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Intelligence, Social Mobility, and Growth

By JOHN HASSLER AND JOSÉ V. RODRÍGUEZ MORA*

We develop a model where the allocation of human resources, intergenerational social mobility, and technological growth are jointly determined. High growth endogenously increases the equilibrium return to innate cognitive ability and makes the allocation of individuals depend more on innate ability and less on social background. Individuals with a higher level of innate cognitive ability can deal better with less known, but more productive, technologies and thus choose a higher rate of technological growth. A social allocation based on innate ability and high growth will thus reinforce each other, implying the possibility of multiple endogenous growth equilibria. (JEL J62, O1)

Intelligence is what you use when you don't know what to do.
——Jean Piaget

Individuals are not born equal. Society and nature endow different individuals with different abilities. An individual's upbringing is determined by his or her social background and affects the individual's future ability to respond adequately to the problems faced by economic agents. Other differences between individuals are the result of nature—some individuals are more talented than others. Using economic jargon, we may say that an individual is born with two types of valuable assets: innate and social.

The distribution of innate and social assets among individuals is not independent between generations. In the game of allocating intellectual ability, Mother Nature stacks the cards in favor of individuals with gifted parents, which

we call the *genetic heritage*. Similarly, the upbringing of one's offspring provides a powerful mechanism for transferring social advantages between generations. We call this mechanism the *social heritage*.

In this paper, we assume that genetic heritage is weaker than social heritage. In other words, an individual's amount of innate assets depends less on his/her parents, and more on chance, than does his/her amount of social assets. More specifically, we assume innate intellectual ability to be less than perfectly correlated between generations, whereas the social advantages that result from a particular upbringing are fully determined by the parents' social position. Intergenerational social mobility, defined as the extent to which factors other than parental social position affect the individual's own social position, will then depend on whether the social sorting mechanism emphasizes traits and abilities determined by innate or social assets. If innate intellectual ability is important for an individual's social position, social mobility will be high. If the individual's upbringing, determined by one's parental background, is of greater importance, social mobility will be low.

Our first goal is to demonstrate that economic mechanisms determine the relative importance of innate abilities and social heritage when individuals are allocated over different economic roles in society. For this purpose, we construct a stylized economy, in which each individual chooses whether to become an entrepreneur or a worker. Workers are paid a common wage, determined on a Walrasian labor market.

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Entrepreneurs, on the other hand, will be re-warded on the basis of their ability to take the correct action in difficult situations. There are no barriers to this career choice—individuals are free to choose the option giving maximum expected lifetime utility. A key result is that the equilibrium allocation of individuals over social positions depends critically on the level of en-trepreneurial difficulty, driven by the rate of technological growth.

We model the social assets of children of entrepreneurs as information given by the par-ents about the optimal actions of an entrepre-neur, at the time when the parents were working. This gives children of entrepreneurs an informational advantage over children of workers, provided that the world has not changed a great deal since their parents were entrepreneurs. We show that low growth im-plies that the world changes slowly—the right actions yesterday are also likely to be the right ones today.

If, instead, the rate of growth is high, the economic environment for entrepreneurs changes rapidly, making the information inher-ited by children of entrepreneurs from their parents less valuable. Consequently, they will not enjoy as great an advantage over children of workers as in the low-growth case. The "ability to learn or understand or to deal with new or trying situations" becomes more important for social selection when all individuals stand on more equal grounds regarding their ex ante in-formation. The quote is a standard definition of intelligence.[1] We therefore use the term intelli-gence for the type of innate ability we are fo-cusing on.[2] When growth is high, the world changes rapidly between generations, and the environment is "new and trying" for everybody. Thus, intelligence is a more important determi-nant of career choices in this case. Since social

background becomes less important, intergen-erational social mobility increases.

Our second goal is to embed the social selec-tion mechanism in an endogenous growth model. Here, growth is driven by a cumulative process in which knowledge about how to use increasingly productive technologies is accu-mulated. Thus, future generations will benefit from the technological advances by previous entrepreneurs. This is not taken into account, however, when individuals are rewarded on the labor market and there is thus no guarantee that individuals will be allocated over jobs in a way that fosters growth.

In our model, entrepreneurs face a trade-off between productivity and difficulty. A great in-novation, giving a large productivity gain over the technology previously used, implies a higher degree of ignorance about its optimal use and a larger scope for mistakes. As we will see, the higher the individual entrepreneur's ability to learn or understand or to deal with new or trying situations, the larger will the individual's technological innovations be. This will create a feedback mechanism whereby rapid technolog-ical growth creates an environment in which the sorting of individuals to entrepreneurial posi-tions is based on intelligence, not on social background. From a growth perspective, this increases the efficiency in the allocation of in-dividuals and fosters future rapid growth.

The feedback mechanism in our model can produce multiple steady-state endogenous growth equilibria. To illustrate this, consider two ex ante identical societies, Richland and Poorland. They have access to the same re-sources (both human and physical), but for his-torical reasons, they have different social structures. The entrepreneurial class of Poor-land consists mainly of the children of previous entrepreneurs. From an intellectual point of view, the entrepreneurs are a random sample of society's entire population and, consequently, quite average with regard to their intellectual abilities. Thus, they are not very innovative and do not substantially change the world; neverthe-less, they confront economic challenges and learn from these. They can explain to their children what actions were the best to take during their working life, which is sufficient to give the children of entrepreneurs the upper hand—they will become the entrepreneurs of

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[1] Merriam-Webster's Collegiate, Tenth Edition, from Encyclopedia Britannica On-Line Edition.

[2] We are aware of the sensitivity involved in classifying individuals as having higher or lower levels of intelligence. We certainly do not want to be judgmental, and hope that the reader is willing to disregard the depreciatory values sometimes associated with our terminology. It should also be noted that we do not need to assume that some individ-uals are better in all dimensions. It is sufficient that different individuals have different talents.

the next generation. Consequently, the intelligence of the entrepreneurial class of Poorland will remain average and Poorlandians will have little or no growth for generations to come.

In Richland, the situation is different: the entrepreneurs are the most intelligent individuals in society and they innovate and generate growth. Thus, they make the world change rapidly, and the information they can pass on to their children thus becomes irrelevant so quickly that it is of no or little value. The next generation of entrepreneurs will thus be formed by the intellectually gifted and the people of Richland will enjoy consistent high growth.

There is empirical evidence in line with our results. Our first key result is that rapid technological progress increases the relative return to (intellectual) ability and diminishes the relative return to family background. This result is a key assumption in both Oded Galor and Daniel Tsiddon (1997) and Galor and Omer Moav (2000), who also discuss some empirical support consistent with their assumptions. For example, Chinhui Juhn et al. (1993) show that the return to unobserved ability and the return to observed characteristics (e.g., education) show different patterns over time. Increases in the return to unobserved ability can account for most of the increase in inequality in the 1970's, but only for a smaller share in the 1980's. The interpretation is that these patterns are driven by rapid technological growth in the 1970's.

Ann P. Bartel and Nachum Sicherman (1999) use a panel of young workers observed between 1979 and 1993 to analyze wages in industries with different rates of technological growth. They also provide evidence of a positive relation between the rate of technological growth and the return to ability. In addition, they find evidence that variation in the return to ability affects sorting. At high rates of technological change, ability becomes more important than formal background, which leads to a concentration of high-ability individuals in industries with rapid technological growth.

Theodore W. Schultz (1964, 1975) also provides evidence in line with our model. In an analysis of the agricultural sector, it is shown that when technological advancements occur slowly, traditional ways of transferring information between generations—like parents teaching their children—are economically efficient. As

technological growth increases, this source of information becomes relatively less valuable.

Our second main result, that the allocation of intellectual resources matters for growth, also finds support in the existing literature and is the main point in Kevin M. Murphy et al. (1991). They also show empirical evidence that talented individuals are more important for growth if they are engineers rather than lawyers. Similarly, William J. Baumol (1990) uses historical evidence to support the idea that growth increases if society directs more entrepreneurial talent to productive, rather than to rent-seeking, activities.

Robert Eriksson and John Goldthorpe (1992) provide another piece of evidence consistent with our results. They construct an index of intergenerational social mobility for nine countries and find that the sample can be divided into one group with relatively low mobility, consisting of The Netherlands, France, Germany, Italy, and the United Kingdom, and another with higher mobility, consisting of Sweden, Japan, the United States, and Australia. The average long-run growth rate has also been lower in the former group; the average growth rate per year between 1870 and 1979 was 1.77 percent in the former group, versus 2.43 percent in the latter.[3]

Most of the literature dealing with growth, social mobility, and income distribution has focused on the effects of financial market imperfections on human capital accumulation [Galor and Joseph Zeira (1993) and Roland Benabou (1996), for instance]. The central issue in our paper—how growth affects the sorting efficiency of the labor market and how this, in turn, affects growth—has not been considered in this line of literature. Galor and Tsiddon (1997) is the paper closest to ours. By defining two different types of technological change, major technological breakthroughs ("inventions") and gradual technological progress ("innovation"), they model the effects of technological change on intergenerational mobility. After an invention has been made, the return to ability is assumed to increase. This increases the social sorting efficiency in a way that produces a

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[3] Our own calculations from Angus Maddison (1982). Note also that, if the somewhat exceptional case of Australia is removed from the latter group, the difference in growth rate becomes greater.

"burst" of economic growth, followed by innovations and a return to lower growth and less-efficient social sorting, which implies cycles in growth and mobility.

In contrast to Galor and Tsiddon (1997), we construct a model that allows us to endogenize the relative returns to the two types of human capital as equilibrium outcomes on the labor market. A contribution of this paper is to provide an explicit theoretical model for how the empirical relation between technological growth and the relative return to intellectual ability may result as a market outcome. In addition, we provide a formal model that explains how social sorting affects the rate of technological growth. Rather than competing with Galor and Tsiddon (1997), we believe that our paper provides important building blocks to their foundational story.

Our model explains technological growth and social mobility as fully determined by the endogenous allocation and accumulation of information. We certainly realize that other factors—notably the distribution and accumulation of physical capital—are also of importance for growth and social mobility. Nevertheless, we believe information to be sufficiently important to motivate an attempt to build a purely information-based model.

The paper is structured in the following way: Section I describes the basic model with exogenous growth, which allows us to analyze the social sorting mechanism as a function of growth. Section II endogenizes growth by introducing a link between the allocation of innate assets, generated by the social sorting mechanism, and growth. Section III summarizes and concludes.

I. A Model of Human Resource Allocation

A. Entrepreneurs and Workers

In each discrete time period, there is a continuum of mass 1 of individuals. Each individual lives one period only, and the common utility function is logarithmic.[4]

Each individual chooses whether to be a worker or an entrepreneur. If an individual chooses to be a worker, she or he gets the known market wage at time t, denoted w_t. If an individual chooses to be an entrepreneur, she or he creates a firm and is the residual claimant to firm profits.

An entrepreneur must make two decisions: (i) to choose the number of workers to hire, denoted l_t; and (ii) to take an entrepreneurial decision, $a \in R$. The task of the entrepreneur is to set a as close to an unobservable stochastic variable x_t as possible. The larger the distance between a and x_t, the lower the profits in t. More specifically, the profits of the firm are:[5]

$$(1) \qquad \Pi = e^{-(x_t - a)^2}(2 e^{r_t} l_t^{1/2} - w_t l_t),$$

where r_t is the log(level of productivity) at period t. We can think of x_t as the "best way" of running a firm. In other words, x_t represents the optimal location of an entrepreneur in the space of possible products, firm organizations, geographic locations, and so on. Obviously, x_t is a multidimensional object in the real world. To simplify, however, we assume that it is unidimensional; making it multidimensional would be a straightforward generalization.

Profits are clearly maximized *ex post* if $a = x_t$. However, no individual knows the value of x_t *ex ante*. Furthermore, individuals differ in their beliefs about x_t, although all agents have rational expectations. Below, we describe how these expectations are formed. Now, take the distribution of x_t as given and consider an individual j who believes that x_t is normally distributed, with mean $\mu(j)$ and variance $1/[P(j)]$. $P(j)$ is thus the precision of j's beliefs, and $1/[P(j)]$ is the expected (squared) error of an entrepreneur when running a firm.

It is straightforward to show that all entrepreneurs will hire $l_t = (e^{r_t}/w_t)^2$ workers, regardless of their beliefs. The best action, of course, does depend on beliefs and is $a = \mu(j)$. An

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[4] The degree of risk aversion is not important for the results. The logarithmic utility function facilitates the expo-

sition. In Appendix B, we present a model with risk-neutral agents, which produces qualitatively similar results.

[5] The somewhat peculiar profit function used is not of qualitative importance for the results, but greatly simplifies the algebra. The model in Appendix B uses a profit function in which the entrepreneurial decision affects gross production.

entrepreneur's utility will be stochastic, with an expected value given by

$$(2) \qquad V^e(j) = E \ln(\Pi)$$

$$= 2r_t - \ln w_t - \frac{1}{P(j)},$$

which, of course, increases in the precision $P(j)$.

If the individual instead chooses to be a worker, that individual's utility will be certain and equal to $\log(w_t)$, independent of his/her beliefs about, and the realization of, x_t. If $\log w_t \geq r_t$, obviously, nobody will choose to be an entrepreneur. For lower wages, an individual with precision $P(j)$ chooses to be an entrepreneur if

$$(3) \qquad P(j) > \frac{1}{2(r_t - \ln w_t)} \equiv z_t.$$

Thus, z_t is the threshold precision such that an individual is indifferent between being an entrepreneur and a worker. This threshold is a monotonically increasing function of the wage. When deriving the equilibrium conditions of the model below, we use z_t rather than the wage, which simplifies the notation. Note also that the labor demand can be written

$$(4) \qquad l_t = e^{1/z_t},$$

which is decreasing in z_t and thus in the wage.

If Condition (3) holds with equality, the agent is indifferent between being an entrepreneur and being a worker. If the agent's precision is smaller than z_t, she or he chooses to be a worker.

B. *Information, Technology, and Intelligence*

Now, turn to the distribution of x_t and the information of different individuals. The distribution of x_t will depend on the particular technology used in period t. We assume that technologies are pairs denoted $[r, x_r]$ with r, $x_r \in \Re$. For any particular technology, r represents (the log of) its maximal productivity and x_r the associated optimal decision. It is common knowledge that for any technology $[r, x_r]$, the optimal decision for technologies with higher

productivity follows a Brownian motion with instantaneous variance σ. In other words, given $[r, x_r]$ and $g \geq 0$,

$$(5) \qquad x_{r+g} = x_r + \int_0^g \sqrt{\sigma} \, dz,$$

where dz is a standard Wiener increment. Thus, the variance of x_{r+g} conditional on x_r is equal to $g\sigma$, whereas the unconditional variance is infinite. The interpretation of condition (5) is that each marginal technological improvement requires new information on how to use the improved technology. Given knowledge about how to use a particular technology, the more new information is required, the higher the targeted productivity increase. Consequently, the margin for errors increases in the amount of technological improvements. In this section, we treat g as exogenous and common to everybody, whereas it is a matter of individual choice in Section II. Fixing the growth rate between time periods to g, we can rewrite (5) as follows

$$(6) \qquad x_t = x_{t-1} + \sqrt{g\sigma}\varepsilon_t,$$

where ε_t is distributed as a standard normal and independent over time.[6] The value of $g\sigma$ is the amount of new information required to fully utilize the newly introduced technology. It can be viewed as an index of the flow rate of new ideas and technological innovations. If $g\sigma$ is high, the flow is high, and the "best way" of running a firm thus changes quickly. A high level of $g\sigma$ might be the result of a rate of growth, which implies that new methods must be learned at a quick rate. It might also be the result of a high value of the parameter σ; in that case, the amount of information required to increase productivity by a particular amount is large. In any case, a high level of $g\sigma$ implies that the intrinsic difficulty of being an entrepreneur is high.

We now specify the private information regarding the distribution of x_t. Each individual has two sources of private information. One refers to x_{t-1} and depends on the individual's

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[6] This formulation is close to the one in Bojan Jovanovic and Yao Nyarko (1996).

social background; the other is the result of an individual's intelligence and provides information on x_t directly.

Let us first describe the information on x_{t-1}—the individual's social assets. We will use subindex $p \in \{e, w\}$ to denote whether the individual's parent is an entrepreneur or a worker. We assume that children of entrepreneurs know more about the optimal decision in the previous period, and this information serves as guidance for what decision to take today. We let γ_p denote the variance in this information.[7] To save on notation, we let $\gamma_e = 0$ and denote $\gamma \equiv \gamma_w > 0$. Our assumptions can be interpreted in the following way: entrepreneurs learn about x_t *ex post* (i.e., after deciding a). This information is then transferred to their descendants without costs. Children of workers, on the other hand, have access only to public information, which is less precise.[8]

We assume that social assets, that is, the pieces of information given by entrepreneurs to their children, are nontradeable. This assumption, of course, is crucial for our results. Our motivation for this is that we focus on the type of human capital that can be transferred only through the (slow) process of upbringing in the parental household.[9]

Now, consider the second source of private information, which is the result of the individual's innate intellectual ability. In this paper, we focus on the ability to learn, understand, and deal with new or trying situations. Since this corresponds to the dictionary definition of intelligence, we use this term and assume that individuals are different with respect to their level of intelligence.

More specifically, by the term *intelligence,* we mean *any* trait or characteristic with the following properties:

1. It helps in solving a problem that the individual has not faced before.
2. It is not a choice variable of the individual or the individual's parents.
3. It is less than perfectly correlated between generations.

We are not neuroscientists, and this paper is not about psychology or neuroscience,[10] so we do not want to imply that we know what intelligence really is or, for example, whether it is affected by genetics. We need to assume only that an individual characteristic with the preceding properties exists in the population with a nondegenerate distribution.[11]

Given the above-mentioned properties, we model the level of intelligence as the precision in an unbiased Gaussian private signal on the right action in a particular situation faced by the individual. Each individual rationally combines his/her private information with information received from other sources. Rational Bayesian updating with normally distributed signals implies that posterior beliefs are normally distributed with a precision equal to the sum of the precisions in the prior and the signal. We should note that intelligence is of no importance in a situation with full information; since the variance of the prior is zero, adding another signal (due to intelligence) cannot further reduce the

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[7] It would be straightforward to let γ be a function of the growth rate in previous periods. This would have an impact only on the dynamic behavior of the economy around its steady states in the case of nonconstant growth rates and would make it possible to analyze growth cycles along the lines in Galor and Tsiddon (1997).

[8] A different, but clearly related, social heritage is modeled in Anna Sjögren (1998), where it is assumed that an individual knows his/her ability in the trade of his/her parents but is unsure of his/her ability in other occupations. This is also the way in which Galor and Tsiddon (1997) model social heritage. Under this assumption, social barriers result from risk aversion.

[9] In this extremely stylized model, "social assets" are simply the knowledge of a particular number x_{t-1}, i.e., something that could, in theory, be bought and sold. In reality, it is inconceivable that the knowledge and the experience acquired by growing up in the "right" family could be bought and sold at a perfect market. In our view, this is not because this kind of knowledge cannot be represented by numbers associated with particular stochastic variables. Rather it is because human limitations imply that this knowledge is so complex (multidimensional) that it can be transferred only if the individual grows up in the right circumstances. We represent this knowledge by a univariate variable only for simplicity.

[10] See William H. Calvin (1996) for a nontechnical overview on how neuroscientists deal with intelligence.

[11] Note that our working definition of intelligence is not equivalent to the scores of IQ tests, which is known to be at odds with the second property. Note also that we do *not* assume that the ability to solve new problems is fully determined by intelligence; however—as we must assume—nor is it perfectly determined by factors within the control of the individual or his/her parents.

TABLE 1—PRECISION OF DIFFERENT AGENTS

Type	Number at t	Precision ($P_{p,i}$)
$\{e, h\}$	$m_{t-1}q$	$\dfrac{1}{g\sigma} + \alpha$
$\{e, l\}$	$m_{t-1}(1 - q)$	$\dfrac{1}{g\sigma} + 1$
$\{w, h\}$	$(1 - m_{t-1})q$	$\dfrac{1}{\gamma + g\sigma} + \alpha$
$\{w, l\}$	$(1 - m_{t-1})(1 - q)$	$\dfrac{1}{\gamma + g\sigma} + 1$

variance. Thus, the more difficult a situation, the more important is intelligence, which seems to be well in line with the dictionary definition of intelligence quoted earlier. Finally, we assume that the private information set is not transferable between individuals and that it cannot be affected by the individual (for example, through training or education).[12]

We now proceed by making the simplification that there are only two levels of intelligence: high or low. We use subindex $i \in \{h, l\}$ to denote these two levels.[13] An individual of intelligence type h receives an unbiased signal on x_t that is distributed as a normal with precision $\alpha > 1$. Individuals of intelligence type l also get an unbiased signal on x_t, but with a precision equal to 1. Furthermore, we assume the intelligence type to be uncorrelated between parents and children.[14] The share of individuals of intelligence type h is q in every period.

All individuals in the economy belong to one of four types: $\{e, h\}$, $\{e, l\}$, $\{w, h\}$, or $\{w, l\}$. The distribution of these types in period t and their corresponding precisions are given in Table 1, where m_{t-1} denotes the number (share) of entrepreneurs in the previous period. Clearly, individuals of type $\{e, h\}$ are always the group

best suited to be entrepreneurs. Similarly, types $\{w, l\}$ make the worst entrepreneurs. Consider the two intermediate groups, types $\{e, l\}$ and $\{w, h\}$. The intrinsic difficulty (the complexity) of being an entrepreneur is the factor determining who will be the best entrepreneurs. For low values of the standard deviation of x_t, given x_{t-1}, which in turn is given by $g\sigma$, type $\{e, l\}$ individuals are better fitted for entrepreneurial tasks than type $\{w, h\}$ individuals, whereas the opposite applies for high values of $g\sigma$. Clearly, there is always a unique positive value of $g\sigma$, such that the two groups are equally good entrepreneurs:

RESULT 1 Threshold Difficulty: *There is a unique value, denoted D^*, such that if $g\sigma$ is larger (smaller) than D^*, $P_{e,l}$ is smaller (larger) than $P_{w,h}$.*[15]

$$(7) \quad D^* = -\frac{\gamma}{2} + \frac{\sqrt{\gamma^2 + 4[\gamma/(\alpha - 1)]}}{2}.$$

C. Equilibrium Conditions

The model has two equilibrium conditions. First, given individual career choices, the labor market must clear. Second, given the wage established in the labor market, each individual chooses the career that maximizes his or her expected utility.

First, consider the labor-market equilibrium. This is a price of labor, which we express in terms of z_t, and a number of entrepreneurs m_t, such that labor supply equals labor demand. For a given number of entrepreneurs at t, labor supply is completely inelastic and equal to $(1 - m_t)$. Labor demand is a function of z_t, since each entrepreneur will hire e^{1/z_t} workers. In equilibrium, the number of workers demanded must equal the fixed supply of labor, that is,

$$(8) \quad m_t e^{1/z_t} = 1 - m_t \Rightarrow$$

$$m_t = \frac{1}{1 + e^{1/z_t}} \equiv L(z_t)$$

$$L'(z_t) > 0.$$

[12] Certainly, training makes it easier to solve problems, but by reducing the variance in the prior distribution, not by increasing the precision in the private signal. In other words, training makes problems easier.

[13] The model in Appendix B assumes a continuum of intelligence levels. The results do not change, but we can no longer provide analytical results.

[14] Once more, this is only for simplicity. As long as this correlation is smaller than unity, the qualitative results will hold.

[15] Missing proofs are available at request from the authors.
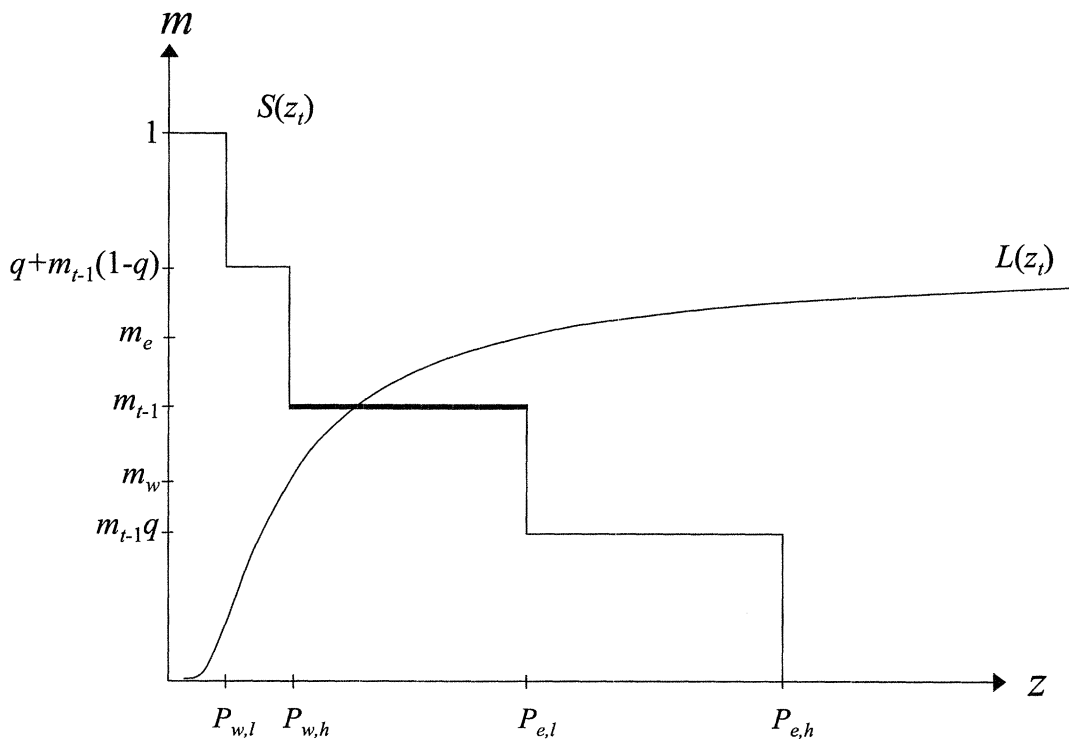
FIGURE 1. AN EQUILIBRIUM IF $g\sigma < D^*$

The labor-market equilibrium thus establishes a positive relationship, denoted $m_t = L(z_t)$, between the wage and the number of entrepreneurs.

The second equilibrium condition is that all agents choose the career that maximizes their individual expected utility. This condition also implies that the number of entrepreneurs is related to the wage. At sufficiently low wage levels, everybody prefers to be an entrepreneur, whereas at higher wage levels, an increasing number of types will come to prefer being workers. Individuals of type $\{w, l\}$ are always the first to prefer being workers as the wage increases and type $\{e, h\}$ individuals are always the last. As to which of the intermediate groups comes first hinges on whether $g\sigma$ is smaller than D^*. If $g\sigma$ is small, the intrinsic difficulty of being an entrepreneur is relatively low and the value of knowing the previous period's "best way" is high, which means that the type $\{w, h\}$ individuals are the first to prefer to be workers as the wage increases. We write the equilibrium sorting condition as

$$(9) \qquad m_t = S(z_t),$$

where $S(\cdot)$ is a step function with $S(0) = 1$ and $S(z) = 0$ if $z > P_{e,n}$. Each discontinuity occurs at the wage at which a category of individuals is indifferent between the two career choices.[16]

Clearly, Conditions (8) and (9) together are necessary and sufficient for equilibrium. The relations between m_t and z_t given by L and S are depicted in Figures 1 and 2. In both cases, the function $L(z_t)$ monotonically increases from zero and converges asymptotically to $\frac{1}{2}$, whereas $S(z_t)$ decreases in steps, from one to zero. This ensures the existence of an (unique) equilibrium (m_t and w_t) at t for any possible m_{t-1} and $g\sigma$.

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[16] $S(z)$ is fully specified in Appendix A.
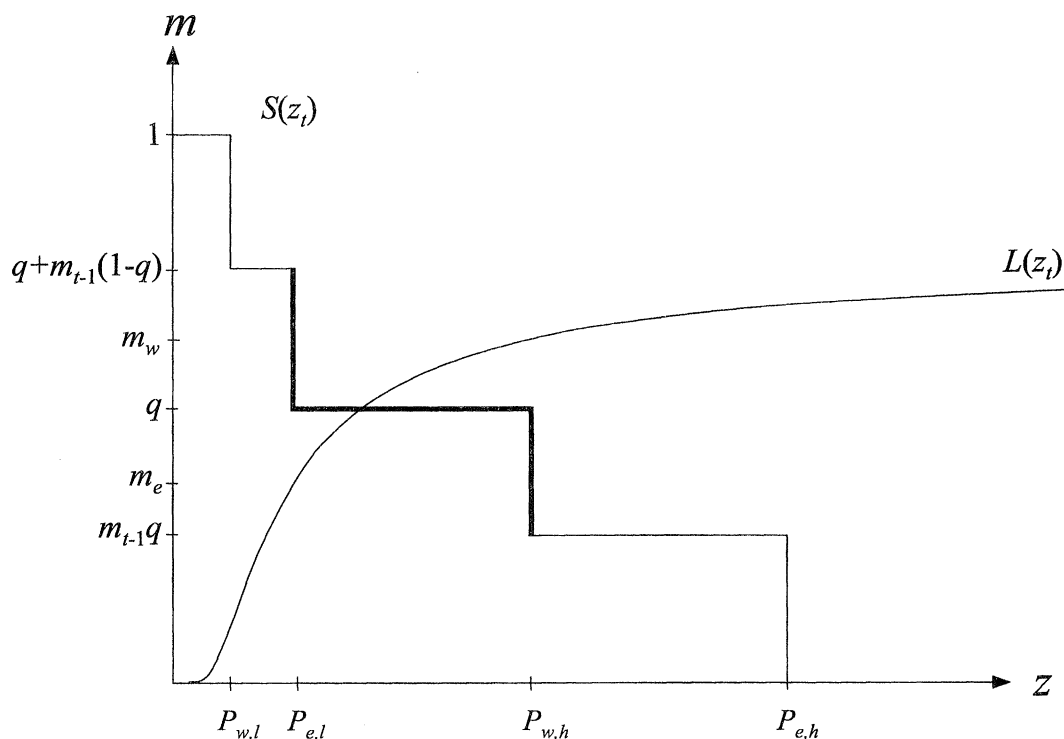
FIGURE 2. AN EQUILIBRIUM IF $g\sigma > D^*$

D. Steady State

Let us now focus on the steady state in the model. For this purpose, it is convenient to make two definitions:

Definition 1:

$$m_w(g\sigma) \equiv L(P_{w,h}) \equiv \frac{1}{1 + e^{\{[1/(\gamma + g\sigma)] + \alpha\}^{-1}}},$$

$$m_e(g\sigma) \equiv L(P_{e,l}) \equiv \frac{1}{1 + e^{[(1/g\sigma) + 1]^{-1}}}.$$

In words, $m_w(g\sigma)$ ($m_e(g\sigma)$) denotes the number of entrepreneurs that results in a labor-market equilibrium wage, making type $\{w, h\}$ ($\{e, l\}$) individuals indifferent between career choices. Clearly, if $g\sigma < D^*$, $m_w(g\sigma) > m_e(g\sigma)$, because if type $\{e, l\}$ individuals make better entrepreneurs than type $\{w, h\}$ individuals, they can accept a larger number of entrepreneurs before getting deterred from be-

ing entrepreneurs by too high a wage. If $g\sigma > D^*$, the opposite is true.

In a steady-state equilibrium, we require the number of entrepreneurs to be constant: $m_{t-1} = m_t = m$. Consider first the case when $g\sigma < D^*$. The two equilibrium relations $m_t = L(z_t)$ and $m_t = S(z_t)$ for this case are depicted in Figure 1. We see that one segment of $S(z_t)$ (marked with a thicker line) equals m_{t-1}. In a steady-state equilibrium, $L(z_t)$ must thus cross $S(z_t)$ at that segment. If $L(z_t)$ crosses $S(z_t)$ at any other segment, the resulting equilibrium value of m_t differs from m_{t-1}. At the steady-state equilibria, the equilibrium wage belongs to the interval $[P_{w,h}, P_{e,l}]$.

RESULT 2: *If $g\sigma < D^*$, m and z are a steady-state equilibrium if and only if $m \in [m_w(g\sigma), m_e(g\sigma)]$ and $m = L(z)$. There is no intergenerational mobility in these equilibria and the set of equilibria is dynamically stable.*

Now, consider the case when $g\sigma > D^*$. In a steady state, the wage can clearly not exceed

$P_{w,l}$. If it did, only type $\{e, h\}$ individuals would be entrepreneurs in the next period (i.e., m_t would equal qm_{t-1}, which is smaller than m_{t-1}). Furthermore, the steady-state wage cannot be lower than $P_{e,l}$. If it was, *both* all types $\{e, h\}$ and $\{e, l\}$ *and* all type $\{w, h\}$ individuals would become entrepreneurs in the next period [i.e., m_t would equal $m_{t-1} + q(1 - m_{t-1})$, which is larger than m_{t-1}].

We now have three cases, depending on the value of the parameter q. The first case arises if q is within the interval $[m_e, m_w]$. This case is depicted in Figure 2, where the steady-state equilibrium is at the point where $L(z_t)$ crosses $S(z_t)$ at the horizontal thick segment where $q = S(z_t)$. This is clearly the unique steady-state equilibrium in this case. All individuals with a high level of intelligence, but no other, are entrepreneurs [i.e., $m = q$ and $z = L^{-1}(q)$].

The second case arises if $q > m_w$. In this case, $L(z_t)$ crosses $S(z_t)$ at the vertical segment above $P_{w,h}$. This equilibrium replicates itself and is the unique steady state. In this case, the share of individuals of intelligence type h is so large that they cannot all become entrepreneurs in equilibrium. The wage is thus $P_{w,h}$, which makes type $\{w, h\}$ individuals indifferent between the career choices and some of them become workers and some entrepreneurs.

The third, and last, case arises if $q < m_e$. In this case, the total number of individuals of intelligence type h is so small that also some individuals with $i = l$ will be entrepreneurs. The wage is $P_{w,l}$, so that type $\{e, l\}$ individuals are indifferent between the two careers and some of them become workers and some entrepreneurs.

Our conclusions for the case when $g\sigma \geq D^*$ can now be summarized as follows:

RESULT 3: *If $g\sigma > D^*$, there is only one steady-state equilibrium for each value of $g\sigma$, with m and z given by:*

$$[m, z] = \begin{cases} [m_w(g\sigma), L^{-1}(m_w(\sigma))], \\ \qquad\qquad\text{if } q > m_w(g\sigma) \\ [q, L^{-1}(q)], \\ \qquad\text{if } m_w(g\sigma) \geq q \geq m_e(g\sigma) \\ [m_e(g\sigma), L^{-1}(m_e(g\sigma))], \\ \qquad\qquad\text{if } m_e(g\sigma) > q. \end{cases}$$

Intergenerational mobility is positive and the equilibrium is dynamically stable.

The intuition behind our results in this section is straightforward. When $g\sigma$ is low, parental upbringing (social assets) is more valuable than intelligence, for two reasons. First, low growth means that the world changes slowly. The information inherited by a child of an entrepreneur is thus highly relevant and useful. Second, the fact that the world is changing slowly implies that the inherent complexity of being an entrepreneur is low. Under such circumstances, a type $\{e, l\}$ individual will be able to make good decisions, which results in a steady state with zero intergenerational social mobility. However, the number of individuals with the advantage of being born to entrepreneurs obviously depends on the number of entrepreneurs in the previous period. The sorting and the labor-market equilibrium restrict the number of entrepreneurs in a steady state. If the number of entrepreneurs is larger than a certain value, the wage will be so high that not all children of entrepreneurs will want to be entrepreneurs themselves. If, on the other hand, the number of entrepreneurs is lower than a certain value, the wage will be so low that some children of workers also will want to be entrepreneurs. Any value of the number of entrepreneurs in this range is a steady state.

When $g\sigma$ is large, it is difficult to be an entrepreneur and the inherited information is of little value. Consequently, intelligent individuals have the upper hand. All intelligent individuals, but no others, will become entrepreneurs unless

- there are so many intelligent individuals that, if they were all entrepreneurs, the wage paid to workers would be so high that the children of workers with intelligence type h would be unwilling to become entrepreneurs, or if
- there are so few intelligent individuals that, if they were the only entrepreneurs, there would be so many workers that the equilibrium wage would be so low that the children of entrepreneurs with intelligence type l would prefer to be entrepreneurs.
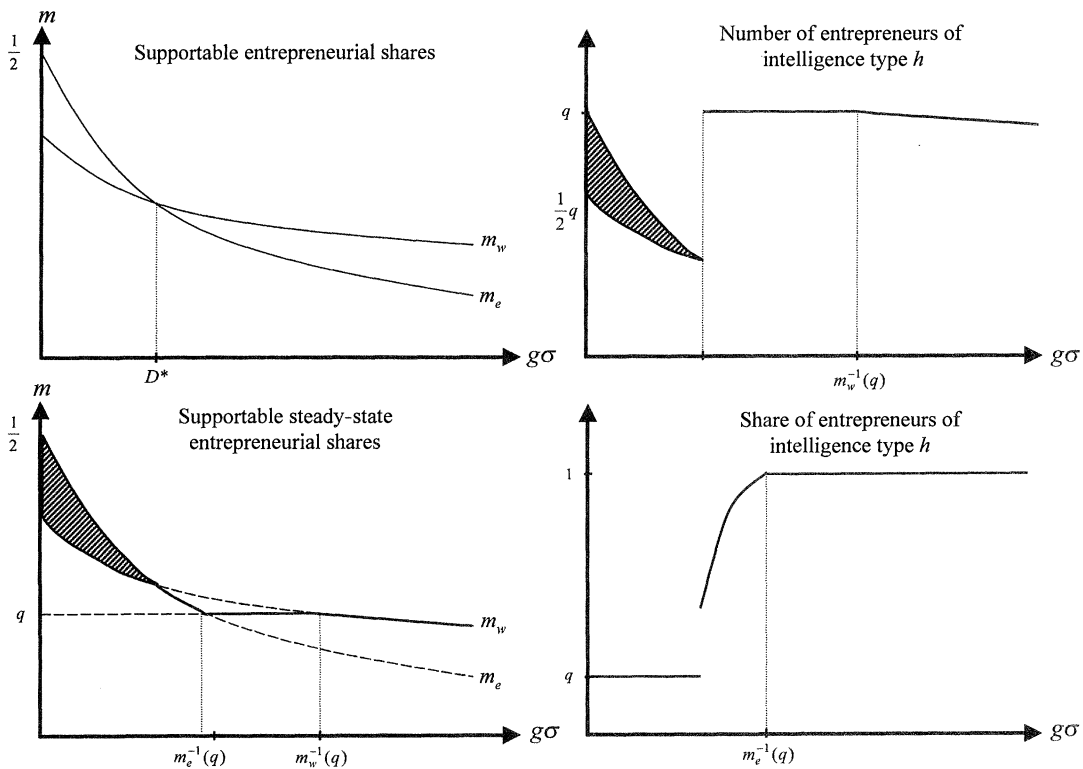
FIGURE 3. CORRESPONDENCES BETWEEN THE STEADY STATE EQUILIBRIUM SHARE OF ENTREPRENEURS, THE NUMBER AND SHARE OF ENTREPRENEURS OF TYPE h AND $g\sigma$

In the former (latter) case, the wage must be such that the type $\{w, h\}$ ($\{e, l\}$) individuals are indifferent between the career choices.

