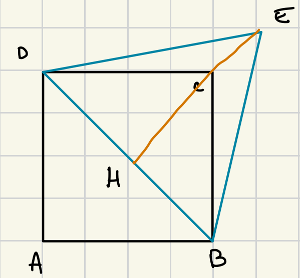
Pag 829 n 59



BDE equilatero

$A_{BDE} = 24\sqrt{3}$

$EH = 6\sqrt{2}$

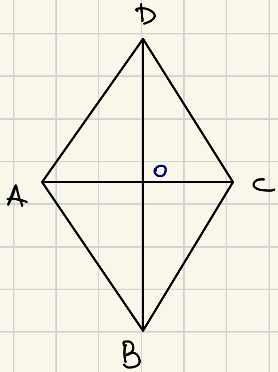
$$\frac{A_{BDE}}{A_{ABCD}} = ?$$

BD:
$$\frac{BD \cdot EH}{2} = A_{BDE} \rightsquigarrow BD = \frac{2\,A_{BDE}}{EH} = \frac{\cancel{2}^{\sqrt{2}} \cdot \cancel{24}^{4}\sqrt{3}}{\cancel{6\sqrt{2}}} = 4\sqrt{6}$$

AB: Per teo Pitagora
$$AB^2 + AD^2 = BD^2$$
$$2AB^2 = BD^2 \rightsquigarrow \cancel{2}^{1}\,AB^2 = \cancel{16}^{8} \cdot 6$$
$$\rightsquigarrow AB = \sqrt{48} = 4\sqrt{3}$$

A_{ABCD}:
$$A_{ABCD} = AB^2 = 48$$
$$\rightsquigarrow \frac{A_{BDE}}{A_{ABCD}} = \frac{\cancel{24}\sqrt{3}}{\cancel{48}\ 2} = \frac{\sqrt{3}}{2}$$

n 61:



BD è la sol di:
$$1 - \frac{2+\sqrt{2}-x}{2+\sqrt{2}} = \frac{x-2+\sqrt{2}}{2-\sqrt{2}}$$

$$\frac{BD}{AC} = 3$$

$P = 2p = ?$

$A = ?$

Risolvo l'equazione

$$\frac{2-[4+\cancel{2\sqrt{2}}-2x-\cancel{2\sqrt{2}}-2+\sqrt{2}\,x]}{\cancel{(2-\sqrt{2})(2+\sqrt{2})}} = \frac{2x-4+\cancel{2\sqrt{2}}+\sqrt{2}\,x-\cancel{2\sqrt{2}}+2}{\cancel{(2-\sqrt{2})(2+\sqrt{2})}}$$

$$2-\cancel{4}+\cancel{2x}+\cancel{2}-\sqrt{2}\,x = \cancel{2x}-\cancel{4}+\sqrt{2}\,x+\cancel{2} \qquad 2\sqrt{2}\,x = 2$$

$$\rightsquigarrow x = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \rightsquigarrow \boxed{BD = \frac{\sqrt{2}}{2}}$$

$$\triangleright \frac{BD}{3} = AC \quad \rightsquigarrow \quad \boxed{AC = \frac{\sqrt{2}}{6}}$$

$$\triangleright A_{ABCD} = \frac{AC\cdot BD}{2} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{6}\cdot\frac{1}{\cancel{2}} = \frac{1}{12}$$

$$\triangleright AB^2 = AO^2 + OB^2 = \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{6}\cdot\frac{1}{2}\right)^2 = \frac{2}{16}+\frac{2}{144} =$$

$$= \frac{1}{8}+\frac{1}{72} = \frac{9+1}{72} = \frac{10}{72} = \frac{5}{36}$$

$$\boxed{AB = \sqrt{\frac{5}{36}} = \frac{\sqrt{5}}{6}} \rightsquigarrow P = 4AB = \overset{2}{\cancel{4}}\cdot\frac{\sqrt{5}}{\underset{3}{\cancel{6}}} = \frac{2}{3}\sqrt{5}$$

Radici come potenze con esponente razionale

Idea: Tratto le radici come esponenti che sono frazioni:

$$\left[\sqrt[5]{2}\,\right]^5 = 2 \qquad \text{In un certo senso il 5 si è "semplificato"}$$

Un modo che conosciamo già di semplificare è con le frazioni. E perché allora non far combaciare i due metodi?

$$\boxed{\sqrt[5]{2} = 2^{\frac{1}{5}}} \longleftrightarrow \left(2^{\frac{1}{5}}\right)^5 \overset{\text{Propr. pot.}}{=} 2^{\frac{1}{\cancel{5}}\cdot\cancel{5}} = 2$$

↓

Def: Dato un radicale $\sqrt[m]{a^n}$, introduco la notazione $a^{\frac{n}{m}}$ in modo che

$$\left.\sqrt[m]{a^n} = a^{\frac{n}{m}}\right]$$ Potenza da esponente frazionario

Warning: Ricordate le condizioni per m che è $m \geq 1$, $m \in \mathbb{N}$ e $a \geq 0$, $n \in \mathbb{N}$

Chi è però il numero $a^{-\frac{n}{m}}$ con $n, m \in \mathbb{N}$ e $a > 0$?
Facciamo così:

$$a^{-\frac{n}{m}} = \left(\frac{1}{a}\right)^{\frac{n}{m}} \overset{\text{Def sopra}}{\boxed{=}} \sqrt[m]{\left(\frac{1}{a}\right)^n}$$

Esempi: $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$ $\qquad$ $\sqrt[7]{2} = 2^{\frac{1}{7}}$

$3^{\frac{3}{4}} = \sqrt[4]{3^3}$

$2^{-\frac{5}{7}} = \sqrt[7]{\left(\frac{1}{2}\right)^5}$ $\qquad$ $\sqrt[3]{-3}$ = Non si trasforma, non esiste vedi le C.E.

Proposizione: Valgono tutte le proprietà delle potenze per le potenze ad esp. frazionario

(1) $a^{\frac{n}{m}} \cdot a^{\frac{p}{q}} = a^{\frac{n}{m} + \frac{p}{q}}$

(2) $a^{\frac{n}{m}} / a^{\frac{p}{q}} = a^{\frac{n}{m} - \frac{p}{q}}$

(3) $\left(a^{\frac{n}{m}}\right)^{\frac{p}{q}} = a^{\frac{np}{mq}}$

(4) $a^{\frac{n}{m}} \cdot b^{\frac{n}{m}} = (ab)^{\frac{n}{m}}$

(5) $a^{\frac{n}{m}} / b^{\frac{n}{m}} = \left(\frac{a}{b}\right)^{\frac{n}{m}}$

Dim

$\left(a^{\frac{n}{m}}\right)^{\frac{p}{q}}$ (3)

$$\left(a^{\frac{n}{m}}\right)^{\frac{p}{q}} = \sqrt[q]{\left(\sqrt[m]{a^n}\right)^p} = \sqrt[q]{\sqrt[m]{a^{np}}} = \sqrt[mq]{a^{np}} = a^{\frac{np}{mq}}$$

Vanno tutte nello stesso modo:
Esp fraz ⤳ Radici ⤳ Prop. radici ⤳ Esp fraz.

Pag 823 n 638

$$\triangleright\ 5^{\frac{1}{2}} \cdot 5^{-\frac{2}{3}} = 5^{\frac{1}{2}+\left(-\frac{2}{3}\right)} = 5^{\frac{3-4}{6}} = 5^{-\frac{1}{6}}$$

$$\sqrt{5} \cdot \sqrt[3]{\frac{1}{5^2}} = \sqrt[6]{5^3 \cdot \frac{1}{5^4}} = \sqrt[6]{\frac{1}{5}}$$

$$\triangleright\ \left(3^{\frac{3}{4}}\right)^{-\frac{2}{9}} = 3^{\frac{\cancel{3}^{\,1}}{\cancel{4}_{\,2}}\left(-\frac{\cancel{2}^{\,1}}{\cancel{9}_{\,3}}\right)} = 3^{-\frac{1}{6}}$$

643 $$\left[\left(4^{\frac{1}{3}}\right)^{\frac{2}{5}} : 4^{-\frac{1}{5}}\right]^{\frac{1}{2}} = \left[4^{\frac{2}{15}} : 4^{-\frac{1}{5}}\right]^{\frac{1}{2}} = \left(4^{\frac{2}{15}+\frac{1}{5}}\right)^{\frac{1}{2}} = \left(4^{\frac{\cancel{5}^{\,1}}{\cancel{15}_{\,3}}}\right)^{\frac{1}{2}} = 4^{\frac{1}{6}}$$

$$\hookrightarrow \sqrt{\sqrt[5]{\left(\sqrt[3]{4}\right)^2} : \sqrt[5]{\frac{1}{4}}}$$

653 $$\left[5^{-\frac{4}{3}} \cdot 15^{\frac{2}{3}} : 45^{\frac{4}{3}}\right] \cdot \left(3^{-4}\right)^{-\frac{1}{2}} =$$

Scompongo

$$\left[5^{-\frac{4}{3}} \cdot \left(5^{\frac{2}{3}} \cdot 3^{\frac{2}{3}}\right) : \left(3^2 \cdot 5\right)^{\frac{4}{3}}\right] \cdot 3^2 =$$

riordino

$$\frac{5^{-\frac{4}{3}} \cdot 5^{\frac{2}{3}} \cdot 3^{\frac{2}{3}}}{3^{\frac{8}{3}} \cdot 5^{\frac{4}{3}}} \cdot 3^2 =$$

Uso le proprietà delle potenze

$$5^{-\frac{4}{3}+\frac{2}{3}-\frac{4}{3}} \cdot 3^{-\frac{8}{3}+\frac{2}{3}+2} =$$

Faccio conti

$$5^{-\frac{6}{3}} \cdot \underbrace{3^{\overset{=0}{\boxed{\frac{-8+2+6}{3}}}}}_{=1} = \frac{1}{25}$$

$\underline{663}$ ▷ $\dfrac{\sqrt[3]{4x\sqrt[3]{x}}}{x} = \dfrac{\left[4x^{1}\cdot x^{1/3}\right]^{1/3}}{x} = \dfrac{\left[4\cdot x^{4/3}\right]^{1/3}}{x} = \dfrac{4^{1/3}\cdot x^{\frac{4}{9}}}{x} = 4^{1/3}\cdot x^{\frac{4}{9}-1} = 4^{1/3}\cdot x^{-\frac{5}{9}} = \dfrac{\sqrt[3]{4}}{\sqrt[9]{x^5}}$

▷ $\dfrac{x\sqrt[5]{2x}}{\sqrt[10]{4x^7}} = \dfrac{x(2x)^{\frac{1}{5}}}{(4x^7)^{\frac{1}{10}}} = \dfrac{\cancel{2^{\frac{1}{5}}}\cdot x\cdot x^{\frac{1}{5}}}{\underbrace{\cancel{4^{\frac{1}{10}}}}_{(2^2)^{\frac{1}{10}}=2^{\frac{1}{5}}}\cdot x^{\frac{7}{10}}} = x^{1+\frac{1}{5}-\frac{7}{10}} = x^{\frac{10+2-7}{10}} = x^{\frac{5}{10}} = x^{\frac{1}{2}} = \sqrt{x}$

$\underline{678}$ $\left(x^{\frac{3}{4}} - x^{\frac{7}{4}}\right)x^{\frac{1}{4}} : (1-x)^{\frac{1}{2}}\cdot x^{\frac{3}{2}}$

$\left[x^{\frac{3}{4}+\frac{1}{4}} - x^{\frac{7}{4}+\frac{1}{4}}\right] : (1-x)^{\frac{1}{2}}\cdot x^{\frac{3}{2}}$

$(x - x^2) : (1-x)^{\frac{1}{2}}\cdot x^{\frac{3}{2}}$

$\dfrac{(x-x^2)}{(1-x)^{1/2}}\cdot x^{\frac{3}{2}} = \dfrac{x(1-x)}{(1-x)^{1/2}}\cdot x^{\frac{3}{2}} = \overbrace{x^{\frac{5}{2}}(1-x)^{\frac{1}{2}}} = \sqrt{x^5(1-x)} = x^2\sqrt{x(1-x)}$

Non potete fare altro. Occhio!

$\underline{683}$ $\left[y^{\frac{3}{2}} + \left(y^{\frac{1}{2}}-1\right)\right]\cdot\left[y^{\frac{3}{2}} - \left(y^{\frac{1}{2}}-1\right)\right]$

$y^{\frac{1}{2}}\left(2y - y^{1/2}\right) + \left(y^{\frac{3}{2}} + y^{\frac{1}{2}} - 1\right)\cdot\left(y^{\frac{3}{2}} - y^{\frac{1}{2}} + 1\right) - \left(y^{\frac{3}{2}}+1\right)^2 + 2(y+1)$

$2y^{\frac{3}{2}} - y + \left(y^{\frac{3}{2}}\right)^2 - \left(y^{\frac{1}{2}}-1\right)^2 - \left(y^3 + 1 + 2y^{\frac{3}{2}}\right) + 2y + 2$

$\cancel{2y^{\frac{3}{2}}} - \cancel{y} + \cancel{y^3} - \cancel{y} - \cancel{1} + 2y^{\frac{1}{2}} - \cancel{y^3} - \cancel{1} - \cancel{2y^{\frac{3}{2}}} + \cancel{2y} + \cancel{2} = 2y^{\frac{1}{2}} = 2\sqrt{y}$