The goal of this derivation is to determine if it is better to have the larger reduction closer or farther from the motor.

The moment of inertia relation in a reduction. A derivation will eventually (hopefully) be posted on torque learn.

$$I_{in} = \left(\frac{T_1}{T_2}\right)^2 (I_{out} + I_{G2}) + I_{G1}$$

Applying this twice yields:

$$I_{in} = \left(\frac{T_1}{T_2}\right)^2 \left(\left(\frac{T_3}{T_4}\right)^2 (I_{out} + I_{G4}) + I_{G3} + I_{G2}\right) + I_{G1}$$

$$I_{in} = I_{out}\left(\frac{T_1}{T_2}\right)^2 \left(\frac{T_3}{T_4}\right)^2 + I_{G4}\left(\frac{T_1}{T_2}\right)^2 \left(\frac{T_3}{T_4}\right)^2 + I_{G3}\left(\frac{T_1}{T_2}\right)^2 + I_{G2}\left(\frac{T_1}{T_2}\right)^2 + I_{G1}$$

The moment of inertia is approximated as:

$$I_{gear} \lesssim \left(\frac{T}{28.7[dp]}\right)^4$$

$$I_{gear} \lesssim \left(\frac{T}{574}\right)^4$$

$$I_{gear} \lesssim \frac{T^4}{1.086 * 10^{11}}$$

Let $k = 1.086 * 10^{11}$

$$I_{in} = I_{out}\left(\frac{T_1}{T_2}\right)^2 \left(\frac{T_3}{T_4}\right)^2 + \frac{T_4^4}{k}\left(\frac{T_1}{T_2}\right)^2 \left(\frac{T_3}{T_4}\right)^2 + \frac{T_3^4}{k}\left(\frac{T_1}{T_2}\right)^2 + \frac{T_2^4}{k}\left(\frac{T_1}{T_2}\right)^2 + \frac{T_1^4}{k}$$

$$I_{in} = I_{out}\frac{T_1^2}{T_2^2}\frac{T_3^2}{T_4^2} + \frac{T_4^4}{k}\frac{T_1^2}{T_2^2}\frac{T_3^2}{T_4^2} + \frac{T_3^4}{k}\frac{T_1^2}{T_2^2} + \frac{T_2^4}{k}\frac{T_1^2}{T_2^2} + \frac{T_1^4}{k}$$

$$I_{in} = I_{out}\frac{T_1^2}{T_2^2}\frac{T_3^2}{T_4^2} + \frac{T_4^2}{k}\frac{T_1^2}{T_2^2}\frac{T_3^2}{1} + \frac{T_3^4}{k}\frac{T_1^2}{T_2^2} + \frac{T_2^2}{k}\frac{T_1^2}{1} + \frac{T_1^4}{k}$$

$$I_{in} = I_{out}\frac{A^2}{B^2}\frac{C^2}{D^2} + \frac{D^2}{k}\frac{A^2}{B^2}\frac{C^2}{1} + \frac{C^4}{k}\frac{A^2}{B^2} + \frac{B^2}{k}\frac{A^2}{1} + \frac{A^4}{k}$$

Let:

$$\frac{B}{A} = t_1$$

$$B = At_1$$

$$\frac{D}{C} = t_2$$

$$D = Ct_2$$

$$I_{in_1} = I_{out}\frac{A^2}{A^2t_1^2}\frac{C^2}{D^2} + \frac{D^2}{k}\frac{A^2}{A^2t_1^2}\frac{C^2}{1} + \frac{C^4}{k}\frac{A^2}{A^2t_1^2} + \frac{A^2t_1^2}{k}\frac{A^2}{1} + \frac{A^4}{k}$$

$$I_{in_1} = I_{out}\frac{C^2}{t_1^2D^2} + \frac{D^2}{k}\frac{C^2}{t_1^2} + \frac{C^4}{kt_1^2} + \frac{A^4t_1^2}{k} + \frac{A^4}{k}$$

$$I_{in_2} = I_{out}\frac{A^2}{B^2}\frac{C^2}{C^2 t_2^2} + \frac{C^2 t_2^2}{k}\frac{A^2}{B^2}\frac{C^2}{1} + \frac{C^4}{k}\frac{A^2}{B^2} + \frac{B^2}{k}\frac{A^2}{1} + \frac{A^4}{k}$$

$$I_{in_2} = I_{out}\frac{A^2}{t_2^2 B^2} + \frac{C^4 t_2^2}{k}\frac{A^2}{B^2} + \frac{C^4}{k}\frac{A^2}{B^2} + \frac{B^2}{k}\frac{A^2}{1} + \frac{A^4}{k}$$

Compare growth rates:

$$\lim_{t_1,t_2\to\infty}\frac{I_{in_1}}{I_{in_2}} = \frac{I_{out}\frac{C^2}{t_1^2 D^2} + \frac{D^2}{k}\frac{C^2}{t_1^2} + \frac{C^4}{k t_1^2} + \frac{A^4 t_1^2}{k} + \frac{A^4}{k}}{I_{out}\frac{A^2}{t_2^2 B^2} + \frac{C^4 t_2^2}{k}\frac{A^2}{B^2} + \frac{C^4}{k}\frac{A^2}{B^2} + \frac{B^2}{k}\frac{A^2}{1} + \frac{A^4}{k}}$$

$$\lim_{t_1,t_2\to\infty}\frac{I_{in_1}}{I_{in_2}} = \frac{0 + 0 + 0 + \infty + \frac{A^4}{k}}{0 + \infty + \frac{C^4}{k}\frac{A^2}{B^2} + \frac{B^2}{k}\frac{A^2}{1} + \frac{A^4}{k}} = \frac{\infty}{\infty}$$

L Hospitals Rule:

$$I_{in_1}' = \frac{d}{dt_1}\left[I_{out}\frac{C^2}{t_1^2 D^2} + \frac{D^2}{k}\frac{C^2}{t_1^2} + \frac{C^4}{k t_1^2} + \frac{A^4 t_1^2}{k} + \frac{A^4}{k}\right]$$

Long story short: I can see that I'll need to apply L Hospitals again, so I'll just take the derivative again to save time.

$$I_{in_1}'' = \frac{d}{dt_1}\left[I_{out}\frac{-2C^2}{t_1^3 D^2} + \frac{-2D^2}{k}\frac{C^2}{t_1^3} + \frac{-2C^4}{k t_1^3} + \frac{2A^4 t_1}{k}\right]$$

$$I_{in_1}'' = I_{out}\frac{6C^2}{t_1^4 D^2} + \frac{6D^2}{k}\frac{C^2}{t_1^4} + \frac{6C^4}{k t_1^4} + \frac{2A^4}{k}$$

$$I_{in_2}' = \frac{d}{dt_2}\left[I_{out}\frac{-2A^2}{t_2^3 B^2} + \frac{2C^4 t_2}{k}\frac{A^2}{B^2}\right]$$

$$I_{in_2}'' = I_{out}\frac{6A^2}{t_2^4 B^2} + \frac{2C^4}{k}\frac{A^2}{B^2}$$

$$\lim_{t_1,t_2\to\infty}\frac{I_{in_1}''}{I_{in_2}''} = \frac{I_{out}\frac{6C^2}{t_1^4 D^2} + \frac{6D^2}{k}\frac{C^2}{t_1^4} + \frac{6C^4}{k t_1^4} + \frac{2A^4}{k}}{I_{out}\frac{6A^2}{t_2^4 B^2} + \frac{2C^4}{k}\frac{A^2}{B^2}}$$

$$\lim_{t_1,t_2\to\infty}\frac{I_{in_1}''}{I_{in_2}''} = \frac{0 + 0 + 0 + \frac{2A^4}{k}}{0 + \frac{2C^4}{k}\frac{A^2}{B^2}} = \frac{\frac{2A^4}{k}}{\frac{2A^2C^4}{kB^2}}$$

$$\lim_{t_1,t_2\to\infty}\frac{I_{in_1}''}{I_{in_2}''} = \frac{2A^4}{k} * \frac{kB^2}{2A^2C^2} = \frac{A^2B^2}{C^2}$$

$$\frac{A^2B^2}{C^2} > 1$$

Center distance constraints:

$$dA = -dB$$
$$dC = -dD$$

$$\lim_{t_1,t_2\to\infty}\frac{I''_{in_1}}{I''_{in_2}}=\frac{(A-t)^2(B+t)^2}{(C-t)^2}$$
$$\lim_{t_1,t_2\to\infty}\frac{I'''_{in_1}}{I'''_{in_2}}=\frac{2(A-t)(B+t)^2+2(B+t)(A-t)^2}{2(C-t)}$$
$$\lim_{t_1,t_2\to\infty}\frac{I^{(4)}_{in_1}}{I^{(4)}_{in_2}}=\frac{D[2(A-t)(B+t)^2+2(B+t)(A-t)^2]}{-2}$$

$$I^{(4)}_{in_1}=D[(2A-2t)(B+t)^2+(2B+2t)(A-t)^2]$$
$$I^{(4)}_{in_1}=D[(2A-2t)(B^2+2Bt+t^2)+(2B+2t)(A^2-2At+t^2)]$$
$$I^{(4)}_{in_1}=-2(B^2+2Bt+t^2)+(2B+2t)(2A-2t)+2(A^2-2At+t^2)$$
$$+(-2A+2t)(2B+2t)$$
$$I^{(4)}_{in_1}=-2B^2-4Bt-2t^2+4AB-4Bt+4At-4t^2+2A^2-4At+2t^2-4AB-4At+4Bt$$
$$+4t^2$$
$$I^{(4)}_{in_1}=-2B^2-4Bt-4At+2A^2$$
$$\lim_{t_1,t_2\to\infty}\frac{I^{(4)}_{in_1}}{I^{(4)}_{in_2}}=\frac{-2B^2-4Bt-4At+2A^2}{-2}=\infty$$

The larger reduction should be placed at the end of the gear box ie. Farthest away from the motor.