E. The Allocation of Intelligence

In the previous subsection, we established that for each level of intrinsic entrepreneurial difficulty, there is a corresponding set of steady-state equilibrium values of the number of entrepreneurs. Now, let us take a closer look at how the allocation of society's intellectual resources varies with the level of entrepreneurial difficulty, as measured by $g\sigma$. The upper left panel of Figure 3 depicts the functions $m_w(g\sigma)$ and $m_e(g\sigma)$. These are both decreasing, for as $g\sigma$ increases, it becomes more difficult to be an entrepreneur. A higher wage is then required to make individuals indifferent between the career choices: m_w and m_e asymptotically converge to $1/(1 + e^{\alpha})$ and $1/(1 + e)$, respectively.

Consider first the value of $g\sigma < D^*$. Then, m_e exceeds m_w, implying that intelligence is irrelevant for social sorting. All values of m such that $m_e \geq m \geq m_w$ constitute an equilibrium, which is represented by the shadowed area between the two curves in the bottom-left panel. In the panels to the right, it is seen that the number (share) of entrepreneurs of type h equals qm (q).

When $g\sigma$ equals D^*, m_w and m_e cross, there is a jump in social mobility, and all individuals of intelligence type h become entrepreneurs.[17] At D^*, the number of entrepreneurs of intelligence type h jumps from qm_e to q. The share of entrepreneurs of intelligence type h also jumps.

[17] In the depicted case, $q < m_w(D^*)$. If $q > m_w(D^*)$, the steady-state value of m is given by m_w to the right of the crossing, since type $\{w, h\}$ individuals must now be indifferent between the career choices. All entrepreneurs are then of intelligence type h.

The number of entrepreneurs is given by m_e, since the type $\{e, l\}$ individuals must be indifferent between the two career choices.

As $g\sigma$ increases further, the number of type $\{e, l\}$ individuals who become entrepreneurs falls and the share of entrepreneurs of intelligence type h thus increases. As long as some type $\{e, l\}$ individuals choose to be entrepreneurs, the number of entrepreneurs will equal m_e, which falls as $g\sigma$ continues to increase. Eventually, no type $\{e, l\}$ individual prefers to be an entrepreneur and only individuals of intelligence type h become entrepreneurs. From this value of $g\sigma$, denoted $m_e^{-1}(q)$, the number of entrepreneurs equals q. Eventually, $m_w(g\sigma)$ equals q [the value of $g\sigma$ at which this occurs is denoted $m_w^{-1}(q)$], unless $q < 1/(1 + e^\alpha)$. For larger values of $g\sigma$, all type $\{e, h\}$, but only some type $\{w, h\}$, individuals are entrepreneurs. Type $\{w, h\}$ individuals must thus be indifferent between career choices, so $m = m_w$. No individual of intelligence type l is an entrepreneur, but the number of entrepreneurs is falling because a higher wage is required to motivate individuals to take on the increasingly difficult task of being entrepreneur.

Two mechanisms create a link between the allocation of individuals of intelligence type h and entrepreneurial difficulty. The first is that, when it becomes more difficult to be an entrepreneur, a smaller share of the population will become entrepreneurs. This mechanism is responsible for the nonincreasing regions of the correspondence between the number of entrepreneurs of type h and the complexity ($g\sigma$).

The second mechanism results from the relation between entrepreneurial complexity and intergenerational social mobility. As we have seen, an increase in the former reduces the relative advantage of having an entrepreneur as a parent. This increases social mobility, the number of entrepreneurs of intelligence type h, and the average intelligence of entrepreneurs. This mechanism is responsible for a positive relation between the number of entrepreneurs of intelligence type h and entrepreneurial difficulty. In our model, this relation takes the form of a jump at $g\sigma = D^*$. The discontinuity results from the assumption of discrete levels of intelligence. In Appendix B we present a model with a continuous distribution of intelligence, in which case this mechanism produces a positive and continuous relation between entrepreneurial difficulty and the intelligence of entrepreneurs.

II. Mobility and Endogenous Growth

We have now established some relationships between entrepreneurial difficulty, driven by exogenous technological growth, and how the social sorting mechanism allocates individuals over different jobs. In the introduction, we argued that this allocation has important implications for technological growth. To analyze this, we need to endogenize technological growth—the purpose of this section. We leave the mechanisms introduced in the previous section, whereby individuals sort themselves over jobs, unchanged.

To endogenize growth, we assume that the average technology used in a period becomes the base for further technological advancements in the following period. Each entrepreneur can choose to use this base technology or any level of improvement. This choice is made under a trade-off between productivity and difficulty—more technological advancements also mean a higher margin for mistakes. As we will see, this trade-off is affected by the intelligence type of the entrepreneur—more intelligent entrepreneurs choose more innovation. Aggregate growth thus depends on the outcome of the social sorting mechanism. More specifically, as entrepreneurial difficulty increases, driven by previous high growth, the social sorting mechanism tends to pick intelligent individuals to be entrepreneurs. This, in turn, implies that society will also enjoy high growth in the future.

A. The Choice of Technology

Let us use b to denote the base technology in a period. This is established as the average technology used by the previous generation of entrepreneurs.[18] In analogy with the assumptions in the previous section, we assume that the social heritage provides information regarding

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[18] Any convex combination of the technologies previously used would do. A more explicit model could involve multiple sectors. Then, each entrepreneur may have an aggregate of the different technologies used by the previous generation as input in the production/innovation process. Such an extension is left for future work.

the right decision associated with technology b, denoted x_b. In addition, intelligence helps by providing a signal on the right decision associated with technology b. Below, we specify how the information individuals have on x_b is related to the average technological advancements in the previous period. Basically, we assume that if only small technological advancements were made in the previous period, entrepreneurs have learned how to use the technology well and can thus provide accurate information on x_b to their children. For the time being, we take the beliefs on x_b by an individual of social background p and intelligence of type i as given and assume that these are normally distributed with a precision $(V_{p,i})^{-1}$.[19]

We then turn to how the beliefs about the optimal actions associated with more productive technologies are formed. Each increment in productivity adds uncertainty to the optimal decision. This addition is given by the Wiener increment dz, as specified in equation (5). If a particular entrepreneur chooses to increase the log(productivity) from b to $b + g$, the added variance is given by $g\sigma$ as in equation (6).

Entrepreneurs use their intelligence to infer further information about each dz. This inference is represented by a signal that is normally distributed around dz, with a precision that depends on individual intelligence. Regardless of intelligence, the precision decays in g at the rate $\beta > 0$.[20] More precisely, the precision for an individual with intelligence type i on a dz at a distance h from b is

$$(10) \qquad\qquad e^{\alpha_i - \beta h}.$$

We set $\alpha_l = 1$ and denote $\alpha_h \equiv \alpha > 1$. Combining equations (5) and (10), we can express the beliefs about x_{b+g} held by an individual with social background p and intelligence i as follows: x_{b+g} is normally distributed with a variance given by

$$(11) \quad V_{p,i} + \int_0^g \left(\frac{1}{\sigma} + e^{\alpha_i - \beta h} \right)^{-1} dh$$

$$= V_{p,i} + \frac{\sigma}{\beta} \ln\left(\frac{\sigma e^{\alpha_i} + e^{\beta g}}{1 + \sigma e^{\alpha_i}} \right).$$

Using the utility and production functions of the previous section, we can now state the utility of an entrepreneur as

$$(12) \quad U^e(p, i) = \max_{l,a,g}\{\ln(2(e^{b+g}l))^{1/2} - wl$$

$$- E(x_{b+g} - a)^2\}$$

$$= \max_g\bigg\{ g + b - \ln w - V_{p,i}$$

$$- \frac{\sigma}{\beta} \ln\left(\frac{\sigma e^{\alpha_i} + e^{\beta g}}{1 + \sigma e^{\alpha_i}} \right)\bigg\}.$$

The first-order condition for g and l implies that

$$(13) \quad g = \frac{\alpha_i + \ln[\sigma/(\sigma - 1)]}{\beta} \equiv g(\alpha),$$

$$l_d = \frac{e^{b + g(\alpha_i)}}{w^2} \equiv l_d(w; \alpha).$$

with $\partial g/\partial\alpha > 0$ and $\partial l_d/\partial\alpha > 0$. In words, a higher level of intelligence increases the chosen level of technological innovation and labor demand. On the other hand, neither the amount of innovations nor labor demand depends on social background. In equation (13), we see that we need to impose the restriction $\sigma > 1$ to bound growth. Otherwise, further innovation would always increase the expected return more than the cost of the associated increase in risk.

