

1. (Simon & Blume - Exercício 8.19) Use o método de Gauss-Jordan para inverter cada matriz abaixo ou para provar que ela é singular:

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

1a $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 1/2 & 1/2 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right]$
 $\Rightarrow \left[\begin{array}{cc|cc} 1 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1/2 & -1/2 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array} \right]$

(b) $\begin{bmatrix} 4 & 5 \\ 2 & 4 \end{bmatrix}$

1b $\begin{bmatrix} 4 & 5 \\ 2 & 4 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|cc} 4 & 5 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 5/4 & 1/4 & 0 \\ 2 & 4 & 0 & 1 \end{array} \right] \Rightarrow$
 $\Rightarrow \left[\begin{array}{cc|cc} 1 & 5/4 & 1/4 & 0 \\ 0 & 6/4 & -2/4 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 5/4 & 1/4 & 0 \\ 0 & 1 & -2/6 & 4/6 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 2/3 & -5/6 \\ 0 & 1 & -2/6 & 4/6 \end{array} \right]$

(c) $\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$

1c $\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ -4 & -2 & 0 & 1 \end{array} \right] \Rightarrow$
 $\Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array} \right] = \text{Singular.}$

(d) $\begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{bmatrix}$

$$7d \begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 3 \\ -6 & -10 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 4 & 0 & | & 1 & 0 & 0 \\ 4 & 6 & 3 & | & 0 & 1 & 0 \\ -6 & -10 & 0 & | & 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 & | & 1/2 & 0 & 0 \\ 4 & 6 & 3 & | & 0 & 1 & 0 \\ -6 & -10 & 0 & | & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & | & 1/2 & 0 & 0 \\ 0 & -2 & 3 & | & -2 & 1 & 0 \\ 0 & 2 & 0 & | & +3 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 & | & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & | & 1 & -1/2 & 0 \\ 0 & 2 & 0 & | & 3 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & | & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & | & 1 & -1/2 & 0 \\ 0 & 0 & 3 & | & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 & | & 1/2 & 0 & 0 \\ 0 & 1 & -3/2 & | & 1 & -1/2 & 0 \\ 0 & 0 & 1 & | & 1/3 & 1/3 & 1/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & | & 1/2 & 0 & 0 \\ 0 & 1 & 0 & | & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & | & 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -5/2 & 0 & -1 \\ 0 & 1 & 0 & | & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & | & 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$(e) \begin{bmatrix} 2 & 6 & 0 & 5 \\ 6 & 21 & 8 & 17 \\ 4 & 12 & -4 & 13 \\ 0 & -3 & -12 & 2 \end{bmatrix}$$

$$1e \begin{bmatrix} 2 & 6 & 0 & 5 & | & 1 & 0 & 0 & 0 \\ 6 & 21 & 8 & 17 & | & 0 & 1 & 0 & 0 \\ 4 & 12 & -4 & 13 & | & 0 & 0 & 1 & 0 \\ 0 & -3 & -12 & 2 & | & 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 3 & 0 & 5/2 & | & 1/2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 & | & -3 & 1 & 0 & 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 0 & 3 & 8 & 2 & -2 & 0 & 1 & 0 \\ 0 & 0 & -4 & 3 & 0 & 0 & 1 & 0 \\ 0 & -3 & -12 & 2 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 5/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 8/3 & 2/3 & -1 & 1/3 & 0 & 0 \\ 0 & 0 & -4 & 3 & -2 & 0 & 1 & 0 \\ 0 & 0 & -4 & 4 & -3 & 1 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 5/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 8/3 & 2/3 & -1 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & -3/4 & 1/2 & 0 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 0 & 3 & -5/2 & 5/2 & -5/2 \\ 0 & 1 & 8/3 & 0 & -1/3 & -1/3 & 2/3 & -2/3 \\ 0 & 0 & 1 & 0 & -1/4 & 3/4 & -1 & 3/4 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & 0 & 3 & -5/2 & 5/2 & -5/2 \\ 0 & 1 & 0 & 0 & 1/3 & -7/3 & 10/3 & -8/3 \\ 0 & 0 & 1 & 0 & -1/4 & 3/4 & -1 & 3/4 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{bmatrix} \Rightarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 9/2 & -15/2 & 11/2 \\ 0 & 1 & 0 & 0 & 1/3 & -7/3 & 20/3 & -8/3 \\ 0 & 0 & 1 & 0 & -1/4 & 3/4 & -1 & 3/4 \\ 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \end{array} \right]$$

In []: