Settimana: 20

Materia: Matematica
Classe: 5A
Data: 23/02/26

Integrali:

Esempio: $\int \frac{1}{1+x}\,dx = \ln(|1+x|) + c$

$\int \frac{1}{1+\sqrt{x}}\,dx$ (Sost.: $\sqrt{x} = t$)

Orango – Tango
▷ Si ricava la x in f_z della t

$$\int \frac{1}{1+t}\,dx = \int \frac{1}{1+t} \cdot 2t\,dt =$$

$x = t^2$

▷ Si deriva a sinistra in x, a destra in t e si aggiunge rispettiv. dx e dt

Linearità

$$\int \frac{2t}{1+t}\,dt \boxed{=} 2\int \frac{t}{1+t}\,dt =$$

$1 \cdot dx = 2t\,dt$

$$\frac{t+1}{t+1} - \frac{1}{t+1}$$

▷ Ho ricavato dx, lo sostituisco

$$2\int \frac{t+1-1}{t+1}\,dt$$

$$2\int \underbrace{\frac{t+1}{t+1}}_{1} - \frac{1}{t+1}\,dt$$

$$2\int\left(1 - \frac{1}{t+1}\right)dt = 2\left[\int 1\,dt - \int \frac{1}{1+t}\,dt\right] =$$

$$2\left[t - \ln|1+t|\right] + c = 2\left(\sqrt{x} - \ln|1+\sqrt{x}|\right) + c$$

Fatto: L'integrale è Lineare, cioè

▷ $\int k f(x)\,dx = k\int f(x)\,dx \qquad k \in \mathbb{R}$

▷ $\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

Dim: Si comporta come la derivata

Fatto: È lecito fare sostituzioni come per i limiti. Ricordarsi che il dx (differenziale) va modificato in modo opportuno seguendo il metodo Orango-tango

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Tip: Sostituire TUTTA la roba che non vi piace

$$\int \frac{x}{\sqrt{x-1}}\, dx =$$

Sost: $\sqrt{x-1} = t$

$x-1 = t^2 \rightsquigarrow dx = 2t\, dt$

$x = t^2+1$

$$= \int \frac{t^2+1}{\cancel{t}}\, 2\cancel{t}\, dt = 2\int (t^2+1)\, dt = 2\left[\frac{t^3}{3} + t\right] + c$$

$$\frac{2}{3}t^3 + 2t + c = \frac{2}{3}\sqrt{(x-1)^3} + 2\sqrt{x-1} + c$$

355: $\int \frac{1}{x-\sqrt{x}}\, dx =$

Sost: $\sqrt{x} = t \rightsquigarrow dx = 2t\, dt$

$x = t^2$

$$= \int \frac{1}{t^2-t} \cdot 2t\, dt = \int \frac{1}{\cancel{t}(t-1)}\, 2\cancel{t}\, dt = 2\int \frac{1}{t-1}\, dt$$

$$= 2\ln|t-1| + c = 2\ln|\sqrt{x}-1| + c$$

356: $\int \frac{1}{e^x + e^{-x}}\, dx =$

Sost $e^x = t \rightsquigarrow dx = \frac{1}{t}\, dt$

$x = \ln(t)$

$$= \int \frac{1}{t+\frac{1}{t}} \cdot \frac{1}{t}\, dt = \int \frac{\cancel{t}}{t^2+1} \cdot \frac{1}{\cancel{t}}\, dt = \operatorname{arctg}(e^x) + c$$

Alternat. $\int \frac{1}{e^x+\frac{1}{e^x}}\,dx = \int \frac{e^x}{e^{2x}+1}\,dx =$ Sost: $e^x = t$

$e^x\,dx = dt$

$$\int \frac{1}{t^2+1}\,dt = \dots = \operatorname{arctg}(e^x)+c$$

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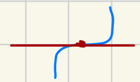
$f(x) = \frac{ax^2-1}{x+2}$ trova a in modo che $x=1$ max

Si impone $f'(1)=0$

$$f'(x) = \frac{(2ax)(x+2) - (ax^2-1)(1)}{(x+2)^2} \qquad f'(1)=0$$

$$0 = \frac{2a\cdot 3 - (a-1)}{3^2} \rightsquigarrow 6a - a + 1 = 0$$

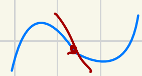
$$\rightsquigarrow 5a = -1 \rightsquigarrow \boxed{a = -\frac{1}{5}}$$



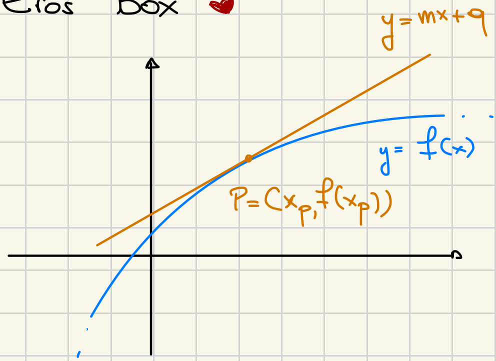
356 $f(x) = e^{\frac{ax-b}{x+c}}$ nel pto di ascisse 0

(1) Ha un flesso $\rightsquigarrow f''(0)=0$

(2) La tangente vale $y = \frac{2}{e}x + \frac{1}{e}$



Eros' Box ❤



(1) $m = f'(x_p)$

(2) $P \in$ Retta, dunque vale:

$y_p = m x_p + q$ cioè

$f(x_p) = f'(x_p)\cdot x_p + q$

$$f(x) = e^{\frac{ax-b}{x+c}}$$

$$f'(x) = e^{\frac{ax-b}{x+c}} \cdot \frac{a(\cancel{x}+c) - 1(\cancel{a}x-b)}{(x+c)^2} = e^{\frac{ax-b}{x+c}} \cdot \boxed{\frac{ac+b}{(x+c)^2}} \rightarrow (ac+b)(x+c)^{-2}$$

$$f''(x) = \underbrace{e^{\frac{ax-b}{x+c}} \cdot \frac{ac+b}{(x+c)^2}}\cdot \underbrace{\frac{ac+b}{(x+c)^2}} + \underbrace{e^{\frac{ax-b}{x+c}}} \underbrace{\cdot (ac+b) \cdot (-2)(x+c)^{-3}}$$

$$\triangleright\ f''(0) = e^{-\frac{b}{c}} \cdot \frac{(ac+b)^2}{c^4} - 2e^{-\frac{b}{c}} \cdot \frac{(ac+b)}{c^3} = 0$$

$$\boxed{\frac{\cancel{e^{-\frac{b}{c}}} \cdot (ac+b)}{c^3} \left(\frac{ac+b}{c} - 2\right) = 0}$$

$$\triangleright\ f'(0) = \frac{2}{e} \rightsquigarrow \boxed{e^{-\frac{b}{c}} \frac{(ac+b)}{c^2} = \frac{2}{e}}$$

$$\triangleright\ f(0) = f'(0) \cdot 0 + \frac{1}{e} \rightsquigarrow e^{\frac{-b}{c}} = \frac{1}{e} \rightsquigarrow \boxed{b = c}$$

$$\begin{cases} \dfrac{ac+c}{c^3}\left(\dfrac{ac+c}{c} - 2\right) = 0 \\ \rightarrow \dfrac{ac+c}{c^2} = 2 \end{cases} \rightsquigarrow \begin{cases} \dfrac{a+1}{c^2}(a-1) = 0 \\ \dfrac{a+1}{c} = 2 \rightsquigarrow a = 2c - 1 \end{cases}$$

$$\frac{2c}{c^2}(2c - 1 - 1) = 0$$

$$\frac{2}{c} \cdot 2(c-1) = 0 \rightsquigarrow \boxed{c = 1} \begin{cases} \rightarrow \boxed{b = 1} \\ \rightarrow \boxed{a = 1} \end{cases}$$