1. (Simon & Blume - Exercício 10.5) Seja $\boldsymbol{u} = (1, 2)$, $\boldsymbol{v} = (0, 1)$, $\boldsymbol{w} = (1, -3)$, $\boldsymbol{x} = (1, 2, 0)$, e $\boldsymbol{z} = (0, 1, 1)$. Compute os seguintes vetores, sempre que eles estiverem definidos: $\boldsymbol{u} + \boldsymbol{v}$, $-4\boldsymbol{w}$, $\boldsymbol{u} + \boldsymbol{z}$, $3\boldsymbol{z}$, $2\boldsymbol{v}$, $\boldsymbol{u} + 2\boldsymbol{v}$, $\boldsymbol{u} - \boldsymbol{v}$, $3\boldsymbol{x} + \boldsymbol{z}$, $-2\boldsymbol{x}$, $\boldsymbol{w} + 2\boldsymbol{x}$.

$8 = 64$

1 $u + v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$-4w = -4 \cdot \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$

$u + z = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} =$ Indefinido

$3z = 3 \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$

$2v = 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$u + 2v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$u - v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$3x + z = 3\begin{bmatrix}1\\2\\0\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}3\\7\\1\end{bmatrix}$$

$$-2x = -2\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-2\\-4\\0\end{bmatrix}$$

$$w + 2x = \begin{bmatrix}1\\-3\end{bmatrix} + 2\begin{bmatrix}1\\2\\0\end{bmatrix} = \text{Indefinido}$$

2. (Simon & Blume - Exercício 10.10) Encontre o comprimento dos seguintes vetores. Desenhe os vetores de (a) até (g):

(a) $\boldsymbol{v}_a = (3, 4)$

(b) $\boldsymbol{v}_b = (0, -3)$

(c) $\boldsymbol{v}_c = (1, 1, 1)$

(d) $\boldsymbol{v}_d = (3, 3)$

(e) $\boldsymbol{v}_e = (-1, -1)$

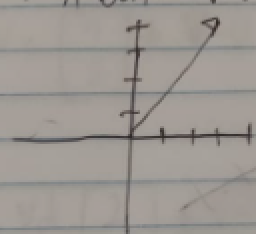
(f) $\boldsymbol{v}_f = (1, 2, 3)$

(g) $\boldsymbol{v}_g = (2, 0)$

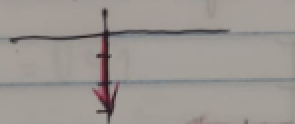
(h) $\boldsymbol{v}_h = (1, 2, 3, 4)$

(i) $\boldsymbol{v}_i = (3, 0, 0, 0, 0)$

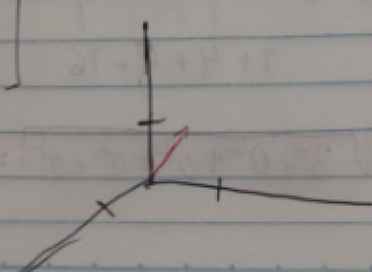
2a) $v_a = \begin{bmatrix}3\\4\end{bmatrix} \Rightarrow \|v_a\| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$



b) $v_b = \begin{bmatrix}0\\-3\end{bmatrix} \Rightarrow \|v_b\| = \sqrt{0^2 + (-3)^2} \Rightarrow \sqrt{9} = 3$

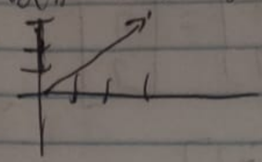


c) $v_c = \begin{bmatrix}1\\1\\1\end{bmatrix} \Rightarrow \|v_c\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

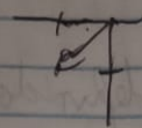


/ /   Seg Ter Qua Qui Sex Sáb Dom

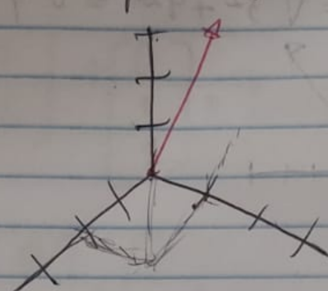
d) $v_d = \begin{bmatrix}3\\3\end{bmatrix} \Rightarrow \|v_d\| = \sqrt{3^2+3^2} \Rightarrow \sqrt{18}$



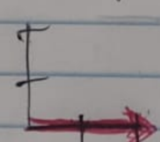
e) $v_e = \begin{bmatrix}-1\\-1\end{bmatrix} \Rightarrow \|v_e\| = \sqrt{(-1)^2+(-1)^2} \Rightarrow \sqrt{2}$



f) $v_f = \begin{bmatrix}1\\2\\3\end{bmatrix} \Rightarrow \|v_f\| = \sqrt{1^2+2^2+3^2} \Rightarrow \sqrt{14}$



g) $v_g = \begin{bmatrix}2\\0\end{bmatrix} \Rightarrow \|v_g\| = \sqrt{2^2+0^2} \Rightarrow \sqrt{4} = 2$



h) $v_h = \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \Rightarrow \|v_h\| = \sqrt{1^2+2^2+3^2+4^2} \Rightarrow \sqrt{30}$

$1+4+9+16$

i) $v_i = \begin{bmatrix}3\\0\\0\\0\\0\end{bmatrix} \Rightarrow \|v_i\| = \sqrt{3^2+0^2+0^2+0^2+0^2} = \sqrt{9} = 3$

3. (Simon & Blume - Exercício 10.11) Encontre a distância entre os pares de vetores abaixo, representando graficamente sempre que possível:

   (a) $\boldsymbol{u}_a = (0, 0)$, $\boldsymbol{v}_a = (3, -4)$
   (b) $\boldsymbol{u}_b = (1, -1)$, $\boldsymbol{v}_b = (7, 7)$
   (c) $\boldsymbol{u}_c = (5, 2)$, $\boldsymbol{v}_c = (1, 2)$
   (d) $\boldsymbol{u}_d = (1, 1, -1)$, $\boldsymbol{v}_d = (2, -1, 5)$
   (e) $\boldsymbol{u}_e = (1, 2, 3, 4)$, $\boldsymbol{v}_e = (1, 0, -1, 0)$

(3) $\|w\| = \|v-u\|$

(a) $u_a = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, v_a \begin{bmatrix} 3 \\ -4 \end{bmatrix} \Rightarrow \|v-u\| = \begin{bmatrix} 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

$\Rightarrow \sqrt{3^2+(-4)^2} \Rightarrow \sqrt{9+16} = 5$

(b) $u_b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, v_b = \begin{bmatrix} 7 \\ 7 \end{bmatrix} = v-u = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{6^2+8^2} = \sqrt{36+64}$

$\Rightarrow \sqrt{100} = 10$

(c) $u_c = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_c \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow v-u = \begin{bmatrix} -4 \\ 0 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{0+(-4)^2} = \sqrt{16}$

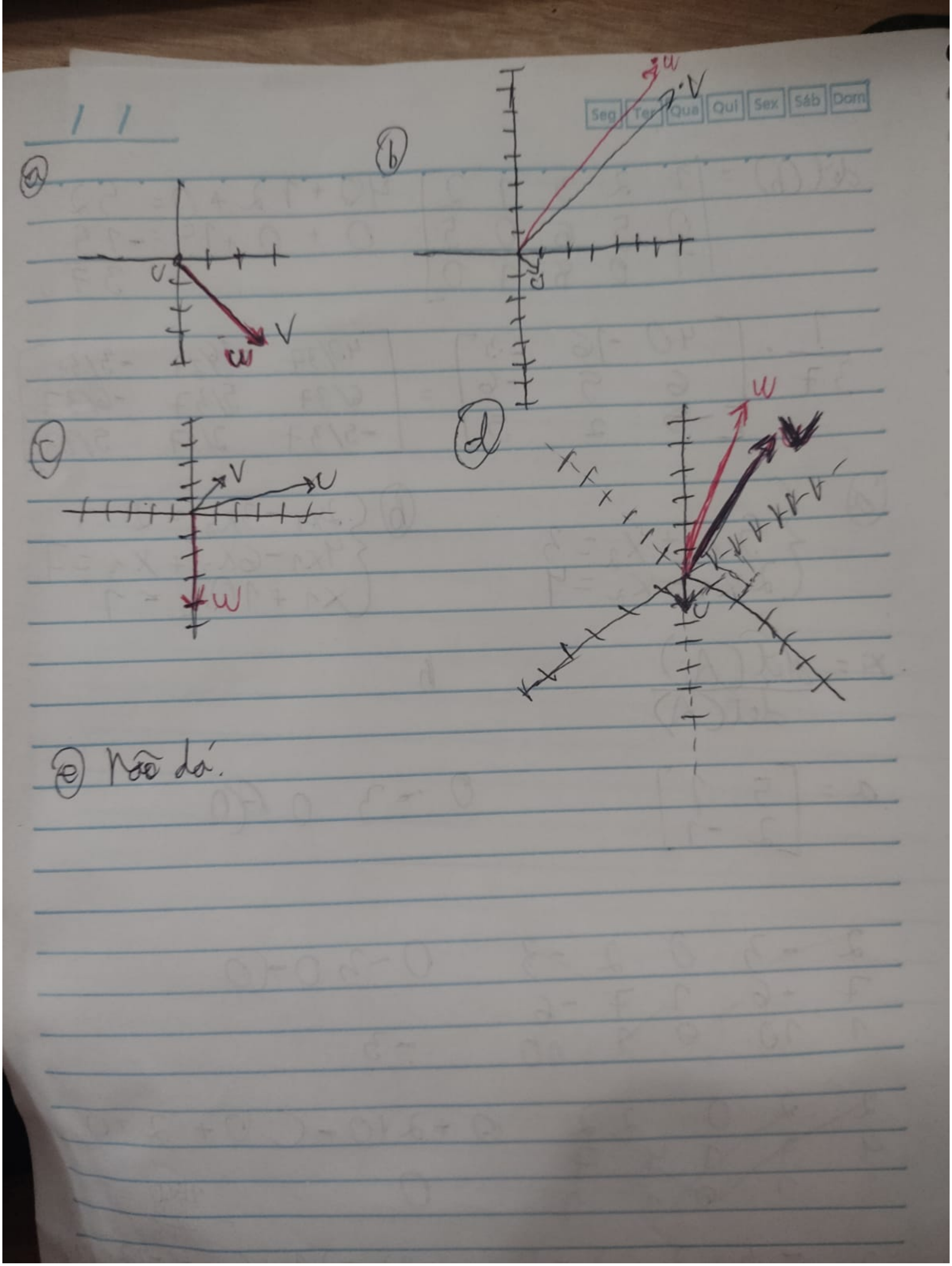
$\Rightarrow 4$

(d) $u_d = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, v_d \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \Rightarrow v-u = \begin{bmatrix} 1 \\ -2 \\ 6 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{1^2+(-2)^2+6^2} \Rightarrow$

$\Rightarrow \sqrt{1+4+36} = \sqrt{41}$

(e) $u_e = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, v_d \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \Rightarrow v-u = \begin{bmatrix} 0 \\ -2 \\ -4 \\ -4 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{0^2+(-2)^2+(-4)^2+(-4)^2}$

$\Rightarrow \sqrt{0+4+16+16} \Rightarrow \sqrt{36} \Rightarrow 6$



Fazer gráficos 3d é bem difícil

4. (Simon & Blume - Exercício 10.12) Para cada um dos seguintes pares de vetores, primeiro determine se o ângulo entre eles é agudo, obtuso ou reto e então calcule esse ângulo:

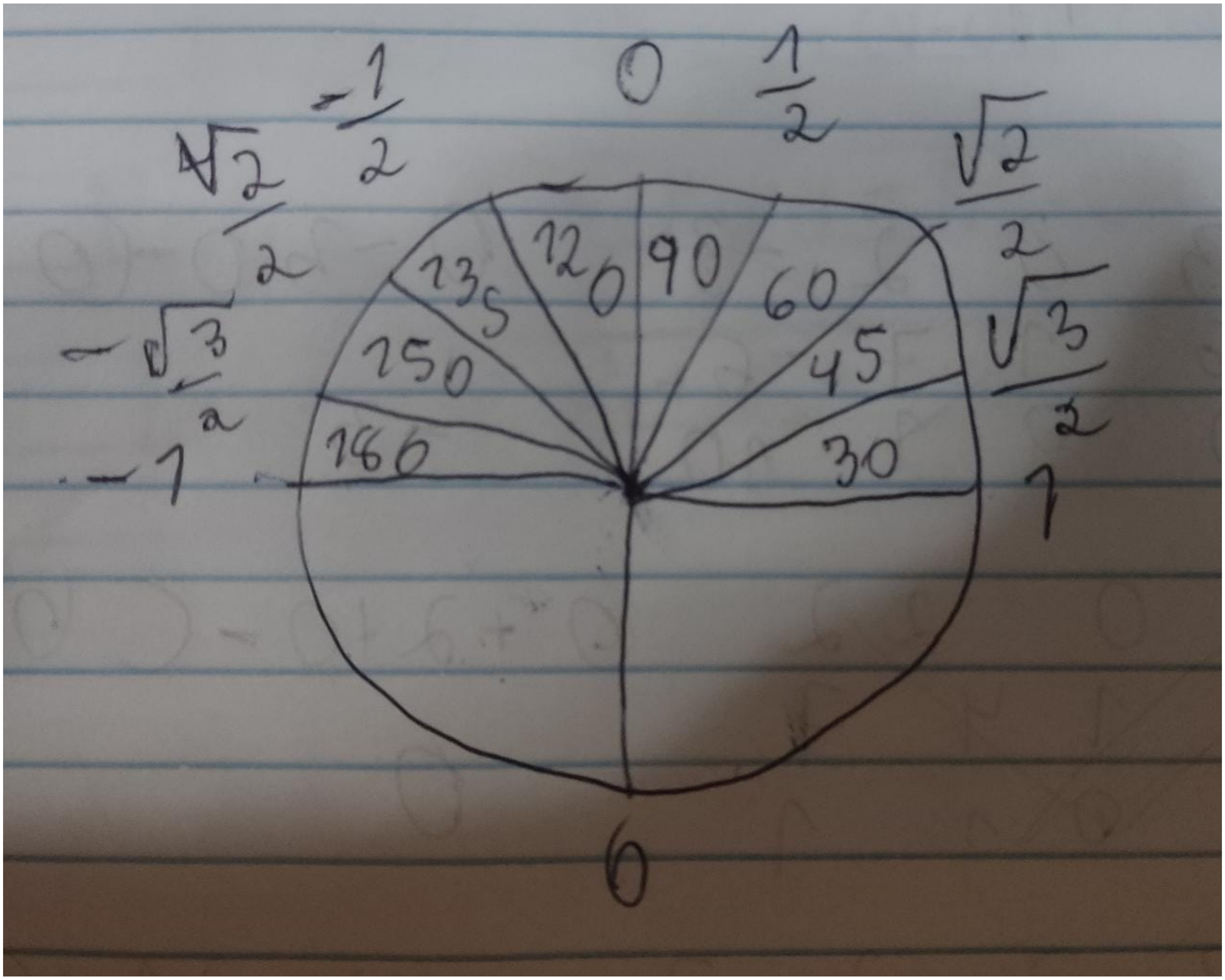
(a) $\boldsymbol{u}_a = (1, 0)$, $\boldsymbol{v}_a = (2, 2)$

(b) $\boldsymbol{u}_b = (4, 1)$, $\boldsymbol{v}_b = (2, -8)$

(c) $\boldsymbol{u}_c = (1, 1, 0)$, $\boldsymbol{v}_c = (1, 2, 1)$

(d) $\boldsymbol{u}_d = (1, -1, 0)$, $\boldsymbol{v}_d = (1, 2, 1)$

(e) $\boldsymbol{u}_e = (1, 0, 0, 0, 0)$, $\boldsymbol{v}_e = (1, 1, 1, 1, 1)$



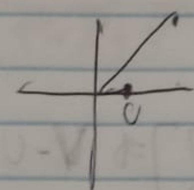
4)

a) $u = (1,0)$ $\cos\theta = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

$u_a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; v_a = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $\sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$

$\dfrac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix}}{1 \cdot \sqrt{8}} \Rightarrow \dfrac{(1 \cdot 2)+(0 \cdot 2)}{\sqrt{8}} = \dfrac{2}{\sqrt{8}} = \dfrac{1}{\sqrt{2}}$

$\cos \dfrac{1}{\sqrt{2}} \Rightarrow \cos \dfrac{\sqrt{2}}{2} = 45^\circ$



$\boxed{\text{Agudo}}$

$u \cdot v > 0$ = Agudo
$u \cdot v < 0$ = Obtuso
$u \cdot v = 0$ = Reto

b) $u_b = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad v_b = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -8 \end{bmatrix} = 4 \cdot 2 + 1 \cdot -8 = 0$

0 $\cos 0 = 90^\circ$ $\boxed{\text{Reto}}$

c) $u_c = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad v_c \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $1 \cdot 1 + 1 \cdot 2 + 0 \cdot 1 = 3$

$\boxed{\text{Agudo}}$

$\dfrac{3}{\sqrt{1^2+1^2+0^2} \cdot \sqrt{1^2+2^2+1^2}} \Rightarrow \dfrac{3}{\sqrt{2} \cdot \sqrt{6}} \Rightarrow \dfrac{3}{\sqrt{2}\sqrt{2}\sqrt{3}} \Rightarrow \dfrac{\sqrt{3}}{2}$

$\cos \dfrac{\sqrt{3}}{2} = 30^\circ$

(d) $u_d = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $V_d = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $1\cdot 1 + -1\cdot 2 + 0\cdot 1 = -1$

$1 \quad -2 \quad 0$

$\boxed{\text{Obtuso}}$

$\frac{-1}{|U|\cdot|V|} \Rightarrow \frac{-1}{\sqrt{1^2+(-1)^2+0^2}\cdot\sqrt{1^2+2^2+1^2}} \Rightarrow \frac{-1}{\sqrt{2}\cdot\sqrt{6}} \Rightarrow \frac{-1}{2\sqrt{3}} \Rightarrow$

$-\frac{\sqrt{3}}{2} \Rightarrow \cos -\frac{\sqrt{3}}{2} = 150^\circ$

(e) $U_e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $V_e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $1\cdot 1 + 0\cdot 1 + 0\cdot 1 + 0\cdot 1 + 0\cdot 1 = 1$

$\boxed{\text{Agudo}}$

$\frac{1}{1\cdot\sqrt{5}} \Rightarrow \frac{1}{\sqrt{5}} \Rightarrow \frac{\sqrt{5}}{5}$

$\Rightarrow \frac{2\sqrt{5}}{10} \Rightarrow \frac{2\cdot\sqrt{5}}{2\cdot 5} \Rightarrow \frac{2\cdot\sqrt{5}}{2\cdot\sqrt{5}\cdot\sqrt{5}} \quad \frac{2}{2\sqrt{5}} \quad \frac{1}{1}$

Não sei expressar $\cos\frac{1}{\sqrt{5}}$ em ângulos.

5. (Simon & Blume - Exercício 10.13) Para cada um dos seguintes vetores, encontre o vetor de comprimento 1 que aponta na mesma direção:

(a) $\boldsymbol{v}_a = (3, 4)$

(b) $\boldsymbol{v}_b = (6, 0)$

(c) $\boldsymbol{v}_c = (1, 1, 1)$

(d) $\boldsymbol{v}_d = (-1, 2, -3)$

Vetor unitário $u = \frac{1}{\|v\|} v$

(5) (a) $v_a = \begin{bmatrix}3\\4\end{bmatrix} \Rightarrow u = \frac{1}{\sqrt{3^2+4^2}} \cdot \begin{bmatrix}3\\4\end{bmatrix} \Rightarrow \frac{1}{5} \cdot \begin{bmatrix}3\\4\end{bmatrix} =$

$\Rightarrow \begin{bmatrix}3/5\\4/5\end{bmatrix} = u \Rightarrow \|u\| = \sqrt{\frac{3^2}{5} + \frac{4^2}{5}} \Rightarrow \sqrt{\frac{9}{25} + \frac{16}{25}}$

$\Rightarrow \sqrt{\frac{25}{25}} \Rightarrow 1$

(b) $v_b = \begin{bmatrix}6\\0\end{bmatrix} \Rightarrow u = \frac{1}{\sqrt{6^2+0^2}} \begin{bmatrix}6\\0\end{bmatrix} \Rightarrow \frac{1}{6} \begin{bmatrix}6\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = u$

(c) $v_c = \begin{bmatrix}1\\1\\1\end{bmatrix} \Rightarrow u = \frac{1}{\sqrt{1^2+1^2+1^2}} \begin{bmatrix}1\\1\\1\end{bmatrix} \Rightarrow \frac{1}{\sqrt{3}} \begin{bmatrix}1\\1\\1\end{bmatrix} \Rightarrow$

$\Rightarrow u = \begin{bmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{bmatrix}$

(d) $v_d = \begin{bmatrix}-1\\2\\-3\end{bmatrix} \Rightarrow u = \frac{1}{\sqrt{(-1)^2+2^2+(-3)^2}} \begin{bmatrix}-1\\2\\-3\end{bmatrix} \Rightarrow \frac{1}{\sqrt{14}} \begin{bmatrix}-1\\2\\-3\end{bmatrix} \Rightarrow$

$u = \begin{bmatrix}-1/\sqrt{14}\\2/\sqrt{14}\\-3/\sqrt{14}\end{bmatrix}$

6. (Simon & Blume - Exercício 10.15) Prove que $\|\boldsymbol{u} - \boldsymbol{v}\|^2 = \|\boldsymbol{u}\|^2 - 2\boldsymbol{u} \cdot \boldsymbol{v} + \|\boldsymbol{v}\|^2$.

(6) $\|u - v\|^2 = \|u\|^2 - 2u \cdot v + \|v\|^2$

$\Rightarrow \|u - v\|^2 \Rightarrow \|u - v\| \cdot \|u - v\| \Rightarrow \|u\|^2 - 2u \cdot v + \|v\|^2$

7. (Simon & Blume - Exercício 10.16) Seja $\boldsymbol{u} = (u_1, u_2)$ um vetor do $\mathbb{R}^2$. Prove que cada uma das seguintes funções é uma norma no $\mathbb{R}^2$:

(a) $\rho(u_1, u_2) = |u_1| + |u_2|$

(b) $\rho(u_1, u_2) = \max\{|u_1|, |u_2|\}$

A informação disponibilizada pelo professor na lista não é o suficiente pra resolver essa questão. Tive que olhar no livro como ela se encontrava, print abaixo:

10.16 *a*) In view of the last paragraph in this section, prove that each of the following ii a norm in $\mathbf{R}^2$:

$$|||(u_1, u_2)||| = |u_1| + |u_2|,$$

$$|||(u_1, u_2)||| = \max\{|u_1|, |u_2|\}.$$

b) What arc the analogous norms in R”‘!

O exercício referencia um parágrafo no livro, que é o seguinte:

The three basic properties of Euclidean length are:

(1) $\|\mathbf{u}\| \geq 0$ and $\|\mathbf{u}\| = 0$ only when” = 0.
(2) $\|r\mathbf{u}\| = |r|\|\mathbf{u}\|$
(3) $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$.

Any assignment of a real number to a vector that satisfies these three properties is called a norm. Exercise 10.16 lists other norms that arise naturally in applications. We will say more about norms in the last section of Chapter 29.

