Budapest University of Technology and Economics
Department of Electron Devices

# Microelectronics, BSc course

## Basic semiconductor physics

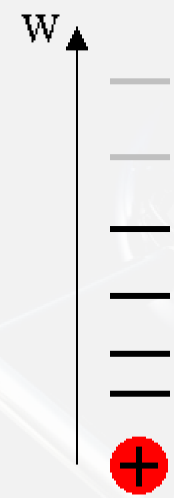
# Summary of the essential semiconductor physics

- Charge carriers in semiconductors
- Currents in semiconductors
- Generation, recombination; continuity equation

# Energy bands in the lattice

- Basic consequences of quantum mechanics:
  - Positive nucleus ($p^+$, $n^0$) – Negative electrons
  - **Pauli theorem**: two $e^-$ cannot be on the same state; in one electron orbital max two $e^-$ with opposite spin
  - Energy minimum: lowest shells, orbitals are filled
  - Switching between shells: energy gained (equals the energy between the actual state and the unfilled shells)
  - It is most likely that the $e^-$ with upper most state

    will gain more energy




Discrete energy levels

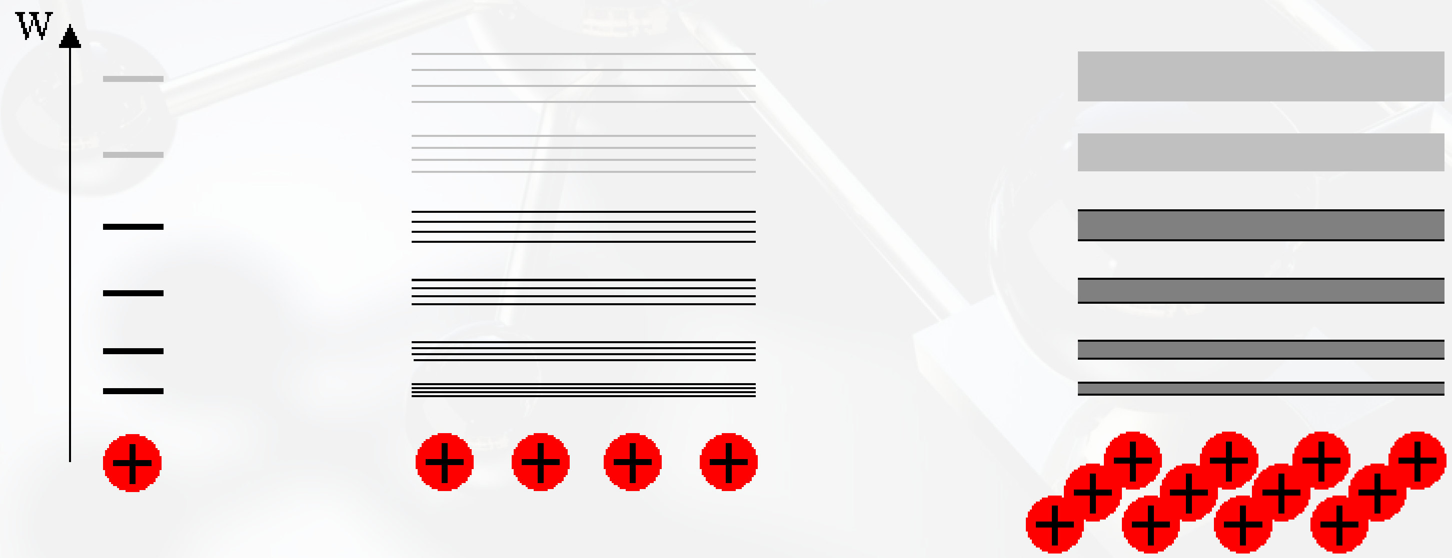
# Energy bands

- Resulting from principles of quantum physics

Discrete energy levels:

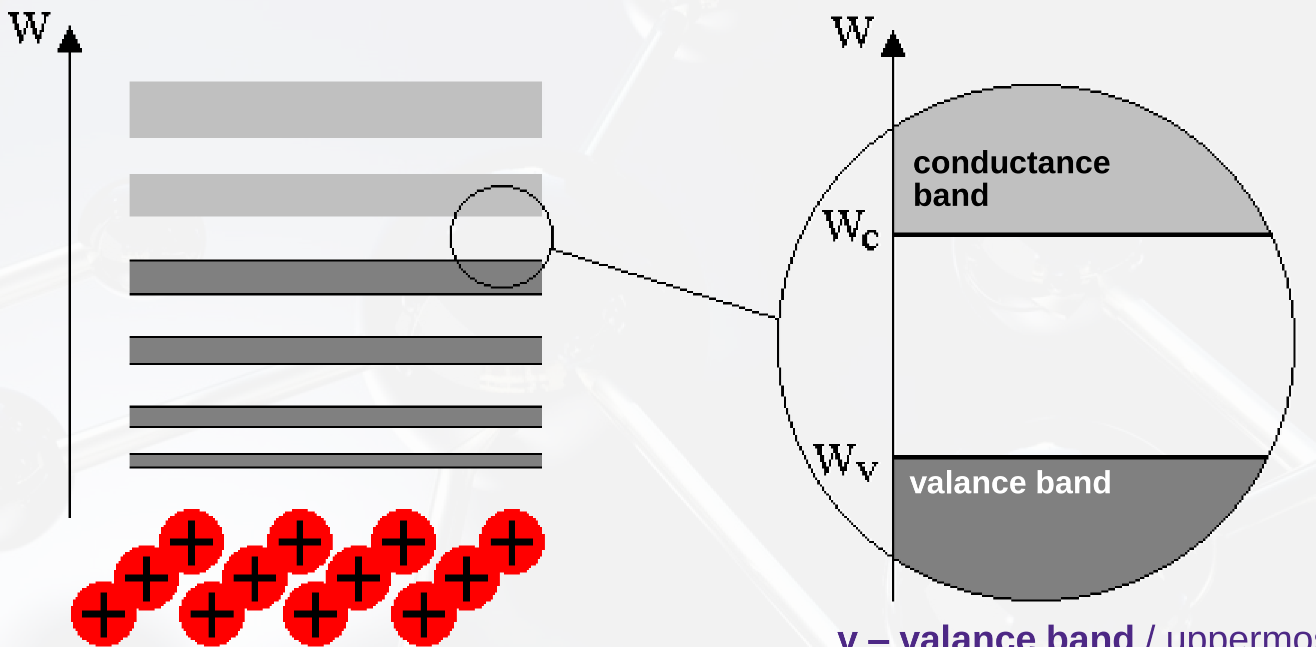
More atoms – more energy levels of electrons:

There are so many of them that they form energy bands:




The discrete (allowed) energy levels of an atom become energy bands in a crystal lattice

# Valance band, conductance band

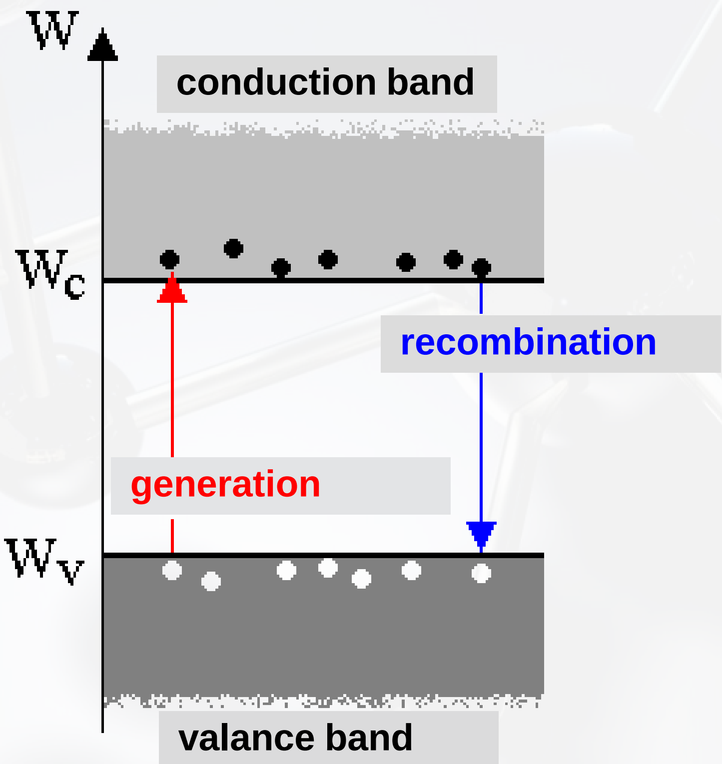



**v – valance band** / uppermost full

**c – conductance band** / lowermost empty

- Valance band – these electrons form the chemical bonds
  - almost full
- Conductance band – electrons here can move freely
  - almost empty

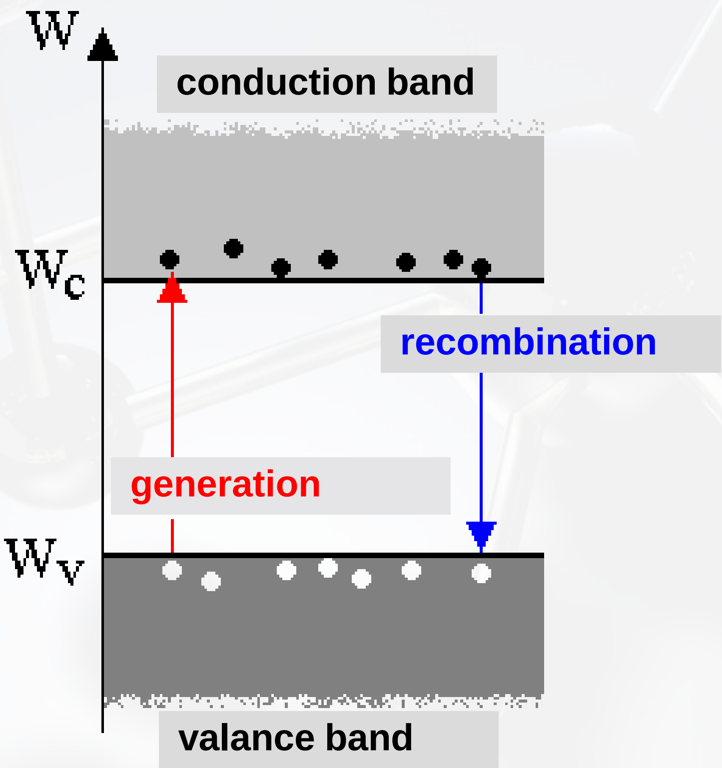
# Electrons and holes




- Generation: using the average thermal energy
- **Electrons:** in the bottom of the conduction band
- **Holes:** in the top of the valance band
- Both the electrons and the holes form the electrical current

| | |
|---|---|
| Electron: | negative charge, positive eff. mass |
| Hole: | positive charge, positive eff. mass |

# Electrons and holes

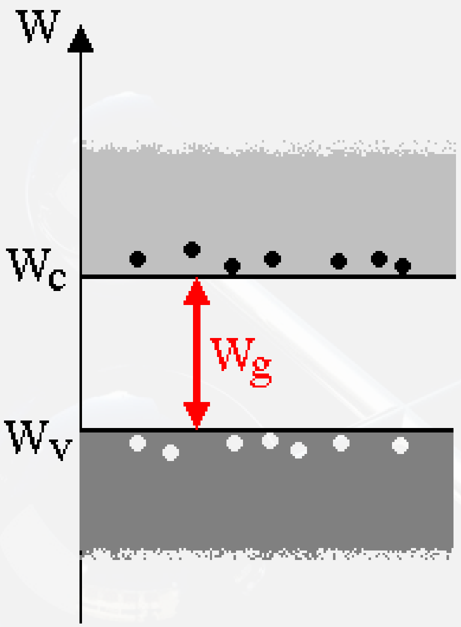



How does the electrons gain energy?

- Thermal excitation
  - Energy of lattice resonance transfered to the $e^-$
- Photon excitation
  - Incident light with energy greater than $W_g$
  - $E = h \cdot \upsilon$

The electron density of the conductance band is determined by the dynamic balance of the generation and the recombination

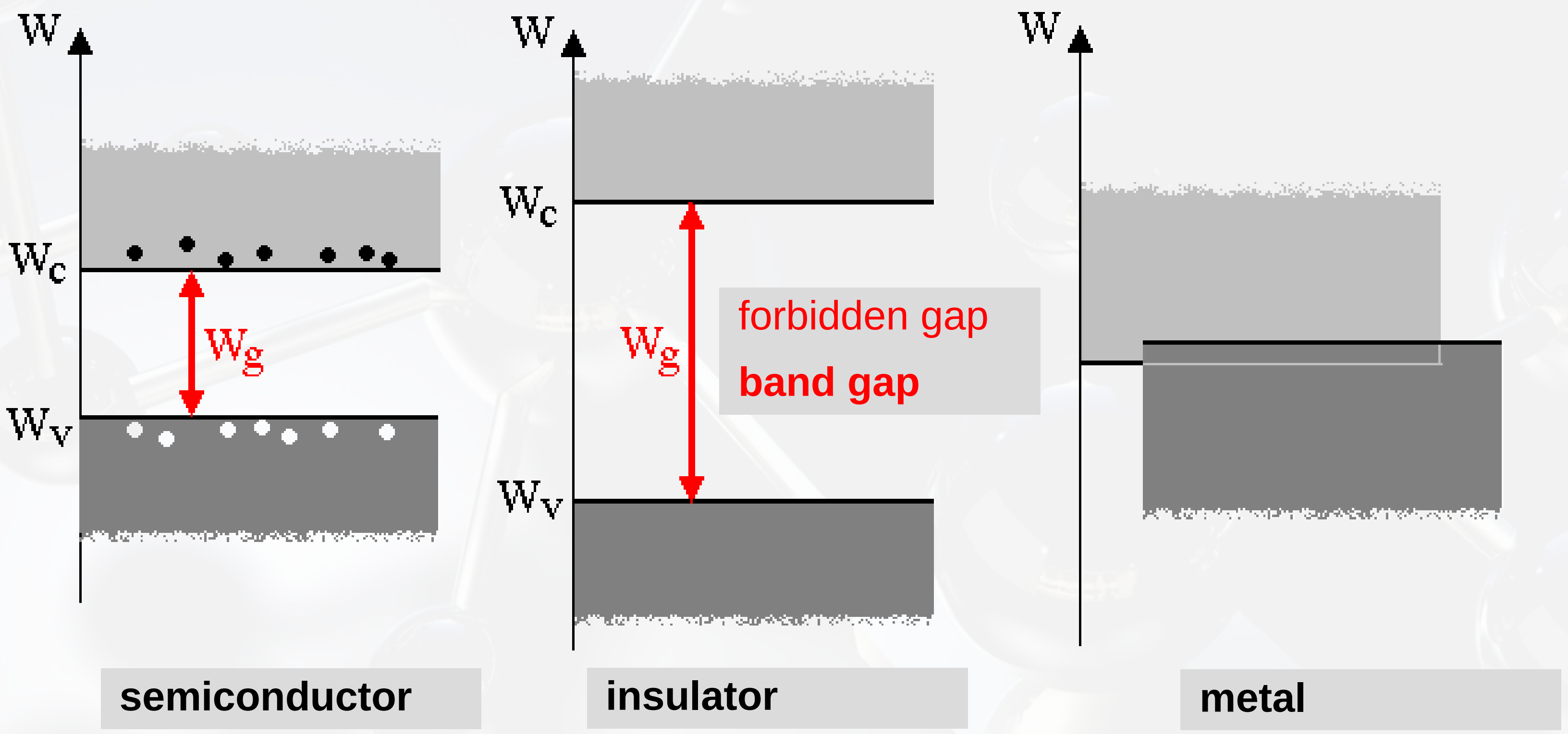
# Conductors and insulators


semiconductor

**Electron cannot be located in the forbidden gap!**

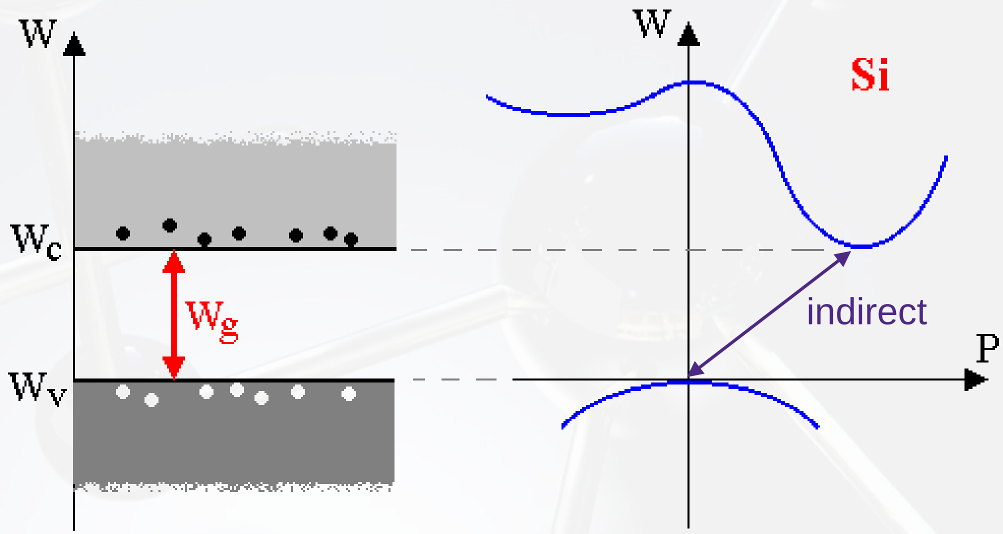
# Conductors and insulators

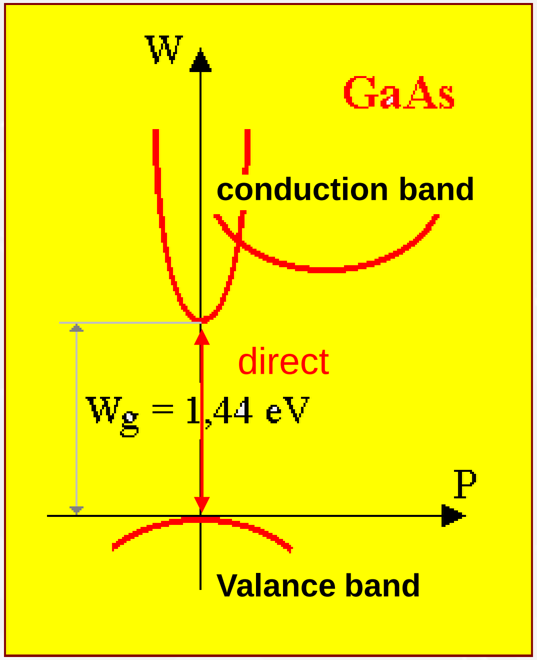



**For Si: $W_g$ = 1.12 eV**  **for $SiO_2$: $W_g$ = 4.3 eV**

1 eV = 0.16 aJ = 0.16 $\cdot 10^{-18}$ J

# Band structure of semiconductors




**In case of indirect band semiconductor the laws of energy and impluse conservations must be fulfilled.**

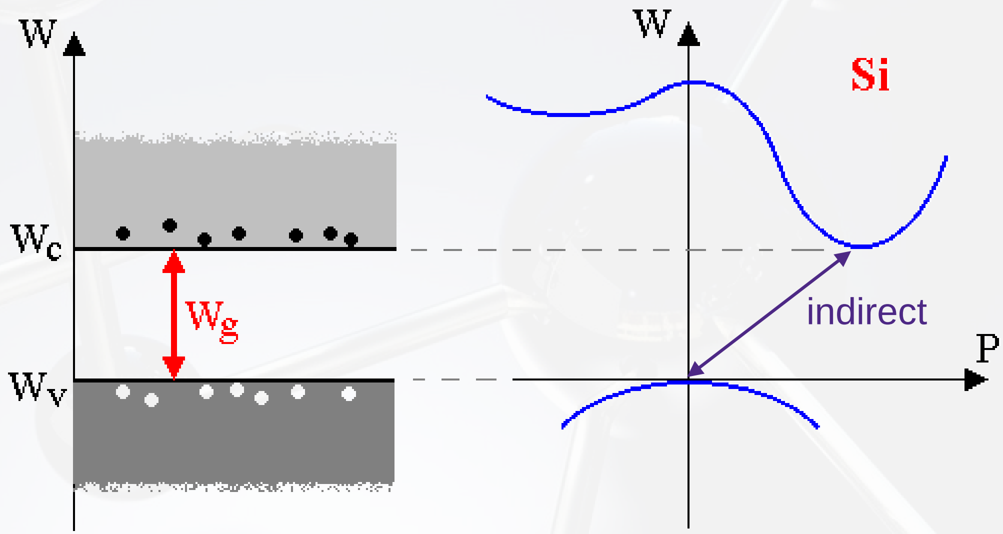
$$F = \frac{dP}{dt}$$

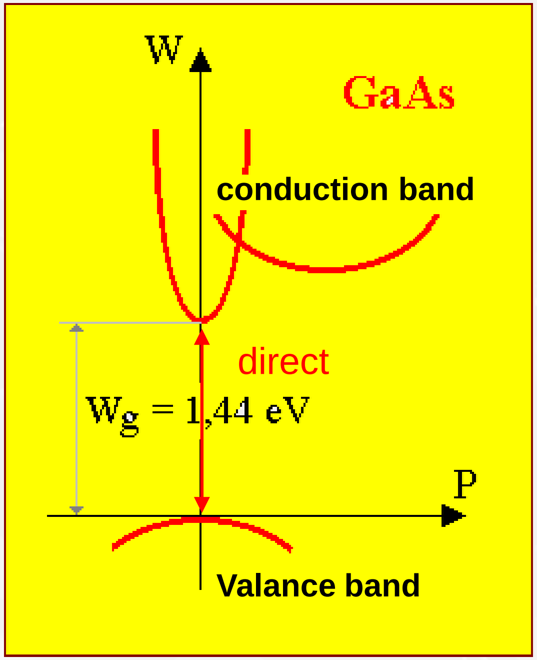
$$P = \frac{h}{2\pi} k$$

**GaAs: direct band ⇒ opto-electronic devices (LEDs)**

**Si: indirect band**

# Band structure of semiconductors




$$W = \frac{1}{2m} p^2 \rightarrow W = \frac{1}{2m_{eff}} P^2$$

$$F = \frac{dP}{dt}$$

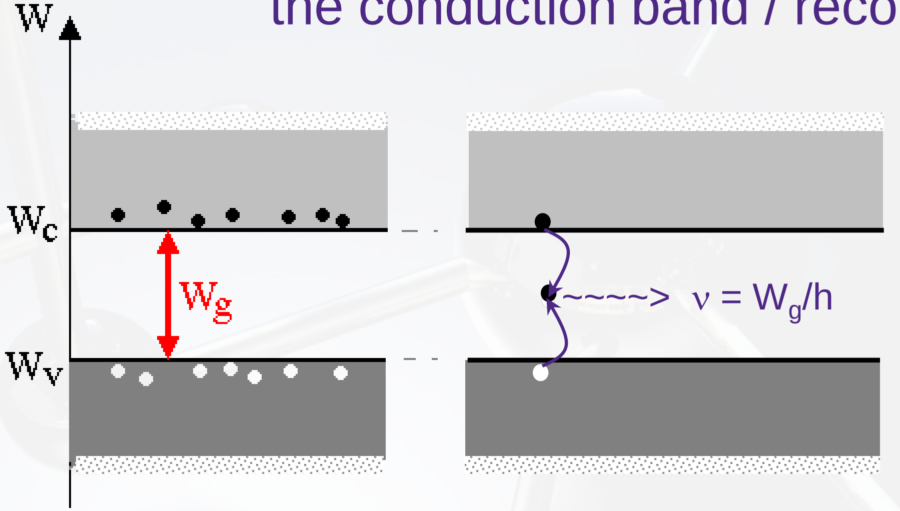
$$P = \frac{h}{2\pi} k$$

**GaAs: direct band ⇒ opto-electronic devices (LEDs)**

**Si: indirect band**

# Generation / recombination

Spontaneous process: thermal excitation – jump into the conduction band / recombination ➔ equilibrium



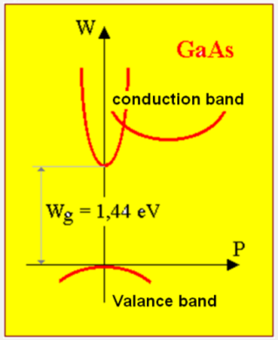

$$\nu = \frac{W_g}{h}$$

$$\lambda = \frac{c}{\nu}$$

**In case of red laser**

**$W_g \approx 1.5$eV**

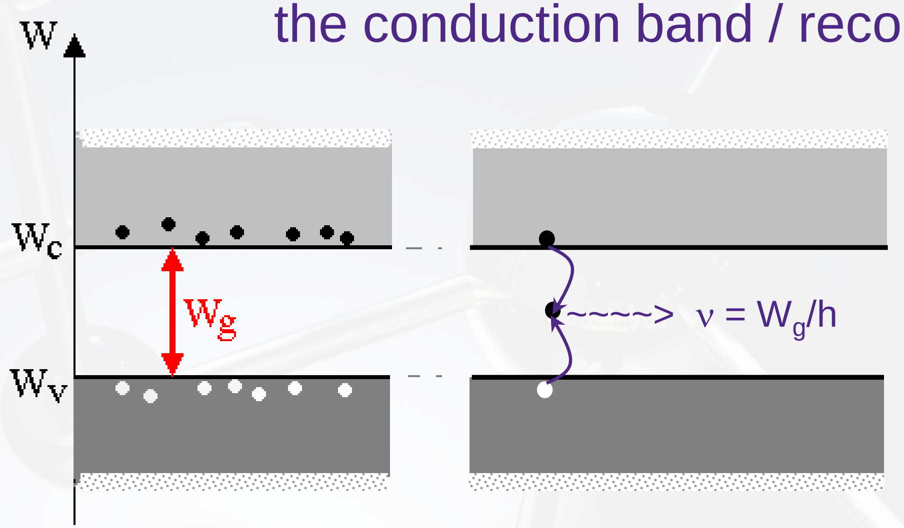
Direct **recombination** may result in light emission (LEDs)

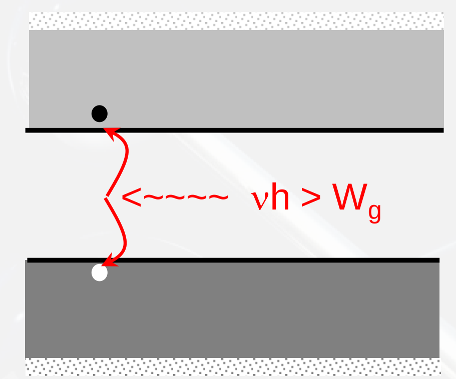



Indirect **recombination** is not radioative, but always accompanied by heat dissipation (Phonon – lattice vibration)

# Generation / recombination

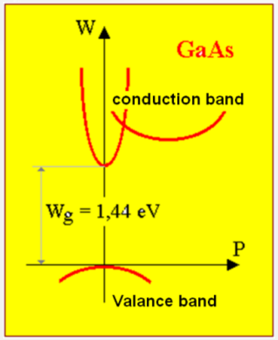
Spontaneous process: thermal excitation – jump into the conduction band / recombination ➔ equilibrium




Direct **recombination** may result in light emission (LEDs)

Light absorption results **in generation** (solar cells)

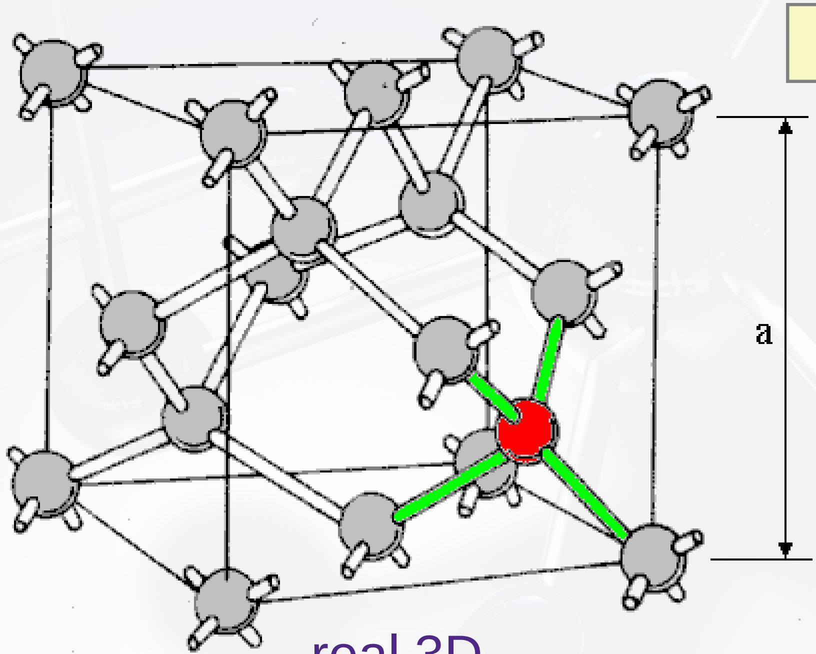



**Experiment**

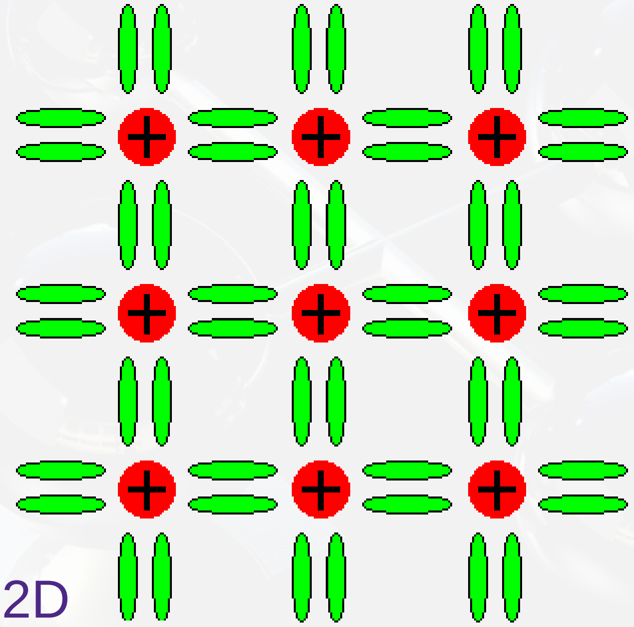
# Crystalline structure of Si

- Si N=14 4 bonds, *IV*-th column of the periodic table

*undoped or* ***intrinsic*** semiconductor



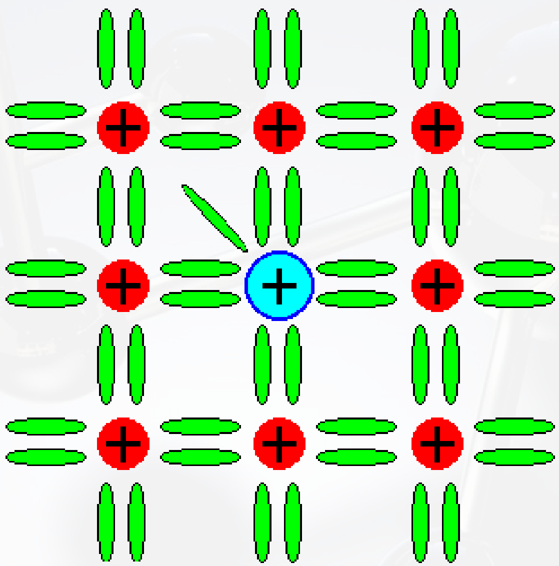

real 3D

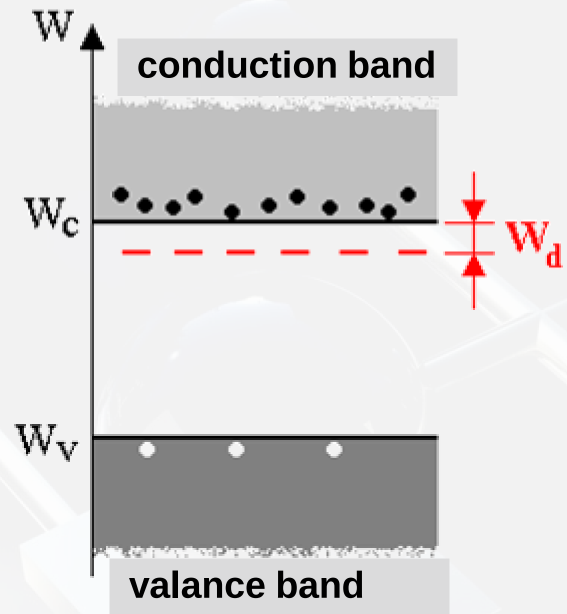


simplified 2D

- Diamond lattice, *lattice constant* **a**=0.543 nm
- Each atom has 4 nearest neighbor

# 5 valance dopant: donor (As, P, Sb)

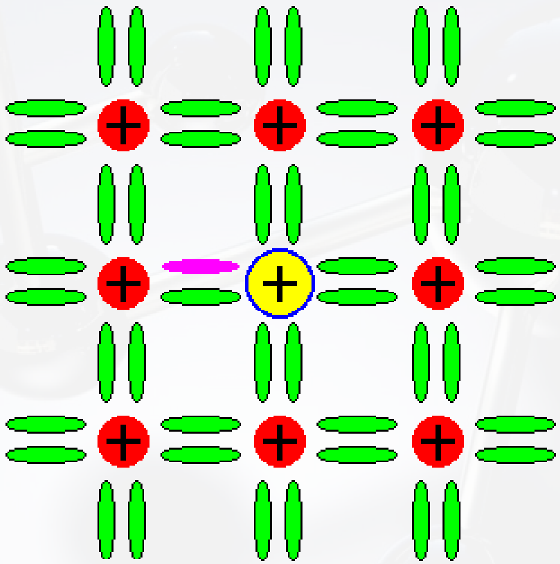


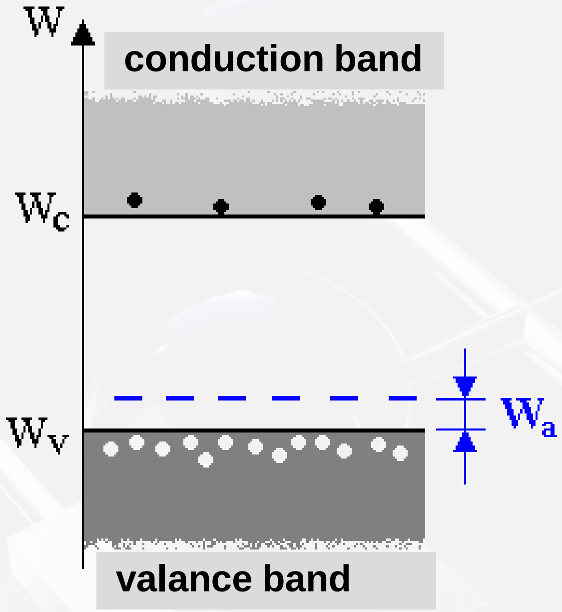



- **Electron:** *majority* carrier
- Hole: *minority* carrier

n-type semiconductor

# 3 valance dopant: acceptor (B, Ga, In)

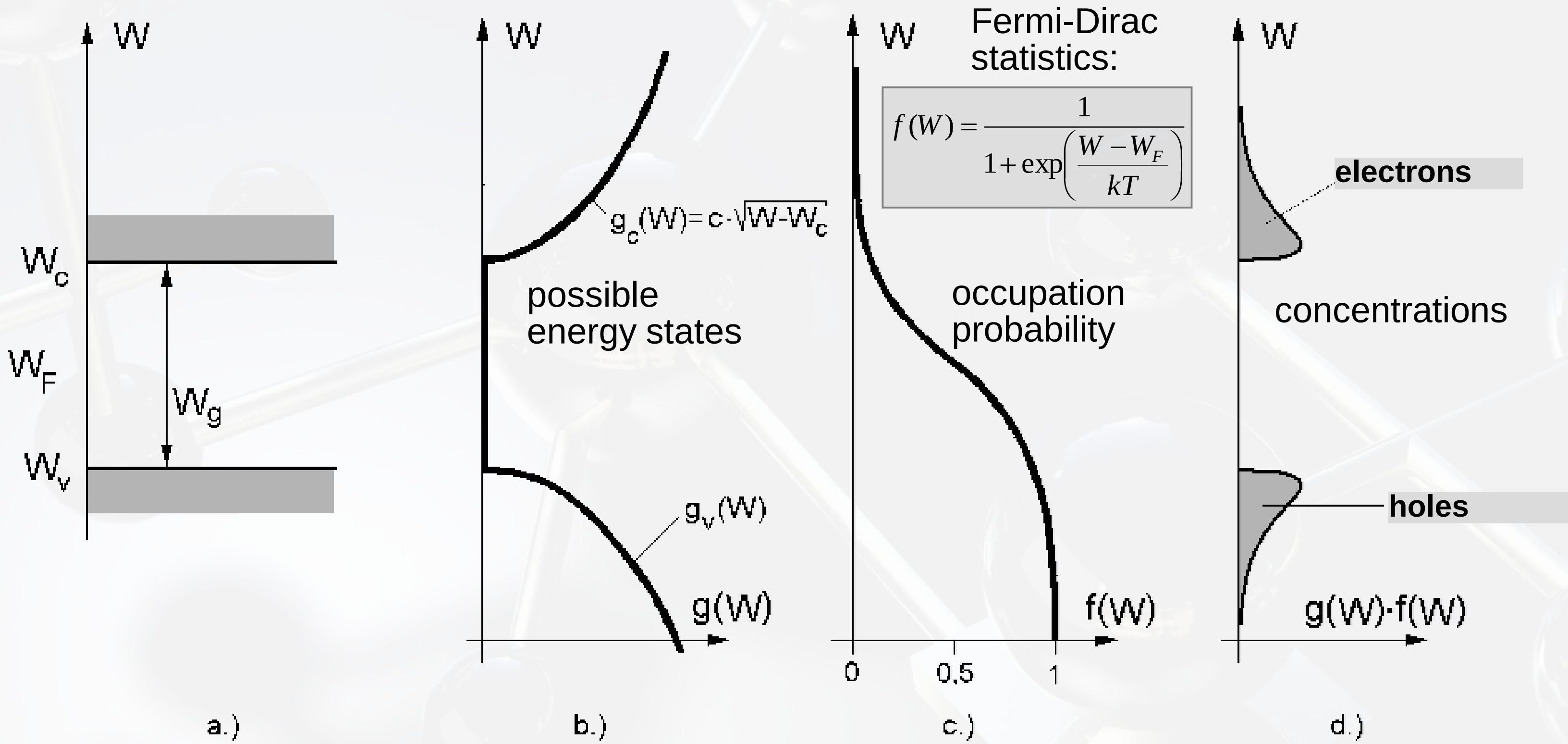





- Electron: *minority* carrier
- **Hole:** *majority* carrier

p-type semiconductor

# Calculation of carrier concentration




$$n=\int_{W_c}^{\infty} g_c(W)\; f(W)\, dW$$

$$p=\int_{0}^{W_v} g_v(W)\left[1-f(W)\right] dW$$

# Calculation of carrier concentration

- The results is:

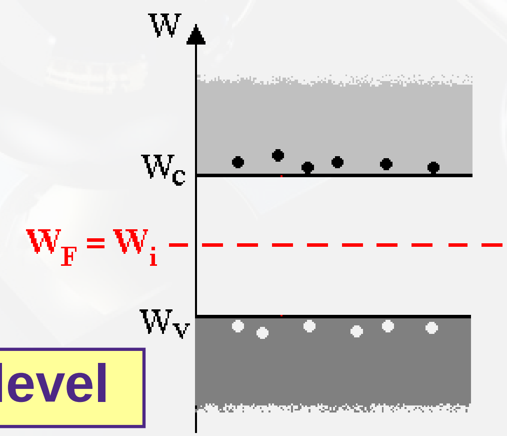
$$n = const\, T^{3/2} \exp\left(-\frac{W_c - W_F}{kT}\right)$$

$$p = const\, T^{3/2} \exp\left(-\frac{W_F - W_v}{kT}\right)$$

- If there is no doping: n = p = $n_i$
  - it is called ***intrinsic*** material

$$\boxed{W_c - W_F = W_F - W_v}$$

$$\boxed{W_F = \frac{W_c + W_v}{2}} = W_i$$



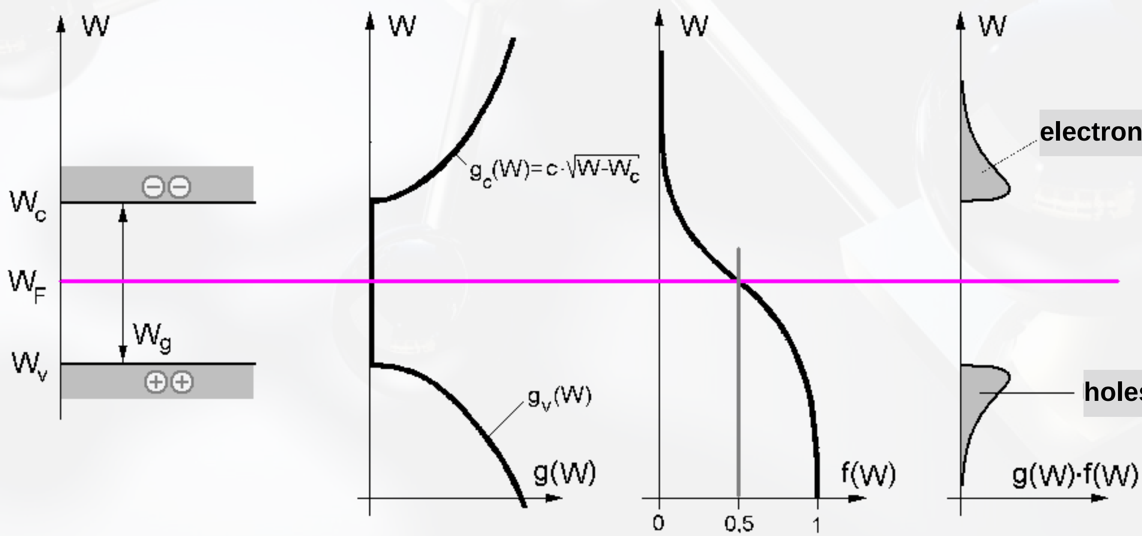

**$W_F$: Fermi-level**

# The Fermi-level

- Formal definition of the Fermi-level: the energy level where the probability of occupancy is 0.5:

$$f(W) = \frac{1}{1 + \exp\left(\frac{W - W_F}{kT}\right)} = 0.5$$

- In case of intrinsic semiconductor this is in the middle of the band gap:




$$\boxed{W_F = \frac{W_c + W_v}{2}}$$

- This is the intrinsic Fermi-level $W_i$

# Carrier concentrations

$$n = const\, T^{3/2} \exp\left(-\frac{W_c - W_F}{kT}\right)$$

$$p = const\, T^{3/2} \exp\left(-\frac{W_F - W_v}{kT}\right)$$

$$n \cdot p = const \cdot T^3 \exp\left(-W_g / kT\right)$$

- **Depends on temperature only**, does not depend on doping concentration

$$n \cdot p = n_i^2$$

Mass action law

For silicon at 300 K absolute temperature

$n_i = 10^{10}$ /cm$^3$

($10^{10}$ electrons in a cube of size 0.01 mm x 0.01 mm x 0.01 mm)

# Carrier concentrations

**Problem**

- Si, T = 300 K, donor concentration $N_D = 10^{17}\ /cm^3$

- **What are the hole and the electron concentrations?**
  - Donor doping $\Rightarrow$ $n \approx N_D =$ $\underline{\mathbf{10^{17}\ /cm^3}}$
  - Hole concentration: $p = n_i^2/n =$ $\mathbf{10^{20}/10^{17}} =$ $\underline{\mathbf{10^3/cm^3}}$

- **What is the relative density of the dopants?**
  - 1 $cm^3$ Si contains $5{\cdot}10^{22}$ atoms
  - thus, $10^{17}/\ 5{\cdot}10^{22}$ = $\underline{\mathbf{2{\cdot}10^{-6}}}$

