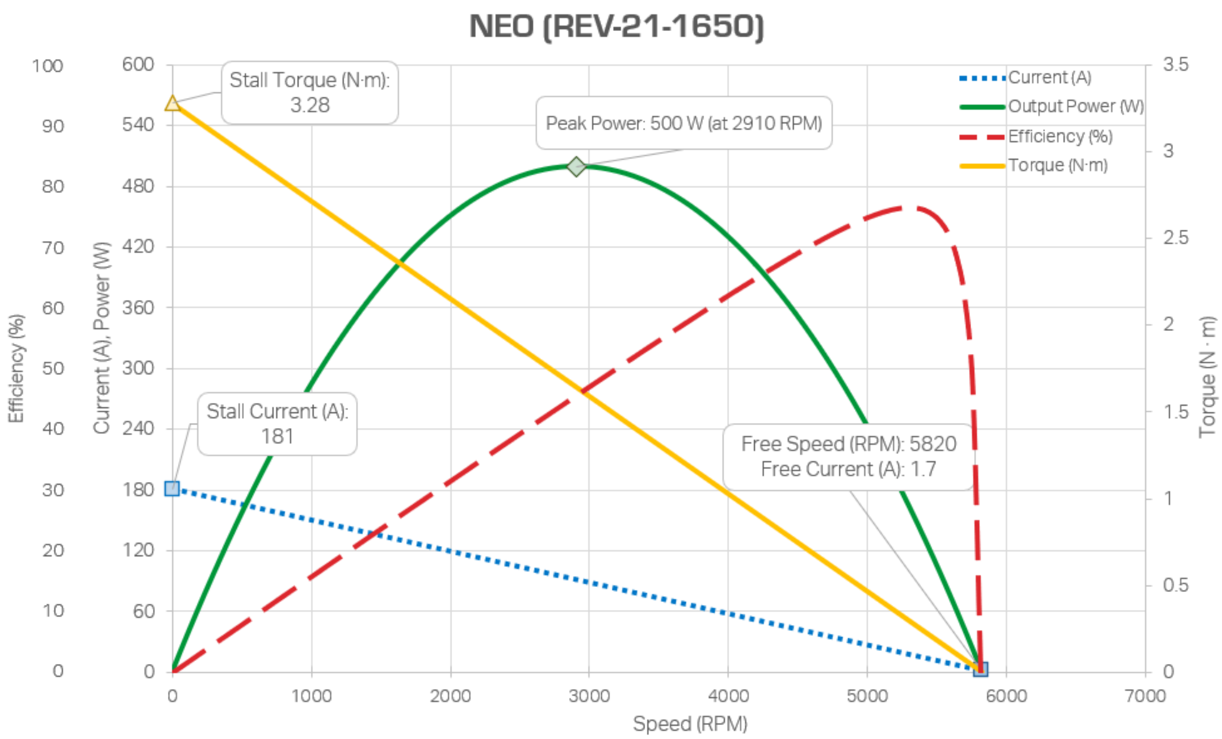

The goal of this derivation is to find an overall reduction that optimizes load speed (aka. Gearbox output speed) based on motor curves and load torque.

From last time:

$$\frac{\omega_{motor}}{\omega_{max}} + \frac{\tau_{motor}}{\tau_{max}} = 1$$

$$\tau_{motor} = \left(1 - \frac{\omega_{motor}}{\omega_{max}}\right)\tau_{max}$$

Properties of reductions (Note: a reduction, r, value of 3 means that the input will spin 3 times to spin the output once):

$$\omega_{load} = \frac{\omega_{motor}}{r}$$

$$\tau_{load} = r\tau_{motor}$$

$$\tau_{motor} = \frac{\tau_{load}}{r}$$

Substitution:

$$\left(1 - \frac{\omega_{motor}}{\omega_{max}}\right)\tau_{max} = \frac{\tau_{load}}{r}$$

Solving for ω_{motor}:

$$1 - \frac{\omega_{motor}}{\omega_{max}} = \frac{\tau_{load}}{r\tau_{max}}$$

$$\frac{\omega_{motor}}{\omega_{max}} = 1 - \frac{\tau_{load}}{r\tau_{max}}$$

$$\omega_{motor} = \omega_{max}\left(1 - \frac{\tau_{load}}{r\tau_{max}}\right)$$

Substitution:

$$\omega_{load} = \frac{\omega_{max}}{r}\left(1 - \frac{\tau_{load}}{r\tau_{max}}\right)$$

$$\omega_{load} = \frac{\omega_{max}}{r} - \frac{\tau_L \omega_{max}}{r^2 \tau_{max}}$$

Yay, now that we have an equation for the angular speed of the load in terms of reduction, we can optimize it.

$$\omega_{load}' = \frac{-\omega_{max}}{r^2} + \frac{2\tau_L \omega_{max}}{r^3 \tau_{max}} = 0$$

$$\frac{\omega_{max}}{r^2} = \frac{2\tau_L \omega_{max}}{r^3 \tau_{max}}$$

Solving for r:

$$r = \frac{2\tau_{load}}{\tau_{max}}$$

From the formula above we can check if it makes partial sense via proportional reasoning. If the gearbox needs to output more torque, a larger reduction is needed. If a motor is more capable of outputting torque, a smaller reduction would be used. Both of those make sense intuitively.

An interesting observation arises by plotting the reduction, r, against the load angular velocity, ω_{load}.

The black line is the reduction versus load angular velocity. The green line is the optimal reduction. The blue line is the efficient reduction.

In hindsight, it derived formula makes a lot of sense as that is where peak power on the motor curve occurs (when $\tau_{motor} = \frac{\tau_{max}}{2}$).

Ideally, a reduction between the peak power line and the efficient line would be chosen. If speed is crucial, choose closer to the peak power line. If cool operation is more important, choose the efficient line.