1. (Simon & Blume - Exercício 8.19) Use o método de Gauss-Jordan para inverter cada matriz abaixo ou para provar que ela é singular:

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

1a $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 1/2 & 1/2 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|cc} 1 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1/2 & -1/2 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right]$

(b) $\begin{bmatrix} 4 & 5 \\ 2 & 4 \end{bmatrix}$

1b $\begin{bmatrix} 4 & 5 \\ 2 & 4 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|cc} 4 & 5 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 5/4 & 1/4 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right] \Rightarrow$

$\Rightarrow \left[\begin{array}{cc|cc} 1 & 5/4 & 1/4 & 0 \\ 0 & 6/4 & -2/4 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 5/4 & 1/4 & 0 \\ 0 & 1 & -2/6 & 4/6 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 2/3 & -5/6 \\ 0 & 1 & -2/6 & 4/6 \end{array}\right]$

(c) $\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$

1c $\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ -4 & -2 & 0 & 1 \end{array}\right] \Rightarrow$

$\Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right]$ = Singular.

(d) $\begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{bmatrix}$

1d
$$\begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 0 & 0 \\ 4 & 6 & 3 & 0 & 1 & 0 \\ -6 & -10 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 4 & 6 & 3 & 0 & 1 & 0 \\ -6 & -10 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & -2 & 3 & -2 & 1 & 0 \\ 0 & 2 & 0 & +3 & 0 & 1 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 2 & 0 & 3 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/2 & 0 & -1 \\ 0 & 1 & 0 & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right]$$

(e) $\begin{bmatrix} 2 & 6 & 0 & 5 \\ 6 & 21 & 8 & 17 \\ 4 & 12 & -4 & 13 \\ 0 & -3 & -12 & 2 \end{bmatrix}$

1e
$$\left[\begin{array}{cccc|cccc} 2 & 6 & 0 & 5 & 1 & 0 & 0 & 0 \\ 6 & 21 & 8 & 17 & 0 & 1 & 0 & 0 \\ 4 & 12 & -4 & 13 & 0 & 0 & 1 & 0 \\ 0 & -3 & -12 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 5/2 & 1/2 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 0 & 3 & 8 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -4 & 3 & -2 & 0 & 1 & 0 \\ 0 & -3 & -12 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 5/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 8/3 & 2/3 & -1 & 1/3 & 0 & 0 \\ 0 & 0 & -4 & 3 & -2 & 0 & 1 & 0 \\ 0 & 0 & -4 & 4 & -3 & 1 & 0 & 1 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 5/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 8/3 & 2/3 & -1 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & -3/4 & +1/2 & 0 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 3 & -5/2 & 5/2 & -5/2 \\ 0 & 1 & 8/3 & 0 & -1/3 & -1/3 & 2/3 & -2/3 \\ 0 & 0 & 1 & 0 & -1/4 & 3/4 & -1 & 3/4 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 3 & -5/2 & 5/2 & -5/2 \\ 0 & 1 & 0 & 0 & 1/3 & -7/3 & 10/3 & -8/3 \\ 0 & 0 & 1 & 0 & -1/4 & 3/4 & -1 & 3/4 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 9/2 & -15/2 & 11/2 \\ 0 & 1 & 0 & 0 & 1/3 & -7/3 & 10/3 & -8/3 \\ 0 & 0 & 1 & 0 & -1/4 & 3/4 & -1 & 3/4 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{array}\right]$$

2. (Simon & Blume - Exercício 8.20) Inverta a matriz de coeficientes para resolver os seguintes sistemas de equações:

   (a) $\begin{cases} 2x_1 + x_2 = 5 \\ x_1 + x_2 = 3 \end{cases}$

2 a $\begin{cases} 2x_1 + x_2 = 5 \\ x_1 + x_2 = 3 \end{cases}$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \begin{matrix} 11 & 5 - \cancel{3} = 2 \\ 21 & -5 + 2 = 1 \end{matrix}$$

$2 \times 2 \quad 2 \times 1$

$x_1 = 2$
$x_2 = 1$
$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{cases} 2x_1 + x_2 = 4 \\ 6x_1 + 2x_2 + 6x_3 = 20 \\ -4x_1 - 3x_2 + 9x_3 = 3 \end{cases}$

~b $\begin{cases} 2x_1 + x_2 = 1 \\ 6x_1 + 2x_2 + 6x_3 = 20 \\ -4x_1 - 3x_2 + 9x_3 = 3 \end{cases}$

$\begin{bmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{bmatrix}$ $\det \begin{vmatrix} 2 & 1 & 0 \\ 6 & 2 & 6 \\ -4 & -3 & 9 \end{vmatrix} = -2$ não singular

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 6 & 2 & 6 & 0 & 1 & 0 \\ -4 & -3 & 9 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & -1 & 6 & -3 & 1 & 0 \\ 0 & -1 & 9 & 2 & 0 & 1 \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -6 & 3 & -1 & 0 \\ 0 & 0 & 3 & 5 & -1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 13 & -3 & 2 \\ 0 & 0 & 1 & 5/3 & -1/3 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & +3/2 & -1 \\ 0 & 1 & 0 & 13 & -3 & 2 \\ 0 & 0 & 1 & 5/3 & -1/3 & 1/3 \end{array}\right] \Rightarrow \begin{bmatrix} -6 & +3/2 & -1 \\ 13 & -3 & 2 \\ 5/3 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 20 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$11 \quad -20 + 30 - 3 = 7$

$21 \quad 52 - 60 + 6 = -2$

$31 \quad 20/3 - \frac{20}{3} + 1 = 1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 1 \end{bmatrix}$$

(c) $\begin{cases} 2x_1 + 4x_2 = 2 \\ 4x_1 + 6x_2 + 3x_3 = 1 \\ -6x_1 - 10x_2 = -6 \end{cases}$

2c) $\begin{cases} 2x_1 + 4x_2 = 2 \\ 4x_1 + 6x_2 + 3x_3 = 1 \\ -6x_1 - 10x_2 = -6 \end{cases} \Rightarrow \begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{bmatrix} \Rightarrow$

$\det \begin{vmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{vmatrix} = -12$

Não singular $\Rightarrow \left[\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 0 & 0 \\ 4 & 6 & 3 & 0 & 1 & 0 \\ -6 & -10 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow$

$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & -2 & 3 & -2 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/2 & 0 & -1 \\ 0 & 1 & 0 & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right]$

$\begin{bmatrix} -5/2 & 0 & -1 \\ 3/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 = -5 + 0 + 6 = 1 \\ x_2 = 3 + 0 - 3 = 0 \\ x_3 = 2/3 + 1/3 - 2 = -1 \end{array}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

3. (Simon & Blume - Exercício 8.28) Qual é a inversa de uma matriz diagonal $n \times n$

$$D = \begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}?$$

3

$$D = \begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix} \Rightarrow$$

$$\left[\begin{array}{ccccc|ccccc} d_1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & d_3 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & d_n & 0 & 0 & 0 & \cdots & 1 \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \cdots & 0 & 1/d_1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 1/d_2 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 1/d_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1/d_n \end{array}\right] \Rightarrow$$

$$D^{-1} = \begin{bmatrix} 1/d_1 & 0 & 0 & \cdots & 0 \\ 0 & 1/d_2 & 0 & \cdots & 0 \\ 0 & 0 & 1/d_3 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & 1/d_n \end{bmatrix}$$

In [ ]: