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$$f(x) = \arcsin\left|\frac{2x-1}{x}\right| + \sqrt{\arctan(2x-1)}$$

a) Dom(f) : $\begin{cases} \arctan(2x-1) \geq 0 \\ -1 \leq \left|\frac{2x-1}{x}\right| \leq 1 \end{cases}$ Sotto radice pari ≥ 0

Dominio di arcsin

L'arcotangente ha Dominio tutto $\mathbb{R}$ quindi non mi dà condizioni

(I) $\operatorname{arctg}(2x-1) \geq 0 \quad \boxed{\Longleftrightarrow} \quad 2x-1 \geq 0 \qquad x \geq \frac{1}{2}$

Dalla definizione

Dal disegno

Oss : $\operatorname{arctg}(a) \geq 0 \Longleftrightarrow a \geq 0$

(II) $-1 \leq \left|\frac{2x-1}{x}\right|$ Sempre vera poiché il valore assoluto di qualcosa è sempre ≥ 0 con $x \neq 0$

$$\boxed{\left|\frac{2x-1}{x}\right| \leq 1}$$

Caso a : $\frac{2x-1}{x} \geq 0 \qquad \begin{matrix} x \geq \frac{1}{2} \\ x > 0 \end{matrix} \qquad x < 0 \ \vee \ x \geq \frac{1}{2}$

$\frac{2x-1}{x} \leq 1 \rightsquigarrow \frac{2x-x-1}{x} \leq 0 \rightsquigarrow \frac{x-1}{x} \boxed{\leq} 0 \qquad \begin{matrix} x \geq 1 \\ x > 0 \end{matrix} \qquad 0 < x \leq 1$

Sol caso a : $\boxed{\frac{1}{2} \leq x \leq 1}$

Caso b : $\frac{2x-1}{x} \leq 0 \qquad 0 < x \leq \frac{1}{2}$

$-\frac{2x-1}{x} \leq 1 \rightsquigarrow \frac{-2x+1-x}{x} \leq 0 \qquad \frac{3x-1}{x} \geq 0$

$\begin{matrix} x \geq \frac{1}{3} \\ x > 0 \end{matrix} \qquad x < 0 \ \vee \ x \geq \frac{1}{3}$

$\underline{\text{Sol caso b}}$: $\boxed{\frac{1}{3} \le x \le \frac{1}{2}}$

$\underline{\text{Sol totale II}}$: Unione tra a e b $\leadsto$ $\boxed{\frac{1}{3} \le x \le 1}$

$$\begin{cases} x \ge \frac{1}{2} \\ \frac{1}{3} \le x \le 1 \end{cases} \leadsto \boxed{\frac{1}{2} \le x \le 1} \longleftrightarrow \left[\frac{1}{2} ; 1\right]$$

b) Calcolare $f(\frac{1}{2})$, $f(1)$

$$f(\tfrac{1}{2}) = \arcsin\left|\frac{2(\frac{1}{2})-1}{\frac{1}{2}}\right| + \sqrt{\arctan(2 \cdot \tfrac{1}{2} - 1)}$$

$$= \underbrace{\arcsin(0)}_{\text{è l'angolo tra } [-\frac{\pi}{2}, \frac{\pi}{2}] \text{ il cui seno è } 0} + \sqrt{\arctan(0)} =$$

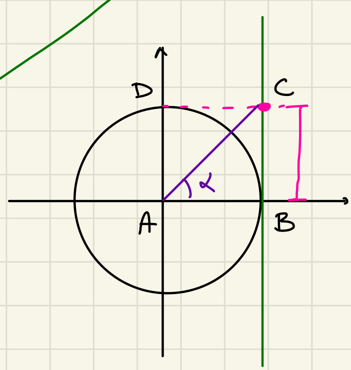
$$= 0 + 0 = 0$$

Devo risolvere $tg(\alpha) = 1$

$$f(1) = \arcsin\left|\frac{2-1}{1}\right| + \sqrt{\arctan(2-1)}$$

$\sin\alpha = 1$

$$= \underbrace{\arcsin(1)}_{\text{è l'angolo tra } [-\frac{\pi}{2}; \frac{\pi}{2}] \text{ il cui seno è } 1} + \sqrt{\underbrace{\arctan(1)}}$$



$$= \frac{\pi}{2} + \sqrt{\frac{\pi}{4}} = \frac{\pi + \sqrt{\pi}}{2}$$

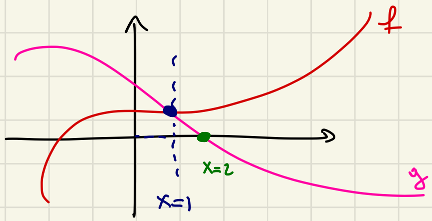
c) $g(x) = a + b \arccos\sqrt{\frac{x-1}{x}}$

▷ Graf(g) interseca Graf(f) quando $x=1$

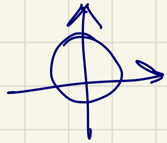
▷ Quando $x=2$, interseca asse y

▷ $g(1) = f(1)$

▷ $g(2) = 0$



$$\begin{cases} a+b\arccos(0) = \dfrac{\pi+\sqrt{\pi}}{2} \\ a+b\arccos\sqrt{\dfrac{1}{2}} = 0 \end{cases}$$



$$\cos\alpha = 0 \iff \alpha = \frac{\pi}{2}$$

$$\arccos\sqrt{\frac{1}{2}} = 45^\circ = \frac{\pi}{4}$$

$$\begin{cases} a+b\cdot\dfrac{\pi}{2} = \dfrac{\pi+\sqrt{\pi}}{2} \\ a+b\cdot\dfrac{\pi}{4} = 0 \end{cases} \rightsquigarrow \begin{cases} -\dfrac{\pi}{4}b+\dfrac{\pi}{2}b = \dfrac{\pi+\sqrt{\pi}}{2} \\ a = -\dfrac{\pi}{4}b \end{cases}$$

$$\rightsquigarrow \begin{cases} b = \dfrac{\pi+\sqrt{\pi}}{\pi}2 = 2\left(1+\dfrac{\sqrt{\pi}}{\pi}\right) \\ a = -\dfrac{\pi}{2}\left(1+\dfrac{\sqrt{\pi}}{\pi}\right) = -\dfrac{1}{2}\left(\pi+\sqrt{\pi}\right) \end{cases}$$