Settimana: 1

Materia: Matematica
Classe: 5A
Data: 15/09/25

Argomenti:

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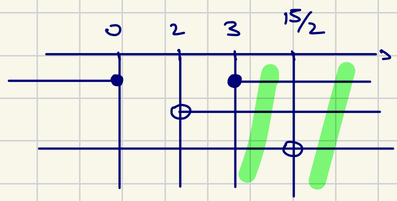
$$f(x) = \frac{\sqrt{x \ln(x-2)}}{\sqrt{8} - 2^{x-6}}$$

(1) Dominio

$$\begin{cases} \text{I)} \quad x \cdot \ln(x-2) \geq 0 \\ \text{II)} \quad x-2 > 0 \\ \text{III)} \quad \sqrt{8} - 2^{x-6} \neq 0 \end{cases}$$



I) f_1: $x \geq 0$ $\rightsquigarrow$ $x \leq 0 \ \vee \ x \geq 3$

f_2: $\ln(x-2) \geq 0 \rightsquigarrow x-2 \geq 1 \rightsquigarrow x \geq 3$

$\ln(1)$

II) $x-2 > 0 \rightsquigarrow x > 2$

III) $\sqrt{8} - 2^{x-6} \neq 0 \rightsquigarrow 2^{\frac{3}{2}} \neq 2^{x-6} \rightsquigarrow x-6 \neq \frac{3}{2} \quad x \neq \frac{15}{2}$

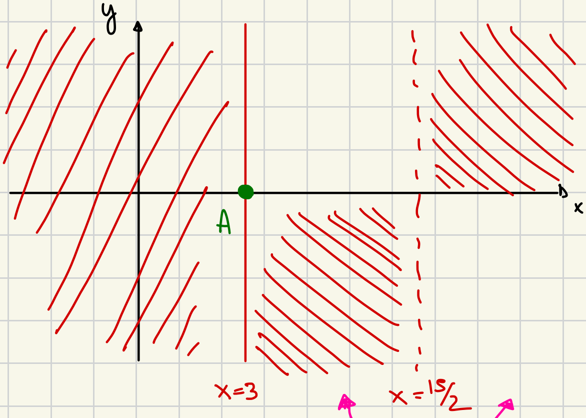
$$\text{Dom}(f) = \left\{ x \geq 3 \ , \ x \neq \frac{15}{2} \right\}$$

(2) **Asse y:** $x=0$ Impossibile

Asse x: $f(x) = y = 0 \quad \frac{\sqrt{x \ln(x-2)}}{\cancel{\sqrt{8} - 2^{x-6}}} = 0$ $\quad A = (3,0)$

$\rightsquigarrow x \ln(x-2) = 0 \rightsquigarrow x = 0$ Non accett.

$\ln(x-2) = 0 \rightsquigarrow x = 3$ Accett.



(3) Segno:

$f(x) \geq 0$

$$\frac{\sqrt{x \ln(x-2)}}{\sqrt{8} - 2^{x-6}} \geq 0$$

$N \geq 0$ $\quad \sqrt{x \ln(x-2)} \geq 0$

Sempre vero in Dom(f)

$D > 0$: $\quad 2^{\frac{3}{2}} - 2^{x-6} > 0$

$x - 6 < \frac{3}{2}$

$x < \frac{15}{2}$

$\leadsto$ Grafico segni $\leadsto$ $f(x) > 0$ se $x < \frac{15}{2}$

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$f(x) = e^{x^2}$ $\qquad g(x) = x + 2$

Composizione: Applicazione successive

Determina (1) $h(x) = (f \circ g)(x)$

(2) $h(x) \leq 1$

Applico f con argomento x+2

(1) $h(x) = (f \circ g)(x) = f\big(g(x)\big) = f(x+2)$

$$= e^{(x+2)^2} = e^{x^2+4x+4}$$

(2) $e^{x^2+4x+4} \leq 1 = e^0$ $\quad \leadsto$ inj $\leadsto$

$x^2 + 4x + 4 \leq 0 \leadsto (x+2)^2 \leq 0$

$\leadsto x = -2$ (Un $\square$ è sempre ≥ 0)

462: $f(x) = \sqrt{x} + 3$ $\qquad g(x) = \ln(x) + 1$

(1) $(f \circ g)(x)$ $\qquad (g \circ f)(x)$ e vedere che sono diverse

(2) $f \circ f^{-1}$ $\qquad (f \circ g)^{-1}$

(1) $(f \circ g)(x) = f(g(x)) = f(\ln(x)+1) = \sqrt{\ln(x)+1} + 3$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}+3) = \ln(\sqrt{x}+3) + 1$

(2) Per calcolare f^{-1} (Dove esiste) si pone $y = f(x)$
e si ricava la x

$y = \sqrt{x} + 3 \qquad \sqrt{x} = y-3 \quad \rightsquigarrow \quad x = (y-3)^2$

Dunque $f^{-1}(y) = (y-3)^2$

cond. es. corrette

$(f \circ f^{-1})(y) = f((y-3)^2) = \sqrt{(y-3)^2} + 3 = y-3+3 = y$

Calcolare $(f \circ g)^{-1}$ dove esiste

$\sqrt{\ln(x)+1} + 3 = y \quad \rightsquigarrow \quad y-3 = \sqrt{\ln(x)+1}$

$(y-3)^2 = \ln(x)+1 \quad \rightsquigarrow \quad (y-3)^2 - 1 = \ln(x)$

$\rightsquigarrow \quad e^{(y-3)^2-1} = x$

$$\boxed{(f \circ g)^{-1}(y) = e^{y^2-6y+8}}$$

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$f(x) = \sqrt{2\log_2^2 x - 7\log_2 x - 4}$

(1) Dom(f). $\begin{cases} (I) & x>0 \\ (II) & 2\log_2^2 x - 7\log_2 x - 4 \geq 0 \end{cases}$ $\begin{cases} x>0 \\ x \leq \frac{\sqrt{2}}{2} \quad \vee \quad x \geq 16 \end{cases}$

(II) $2v^2 - 7v - 4 \geq 0$ $\qquad \log_2 x = v$

$2v^2 - 8v + v - 4 \geq 0$

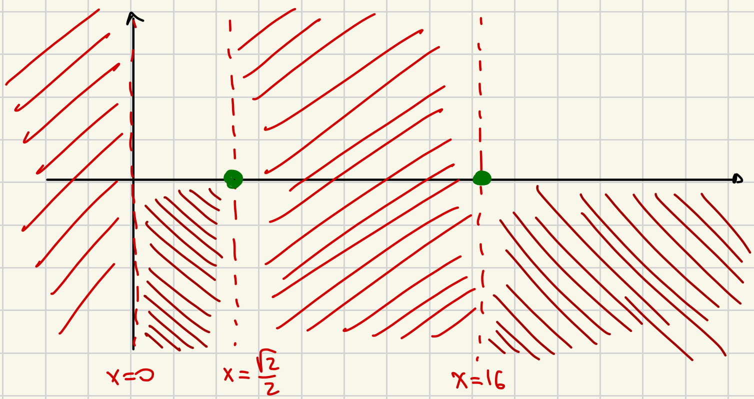
$2v(v-4) + v - 4 \geq 0$

$(2v+1)(v-4) \geq 0 \qquad v = -\frac{1}{2}, \ v = 4 \qquad v \leq -\frac{1}{2} \quad \vee \quad v \geq 4$

$\log_2 x \leq -\frac{1}{2} \rightsquigarrow \log_2 x \leq \log_2 2^{-1/2} \rightsquigarrow x \leq \frac{\sqrt{2}}{2}$

$\log_2 x \geq 4 \rightsquigarrow \qquad \rightsquigarrow x \geq 16$

$\mathrm{Dom}(f) = \{ 0 < x \leq \frac{\sqrt{2}}{2} \quad \vee \quad x \geq 16 \}$



(2) Int assi: Asse y: $x=0$ Imp. per dominio

Asse x: $y=0 \qquad 0 = \sqrt{2\log_2^2 x - 7\log_2 x - 4}$

$\rightsquigarrow 2\log_2^2 x - 7\log_2 x - 4 = 0 \rightsquigarrow$ Già fatto $\rightsquigarrow x = \frac{\sqrt{2}}{2}, \; x = 16$

$A = \left(\frac{\sqrt{2}}{2}; 0\right) \qquad B = (16, 0)$

(3) Segno: $f(x) = \sqrt{2\log_2^2 x - 7\log_2 x - 4} \geq 0 \qquad \forall x \in Dom(f)$

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$$S(x) = \frac{\sqrt{2x+3-\sqrt{x-1}}}{\ln(4-x)}$$

Mannaggia Gambo

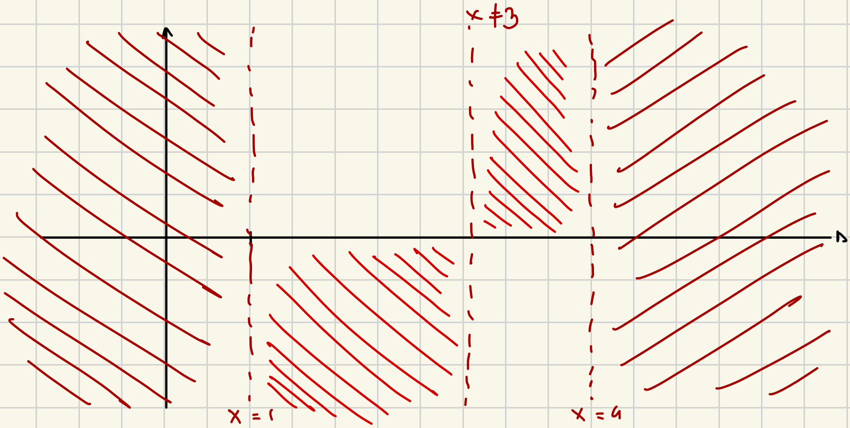
(1) Dom(S): $\begin{cases} x-1 \geq 0 \\ 4-x > 0 \\ 2x+3-\sqrt{x-1} \geq 0 \\ \ln(4-x) \neq 0 \end{cases} \quad \begin{cases} x \geq 1 \\ x < 4 \\ 2x+3 \geq \sqrt{x-1} \\ x \neq 3 \end{cases} \quad \begin{cases} x \geq 1 \\ x < 4 \\ x \geq 1 \end{cases} \; x \neq 3$

(III) $\sqrt{x-1} \leq 2x+3$

$\begin{cases} x-1 \geq 0 \\ 2x+3 \geq 0 \\ x-1 \leq (2x+3)^2 \end{cases} \quad \begin{cases} x \geq 1 \\ x \geq -\frac{3}{2} \\ x-1 \leq 4x^2+9+12x \end{cases} \quad (c) \; \boxed{\begin{cases} x \geq 1 \\ x \geq -\frac{3}{2} \\ \forall x \in \mathbb{R} \end{cases}}$

(c) $4x^2+11x+10 \geq 0 \qquad \Delta = 121-160 < 0$

$Dom(S) = \{\, 1 \leq x < 4, \; x \neq 3 \,\}$



(2) Int Assi : Asse y: $x=0$ Imp per dominio

Asse x $y=0$. $\sqrt{2x+3-\sqrt{x-1}}=0$ ~> elevo al □

$$2x+3-\sqrt{x-1}=0$$

... Per i conti fatti prima è impossibile

↳ Spoiler di argomenti futuri.

(3) Segno: PER CONTINUITÀ: Se $S(2)>0$, allora tutto il grafico sarà >0 nella zona $x\in[1,3)$

In calcolatrice $S(2)\approx 3,5>0 \Rightarrow S(x)>0$ $\forall x\in[1,3)$

E dato che $S(3,5)<0 \Rightarrow S(x)<0 \quad \forall x\in(3,4)$