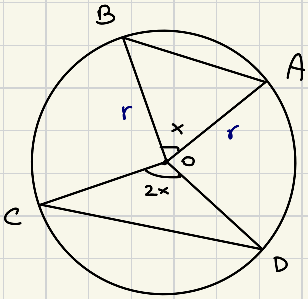
Pag 961 n 348



$r = 4$

$A\hat{O}B = x$

$AB < CD$

▷ Per quale x vale che $CD = \sqrt{3}\,AB$?

▷ $f(x) = \dfrac{AB + CD}{8}$ e il suo periodo.

$$AB^2 = AO^2 + OB^2 - 2AO \cdot OB \cos x$$
$$= r^2 + r^2 - 2r^2 \cos x$$
$$= 2r^2(1 - \cos x)$$

$$CD^2 = \ldots\ldots = 2r^2(1 - \cos 2x)$$

Dato che $AB, CD > 0$ elevo al ^{2} $CD = \sqrt{3}\,AB$

$$CD^2 = 3AB^2 \qquad \cancel{2r^2}(1 - \cos 2x) = 3 \cdot \cancel{2r^2}(1 - \cos x)$$

$$1 - \cos 2x = 3 - 3\cos x$$

↓ duplicazione

$$1 - (2\cos^2 x - 1) = 3 - 3\cos x$$

$-2\cos x - \cos x$

$$2\cos^2 x - 3\cos x + 1 = 0 \quad \rightsquigarrow \quad 2t^2 - 3t + 1 = 0 \qquad \Delta = 1 \quad \sqrt{\Delta} = 1$$
$$2\cos x(\cos x - 1) - (\cos x - 1) = 0$$
$$(2\cos x - 1)(\cos x - 1) = 0$$

$$\left.\begin{array}{lll} \cos x = \frac{1}{2} & \rightsquigarrow & x = \frac{\pi}{3} \\ \cos x = 1 & \rightsquigarrow & x = 0 \end{array}\right] \rightsquigarrow \text{ solo } \boxed{x = \tfrac{\pi}{3} \text{ accettabile}}$$

▷ $f(x) = \dfrac{AB + CD}{8}$

$$AB^2 = 2r^2(1-\cos x)$$

$2\cos^2 x - 1 = \cos 2x$

Uso la formula con $\frac{x}{2}$

$2\cos^2 \frac{x}{2} - 1 = \cos x$

$$AB^2 = 2r^2\left(1-\left(2\cos^2\frac{x}{2}-1\right)\right)$$

$$AB^2 = 4r^2\left(1-\cos^2\frac{x}{2}\right) = 4r^2\sin^2\frac{x}{2} \rightsquigarrow \boxed{AB = 2r\sin\frac{x}{2}}$$

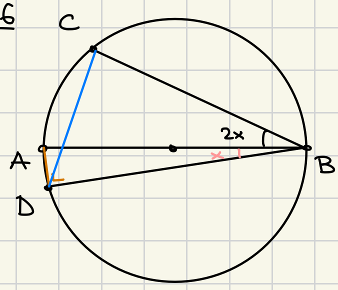
$$CD = \dots\dots = 2r\sin x$$

$$f(x) = \frac{AB+CD}{8} = \frac{\cancel{2r}\left(\sin\frac{x}{2}+\sin x\right)}{\cancel{8}} = \sin\frac{x}{2}+\sin x$$

$$= \sin\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$= \sin\frac{x}{2}\left(1+2\cos\frac{x}{2}\right) \rightsquigarrow \text{Periodo } 4\pi$$

346



$OA = r$

$C\hat{B}A = 2A\hat{B}D$

$A\hat{B}D = x$

- $f(x) = \frac{CD}{AD}$
- Rappresenta $f(x)$

$AD = 2r\sin x$

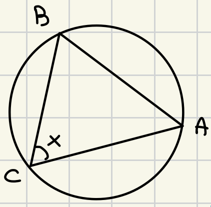
$CD = 2r\sin 3x$

$$f(x) = \frac{CD}{AB} = \frac{\sin 3x}{\sin x} = \frac{\sin(2x+x)}{\sin x}$$

$$= \frac{\sin(2x)\cos x + \sin x\cos(2x)}{\sin x}$$

$$= \frac{2\cancel{\sin x}\cos^2 x + \cancel{\sin x}(\cos^2 x - \sin^2 x)}{\cancel{\sin x}} = 4\cos^2 x - 1$$

Teorema di Mirco (della corda): Dato ABC inscritto e circonferenza di raggio r. Vale che



$$\boxed{AB = 2r\sin x}$$

Dim: Esercizio