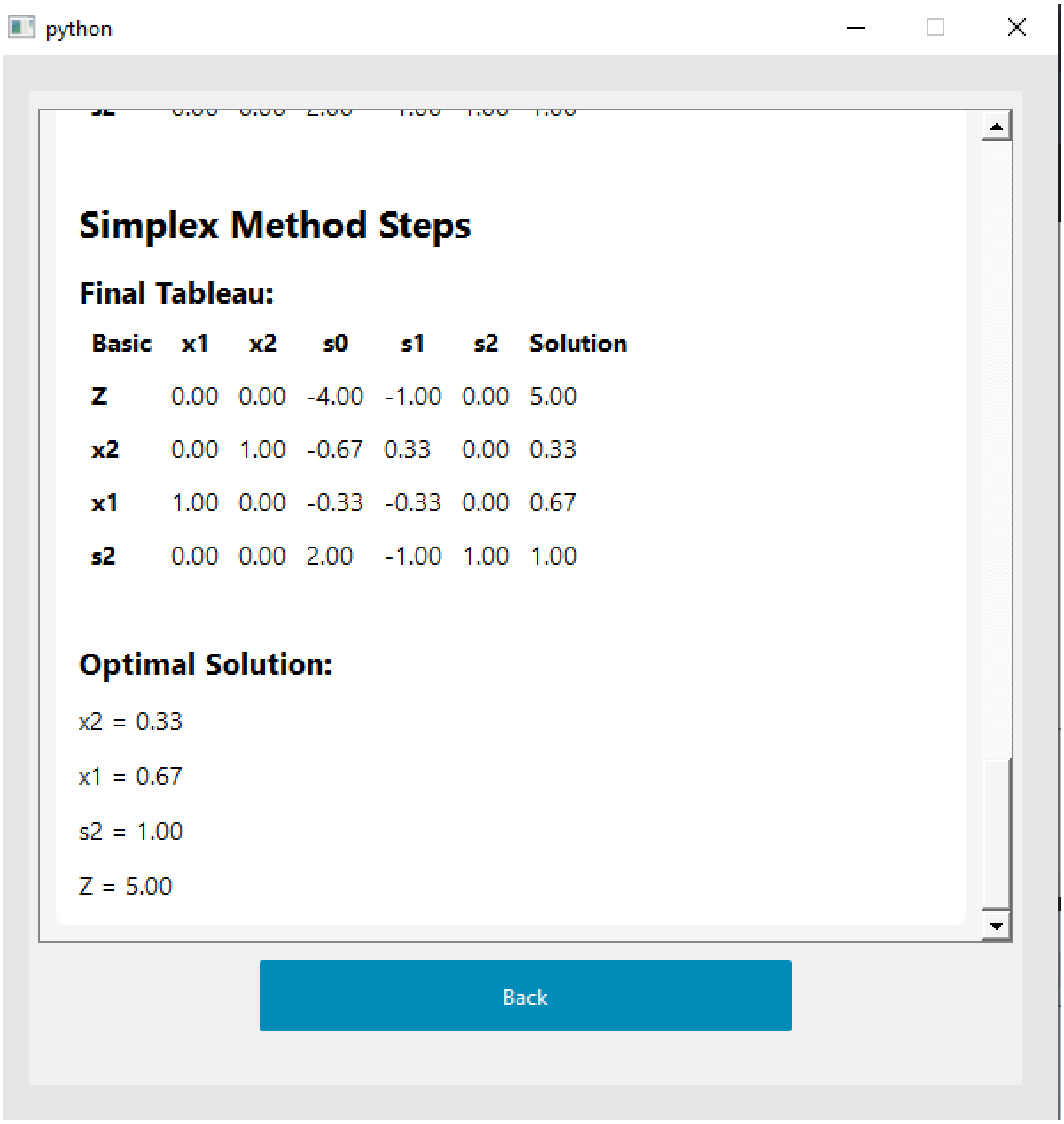
Basic	x1	x2	s0	s1	s2	Solution
Z	-6.00	-3.00	0.00	0.00	0.00	0.00
x2	0.00	1.00	-0.67	0.33	0.00	0.33
x1	1.00	0.00	-0.33	-0.33	0.00	0.67
s2	0.00	0.00	2.00	-1.00	1.00	1.00

Reset the basic in objective function

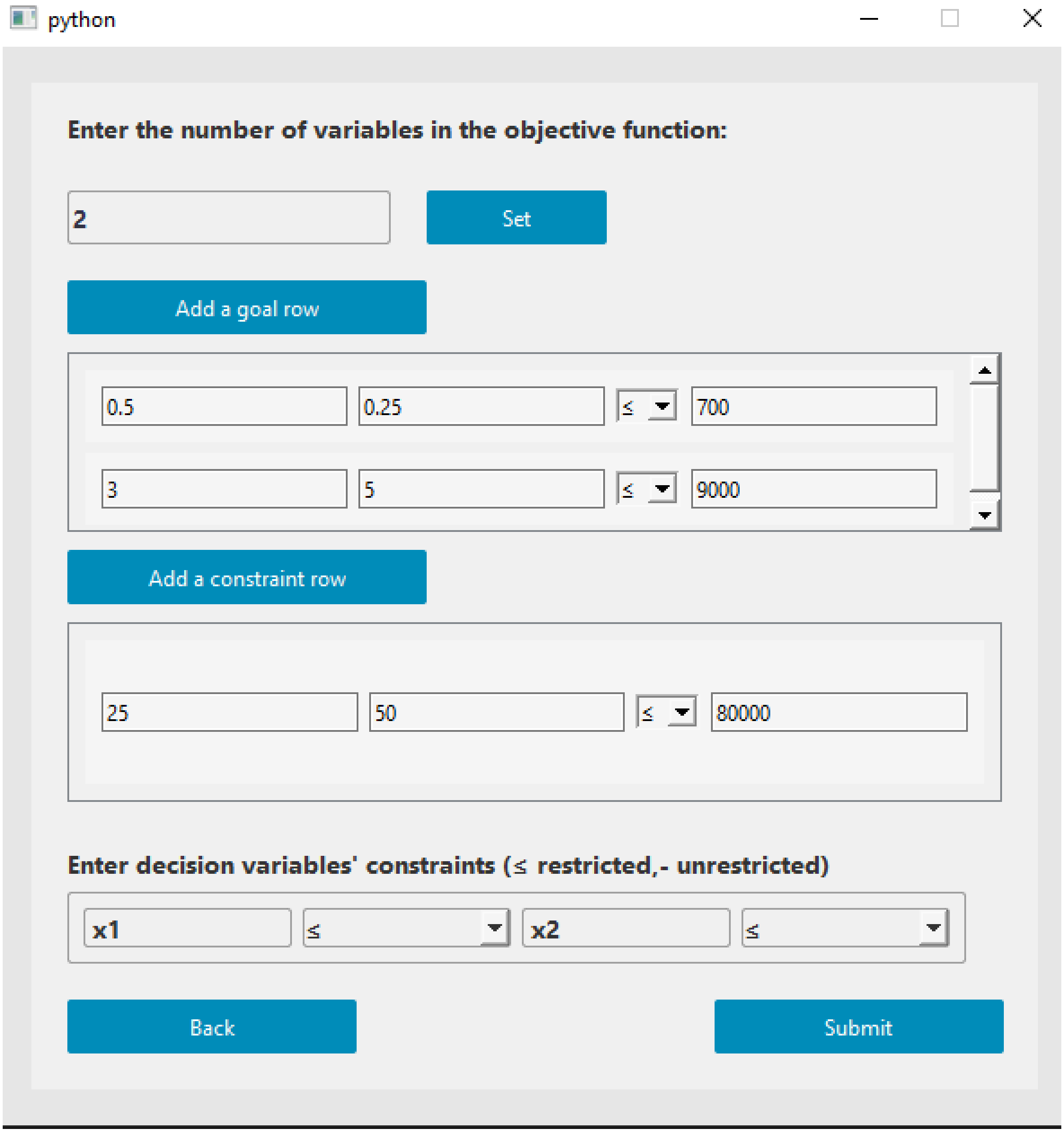
Basic	x1	x2	s0	s1	s2	Solution
Z	0.00	0.00	-4.00	-1.00	0.00	5.00
x2	0.00	1.00	-0.67	0.33	0.00	0.33
x1	1.00	0.00	-0.33	-0.33	0.00	0.67
s2	0.00	0.00	2.00	-1.00	1.00	1.00

Simplex Method Steps

Back



Goal-Programming


python
Enter the number of variables in the objective function:
2
Set
Add a goal row
0.5
0.25
≤
700
3
5
≤
9000
Add a constraint row
25
50
≤
80000
Enter decision variables' constraints (≤ restricted,- unrestricted)
x1
≤
x2
≤
Back
Submit

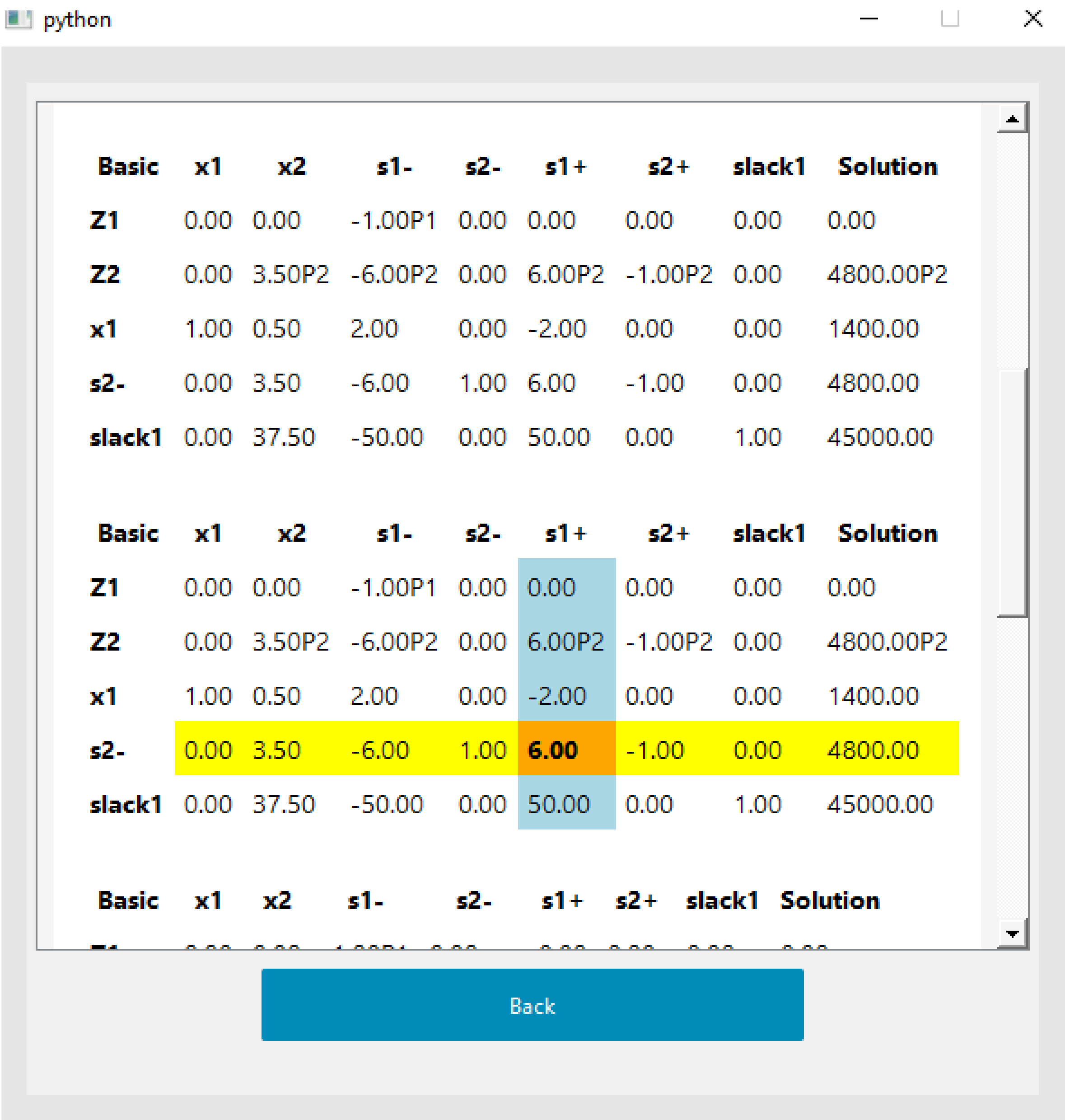
python

Goal Method

Basic	x1	x2	s1-	s2-	s1+	s2+	slack1	Solution
Z1	0.00	0.00	-1.00P1	0.00	0.00	0.00	0.00	0.00
Z2	0.00	0.00	0.00	-1.00P2	0.00	0.00	0.00	0.00
s1-	0.50	0.25	1.00	0.00	-1.00	0.00	0.00	700.00
s2-	3.00	5.00	0.00	1.00	0.00	-1.00	0.00	9000.00
slack1	25.00	50.00	0.00	0.00	0.00	0.00	1.00	80000.00

Basic	x1	x2	s1-	s2-	s1+	s2+	slack1	Solution
Z1	0.50P1	0.25P1	0.00	0.00	-1.00P1	0.00	0.00	700.00P1
Z2	3.00P2	5.00P2	0.00	0.00	0.00	-1.00P2	0.00	9000.00P2
s1-	**0.50**	0.25	1.00	0.00	-1.00	0.00	0.00	700.00
s2-	3.00	5.00	0.00	1.00	0.00	-1.00	0.00	9000.00
slack1	25.00	50.00	0.00	0.00	0.00	0.00	1.00	80000.00

