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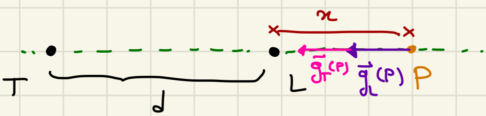

$d = TL = 3{,}633 \cdot 10^8\,m$

P da determinare in modo che il campo $g(P)$ sia il doppio di quello generato solo dalla Terra.

La condizione del problema $\quad g_T(P) + g_L(P) = 2g_T(P)$

Calcolo i campi gravitazionali:

(1) Si mette una massa di prova in P.

(2) Si calcola la forza $\vec F$ fatta su tale massa di prova

(3) $\vec g = \dfrac{\vec F}{m}$

$x > 0$

Terra: $\quad F_T = G\dfrac{M_T m}{(d+x)^2} \qquad\qquad g_T(P) = G\dfrac{M_T}{(d+x)^2}$

Luna: $\quad F_L = G\dfrac{M_L m}{x^2} \qquad\qquad g_L(P) = G\dfrac{M_L}{x^2}$

Impongo la condizione

$$g_T(P) + g_L(P) = 2g_T(P) \quad \rightsquigarrow$$

$$g_L(P) = g_T(P)$$

$$\cancel{G}\frac{M_L}{x^2} = \cancel{G}\frac{M_T}{(d+x)^2} \quad \rightsquigarrow \quad (d+x)^2 M_L = x^2 M_T$$

$$d^2 M_L + 2dx M_L + x^2 M_L = x^2 M_T$$

$$x^2(M_T - M_L) - 2x\,d M_L - d^2 M_L = 0$$

$$\Delta = (2d M_L)^2 - 4(M_T - M_L)(-d^2 M_L) =$$
$$= \cancel{4d^2 M_L^2} + 4d^2 M_T M_L - \cancel{4d^2 M_L^2} \qquad \sqrt{\Delta} = 2d\sqrt{M_T M_L}$$

$$x = \frac{2dM_L \mp 2d\sqrt{M_T M_L}}{2(M_T - M_L)} = d\,\frac{M_L \mp \sqrt{M_T M_L}}{M_T - M_L}$$

$\leadsto$ si sceglie il + trovando

$$x = d\,\frac{M_L + \sqrt{M_T M_L}}{M_T - M_L} = d\,\frac{\sqrt{M_L}\,(\cancel{\sqrt{M_L} + \sqrt{M_T}})}{(\sqrt{M_T} - \sqrt{M_L})(\cancel{\sqrt{M_T} + \sqrt{M_L}})} = d\,\frac{\sqrt{M_L}}{\sqrt{M_T} - \sqrt{M_L}}$$

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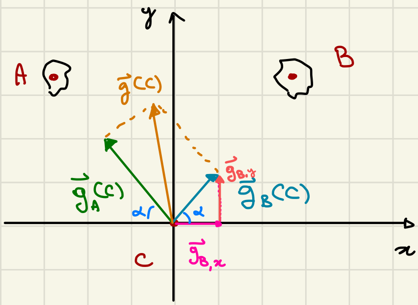

$AB = BC = CA = \ell = 25{,}3$ km

$m_A = 6{,}87 \cdot 10^{10}$ kg

$m_B = 3{,}26 \cdot 10^{10}$ kg

$\vec{g}(C) = ?$

$$g_A(C) = G\,\frac{m_A}{AC^2} = G\,\frac{m_A}{\ell^2}$$

$$g_B(C) = G\,\frac{m_B}{\ell^2}$$

Per come è messo il sdr, $\alpha = \frac{180° - 60°}{2} = 60°$

$g_{B,x} = g_B \cos\alpha$, $g_{B,y} = g_B \cdot \sin\alpha$

$g_{A,x} = g_A \cdot \cos\overset{(120-\alpha)}{\alpha}$, $g_{A,y} = g_A \cdot \sin\overset{(120-\alpha)}{\alpha}$

$\vec{g}_B = (g_{B,x}\,;\, g_{B,y})$ $\qquad \vec{g}_A = (-g_{A,x}\,,\, g_{A,y})$

$$\vec{g} = \vec{g}_A + \vec{g}_B = (g_{B,x} - g_{A,x}\,;\, g_{B,y} + g_{A,y})$$

$$= \left(\frac{Gm_B}{\ell^2}\cos\alpha - \frac{Gm_A}{\ell^2}\cos\overset{(120-\alpha)}{\alpha}\,;\, \frac{G}{\ell^2}m_B \cdot \sin\alpha + \frac{G}{\ell^2}m_A \cdot \sin\overset{(120-\alpha)}{\alpha}\right)$$

$$= \frac{G}{\ell^2} \left(\cos\alpha \, (m_B - m_A) \; ; \; \sin\alpha \, (m_B + m_A) \right)$$

$$\approx \left(-1{,}88 \cdot 10^{-9} \; ; \; 9{,}14 \cdot 10^{-9} \right) \frac{N}{kg}$$

Per il modulo di $\vec{g}$ uso il teorema di Pitagora (GF)

$$g^2 = \frac{G^2}{\ell^4} \left[\cos^2\alpha \, (m_B - m_A)^2 + \sin^2\alpha \, (m_B + m_A)^2 \right]$$

$$= \frac{G^2}{\ell^4} \left[\begin{array}{l} \cos^2\alpha \cdot m_B^2 + \cos^2\alpha \cdot m_A^2 - 2 m_A m_B \cos^2\alpha + \\ \sin^2\alpha \; m_B^2 + \sin^2\alpha \; m_A^2 + 2 m_A m_B \sin^2\alpha \end{array} \right]$$

$\sin^2\alpha + \cos^2\alpha = 1$

$$= \frac{G^2}{\ell^4} \left[m_B^2 + m_A^2 + 2 m_A m_B \left(\sin^2\alpha - \cos^2\alpha \right) \right]$$