1. (Simon & Blume - Exercício 11.2) Determine se cada conjunto de vetores abaixo é linearmente independente.

   (a) $\boldsymbol{u}_a = (2, 1)$, $\boldsymbol{v}_a = (1, 2)$

   (b) $\boldsymbol{u}_b = (2, 1)$, $\boldsymbol{v}_b = (-4, -2)$

   (c) $\boldsymbol{u}_c = (1, 1, 0)$, $\boldsymbol{v}_c = (0, 1, 1)$

   (d) $\boldsymbol{u}_d = (1, 1, 0)$, $\boldsymbol{v}_d = (0, 1, 1)$, $\boldsymbol{w}_d = (1, 0, 1)$

(1) $U_a \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $V_a \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $aU + bV = 0 \Rightarrow a + b = 0$

$a \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 2a + 1b = 0 \\ 1a + 2b = 0 \end{matrix}$

$\Rightarrow \begin{matrix} 2a + 1b = 0 \\ 0 + 1{,}5b = 0 \end{matrix} \Rightarrow b = 0 \Rightarrow a = 0$ LI

(b) $U_b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $V_b = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$ $a \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b \begin{bmatrix} -4 \\ -2 \end{bmatrix}$

$\Rightarrow \begin{matrix} 2a - 4b = 0 \\ 1a - 2b = 0 \end{matrix} \Rightarrow \begin{matrix} a = 2b \\ b = 1/2\, a \end{matrix}$ LD

(c) $U_c = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $V_c = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $\begin{matrix} 1a + 0b = 0 \\ 1a + 1b = 0 \\ 0a + 1b = 0 \end{matrix}$ $\begin{matrix} a = 0 \\ b = 0 \end{matrix}$ LI

(d) $U_d = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $V_d = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $W_d = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{matrix} a + 0 + c = 0 \\ a + b + 0 = 0 \\ 0 + b + c = 0 \end{matrix}$

$a = -c = -b$

$b = -a = c$ e $b = -c = a$ LI

$c = -a$ e $c = -b = -c = b = a$

2. (Simon & Blume - Exercício 11.6) Prove que qualquer coleção de vetores que inclui o vetor zero não pode ser linearmente independente.

(2) $\{V_1, \ldots, V_N\} \Rightarrow V_1 = 0 \Rightarrow 1V_1 + 0V_2 + 0V_3 + \ldots + 0V_N$

1v1+0v2+...+0vn=0

3. (Simon & Blume - Exercício 11.10) Os vetores $\boldsymbol{v}_1 = (1, 2, 3)$, $\boldsymbol{v}_2 = (4, 5, 12)$ e $\boldsymbol{v}_3 = (0, 8, 0)$ geram o $\mathbb{R}^3$? Explique.

(3) $V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad V_2 = \begin{bmatrix} 4 \\ 5 \\ 12 \end{bmatrix} \quad V_3 = \begin{bmatrix} 0 \\ 8 \\ 6 \end{bmatrix}$

$$a \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + b \begin{bmatrix} 4 \\ 5 \\ 12 \end{bmatrix} + c \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} a \\ 2a \\ 3a \end{bmatrix} + \begin{bmatrix} 4b \\ 5b \\ 12b \end{bmatrix} + \begin{bmatrix} 0 \\ 8c \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{aligned} a + 4b + 0 &= x \\ 2a + 5b + 8c &= y \\ 3a + 12b + 0 &= z \end{aligned} \Rightarrow \begin{aligned} a + 4b + 0 &= x \\ 0 - 3b + 8c &= y - x \\ 0 + 0 + 0 &= z - 3x \end{aligned}$$

Existe b $(z - 3x)$ sem solução.

4. (Simon & Blume - Exercício 11.14) Quais conjuntos de vetores abaixo são uma base no $\mathbb{R}^3$?

   (a) $\boldsymbol{u}_a = (1, 1, 1)$, $\boldsymbol{v}_a = (1, 2, 1)$

   (b) $\boldsymbol{u}_b = (1, 1, 1)$, $\boldsymbol{v}_b = (1, 2, 1)$, $\boldsymbol{w}_b = (1, 0, 1)$

   (c) $\boldsymbol{u}_c = (6, 3, 9)$, $\boldsymbol{v}_c = (5, 2, 8)$, $\boldsymbol{w}_c = (4, 1, 7)$

   (d) $\boldsymbol{u}_d = (1, 1, 1)$, $\boldsymbol{v}_d = (1, 2, 1)$, $\boldsymbol{w}_d = (1, 0, 0)$

   (e) $\boldsymbol{u}_e = (1, 1, 1)$, $\boldsymbol{v}_e = (1, 2, 1)$, $\boldsymbol{w}_e = (1, 0, 0)$, $\boldsymbol{x}_e = (0, 1, 0)$

(4) a) $v_a = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\quad V_a = \begin{bmatrix}1\\2\\1\end{bmatrix}$

$$\begin{aligned} a+b &= x \\ a+2b &= y \\ a+b &= z \end{aligned} \qquad \begin{aligned} b &= y-x \\ \\ 0 &= z-x \end{aligned}$$

b)
$$\begin{aligned} a+b &= x \\ 0+b &= y-x \\ 0+0 &= z-x \end{aligned}$$

(b) $u_b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad w_b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{cases} a + b + c = x \\ a + 2b + 0 = y \\ a + b + c = z \end{cases} \Rightarrow \begin{cases} a + b + c = x \\ 0 + b + 0 = y - x \\ 0 + 0 + 0 = z - x \end{cases}$$

(c) $u_c = \begin{bmatrix} 6 \\ 3 \\ 9 \end{bmatrix} \quad v_c = \begin{bmatrix} 5 \\ 2 \\ 8 \end{bmatrix} \quad w_c = \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix} \Rightarrow \begin{cases} 6a + 5b + 4c = x \\ 3a + 2b + 1c = y \\ 9a + 8b + 7c = z \end{cases}$

$$\begin{cases} 6a + 5b + 4c = x \\ 0 - \frac{1}{2}b - 1c = y - \frac{1}{2}x \\ 0 + \frac{1}{2}b + 1c = z - \frac{3}{2}x \end{cases} \Rightarrow \begin{cases} 6a + 5b + 4c = x \\ 0 - \frac{1}{2}b - 1c = y - \frac{1}{2}x \\ 0 + 0 + 0 = z + y - 2x \end{cases}$$

(d) $u_d = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_d = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad w_d = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$a + b + c = x$
$a + 2b + 0 = y$
$a + b = z$

$c = x - z$
$b = y - z$
$a + y - z + x - z = x$
$a + x - y + 2z - x$
$a = -y + 2z$

$x = c + z$
$z = y - b$
$y = 2z + a$

$$\cancel{x-z}\; -y + 2z \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + y - z \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x - z \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -y + 2z + y - z + x - z = x \\ -y + 2z + 2y - 2z + 0 = y \\ -y + 2z + y - z + 0 = z \end{bmatrix}$$

u_d, v_d, w_d geram $\mathbb{R}^3$

$$a\begin{bmatrix}1\\1\\1\end{bmatrix} + b\begin{bmatrix}1\\2\\1\end{bmatrix} + c\begin{bmatrix}1\\0\\6\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{matrix}a+b+c=0\\ a+2b+0=0\\ a+b+0=0\end{matrix}$$

$\Rightarrow c=0,\ b=0 \Rightarrow a+0+0=0 \Rightarrow a=0$

u_d, v_d, w_d é LI

Logo $\{u_d, v_d, w_d\}$ é uma base $\mathbb{R}^3$

(e) $u_e = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $v_e = \begin{bmatrix}1\\2\\1\end{bmatrix}$ $w_e = \begin{bmatrix}1\\0\\6\end{bmatrix}$ $x_e = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$a+b+c+0=x$ $\qquad c=x-z$

$a+2b+0+d=y$ $\qquad b=y-z-d$

$a+b+0+0=z$ $\qquad a=-y+z+d-x+z+x$

$a=-y+2z+d$

In [ ]: