1. (Simon & Blume - Exercício 7.2) Resolva os seguintes sistemas por substituição, por eliminação Gaussiana e por eliminação de Gauss-Jordan:

(a) $\begin{cases} x - 3y + 6z = -1 \\ 2x - 5y + 10z = 0 \\ 3x - 8y + 17z = 1 \end{cases}$

Substituição

$1a \begin{cases} x - 3y + 6z = -1 \\ 2x - 5y + 10z = 0 \Rightarrow \\ 3x - 8y + 17z = 1 \end{cases} \quad x = -1 + 3y - 6z$

$\Rightarrow 2(-1 + 3y - 6z) - 5y + 10z = 0 \Rightarrow -2 + 6y - 12z - 5y + 10z = 0$

$\Rightarrow y - 2z = 2 \Rightarrow y = 2 + 2z \Rightarrow x = -1 + 3(2 + 2z) - 6z \Rightarrow$

$x = -1 + 6 + 6z - 6z \Rightarrow x = 5 \Rightarrow 3(5) - 8(2 + 2z) + 17z = 1$

$\Rightarrow 15 - 16 - 16z + 17z = 1 \Rightarrow -1 + 1z = 1 = z = 2$

$x - 3y + 6z = -1$

$5 - 3y + 6 \cdot (2) = -1 \Rightarrow 17 - 3y = -1 \Rightarrow -3y = -18 \Rightarrow y = \frac{-18}{-3}$

$\Rightarrow y = 6$

Gaussiano

1a $\begin{cases} x - 3y + 6z = -1 \\ 2x - 5y + 10z = 0 \\ 3x - 8y + 17z = 1 \end{cases} \Rightarrow \begin{cases} x - 3y + 6z = -1 & L1 \\ 0 + 1y - 2z = 2 & L2 \leftarrow L2 - 2L1 \\ 0 + 1y - 1z = 4 & L3 \leftarrow L3 - 3L1 \end{cases}$

$\Rightarrow \begin{cases} x - 3y + 6z = -1 & L1 \\ 0 + 1y - 2z = 2 & L2 \\ 0 + 0 + 1z = 2 & L3 \leftarrow L3 - L2 \end{cases}$

$\boxed{z = 2 \Rightarrow y - 4 = 2 \Rightarrow y = 6 \Rightarrow x - 18 + 12 = -1 \quad x = 5}$

Gauss-Jordan

1a $\begin{cases} x - 3y + 6z = -1 \\ 2x - 5y + 10z = 0 \\ 3x - 8y + 17z = 1 \end{cases} \Rightarrow \begin{cases} x - 3y + 6z = -1 & L1 \\ 0 + y - 2z = 2 & L2 \leftarrow L2 - 2L1 \\ 0 + y - z = 4 & L3 \leftarrow L3 - 3L1 \end{cases}$

$\Rightarrow \begin{cases} x - 3y + 6z = -1 & L1 \\ 0 + y - 2z = 2 & L2 \\ 0 + 0 + z = 2 & L3 \leftarrow L3 - L2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x - 3y + 0 = -13 & L1 \leftarrow L1 - 6L3 \\ 0 + y + 0 = 6 & L2 \leftarrow L2 + 2L3 \\ 0 + 0 + z = 2 & L3 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x + 0 + 0 = 5 & L1 \leftarrow L1 + 3L2 \\ 0 + y + 0 = 6 & L2 \\ 0 + 0 + z = 2 & L3 \end{cases} \Rightarrow \quad x = 5, \; y = 6, \; z = 2$

(b) $\begin{cases} x_1 + x_2 + x_3 = 0 \\ 12x_1 + 2x_2 - 3x_3 = 5 \\ 3x_1 + 4x_2 + x_3 = -4 \end{cases}$

Substituição

$$1b\ \begin{cases} x_1 + x_2 + x_3 = 0 \\ 12x_1 + 2x_2 - 3x_3 = 5 \\ 3x_1 + 4x_2 + x_3 = -4 \end{cases}$$

$$x_1 = -x_2 - x_3 \Rightarrow 12(-x_2 - x_3) + 2x_2 - 3x_3 = 5 \Rightarrow$$

$$-12x_2 - 12x_3 + 2x_2 - 3x_3 = 5 \Rightarrow$$

$$-10x_2 - 15x_3 = 5 \Rightarrow$$

$$\Rightarrow -10x_2 = 5 + 15x_3 \Rightarrow x_2 = -\left(\frac{5 + 15x_3}{10}\right) \Rightarrow$$

$$3\left(\frac{5 + 15x_3}{10} - x_3\right) + 4\left(-\left(\frac{5 + 15x_3}{10}\right)\right) + x_3 = -4 \Rightarrow$$

$$3\left(\frac{5 + 5x_3}{10}\right) - 4\left(\frac{5 + 15x_3}{10}\right) + x_3 = -4 \Rightarrow x_3 + \frac{15 + 15x_3}{10} - \frac{20 - 60x_3}{10} +$$

$$\frac{15 + 15x_3}{10} - \frac{20 - 60x_3}{10} + x_3 = -4 \Rightarrow$$

$$\frac{-5 - 45x_3}{10} + x_3 = -4 \Rightarrow \frac{-5 - 35x_3}{10} = -4 \Rightarrow$$

$$-5 - 35x_3 = -40 \Rightarrow -35x_3 = -35 \Rightarrow x_3 = 1$$

$$x_1 = -\left(-\left(\frac{5 + 15(1)}{10}\right) - (1)\right) = -(-2) - 1 \Rightarrow x_1 = 1$$

$$x_2 = -\left(\frac{5 + 15(1)}{10}\right) \Rightarrow -(2) \Rightarrow x_2 = -2 \Rightarrow \boxed{\begin{matrix} x_1 = 1 \\ x_2 = -2 \\ x_3 = 1 \end{matrix}}$$

Gaussiano

1b $\begin{cases} x_1 + x_2 + x_3 = 0 \\ 12x_1 + 2x_2 - 3x_3 = 5 \\ 3x_1 + 4x_2 + x_3 = -4 \end{cases} \Rightarrow \begin{cases} x_1 + x_2 + x_3 = 0 & L1 \\ 0 - 10x_2 - 15x_3 = 5 & L2 \leftarrow L2 - 12L1 \\ 0 \quad 1x_2 - 2x_3 = -4 & L3 \leftarrow L3 - 3L1 \end{cases}$

$\Rightarrow \begin{cases} x_1 + x_2 + x_3 = 0 & L1 \\ 0 - 10x_2 - 15x_3 = 5 & L2 \\ 0 + 0 \quad -\frac{7}{2}x_3 = -\frac{7}{2} & L3 \leftarrow L3 + \frac{1}{10}L2 \end{cases}$

$$x_3 = 1$$

$$-10x_2 - 15(1) = 5$$

$$-10x_2 = 20$$

$$x_2 = -2$$

$$x_1 + (-2) + 1 = 0$$

$$x_1 = 1$$

Gauss-Jordan

1b

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 12x_1 + 2x_2 - 3x_3 = 5 \\ 3x_1 + 4x_2 + x_3 = -4 \end{cases} \Rightarrow \begin{cases} x_1 + x_2 + x_3 = 0 & L1 \\ 0 - 10x_2 - 15x_3 = 5 & L2 \leftarrow L2 - 12L1 \\ 0 \quad 1x_2 - 2x_3 = -4 & L3 \leftarrow L3 - 3L1 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 + x_2 + x_3 = 0 & L1 \\ 0 - 10x_2 - 15x_3 = 5 & L2 \\ 0 + 0 \quad -\frac{7}{2}x_3 = -\frac{7}{2} & L3 \leftarrow L3 + \frac{1}{10}L2 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 + x_2 + x_3 = 0 & L1 \\ 0 + x_2 + \frac{3}{2}x_3 = -\frac{1}{2} & L2 \leftarrow -\frac{1}{10}L2 \\ 0 + 0 + x_3 = 1 & L3 \leftarrow -\frac{7}{2}L3 \end{cases} \Rightarrow$$

$$\begin{cases} x_1 + x_2 + 0 = -1 & L1 \leftarrow L1 - L3 \\ 0 + x_2 + 0 = -2 & L2 \leftarrow L2 - \frac{3}{2}L3 \\ 0 + 0 + x_3 = 1 & L3 \end{cases} \Rightarrow$$

$$\begin{cases} x_1 + 0 + 0 = 1 & L1 \leftarrow L1 - L2 \\ 0 + x_2 + 0 = -2 & L2 \\ 0 + 0 + x_3 = 1 & L3 \end{cases} \Rightarrow \boxed{\begin{matrix} x_1 = 1 \\ x_2 = -2 \\ x_3 = 1 \end{matrix}}$$

2. (Simon & Blume - Exercício 7.3) Resolva os seguintes sistemas por eliminação de Gauss-Jordan.

   (a) $\begin{cases} 3x + 3y = 4 \\ x - y = 10 \end{cases}$

2a $\begin{cases} 3x + 3y = 4 \\ x - y = 10 \end{cases} \Rightarrow \begin{cases} x + y = 4/3 & L1 \leftarrow \frac{1}{3} L1 \\ x - y = 10 & L2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x + y = 4/3 & L1 \\ 0 - 2y = 26/3 & L2 \leftarrow L2 - L1 \end{cases} \Rightarrow \begin{cases} x + y = 4/3 & L1 \\ 0 + y = -13/3 & L2 \leftarrow \frac{-1}{2} L2 \end{cases}$

$\Rightarrow \begin{cases} x + 0 = 17/3 & L1 \leftarrow L1 - L2 \\ 0 + y = \frac{-13}{3} \end{cases}$

(b) $\begin{cases} 4x + 2y - 3z = 1 \\ 6x + 3y - 5z = 0 \\ x + y + 2z = 9 \end{cases}$

2b $\begin{cases} 4x + 2y - 3z = 1 \\ 6x + 3y - 5z = 0 \\ x + y + 2z = 9 \end{cases} \Rightarrow \begin{cases} x + 1/2 y - 3/4 z = 1/4 & L1 \leftarrow \frac{1}{4} L1 \\ 6x + 3y - 5z = 0 & L2 \\ x + y + 2z = 9 & L3 \end{cases} \Rightarrow$

$\begin{cases} x + 1/2 y - 3/4 z = 1/4 \\ 0 + 0 - 1/2 z = -3/2 & L2 \leftarrow L2 - 6L1 \\ 0 + 1/2 y + 11/4 z = 35/4 & L3 \leftarrow L3 - L1 \end{cases} \Rightarrow$

$\begin{cases} x + 1/2 y - 3/4 z = 1/4 & L1 \\ 0 + y + 11/2 z = 35/2 & L2 \leftarrow 2L3 \\ 0 + 0 + z = 3 & L3 \leftarrow -2 \cdot L2 \end{cases} \Rightarrow$

$$\begin{cases} x + \frac{1}{2}y + 0 = \frac{5}{2} & L1 \leftarrow L1 + \frac{3}{4}L3 \\ 0 + y + 0 = 1 & L2 \leftarrow L2 - \frac{11}{2}L3 \\ 0 + 0 + z = 3 & L3 \end{cases} \Rightarrow$$

$$\begin{cases} x + 0 + 0 = 2 & L1 \leftarrow L1 - \frac{1}{2}L2 \\ 0 + y + 0 = 1 & L2 \\ 0 + 0 + z = 3 & L3 \end{cases}$$

(c) $\begin{cases} 2x + 2y - z = 2 \\ x + y + z = -2 \\ 2x - 4y + 3z = 0 \end{cases}$

2c

$$\begin{cases} 2x + 2y - z = 2 \\ x + y + z = -2 \\ 2x - 4y + 3z = 0 \end{cases} \Rightarrow \begin{cases} x + y - z/2 = 1 & L1 \leftarrow \frac{1}{2}L1 \\ x + y + z = -2 & L2 \\ 2x - 4y + 3z = 0 & L3 \end{cases}$$

$$\Rightarrow \begin{cases} x + y - z/2 = 1 & L1 \\ 0 \quad 0 + 3z/2 = -3 & L2 \leftarrow L2 - L1 \\ 0 - 6y + 4z = -2 & L3 \leftarrow L3 - 2L1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x + y - z/2 = 1 & L1 \\ 0 + y - \frac{2}{3}z = 1/3 & L2 \leftarrow -\frac{1}{6}L3 \\ 0 + 0 + z = -2 & L3 \leftarrow \frac{2}{3}L2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x + y + 0 = 0 & L1 \leftarrow L1 + \frac{1}{2}L3 \\ 0 + y + 0 = -1 & L2 \leftarrow L2 + \frac{2}{3}L3 \\ 0 + 0 + z = -2 & L3 \end{cases}$$

$$L1 \leftarrow L1 - L2$$

$$\Rightarrow \begin{cases} x + 0 + 0 = 1 & L1 \\ 0 + y + 0 = -1 & L2 \\ 0 + 0 + z = -2 & L3 \end{cases}$$

3. (Simon & Blume - Exercício 7.7) Use a eliminação Gaussiana para resolver

$$\begin{cases} 3x + 3y = 4 \\ -x - y = 10 \end{cases}$$

O que acontece e por quê?

3

$$\begin{cases} 3x + 3y = 4 \\ -x - y = 10 \end{cases} \Rightarrow \begin{cases} 3x + 3y = 4 \\ 0 \quad 0 = 34/3 \end{cases}$$

$$0x + 0y = 34/3 = 0 + 0 = \frac{34}{3} \Rightarrow \text{Sem solução}$$

4. (Simon & Blume - Exercício 7.11) Escreva os três sistemas no Exercício 7.3 na forma matricial. Então use operações sobre as linhas para encontrar as suas formas escalonada e reduzida correspondentes e calcular a solução.

