1. (Simon & Blume - Exercício 8.1) Sejam

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix},\ D = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix},\ E = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(a) Compute cada uma das seguintes matrizes se ela for definida:

$A + B$, $B + C$, $B - A$, $3B$, AB, CA, CE, EC, B^T, $(CA)^T$, $A^T C^T$.

1a $A+B$ $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 \\ 4 & -2 & 4 \end{bmatrix}$

$B + C$ = Indefinido

$B - A$ $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 & -2 \\ 4 & 0 & 0 \end{bmatrix}$

$3 \cdot B$ $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix} \cdot 3 = \begin{bmatrix} 0 & 3 & -3 \\ 12 & -3 & 6 \end{bmatrix}$

$A \cdot B$ = Indefinido

2×3 $\quad 2\times 3$

$$C \cdot A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix} \Rightarrow$$

$2\times2 \quad 2\times3$

$1\cdot2+2\cdot0=2$ CA_{11}

$3\cdot1+2\cdot-1=1$ 12

$1\cdot1+2\cdot2=5$ 13

$3\cdot2-1\cdot0=6$ 21

$3\cdot3-1\cdot-1=10$ 22

$3\cdot1-1\cdot2=1$ 23

$$\begin{bmatrix} 2 & 1 & 5 \\ 6 & 10 & 1 \end{bmatrix}$$

$$C \cdot E = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow$$

$2\times2 \quad 2\times1$

$1\cdot1+2\cdot-1=-1$ 11

$3\cdot1-1\cdot-1=4$ 21

$$CE = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$E \cdot C$ = Indefinido

$2\times1 \quad 2\times2$

$$B^T = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 4 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$(CA)^T = \begin{bmatrix} 2 & 1 & 5 \\ 6 & 10 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 \\ 1 & 10 \\ 5 & 1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \Rightarrow$$

$$\underset{3\times 2}{\begin{bmatrix} 2 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}} \Rightarrow$$

$2\cdot 1 + 0\cdot 2 = 2_{11}$

$2\cdot 3 + 0\cdot -1 = 6_{12}$

$3\cdot 1 + -1\cdot 2 = 1_{21}$

$3\cdot 3 + -1\cdot -1 = 10_{22}$

$1\cdot 1 + 2\cdot 2 = 5_{31}$

$1\cdot 3 + 2\cdot -1 = 1_{32}$

$$\begin{bmatrix} 2 & 6 \\ 1 & 10 \\ 5 & 1 \end{bmatrix} = A^T C^T$$

(b) Verifique que $(DA)^T = A^T D^T$.

1b $(DA)^T = A^T D^T \Rightarrow$

$$\underset{2\times 2 \;\; 2\times 3}{DA} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 4 \\ 2 & 2 & 3 \end{bmatrix}$$

$$(DA)^T = \begin{bmatrix} 4 & 2 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} = \underset{3\times 2}{\begin{bmatrix} 2 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}} \Rightarrow$$

$$\begin{bmatrix} 4 & 2 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} = \underset{3\times 2}{\begin{bmatrix} 2 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}} \Rightarrow \begin{array}{ll} 2\cdot 2 + 0\cdot 1 = 4 & 11 \\ 2\cdot 1 + 0\cdot 1 = 2 & 12 \\ 3\cdot 2 - 1\cdot 1 = 5 & 21 \\ 3\cdot 1 - 1\cdot 1 = 2 & 22 \\ 1\cdot 2 + 2\cdot 1 = 4 & 31 \\ 1\cdot 1 + 2\cdot 1 = 3 & 32 \end{array}$$

$$\begin{bmatrix} 4 & 2 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \checkmark$$

(c) Verifique que $CD \neq DC$.

