Chapter 3: Operators

Author: Meng-Gen Tsai
Email: plover@gmail.com

Problem 3.6. *Prove that* $\hat{A} + \hat{B} = \hat{B} + \hat{A}$.

Two operators $\hat{A}$ and $\hat{B}$ are said to be equal if $\hat{A}f = \hat{B}f$ for all functions f.

Proof.

$$(\hat{A} + \hat{B})f = \hat{A}f + \hat{B}f = \hat{B}f + \hat{A}f = (\hat{B} + \hat{A})f$$

holds for any function f. By definition, $\hat{A} + \hat{B} = \hat{B} + \hat{A}$. □

Problem 3.7. *Let* $\hat{D} = d/dx$. *Verify that* $(\hat{D} + x)(\hat{D} - x) = \hat{D}^2 - x^2 - 1$.

Proof.

$$\begin{aligned}((\hat{D} + x)(\hat{D} - x))f &= (\hat{D} + x)((\hat{D} - x)f) \\ &= (\hat{D} + x)(f' - xf) \\ &= (f' - xf)' + x(f' - xf) \\ &= (f'' - f - xf') + (xf' - x^2 f) \\ &= f'' - f - x^2 f \\ &= (\hat{D}^2 - x^2 - 1)f\end{aligned}$$

holds for any function f. By definition, $(\hat{D} + x)(\hat{D} - x) = \hat{D}^2 - x^2 - 1$. □