1. (Simon & Blume - Exercício 10.5) Seja $\boldsymbol{u} = (1, 2)$, $\boldsymbol{v} = (0, 1)$, $\boldsymbol{w} = (1, -3)$, $\boldsymbol{x} = (1, 2, 0)$, e $\boldsymbol{z} = (0, 1, 1)$. Compute os seguintes vetores, sempre que eles estiverem definidos: $\boldsymbol{u} + \boldsymbol{v}$, $-4\boldsymbol{w}$, $\boldsymbol{u} + \boldsymbol{z}$, $3\boldsymbol{z}$, $2\boldsymbol{v}$, $\boldsymbol{u} + 2\boldsymbol{v}$, $\boldsymbol{u} - \boldsymbol{v}$, $3\boldsymbol{x} + \boldsymbol{z}$, $-2\boldsymbol{x}$, $\boldsymbol{w} + 2\boldsymbol{x}$.

$8 = 64$

1 $$u + v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$-4w = -4 \cdot \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$$

$$u + z = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \text{Indefinido}$$

$$3z = 3 \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

$$2v = 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$u + 2v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$u - v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$3x + z = 3\begin{bmatrix}1\\2\\0\end{bmatrix} + \begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}3\\7\\1\end{bmatrix}$$

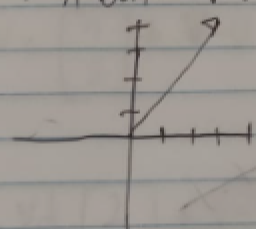
$$-2x = -2\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-2\\-4\\0\end{bmatrix}$$

$$w + 2x = \begin{bmatrix}1\\-3\end{bmatrix} + 2\begin{bmatrix}1\\2\\0\end{bmatrix} = \text{Indefinido}$$

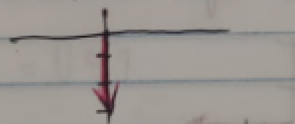
2. (Simon & Blume - Exercício 10.10) Encontre o comprimento dos seguintes vetores. Desenhe os vetores de (a) até (g):

   (a) $\boldsymbol{v}_a = (3, 4)$
   (b) $\boldsymbol{v}_b = (0, -3)$
   (c) $\boldsymbol{v}_c = (1, 1, 1)$
   (d) $\boldsymbol{v}_d = (3, 3)$
   (e) $\boldsymbol{v}_e = (-1, -1)$
   (f) $\boldsymbol{v}_f = (1, 2, 3)$
   (g) $\boldsymbol{v}_g = (2, 0)$
   (h) $\boldsymbol{v}_h = (1, 2, 3, 4)$
   (i) $\boldsymbol{v}_i = (3, 0, 0, 0, 0)$

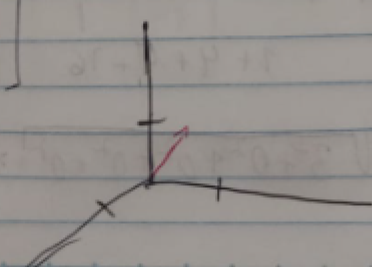
(2a) $v_a = \begin{bmatrix}3\\4\end{bmatrix} \Rightarrow \|v_a\| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$



(b) $v_b = \begin{bmatrix}0\\-3\end{bmatrix} \Rightarrow \|v_b\| = \sqrt{0^2 + (-3)^2} \Rightarrow \sqrt{9} = 3$

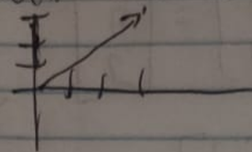


(c) $v_c = \begin{bmatrix}1\\1\\1\end{bmatrix} \Rightarrow \|v_c\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

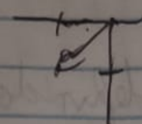


/ /   Seg Ter Qua Qui Sex Sáb Dom

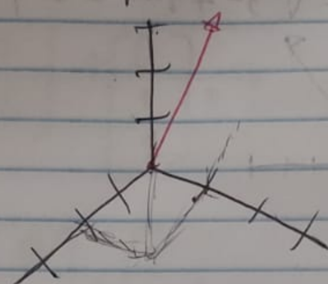
(d) $V_d = \begin{bmatrix}3\\3\end{bmatrix} \Rightarrow ||V_d|| = \sqrt{3^2+3^2} \Rightarrow \sqrt{18}$



