Pag 616 e seguenti

n 303: $4^{2x-1} - 10 \cdot 4^{x-1} + 4 > 0$ $\qquad \dfrac{4^{2x}}{4} - \dfrac{10 \cdot 4^{x}}{4} + 4 > 0$

$\dfrac{4^{2x} - 10 \cdot 4^{x} + 16}{\cancel{4}} > 0 \qquad 4^x = t$

$t^2 - 10t + 16 > 0 \qquad \Delta = 100 - 64 = 36 \qquad \sqrt{\Delta} = 6$

$t_{1,2} = \dfrac{10 \pm 6}{2} \begin{matrix} \longrightarrow 2 \\ \longrightarrow 8 \end{matrix} \quad \rightsquigarrow \quad t < 2 \ \vee \ t > 8$

$4^x < 2 \quad \rightsquigarrow \quad 2^{2x} < 2 \quad \rightsquigarrow \quad x < \frac{1}{2}$

$4^x > 8 \quad \rightsquigarrow \quad 2^{2x} > 2^3 \quad \rightsquigarrow \quad x > \frac{3}{2}$

$$\boxed{x < \frac{1}{2} \quad \vee \quad x > \frac{3}{2}}$$

n 306: $\dfrac{5^x - 125}{(1-2^x)(3^x-3)} \geq 0$

$N \geq 0 \qquad 5^x - 125 \geq 0 \qquad 5^x - 5^3 \geq 0 \qquad 5^x \geq 5^3 \quad \rightsquigarrow \quad x \geq 3$

$D_1 > 0 \qquad 1 - 2^x > 0 \qquad -2^x > -1$

$2^x < 1 = 2^0 \quad \rightsquigarrow \quad x < 0$

$D_2 > 0 \qquad 3^x - 3 > 0 \qquad 3^x > 3 \quad \rightsquigarrow \quad x > 1$

		0		1		3	
N	$-$		$-$		$-$	•	$+$
D_1	$+$	0	$-$		$-$		$-$
D_2	$-$		$-$	0	$+$		$+$
	$+$		$-$		$+$		$-$

$$\boxed{x < 0 \quad \vee \quad 1 < x \leq 3}$$

n 330 $\dfrac{5^{\frac{4}{3}x+3}}{\sqrt{49^{x+2}}} \leq \dfrac{7\cdot\sqrt[3]{25^x}}{\sqrt[3]{7^x}}$ (Vedi 184)

$$\frac{5^{\frac{4}{3}x+3}}{\left[(7^2)^{x+2}\right]^{\frac{1}{2}}} \leq \frac{7\cdot 5^{\frac{2}{3}x}}{7^{\frac{x}{3}}}$$

⇝ Porto tutto in un'unica base $\left(\frac{5}{7}\right)$ o $\left(\frac{7}{5}\right)$ mettendo tutto a sinistra

$$\frac{5^{\frac{4}{3}x+3}\cdot 7^{\frac{x}{3}}}{7^{x+2}\cdot 5^{\frac{2}{3}x}\cdot 7} \leq 1$$

⇝ Uso proprietà pot.

$$\frac{5^{\frac{4}{3}x+3-\frac{2}{3}x}}{7^{x+2+1-\frac{x}{3}}} \leq 1 \rightsquigarrow \frac{5^{\frac{2}{3}x+3}}{7^{\frac{2}{3}x+3}} \leq 1 \rightsquigarrow \left(\frac{5}{7}\right)^{\frac{2}{3}x+3} \leq \left(\frac{5}{7}\right)^0$$

base < 1 fa invertire il segno

$$\frac{2}{3}x+3 \geq 0$$

$$\frac{2}{3}x \geq -3 \rightsquigarrow \boxed{x \geq -\frac{9}{2}}$$

Pag 621 n 27

$$3\cdot \underbrace{5^{2(x-2)}}_{5^{2x}\ 5^{-4}} + 5^x \geq 13\cdot 5^{x-2} + 15 \qquad 5^x = t$$

$$\frac{3\cdot t^2}{5^4} + t \geq \frac{13\cdot t}{5^2} + 15$$

$$3\cdot t^2 + 5^4\cdot t \geq 5^2\cdot 13\cdot t + 15\cdot 5^4$$

$$3t^2 + 625t \geq 325t + 9375$$

$$3t^2 + 300t - 9375 \geq 0$$

$$t^2 + 100t - 3125 \geq 0$$

$$\Delta = 100^2 + 4\cdot 3125 =$$
$$= (2^2\cdot 5^2)^2 + 2^2\cdot 5^5 =$$
$$= 2^4\cdot 5^4 + 2^2\cdot 5^5 =$$
$$= 2^2\cdot 5^4\,(2^2+5) =$$
$$= (2\cdot 5^2\cdot 3)^2 = (150)^2$$

$$t_{1,2} = \frac{-100 \pm 150}{2} \quad \begin{matrix} -125 \\ 25 \end{matrix}$$

$$\boxed{t \leq -125 \quad \vee \quad t \geq 25}$$

$5^x \leq -125$ Impossibile

$5^x \geq 25 \rightsquigarrow x \geq 2$

Pag 644 e seguenti:

19: $\log_{11}(121) = 2$ $\qquad\qquad$ $\log_7 343 = 3$

22: $\log_2 \frac{4}{\sqrt{2}} = \log_2\left(\frac{2^2}{2^{1/2}}\right) = \frac{3}{2}$

$\log_6 (6\sqrt[3]{6}) = \log_6 (6 \cdot 6^{\frac{1}{3}}) = \frac{4}{3}$

49: $\log_a 100 = 2 \longleftrightarrow a^2 = 100 \longrightarrow a = 10$

$\log_a 2 = 1 \longleftrightarrow a^1 = 2 \longrightarrow a = 2$

67: $\log 1 + \log 10 + \log 100 + \log 1000$

$0 + 1 + 2 + 3 = 6$

71: $\log_2 \underbrace{32}_{2^5} - 4\log_4 16 + \underbrace{\log\left[\underbrace{\log_2 (\overbrace{\log 100}^{2})}_{1}\right]}$

$5 - 4 \cdot 2 + 0 = 5 - 8 = -3$

70: $\log_{\frac{1}{7}}\left(\underbrace{6^{\log_6 7}}\right) + \log_4 \frac{1}{2} - \left(\log_3 \sqrt{3} - \log 1\right)$

$\downarrow$

$\log_6 7 = x \longleftrightarrow 6^x = 7 \longleftrightarrow 6^{\log_6 7} = 7$

$-1 + \left(-\frac{1}{2}\right) - \left(\frac{1}{2} - 0\right) = -1 - \frac{1}{2} - \frac{1}{2} = -2$ (torte)

89: $\log_5\left(\frac{3\sqrt[6]{a}}{\sqrt[27]{b}}\right) = \log_5 (3 \cdot \sqrt[6]{a}) - \log_5 (\sqrt[27]{b}) =$

$= \log_5 (3) + \log_5 (a^{\frac{1}{6}}) - \log_5 (b^{\frac{1}{27}}) =$

$= \log_5 (3) + \frac{1}{6}\log_5 (a) - \frac{1}{27}\log_5 (b)$

85 $\log_{\sqrt{2}} \dfrac{\sqrt[5]{4}}{8\sqrt{2}} = \dfrac{\log_2\left(\frac{\sqrt[5]{4}}{8\sqrt{2}}\right)}{\log_2(\sqrt{2})} =$

$= 2 \log_2 \left(\dfrac{(2^2)^{\frac{1}{5}}}{2^3 \cdot 2^{1/2}} \right) = 2 \log_2 \left(2^{\frac{2}{5} - \left(\frac{7}{2}\right)} \right)$

$= 2 \log_2 \left(2^{-\frac{31}{10}} \right) = 2 \cdot \left(-\frac{31}{10} \right) = -\frac{31}{5}$

290: $\log_2 \left(\sqrt{5-x^2} - x \right) = 0$ C.E. $\begin{cases} 5-x^2 \geq 0 \\ \sqrt{5-x^2} - x > 0 \end{cases}$

$\sqrt{5-x^2} - x = 1$

$\sqrt{5-x^2} = 1+x$ $\leadsto$ Elevo alla 2 sotto le condizioni $\begin{cases} 5-x^2 \geq 0 \\ 1+x \geq 0 \end{cases}$

Se $1+x$ negativo, sicuram. ~~Non~~ ha soluzione

$5 - x^2 = 1 + x^2 + 2x$ $2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$ $\Delta = 9$ $\sqrt{\Delta} = 3$ $x_1 x_2 = \dfrac{-1 \pm 3}{2} < \begin{matrix} 1 \\ -2 \end{matrix}$

$\boxed{x = 1 \quad \text{Accettabile}}$

$x = -2$ Non Accettabile per

352 $3 = \dfrac{14}{\log_5 x + 2} + \dfrac{4}{\log_5 x - 1}$ C.E. $\begin{cases} x > 0 \\ \log_5 x + 2 \neq 0 \\ \log_5 x - 1 \neq 0 \end{cases}$

$\log_5 x = t$ $3 = \dfrac{14}{t+2} + \dfrac{4}{t-1}$

$\dfrac{(3t-3)(t+2)}{\cancel{(t-1)(t+2)}} = \dfrac{14t - 14 + 4t + 8}{\cancel{(t-1)(t+2)}}$

$3t^2 + 6t - 3t - \cancel{6} = 18t - \cancel{6}$ $\leadsto$ $3t^2 - 15t = 0$ $\leadsto$ $3t(t-5) = 0$

$t=0 \rightsquigarrow \log_5 x = 0 \rightsquigarrow x = 1$ Accett.

$t=5 \rightsquigarrow \log_5 x = 5 \rightsquigarrow x = 5^5 = 3125$ Accett.

n 331

$$\log_2(x^2-4) + 2\log_2 x = \overset{\log_2(2)}{1} + \log_2(5x^2+16)$$

C.E. $\begin{cases} x^2-4>0 \\ x>0 \\ 5x^2+16>0 \end{cases}$

$$\log_2\left[(x^2-4)(x^2)\right] = \log_2\left[2\cdot(5x^2+16)\right] \rightsquigarrow \text{inj}$$

$$x^4-4x^2 = 10x^2+32$$

$$x^4-14x^2-32=0 \qquad x^2=f$$

$$f^2-14f-32=0 \qquad (f-16)(f+2)=0$$

$f=16 \qquad x^2=16 \rightsquigarrow x=\pm 4 \rightsquigarrow x=4$ Accett

$f=-2 \qquad x^2=-2 \rightsquigarrow$ Impossibile