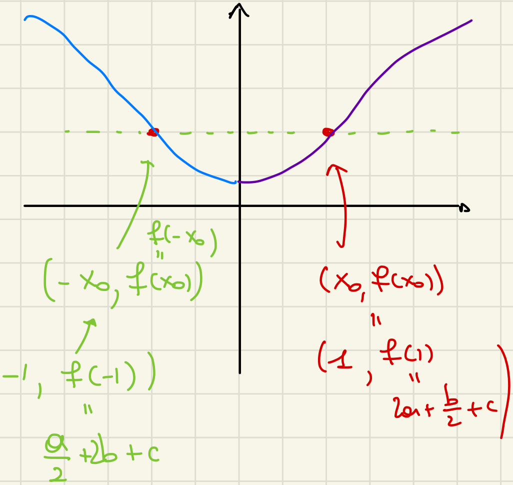
Pag 624 n 56

$f(x) = a2^x + b2^{-x} + c$

(a) trova a, b, c con 3 condizioni

(1) Grafico simmetrico rispetto $x=0$

$$\frac{a}{2} + 2b + c = 2a + \frac{b}{2} + c$$



(2) Passi per $\left(1; \frac{7}{2}\right)$

$$\frac{7}{2} = 2a + \frac{b}{2} + c$$

(3) $f(0) = 4$ $\quad 4 = a + b + c$

$$\begin{cases} a+b+c=4 \\ 7 = 4a + b + 2c \\ a + 4b = 4a + b \end{cases} \quad \begin{cases} 2a + c = 4 \\ 7 = 5a + 2c \\ \cancel{3}b = \cancel{3}a \end{cases} \quad \begin{cases} 2a + c = 4 \\ 7 = a + \underbrace{2(2a+c)}_{8} \\ a = b \end{cases}$$

$$\begin{cases} c = 6 \\ a = -1 \\ b = -1 \end{cases} \qquad f(x) = -2^x - 2^{-x} + 6$$

(b) $f(x) \geq \frac{9}{2}$

$$-2^x - 2^{-x} + 6 \geq \frac{9}{2} \quad \rightsquigarrow \quad -2^x - \frac{1}{2^x} + \frac{3}{2} \geq 0$$

$$\frac{-2\,2^{2x} - 2 + 3 \cdot 2^x}{2 \cdot 2^x} \geq 0$$

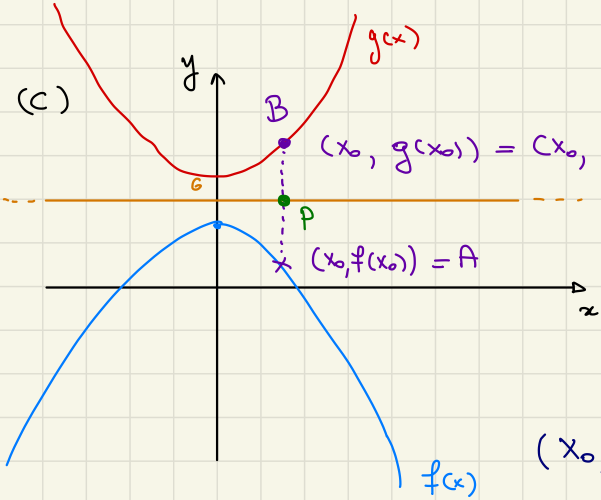
Dato che $D > 0$ sempre
Non lo considero.

$$-2\cdot 2^{2x}+3\cdot 2^{x}-2\geq 0 \qquad 2^x = d$$

$$-2d^2+3d-2\geq 0$$

$$2d^2-3d+2\leq 0 \qquad \Delta = 9-16<0 \qquad \text{Impossibile}$$

(c)



$$P=(x_0, 6)$$

P è il pto medio tra A e B

$$P=\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)$$

$$(x_0, 6) = \left(\underbrace{\frac{x_0+x_0}{2}}_{x_0}, \frac{f(x_0)+g(x_0)}{2}\right)$$

Coordinata y, $6 = \dfrac{f(x_0)+g(x_0)}{2}$. Dato che questa relazione vale $\forall x_0$, la scrivo in maniera generica

$$12 = f(x)+g(x)$$

$$12 = -2^x-2^{-x}+6+g(x)$$

$$g(x) = 2^x+2^{-x}+6$$

(d) Trovo intersezione tra f(x) e g(x).

Formalmente $\begin{cases} y=-2^x-2^{-x}+6 \\ y=2^x+2^{-x}+6 \end{cases}$ e risolvo

Operativamente $f(x) = g(x)$ e risolvo

$$2^x + 2^{-x} + \cancel{6} = -2^x - 2^{-x} + \cancel{6}$$

$$\cancel{2} \cdot 2^x + \cancel{2} \cdot 2^{-x} = 0$$

$$2^x = -2^{-x}$$

Impossibile: LHS positivo
RHS negativo.

Pag 687 n93

$$f(x) = \sqrt{3^{\frac{x}{2}} + 3^x - 2}$$

$$\log_3(4) = \log_3 2^2 = 2\log_3 2$$

$$f(\log_3 4) = \sqrt{3^{\frac{\cancel{2}\log_3 2}{\cancel{2}}} + 3^{\log_3 4} - 2} =$$

$$= \sqrt{\cancel{2} + 4 - \cancel{2}} = 2$$