Settimana: 10

Materia: Matematica
Classe: 5A
Data: 17/11/25

Pag 1637 n 319

$$f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$$

$$\sqrt{x} = x^{\frac{1}{2}}$$
$$(\sqrt{x})' = \frac{1}{2}x^{-\frac{1}{2}}$$

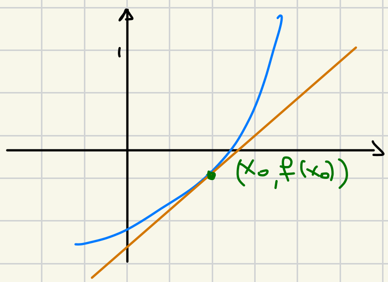
$$f'(x) = \frac{1}{\frac{\sqrt{4+x^2}}{x}} \cdot \left(\frac{\sqrt{4+x^2}}{x}\right)' =$$

$$= \frac{x}{\sqrt{4+x^2}} \cdot \frac{[(\sqrt{4+x^2})' x - \sqrt{4+x^2}]}{x^2} =$$

$$= \frac{\cancel{x}}{\sqrt{4+x^2}} \cdot \frac{1}{x^{\cancel{2}}} \cdot \left(\frac{1}{\cancel{2}}(4+x^2)^{-\frac{1}{2}} \cdot \cancel{2}x \cdot x - \sqrt{4+x^2}\right)$$

$$= \frac{\frac{x^2}{\sqrt{4+x^2}} - \sqrt{4+x^2}}{\sqrt{4+x^2} \cdot x} = \frac{\cancel{x^2} - 4 - \cancel{x^2}}{x(4+x^2)} = \frac{-4}{x(4+x^2)}$$

Proposizione: Sia $f: (a,b) \longrightarrow \mathbb{R}$ una funzione, e sia $x_0 \in (a,b)$. Supponiamo f derivabile. Allora la retta tangente al grafico di f in x_0 è:



$$y - f(x) = f'(x_0) \cdot (x - x_0)$$

$$y - y_A = m\,(x - x_A)$$

Dim: $m = f'(x_0)$; passa per $(x_0, f(x_0))$.

Esercizio: $f(x) = \ln(2x+3) \cdot \sin(x^2) \qquad x_0 = 2$

$$f(2) = \ln(7) \cdot \sin(4) \approx 0{,}13 \qquad P = (2; \ln(7)\sin(4))$$

$$f'(x) = [\ln(2x+3)]' \sin(x^2) + \ln(2x+3) \cdot [\sin(x^2)]'$$

$$= \frac{1}{2x+3} \cdot 2 \cdot \sin(x^2) + \ln(2x+3) \cdot \cos(x^2) \cdot 2x$$

$$f'(2) = \frac{2}{7} \sin(4) + \ln(7) \cos(4) \cdot 4 \approx 7{,}78$$

$$\boxed{y - 0{,}13 = 7{,}78\,(x-2)}$$

Dim dell'algebra delle derivate:

(1) $D(f+g) = Df + Dg$

$$[D(f+g)](x) = \lim_{h \to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h} =$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h} =$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} = Df(x) + Dg(x)$$

(2) $D(fg) = (Df) \cdot g + f \cdot Dg$

$$D(fg)(x) = \lim_{h \to 0} \frac{(fg)(x+h) - (fg)(x)}{h} =$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} + \frac{f(x+h)g(x) - f(x+h)g(x)}{h}$$

$$= \lim_{h \to 0} \underbrace{\frac{[f(x+h) - f(x)]}{h}}_{= f'(x)} g(x) + \underbrace{f(x+h)}_{\downarrow f(x)} \underbrace{\frac{[g(x+h) - g(x)]}{h}}_{g'(x)}$$

$$= (Df)(x) \cdot g(x) + f(x) \cdot Dg(x)$$

(3) Derivata del quoziente ⇝ Esercizio

(4) Derivata della fz composta ⇝ Facoltativa.

Derivata della funzione inversa:

Teorema: Let $f : (a;b) \longrightarrow \mathbb{R}$ bijective, then the inverse function exists, and we have (dove ha senso la scrittura)

$$\left(f^{-1}\right)'(y) = \frac{1}{f'(x)} \qquad \text{where } f(x) = y$$

Dim: Dato che $(f \circ f^{-1})(y) = y$ posso ricavare la derivata di f^{-1} usando la funzione composta

$$(f \circ f^{-1})'(y) = (y)'$$

$$f'\left(f^{-1}(y)\right) \cdot \left(f^{-1}\right)'(y) = 1$$

$$\Rightarrow \left(f^{-1}\right)'(y) = \frac{1}{f'\left(f^{-1}(y)\right)} = \frac{1}{f'(x)}$$

$\square$

Derivate delle funzioni trigonometriche inverse

$$(D \arccos)(y) = \frac{1}{D\cos(x)} \qquad \text{con } x = \arccos(y), \quad \cos x = y$$

$$= \frac{1}{-\sin(\arccos y)} = -\frac{1}{\sin(\arccos y)}$$

$$= -\frac{1}{\sqrt{1-\cos^2(\arccos y)}} = -\frac{1}{\sqrt{1-y^2}}$$

$$(D\arccos)(x) = -\frac{1}{\sqrt{1-x^2}}$$

$(D\arcsin)(x) =$ Esercizio $= \dfrac{1}{\sqrt{1-x^2}}$

$$(D\,\mathrm{arctg})(y) = \frac{1}{(D\,\mathrm{tg})(x)}$$

$\mathrm{tg}(x) = y$

$x = \mathrm{arctg}(y)$

$$(D\,\mathrm{tg})(x) = D\frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \frac{1}{\frac{1}{\cos^2(x)}} =$$

$= \cos^2(\mathrm{arctg}\, y) =$ specchietto

$$= \frac{1}{1+x^2}$$

Calcolo $\cos^2(\mathrm{arctg}(x))$. Pongo $\mathrm{tg}\,\alpha = x$

$\Rightarrow \sin\alpha = x \cdot \cos\alpha$

$$\begin{cases} \sin^2\alpha = x^2 \cdot \cos^2\alpha \\ \sin^2\alpha + \cos^2\alpha = 1 \end{cases}$$

$\rightsquigarrow \cos^2\alpha\,(x^2+1) = 1$

$\rightsquigarrow \cos^2(\mathrm{arctg}(x)) = \dfrac{1}{1+x^2}$

$$(D\,\mathrm{arctg})(x) = \frac{1}{1+x^2}$$

Pag 1646 n 567

$$(tg\,x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\overbrace{\cos x \cdot \cos x - (-\sin x)\cdot \sin x}^{1}}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$f(x) = tg\,x \cdot \ln\cos x + tg\,x - x$$

$$f'(x) = (tg\,x)' \cdot \ln\cos x + tg(x) \cdot (\ln\cos x)' + (tg\,x)' - 1$$

$$= \frac{1}{\cos^2 x}\ln(\cos x) + tg\,x \cdot \frac{1}{\cos x} \cdot (-\sin x) + \frac{1}{\cos^2 x} - 1$$

$$= \frac{\ln(\cos x) \overbrace{- \sin^2 x + 1 - \cos^2 x}^{=0}}{\cos^2 x} = \frac{\ln(\cos x)}{\cos^2 x}$$

600

$$f(x) = \ln\frac{1-\sin x}{1+\sin x}$$

$$f'(x) = \frac{1}{\frac{1-\sin x}{1+\sin x}} \frac{(-\cos x)(1+\sin x) - (1-\sin x)\cdot\cos x}{(1+\sin x)^2}$$

$$= \underbrace{\frac{\cancel{1+\sin x}}{1-\sin x} \cdot \frac{-\cos x - \sin x\,\cancel{\cos x} - \cos x + \sin x\,\cancel{\cos x}}{(1+\sin x)^{\cancel{2}}}}_{1-\sin^2 x = \cos^2 x}$$

$$= \frac{-2\cancel{\cos x}}{\cos^{\cancel{2}} x} = -\frac{2}{\cos x}$$

594: $f(x) = arctg\left(\frac{x^3-x}{1+x^4}\right) + arctg(x)$ $\qquad D(arctg)(x) = \frac{1}{1+x^2}$

$$f'(x) = \frac{1}{1+\left(\frac{x^3-x}{1+x^4}\right)^2} \cdot \frac{(3x^2-1)(1+x^4) - (x^3-x)(4x^3)}{(1+x^4)^2} + \frac{1}{1+x^2}$$

$$= \frac{\cancel{(1+x^4)^2}}{\underbrace{(1+x^4)^2+(x^3-x)^2}_{1+x^8+2\cancel{x^4}+x^6+x^2-2\cancel{x^4}}} \cdot \frac{3x^2+3x^6-1-x^4-4x^6+4x^4}{\cancel{(1+x^4)^2}} + \frac{1}{1+x^2}$$

$$= \frac{-x^6+3x^4+3x^2-1}{\underbrace{x^8+x^6+x^2+1}_{x^6(x^2+1)+(x^2+1)}} + \frac{1}{1+x^2}$$

$$= \frac{-(x^2-1)^3}{(x^6+1)(x^2+1)} + \frac{1}{1+x^2} = \frac{-\cancel{x^6}+3x^4+3x^2-\cancel{1}+\cancel{x^6}+\cancel{1}}{(x^6+1)(1+x^2)} =$$

$$= \frac{3x^2\cancel{(x^2+1)}}{(x^6+1)\cancel{(1+x^2)}} = \frac{3x^2}{x^6+1}$$

Studio di funzione

Pag 1786 n 159

$f(x) = 2x\sqrt{x+1}$

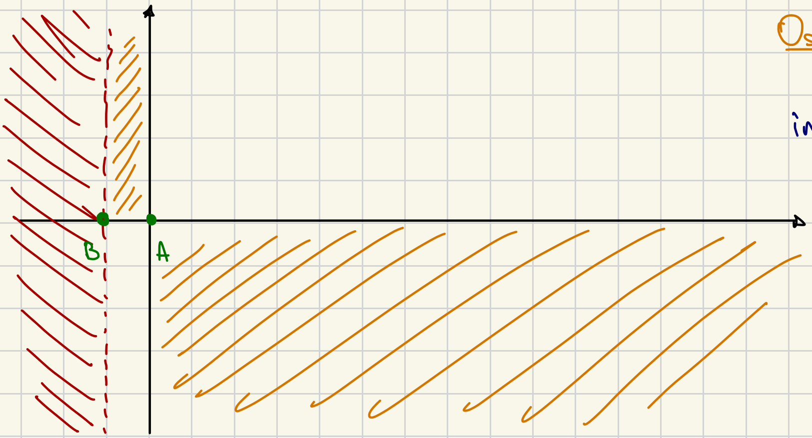
1) Dom f : $x \geq -1$ $\qquad f: \{x \geq -1\} \longrightarrow \mathbb{R}$

2) Int assi : $\begin{cases} x=0 \\ y=0 \end{cases}$ $\quad \begin{cases} y=0 \\ x=0 \\ x=-1 \end{cases}$ $\qquad A=(0,0)$ $\quad B=(-1;0)$

3) Segno : $f(x) \geq 0$ se $x \geq 0$

4) limiti : $\lim_{x \to -1} f(x) = 0$ $\qquad$ As. obl : $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} 2\sqrt{x+1} = \infty$

$\lim_{x\to+\infty} f(x) = \infty$



Oss: Il pto "più basso" lo trovo imponendo $f'(x)=0$ poiché la tangente mi deve dare il coeff. angolare di una retta orizzontale

5) $f(x) = 2x\underbrace{\sqrt{x+1}}_{(x+1)^{1/2}}$

$$f'(x) = 2\left[1\sqrt{x+1} + x\cdot\frac{1}{2}\frac{1}{\sqrt{x+1}}\right] =$$

$$= \frac{2}{\sqrt{x+1}}\,(2x+2+x) = \frac{2}{\sqrt{x+1}}\,(3x+2)$$

$$f'(x) = 0 \implies 3x+2=0 \implies \boxed{x = -\frac{2}{3}}$$