Polar:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$

$$r^2 = x^2 + y^2$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r\cos\theta + r'\sin\theta}{r'\cos\theta - r\sin\theta}$$

If continuous and non-negative on $[\theta_1, \theta_2]$:

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta$$

$$\frac{\pi}{6} = 0.524$$

$$\frac{\pi}{4} = 0.785$$

$$\frac{\pi}{3} = 1.047$$

$$\frac{\pi}{2} = 1.571$$

$$\pi = 3.142$$

$$s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Use sin for revolutions about θ=0 and cos for $\theta = \frac{\pi}{2}$:

$$S.A. = 2\pi \int_{\theta_1}^{\theta_2} r\sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Taylor Polynomials:

Nth Taylor polynomial for f at c:

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

Remainder:

$$R_n(x) = f(x) - P_n(x)$$

Lagrange Error Bound:

$$|R_n(x)| = |f(x) - P_n(x)|$$

Taylor's Theorem:

If differentiable through n+1 on an interval I containing c, then for each x in I, there exists a z between x and c such that f equals the Taylor polynomial plus the remainder.

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$$

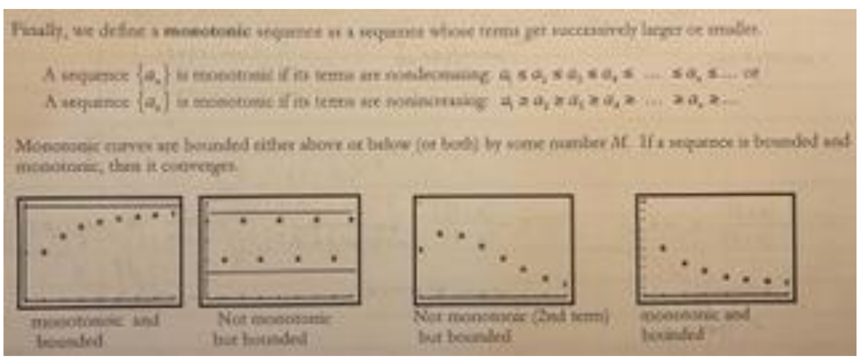
Z should not need to be found instead find the maximum value of the (n+1)th derivative. Lagrange error is the maximum error between the exact value and the approximation.

Sequences:

$$\lim_{x\to\infty} f(x) = L, f(n) = \{a_n\} \therefore \lim_{n\to\infty} a_n = L$$

Squeeze theorem:

$$\lim_{x\to\infty} a_n = L, \lim_{x\to\infty} b_n = L, a_n \le c_n \le b_n \therefore \lim_{x\to\infty} c_n = L$$


Finally, we define a **monotonic** sequence as a sequence whose terms get successively larger or smaller.

A sequence $\{a_n\}$ is monotonic if its terms are nondecreasing: $a_1 \le a_2 \le a_3 \le a_4 \le \ldots \le a_n \le \ldots$ or

A sequence $\{a_n\}$ is monotonic if its terms are nonincreasing: $a_1 \ge a_2 \ge a_3 \ge a_4 \ge \ldots \ge a_n \ge \ldots$

Monotonic curves are bounded either above or below (or both) by some number M. If a sequence is bounded and monotonic, then it converges.

Maclaurin Series:

Summary of Common Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots \qquad R = \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots \qquad R = \infty$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n x^n \left(\frac{1}{n}\right) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + \frac{kx}{1!} + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$

Series:

Test	Series Form	Converges	Diverges	Comment
nth term	$\sum_{n=1}^{\infty} a_n$	Cannot be used to show convergence	$\lim_{n\to\infty} a_n \neq 0$	
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^n a_n$	$0 < a_{n+1} < a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_n \rvert \leq a_{n+1}$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$ (exists)		Sum: $S = b_1 - L$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	$0 < R_n < \int_n^{\infty} f(x)\,dx$
Limit Comparison $(a_n, b_n) > 0$	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ (exists) and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ (exists) and $\sum_{n=1}^{\infty} b_n$ diverges	
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} > 1$	Inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = 1$
Direct Comparison $(a_n, b_n) > 0$	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	

Power series:

Power series = normal series combined with a variable

Assume: $0^0 = 1$

Use the ratio test to find interval of convergence. Check endpoints.

Geometric Power Series centered at c:

$$f(x) = \frac{a}{x-b}$$

$$f(x) = -\frac{a}{b-c}\sum_{n=0}^{\infty}\left(\frac{x-c}{b-c}\right)^n$$

R=|b-c|

Function	Interval of Convergence
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots + x^n + \ldots$	$-1 < x < 1$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots$	$-\infty < x < \infty$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots$	$-\infty < x < \infty$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$	$-\infty < x < \infty$
$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 + \ldots + (-1)^n (x-1)^n + \ldots$	$0 < x < 2$
$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 + \ldots + (-1)^n x^n + \ldots$	$-1 < x < 1$
$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \ldots + \frac{(-1)^{n-1}(x-1)^n}{n} + \ldots$	$0 < x \le 2$
$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + \frac{(-1)^n x^{2n+1}}{2n+1} + \ldots$	$-1 \le x \le 1$

Vectors:

Vectors are parallel if scalars apart

$$v \cdot v = \|v\|^2$$

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

If dot product = 0, then vectors are orthogonal.

Direction cosines:

$$\cos\alpha = \frac{v_1}{\|v\|}, \cos\beta = \frac{v_2}{\|v\|}, \cos\gamma = \frac{v_3}{\|v\|}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$u = w_1 + w_2$$

$$w_1 = \mathrm{proj}_v u = \left(\frac{u \cdot v}{\|v\|}\right)\frac{v}{\|v\|} \ = \text{projection of u onto v}$$

$$w_2 = u - w_1 = \text{component of u ortho to v}$$

Cross products:

$$\|u \times v\| = \|u\|\|v\| \sin\theta$$

u x v = 0 if u and v are parallel

$\|u \times v\|$ is the area of a parallelogram with u and v sides

Triple scalar product: Volume of a parallelepiped: $V = |u \cdot (v \times w)|$

Lines in space:

Parametric:

$$x = x_1 + at, y = y_1 + bt, z = z_1 + ct$$

Symmetric:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Planes in space:

$$n\langle a, b, c\rangle$$

Standard form:

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

Angle between planes = angle between normal vectors:

$$\cos\theta = \frac{|n_1 \cdot n_2|}{\|n_1\|\|n_2\|}$$

Find line of intersection by eliminating z, setting x to t and solving for y and z

Planes are parallel to missing variables.

Shortest distance between point and plane:

$$D = \frac{|\overrightarrow{PQ} \cdot n|}{\|n\|}, \text{Q is given point, P is any point on plane}$$

Distance between point and a line:

$$D = \frac{\|\overrightarrow{PQ} \times u\|}{\|u\|}, \text{Q is point, P is point on line}$$

Quadric Surfaces

Ellipsoid

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1}$$

Ellipse Trace - Parallel to xy – plane

Ellipse Trace - Parallel to xz – plane

Ellipse Trace - Parallel to yz – plane

If $a=b=c\neq 0$, the surface is a sphere

Hyperboloid of One Sheet

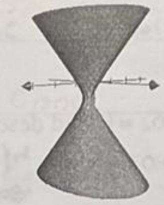
$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1}$$

Ellipse Trace - Parallel to xy – plane

Hyperbola Trace - Parallel to xz – plane

Hyperbola Trace - Parallel to yz – plane

Axis - variable whose coefficient <0



Hyperboloid of Two Sheets

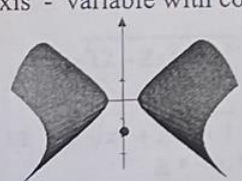
$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1}$$

Ellipse - Parallel to xy – plane

Hyperbola Trace - Parallel to xz – plane

Hyperbola Trace - Parallel to yz – plane

Axis - variable with coefficient >0



Elliptic Cone

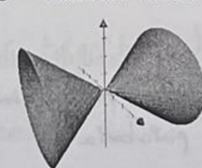
$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0}$$

Ellipse Trace - Parallel to xy – plane

Hyperbola Trace - Parallel to xz – plane

Hyperbola Trace - Parallel to yz – plane

Axis - variable whose coefficient <0



Elliptic Paraboloid

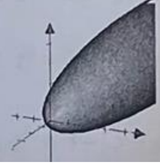
$$\boxed{z=\frac{x^2}{a^2}+\frac{y^2}{b^2}}$$

Ellipse Trace - Parallel to xy – plane

Parabola Trace - Parallel to xz – plane

Parabola Trace - Parallel to yz – plane

Axis - variable whose power is one



Hyperbolic Paraboloid

$$\boxed{z=\frac{y^2}{b^2}-\frac{x^2}{a^2}}$$

Hyperbola Trace - Parallel to xy – plane

Parabola Trace - Parallel to xz – plane

Parabola Trace - Parallel to yz – plane

Axis - variable whose power is one

Coordinates:

$$\rho=\sqrt{x^2+y^2+z^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\phi = \cos^{-1}\frac{z}{\rho}$$

$$x = \rho\sin\phi\cos\theta$$

$$y = \rho\sin\phi\sin\theta$$

$$z = \rho\cos\phi$$

Vector Valued Functions

$$T(t) = \frac{r'(t)}{\|r'(t)\|}$$

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

$$s = \int_a^b \|r'(t)\|\,dt$$

$$K = \|T'(s)\| = \text{big headache}$$

$$K = \frac{\|T'(t)\|}{\|r'(t)\|} = \frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|^3}$$

$$K_{circ} = \frac{1}{r}, K_{line} = 0$$

$$K = \frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}$$

Polars and Trig *Double angle, log trig, power reducing* *Derivaative, Integral, Arc Length*	*Series Convergence*
Power Series *Taylor's theorem*	*Vectors and Planes and Coordinate Systems* *Dot/cross app formulas* *Equation of line, plane, and spherical coords*
Surfaces *and* *Types of quartic surfaces*	*Vector Valued Functions* *Tanget, normal, curvature*