The *(virtually)* COMPLETE Guide to Spiral Wrapping a Pipe

Goal

To learn how to wrap a pipe in a spiral.

Knowns

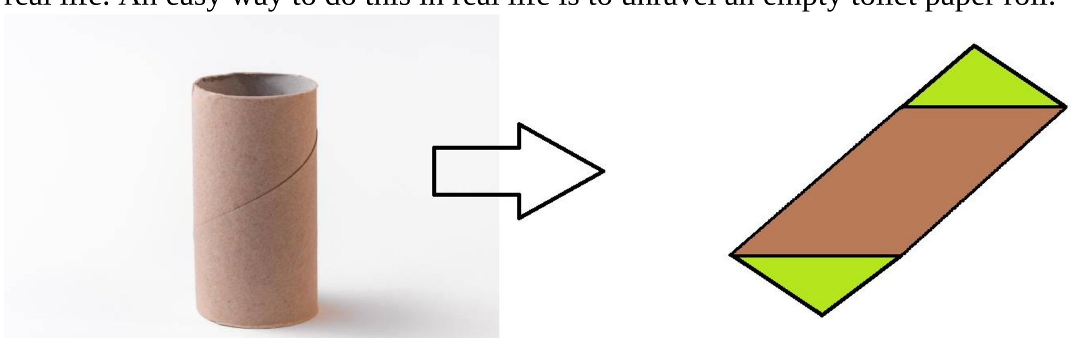
- The length of the section of pipe we want wrapped
- The width of the ribbon/tape
- The radius of the pipe to be wrapped

Wants

- How to cut the tape to perfectly wrap the pipe

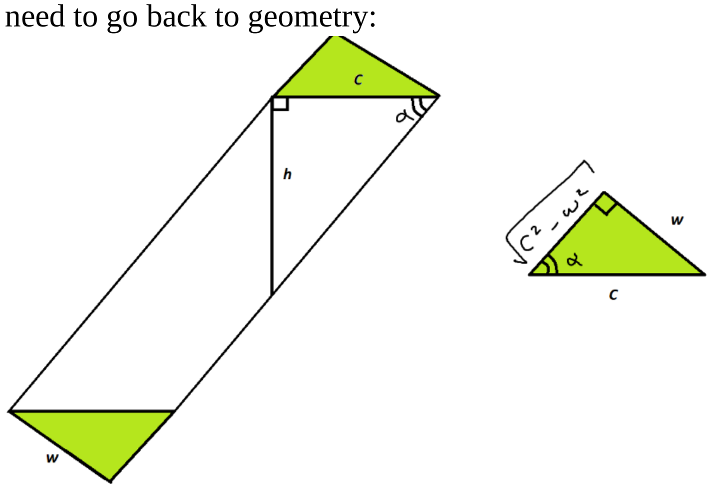
Thought Process and Observations

1. When a ribbon is wrapped in a spiral around a pipe, the ends of the ribbon stick off. These ends must be cut to form a perfect wrap. The shape that the tape must be cut into can be determined by drawing it out and thinking about it or by unwrapping a cylinder in real life. An easy way to do this in real life is to unravel an empty toilet paper roll.



2. The unraveled toilet paper roll forms a parallelogram. So, the green sections of the ribbon must be removed to form a spiral. To determine how to cut the tape, the dimensions of the parallelogram need to be found from our knowns.

3. By observation, we can see that the top and bottom of the parallelogram have the same length as the circumference of the cylinder it forms.
4. To determine the slant height of the parallelogram is a bit trickier that uses some “advanced” math concepts. “Advanced” is in quotes as the math simplifies nicely so that I hope that readers will be able to take away some main ideas from this process.
5. The general idea to find the slant height, is to determine the arc length of an equation of a helix that fits our constraints. It sounds like a lot but let’s break it down.
6. Vector Valued Functions:
    a. Our helix equation will be determined in this form so it’s important to generally understand what they are.
    b. Most equations taught in math before pre-calc are scalar functions. This means that functions take one or more inputs and give a **single output**. Vector valued functions, in contrast, take one or more inputs and give a **vector output** and vectors can represent multiple numbers. This is helpful to describe our helix which exists in **Three** dimensions.
    c. Parametric equations could be considered a class of vector valued functions.
    d. For our helix, the parameterized variable (aka. Input variable) will the angle from the positive x axis, θ.
    e. For the parametric equation of a circle or a helix, the x and y components are identical:
        i. $x = r\cos\theta$
        ii. $y = r\sin\theta$
    f. Unlike a circle, our helix extends into the z axis at a constant rate:
        i. $z = k\theta$, where *k* governs how stretched or compressed the helix is
7. So far, we have an equation for our helix as *r* is known, and θ is a variable, yet *k* is unknown.
8. To find *k*, we need to go back to geometry:



    a. Above shows a diagram of our unravels wrap with labeled knowns.
    b. *h* refers to the height of one wrap, *w* the width of the ribbon, and *C* the circumference of the pipe.

   c. From the z component of our helix equation, $h = 2\pi k$ since one wrap is 2π radians.
   d. Some Trig Stuff:
      i. $\tan\alpha = \frac{h}{C}$
      ii. $h = C\tan\alpha$
      iii. $\sin\alpha = \frac{w}{C}$
      iv. $\tan\alpha = \frac{w}{\sqrt{C^2-w^2}}$
      v. $h = \frac{Cw}{\sqrt{C^2-w^2}}$
   e. $k = \frac{Cw}{2\pi\sqrt{C^2-w^2}}$
9. Now that we have *k*, our helix is fully defined, and we just need to find its arc length.
   a. Arc length can be thought of the length of an unraveled string. The arc length of a circle is its circumference; of a rectangle, perimeter; of a helix, the length of the helix pulled into a straight line.
   b. In multivariable calculus, the formula for arc length is the integral of the magnitude of the derivative of a vector valued function. This can be a lot BUT thankfully it's simple here.
10. The derivative of a vector valued function is the derivative of each individual component:
    a. $\frac{dx}{d\theta} = -r\sin\theta$
    b. $\frac{dy}{d\theta} = r\cos\theta$
    c. $\frac{dz}{d\theta} = k$
11. The magnitude of a vector is the square root of the sum of squares of each component (much like the Pythagorean theorem):
12. $\|f'\| = \sqrt{(-r\sin\theta)^2 + (r\cos\theta)^2 + (k)^2}$
    a. $\|f'\| = \sqrt{r^2\sin^2\theta + r^2\cos^2\theta + k^2}$
    b. $\|f'\| = \sqrt{r^2(\sin^2\theta + \cos^2\theta) + k^2}$
    c. $\|f'\| = \sqrt{r^2 + k^2}$, Note: $\sin^2\theta + \cos^2\theta = 1$ is a famous trig identity
13. Putting it all together, our slant height, *l*:
    a. $l = \int_0^{\theta_f} \sqrt{r^2 + k^2}\, d\theta$
14. Pretty good. But we still need to find θ_f.
15. From the z component of our helix equation, $k\theta = H$, where *H* is the total height of the wrap since *H* and *k* are both known: $\theta_f = \frac{H}{k}$.
16. $l = \int_0^{\frac{H}{k}} \sqrt{r^2 + k^2}\, d\theta = \sqrt{r^2 + k^2}\left(\frac{H}{k}\right)$
17. Yay! We now know how to find the length of all sides of the parallelogram we need to wrap our pipe in tape.

For Our Intake

- $w = 3.93$ in
- $r = 1.25$ in
- $C = 7.85$ in
- $k = 0.722$ in/rad
- $l = 2.508 * H$