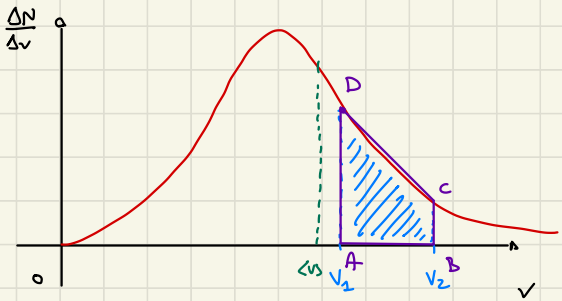
Distribuzione di Maxwell delle velocità molecolari

Def: La distribuzione delle velocità molecolari $\frac{\Delta N}{\Delta v}(v)$ di un gas è data dalle funzione

$$\frac{\Delta N}{\Delta v} = \frac{4N}{\sqrt{\pi}} \left(\frac{m}{2k_B T}\right)^{\frac{3}{2}} v^2 \, e^{-\frac{mv^2}{2k_B T}}$$

- m massa singola molecola
- N numero di molecole
- T temperature
- v velocità.



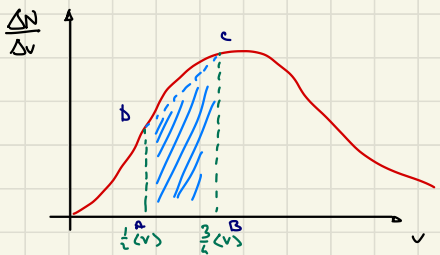
Oss: L'area azzurre calcola quante molecole del gas hanno velocità compresa tra v_1 e v_2

Q: Quanto vale l'area sottesa da tutto il grafico? Vale N because each atom has a specific speed between 0 to ∞

Q: How can I compute the Area between v_1 to v_2?
Well, We can't; but we can do an approximation of it.
Posso costruire il trapezio ABCD e l'area del trapezio è circa il valore che sto cercando

---

Pag 398 n 146



N molecole
Quante molecole hanno v compresa tra $\frac{1}{2}\langle v\rangle$ e $\frac{3}{4}\langle v\rangle$

$$AB = \frac{3}{4}\langle v\rangle - \frac{1}{2}\langle v\rangle = \boxed{\frac{1}{4}\langle v\rangle}$$

$$\langle v\rangle^2 = \frac{3k_BT}{m} \quad \Rightarrow \quad \frac{1}{3}\langle v\rangle^2 = \frac{k_BT}{m}$$

$$AD = \frac{\Delta N}{\Delta v}\left(\frac{1}{2}\langle v\rangle\right) = \frac{4N}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{\frac{3}{2}}\left(\frac{1}{2}\langle v\rangle\right)^2 e^{-\frac{m}{2k_BT}\left(\frac{1}{2}\langle v\rangle\right)^2}$$

$$= \frac{4N}{\sqrt{\pi}}\left(\frac{3}{2\langle v\rangle^2}\right)^{\frac{3}{2}} \cdot \left(\frac{1}{2}\langle v\rangle\right)^2 e^{-\frac{3}{2\langle v\rangle^2}\left(\frac{1}{2}\langle v\rangle\right)^2}$$

$$= \frac{4N}{\sqrt{\pi}}\left(\frac{3}{2}\right)^{\frac{3}{2}} \cdot \frac{1}{\langle v\rangle^3} \cdot \frac{1}{4} \cdot \langle v\rangle^2 \; e^{-\frac{3}{8}}$$

$$\boxed{= \frac{4N}{\sqrt{\pi}}\left(\frac{3}{2}\right)^{\frac{3}{2}} \cdot \frac{1}{4} e^{-\frac{3}{8}} \cdot \frac{1}{\langle v\rangle}}$$

$$BC = \frac{\Delta N}{\Delta v}\left(\frac{3}{4}\langle v\rangle\right) = \frac{4N}{\sqrt{\pi}}\left(\frac{3}{2\langle v\rangle^2}\right)^{\frac{3}{2}} \cdot \left(\frac{3}{4}\langle v\rangle\right)^2 e^{-\frac{3}{2\langle v\rangle^2}\left(\frac{3}{4}\langle v\rangle\right)^2}$$

$$= \boxed{\frac{4N}{\sqrt{\pi}}\left(\frac{3}{2}\right)^{\frac{3}{2}} \cdot \frac{9}{16} \cdot e^{-\frac{27}{32}} \cdot \frac{1}{\langle v\rangle}}$$

Area Trapezio: $\frac{(AD+BC)\cdot AB}{2} =$

$$\frac{\frac{4N}{\sqrt{\pi}}\left(\frac{3}{2}\right)^{\frac{3}{2}} \frac{1}{\langle v\rangle^3}\left(\frac{1}{4}e^{-\frac{3}{8}} + \frac{9}{16}e^{-\frac{27}{32}}\right)\cdot\frac{1}{4}\langle v\rangle}{2} \overset{\text{calc}}{\simeq} N \cdot 0{,}21$$

Area Percentuale: $\frac{\text{Area Trap}}{\text{Area Tot}} = \frac{N\cdot 0{,}21}{N} \longrightarrow$ circa 21%.