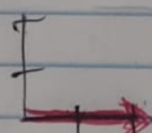
(e) $V_e = \begin{bmatrix}-1\\-1\end{bmatrix} \Rightarrow ||V_e|| = \sqrt{(-1)^2+(-1)^2} \Rightarrow \sqrt{2}$



(f) $V_f = \begin{bmatrix}1\\2\\3\end{bmatrix} \Rightarrow ||V_f|| = \sqrt{1^2+2^2+3^2} \Rightarrow \sqrt{14}$



(g) $V_g = \begin{bmatrix}2\\0\end{bmatrix} \Rightarrow ||V_g|| = \sqrt{2^2+0^2} \Rightarrow \sqrt{4} = 2$



(h) $V_h = \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \Rightarrow ||V_h|| = \sqrt{1^2+2^2+3^2+4^2} \Rightarrow \sqrt{30}$

$1+4+9+16$

(i) $V_i = \begin{bmatrix}3\\0\\0\\0\\0\end{bmatrix} \Rightarrow ||V_i|| = \sqrt{3^2+0^2+0^2+0^2+0^2} = \sqrt{9} = 3$

3. (Simon & Blume - Exercício 10.11) Encontre a distância entre os pares de vetores abaixo, representando graficamente sempre que possível:

(a) $\boldsymbol{u}_a = (0, 0)$, $\boldsymbol{v}_a = (3, -4)$

(b) $\boldsymbol{u}_b = (1, -1)$, $\boldsymbol{v}_b = (7, 7)$

(c) $\boldsymbol{u}_c = (5, 2)$, $\boldsymbol{v}_c = (1, 2)$

(d) $\boldsymbol{u}_d = (1, 1, -1)$, $\boldsymbol{v}_d = (2, -1, 5)$

(e) $\boldsymbol{u}_e = (1, 2, 3, 4)$, $\boldsymbol{v}_e = (1, 0, -1, 0)$

(3) $\|w\| = \|v - u\|$

(a) $u_a = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $v_a \begin{bmatrix} 3 \\ -4 \end{bmatrix} \Rightarrow \|v-u\| = \begin{bmatrix} 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

$\Rightarrow \sqrt{3^2 + (-4)^2} \Rightarrow \sqrt{9 + 16} = 5$

(b) $u_b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $v_b = \begin{bmatrix} 7 \\ 7 \end{bmatrix} = v - u = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$

$\Rightarrow \sqrt{100} = 10$

(c) $u_c = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $v_c \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow v - u = \begin{bmatrix} -4 \\ 0 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{0 + (-4)^2} = \sqrt{16}$

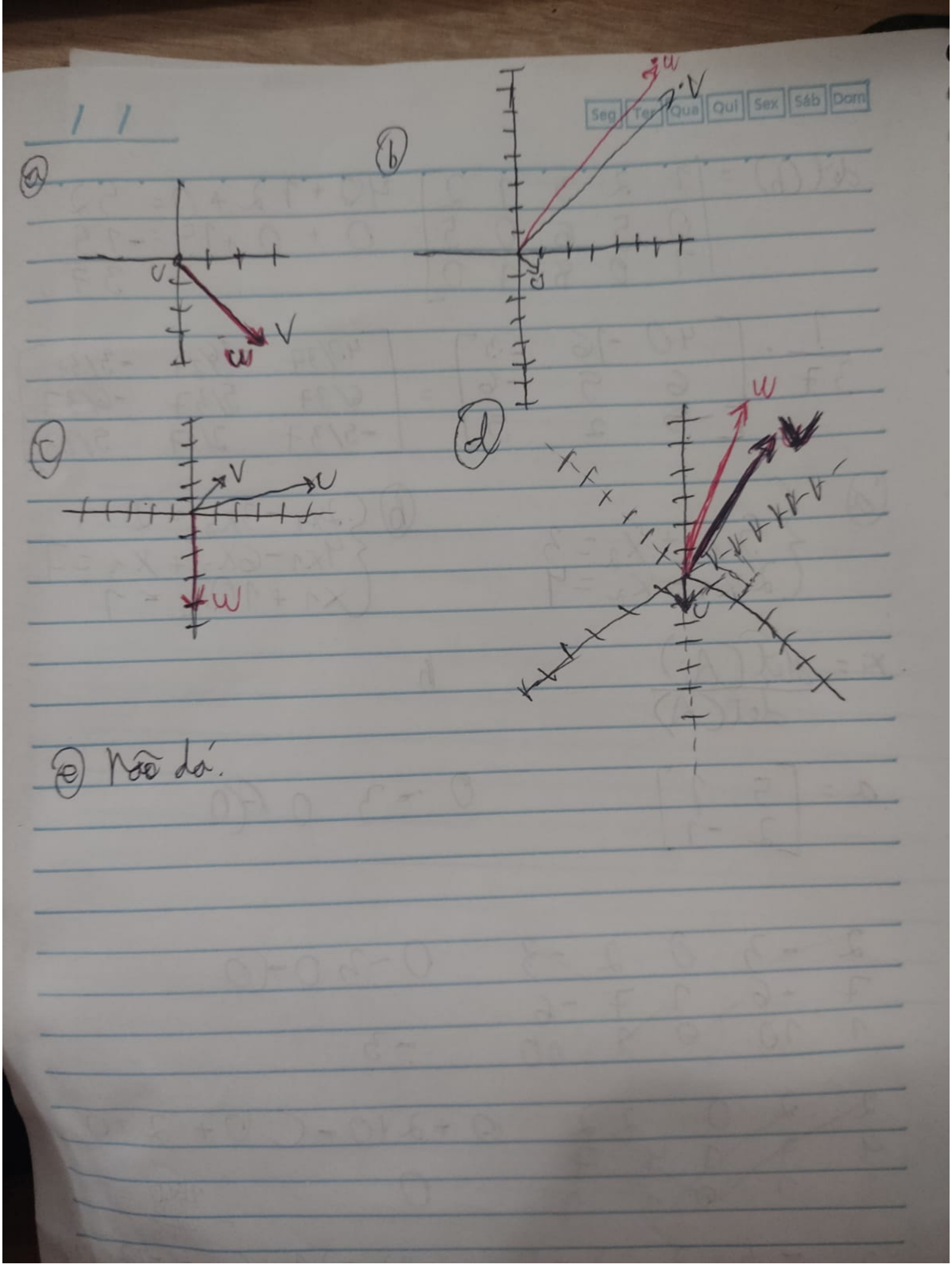
$\Rightarrow 4$

(d) $u_d = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $v_d \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \Rightarrow v - u = \begin{bmatrix} 1 \\ -2 \\ 6 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{1^2 + (-2)^2 + 6^2} \Rightarrow$

$\Rightarrow \sqrt{1 + 4 + 36} = \sqrt{41}$

(e) $u_e = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $v_d \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \Rightarrow v - u = \begin{bmatrix} 0 \\ -2 \\ -4 \\ -4 \end{bmatrix} \Rightarrow \|v-u\| = \sqrt{0^2 + (-2)^2 + (-4)^2 + (-4)^2}$

$\Rightarrow \sqrt{0 + 4 + 16 + 16} \Rightarrow \sqrt{36} \Rightarrow 6$



Fazer gráficos 3d é bem difícil

4. (Simon & Blume - Exercício 10.12) Para cada um dos seguintes pares de vetores, primeiro determine se o ângulo entre eles é agudo, obtuso ou reto e então calcule esse ângulo:

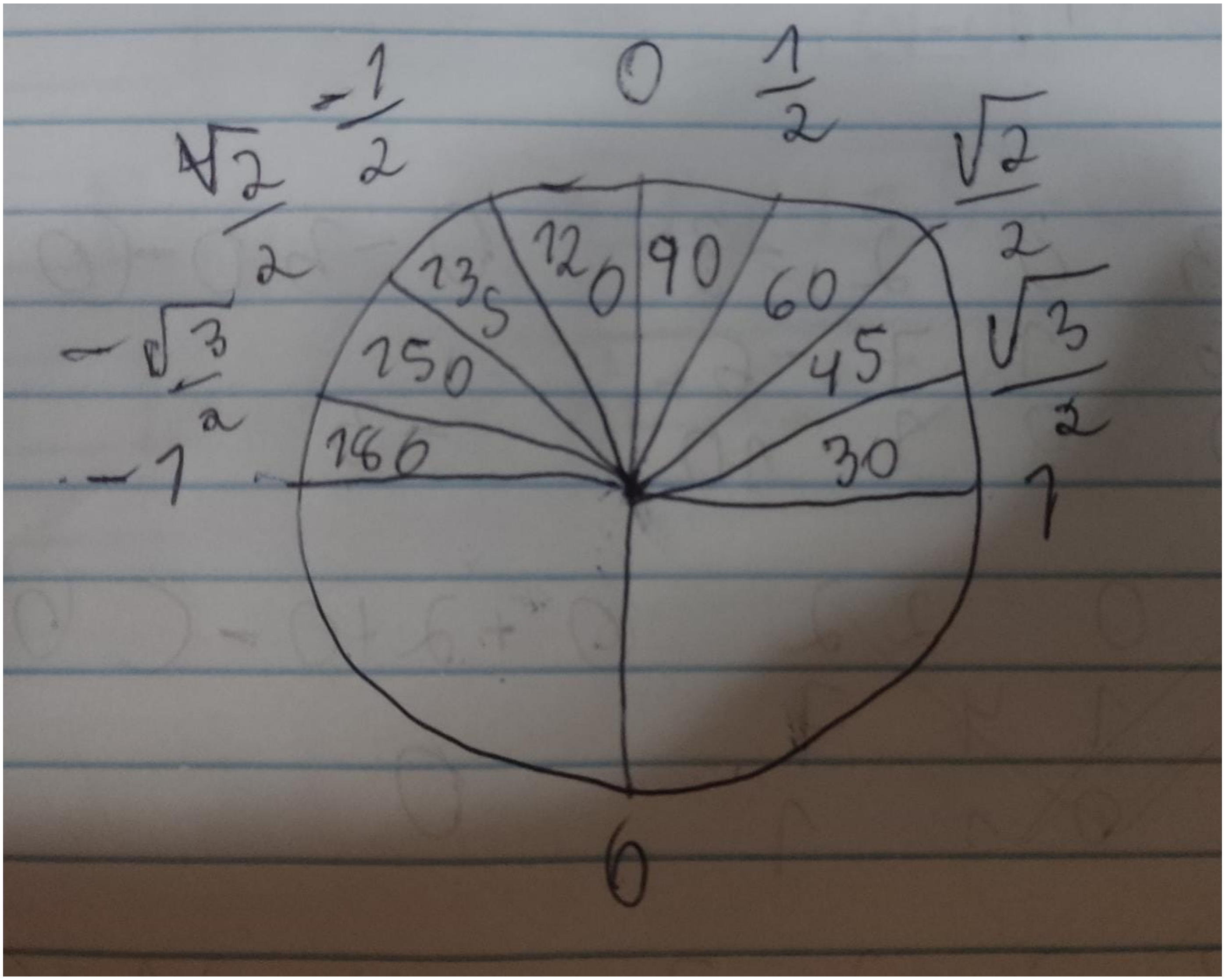
(a) $\boldsymbol{u}_a = (1, 0)$, $\boldsymbol{v}_a = (2, 2)$

(b) $\boldsymbol{u}_b = (4, 1)$, $\boldsymbol{v}_b = (2, -8)$

(c) $\boldsymbol{u}_c = (1, 1, 0)$, $\boldsymbol{v}_c = (1, 2, 1)$

(d) $\boldsymbol{u}_d = (1, -1, 0)$, $\boldsymbol{v}_d = (1, 2, 1)$

(e) $\boldsymbol{u}_e = (1, 0, 0, 0, 0)$, $\boldsymbol{v}_e = (1, 1, 1, 1, 1)$



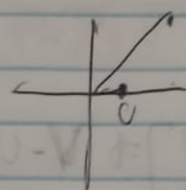
4

(a) $\cos\theta = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

$u_a = \begin{bmatrix}1\\0\end{bmatrix}, v_a = \begin{bmatrix}2\\2\end{bmatrix}$ $\quad \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$