Ou seja, o que ele quer é que se provem essas 3 propriedades tanto para |u1| + |u2| quanto para max{|u1|,|u2|}. Olhando o livro e as notas de aula do professor, achei uma forma de provar para o primeiro caso, mas não para o segundo (essa função max).

(1) $||U|| \geq 0$ e $||U|| = 0$ quando $U = 0$

(2) $||rU|| = |r| \, ||U||$

(3) $||U+V|| \leq ||U|| + ||V||$

$1 = ||U|| = \sqrt{U_1^2 + U_2^2 + \dots U_N^2} \geq 0$. Só será 0 se todos os U's forem 0.

$2 = ||rU|| = \sqrt{(rU_1)^2 + (rU_2)^2 + \dots + (rU_N)^2} \Rightarrow$
$\Rightarrow \sqrt{r^2 \cdot (U_1^2 + U_2^2 + \dots + U_N^2)} \Rightarrow \sqrt{r^2} \cdot \sqrt{U_1^2 + U_2^2 + \dots + U_N^2} \Rightarrow |r| \, ||U||$

$3 = \|U+V\| \leq \|U\| + \|V\| \Rightarrow$

$\frac{U\cdot V}{\|U\|\|V\|} = \cos\theta \leq 1 \Rightarrow U\cdot V \leq \|U\|\cdot\|V\| \Rightarrow$

$\Rightarrow (U\cdot V)^2 \leq (\|U\|^2\cdot\|V\|)^2 \Rightarrow \|U\|^2 + 2U\cdot V + \|V\|^2 \leq (\|U\|^2\cdot\|V\|)^2$

$\Rightarrow \|U\|^2 + 2U\cdot V + \|V\|^2 \leq \|U\|^2 + 2\|U\|\|V\| + \|V\|^2$

$\Rightarrow U\cdot U + U\cdot V + V\cdot U + V\cdot V \leq (\|U\|+\|V\|)^2 \Rightarrow$

$\Rightarrow U(U+V) + V(U+V) \leq (\|U\|+\|V\|)^2 \Rightarrow$

$\Rightarrow (U+V)\cdot(U+V) \leq (\|U\|+\|V\|)^2 \Rightarrow$

$\Rightarrow \|U+V\|^2 \leq (\|U\|+\|V\|)^2 \Rightarrow \|U+V\| \leq \|U\|+\|V\|$

(b) Menor ideia

8. (Simon & Blume - Exercício 10.21) Prove as seguintes identidades:

   (a) $\|\boldsymbol{u}+\boldsymbol{v}\|^2 + \|\boldsymbol{u}-\boldsymbol{v}\|^2 = 2\|\boldsymbol{u}\|^2 + 2\|\boldsymbol{v}\|^2$

   (b) $\boldsymbol{u}\cdot\boldsymbol{v} = \frac{1}{4}\|\boldsymbol{u}+\boldsymbol{v}\|^2 - \frac{1}{4}\|\boldsymbol{u}-\boldsymbol{v}\|^2$

(8) (a) $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$

$\Rightarrow \|u+v\|^2 + \|u-v\|^2 \Rightarrow \|u\|^2 + 2uv + \|v\|^2 + \|u\|^2 - 2uv + \|v\|^2$

$\Rightarrow 2\|u\|^2 + 2\|v\|^2$.

8(b) $u \cdot v = \frac{1}{4}\|u+v\|^2 - \frac{1}{4}\|u-v\|^2$

$\Rightarrow \frac{1}{4}\|u+v\|^2 - \frac{1}{4}\|u-v\|^2 \Rightarrow \frac{1}{4}\left((u+v)(u+v) - (u-v)(u-v)\right)$

$\Rightarrow \frac{1}{4}\left(u^2 + 2uv + v^2 - (u^2 - 2uv + v^2)\right) \Rightarrow$

$\Rightarrow \frac{1}{4}\left(0 + 4uv + 0\right) \Rightarrow \frac{4}{4}uv = u \cdot v$