Pag 824 n82

(a) $f(x) = 2\cos^2\frac{x}{2} + \sin x$ e determina il periodo. Usa le formule goniometriche per semplificare l'espressione

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$ Mi piace poco ← Mirco osserva che funzione

$\cos(2x) = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$

$\cos x = 2\cos^2\frac{x}{2} - 1$
$\downarrow$
$2\cos^2\frac{x}{2} = 1 + \cos x$

$f(x) = 1 + \cos x + \sin x$

Qua si usa la formula dell'angolo aggiunto

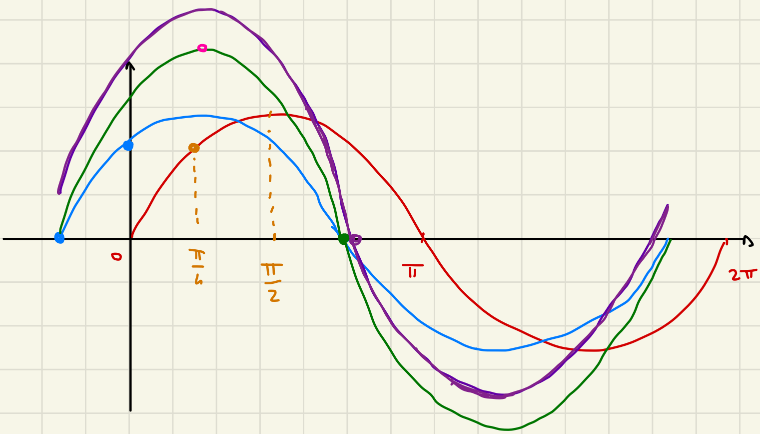
$$a\sin x + b\cos x = r\sin(x+\alpha) \qquad r = \sqrt{a^2+b^2}, \quad \alpha = \text{arctg}\left(\frac{b}{a}\right)$$

$a=1,\ b=1 \rightsquigarrow r = \sqrt{2}$
$\alpha = \text{arctg}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$f(x) = 1 + \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

Periodo di $f(x)$ coincide con il periodo di $\sin x$, cioè $T = 2\pi$

Disegno la funzione:



$\sin(x)$

$\sin\left(x + \frac{\pi}{4}\right)$
Spostato indietro di $\frac{\pi}{4}$

$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$
Rende i pti più alti in modulo

$1+\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$
Alzo di 1 tutti i pti

$f(x) = 1 + \sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right)$ $\qquad$ $Im(f) = ?$

$\sin\left(x + \frac{\pi}{4}\right) \in [-1, 1]$

$\sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right) \in [-\sqrt{2}; \sqrt{2}]$

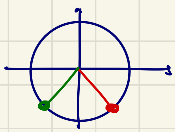
$1 + \sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right) \in [1 - \sqrt{2}; 1 + \sqrt{2}]$

Risolvo $f(x) = 0$

$1 + \sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right) = 0$

$\sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right) = -1 \rightsquigarrow \sin\left(x + \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\rightsquigarrow x + \frac{\pi}{4} = -\frac{\pi}{4}$ $\qquad \rightsquigarrow x = -\frac{\pi}{2}$



$\rightsquigarrow x + \frac{\pi}{4} = -\frac{3}{4}\pi$ $\qquad \rightsquigarrow x = -\pi$