$\dfrac{\begin{bmatrix}1\\0\end{bmatrix}\cdot\begin{bmatrix}2\\2\end{bmatrix}}{1\cdot\sqrt{8}} \Rightarrow \dfrac{(1\cdot 2)+(0\cdot 2)}{\sqrt{8}} = \dfrac{2}{\sqrt{8}} = \dfrac{1}{\sqrt{2}}$

$\cos \dfrac{1}{\sqrt{2}} \Rightarrow \cos \dfrac{\sqrt{2}}{2} = 45^0$



$\boxed{\text{Agudo}}$

$u \cdot v > 0 = \text{Agudo}$

$u \cdot v < 0 = \text{Obtuso}$

$u \cdot v = 0 = \text{Reto}$

(b) $u_b = \begin{bmatrix}4\\1\end{bmatrix} \quad v_b = \begin{bmatrix}2\\-8\end{bmatrix}$

$\begin{bmatrix}4\\1\end{bmatrix}\cdot\begin{bmatrix}2\\-8\end{bmatrix} = 4\cdot 2 + 1\cdot -8 = 0$

$\underline{0} \qquad \cos 0 = 90^0$

$\boxed{\text{Reto}}$

(c) $u_c = \begin{bmatrix}1\\1\\0\end{bmatrix} \quad v_c \begin{bmatrix}1\\2\\1\end{bmatrix}$ $\quad 1\cdot 1 + 1\cdot 2 + 0\cdot 1 = 3$

$\boxed{\text{Agudo}}$

$\dfrac{3}{\sqrt{1^2+1^2+0^2}\cdot\sqrt{1^2+2^2+1^2}} \Rightarrow \dfrac{3}{\sqrt{2}\cdot\sqrt{6}} \Rightarrow \dfrac{3}{\sqrt{2}\sqrt{2}\sqrt{3}} \Rightarrow \dfrac{\sqrt{3}}{2}$

$\cos \dfrac{\sqrt{3}}{2} = 30^0$

(d) $v_d = \begin{bmatrix}1\\-1\\0\end{bmatrix}$ $V_d = \begin{bmatrix}1\\2\\1\end{bmatrix}$ $1\cdot1 + -1\cdot2 + 0\cdot1 = -1$

$1 \quad -2 \quad 0$

$\boxed{\text{Obtuso}}$

$$\frac{-1}{|U|\cdot|V|} \Rightarrow \frac{-1}{\sqrt{1^2+(-1)^2+0^2}\cdot\sqrt{1^2+2^2+1^2}} \Rightarrow \frac{-1}{\sqrt{2}\cdot\sqrt{6}} \Rightarrow \frac{-1}{2\sqrt{3}} \Rightarrow$$

$$\frac{-\sqrt{3}}{2} \Rightarrow \cos\frac{-\sqrt{3}}{2} = 150^\circ$$

(e) $U_e = \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$ $V_e = \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$ $1\cdot1+0\cdot1+0\cdot1+0\cdot1+0\cdot1=1$

$\boxed{\text{Agudo}}$

$$\frac{1}{1\cdot\sqrt{5}} \Rightarrow \frac{1}{\sqrt{5}} \Rightarrow \frac{\sqrt{5}}{5}$$

$$\Rightarrow \frac{2\sqrt{5}}{10} \Rightarrow \frac{2\cdot\sqrt{5}}{2\cdot5} \Rightarrow \frac{2\cdot\sqrt{5}}{2\cdot\sqrt{5}\cdot\sqrt{5}} \quad \frac{2}{2\sqrt{5}} \quad \frac{1}{1}$$

Não sei expressar $\cos\frac{1}{\sqrt{5}}$ em ângulos.

In [ ]: