

1. (Simon & Blume - Exercício 7.16) Use a eliminação de Gauss-Jordan para resolver os seguintes quatro sistemas de equações lineares. Quais variáveis são livres e quais são básicas em cada solução?

(a)

$$\begin{cases} w + 2x + y - z = 1 \\ 3w - x - y + 2z = 3 \\ -x + y - z = 1 \\ 2w + 3x + 3y - 3z = 3 \end{cases}$$

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$$\left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 & 3 \\ 0 & -1 & 1 & 1 & 1 \\ 2 & 3 & 3 & -3 & 3 \end{array} \right\} \xrightarrow{\text{R2} \leftarrow R2 - 3R1} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & -7 & -4 & 5 & 0 \\ 0 & -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & -1 & 1 \end{array} \right\} \xrightarrow{\text{L4} \leftarrow L4 - 2L1}$$

$$\xrightarrow{\text{R1} \leftarrow R1 - R3} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 7 & 4/7 & -5/7 & 0 \\ 0 & -1 & 1 & -1 & 1 \\ 0 & -1 & 2 & -1 & 1 \end{array} \right\} \xrightarrow{\text{L2} \leftarrow -\frac{1}{7}L2} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & -1/7 & 5/7 & 0 \\ 0 & -1 & 1 & -1 & 1 \\ 0 & -1 & 2 & -1 & 1 \end{array} \right\} \xrightarrow{\text{L3} \leftarrow L3 + L2}$$

$$\xrightarrow{\text{R1} \leftarrow R1 - R3} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & 4/7 & -5/7 & 0 \\ 0 & 0 & 1/7 & -4/7 & 1 \\ 0 & 0 & 11/7 & -12/7 & 1 \end{array} \right\} \xrightarrow{\text{L4} \leftarrow L4 + L2} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & 4/7 & -5/7 & 0 \\ 0 & 0 & 11/7 & -12/7 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - L2}$$

$$\xrightarrow{\text{R1} \leftarrow R1 - R3} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & 4/7 & -5/7 & 0 \\ 0 & 0 & 11/7 & -12/7 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L4} \leftarrow L4 - L3} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & 4/7 & -5/7 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - L2}$$

$$\xrightarrow{\text{R1} \leftarrow R1 - R3} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & 4/7 & -5/7 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L3} \leftarrow L3 - L1} \left\{ \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & 4/7 & -5/7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - L2}$$

$$\xrightarrow{\text{R1} \leftarrow R1 - R2} \left\{ \begin{array}{cccc|c} 1 & 2 & 0 & 1/11 & 4/11 \\ 0 & 1 & 0 & -1/11 & -4/11 \\ 0 & 0 & 1 & -1/11 & 7/11 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - L2} \left\{ \begin{array}{cccc|c} 1 & 2 & 0 & 1/11 & 4/11 \\ 0 & 1 & 0 & -1/11 & -4/11 \\ 0 & 0 & 1 & -1/11 & 7/11 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - 2L2} \left\{ \begin{array}{cccc|c} 1 & 0 & 0 & 3/11 & 12/11 \\ 0 & 1 & 0 & -1/11 & 4/11 \\ 0 & 0 & 1 & -1/11 & 7/11 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L3} \leftarrow L3 - L1} \left\{ \begin{array}{cccc|c} 1 & 0 & 0 & 3/11 & 12/11 \\ 0 & 1 & 0 & -1/11 & 4/11 \\ 0 & 0 & 1 & -1/11 & 7/11 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - L2} \left\{ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/11 & 4/11 \\ 0 & 0 & 1 & -1/11 & 7/11 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{L1} \leftarrow L1 - L2} \left\{ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\} \xrightarrow{\text{W}, \text{X}, \text{Y} = \text{Basic vars}} \text{Z} = \text{Linear.}$$

$$(b) \begin{cases} w - x + 3y - z = 0 \\ w + 4x - y + z = 3 \\ 3w + 7x + y + z = 6 \\ 3w + 2x + 5y - z = 3 \end{cases}$$

