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$$f(x) = \frac{\ln^2 x - \ln x}{\ln\sqrt{x-1}}$$

▷ Dom(f) :
$$\begin{cases} x>0 \\ \sqrt{x-1}>0 \\ \ln\sqrt{x-1} \neq 0 \; \underset{\ln(1)}{\overset{\shortparallel}{}} \end{cases} \qquad \begin{cases} x>0 \\ x-1>0 \\ \sqrt{x-1} \neq 1 \end{cases} \qquad \begin{cases} x>0 \\ x>1 \\ x\neq 2 \end{cases}$$

$$\text{Dom}(f) = \{ x>1 \;\wedge\; x\neq 2 \}$$

▷ $f(x)=0$ $\qquad \dfrac{\ln^2 x - \ln x}{\ln(\cancel{\sqrt{x-1}})} = 0$

$\ln x\,(\ln x - 1) = 0$ $\qquad \ln x = 0 \rightsquigarrow \boxed{x=1}$ Non Acc
$\qquad\qquad\qquad\qquad\qquad \ln x - 1 = 0 \quad \ln x = \ln e \rightsquigarrow \boxed{x=e}$ Acc. | confronto col dom.

$P = (e, 0)$

Remind: $\ln(\;) = \log_e(\quad)$

▷ $f(x) \geq 0$ $\qquad \dfrac{\ln^2 x - \ln x}{\ln(\sqrt{x-1})} \geq 0$

$N \geq 0 \qquad \ln^2(x) - \ln(x) \geq 0$ $\qquad$ soluzioni $x=1$, $x=e$

$x \leq 1 \;\vee\; x \geq e$

$D > 0 \qquad \ln(\sqrt{x-1}) > 0 \;(=\ln(1))$

$\sqrt{x-1} > 1 \rightsquigarrow x > 2$

Non importa sistemi perché ho fatto Dom f

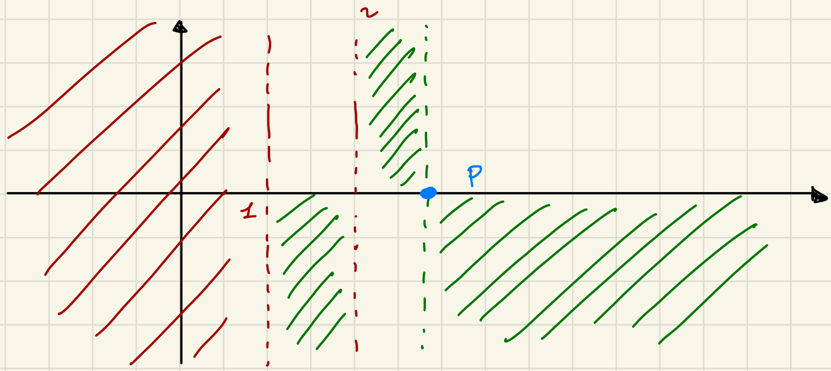
	1		2		e	
N	+	•	−		−	•	+
D	−		−	o	+		+
	−		+		−		+

$1 \leq x < 2 \;\vee\; x \geq e$

$[1;2) \cup [e;+\infty)$

Devo intersecare con il $\text{Dom}(f) = \{x > 1 \; ; \; x \neq 2\}$

$\Rightarrow$ $f(x)$ positiva se $\quad 1 \boxed{<} x < 2 \quad \vee \quad x \geq e$



Es 85 $\quad f(x) = \dfrac{2\ln(x)}{1+\ln(x)}$

(a) Dom(f): $\begin{cases} x > 0 \\ 1+\ln(x) \neq 0 \end{cases}$ $\quad \begin{cases} x > 0 \\ \ln(x) \neq -1 \end{cases}$ $\quad \begin{cases} x > 0 \\ x \neq e^{-1} \end{cases}$

$\ln''(e^{-1})$

$$\text{Dom}(f) = \left\{ x > 0 \; ; \; x \neq \frac{1}{e} \right\}$$

$f(x) = 0 \qquad 2\ln(x) = 0 \qquad \ln(x) = 0 \qquad x = 1$ Acc.

$f(x) \geq 0 \qquad \dfrac{2\ln(x)}{1+\ln(x)} \geq 0$

$N \geq 0 \qquad 2\ln(x) \geq 0 \quad \leadsto \quad x \geq 1$

$D > 0 \qquad 1+\ln(x) > 0 \quad \leadsto \quad x > \frac{1}{e}$

		$1/e$		1	
N	$-$		$-$	$\bullet$	$+$
D	$-$	$\circ$	$+$		$+$
	$+$		$-$		$+$

Sol: $\quad x < \frac{1}{e} \quad \vee \quad x \geq 1 \quad \leadsto$ Intersecando con Dom(f) $\quad 0 < x < \frac{1}{e} \quad \vee \quad x \geq 1$

$$\left(0 ; \frac{1}{e}\right) \cup [1 ; +\infty)$$

(b) Per quali x $\quad f(x) \geq 1$?

$$\frac{2\ln(x)}{1+\ln(x)} \geq 1 \quad \rightsquigarrow \quad \frac{\ln(x)-1}{\ln(x)+1} \geq 0 \qquad \text{.... conti ...}$$

.... Sol: $\quad x < \frac{1}{e} \quad \vee \quad x \geq e$

(c) $f(x) = \dfrac{2\ln(x)}{1+\ln(x)}$. Trova $f^{-1}(x)$ funzione inversa.

f iniettiva: $\quad f(x_1) = f(x_2) \overset{?}{\Longrightarrow} x_1 = x_2$

$$\frac{\cancel{2}\ln(x_1)}{1+\ln(x_1)} = \frac{\cancel{2}\ln(x_2)}{1+\ln(x_2)}$$

$$(1+\ln(x_2))\ln(x_1) = (1+\ln(x_1))\ln(x_2)$$

$$\ln(x_1) + \cancel{\ln(x_1)\ln(x_2)} = \ln(x_2) + \cancel{\ln(x_1)\ln(x_2)}$$

$\ln(x_1) = \ln(x_2) \overset{\text{inj}}{\underset{\log}{\rightsquigarrow}} x_1 = x_2 \qquad$ Dunque f iniettiva

f suriettiva: Se non specifichiamo il codominio, non ha senso porsi la domanda.
Per ovviare al problema scriviamo

$$f: \{x>0;\ x \neq \tfrac{1}{e}\} \longrightarrow \operatorname{Im}(f)$$

$\rightsquigarrow$ Posso scrivere la funzione inversa:

Si pone $\quad y = \dfrac{2\ln(x)}{1+\ln(x)} \quad$ e Ricavo la x in funzione della y

$$[1+\ln(x)]\,y = 2\ln(x)$$

$$y + y\ln(x) = 2\ln(x) \rightsquigarrow y = 2\ln(x) - y\ln(x)$$

$$\rightsquigarrow \ln(x)\,(2-y) = y$$

$$\rightsquigarrow \ln(x) = \frac{y}{2-y}$$

$$\rightsquigarrow x = e^{\frac{y}{2-y}}$$

$\longrightarrow$ g funzione inversa $\quad g(y) = e^{\frac{y}{2-y}}$