Chapter 10: Integration of Differential Forms

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Exercise 10.1. ...

Proof.

(1)

(2)

□

Exercise 10.2. ...

Proof.

(1)

(2)

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Exercise 10.3. ...

Proof.

(1)

(2)

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Exercise 10.4. ...

Proof.

(1)

(2)

$\square$

Exercise 10.5. ...

Proof.

(1)

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$\square$

Exercise 10.6. ...

Proof.

(1)

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$\square$

Exercise 10.7. ...

Proof.

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$\square$

Exercise 10.8. ...

Proof.

(1)

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$\square$

Exercise 10.9. ...

Proof.

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Exercise 10.10. ...

Proof.

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Exercise 10.11. ...

Proof.

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Exercise 10.12. ...

Proof.

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Exercise 10.13. ...

Proof.

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Exercise 10.14. *...*

Proof.

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Exercise 10.15. *If ω and λ are k- and m-forms, respectively, prove that*

$$\omega \wedge \lambda = (-1)^{km} \lambda \wedge \omega.$$

Proof.

(1) Write

$$\omega = \sum_I b_I(\mathbf{x}) dx_I, \qquad \lambda = \sum_J c_J(\mathbf{x}) dx_J$$

in the stardard presentations, where I and J range over all increasing k-indices and over all increasing m-indices taken from the set $\{1, \ldots, n\}$.

(2) So

$$\omega \wedge \lambda = \sum_{I,J} b_I(\mathbf{x}) c_J(\mathbf{x}) dx_I \wedge dx_J.$$

Here

$$\begin{aligned}
dx_I \wedge dx_J &= dx_{i_1} \wedge \cdots \wedge dx_{i_k} \wedge dx_J \\
&= (-1)^m dx_{i_1} \wedge \cdots \wedge dx_{i_{k-1}} \wedge dx_J \wedge dx_{i_k} \\
&= (-1)^{2m} dx_{i_1} \wedge \cdots \wedge dx_{i_{k-2}} \wedge dx_J \wedge dx_{i_{k-1}} \wedge dx_{i_k} \\
&\cdots \\
&= (-1)^{km} dx_J \wedge dx_{i_1} \wedge \cdots \wedge dx_{i_k} \\
&= (-1)^{km} dx_J \wedge dx_I.
\end{aligned}$$

□

Exercise 10.16. *...*

Proof.

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Exercise 10.17. ...

Proof.

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Exercise 10.18. ...

Proof.

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Exercise 10.19. ...

Proof.

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Exercise 10.20. ...

Proof.

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Exercise 10.21. ...

Proof.

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Exercise 10.22. ...

Proof.

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Exercise 10.23. ...

Proof.

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Exercise 10.24. ...

Proof.

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Exercise 10.25. ...

Proof.

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Exercise 10.26. ...

Proof.

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Exercise 10.27. ...

Proof.

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Exercise 10.28. ...

Proof.

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Exercise 10.29. ...

Proof.

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Exercise 10.30. ...

Proof.

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Exercise 10.31. ...

Proof.

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Exercise 10.32. ...

Proof.

(1)

(2)

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