

1. (Simon & Blume - Exercício 9.14) Para os seguintes pares de matrizes, verifique que (i) $\det(A^T) = \det(A)$; (ii) $\det(A \cdot B) = \det(A) \det(B)$; e (iii) $\det(A + B) \neq \det(A) + \det(B)$ em geral.

(a) $A = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$

1. a

$$A = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 4 & 1 \\ 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$
$$\det \begin{vmatrix} 4 & 5 \\ 1 & 1 \end{vmatrix} = -1 \quad \det \begin{vmatrix} 4 & 1 \\ 5 & 1 \end{vmatrix} = -1$$

i.

$$AB = \begin{bmatrix} 17 & 21 \\ 4 & 5 \end{bmatrix} \quad \det \begin{vmatrix} 17 & 21 \\ 4 & 5 \end{vmatrix} = 1 \quad \det \begin{vmatrix} 7 & 9 \\ 2 & 2 \end{vmatrix} = -4$$
$$\det(A) \cdot \det(B) = -1 \cdot -1 = 1$$
$$\det(AB) = 1$$
$$A + B = C \Rightarrow \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 2 & 2 \end{bmatrix}$$
$$\det \begin{vmatrix} 7 & 9 \\ 2 & 2 \end{vmatrix} = -4 \quad \det(A) + \det(B) = -2$$

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$

16 ;

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 21 & 18 \\ 24 & 37 & 30 \\ 24 & 30 & 36 \end{bmatrix}$$

$$A+B=C = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 7 & 5 \\ 4 & 5 & 12 \end{bmatrix}$$

i $\det(A) = 24 \quad \det(A^T) = 24$

ii $\det(B) = 18$

$\det(AB) = 432 \quad \det(A) \cdot \det(B) = 24 \cdot 18 = 432$

iii $\det(C) = 56 \quad \det(A) + \det(B) = 24 + 18 = 42$

In []: