Settimana: 6

Materia: Matematica

Classe: 5C

Data: 20/10/2025

Argomenti:

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$$\lim_{x\to 0^+}\left(\frac{x^2}{4}\right)^{\frac{1}{3\ln x}} = \lim_{x\to 0^+} e^{\frac{1}{3\ln x}\cdot\ln\left(\frac{x^2}{4}\right)} =$$

$$= \lim_{x\to 0^+} e^{\frac{2\cdot\ln\left(\frac{x}{2}\right)}{3\ln x}} =$$

$$= \lim_{x\to 0^+} e^{\frac{2\ln x}{3\ln x} - \frac{2\ln(2)}{\ln(x)}} = e^{\frac{2}{3}}$$

533 $\lim_{x\to 0}\left(1+tg^2x\right)^{\frac{1}{x\ln(x+1)}} =$

$$\lim_{x\to 0} e^{\frac{1}{x\ln(x+1)}\ln(1+tg^2(x))\cdot\frac{x}{x}} =$$

$$\lim_{x\to 0} e^{\frac{x}{\ln(x+1)}\cdot\frac{\ln(1+tg^2(x))}{x^2}\cdot\frac{tg^2(x)}{tg^2(x)}} =$$

$$\lim_{x\to 0} e^{\frac{x}{\ln(x+1)}\cdot\frac{\ln(1+tg^2(x))}{tg^2(x)}\cdot\frac{tg^2(x)}{x^2}} =$$

Tenderà a 1 poiché sost $tg^2x = t$

$$\lim_{x\to 0} e^{\underbrace{\frac{x}{\ln(x+1)}}_{1}\cdot\underbrace{\boxed{\frac{\ln(1+tg^2(x))}{tg^2(x)}}}_{1}\cdot\underbrace{\boxed{\left(\frac{\sin x}{x}\right)^2}}_{1}\cdot\underbrace{\boxed{\frac{1}{\cos^2 x}}}_{1}} = e$$

$$\lim_{x\to 0}\frac{\ln(x+1)}{x} = 1$$

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$$f(x) = \begin{cases} \dfrac{\sin 2x}{x} & x<0 \\ 2e^{\frac{ax+b}{x-c}} & x \geq 0 \quad x \neq c \end{cases} \qquad a,b \in \mathbb{R}, \ c \in \mathbb{R}^+$$

(1) Trova a,b,c sapendo

i) f è continua in 0

ii) $\lim_{x \to +\infty} f(x) = 2e$

iii) $\lim_{x \to 3^-} f(x) = 0$

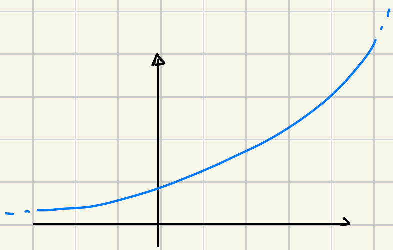
i) $$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \overset{1}{\frac{\sin 2x}{2x}} \cdot 2 = 2$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2e^{\frac{ax+b}{x-c}} = 2e^{-\frac{b}{c}}$$

$$f(0) = 2e^{-\frac{b}{c}} \rightsquigarrow \boxed{2 = 2e^{-\frac{b}{c}}}$$

ii) $$\lim_{x \to +\infty} 2e^{\frac{ax+b}{x-c}} = \lim_{x \to +\infty} 2e^{\frac{\cancel{x}\left(a+\frac{b}{x}\right)}{\cancel{x}\left(1-\frac{c}{x}\right)}} = \boxed{2e^{a} = 2e}$$

iii) $$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} 2e^{\frac{ax+b}{x-c}} = 0$$



Per fare in modo che l'exp vada a 0, l'esponente deve essere $-\infty$

$$\Rightarrow \lim_{x \to 3^-} \frac{ax+b}{x-c} = -\infty$$

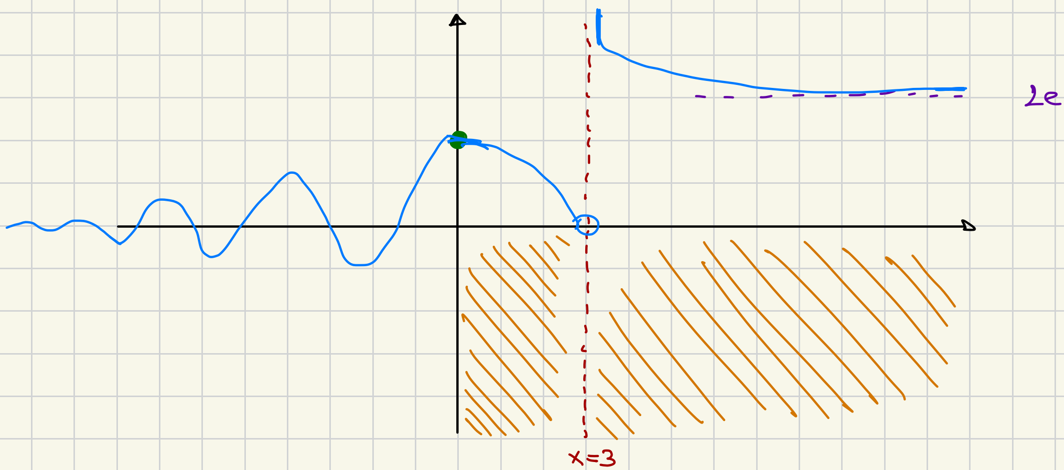
$\Rightarrow$ Al denominatore, quando $x \to 3^-$, deve venire 0 $\Rightarrow$ $\boxed{c=3}$

Condizioni.
$$\begin{cases} 2 = 2e^{-\frac{b}{c}} \\ a = 1 \\ c = 3 \end{cases} \qquad \begin{cases} \frac{b}{3} = 0 \\ a = 1 \\ c = 3 \end{cases} \qquad \begin{cases} b = 0 \\ a = 1 \\ c = 3 \end{cases}$$

$$f(x) = \begin{cases} \frac{\sin 2x}{x} & x<0 \\ 2e^{\frac{x}{x-3}} & x \geq 0 \quad x \neq 3 \end{cases}$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2e^{\overbrace{\frac{x}{\underbrace{x-3}_{\to 0^+}}}^{\to +\infty}} = \infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sin 2x}{x} \overset{\text{Teo Confronto}}{\boxed{=}} 0$$



$$\lim_{x \to -\infty} x f(x) = \lim_{x \to -\infty} \cancel{x} \cdot \frac{\sin 2x}{\cancel{x}} = \lim_{x \to -\infty} (\sin 2x)$$

Il limite NON esiste, perché a $-\infty$ continua a oscillare

$$\lim_{x \to 0^-} \frac{f(x)}{x} = \lim_{x \to 0^-} \frac{\sin 2x}{x^2} = \lim_{x \to 0^-} \overbrace{\frac{\sin 2x}{2x}}^{\to 1} \cdot \overbrace{\frac{2}{x}}^{\to -\infty} = -\infty$$

n. 100 $\quad f(x) = \dfrac{e^{ax} - a}{e^{bx} + b} \qquad a,b \in \mathbb{R}$

Trova a,b t.c. $\quad$ i) $\lim_{x \to +\infty} f(x) = 1$

ii) $\lim_{x \to -\infty} f(x) = -1$

(i) $\lim_{x \to +\infty} \dfrac{e^{ax} - a}{e^{bx} + b} = \left(\begin{array}{l} \text{sost: } e^x = t \\ x \to +\infty, \ t \to +\infty \end{array} \right) =$

$$\lim_{t \to +\infty} \frac{t^a - a}{t^b + b} = \lim_{t \to +\infty} \frac{t^a \left(1 - \frac{a}{t^a}\right)}{t^b \left(1 + \frac{b}{t^b}\right)} =$$

$$\lim_{t \to +\infty} \underbrace{t^{a-b}}_{\to 1} \underbrace{\left(\frac{1 - \frac{a}{t^a}}{1 + \frac{b}{t^b}}\right)}_{\to 1} = 1$$

⇝ Dunque $a = b$

(c'è un piccolo problema: funzione solamente se $a,b > 0$, altrimenti non va)

⇝ Quello che si deve fare è analizzare tutte le casistiche

(1) $a,b > 0$ $\qquad$ (3) $a < 0$, $b > 0$

(2) $a > 0$, $b < 0$ $\qquad$ (4) $a,b < 0$

Per case con cond. 2

b) Pongo $a = b = 1$ e traccio il grafico probabile

$$f(x) = \frac{e^x - 1}{e^x + 1}$$

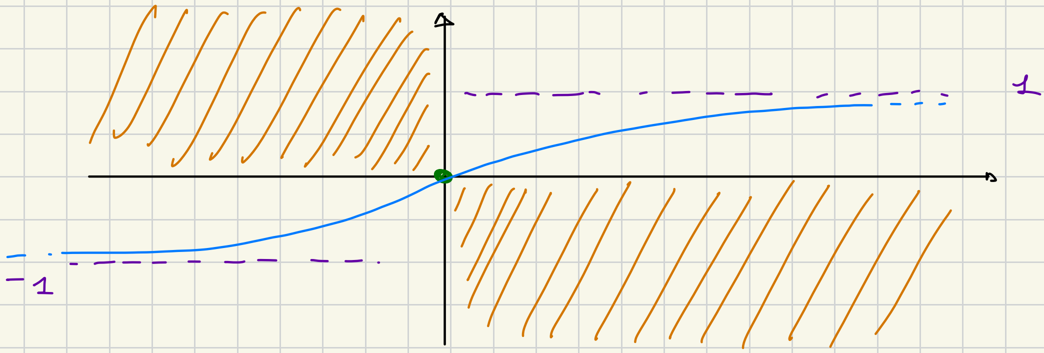
1) Dom(f): $\quad e^x + 1 \neq 0 \quad$ Sempre $\quad f: \mathbb{R} \longrightarrow \mathbb{R}$

2) Int. Assi. $\quad$ Asse x : $\quad y = 0 \quad e^x - 1 = 0 \quad x = 0 \qquad A = (0;0)$

Asse y : $\quad x = 0 \quad ⇝ \quad y = 0$

(3) Segno: $f(x) \geq 0 \quad \frac{e^x-1}{e^x+1} \geq 0 \leadsto e^x > 1 \leadsto \boxed{x>0}$

(4) limiti: $\lim_{x\to+\infty} f(x) = 1 \qquad \lim_{x\to-\infty} f(x) = -1$



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$$f(x) = \frac{3x - e^{\sin x}}{5 + e^{-x} - \cos x}$$

Dire se esiste $\lim_{x\to+\infty} f(x)$ e $\lim_{x\to-\infty} f(x)$

$$\lim_{x\to+\infty} \frac{3x - e^{\sin x}}{5+e^{-x} - \cos x} =$$ Guess $+\infty$

$$\frac{3x - e}{5+e^{-x}+1} \leq \frac{3x - e^{\sin x}}{5+e^{-x} - \cos x} \leq \frac{3x - \frac{1}{e}}{5+e^{-x} - 1}$$

$x \to +\infty$ $\qquad$ $x \to +\infty$

$$\infty \leq \lim_{x\to+\infty} f(x) \leq \infty$$

Teorema dei Carabinieri

esiste ed è ∞

$$\lim_{x \to -\infty} \frac{3x - e^{\sin x}}{5 + e^{-x} - \cos x} =$$

Gerarchie infiniti

$\triangleright \lim_{x \to +\infty} \frac{x}{e^x} = 0$

$\triangleright \lim_{x \to +\infty} \frac{x^n}{e^x} = 0 \quad \forall n \in \mathbb{N}$

Dim più avanti

Sost. $x = -t$ $\quad x \to -\infty$, $t \to +\infty$

$$\lim_{t \to +\infty} \frac{-3t - e^{-\sin t}}{5 + e^t - \cos(t)}$$

$$\lim_{t \to +\infty} \frac{t\left(-3 - \frac{e^{-\sin t}}{t}\right)}{e^t\left(\frac{5}{e^t} + 1 - \frac{\cos(t)}{e^t}\right)} = 0$$

Limite esiste e fa 0

$\frac{t}{e^t} \to 0$, $\frac{e^{-\sin t}}{t} \to 0$, $\frac{5}{e^t} \to 0$, $\frac{\cos(t)}{e^t} \to 0$, $\to -3$

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$$\lim_{x \to 0} (1+x^2)^{\frac{1}{\sin^2 x}} = \lim_{x \to 0} e^{\frac{1}{\sin^2 x} \ln(1+x^2)}$$

$$= \lim_{x \to 0} e^{\frac{1}{\sin^2 x} \cdot \frac{x^2}{x^2} \cdot \ln(1+x^2)} = e$$

$1 = 1^2$ $\quad$ 1

Sto usando $\lim_{t \to 0} \frac{\ln(1+t)}{t} = 1$

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$$\lim_{x \to 2} \frac{\ln(x+3) - \ln(2x+1)}{x^2 + x - 6} = \lim_{x \to 2} \frac{\ln\left(\frac{x+3}{2x+1}\right)}{(x-2)(x+3)} =$$

So che $\lim_{t \to 0} \frac{\ln(1+t)}{t} = 1$

Sost. $1 + t = \frac{x+3}{2x+1}$

$2x + 1 + 2xt + t = x + 3$

$x(1+2t) = 2 - t$

$$\left(x = \frac{2-t}{1+2t} \qquad \begin{array}{l} x \to 2 \\ t \to 0 \end{array} \right)$$

$$\lim_{t\to 0} \frac{\ln(1+t)}{\left(\frac{2-t}{1+2t}-2\right)\left(\frac{2-t}{1+2t}+3\right)} = \lim_{t\to 0} \frac{\ln(1+t) \quad (1+2t)^2}{(2-t-2-4t)(2-t+3+6t)}$$

$$= \lim_{t\to 0} \underbrace{\frac{\ln(1+t)}{-5t}}_{1} \cdot \frac{(1+2t)^2}{(5+5t)} = -\frac{1}{25}$$

Q1.1

$$\lim_{x\to 0} (1-\sin x)^{\frac{\cos x}{x}} = \lim_{x\to 0} e^{\frac{\cos x}{x} \ln(1-\sin x) \cdot \frac{\sin x}{\sin x}}$$

$$= \lim_{x\to 0} e^{\frac{\ln(1-\sin x)}{\sin x} \cdot \frac{\sin x}{x} \cdot \cos x} = e$$

Q1.2

$$\lim_{x\to 0} \frac{\cos x - e^x}{\sin x} = \lim_{x\to 0} \left(\frac{\cos x - 1}{\sin x} + \frac{1-e^x}{\sin x} \right)$$

$$= \lim_{x\to 0} \left(-\frac{1-\cos x}{\sin x} \cdot \frac{x^2}{x^2} + (-1) \frac{e^x - 1}{\sin x} \cdot \frac{x}{x} \right)$$