

## Chapter 1: Unique Factorization

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**Exercise 1.31.** *Show that 2 is divided by  $(1+i)^2 \in \mathbb{Z}[i]$ .*

The ring morphism  $\mathbb{Z} \rightarrow \mathbb{Z}[i]$  corresponds to a map of schemes  $f : \operatorname{Spec}(\mathbb{Z}[i]) \rightarrow \operatorname{Spec}(\mathbb{Z})$ . Suppose  $(p)$  is a prime ideal of  $\mathbb{Z}$ . Might find the points of  $f^{-1}(p) \in \operatorname{Spec}(\mathbb{Z}[i])$ .

*Proof.*  $(1+i)^2 = 2i \in \mathbb{Z}[i]$ . Thus  $2 \mid (1+i)^2 \in \mathbb{Z}[i]$ .  $\square$