Chapter 2: Applications of Unique Factorization

Exercise. If $\frac{a}{b} \in \mathbb{Z}_p$ is not a unit, prove that $\frac{a}{b} + 1$ is a unit.

Proof:
$\frac{a}{b} \in \mathbb{Z}_p$ is not a unit iff $p \mid a$ and $p \nmid b$. Thus $p \nmid (a+b)$. That is, $\frac{a}{b} + 1 = \frac{a+b}{b} \in \mathbb{Z}_p$ is a unit. $\square$