  - **The purity of doped Si is 0.999998**

# Carrier concentrations

$$n = const\,T^{3/2}\exp\left(-\frac{W_c - W_F}{kT}\right)$$

$$n_i = const\,T^{3/2}\exp\left(-\frac{W_c - W_i}{kT}\right)$$

$$\frac{n}{n_i} = \exp\left(\frac{W_F - W_i}{kT}\right)$$

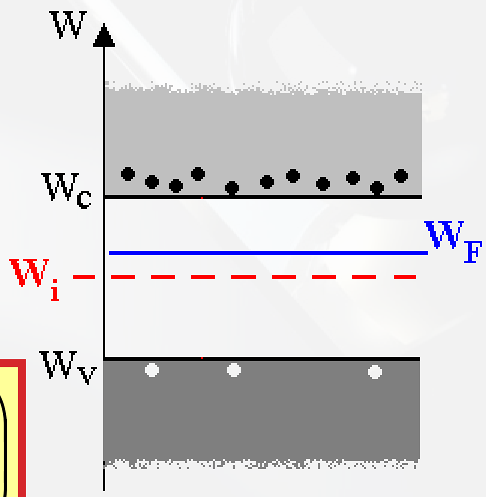
$$\boxed{n = n_i\,\exp\left(\frac{W_F - W_i}{kT}\right)}$$

$$\boxed{p = n_i\,\exp\left(-\frac{W_F - W_i}{kT}\right)}$$

- Just re-order the equations

**kT = 1.38·10$^{-23}$ VAs/K · 300 K =4,14 ·10$^{-21}$ J = 0.026 eV = 26 meV**

***Thermal energy***




**In doped semiconductors the Fermi-level is shifted wrt the intrinsic Fermi-level**

# Temperature dependence

$n_i^2$ = $$\boxed{n \cdot p = const \cdot T^3 \exp\left(-W_g / kT\right)}$$

$$\frac{d}{dT} n_i^2 = n_i^2 \left( \frac{3}{T} + \frac{W_g}{kT^2} \right)$$

$$\frac{d\, n_i^2}{n_i^2} = \left( 3 + \frac{W_g}{kT} \right) \frac{d\,T}{T}$$

- How strong is it for Si? **Problem**

$$\boxed{\frac{d\, n_i^2}{n_i^2} = \left( 3 + \frac{1{,}12}{0{,}026} \right) \frac{d\,T}{300} \cong 0.15\, d\,T}$$

**≈ 15% / ºC**

# Temperature dependence of carrier concentrations

**Problem**

**Si, T = 300 K, donor dopant concentration $N_D = 10^{17}$ /cm$^3$**

**$n \cong N_D = 10^{17}$ /cm$^3$**

**$p = n_i^2 / n = 10^{20}/10^{17} = 10^3$/cm$^3$** $\Leftarrow \; n \cdot p = n_i^2$

**How do n and p change, if T is increased by 25 degrees?**

**$n \cong N_D = 10^{17}$ /cm$^3$ – unchanged**

**$n_i^2 = 10^{20} \cdot 1.15^{25} = 33 \cdot 10^{20}$**

**$\Rightarrow p = n_i^2 / n = 33 \cdot 10^{20}/10^{17} = 3.3 \cdot 10^4$/cm$^3$**

**Only the minority carrier concentration increased!**

**$\Delta T = 16.5\ ^oC \rightarrow 10\times$**

# Currents in semiconductors

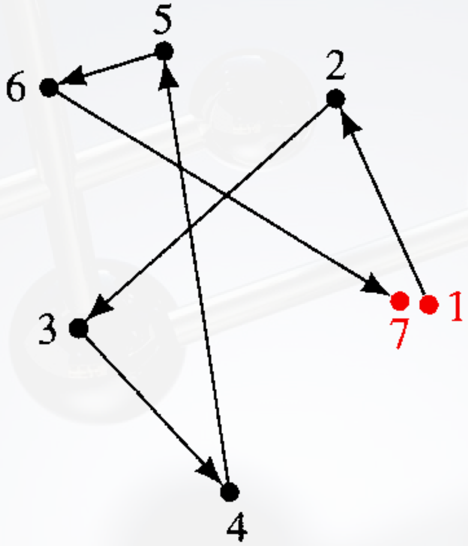
- Drift current
- Diffusion current

Not discussed:

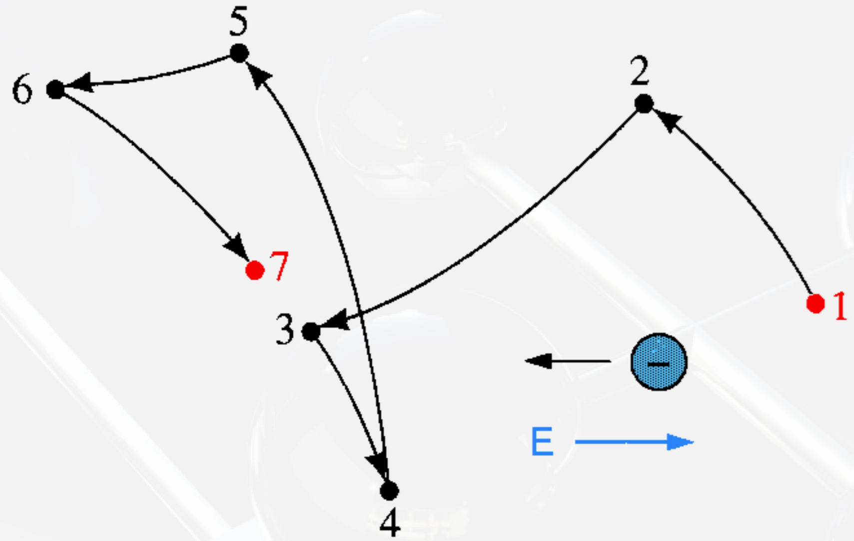
- currents due to temperature gradients
- currents induced by magnetic fields
- energy transport besides carrier transport
- combined transport phenomena

