

Settimana: 1

Materia: **Matematica**

Classe: **3D**

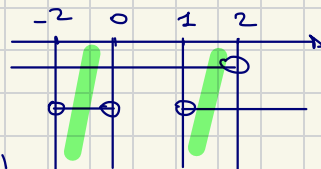
Data: 15/09/25

Argomenti: Ripasso argomenti II superiore: eq, diseq., radicali, parametriche, sistemi di diseq. Eq. e diseq. irrazionali; definizioni. Metodo risolutivo di $\sqrt{A(x)} = B(x)$. Esercizi omessi.

Es 461 Pag 50

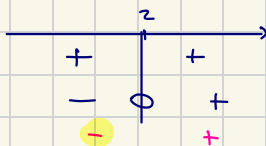
$$\begin{cases} \frac{x}{x-2} < 1 \\ \frac{4-2x-2x^2}{x^3+x^2+x} < 0 \end{cases} \quad \textcircled{\text{I}}$$

$$\begin{cases} x < 2 \\ -2 < x < 0 \vee x > 1 \end{cases}$$



$$\textcircled{\text{I}} \quad \frac{x-x+2}{x-2} < 0 \rightsquigarrow \frac{2}{x-2} < 0$$

$$\begin{aligned} 2 > 0 & \quad \forall x \in \mathbb{R} \\ x-2 > 0 & \quad x > 2 \end{aligned}$$



Sol finale $\begin{matrix} -2 < x < 0 \\ \vee \\ 1 < x < 2 \end{matrix}$

$x < 2$

$$\textcircled{\text{II}} \quad \frac{2x^2+2x-4}{x(x^2+x+1)} > 0$$

$$\frac{2(x^2+x-2)}{x(x^2+x+1)} > 0$$

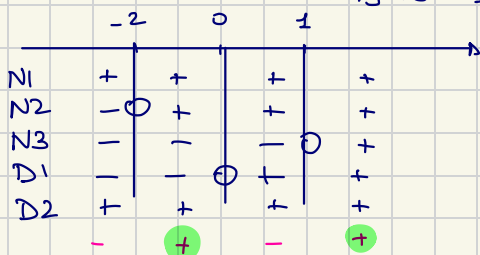
$$\frac{2(x+2)(x-1)}{x(x^2+x+1)} > 0$$

$$N1 > 0 \quad 2 > 0 \quad \forall x \in \mathbb{R} \quad N2 > 0 \quad x+2 > 0 \quad x > -2$$

$$N3 > 0 \quad x-1 > 0 \quad x > 1 \quad D1 > 0 \quad x > 0 \quad x > 0$$

$$D2 > 0 \quad x^2+x+1 > 0 \quad \Delta = b^2-4ac = 1^2-4 \cdot 1 \cdot 1 = -3$$

$$\Delta < 0 \Rightarrow \forall x \in \mathbb{R}$$



$\begin{matrix} -2 < x < 0 \\ \vee \\ x > 1 \end{matrix}$

Es 595

$$\left| \frac{x-3}{x+2} \right| > 6$$

Caso a: Ciò che è dentro il val. assoluto ≥ 0

$$\frac{x-3}{x+2} \geq 0$$

N: $x \geq 3$
D: $x > -2$

$$x < -2 \vee x \geq 3$$

Tolgo il val. assoluto senza far nulla

$$\frac{x-3}{x+2} > 6$$

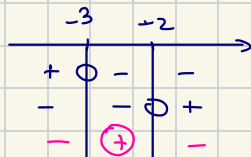
$$\frac{x-3-6x-12}{x+2} > 0$$

$$\frac{-5x-15}{x+2} > 0$$

N: $-5x-15 > 0$

$$5x < -15$$

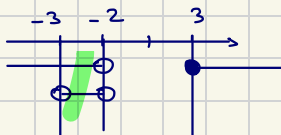
$$x < -3$$



D: $x > -2$

$$-3 < x < -2$$

Sistema:



$$-3 < x < -2 \text{ sol. caso A.}$$

Caso b: Ciò che è dentro il val. Assoluto ≤ 0

$$-2 < x \leq 3$$

Tolgo il val. Assoluto cambiando di segno

$$-\frac{x-3}{x+2} > 6$$

$$\frac{-x+3-6x-12}{x+2} > 0$$

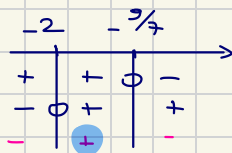
$$\frac{-7x-9}{x+2} > 0$$

N: $-7x-9 > 0$

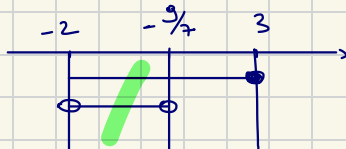
$$-7x > 9$$

$$x < -\frac{9}{7}$$

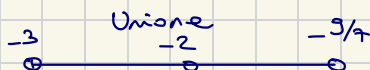
D: $x > -2$



$$-2 < x < -\frac{9}{7}$$



$$-2 < x < -\frac{9}{7} \text{ sol caso B}$$



$$\text{Sol finale } -3 < x < -\frac{9}{7} \quad x \neq -2$$

Eq. di II grado

$$\sqrt{3}x^2 + \sqrt{2}x - \sqrt{3} = 0 \quad \text{da risolvere}$$

$$\Delta = b^2 - 4ac = 2 - 4(\sqrt{3})(-\sqrt{3}) = 2 + 12 = 14$$

$$x_1/x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-\sqrt{2} \pm \sqrt{14}}{2\sqrt{3}} \quad \begin{array}{l} \textcircled{+} \quad \frac{-\sqrt{2} + \sqrt{14}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{6} + \sqrt{42}}{6} \\ \textcircled{-} \quad \frac{-\sqrt{2} - \sqrt{14}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{6} - \sqrt{42}}{6} \end{array}$$

$$ax^2 + (a+1)x + (a+2) = 0 \quad \text{parametro } a$$

(1) Quando ha soluzioni reali e distinte?

$$\Delta > 0 \quad b^2 - 4ac > 0 \quad (a+1)^2 - 4a(a+2) > 0$$

$$a^2 + 2a + 1 - 4a^2 - 8a > 0$$

$$-3a^2 - 6a + 1 > 0$$

$$3a^2 + 6a - 1 < 0$$

$$\Delta = 36 + 12 = 48$$

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

$$a_1/a_2 = \frac{-6 \pm 4\sqrt{3}}{6} \quad \begin{array}{l} \textcircled{+} \quad \frac{-3 + 2\sqrt{3}}{3} \\ \textcircled{-} \quad \frac{-3 - 2\sqrt{3}}{3} \end{array}$$

\leadsto

$$\boxed{\frac{-3 - 2\sqrt{3}}{3} < a < \frac{-3 + 2\sqrt{3}}{3}}$$

Es 480 pag 51

$$\left\{ \begin{array}{l} \frac{x^3+x+2}{x^2+5} > 0 \rightarrow \frac{2}{5} > 0 \text{ ok} \\ \frac{x-x^3}{2(x^3-4x+3)} \leq 0 \rightarrow 0 \leq 0 \text{ ok} \\ x^2 \leq 0 \end{array} \right.$$

$x=0$ \Rightarrow dunque provo $x=0$ nelle
altre. Tutti gli altri casi sono
imp. perché la III è imp.

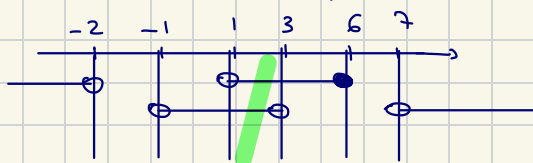
\Rightarrow Quindi sol. finale

$$\boxed{x=0}$$

Es 478

$$\left\{ \begin{array}{l} \frac{x^2-6x}{x^3+x^2-2x} \leq 0 \\ \frac{2}{x+1} > \frac{1}{x-3} \end{array} \right.$$

$$\left\{ \begin{array}{l} x < -2 \vee 1 < x \leq 6 \\ -1 < x < 3 \vee x > 7 \end{array} \right.$$



$$(I) \frac{x(x-6)}{x(x^2+x-2)} \leq 0$$

$$\frac{x(x-6)}{x(x+2)(x-1)} \leq 0$$

Sol. finale

$$1 < x < 3$$

$$\boxed{x < -2 \vee 1 < x \leq 6}$$

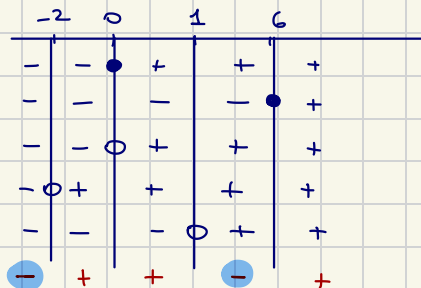
N1: $x \geq 0$

N2: $x \geq 6$

D1: $x > 0$

D2: $x > -2$

D3: $x > 1$

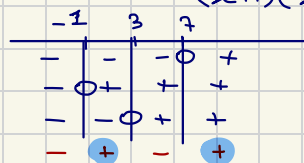


$$(II) \frac{2}{x+1} - \frac{1}{x-3} > 0 \Rightarrow \frac{2x-6-x-1}{(x+1)(x-3)} > 0 \Rightarrow \frac{x-7}{(x+1)(x-3)} > 0$$

N1: $x > 7$

D1: $x > -1$

D2: $x > 3$



$$\boxed{-1 < x < 3 \vee x > 7}$$

Es 658

$$|x^2 - 4| > 4x - 8$$

Caso a: $x^2 - 4 \geq 0$

$$(x-2)(x+2) \geq 0$$

$$x = \pm 2 \text{ } \rightsquigarrow \text{soluzioni eq.}$$

$$x \leq -2 \vee x \geq 2$$

$$x^2 - 4 > 4x - 8$$

$$x^2 - 4x + 4 > 0$$

$$(x-2)^2 > 0$$

$$\forall x \in \mathbb{R}, x \neq 2$$

Sol caso a: $x \leq -2 \vee x \geq 2$

Caso b $x^2 - 4 \leq 0$

$$-2 \leq x \leq 2$$

$$-(x^2 - 4) > 4x - 8$$

$$-x^2 + 4 - 4x + 8 > 0$$

$$-x^2 - 4x + 12 > 0$$

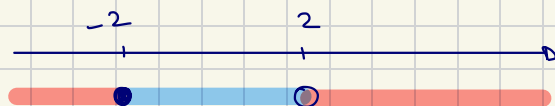
$$x^2 + 4x - 12 < 0$$

$$(x+6)(x-2) < 0 \quad x = -6, 2$$

$$-6 < x < 2$$

Sol caso b: $-2 < x < 2$

Unisco le due soluzioni



$$\forall x \in \mathbb{R}, x \neq 2$$

Es 5 Pag 43

$$x \in \mathbb{R}, x > 0 \quad \text{Dimostrare che: } x + x^{-1} \geq 2$$

$$x + \frac{1}{x} \geq 2 \quad \rightsquigarrow \quad \frac{x^2 + 1 - 2x}{x} \geq 0 \quad \rightsquigarrow \quad \frac{x^2 - 2x + 1}{x} \geq 0$$

$$\rightsquigarrow \frac{(x-1)^2}{x} \geq 0$$

$$N: (x-1)^2 \geq 0 \quad \forall x \in \mathbb{R} \quad \rightsquigarrow \text{Sol: } x > 0$$

$$D: x > 0$$

□

Equazioni e disequazioni irrazionali

Def. Una eq. o diseq. irrazionale è una eq. o diseq. in cui ci sono radicali contenenti un'incognita

Es. $\sqrt{x+3} \leq 3$ diseq. irrazionale

$$\sqrt{x^2+2} + \sqrt{x} = 0 \quad \text{eq. irrazionale}$$

$$\sqrt[4]{x} = \sqrt[12]{x-1} \quad \text{eq. irrazionale.}$$

Prendiamo in considerazione l'eq irrazionale del tipo

$$\sqrt[n]{A(x)} = B(x) \quad \begin{matrix} n \in \mathbb{N} \\ A(x), B(x) \text{ espr. alg.} \end{matrix}$$

▷ Se n è pari:

(1) Impongo le C.E. $A(x) \geq 0$

(2) Impongo che ciò che è uguale alla radice sia positivo
(Perché radice di indice pari mi dà come risultato un num. pos.)

$$B(x) \geq 0$$

(3) Elio alla n

$$A(x) = [B(x)]^n$$

(4) Risolvo e confronto con (1) e (2)

Es Guidi:



$$\sqrt{x+3} = 2x$$

C.E. $x+3 \geq 0$

$2x \geq 0$ (Perché a sx la radice è ≥ 0)

Elio al □ e risolvo

$$x+3 = 4x^2$$

$$4x^2 - x - 3 = 0$$

$$\Delta = 1 + 48 = 49 \quad 1 \quad \text{Accett.}$$

$$x_1/x_2 = \frac{1 \pm 7}{8} < -\frac{3}{4} \quad \text{Non Acc}$$

Consiglio per la vita: Prima di elevare radici a una potenza pari devo controllare che le quantità siano positive

▷ Se n è dispari $\sqrt[n]{A(x)} = B(x)$

(1) Eleva alla n e risolvi

$$A(x) = [B(x)]^n$$

N.B.: Quando n è dispari si può ragionevolmente essere felici ☺

Es. Hormon



$$\sqrt[3]{x^2+x} = x$$

↳ Eleva alla 3

$$x^2+x = x^3$$

$$x^3 - x^2 - x = 0$$

$$x(x^2 - x - 1) = 0$$

$$x = 0$$

$$\Delta = 1 + 4 = 5$$

$$x_1/x_2 = \frac{1 \pm \sqrt{5}}{2}$$

$$\frac{1+\sqrt{5}}{2}$$

$$\frac{1-\sqrt{5}}{2}$$

Es. 7B pag. 63

$$\sqrt{x^2-1-5(x-1)} + 3x = 3$$

$$\sqrt{x^2-1-5(x-1)} = 3-3x$$

$$x^2-1-5x+5 = 9x^2+9-18x$$

$$8x^2-13x+5=0$$

$$\Delta = 169 - 160 = 9 \quad \sqrt{\Delta} = 3$$

$$x_1/x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{13 \pm 3}{16} < \frac{1}{5/8}$$

Acc

Acc

$$\begin{cases} \text{(I)} & x^2-1-5(x-1) \geq 0 \\ \text{(II)} & 3-3x \geq 0 \end{cases} \quad \left\{ \begin{array}{l} x \leq 1 \vee x \geq 4 \\ x \leq 1 \end{array} \right.$$

$$\text{(III)} \quad x \leq 1$$

$$\text{(I)} \quad x^2-1-5x+5 \geq 0$$

$$x^2-5x+4 \geq 0$$

$$(x-4)(x-1) \geq 0$$

$$x=1, 4$$

$$x \leq 1 \vee x \geq 4$$

Esempio, $\sqrt{x+1} = -|x|$

la guardo in faccia:

Impossibile perché a sx c'è un num. positivo
a dx " " " negativo

Es 415:

$$\sqrt{4x + x^2 - 4} = 1 - x - \frac{1}{x}$$

$$x^2 + 4x - 4 = (1-x)^2 + \left(\frac{1}{x}\right)^2 - 2(1-x)\frac{1}{x}$$

$$x^2 + 4x - 4 = 1 + \cancel{x^2} - 2x + \frac{1}{x^2} - \frac{2}{x} + 2$$

$$6x - \frac{4}{x^2} - \frac{2}{x} = 0$$

$$\frac{6x^3 - 4x^2 - 1 + 2x}{x^2} = 0 \quad x \neq 0$$

$$6x^3 - 4x^2 + 2x - 1 = 0$$

$$(x-1)(6x^2 - x - 1) = 0$$

6	-7	2	-1
1	6	-1	1
6	-1	1	-1

$$(x-1)(6x^2 - 3x + 2x - 1) = 0$$

$$(x-1)(3x+1)(2x-1) = 0 \quad \Rightarrow x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

Non Accettabili per c.e.

$$\begin{cases} \text{(I)} & x^2 + 4x - 4 \geq 0 \\ \text{(II)} & \frac{-x^2 + x - 1}{x} \geq 0 \end{cases} \begin{cases} x \leq -2 - 2\sqrt{2} \vee x \geq -2 + 2\sqrt{2} \\ x < 0 \end{cases}$$

$$\text{(I)} \quad \frac{\Delta}{a} = \left(\frac{b}{2}\right)^2 - ac = 4 + 4 = 8$$

$$\sqrt{\frac{\Delta}{a}} = 2\sqrt{2}$$

$$x_1/x_2 = -2 \pm 2\sqrt{2} \begin{cases} -2 + 2\sqrt{2} \\ -2 - 2\sqrt{2} \end{cases}$$

$$\text{(II)} \quad \frac{x^2 - x + 1}{x} \leq 0$$

$$N: x^2 - x + 1 \geq 0$$

$$\Delta = 1 - 4 = -3 \quad \forall x \in \mathbb{R}$$

$$D: x > 0$$