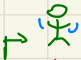
Proposizione: $$\tan(\alpha+\beta) = \frac{tg(\alpha) + tg(\beta)}{1 - tg(\alpha)\, tg(\beta)}$$

$$\tan(\alpha-\beta) = \frac{tg(\alpha) - tg(\beta)}{1 + tg(\alpha)\, tg(\beta)}$$

Dim: $$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)}{\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)}$$



Spirito: Divido per robe per far comparire le tangenti.

Divido Num e Den per $\cos(\alpha)\cos(\beta)$

$$= \frac{\dfrac{\sin(\alpha)\cancel{\cos(\beta)}}{\cos(\alpha)\cancel{\cos(\beta)}} + \dfrac{\sin(\beta)\cancel{\cos(\alpha)}}{\cancel{\cos(\alpha)}\cos(\beta)}}{\dfrac{\cancel{\cos(\alpha)}\cancel{\cos(\beta)}}{\cancel{\cos(\alpha)}\cancel{\cos(\beta)}} - \dfrac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}}$$

$$= \frac{tg(\alpha) + tg(\beta)}{1 - tg(\alpha)\, tg(\beta)}$$

Formula sopra

$$\tan(\alpha-\beta) = tg(\alpha + (-\beta)) \boxed{=} \frac{tg(\alpha) + \overbrace{tg(-\beta)}^{-tg(\beta)}}{1 - tg(\alpha)\underbrace{tg(-\beta)}_{-tg(\beta)}}$$

$$= \frac{tg(\alpha) - tg(\beta)}{1 + tg(\alpha)\, tg(\beta)}$$

$\square$

Proposizione (Formula dell'angolo aggiunto): Se ho una scrittura del tipo $a\sin x + b\cos x$ con $a, b \in \mathbb{R}$, esistono unici $r \geq 0$, α angolo tali che

$$a\sin x + b\cos x = r \cdot \sin(x+\alpha)$$

α è detto ==angolo aggiunto==

Dim: Uso formule di somma in $\sin(x+\alpha)$

$$\sin(x+\alpha) = \sin x \cos\alpha + \sin\alpha \cos x$$

A posteriori dovrà valere che:

$$a \sin x + b \cos x = r\cos\alpha \sin x + r \sin\alpha \cos x$$

Impongo quindi ciò che moltiplica $\sin x$ uguali e
" " $\cos x$ uguali

$$\begin{cases} a = r\cos\alpha & \rightsquigarrow \sin x \\ b = r\sin\alpha & \rightsquigarrow \cos x \end{cases}$$

Due equazioni
Due incognite r, α.

Faccio la II diviso la I $\quad \frac{b}{a} = \operatorname{tg}(\alpha) \rightsquigarrow \boxed{\alpha = \operatorname{arctg}\left(\frac{b}{a}\right)}$

↳ Warning: la posso fare solo se $a \neq 0$. Se $a = 0$, la formula non è valida

Per trovare r faccio il $\square$ delle due equazioni e le sommo.

$$a^2 + b^2 = r^2\cos^2\alpha + r^2\sin^2\alpha$$
$$a^2 + b^2 = r^2 \underbrace{(\sin^2\alpha + \cos^2\alpha)}_{=1}$$
$$a^2 + b^2 = r^2 \rightsquigarrow \boxed{r = \sqrt{a^2 + b^2}}$$

La radice è permessa

$\square$

Esercizio: Trovare x tali che $\sin(x) + \cos(x) = 1$ (in I/IV quadr)

$$\sin x + \cos x = r\sin(x+\alpha) \qquad r = \sqrt{a^2+b^2} = \sqrt{2}$$
$$\alpha = \operatorname{arctg}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

$$\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 1$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \rightsquigarrow x + \frac{\pi}{4} = \frac{\pi}{4} \rightsquigarrow \boxed{x = 0}$$

Proposizione: Formule di Bisezione:

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\operatorname{tg}\left(\frac{x}{2}\right) = \pm\frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{1+\cos x} = \pm\frac{\sin x}{1+\cos x}$$

Dim: $\cos(2\alpha) = \cos^2\alpha - \overbrace{\sin^2\alpha}^{1-\cos^2\alpha} = 2\cos^2\alpha - 1$

Spirito: Dalla formula di duplicazione ho una formula che mette in relazione un angolo con il suo doppio. Se chiamo $\alpha = \frac{x}{2}$ ottengo

$$\cos\left(\cancel{2}\cdot\frac{x}{\cancel{2}}\right) = 2\boxed{\cos^2\frac{x}{2}} - 1 \quad \leftarrow \text{Voglio te}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos(x)}{2} \leadsto \boxed{\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}}$$

▷ $\sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1$

$$\sin^2\frac{x}{2} + \frac{1+\cos(x)}{2} = 1 \leadsto \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$$

▷ $$\operatorname{tg}\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \pm\sqrt{\frac{\frac{1-\cos(x)}{\cancel{2}}}{\frac{1+\cos x}{\cancel{2}}}} \cdot \sqrt{\frac{(1-\cos x)}{(1-\cos x)}}$$

$$= \pm\sqrt{\frac{(1-\cos x)^2}{\sin^2 x}} = \pm\frac{1-\cos x}{\sin x}$$

Posso moltiplicare

$$\sqrt{\frac{1+\cos x}{1+\cos x}} \cdots = \pm\sqrt{\frac{\sin^2 x}{(1+\cos x)^2}} = \pm\frac{\sin x}{1+\cos x}$$

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$$1+\cot^2\alpha + \sin\left(\frac{\pi}{4}+\alpha\right) = \csc^2\alpha + \cos\left(\frac{\pi}{4}-\alpha\right)$$

$$1+\frac{\cos^2\alpha}{\sin^2\alpha} + \cancel{\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\cos\alpha} + \cancel{\sin\alpha\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}} = \frac{1}{\sin^2\alpha} + \cancel{\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\cos\alpha} + \cancel{\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\sin\alpha}$$

$$\frac{\overbrace{\sin^2\alpha+\cos^2\alpha}^{=1}}{\sin^2\alpha} = \frac{1}{\sin^2\alpha}$$

Dunque l'identità è verificata

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$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$

$\left[\sin(\alpha)\cos\left(\frac{3}{4}\pi\right) - \sin\left(\frac{3}{4}\pi\right)\cos(\alpha)\right]^2$

$\left[\sin(\alpha)\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\cos(\alpha)\right]^2$

$\left(\sin(\alpha)\left(-\frac{\sqrt{2}}{2}\right)\right)^2 + \left(-\frac{\sqrt{2}}{2}\cos(\alpha)\right)^2 - 2\sin(\alpha)\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)\cos(\alpha)$

$$\underbrace{\sin^2\left(\alpha-\frac{3}{4}\pi\right)}_{\left[\frac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)\right]^2} - \sin^2\left(\frac{\pi}{4}+\alpha\right) = \underbrace{\cos\left(\frac{2}{3}\pi-\alpha\right)}_{\cos\frac{2}{3}\pi\cos\alpha+\sin\frac{2}{3}\pi\sin\alpha} + \underbrace{\cos\left(\frac{\pi}{3}+\alpha\right)}_{\cos\frac{\pi}{3}\cos\alpha-\sin\frac{\pi}{3}\sin\alpha}$$

$$\frac{1}{2}\left[\cancel{\sin\alpha+\cos\alpha}\right]^2 - \frac{1}{2}\cancel{(\sin\alpha+\cos\alpha)^2} = -\frac{1}{2}\cancel{\cos\alpha} + \frac{\sqrt{3}}{2}\cancel{\sin\alpha} + \frac{1}{2}\cancel{\cos\alpha} - \frac{\sqrt{3}}{2}\cancel{\sin\alpha}$$

$0=0$ identità verificata