Chapter 1: Unique Factorization

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Exercise 1.31. Show that 2 is divided by $(1+i)^2 \in \mathbb{Z}[i]$.

1+i is irreducible in $\mathbb{Z}[i]$.

The ring morphism $\mathbb{Z} \to \mathbb{Z}[i]$ corresponds to a map of schemes $f : \operatorname{Spec}(\mathbb{Z}[i]) \to \operatorname{Spec}(\mathbb{Z})$. Suppose (p) is a prime ideal of \mathbb{Z} . Might find the points of $f^{-1}(p) \in \operatorname{Spec}(\mathbb{Z}[i])$.

Proof. $(1+i)^2 = 2i \in \mathbb{Z}[i]$. Thus $2 \mid (1+i)^2 \in \mathbb{Z}[i]$. \square

Exercise 1.34. Show that 3 is divided by $(1 - \omega)^2 \in \mathbb{Z}[\omega]$.

Proof. $(1-\omega)^2=1-2\omega+\omega^2=(1+\omega+\omega^2)-3\omega=-3\omega\in\mathbb{Z}[\omega]$. Thus $3\mid (1-\omega)^2\in\mathbb{Z}[\omega]$. \square