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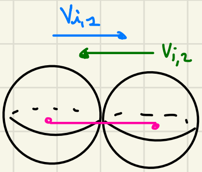


"rho" $= \rho = 2{,}515 \ \frac{g}{cm^3}$ densità

$R = 10$ km

(A) sono all'infinito

(1) Impatto

Volume sfere

$$\boxed{V = \frac{4}{3}\pi r^3}$$

$v_i = ?$, $a_i = ?$

$$E_A = K_A + U_A = 0 + 0 = 0$$

$$E_i = K_i + U_i = \frac{1}{2} m_1 v_{i,1}^2 + \frac{1}{2} m_2 v_{i,2}^2 + \left(-G\frac{m_1 m_2}{2R}\right)$$

$E_A = E_i$

$$0 = \frac{1}{2} m_1 v_{i,1}^2 + \frac{1}{2} m_2 v_{i,2}^2 - \frac{G m_1 m_2}{2R} \quad (*)$$

$$\rho = \frac{m}{V} \rightsquigarrow m = \rho \cdot V = \rho \cdot \frac{4}{3}\pi R^3 \rightsquigarrow m_1 = m_2 = m = \frac{4}{3}\pi R^3 \rho$$

Dato la qdm. si conserva

$$0 = p_A = p_i = \cancel{m} v_{i,1} - \cancel{m} v_{i,2} \rightsquigarrow v_{i,1} = v_{i,2} = v_i$$

Butto tutto in (*) e mi manca solo v_i

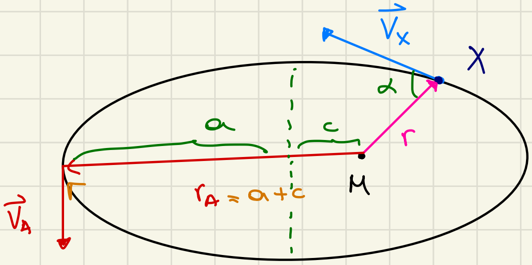
$$0 = \frac{1}{2}\cancel{m} v_i^2 + \frac{1}{2}\cancel{m} v_i^2 - \frac{G m^{\cancel{2}}}{2R} \rightsquigarrow v_i^2 = \frac{Gm}{2R} = \frac{G}{\cancel{2}} \cdot \frac{\overset{2}{\cancel{4}}}{3}\pi \frac{R^{\cancel{3}\,2}\rho}{\cancel{R}}$$

$$\rightsquigarrow v_i^2 = \frac{2}{3} G R^2 \rho \pi$$

II principio: $\vec{F}_s = m\vec{a}_{s,i}$ $\qquad F = G\frac{m^2}{(2R)^2}$

$$a_i = \frac{G m^{\cancel{2}}}{(2R)^2}\frac{1}{\cancel{m}} = \frac{Gm}{4R^2} = \frac{G}{\cancel{4}\cancel{R^2}}\frac{\cancel{4}}{3}\pi R^{\cancel{3}}\rho = \frac{\pi}{3} G R \rho$$

n. 141 (Francesca Malavolti)



$M = 2{,}71 \cdot 10^{35}$ kg
$m = 8{,}96 \cdot 10^{25}$ kg
$a = 5{,}48 \cdot 10^{13}$ m
$e = 0{,}412$
$v_X = 5{,}42 \cdot 10^{5}\ \frac{m}{s}$

$r = ?$, $\alpha = ?$

(1) E_{TOT} si conserva $\quad E_{TOT} = -G\frac{mM}{2a}$

$$E_X = K_X + U_X = \frac{1}{2} m v_X^2 + \left(-G\frac{mM}{r}\right)$$ Energia quando m è su X.

Impongo $E_{TOT} = E_X$ e trovo r:

$$-G\frac{\cancel{m}M}{2a} = \frac{1}{2}\cancel{m}v_X^2 - G\frac{\cancel{m}M}{r}$$

$$\frac{GM}{r} = \frac{1}{2}v_X^2 + \frac{GM}{2a} = \frac{aV_X^2 + GM}{2a}$$

$$\frac{r}{GM} = \frac{2a}{aV_X^2 + GM} \leadsto \boxed{r = \frac{2aGM}{aV_X^2 + GM}} \simeq 5{,}78 \cdot 10^{13}\ m$$

(2) Il momento angolare si conserva. $\quad L_X = L_A \quad$ A afelio

$$L_X = r \cdot m \cdot V_X \ \sin(\alpha)$$
$$L_A = r_A \cdot m \cdot V_A \cdot 1 = (a+c)\cdot m\sqrt{\frac{GM}{a}\left(\frac{a-c}{b}\right)^2}$$

$e = \frac{c}{a} \leadsto c = ea$

$a^2 - b^2 = c^2 \quad b^2 = a^2 - c^2$

$b^2 = a^2(1-e^2)$

$$= a(1+e)\cdot m\sqrt{\frac{GM}{a}\left(\frac{\cancel{a^2}(1-e)^{\cancel{2}}}{\cancel{a^2}(1-e^2)}\right)}$$

$(\cancel{1-e})(1+e)$

$$= m\sqrt{GMa(1+e)(1-e)}$$

$$L_x = L_A \qquad r \cdot \cancel{m} \cdot V_x \cdot \sin\alpha = \cancel{m} \left(GMa(1-e^2)\right)^{\frac{1}{2}}$$

$$\sin\alpha = \frac{\left[GMa(1-e^2)\right]^{1/2}}{r \cdot V_x}$$

$$\rightsquigarrow \quad \alpha = \sin^{-1}\left(\frac{\left[GMa(1-e^2)\right]^{1/2}}{r \cdot V_x}\right) \approx 65{,}9^\circ$$