PROBLEM

Many countries, including third world or developing countries are gaining access to oil drilling techniques. However, the cost associated with expensive computational equipment such as supercomputers may be cost prohibitive to the third world or developing countries. Countries like Somalia now have the capability to drill for oil but they do not have access to the supercomputer required to simulate thousands of particles in case of a spill.

GOAL

Design a simple process to model the dispersion of oil spills. A process simple enough to run on an average computer but accurate enough to model the spill.

BACKGROUND INFORMATION

Formulas

- Shoelace formula[I]
  - $A = \frac{1}{2} abs\left(\det\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}\right)$
- Centroid formula[II]
  - Triangle: $\bar{x} = \frac{x_1+x_2+x_3}{3}, \bar{y} = \frac{y_1+y_2+y_3}{3}$
  - Composite: $\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i}, \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$
- Green's theorem (simplified)[III]
  - $\int_{X_1}^{X_2} \int_{Y_1}^{Y_2} 1\, dy\, dx = \frac{1}{2} \oint_{t_1}^{t_2} x\, dy - y\, dx$

Concepts

- Vector operations (adding and scaling)
- Calculus (derivatives and integrals)
- Polar plane (r=radius, θ=angle from +x-axis)[IV]

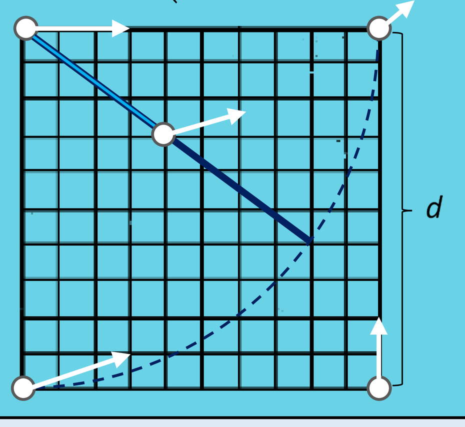
Methods

Cubic Interpolation-Stephen Method[V]

- $f(x) = d_i(x-x_i)^3 + c_i(x-x_i)^2 + y_i'(x-x_i) + y_i$
- $c_i = \frac{3s_i - 2y_i' - y_{i+1}'}{x_{i+1}-x_i}, d_i = \frac{y_i' + y_{i+1}' - 2s_i}{(x_{i+1}-x_i)^2}, s_i = \frac{y_{i+1}-y_i}{x_{i+1}-x}$
- $y_i' = (sign(\Delta_{i-1}) + sign(\Delta_i))min\left(|\Delta_{i-1}|, |\Delta_i|, \left|\frac{h_i\Delta_{i-1}+h_{i-1}\Delta_i}{2(h_{i-1}+h_i)}\right|\right)$
- $h_i = x_{i+1} - x_i, \Delta_i = \frac{y_{i+1}-y_i}{h_i}$

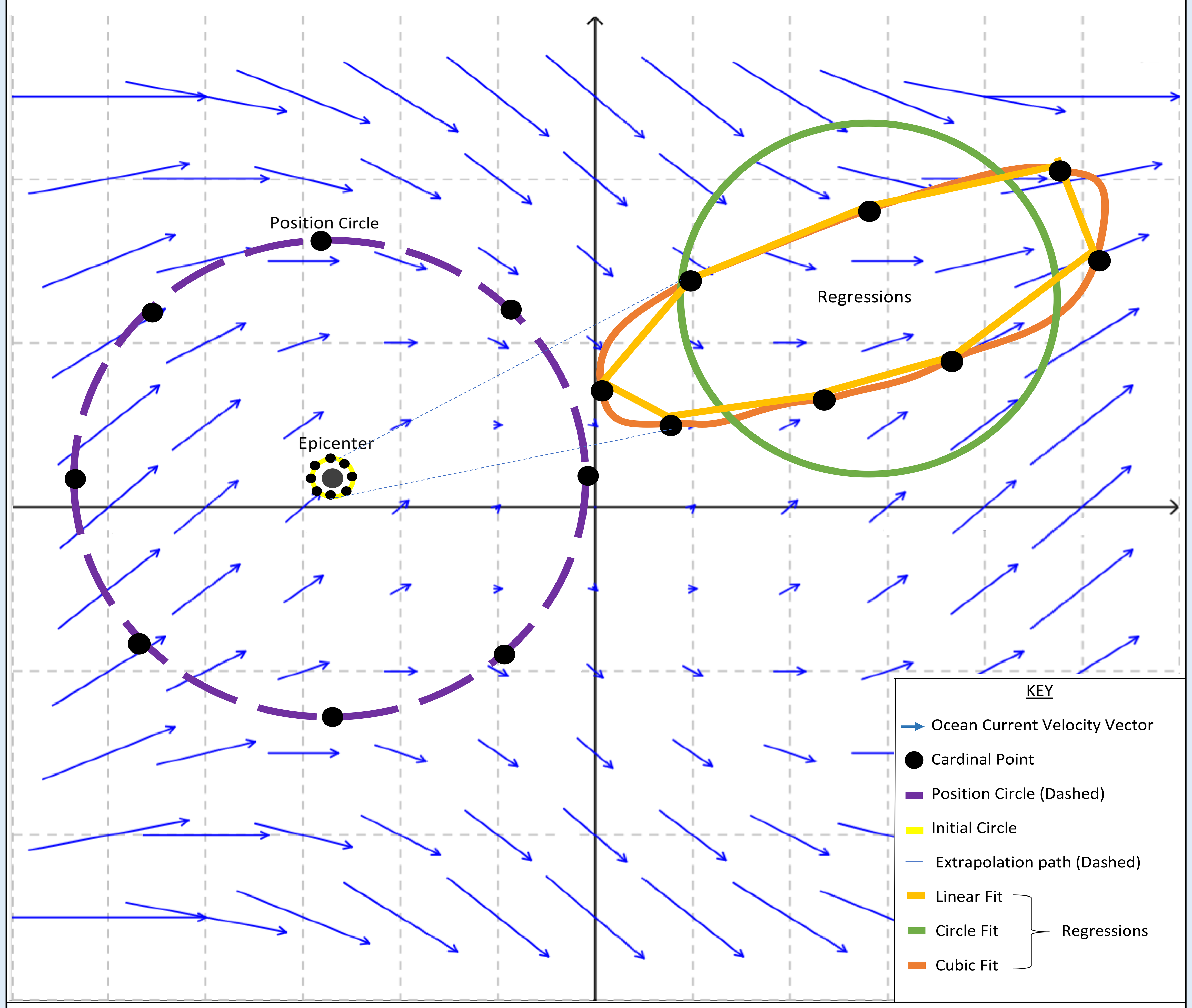
Vector Interpolation Method[VI]

- $u' = \sum u_i w_i, v' = \sum v_i w_i, w = \frac{a}{\sum a}$
- $a = real\left(\sqrt{\frac{d-\sqrt{(x_i-x')^2+(y_i-y')^2}}{d}}\right)^2$



"a" is a raw weight. This weight is based on the distance between two points. Since some vectors in the grid are father than "d" apart they produce a negative weight. This doesn't make sense so the negatives are converted to zero. Then, "w" adjusts the weights since the negatives were removed. Finally, the weights are multiplied by their respective vector producing the interpolated vector.

PROCESS



Set up

- Set up and calculate[VI] ocean current velocity vectors at cardinal points on position circle.

Select

- Select start circle radius- this is an approximate of the radius of the initial spill.

Extrapolate

- Extrapolate to calculate the final position of the cardinal points.
- In other words, multiply the velocity vectors by time. Then, add the resultant distance to the initial cardinal points.

Interpolate

- Interpolate the border of the spill and find the area within the border.
- 3 methods were imployed: linear[I], circular[II,IV], and cubic fits[III,V].

Verify

- Verify the model's integrity by repeating the process.

KEY TERMS

Epicenter: The "point" where the spill started.

Initial Circle: A circle approximating the initial spill.

Position Circle: A theoretical circle that approximates the median set of vectors.

Ocean Surface Current Analysis Real-time (OSCAR): A satellite that measures the velocities of ocean currents at a third of a degree of latitude.

Physical Oceanography Distributed Active Archive Center (PO.DAAC): A web-database archiving data from OSCAR.

RESULTS

Area of Oil Spills and Attributes Boundary Obtainment Methods

BOM	Sanchi	Bonga	Ennore	Avg.	Std.	t	p
Newspaper	101	906	.034	335.678	496.487		
Linear fit	180	944	.002	374.667	501.203	.0957	.932
Circle fit	241	800	.315	347.105	410.265	.0307	.978
Cubic fit	256	668	.013	281.004	346.380	.1564	.944

Columns 2-6 are in km^2. Column 1 contains the Boundary Obtainment Method (BOM) for which the information in the rows they head were obtained. Columns 2-4 are three random spills that had their areas approximated using the methods listed in column 1. Columns 5 and 6 contain averages and standard deviations of columns 2-4, respectively. Column 7 and 8 contain the t and p values, respectively, for t-tests between the 3 interpolation methods and the benchmark-the newspaper.

CONCLUSIONS

All three boundary interpolation methods were shown to not be significantly different from the benchmark-p values were high. This meant that the model was successful in modeling oil spills. Though, there was error in the model. This error was due to the curvature of the Earth, assuming the spills would spread radially, and lack of input data. In addition to errors in the model, there are many factors and anomalies that occur in the physical world. These are what make an accurate model almost impossible.

FUTURE IDEAS

In the future, this project can be improved by increasing the sample size or reducing error. The former is self-explanatory. The latter can be done in two ways. One, perform the whole process in the latitude-longitude plane to account for the curvature of the Earth. Or two, design another model that can accurately model spills that spread linearly too. It would also be interesting to compare this model to other models.

All Figures and Tables were made by the experimenter using Word, Ms Paint, and GeoGebra.