Chapter 1: Unique Factorization

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Exercise 1.31. *Show that* 2 *is divided by* $(1+i)^2 \in \mathbb{Z}[i]$.

The ring morphism $\mathbb{Z} \to \mathbb{Z}[i]$ corresponds to a map of schemes $f : \mathrm{Spec}(\mathbb{Z}[i]) \to \mathrm{Spec}(\mathbb{Z})$. Suppose (p) is a prime ideal of $\mathbb{Z}$. Might find the points of $f^{-1}(p) \in \mathrm{Spec}(\mathbb{Z}[i])$.

Proof. $(1+i)^2 = 2i \in \mathbb{Z}[i]$. Thus $2 \mid (1+i)^2 \in \mathbb{Z}[i]$. $\square$

Exercise 1.34. *Show that* 3 *is divided by* $(1-\omega)^2 \in \mathbb{Z}[\omega]$.

Proof. $(1-\omega)^2 = 1 - 2\omega + \omega^2 = (1+\omega+\omega^2) - 3\omega = -3\omega \in \mathbb{Z}[\omega]$. Thus $3 \mid (1-\omega)^2 \in \mathbb{Z}[\omega]$. $\square$