Back



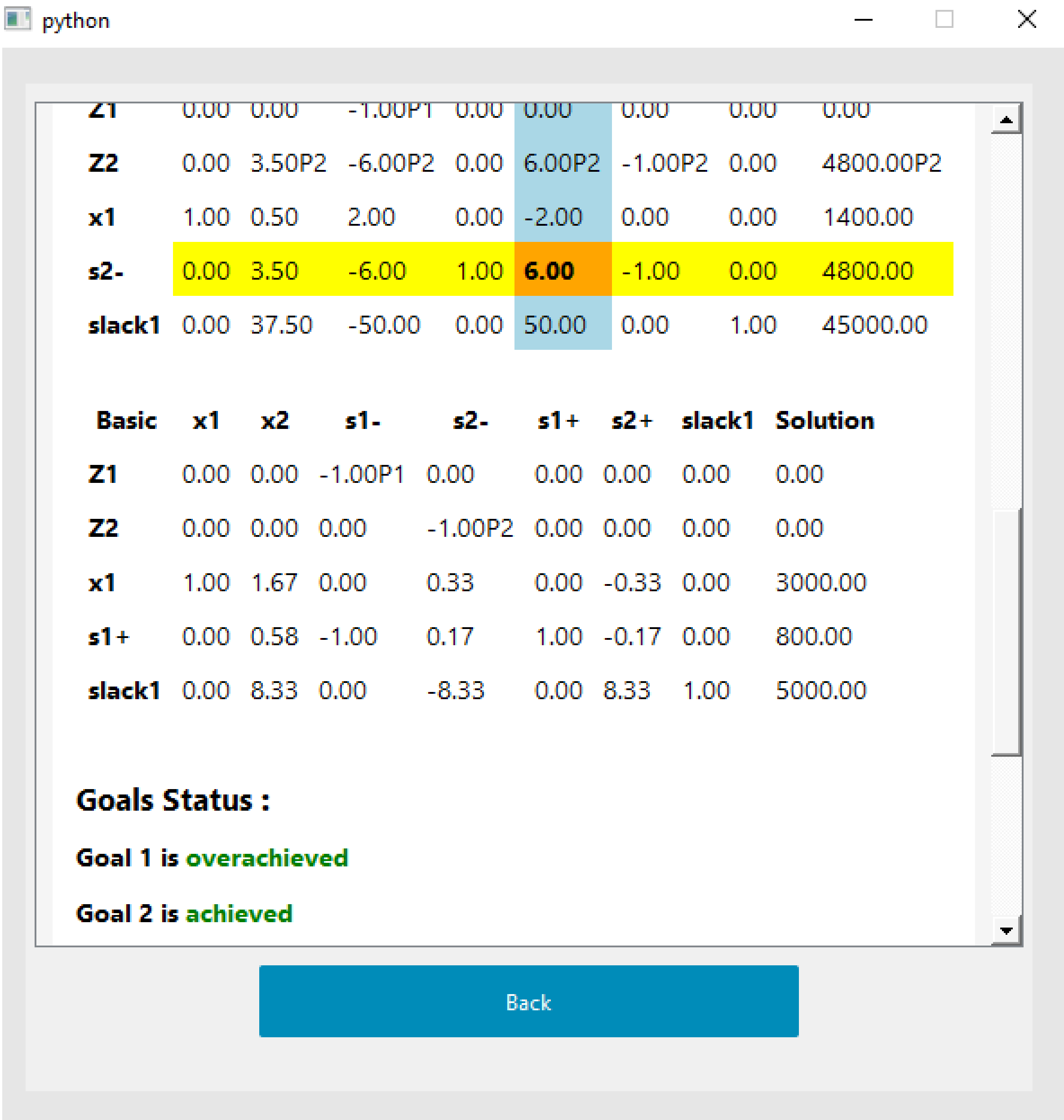
python

Basic	x1	x2	s1-	s2-	s1+	s2+	slack1	Solution
Z1	0.00	0.00	-1.00P1	0.00	0.00	0.00	0.00	0.00
Z2	0.00	3.50P2	-6.00P2	0.00	6.00P2	-1.00P2	0.00	4800.00P2
x1	1.00	0.50	2.00	0.00	-2.00	0.00	0.00	1400.00
s2-	0.00	3.50	-6.00	1.00	6.00	-1.00	0.00	4800.00
slack1	0.00	37.50	-50.00	0.00	50.00	0.00	1.00	45000.00

Basic	x1	x2	s1-	s2-	s1+	s2+	slack1	Solution
Z1	0.00	0.00	-1.00P1	0.00	0.00	0.00	0.00	0.00
Z2	0.00	3.50P2	-6.00P2	0.00	6.00P2	-1.00P2	0.00	4800.00P2
x1	1.00	0.50	2.00	0.00	-2.00	0.00	0.00	1400.00
s2-	0.00	3.50	-6.00	1.00	**6.00**	-1.00	0.00	4800.00
slack1	0.00	37.50	-50.00	0.00	50.00	0.00	1.00	45000.00

Basic	x1	x2	s1-	s2-	s1+	s2+	slack1	Solution

Back


python
Z1 0.00 0.00 -1.00P1 0.00 0.00 0.00 0.00 0.00
Z2 0.00 3.50P2 -6.00P2 0.00 6.00P2 -1.00P2 0.00 4800.00P2
x1 1.00 0.50 2.00 0.00 -2.00 0.00 0.00 1400.00
s2- 0.00 3.50 -6.00 1.00 6.00 -1.00 0.00 4800.00
slack1 0.00 37.50 -50.00 0.00 50.00 0.00 1.00 45000.00
Basic x1 x2 s1- s2- s1+ s2+ slack1 Solution
Z1 0.00 0.00 -1.00P1 0.00 0.00 0.00 0.00 0.00
Z2 0.00 0.00 0.00 -1.00P2 0.00 0.00 0.00 0.00
x1 1.00 1.67 0.00 0.33 0.00 -0.33 0.00 3000.00
s1+ 0.00 0.58 -1.00 0.17 1.00 -0.17 0.00 800.00
slack1 0.00 8.33 0.00 -8.33 0.00 8.33 1.00 5000.00
Goals Status :
Goal 1 is overachieved
Goal 2 is achieved
Back