Pag 1029 n 394 ~ Esercizio guida

$x^3 - 8i = 0$

Posso riscrivere l'equazione come $x^3 = +8i$

$8i$ lo porto in forma trigonometrica

$r = \sqrt{a^2+b^2} = \sqrt{0+8^2} = 8$

α: Dato che non c'è parte reale, $\alpha = \frac{\pi}{2}$ se $b>0$
$\alpha = -\frac{\pi}{2}$ se $b<0$

Pensare ad diag cart.

$8i = 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 8e^{i\frac{\pi}{2}}$

$x = \sqrt[3]{8}\; e^{i\frac{\frac{\pi}{2}+2k\pi}{3}}$ $\quad k = 0,1,2$

Deriva da questo già detto, ma ora non ha senso esplic.

Ha senso perché sto facendo radice di un numero reale

$\rightsquigarrow x = 2e^{i\frac{\pi+4k\pi}{6}}$ $\quad k=0,1,2$

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384 $\quad x^4+64=0$

$x^4 = -64$

$r = \sqrt{a^2+b^2} = \sqrt{(-64)^2+0^2} = 64$

$\alpha = \operatorname{arctg}\left(\frac{b}{a}\right) = \operatorname{arctg}\left(\frac{0}{-64}\right) = 0$

$x^4 = 64\, e^{0i}$

$x = \sqrt[4]{64}\; e^{\frac{0i+2k\pi}{4}} = 2\sqrt{2}\, e^{\frac{2k\pi}{4}}$ $\quad k=0,1,2,3$

369 (variante). $x^5 = 2 + 2\sqrt{3}\, i$

$r = \sqrt{a^2+b^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\alpha = \operatorname{arctg}\left(\frac{2\sqrt{3}}{2}\right) = \operatorname{arctg}(\sqrt{3}) = \frac{\pi}{3}$

$x^5 = 4\, e^{i\frac{\pi}{3}}$

$x = \sqrt[5]{4}\; e^{\frac{\frac{\pi}{3}+2k\pi}{5} i} = \sqrt[5]{4}\; e^{\frac{\pi+6k\pi}{15} i} \qquad k=0,1,2,3,4$

342: Calcolare $z_1^2 - \frac{1}{z_2^2}$ $\qquad z_1 = 2e^{i\frac{\pi}{6}} \qquad z_2 = e^{i\frac{\pi}{4}}$

$\left[2e^{i\frac{\pi}{6}}\right]^2 - \frac{1}{\left(e^{i\frac{\pi}{4}}\right)^2} \boxed{=} 4e^{i\frac{\pi}{3}} - e^{i\left(-\frac{\pi}{2}\right)}$

$= 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) - \left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$= 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) - (0 - i)$

$= 2 + 2\sqrt{3}\, i + i = 2 + i\left(2\sqrt{3}+1\right)$

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465 $\quad x^2 - 2x + 2 = 0$

$\frac{\Delta}{4} = \left(\frac{b}{2}\right)^2 - ac = 1 - 2 = -1 \qquad \sqrt{\frac{\Delta}{4}} = i$

$x_1/x_2 = 1 \pm i$

$1+i = \sqrt{2}\, e^{i\frac{\pi}{4}} \qquad r^2 = a^2 + b^2 = 1+1 = 2 \quad r = \sqrt{2}$

$\alpha = \operatorname{arctg}\left(\frac{b}{a}\right) = \operatorname{arctg}(1) = \frac{\pi}{4}$

$1-i = \sqrt{2}\, e^{-i\frac{\pi}{4}} \qquad \alpha = \operatorname{arctg}\left(\frac{b}{a}\right) = \operatorname{arctg}(-1) = -\frac{\pi}{4}$

Es 6 Prova B

$$x^2 - (2-i)x + 3 - i = 0$$

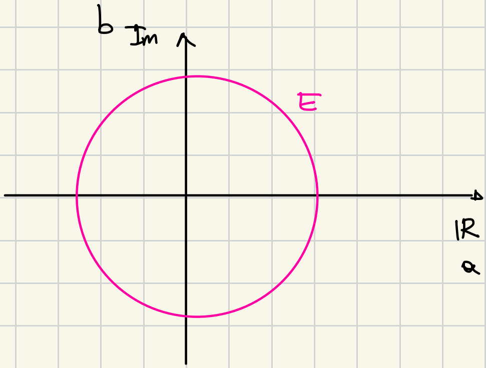
$$\Delta = [-(2-i)]^2 - 4(3-i) = 4 - 1 - \cancel{4i} - 12 + \cancel{4i} = -9 \qquad \sqrt{\Delta} = 3i$$

$$x_1/x_2 = \frac{2-i \pm 3i}{2} \quad \begin{cases} + & 1+i \\ - & 1-2i \end{cases}$$

Es 3 Pag 1040

$$E = \{ z \in \mathbb{C} \mid |z| = 5 \}$$

▷ Rappresentale nel piano di Gauss



$$z = a + ib$$

$$|z| = \sqrt{a^2+b^2} \overbrace{= 5}^{\text{Per stare in } E}$$

$$\Rightarrow \quad a^2 + b^2 = 25$$

⇝ E una circonferenza di centro O e raggio 5.

▷ Posto $z_1 = 4 + 4\sqrt{3}\, i$ $\quad$ Verifica che $\frac{z_1}{z_2} \in E$.

$z_2 = \frac{4\sqrt{3}}{5} - \frac{4}{5} i$

$$\frac{z_1}{z_2} = \frac{4 + 4\sqrt{3}\, i}{\frac{4\sqrt{3}}{5} - \frac{4}{5} i} = \text{Li porto in forma exp} = \frac{8 e^{i\frac{\pi}{3}}}{\frac{8}{5} e^{-i\frac{\pi}{6}}}$$

$$= 5 e^{i\left(\frac{\pi}{3} + \frac{\pi}{6}\right)} = 5 e^{i\frac{\pi}{2}}$$