

$$p = (1 - \lambda s) + \lambda t \quad (1)$$

$$\mathbf{J}(\lambda \mathbf{f}) = \lambda \mathbf{J}(\mathbf{f}) + \mathbf{f} \otimes \nabla \lambda \quad (2)$$

$$p = s - \lambda(s - t) \quad (3)$$

$$\mathbf{J}(p) = \mathbf{J}(s) - \mathbf{J}(\lambda(s - t)) = \mathbf{J}(s) - \lambda \mathbf{J}(s - t) - (s - t) \otimes \nabla \lambda \quad (4)$$

$$\nabla \left(\frac{f}{g} \right) = \frac{g \nabla f - f \nabla g}{g^2} \quad (5)$$

$$\lambda = \frac{\langle s - \tilde{p}, s - t \rangle}{\|s - t\|^2} = \frac{\langle s, s - t \rangle}{\|s - t\|^2} - \frac{\langle \tilde{p}, s - t \rangle}{\|s - t\|^2} \quad (6)$$

$$\nabla \lambda = \nabla \frac{\langle s, s - t \rangle}{\|s - t\|^2} - \nabla \frac{\langle \tilde{p}, s - t \rangle}{\|s - t\|^2} = \frac{\|s - t\|^2 \nabla \langle s, s - t \rangle - \langle s, s - t \rangle \nabla \|s - t\|^2}{\|s - t\|^4} \quad (7)$$

$$- \frac{\|s - t\|^2 \nabla \langle \tilde{p}, s - t \rangle - \langle \tilde{p}, s - t \rangle \nabla \|s - t\|^2}{\|s - t\|^4} \quad (8)$$

$$= \frac{\nabla \langle s, s - t \rangle}{\|s - t\|^2} - \frac{\langle s - \tilde{p}, s - t \rangle \nabla \|s - t\|^2}{\|s - t\|^2 \|s - t\|^2} - \frac{\nabla \langle \tilde{p}, s - t \rangle}{\|s - t\|^2} \quad (9)$$

$$= \frac{\nabla \langle s, s - t \rangle - \lambda \nabla \|s - t\|^2 - \nabla \langle \tilde{p}, s - t \rangle}{\|s - t\|^2} \quad (10)$$

$$\nabla \langle \mathbf{f}, \mathbf{g} \rangle = \mathbf{g}^T \mathbf{J}(\mathbf{f}) + \mathbf{f}^T \mathbf{J}(\mathbf{g}) \quad (11)$$

$$\nabla \langle s, s - t \rangle = (s - t)^T \mathbf{J}(s) + s^T \mathbf{J}(s - t) \quad (12)$$

$$\nabla \langle \tilde{p}, s - t \rangle = (s - t)^T \mathbf{J}(\tilde{p}) + \tilde{p}^T \mathbf{J}(s - t) \quad (13)$$

$$\nabla \|\mathbf{h}\|^2 = 2\mathbf{h}^T \mathbf{J}(\mathbf{h}) \quad (14)$$

$$\nabla \|s - t\|^2 = 2(s - t)^T \mathbf{J}(s - t) \quad (15)$$

$$\nabla \|\mathbf{h}\| = \frac{\mathbf{h}^T \mathbf{J}(\mathbf{h})}{\|\mathbf{h}\|} \quad (16)$$

$$E_d(\theta_i) = \|p_i - \tilde{p}_i\|^2 \quad (17)$$

$$\nabla \|p_i - \tilde{p}_i\| = \frac{(p_i - \tilde{p}_i)^T \mathbf{J}(p_i - \tilde{p}_i)}{\|p_i - \tilde{p}_i\|} = \frac{(p_i - \tilde{p}_i)^T \mathbf{J}(p_i)}{\|p_i - \tilde{p}_i\|} \quad (18)$$

$$E_s^i(\theta_i, \theta_j) = \frac{\|p_i - p'_i\|^2}{\|p_i - p_j\|^2} \quad (19)$$

$$E_s^j(\theta_i, \theta_j) = \frac{\|p_j - p'_j\|^2}{\|p_i - p_j\|^2} \quad (20)$$

$$\nabla \frac{\|p_i - p'_i\|}{\|p_i - p_j\|} = \frac{\|p_i - p_j\| \nabla \|p_i - p'_i\| - \|p_i - p'_i\| \nabla \|p_i - p_j\|}{\|p_i - p_j\|^2} \quad (21)$$

$$= \frac{(p_i - p'_i)^T \mathbf{J}(p_i - p'_i)}{\|p_i - p'_i\| \|p_i - p_j\|} - \frac{\|p_i - p'_i\| (p_i - p_j)^T \mathbf{J}(p_i - p_j)}{\|p_i - p_j\|^3} \quad (22)$$