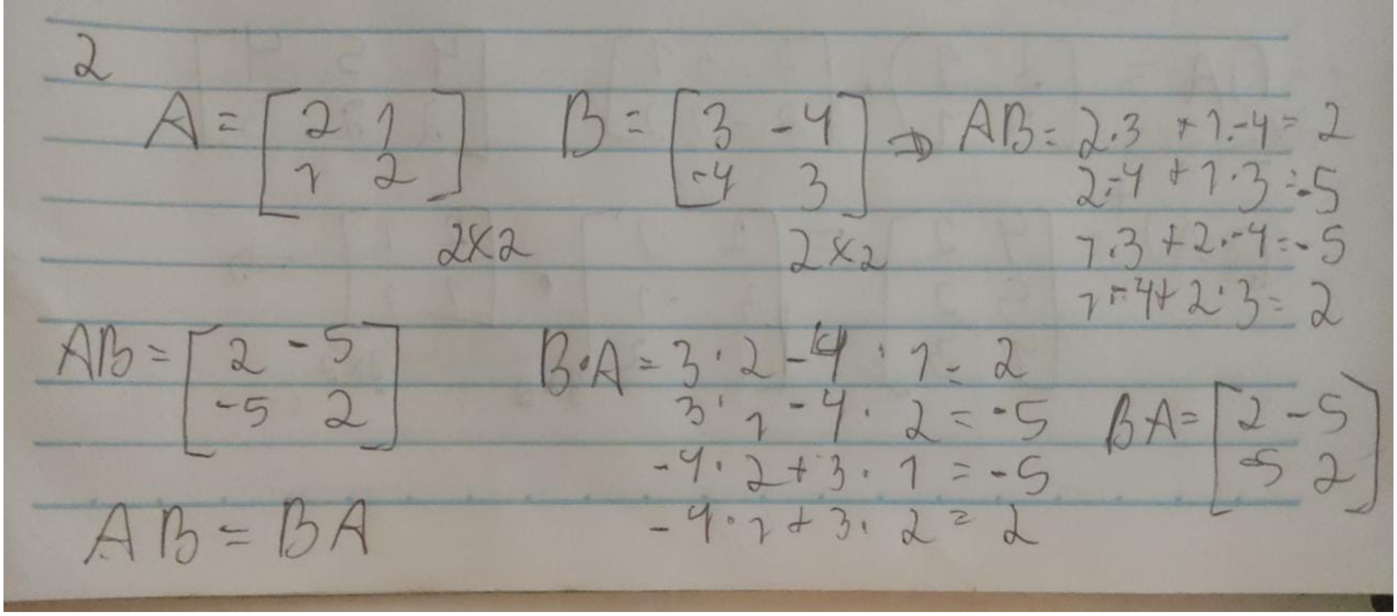
1c $CD \neq DC \Rightarrow C \cdot D = \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}}$

$$\Rightarrow \begin{array}{ll} 11 & 1\cdot 2 + 2\cdot 1 = 4 \\ 12 & 1\cdot 1 + 2\cdot 1 = 3 \\ 21 & 3\cdot 2 - 1\cdot 1 = 5 \\ 22 & 3\cdot 1 - 1\cdot 1 = 2 \end{array} \qquad CD = \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} \Rightarrow$$

$$\Rightarrow DC = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = \begin{array}{ll} 11 & 2\cdot 1 + 1\cdot 3 = 5 \\ 12 & 2\cdot 2 + 1\cdot -1 = 3 \\ 21 & 1\cdot 1 + 1\cdot 3 = 4 \\ 22 & 1\cdot 2 + 1\cdot -1 = 1 \end{array}$$

$$DC = \begin{bmatrix} 5 & 3 \\ 4 & 1 \end{bmatrix} \neq CD.$$

2. (Simon & Blume - Exercício 8.5) Às vezes acontece de $AB = BA$. Cheque isso para $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ e $B = \begin{bmatrix} 3 & -4 \\ -4 & 3 \end{bmatrix}$.



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$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ 2×2 $B = \begin{bmatrix} 3 & -4 \\ -4 & 3 \end{bmatrix}$ 2×2 $\Rightarrow AB = 2 \cdot 3 + 1 \cdot -4 = 2$

$2 \cdot -4 + 1 \cdot 3 = -5$

$1 \cdot 3 + 2 \cdot -4 = -5$

$1 \cdot -4 + 2 \cdot 3 = 2$

$AB = \begin{bmatrix} 2 & -5 \\ -5 & 2 \end{bmatrix}$ $B \cdot A = 3 \cdot 2 - 4 \cdot 1 = 2$

