$$p = (1 - \lambda)s + \lambda t \tag{1}$$

$$\mathbf{J}(\lambda \mathbf{f}) = \lambda \mathbf{J}(\mathbf{f}) + \mathbf{f} \otimes \nabla \lambda \tag{2}$$

$$p = s - \lambda(s - t) \tag{3}$$

$$\mathbf{J}(p) = \mathbf{J}(s) - \mathbf{J}(\lambda(s - t)) = \mathbf{J}(s) - \lambda \mathbf{J}(s - t) - (s - t) \otimes \nabla \lambda \tag{4}$$

$$\nabla\Big(\frac{f}{g}\Big) = \frac{g\nabla f - f\nabla g}{g^2} \tag{5}$$

$$\lambda = \frac{\langle s - \tilde{p}, s - t\rangle}{\|s - t\|^2} = \frac{\langle s, s - t\rangle}{\|s - t\|^2} - \frac{\langle \tilde{p}, s - t\rangle}{\|s - t\|^2} \tag{6}$$

$$\nabla\lambda = \nabla\frac{\langle s, s - t\rangle}{\|s - t\|^2} - \nabla\frac{\langle \tilde{p}, s - t\rangle}{\|s - t\|^2} = \frac{\|s - t\|^2\nabla\langle s, s - t\rangle - \langle s, s - t\rangle\nabla\|s - t\|^2}{\|s - t\|^4} \tag{7}$$

$$- \frac{\|s - t\|^2\nabla\langle \tilde{p}, s - t\rangle - \langle \tilde{p}, s - t\rangle\nabla\|s - t\|^2}{\|s - t\|^4} \tag{8}$$

$$= \frac{\nabla\langle s, s - t\rangle}{\|s - t\|^2} - \frac{\langle s - \tilde{p}, s - t\rangle\nabla\|s - t\|^2}{\|s - t\|^2\|s - t\|^2} - \frac{\nabla\langle \tilde{p}, s - t\rangle}{\|s - t\|^2} \tag{9}$$

$$= \frac{\nabla\langle s, s - t\rangle - \lambda\nabla\|s - t\|^2 - \nabla\langle \tilde{p}, s - t\rangle}{\|s - t\|^2} \tag{10}$$

$$\nabla\langle \mathbf{f}, \mathbf{g}\rangle = \mathbf{g}^{\mathrm{T}}\mathbf{J}(\mathbf{f}) + \mathbf{f}^{\mathrm{T}}\mathbf{J}(\mathbf{g}) \tag{11}$$

$$\nabla\langle s, s - t\rangle = (s - t)^{\mathrm{T}}\mathbf{J}(s) + s^{\mathrm{T}}\mathbf{J}(s - t) \tag{12}$$

$$\nabla\langle \tilde{p}, s - t\rangle = (s - t)^{\mathrm{T}}\mathbf{J}(\tilde{p}) + \tilde{p}^{\mathrm{T}}\mathbf{J}(s - t) \tag{13}$$

$$\nabla\|\mathbf{h}\|^2 = 2\mathbf{h}^{\mathrm{T}}\mathbf{J}(\mathbf{h}) \tag{14}$$

$$\nabla\|s - t\|^2 = 2(s - t)^{\mathrm{T}}\mathbf{J}(s - t) \tag{15}$$

$$\nabla\|\mathbf{h}\| = \frac{\mathbf{h}^{\mathrm{T}}\mathbf{J}(\mathbf{h})}{\|\mathbf{h}\|} \tag{16}$$

$$E_d(\theta_i) = \|p_i - \tilde{p}_i\|^2 \tag{17}$$

$$\nabla\|p_i-\tilde{p_i}\|=\frac{(p_i-\tilde{p_i})^{\mathrm{T}}\mathbf{J}(p_i-\tilde{p_i})}{\|p_i-\tilde{p_i}\|}=\frac{(p_i-\tilde{p_i})^{\mathrm{T}}\mathbf{J}(p_i)}{\|p_i-\tilde{p_i}\|} \tag{18}$$

$$E_s^i(\theta_i,\theta_j)=\frac{\|p_i-p_i'\|^2}{\|p_i-p_j\|^2} \tag{19}$$

$$E_s^j(\theta_i,\theta_j)=\frac{\|p_j-p_j'\|^2}{\|p_i-p_j\|^2} \tag{20}$$

$$\nabla\frac{\|p_i-p_i'\|}{\|p_i-p_j\|}=\frac{\|p_i-p_j\|\nabla\|p_i-p_i'\|-\|p_i-p_i'\|\nabla\|p_i-p_j\|}{\|p_i-p_j\|^2} \tag{21}$$

$$=\frac{(p_i-p_i')^{\mathrm{T}}\mathbf{J}(p_i-p_i')}{\|p_i-p_i'\|\|p_i-p_j\|}-\frac{\|p_i-p_i'\|(p_i-p_j)^{\mathrm{T}}\mathbf{J}(p_i-p_j)}{\|p_i-p_j\|^3} \tag{22}$$