B. Social Mobility

Now, let us turn to the conditions determining who chooses to become an entrepreneur. As in Section I, for sufficiently low (high) wages, all (no) agents would choose to become entrepreneurs. For each type, there is a threshold such that if the wage is higher than that threshold, the individual prefers to be a worker, and

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[19] To reduce notation, we discard time subscripts on variables where this can be done without causing ambiguity. It should be understood that all time-varying variables without time subscripts are measured at time t.

[20] It is straightforward to introduce heterogeneity with respect to β.

we show that this threshold depends on both social background and intelligence type. More important, exactly like in the previous section, the threshold for individuals of type $\{e, l\}$ will be more sensitive to increases in entrepreneurial complexity, driven by previous growth decisions, than will the threshold for individuals of type $\{w, h\}$.

To simplify the exposition, let us use the following definition:

$$(14) \quad X(\alpha_i) \equiv g(\alpha_i) - \frac{\sigma}{\beta} \ln \frac{\sigma e^{\alpha_i} + e^{\beta g(\alpha_i)}}{1 + \sigma e^{\alpha_i}} .$$

$X(\alpha_i)$ represents the net gain in expected utility an entrepreneur can get by choosing g optimally, rather than setting it to zero; of course, this gain is nonnegative and zero if and only if optimal g is zero. More important, it increases in α, since a more intelligent entrepreneur is better able to take advantage of the opportunity to innovate. Furthermore, as optimal growth, $X(\alpha_i)$ is independent of social background.

An individual with social background p and intelligence type i will choose to be an entrepreneur if:

$$(15) \qquad X(\alpha_i) + b - 2 \ln w > V_{p,i}.$$

This expression is the counterpart to equation (3) in Section I. The only difference is that there is an additional advantage of being of intelligence type h—namely, to take advantage of the possibility to choose more advanced technologies. Note that the left-hand side of (15) depends on the intelligence level only, whereas the right-hand side also depends on social background. To determine who will become an entrepreneur, we need to specify the individual's knowledge regarding the utilization of the base technology (measured by $V_{p,i}$).

C. Growth Externalities

As in Section I, the agents' information regarding the best usage of the base technology comes from two sources: their intelligence and their social background. Their intelligence provides them with a signal with a precision denoted $\tilde{\alpha}_i$, and we assume $\tilde{\alpha}_h > \tilde{\alpha}_l$. As in the previous section, we reduce notation by normalizing $\tilde{\alpha}_l$ to unity and denoting $\tilde{\alpha}_h \equiv \tilde{\alpha}$.

We now specify the intergenerational informational spillover, that is, the intergenerational transmission of information on x_b. In analogy to the previous section, we assume that children of entrepreneurs receive full information regarding the base technology used by their parents. Children of workers, on the other hand, get a signal that is potentially imperfect and with a variance denoted $\gamma \geq 0$. Thus, increases in γ increase the parametric informational disadvantage of children of workers.

In period t, let $\bar{g}_{t-1}$ denote the average (aggregate) rate of technological growth in the previous period. Before using the information provided by their intelligence, the previous assumptions imply that children of entrepreneurs and workers believe that x_b is normally distributed with variances $\sigma \bar{g}_{t-1}$ and $\gamma + \sigma \bar{g}_{t-1}$, respectively. Combining this information with the intelligence signal yields

$$(16) \qquad V_{e,i}(\bar{g}_{t-1}) = \left(\frac{1}{\sigma \bar{g}_{t-1}} + \tilde{\alpha}_i \right)^{-1} ,$$

$$V_{w,i}(\bar{g}_{t-1}) = \left(\frac{1}{\gamma + \sigma \bar{g}_{t-1}} + \tilde{\alpha}_i \right)^{-1} .$$

It is now clear that type $\{e, h\}$ individuals are always in the best position to become entrepreneurs and type $\{w, l\}$ individuals are in the worst position. Type $\{e, l\}$ individuals are better entrepreneurs than type $\{w, h\}$ individuals if and only if

$$(17) \qquad X(\alpha) - X(1)$$

$$< V_{w,h}(\bar{g}_{t-1}) - V_{e,l}(\bar{g}_{t-1}).$$

Furthermore,

RESULT 4 Growth Threshold: *There is a level of growth* $g^* \geq 0$ *such that if* $\bar{g}_{t-1}$ *is larger than* g^*, *the expected utility of an individual of type* $\{w, h\}$ *is higher than the expected utility of an individual of type* $\{e, l\}$ *if they are both entrepreneurs and face the same wage. If* $g^* > 0$, *the opposite is true if* $\bar{g}_{t-1}$ *is smaller than* g^*. *Furthermore,* $\partial g^*/ \partial \gamma > 0$, $\partial g^*/\partial \alpha < 0$, *and* $\partial g^*/\partial \tilde{\alpha} < 0$.

It is straightforward to verify this result by noting that an increase in $\bar{g}_{t-1}$ and a reduction in γ reduce the informational disadvantage of children of workers [reducing the right-hand side of (17)], while leaving the benefit of being of intelligence type h [the left-hand side of (17)] unaffected. On the other hand, an increase in α increases the left-hand side and an increase in $\tilde{\alpha}$ decreases the right-hand side. A particularly simple example is the case when $\gamma = \infty$, and $\tilde{\alpha}_l = 0$. In this case, the right-hand side of (17) simplifies to $\tilde{\alpha}^{-1} - \sigma \bar{g}_{t-1}$.

In parallel to the results in Section I, we thus find that if the intrinsic difficulty of being an entrepreneur, driven by $\bar{g}_{t-1}$, is sufficiently high, type $\{w, h\}$ individuals are better suited for the task than type $\{e, l\}$ individuals, whereas the opposite may occur for low levels of difficulty. The reason for the existence of the threshold is identical to the reason in Section I: an increase in previous growth increases the intrinsic difficulty of being an entrepreneur and reduces the relevance of the information inherited by children of entrepreneurs. Thus, the relative advantage of being of intelligence type h versus having a parent who was an entrepreneur increases in growth.[21]

The labor-market equilibrium under endogenous growth will share all important features with the equilibrium described in Section I, subsection C. The same two equilibrium conditions must be satisfied—individuals choose occupation optimally and, given the number of entrepreneurs, the wage clears the labor market. As shown in Appendix B, the occupation choice defines m_t as a decreasing step-function of the wage and the labor-market equilibrium m_t as a strictly increasing continuous function of w_t. This ensures the existence of a unique equilibrium for any combination of growth rates and number of entrepreneurs in the previous period. Furthermore, for any combination of constant values of $g(\alpha)$ and $g(1)$, there exist steady-state values of m and w. As in Section I, there is a set

of steady states for the case of no social mobility and unique values under positive social mobility. In both cases, however, the steady state composition of entrepreneurs is uniquely determined.

We now have the main result of this section.

RESULT 5: *Growth feedback: Let $m(\bar{g}_{t-1})$ denote a steady-state value of the number of entrepreneurs associated with a constant growth rate $\bar{g}_{t-1}$. Then there exists a nondecreasing function $G(\bar{g}_{t-1})$ such that, if m_{t-1} equals $m(\bar{g}_{t-1})$, $\bar{g}_t$ is given by $G(\bar{g}_{t-1})$. $G(\bar{g}_{t-1})$ is discontinuous at $G(g^*)$.*

The previous result is depicted in Figure 4. One curve is simply a 45-degree ray from the origin. The other curve is the function $G(\bar{g}_{t-1})$. Clearly, $G(0)$ equals $qg(\alpha) + (1 - q)g(1)$ if $g^* > 0$ and is constant until $\bar{g}_{t-1} = g^*$, where it jumps. As we know, this results from the fact that the level of entrepreneurial complexity has then increased to the point at which the advantage of being of intelligence type h outweighs the advantage of having entrepreneurial social background. The resulting change in the allocation of individuals over jobs is more efficient from a growth perspective, so growth increases.

As $\bar{g}_{t-1}$ increases from g^*, an increasing number of entrepreneurs of intelligence type l find it too difficult to be entrepreneurs, which raises the average intelligence of the remaining entrepreneurs and aggregate growth increases. This continues until all entrepreneurs are of intelligence type h and $\bar{g}_t$ equals $g(\alpha)$.[22]

Clearly, we have steady states where the lines cross. They are locally stable to variations in g_{t-1} when the second curve intersects the 45-degree ray from above. Note that we know from Result 4 and equation (13) that, by varying the informational disadvantage of children of workers (i.e., changing γ), we can shift g^* without affecting the aggregate growth rate in the no-mobility steady state. This fact, in combination with the discontinuity at g^*, ensures that two stable steady-state endogenous growth equilibria generically exist.

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[21] It is straightforward to extend the analysis by assuming that children of entrepreneurs receive a signal on each *innovation* taken in the previous period. If the precision in these signals is lower than $[(1 + \tilde{\alpha}\gamma)^2 - 1]/\sigma$, the informational disadvantage of individuals of type $\{w, h\}$ monotonically falls in $\bar{g}_{t-1}$. Furthermore, as long as the precision is finite, Result 4 holds.

[22] As in Section I, if q is sufficiently low, only individuals of type h become entrepreneurs if $\bar{g}_{t-1} > g^*$. Then G is simply a step function.
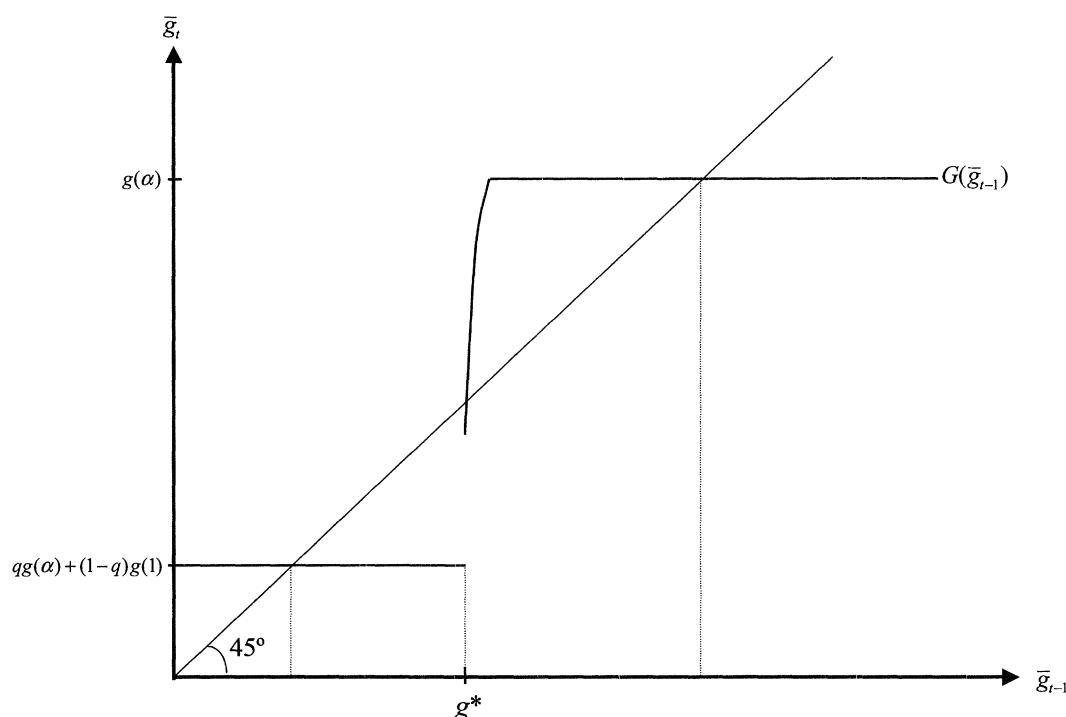
FIGURE 4. MULTIPLE STEADY STATES

III. Conclusion

The presented model is certainly very stylized. Nevertheless, we think it describes important real-world mechanisms relating growth, intergenerational social mobility, and the demands on individuals in different social positions. Different individuals have different aptitudes with regard to creating ideas and finding better ways of production. The extent to which society can take advantage of this depends on the efficiency of the social sorting mechanism. Furthermore, it seems very unlikely that the full social value of creating ideas and inspiring other individuals is captured by those producing the ideas and the inspiration. A Walrasian labor market will thus assign jobs in a way that may hamper economic growth. The inefficiencies of the labor market, however, can be mitigated when the growth rate is higher, since high growth rates reduce the importance of the transmission of social advantages from parents to their

children, thus making individuals compete on more equal grounds.

This paper models a single economy and it must thus be extended to be consistent with the growing body of literature showing that growth in different countries appears to converge while incomes diverge toward a bimodal distribution.[23] In a working paper version of this paper, we make two additional assumptions: (i) the best technology used in the world "trickles down" to other countries with a time lag, and (ii) at that lagged date, it can be used without much need for intelligence. These assumptions will have the single effect of making the steady-state growth rate of all countries coincide. The different steady states instead imply different income levels.

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[23] See Steven N. Durlauf and Paul A. Johnson (1995), Oded Galor (1996), Charles I. Jones (1997), Lant Pritchett (1997), and Danny Quah (1997).

In our model, each entrepreneur is free to choose the level of technological growth in his/her firm. An interesting extension would be to include a policy parameter that affects this choice—for example, taxes on or subsidies to the return to innovation. Such a policy could be made endogenous by introducing a political choice mechanism. As in Per Krusell and José Víctor Ríos-Rull (1996), different individuals would then have different vested interests with regard to this policy.

Another interesting extension would be to include multiple production sectors and allow different levels of technological growth in different sectors, which would render possible a test of the model on cross-sectional data on industries (as in, e.g., Bartel and Sicherman (1999)].

The analysis in this paper has been positive and the outcome of the model is determined by history. To allow policy recommendations, we should include policy parameters in the analysis. Forecasting the results of this, we conjecture that improvements in, for example, the public school system would lead to a reduction in the difference in informational disadvantage of coming from less-privileged homes. The resulting improvement in social selection could prove to be self-enforcing by leading to a more dynamic society in which social background is less important for social sorting. Thus, better public schooling could be a way out of a low-growth trap. The feedback mechanism can also cause multiplier effects, whereby policy changes lead to long-run effects larger than the initial ones. This might also occur in the case when multiple steady states do not exist.

Furthermore, increases in what we have called the intrinsic difficulty of being (and becoming) an entrepreneur may have positive effects on intergenerational social mobility, which could improve the allocation of the nation's stock of innate ability. Such changes could occur in many different ways, of which higher intellectual standards in schools, increased market competition, and opening the economy to trade and to foreign influences are a few examples. Such changes would have not only negative effects on the level of output but also positive effects on growth. The development of a model better suited for making welfare comparisons is left for future research.

APPENDIX A

Complete Description of the Equilibrium Sorting Condition

$S(z)$ is given by

(A1)

$$
\begin{cases}
1 & \text{if } P_{w,l} > z_t \\
[q + m_{t-1}(1-q), 1] & \text{if } z_t = P_{w,l} \\
q + m_{t-1}(1-q) & \text{if } P_{w,h} > z_t > P_{w,l} \\
[m_{t-1}, q + m_{t-1}(1-q)] & \text{if } z_t = P_{w,h} \\
m_{t-1} & \text{if } P_{e,l} > z_t > P_{w,h} \\
[m_{t-1}q, m_{t-1}] & \text{if } z_t = P_{e,l} \\
m_{t-1}q & \text{if } P_{e,h} > z_t > P_{e,l} \\
[0, m_{t-1}q] & \text{if } z_t = P_{e,h} \\
0 & \text{if } z_t > P_{e,h}
\end{cases}
$$

if $g\sigma < D^*$ and by

$$
\begin{cases}
1 & \text{if } P_{w,l} > z_t \\
[q + m_{t-1}(1-q), 1] & \text{if } z_t = P_{w,l} \\
q + m_{t-1}(1-q) & \text{if } P_{e,l} > z_t > P_{w,l} \\
[q, q + m_{t-1}(1-q)] & \text{if } z_t = P_{e,l} \\
q & \text{if } P_{w,h} > z_t > P_{e,l} \\
[m_{t-1}q, q] & \text{if } z_t = P_{w,h} \\
m_{t-1}q & \text{if } P_{e,h} > z_t > P_{w,h} \\
[0, m_{t-1}q] & \text{if } z_t = P_{e,h} \\
0 & \text{if } z_t > P_{e,h}
\end{cases}
$$

if $g\sigma \geq D^*$.

Labor-Market Equilibrium Under Endogenous Growth

The introduction of a choice of technology implies that the labor demand by an individual entrepreneur depends on his or her intelligence type, as shown in equation (13). Thus, the equilibrium wage on the labor market now depends on the share of intelligent entrepreneurs and on social mobility. A larger share of entrepreneurs of intelligence type h implies a higher labor demand and a smaller number of entrepreneurs for each wage.

As in Section I, subsection C, the number of individuals preferring to be entrepreneurs will be a decreasing step function of the labor market wage. If $\bar{g}_{t-1} < g^*$, the horizontal sections, as in (A1), will be given by 1, $q + m_{t-1}(1 -$

q), m_{t-1}, $m_{t-1}q$, and zero. If $\bar{g}_{t-1} \geq g^*$, as in (A1), they will instead be given by 1, $q + m_{t-1}(1 - q)$, q, qm_{t-1}, and zero.

The labor-market equilibrium curve also depends on whether $\bar{g}_{t-1} > g^*$. Consider first the case when $\bar{g}_{t-1} > g^*$. Then, the labor-market equilibrium curve will consist of two segments:

(A2) $1 - m_t$

$$= \begin{cases} m_t l_d(w_t; \alpha), & \text{if } m_t \leq q \\ q l_d(w; \alpha) + (m_t - q) l_d(w_t; 1), \\ \qquad\qquad\qquad \text{if } m_t > q. \end{cases}$$

Clearly, this implies that we can write m_t as a continuous and strictly increasing function of w starting at zero and approaching unity as w goes to infinity.

In the case $\bar{g}_{t-1} < g^*$, we will instead have four segments. In the first, only type $\{e, h\}$ individuals are entrepreneurs. This segment covers values of m_t between zero and qm_{t-1}. The next segment covers values of m_t between qm_{t-1} and m_{t-1}. Here, all type $\{e, h\}$ and some type $\{e, l\}$ individuals are entrepreneurs. For values of m_t between m_{t-1} and $m_{t-1} + q(1 - m_{t-1})$, all type e and some type $\{w, h\}$ individuals are entrepreneurs and, finally, for m_t larger than $m_{t-1} + q(1 - m_{t-1})$ all individuals of intelligence type h and some type $\{w, l\}$ ones are entrepreneurs. Thus, the right-hand side of the previous equation now becomes

(A3)

$$\begin{cases} m_t l_d(w; \alpha), & \text{if } m_t \leq qm_{t-1}, \\ qm_{t-1} l_d(w; \alpha) \\ \quad + (m_t - qm_{t-1}) l_d(w; 1), \\ \qquad\qquad \text{if } m_{t-1} \geq m_t > qm_{t-1}, \\ (m_t - (1 - q)m_{t-1}) l_d(w; \alpha) \\ \quad + (1 - q)m_{t-1} l_d(w; 1), \\ \quad \text{if } m_{t-1} + q(1 - m_{t-1}) \geq m_t > m_{t-1}, \\ q l_d(w; \alpha) + (m_t - q) l_d(w; 1), \\ \qquad\qquad \text{if } m_t > q + q(1 - m_{t-1}), \end{cases}$$

which also produces a continuous and upward-sloping relation between m_t and w_t, starting at zero and approaching unity as w goes to infinity. Thus, the basic properties of labor-market equilibrium in Section II are maintained. It is

also straightforward to show that the basic stability properties carry over to the endogenous growth case. In particular, as in Section I, subsection C, we have three cases for the steady state when $\bar{g}_{t-1} > g^*$, depending on the level of q. If it is high (low), the wage has to make the $\{w, h\}$ ($\{e, l\}$) type indifferent between the career choices. In a middle range, the steady state is such that all individuals of intelligence type h, but no others, are entrepreneurs. In either case, the equilibrium is independent of small variations in m_{t-1}, around the steady state.

APPENDIX B: AN ALTERNATIVE MODEL

Here, we describe a model closely resembling the model in Section I. In this model, we assume that x_t follows an AR(1) process, with autocorrelation $\rho \in [0, 1]$, and σ denoting the variance of the innovation. Setting $\rho = 1$ produces equation (6). We assume children of entrepreneurs know x_{t-1}, whereas children of workers know only the unconditional distribution. This is equivalent to setting $\gamma_w = \sigma/(1 - \rho)$ in Section I. In addition, there are three differences that make it impossible to solve this model analytically. First, instead of having only two levels of intelligence, we assume that the level of intelligence, denoted q, is continuous with a distribution represented by a distribution function $F(q)$. As previously, there is an individual signal on x_t with a precision given by q_i.

Second, we assume that the entrepreneurial error affects output rather than profits. The profit function of a firm is thus

(B1) $\pi = \theta^{-1} e^{-[(x-a)^2/2]} e^r l^\theta - wl,$

where θ is the labor share of income. Let the precision in the beliefs about x by an individual with a level of intelligence q_i and with a parent who had occupation $p \in \{w, e\}$ be denoted $P(p, i)$. Solving for the maximum of expected profits over l, we find that the labor demand in a firm run by an individual of type $\{p, i\}$ is given by

(B2) $l_d(w/e^r, p, q_i; \sigma)$

$$= \left(\sqrt{\frac{1}{1 + P(p, i)^{-1}}} \, \frac{e^r}{w} \right)^{1/(1-\theta)},$$

and expected profit

(B3) $E\pi(e^r, w/e^r, p, q_i; \sigma)$

$$= e^r \left(\sqrt{\frac{1}{1 + P(p, i)^{-1}}} \frac{e^r}{w} \right)^{\theta/(1-\theta)}.$$

Third, we assume risk neutrality. Then, each individual chooses to become a worker (an entrepreneur) if the wage is higher (lower) than the expected profit. The threshold level of intelligence is determined by the condition that the expected profit equals the wage, which makes the (risk-neutral) agent indifferent between the two choices.

(B4) $\pi(e^r, w/e^r, p, q_i; \sigma)$

$$= e^r \left(\sqrt{\frac{1}{1 + P(p, i)^{-1}}} \frac{e^r}{w} \right)^{\theta/(1-\theta)} = w.$$

Solving this for the threshold precision $\bar{P}(w/e^r)$ yields

(B5) $\bar{P}(w/e^r) = \left[\left(\frac{w}{e^r} \right)^{2/\theta} - 1 \right]^{-1}.$

We can then find the threshold intelligence levels for the two types of parental background, denoted $\tilde{q}_e$ and $\tilde{q}_w$:

(B6) $\tilde{q}_e(w/e^r; \sigma) = \bar{P}(w/e^r) - \frac{1}{\sigma},$

and

(B7) $\tilde{q}_w(w/e^r; \sigma) = \bar{P}(w/e^r) - \frac{1-\rho}{\sigma}.$

Now, consider the labor market. The supply of workers is the number of entrepreneurs' children with a level of intelligence lower than $\tilde{q}_m$ and the number of workers' children with a level of intelligence lower than $\tilde{q}_w$. This means that the aggregate labor supply in period t is

(B8) $L_s(w/e^r, m_{t-1}; \sigma)$

$$= m_{t-1} F(\tilde{q}_e) + (1 - m_{t-1}) F(\tilde{q}_w).$$

We also have that

(B9) $m_t = m_{t-1}(1 - F(\tilde{q}_m))$

$$+ (1 - m)(1 - F(\tilde{q}_w)).$$

The aggregate labor demand is given by

(B10) $L^d(w/e^r, m_{t-1}; \sigma)$

$$= m_{t-1} \int_{\tilde{q}_e}^{\infty} l_d(w/e^r, q_i, e; \sigma) \, dF(q)$$

$$+ (1 - m_{t-1}) \int_{\tilde{q}_m}^{\infty} l_d(w/e^r, q_i, w; \sigma) \, dF(q).$$

In a steady state equilibrium, it is required that

(B11) $m_t = m_{t-1},$

$$L^d(w/e^r, m_{t-1}\sigma) = L^s(w/e^r, m_{t-1}; \sigma),$$

which defines a steady-state value of m and a corresponding steady-state level of w/e^r.

Now, let us specify some parameters to illustrate the behavior of the model. We have used $\theta = 0.5$, $\rho = 0.5$, and set $F(q) = q$, so that $q \in [0, 1]$. This means that the aggregate stock of intelligence in the economy is $\frac{1}{2}$.

The results are depicted in Figure 5. The top left panel shows the cutoff level of intelligence such that all individuals with a level of intelligence lower than that level prefer to be workers. We see that for low enough σ, all children of entrepreneurs choose to become entrepreneurs and all children of workers to become workers— intergenerational mobility is zero. As σ increases, innate assets become relatively more important. The cutoff levels of intelligence for the two groups thus become closer and approach the same level at
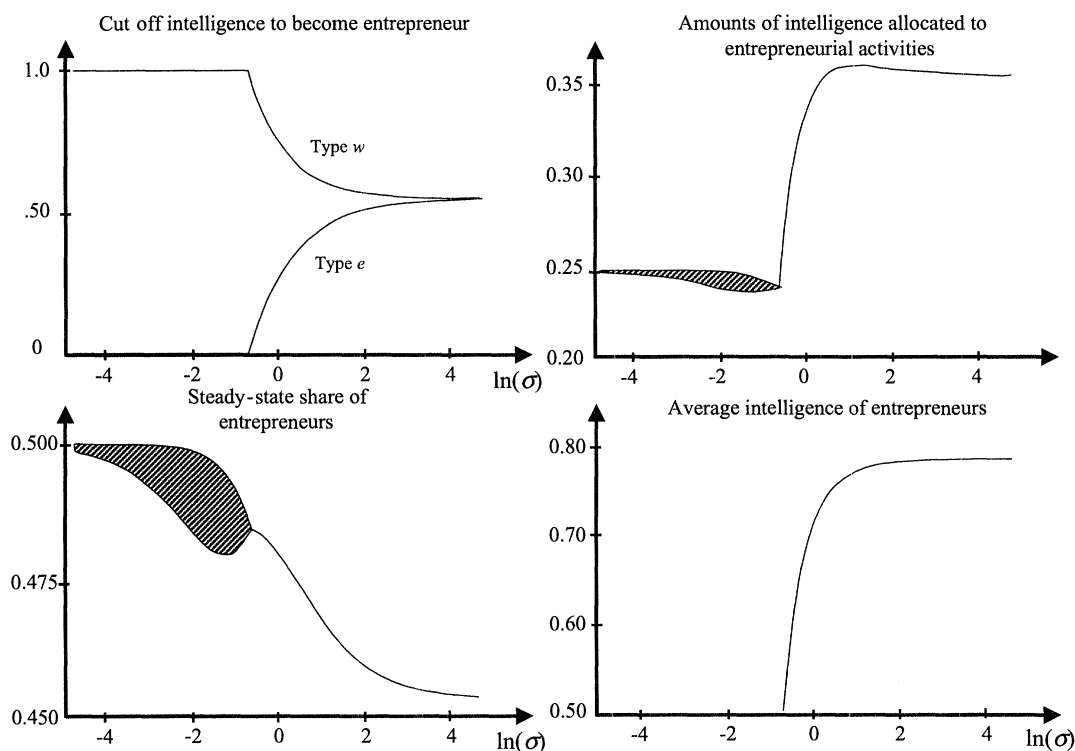
FIGURE 5. GROWTH STEADY STATES

around 0.55. The bottom left panel shows the number of entrepreneurs that can be sustained in a steady state equilibrium. For low values of σ, there is a multiplicity of equilibria for the same reason as in the model in the main text; here, there is a range of wages such that neither the children of entrepreneurs nor the children of workers want to pursue a career different from that of their parents. Any level of m that produces a wage within this range is a steady-state equilibrium. Above the level of σ at which social mobility starts to become operative, there is a single steady-state equilibrium for each level of σ.

The increase in intergenerational social mobility, incurred by an increase in σ, raises the average level of intelligence among entrepreneurs. This is shown in the bottom right panel of Figure 5. The amount of intelligence among entrepreneurs has a shape very similar to the one depicted in Figure 3. For low values of σ, no more than half the stock of intelligence is allocated to entrepreneurial positions. At the point of σ at which social

mobility becomes operative, this share increases quickly, since sorting becomes more efficient. Then, the amount of intelligence allocated to entrepreneurial positions starts falling slowly, reflecting that it becomes more difficult for everybody to be an entrepreneur. Here, as in the model in the main text, two mechanisms working in opposite directions create a nonmonotonous relation between σ and the amount of intelligence allocated to entrepreneurial positions. The first is responsible for the downward slope for low and high levels of σ. The other creates an intermediate range in which it increases rapidly, but not discontinuously, as in the model in Section I.

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