4 a

$$\left[\begin{array}{cc|c} 3 & 3 & 4 \\ 1 & -1 & 10 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 4/3 \\ 1 & -1 & 10 \end{array}\right] \begin{array}{l} L1 \leftarrow \frac{1}{3}L1 \\ L2 \end{array}$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 4/3 \\ 0 & -2 & 26/3 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow L2 - L1 \end{array} \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 4/3 \\ 0 & 1 & -13/3 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow -\frac{1}{2}L2 \end{array}$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 17/3 \\ 0 & 1 & -13/3 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 - L2 \\ L2 \end{array}$$

4b $\left[\begin{array}{ccc|c} 4 & 2 & -3 & 1 \\ 6 & 3 & -5 & 0 \\ 1 & 1 & 2 & 9 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1/2 & -3/4 & 1/4 \\ 6 & 3 & -5 & 0 \\ 1 & 1 & 2 & 9 \end{array}\right] \begin{array}{l} L1 \leftarrow \frac{1}{4}L1 \\ L2 \\ L3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1/2 & -3/4 & 1/4 \\ 0 & 0 & -1/2 & -3/2 \\ 0 & 1/2 & 11/4 & 35/4 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow L2 - 6L1 \\ L3 \leftarrow L3 - L1 \end{array} \Rightarrow$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1/2 & -3/4 & 1/4 \\ 0 & 1/2 & 11/4 & 35/4 \\ 0 & 0 & -1/2 & -3/2 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow L3 \\ L3 \leftarrow L2 \end{array} \Rightarrow$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1/2 & -3/4 & 1/4 \\ 0 & 1 & 11/2 & 35/2 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow 2L2 \\ L3 \leftarrow -2L3 \end{array} \Rightarrow$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1/2 & 0 & 5/2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 + \frac{3}{4}L3 \\ L2 \leftarrow L2 - \frac{11}{2}L3 \\ L3 \end{array} \Rightarrow$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 - \frac{1}{2}L2 \\ L2 \\ L3 \end{array}$

4C

$$\left[\begin{array}{ccc|c} 2 & 2 & -1 & 2 \\ 1 & 1 & 1 & -2 \\ 2 & -4 & 3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1/2 & 1 \\ 1 & 1 & 1 & -2 \\ 2 & -4 & 3 & 0 \end{array}\right] \begin{array}{l} L1 \leftarrow \frac{1}{2} L1 \\ L2 \\ L3 \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1/2 & 1 \\ 0 & 0 & 3/2 & -3 \\ 0 & -6 & 4 & -2 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow L2 - L1 \\ L3 \leftarrow L3 - 2L1 \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1/2 & 1 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} L1 \\ L2 \leftarrow -\frac{1}{6} L3 \\ L3 \leftarrow \frac{2}{3} L2 \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 + \frac{1}{2} L3 \\ L2 \leftarrow L2 + \frac{2}{3} L3 \\ L3 \end{array} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & +1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 - L2 \\ L2 \\ L3 \end{array}$$

5. (Simon & Blume - Exercício 7.12) Use a eliminação de Gauss-Jordan na forma matricial para resolver o sistema

$$\begin{cases} w + x + 3y - 2z = 0 \\ 2w + 3x + 7y - 2z = 9 \\ 3w + 5x + 13y - 9z = 1 \\ -2w + x - z = 0 \end{cases}$$

5
$$\begin{cases} w + x + 3y - 2z = 0 \\ 2w + 3x + 7y - 2z = 9 \\ 3w + 5x + 13y - 4z = 1 \\ -2w + x + 0 - z = 0 \end{cases} \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & -2 & 0 \\ 2 & 3 & 7 & -2 & 9 \\ 3 & 5 & 13 & -4 & 1 \\ -2 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 2 & 4 & -3 & 1 \\ 0 & 3 & 6 & -5 & 0 \end{array}\right] \begin{array}{l} L_1 \\ L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \\ L_4 \leftarrow L_4 + 2L_1 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 2 & -7 & -17 \\ 0 & 0 & 3 & -11 & -27 \end{array}\right] \begin{array}{l} L_1 \\ L_2 \\ L_3 \leftarrow L_3 - 2L_2 \\ L_4 \leftarrow L_4 - 3L_2 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 3 & -11 & -27 \end{array}\right] \begin{array}{l} L_1 \\ L_2 \\ L_3 \leftarrow \frac{1}{2}L_3 \\ L_4 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right] \begin{array}{l} L_1 \\ L_2 \\ L_3 \\ L_4 \leftarrow L_4 - 3L_3 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -2 & 0 \\ 0 & 1 & 1 & 2 & 9 \\ 0 & 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L_1 \\ L_2 \\ L_3 \\ L_4 \leftarrow -\frac{2}{1}L_4 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & 0 & 6 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 + 2L4 \\ L2 \leftarrow L2 - 2L4 \\ L3 \leftarrow L3 + \frac{7}{2}L4 \\ L4 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 - 3L3 \\ L2 \leftarrow L2 - L3 \\ L3 \\ L4 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} L1 \leftarrow L1 - L2 \\ L2 \\ L3 \\ L4 \end{array}$$

In [ ]: