Settimana: 18

Materia: Matematica
Classe: 5A
Data: 9/02/26

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$$f(x) = \sqrt[3]{(x-4)^2}$$

(1) $\mathrm{Dom}(f) = \mathbb{R}$

(2) Assi: $y=0 \rightsquigarrow (x-4)^2 = 0 \rightsquigarrow x = 4$ $\qquad A=(4;0)$

$x=0 \rightsquigarrow y = \sqrt[3]{16} = 2\sqrt[3]{2}$ $\qquad B=(0;\ 2\sqrt[3]{2})$

(3) Segno: $\sqrt[3]{(x-4)^2} \geq 0 \quad \forall x \in \mathbb{R}$

(4) Limiti: $\displaystyle\lim_{x \to \pm\infty} \sqrt[3]{(x-4)^2} = +\infty$

Controllo obliquo:

$$[x(1-\tfrac{4}{x})]^2$$

$$\lim_{x \to \pm\infty} \frac{\sqrt[3]{(x-4)^2}}{x} = \lim_{x \to \pm\infty} \frac{\sqrt[3]{x^2}\,\sqrt[3]{(1-\frac{4}{x})^2}}{x} = 0$$

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$\dfrac{1}{\sqrt[3]{x}}$

Non c'è Asintoto obliquo.

(5) $f(x) = \sqrt[3]{(x-4)^2} = (x-4)^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3}(x-4)^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{(x-4)}}$$

Warning $x=4$ la derivata NON la posso calcolare con la formula sopra e dovrò fare con la def.

Faccio subito:

$$\lim_{h\to 0}\frac{f(4+h)-f(4)}{h} = \lim_{h\to 0}\frac{\sqrt[3]{h^2}-0}{h}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt[3]{h}} \begin{cases} \xrightarrow{h\to 0^+} +\infty \\ \xrightarrow{h\to 0^-} -\infty \end{cases}$$

 cuspide

$f'(x) \geq 0$ $\qquad \frac{2}{3}\cdot\frac{1}{\sqrt[3]{(x-4)}} \geq 0 \quad \rightsquigarrow \quad x > 4$

$$\begin{array}{c|c} & 4 \\ \hline - & + \\ \searrow & \nearrow \end{array}$$

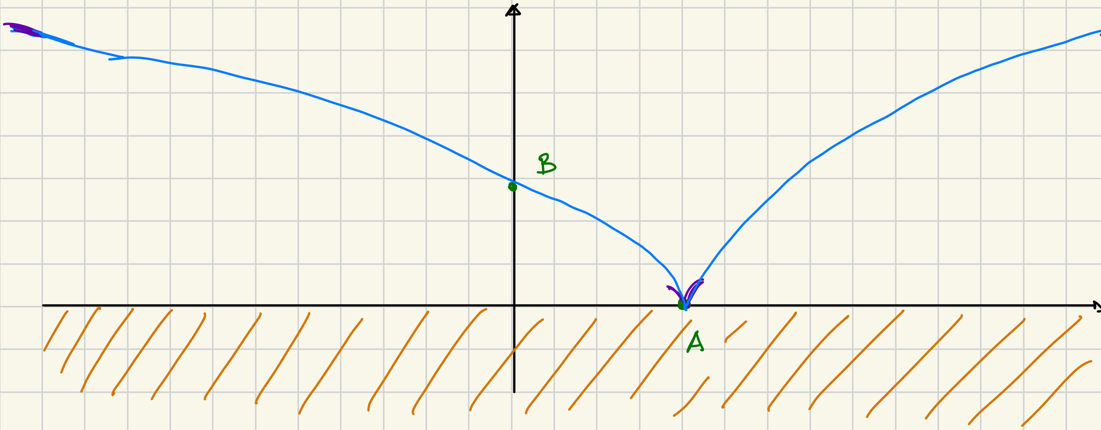
(6) $f''(x)$: $\qquad f'(x) = \frac{2}{3}(x-4)^{-1/3}$

$$f''(x) = \frac{2}{3}\cdot\left(-\frac{1}{3}\right)\cdot(x-4)^{-\frac{4}{3}} = -\frac{2}{9\sqrt[3]{(x-4)^4}}$$

Va posto di nuovo ≥ 0 $\qquad f''(x) = -\frac{2}{9\sqrt[3]{(x-4)^4}} \geq 0$

$\nexists x \in \mathbb{R}$, Mai

$$\begin{array}{c|c} & 4 \\ \hline - & - \\ \frown & \frown \end{array}$$



Esercizio: Tra tutti i rettangoli di Perimetro 24, trovare quello di Area max. ⟶ Si risolvono con le derivate.

a, b i due lati

So che $2a + 2b = 24$

$a + b = 12 \leadsto b = 12 - a$

Definisco la funzione Area $= ab$

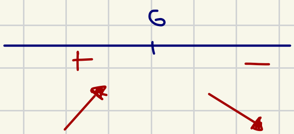
$$f(a) = a(12-a)$$

Adesso la derivo e trovo il minimo: $f(a) = 12a - a^2$

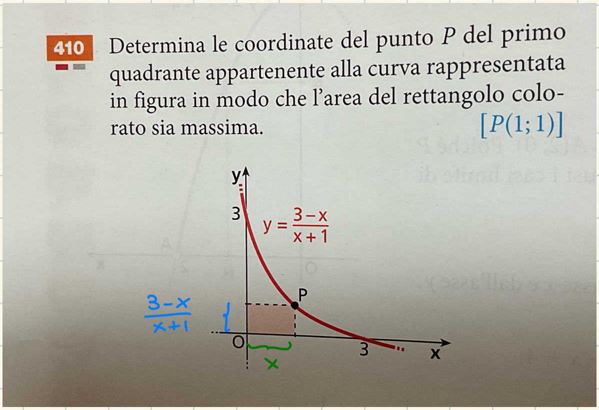
$$f'(a) = 12 - 2a$$

$\boxed{f'(a) \geq 0}$ ⟶ Faccio $\geq$ per vedere se esce max o min

$12 - 2a \geq 0 \qquad 2(6-a) \geq 0 \qquad a \leq 6$



⟶ 6 è un massimo e area max $6(12-6) = 36$



410 Determina le coordinate del punto P del primo quadrante appartenente alla curva rappresentata in figura in modo che l'area del rettangolo colorato sia massima. $[P(1;1)]$

$f(x) = \text{Area}(x) =$

$= x \, \frac{3-x}{x+1}$

Per l'area max.

$\boxed{f'(x) \geq 0}$ e risolvo

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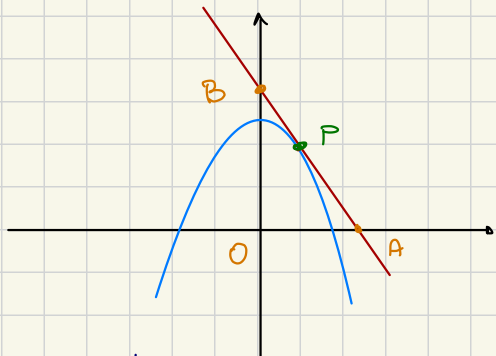
$f(x) = 4-x^2$

Trovo P in modo che l'area del triangolo sia minima



$P = (x_0; 4-x_0^2)$

$f'(x) = -2x$. Valuto nel punto P. $f'(x_0) = -2x_0$

coeff. angolare retta rossa $-2x_0$

retta: $[y-(4-x_0^2)] = -2x_0(x-x_0)$

$y = -2xx_0 + 2x_0^2 + 4 - x_0^2$

$y = -2x_0 \cdot x + 4 + x_0^2$

Trovo A e B in fz di x_0.

A: $y=0$ $\quad 0 = -2x_0 x + 4 + x_0^2 \leadsto x = \frac{4+x_0^2}{2x_0}$

B: $x=0 \qquad y = 4+x_0^2$

$$A = \left(\frac{4+x_0^2}{2x_0} ; 0 \right) \qquad B = \left(0 ; 4+x_0^2\right)$$

$$\text{Area}(x_0) = \frac{4+x_0^2}{2x_0} \left(4+x_0^2\right) \cdot \frac{1}{2}$$

$$f(x) = \frac{(4+x^2)^2}{4x}$$ funzione Area $\quad \text{Dom}(f) = (0;2)$

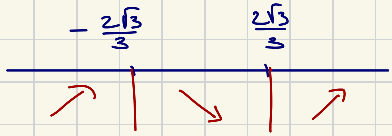
Per trovare il minimo, derivo e pongo ≥ 0

$$f'(x) = \frac{2(4+x^2)\cdot 2x \; 4x - 4(4+x^2)^2}{16x^2}$$

$$= \frac{(4+x^2)}{16x^2}\left(16x^2 - 16 - 4x^2\right)$$

$$= \frac{4+x^2}{16x^2}\left(12x^2-16\right) = \frac{4+x^2}{4x^2}\left(3x^2-4\right)$$

$f'(x) \geq 0$



$\rightsquigarrow x = \frac{2\sqrt{3}}{3} \rightsquigarrow$ Area minima

$$f\left(\frac{2\sqrt{3}}{3}\right) = \frac{\left(4+\frac{4}{3}\right)^2}{4\,\frac{2\sqrt{3}}{3}} = \frac{\overset{8\cdot\cancel{8}\cdot 2\cdot 2}{\cancel{16^2}}}{3^2} \cdot \frac{\cancel{3}^{\sqrt{3}}}{\cancel{8}\cancel{\sqrt{3}}} =$$

$$= \frac{32}{9}\sqrt{3}$$

89 $f(x) = (x-1)^2 + (x-2)^2 + (x-3)^2 + (x-4)^2 + (x-5)^2$

Trova minimo di f.

$$f'(x) = 2(x-1) + 2(x-2) + 2(x-3) + 2(x-4) + 2(x-5)$$

$$= 2(5x - 15) = 0 \rightsquigarrow \boxed{x = 3}$$

$$\min f = f(3) = (3-1)^2 + (3-2)^2 + (3-3)^2 + (3-4)^2 + (3-5)^2$$

$$= 4 + 1 + 0 + 1 + 4$$

$$= 10$$