Settimana: 11

Materia: Matematica
Classe: 5C
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Argomenti

(1) $f(x) = k$ costante — $f'(x) = 0$

(2) $f(x) = x^n$ $\quad n \geq 1, n \in \mathbb{N}$ — $f'(x) = n \cdot x^{n-1}$

(3) $f(x) = \sin x$ — $f'(x) = \cos x$

(4) $f(x) = \cos x$ — $f'(x) = -\sin x$

(5) $f(x) = e^x$ — $f'(x) = e^x$

(6) $f(x) = a^x$ $\quad a>0, a \neq 1$ — $f'(x) = a^x \ln a$

(7) $f(x) = \ln x$ — $f'(x) = 1/x$

(8) $f(x) = \log_a x$ $\quad a>0, a\neq 1$ — $f'(x) = \frac{1}{x} \cdot \log_a e$

(2bis) $f(x) = x^\alpha$, $\alpha \neq 0, \alpha \in \mathbb{R}$ — $f'(x) = \alpha \cdot x^{\alpha-1}$

Dim: (1) $f(x) = k$ costante

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{k-k}{h} = 0$$

(3) $f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin x}{h}$$

$$= \lim_{h\to 0} \sin(x) \underbrace{\frac{\cos(h)-1}{h}}_{\to 0} + \cos x \underbrace{\frac{\sin h}{h}}_{\to 1} = \cos x$$

(5) $f(x) = e^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$$

$$= \lim_{h\to 0} e^x \underbrace{\frac{(e^h-1)}{h}}_{\to 1} = e^x$$

Le altre vedete voi.

Regole di Derivazione (Dim successive)

(1) $D(f \pm g) = Df \pm Dg$

(2) $D(f \cdot g) = (Df)g + f \cdot (Dg)$ (2bis) $D(k \cdot f) = k \cdot Df$

k costante

(3) $D\left(\frac{f}{g}\right) = \frac{(Df)\cdot g - f(Dg)}{g^2}$

(4) $[D(f\circ g)](x) = Df(g(x)) \cdot Dg(x)$ Hard

Dim: (1) $D(f+g) = Df + Dg$ $f(x) = 2x$, $g(x) = e^x$ $(f+g)(x) = 2x+e^x$

$$[D(f+g)](x) = \lim_{h\to 0} \frac{(f+g)(x+h)-(f+g)(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)+g(x+h)-f(x)-g(x)}{h} =$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = Df(x) + Dg(x)$$

(3) $$D\left(\frac{f}{g}\right) = \frac{(Df)\cdot g - f\,Dg}{g^2}$$

$$\left[D\left(\frac{f}{g}\right)\right](x) = \lim_{h\to 0} \frac{\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x) - f(x)\,g(x+h) - f(x)\,g(x) + f(x)\,g(x)}{h\cdot g(x+h)\cdot g(x)}$$

$$= \lim_{h\to 0} \frac{\overbrace{[f(x+h)-f(x)]}^{Df(x)}\,g(x) - \overbrace{f(x)[g(x+h)-g(x)]}^{Dg(x)}}{h\,\underbrace{g(x+h)\cdot g(x)}_{\to\, g(x)^2}}$$

$$= \frac{Df(x)\,g(x) - f(x)\cdot Dg(x)}{g(x)^2}$$

Teorema / Formula: Sia $f: D\subseteq \mathbb{R} \longrightarrow \mathbb{R}$ una funzione biunivoca continua e derivabile. Supponiamo che la fz inversa $f^{-1}: \mathbb{R} \to D$ sia derivabile. Vale che

$$(Df^{-1})(y) = \frac{1}{(Df)(x)} \qquad \text{con} \quad f(x) = y, \quad x = f^{-1}(y)$$

Dim: So che $(f^{-1}\circ f)(x) = x$. Faccio la derivata:

$$\downarrow$$

$$f^{-1}(f(x)) = x$$

$$\downarrow$$

$$Df^{-1}(f(x)) \cdot Df(x) = 1$$

$$\Longrightarrow \boxed{Df^{-1}(y) = \frac{1}{Df(x)}}$$

$\square$

Formule da sapere:

(1) $f(x) = \arcsin x$ $\qquad f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

(2) $f(x) = \arccos x$ $\qquad f'(x) = -\dfrac{1}{\sqrt{1-x^2}}$ $\qquad$ Esercizio

(3) $f(x) = \operatorname{arctg} x$ $\qquad f'(x) = \dfrac{1}{1+x^2}$ $\qquad$!!!

Dim: (1) arcsin y è la funzione inversa di sin x, cioè

$$\arcsin(\sin x) = x \qquad \begin{aligned} \sin x &= y \\ x &= \arcsin y \end{aligned}$$

$$(D\arcsin)(y) = \frac{1}{(D\sin)(x)} = \frac{1}{\cos x} = \frac{1}{\cos(\arcsin y)} =$$

$$\sin^2\alpha + \cos^2\alpha = 1 \rightsquigarrow \cos^2\alpha = 1-\sin^2\alpha \rightsquigarrow \cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$= \frac{1}{\sqrt{1-\sin^2(\arcsin y)}} = \frac{1}{\sqrt{1-y^2}}$$

CIAO Pietro amodeo è stato qui