Chapter 10: Integration of Differential Forms

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Exercise 10.1. *...*

Proof.

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Exercise 10.2. *...*

Proof.

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Exercise 10.3. *...*

Proof.

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Exercise 10.4. *...*

Proof.

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Exercise 10.5. ...

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Exercise 10.6. ...

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Exercise 10.7. ...

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Exercise 10.8. ...

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Exercise 10.9. ...

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Exercise 10.10. *...*

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Exercise 10.11. *...*

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Exercise 10.12. *...*

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Exercise 10.13. *...*

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Exercise 10.14. *...*

Proof.

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Exercise 10.15. *If ω and λ are k- and m-forms, respectively, prove that*

$$\omega \wedge \lambda = (-1)^{km} \lambda \wedge \omega.$$

Proof.

(1) Write

$$\omega = \sum_I b_I(\mathbf{x}) dx_I, \qquad \lambda = \sum_J c_J(\mathbf{x}) dx_J$$

in the stardard presentations, where I and J range over all increasing k-indices and over all increasing m-indices taken from the set $\{1, \ldots, n\}$.

(2) *Show that $dx_I \wedge dx_J = (-1)^{km} dx_J \wedge dx_I$.*

$$\begin{aligned} dx_I \wedge dx_J &= dx_{i_1} \wedge \cdots \wedge dx_{i_k} \wedge dx_J \\ &= (-1)^m dx_{i_1} \wedge \cdots \wedge dx_{i_{k-1}} \wedge dx_J \wedge dx_{i_k} \\ &= (-1)^{2m} dx_{i_1} \wedge \cdots \wedge dx_{i_{k-2}} \wedge dx_J \wedge dx_{i_{k-1}} \wedge dx_{i_k} \\ &\cdots \\ &= (-1)^{km} dx_J \wedge dx_{i_1} \wedge \cdots \wedge dx_{i_k} \\ &= (-1)^{km} dx_J \wedge dx_I. \end{aligned}$$

(3)

$$\begin{aligned} \omega \wedge \lambda &= \sum_{I,J} b_I(\mathbf{x}) c_J(\mathbf{x}) dx_I \wedge dx_J \\ &= (-1)^{km} \sum_{J,I} c_J(\mathbf{x}) b_I(\mathbf{x}) dx_J \wedge dx_I \\ &= (-1)^{km} \lambda \wedge \omega. \end{aligned}$$

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Exercise 10.16. ...

Proof.

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Exercise 10.17. ...

Proof.

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Exercise 10.18. ...

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Exercise 10.19. ...

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Exercise 10.20. ...

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Exercise 10.21. ...

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Exercise 10.22. ...

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Exercise 10.23. ...

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Exercise 10.24. ...

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Exercise 10.25. ...

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Exercise 10.26. *...*

Proof.

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Exercise 10.27. *...*

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Exercise 10.28. *...*

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Exercise 10.29. *...*

Proof.

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Exercise 10.30. *...*

Proof.

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Exercise 10.31. *...*

Proof.

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Exercise 10.32. *...*

Proof.

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