# Drift current

## Thermal movement of electrons




**No electrical field**




**There is E electrical field**

$$\boxed{\overline{v}_s = \mu \overline{E}}$$

**μ = mobility**

**m²/Vs**

# Drift current

$$\overline{J} = \rho\,\overline{v}$$

ρ **charge density**

v **velocity (average)**

$$\overline{v}_s = \mu \overline{E}$$

$$\overline{J}_n = q\,n\,\mu_n\,\overline{E}$$

$$\overline{J}_p = q\,p\,\mu_p\,\overline{E}$$

$$\overline{J} = q\left(n\,\mu_n + p\,\mu_p\right)\overline{E}$$

$$\overline{J} = \sigma_e\,\overline{E}$$

**Differential Ohm's law**

$$\rho_e = \frac{1}{\sigma_e}$$

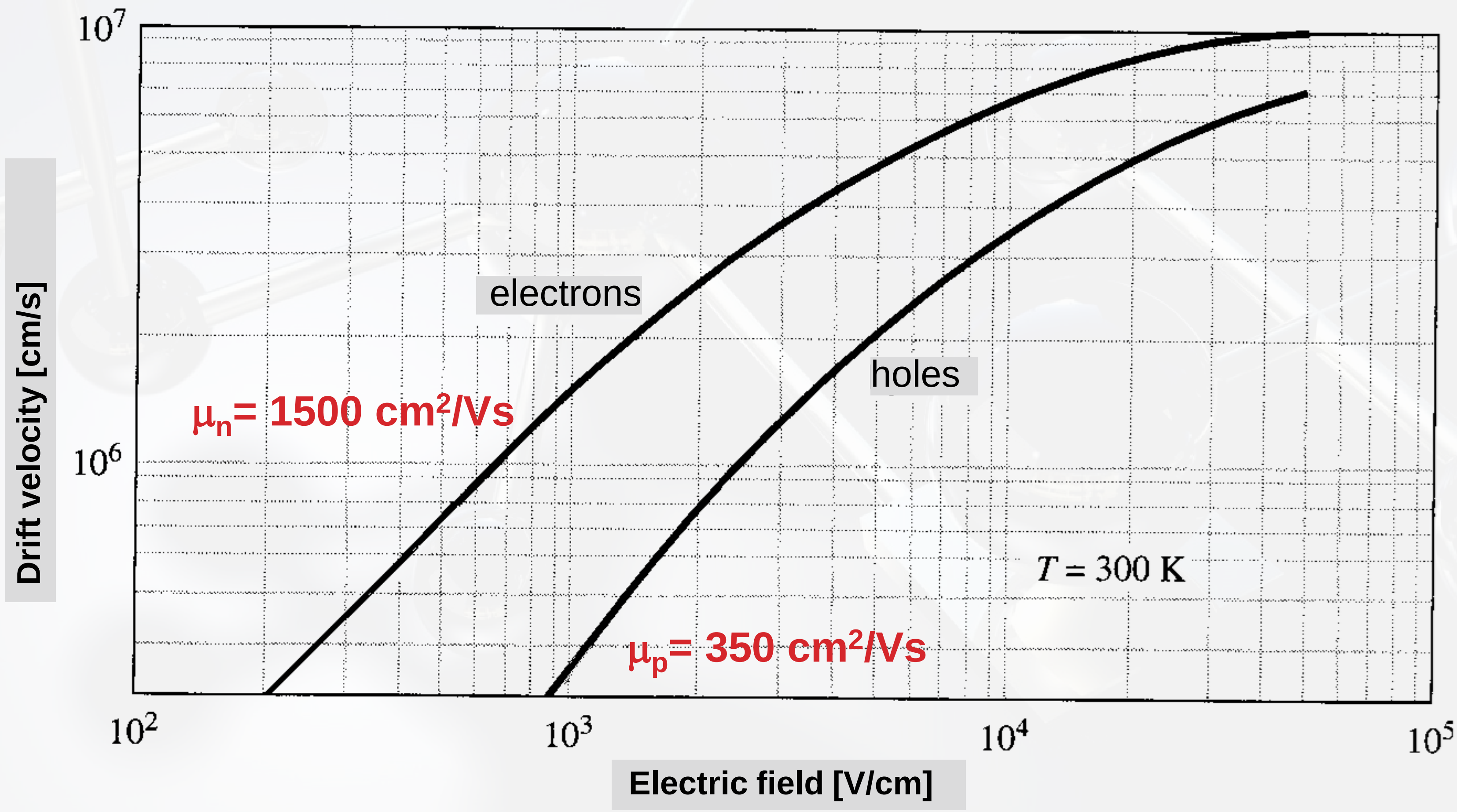
**Specific resistance**

$$\sigma_e = q\left(n\,\mu_n + p\,\mu_p\right)$$

Specific electrical conductivity of the semiconductor

# Carrier mobilities

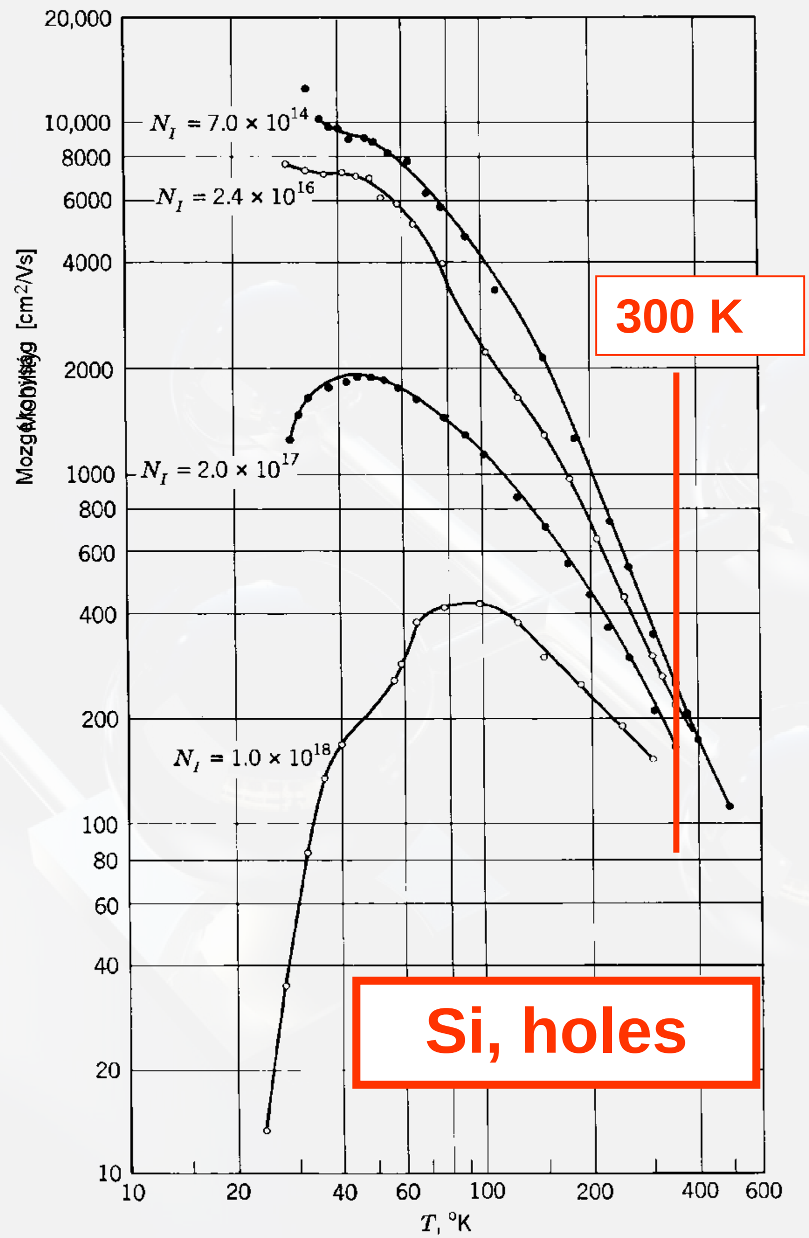
**Si**



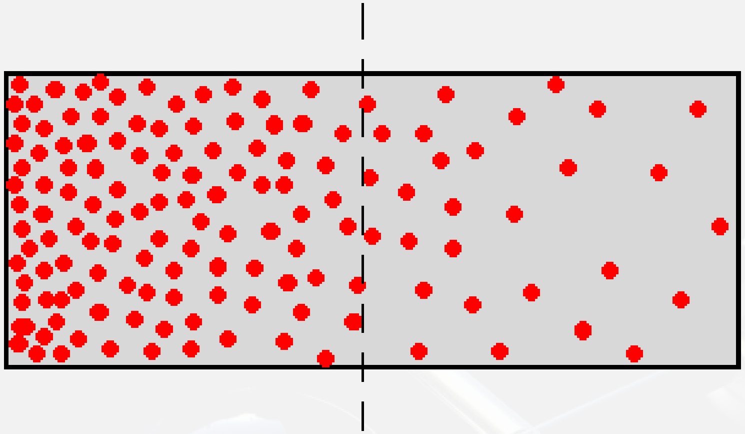
# Carrier mobilities

- Mobility decreases with increasing doping concentration
- At around room temperature mobilities decrease as temperature increases

$$\mu \sim T^{-3/2}$$



# Diffusion current



- Reasons:
  - concentration difference (gradient)
  - thermal movement
- Proportional to the gradient
- D: diffusion constant [$m^2/s$]

$$\overline{J_n} = q\, D_n\, \overline{\mathrm{grad}\; n}$$

$$\overline{J_p} = -q\, D_p\, \overline{\mathrm{grad}\; p}$$

# Total currents

$$\boxed{\overline{J_n} = qn\mu_n \overline{E} + q\,D_n\,\overline{\mathrm{grad}}\; n}$$

$$\boxed{\overline{J_p} = qp\mu_p \overline{E} - q\,D_p\,\overline{\mathrm{grad}}\; p}$$

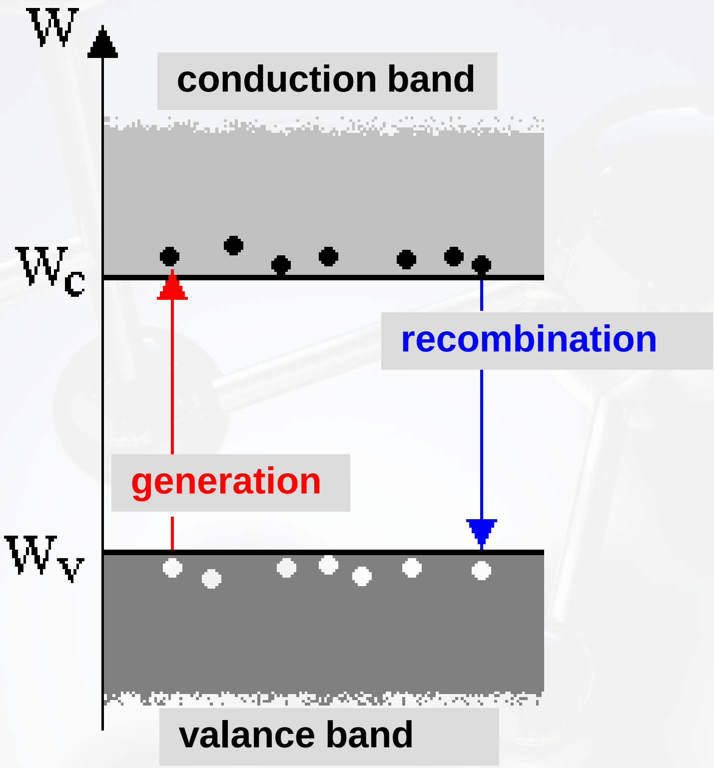
$$\boxed{D = \frac{kT}{q}\mu}$$

**Einstein's relationship**

$$U_T = \left.\frac{kT}{q}\right|_{T=300K} = \frac{1.38\cdot 10^{-23}\,[\mathrm{VAs/K}]\cdot 300[\mathrm{K}]}{1.6\cdot 10^{-19}\,[\mathrm{As}]} \cong 0.026\ \mathrm{V} = 26\,\mathrm{mV}$$

**Thermal voltage**

# Generation, recombination




- ***Life-time:*** average time an electron spends in the conduction band
  - This value is influenced by the impurities (recombination centrals, allowed states in the band gap)

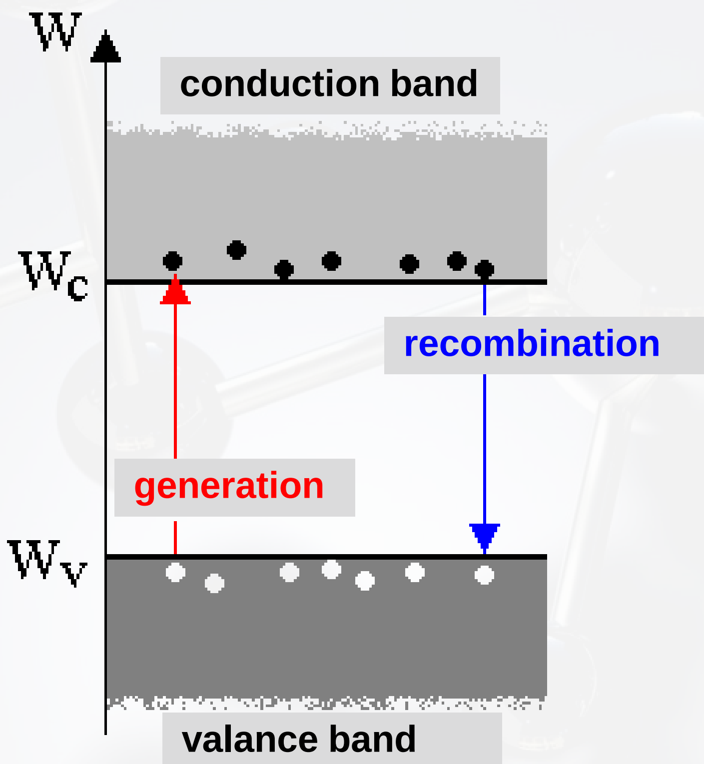
$\boxed{\tau_n, \tau_p}$ **1 ns … 1 μs**

  - If: $\tau_n$ the life-time of an e$^-$→probability of recombination within dt time is dt/$\tau_n$
- ***Recombination rate:*** ***r*** [1/m$^3$s]

Number of recombinated carriers in a unit volume within a unit timeframe.

$$r_n = \frac{n \cdot dt}{\tau_n} \xrightarrow{\text{unit time, unit V}} r_n = \frac{n}{\tau_n}$$

# Generation, recombination




$$r_n = \frac{n}{\tau_n}$$

$$r_p = \frac{p}{\tau_p}$$

- ***Life-time:*** average time an electron spends in the conduction band

$$\boxed{\tau_n, \tau_p}$$

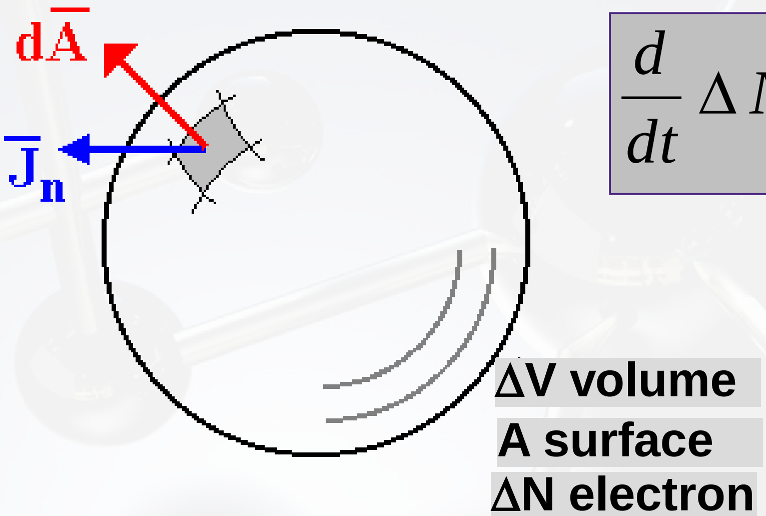
**1 ns … 1 µs**

- ***Generation rate:*** ***g*** [1/m³s]

- ***Recombination rate:*** ***r*** [1/m³s]

$$g_n = r_n\Big|_{equilibrium} = \frac{n_0}{\tau_n}$$

# Continuity equation




$$\frac{d}{dt}\Delta N = -\frac{1}{-q}\oint_A \overline{J_n}\, d\overline{A} + g_n \cdot \Delta V - \frac{n}{\tau_n}\Delta V$$

$$\frac{d}{dt}\frac{\Delta N}{\Delta V} = \frac{1}{q}\frac{1}{\Delta V}\oint_A \overline{J_n}\, d\overline{A} + g_n - \frac{n}{\tau_n}$$

$$\boxed{\frac{dn}{dt} = \frac{1}{q}\operatorname{div}\left(\overline{J_n}\right) + g_n - \frac{n}{\tau_n}}$$

# Diffusion equation

$$\frac{dn}{dt} = \frac{1}{q}\operatorname{div}\left(\overline{J_n}\right) + g_n - \frac{n}{\tau_n}$$

$$\overline{J_n} = qn\mu_n \overline{E} + q D_n \overline{\operatorname{grad}} \, n$$

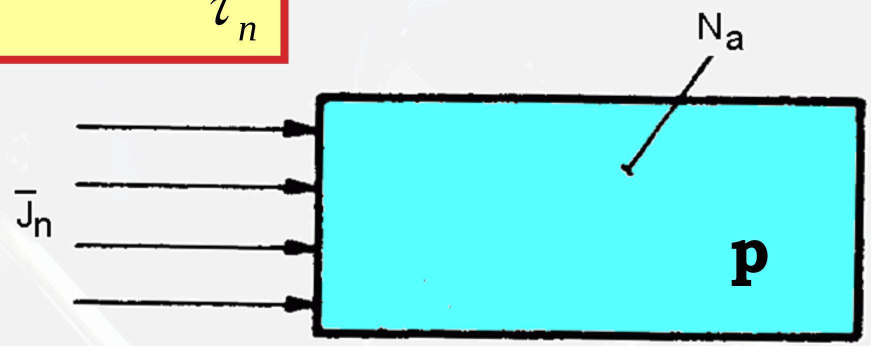
$$\boxed{\frac{dn}{dt} = \mu_n \operatorname{div}\left(n\overline{E}\right) + D_n \operatorname{div}\operatorname{grad} n + g_n - \frac{n}{\tau_n}}$$

$$\boxed{\frac{dp}{dt} = -\mu_p \operatorname{div}\left(p\overline{E}\right) + D_p \operatorname{div}\operatorname{grad} \, p + g_p - \frac{p}{\tau_p}}$$

# Example for the solution of the diff. eq.:

$$\frac{dn}{dt} = \mu_n \,\mathrm{div}\left(n\overline{E}\right) + D_n \,\mathrm{divgrad}\, n + g_n - \frac{n}{\tau_n}$$

- Si block doped homogenously by p
- e⁻ injected with constant current density
- e⁻ are moving according to the diffusion (E=0) while recombinating
- Steady-state (dn/dt=0)



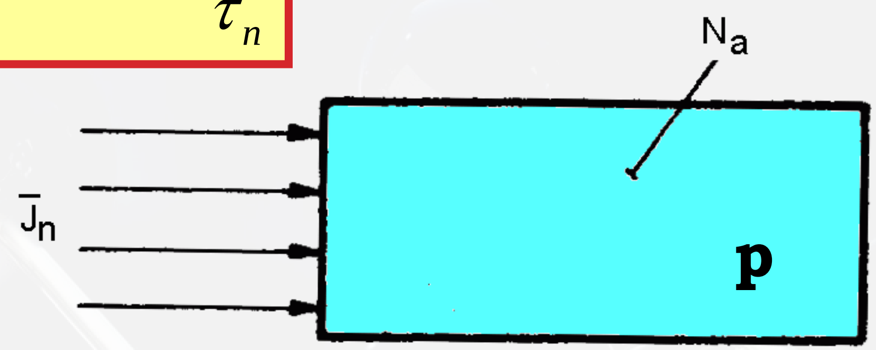

- What is the n(x) distribution of the injected minority carriers?
- What is the average penetration depth, prior to recombination?

# Example for the solition of the diff. eq.:

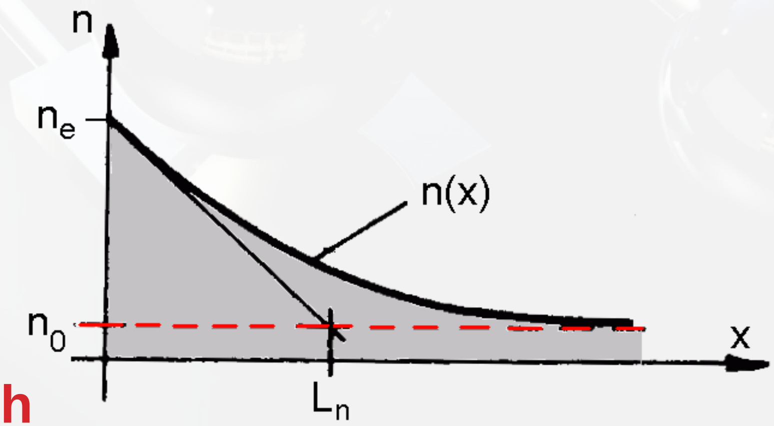
$$\frac{dn}{dt} = \mu_n \operatorname{div}\left(n\overline{E}\right) + D_n \operatorname{divgrad} n + g_n - \frac{n}{\tau_n}$$

$$0 = D_n \frac{d^2 n}{dx^2} + g_n - \frac{n}{\tau_n}$$

$$0 = D_n \frac{d^2 n}{dx^2} + \frac{n_0}{\tau_n} - \frac{n}{\tau_n}$$




$$n(x) = n_0 + (n_e - n_0)\exp(-x/\sqrt{D_n \tau_n})$$




$$L_n = \sqrt{D_n \tau_n}$$

**diffusion length**

# Omne ignotum pro magnifico.

Everything unknown seems magnificent.
Tacitus, 55-120 AD, Roman historian