$$7 \text{ b} \quad \begin{cases} W - X + 3Y - Z = 0 \\ W + 4X - Y + Z = 3 \\ 3W + 7X + Y + Z = 6 \\ 3W + 2X + 5Y - Z = 3 \end{cases} \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 0 \\ 1 & 4 & -1 & 1 & 3 \\ 3 & 7 & 1 & 1 & 6 \\ 3 & 2 & 5 & -1 & 3 \end{array} \right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 0 \\ 0 & 5 & -4 & 2 & 3 \\ 0 & 10 & -8 & 4 & 6 \\ 0 & 5 & -4 & 2 & 3 \end{array} \right] \begin{matrix} L_1 \\ L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - 3L_1 \\ L_4 \leftarrow L_4 - 3L_1 \end{matrix} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 0 \\ 0 & 5 & -4 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} L_1 \\ L_2 \\ L_3 \leftarrow L_3 - 2L_2 \\ L_4 \leftarrow L_4 - L_2 \end{matrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 0 \\ 0 & 1 & -\frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} L_1 \\ L_2 \leftarrow \frac{1}{5}L_2 \\ L_3 \\ L_4 \end{matrix}$$

$$\Rightarrow W = X + (-3Y) + Z \quad W, X = \text{Banks} \\ X = \frac{3}{5} + \frac{4}{5}Y - \frac{2}{5}Z \quad Y, Z = \text{Lives}$$

$$(c) \begin{cases} w + 2x + 3y - z = 1 \\ -w + x + 2y + 3z = 2 \\ 3w - x + y + 2z = 2 \\ 2w + 3x - y + z = 1 \end{cases}$$

$$2C \left\{ \begin{array}{l} w + 2x + 3y - z = 1 \\ w + x + 2y + 3z = 2 \\ 3w - x + y + 2z = 2 \\ 2w + 3x - y + 2z = 1 \end{array} \right. \quad \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 1 & 1 & 2 & 3 & 2 \\ 3 & -1 & 1 & 2 & 2 \\ 2 & 3 & -1 & 1 & 1 \end{array} \right. \Rightarrow$$

$$\Rightarrow \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 3 & 5 & 2 & 3 \\ 0 & -7 & -8 & 5 & -1 \\ 0 & -1 & -7 & 3 & -1 \end{array} \right. \quad \begin{array}{l} L1 \\ L2 \leftarrow L2 - L1 \\ L3 \leftarrow L3 - 3L1 \\ L4 \leftarrow L4 - 2L1 \end{array} \quad \Rightarrow$$

$$\Rightarrow \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 5/3 & 2/3 & 1 \\ 0 & -7 & -8 & 5 & -1 \\ 0 & -1 & -7 & 3 & -1 \end{array} \right. \quad \begin{array}{l} L1 \\ L2 \leftarrow \frac{1}{3}L2 \\ L3 \\ L4 \end{array} \quad \Rightarrow$$

$$\Rightarrow \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 5/3 & 2/3 & 1 \\ 0 & 0 & 11/3 & 29/3 & 6 \\ 0 & 0 & -16/3 & 11/3 & 0 \end{array} \right. \quad \begin{array}{l} L1 \\ L2 \\ L3 \leftarrow L3 + 7L2 \\ L4 \leftarrow L4 + 9L2 \end{array} \quad \Rightarrow$$

$$\Rightarrow \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 5/3 & 2/3 & 1 \\ 0 & 0 & 1 & 29/11 & 18/11 \\ 0 & 0 & -16/3 & 11/3 & 0 \end{array} \right. \quad \begin{array}{l} L1 \\ L2 \\ L3 \leftarrow \frac{1}{11}L3 \\ L4 \end{array} \quad \Rightarrow$$

$$\begin{array}{l} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 5/3 & 2/3 & 1 \\ 0 & 0 & 1 & 29/11 & 18/11 \\ 0 & 0 & -16/3 & 11/3 & 0 \end{array} \right. \quad \begin{array}{l} L1 \\ L2 \\ L3 \\ L4 \leftarrow L4 + \frac{16}{3}L3 \end{array} \\ \hline \end{array}$$

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$$\rightarrow \left[\begin{array}{ccccc|cc} 1 & 2 & 3 & -1 & 7 & L_1 \\ 0 & 1 & 5/3 & 2/3 & 7 & L_2 \\ 0 & 0 & 1 & 29/11 & 78/11 & L_3 \\ 0 & 0 & 0 & 1 & 32/65 & L_4 \end{array} \right] \xrightarrow{\frac{1}{195}} \left[\begin{array}{ccccc|cc} 1 & 2 & 3 & -1 & 7 & L_1 \\ 0 & 1 & 5/3 & 2/3 & 7 & L_2 \\ 0 & 0 & 1 & 29/11 & 78/11 & L_3 \\ 0 & 0 & 0 & 1 & 32/65 & L_4 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccccc|cc} 1 & 2 & 3 & 0 & 97/65 & L_1 \leftarrow L_1 + L_4 \\ 0 & 1 & 5/3 & 0 & 131/65 & L_2 \leftarrow L_2 - \frac{2}{3}L_4 \\ 0 & 0 & 1 & 0 & 242/65 & L_3 \leftarrow L_3 - \frac{29}{11}L_4 \\ 0 & 0 & 0 & 1 & 32/65 & L_4 \end{array} \right] \Rightarrow$$

$$\rightarrow \left[\begin{array}{ccccc|cc} 1 & 2 & 0 & 0 & 37/65 & L_1 \leftarrow L_1 - 3L_3 \\ 0 & 1 & 0 & 0 & 7/65 & L_2 \leftarrow L_2 - \frac{5}{3}L_3 \\ 0 & 0 & 1 & 0 & 22/65 & L_3 \\ 0 & 0 & 0 & 1 & 32/65 & L_4 \end{array} \right] \Rightarrow$$

$$\rightarrow \left[\begin{array}{ccccc|cc} 1 & 0 & 0 & 0 & 17/65 & L_1 \leftarrow L_1 - 2L_2 \\ 0 & 1 & 0 & 0 & 7/65 & L_2 \\ 0 & 0 & 1 & 0 & 22/65 & L_3 \\ 0 & 0 & 0 & 1 & 32/65 & L_4 \end{array} \right] \quad w, x, y, z = \text{Basisvektoren}$$

$$(d) \begin{cases} w + x - y + 2z = 3 \\ 2w + 2x - 2y + 4z = 6 \\ -3w - 3x + 3y - 6z = -9 \\ -2w - 2x + 2y - 4z = -6 \end{cases}$$

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$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 3 \\ 2 & 2 & -2 & 4 & 6 \\ -3 & -3 & 3 & -6 & -9 \\ -2 & -2 & 2 & -4 & -6 \end{array} \right] \begin{matrix} L1 \\ L2 \\ L3 \\ L4 \end{matrix}$$

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$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} L1 \\ L2 \leftarrow L2 - 2L1 \\ L3 \leftarrow L3 + 3L1 \\ L4 \leftarrow L4 + 2L1 \end{matrix} \Rightarrow$$

$w = \text{Básica}$
 $x, y, z = \text{Liberas}$

2. (Simon & Blume - Exercício 7.17)

- (a) Use a flexibilidade da variável livre para encontrar inteiros positivos que satisfazem o sistema

$$\begin{cases} x + y + z = 13 \\ x + 5y + 10z = 61 \end{cases}$$

$$2a \begin{cases} x + y + z = 23 \\ x + 5y + 10z = 61 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 23 \\ 1 & 5 & 10 & 61 \end{array} \right] \xrightarrow{\text{R2} - R1}$$

$$\xrightarrow{\text{R1} \leftarrow R1 - R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 23 \\ 0 & 4 & 9 & 48 \end{array} \right] \xrightarrow{\text{R2} - 4R1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 23 \\ 0 & 1 & \frac{9}{4} & 12 \end{array} \right] \xrightarrow{\text{R1} - R2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{5}{4} & 1 \\ 0 & 1 & \frac{9}{4} & 12 \end{array} \right] \xrightarrow{\text{R1} - R2} \begin{aligned} x &= 1 + \frac{5}{4}z \\ y &= 12 - \frac{9}{4}z \end{aligned}$$

Teoricamente, qualquer valor $z \in \mathbb{R}$ satisfaz o sistema. No entanto, para obter valores x e y inteiros, é necessário que z seja múltiplo de 2 e maior ou igual a 4.

- (b) Suponha que você entregue uma nota de um dólar por um doce de 6 centavos e receba 16 moedas de troco - todas pennies (1 centavo), nickels (5 centavos) e dimes (10 centavos). Quantas moedas de cada tipo você recebe? [Dica: Veja a parte (a)]

$$2b \begin{cases} x + y + z = 16 \\ x + 5y + 10z = 94 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 1 & 5 & 10 & 94 \end{array} \right] \xrightarrow{\text{R2} - R1}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 4 & 9 & 78 \end{array} \right] \xrightarrow{\text{R2} - 4R1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 1 & \frac{9}{4} & \frac{78}{4} \end{array} \right] \xrightarrow{\text{R1} - R2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{5}{4} & \frac{19}{4} \\ 0 & 1 & \frac{9}{4} & \frac{78}{4} \end{array} \right] \xrightarrow{\text{R1} - R2}$$

$$x = \frac{5}{4}z - \frac{19}{4} \Rightarrow \text{se } z = 6 \text{ então } x \text{ e } y \text{ não são inteiros, com valores } 4 \text{ e } 6 \text{ respectivamente, rounded}$$