$3 \cdot 1 - 4 \cdot 2 = -5$

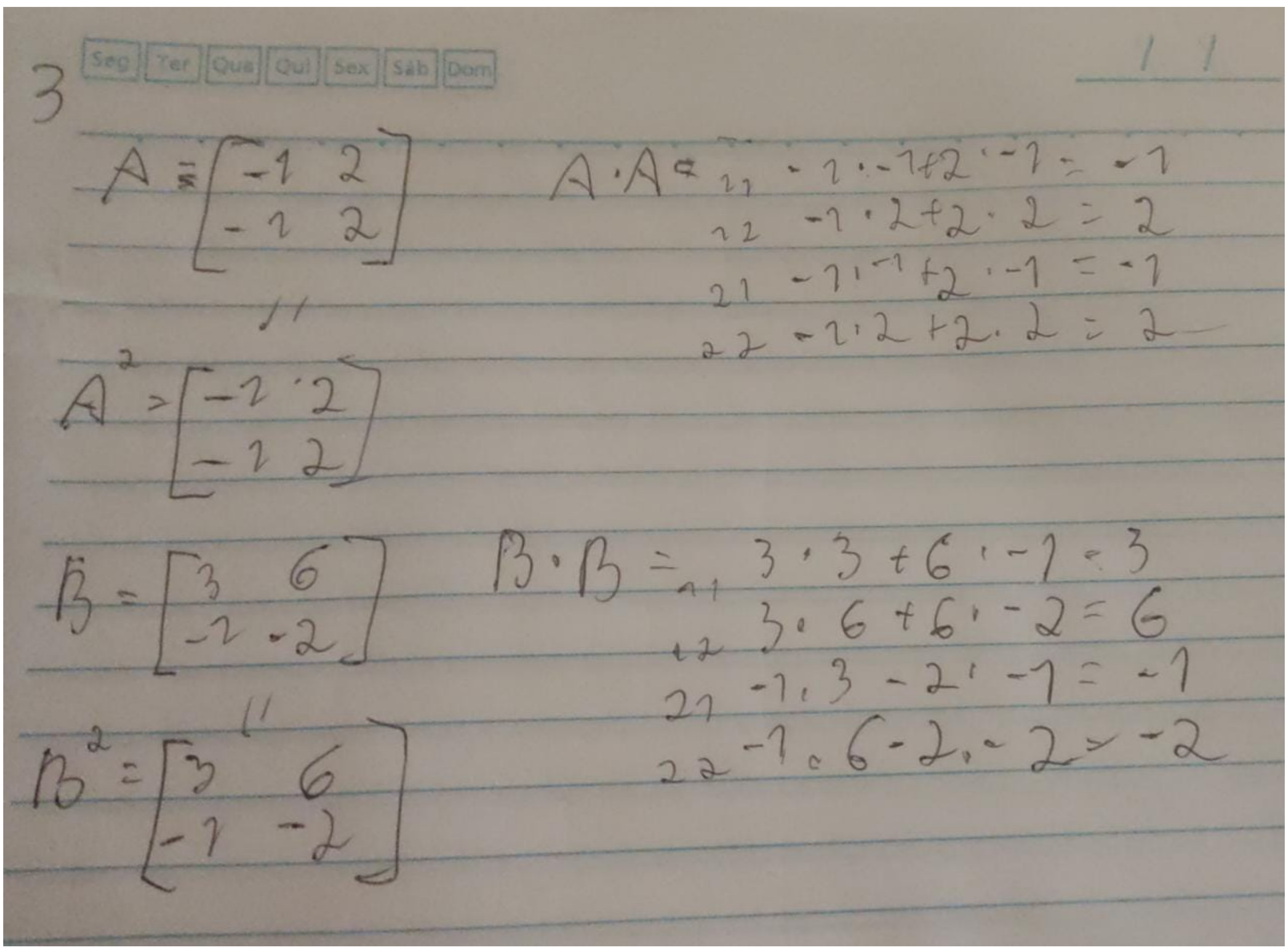
$-4 \cdot 2 + 3 \cdot 1 = -5$

$-4 \cdot 1 + 3 \cdot 2 = 2$

$BA = \begin{bmatrix} 2 & -5 \\ -5 & 2 \end{bmatrix}$

$AB = BA$

3. (Simon & Blume - Exercício 8.7) Mostre que $A = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}$ e $B = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$ são idempotentes.



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$A = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}$ $A \cdot A = {}_{11}\ -1 \cdot -1 + 2 \cdot -1 = -1$

${}_{12}\ -1 \cdot 2 + 2 \cdot 2 = 2$

${}_{21}\ -1 \cdot -1 + 2 \cdot -1 = -1$

${}_{22}\ -1 \cdot 2 + 2 \cdot 2 = 2$

//

$A^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$ $B \cdot B = {}_{11}\ 3 \cdot 3 + 6 \cdot -1 = 3$

${}_{12}\ 3 \cdot 6 + 6 \cdot -2 = 6$

${}_{21}\ -1 \cdot 3 - 2 \cdot -1 = -1$

${}_{22}\ -1 \cdot 6 - 2 \cdot -2 = -2$

//

$B^2 = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

4. (Simon & Blume - Exercício 8.9) Quantas matrizes de permutação $n \times n$ existem?

n!

In [ ]: