Temperature sensors

Industrial control

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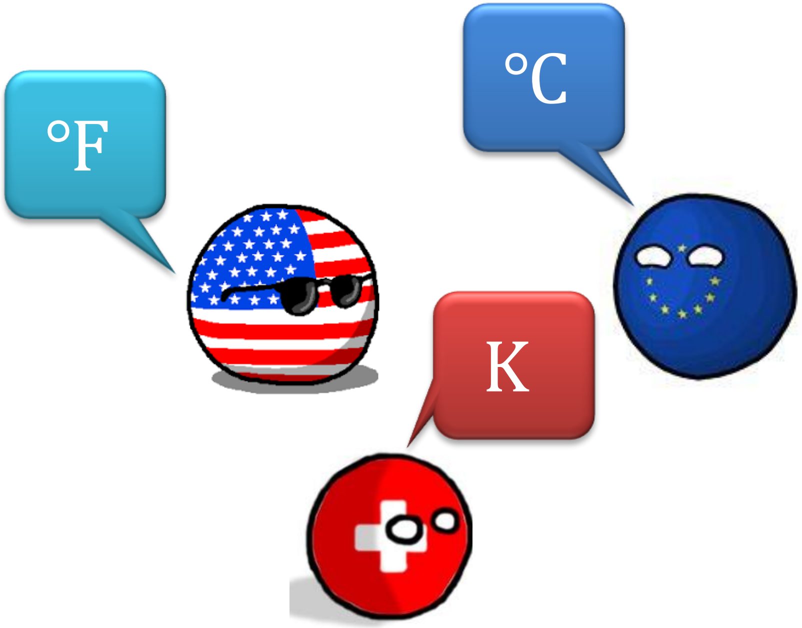
Temperature sensing in practice

- How would you define temperature?
  - Scalar material property
  - Related to the average kinetic energy of particles
  - Statistical concept

- In practice: „temperature is what we measure using a thermometer”

Units

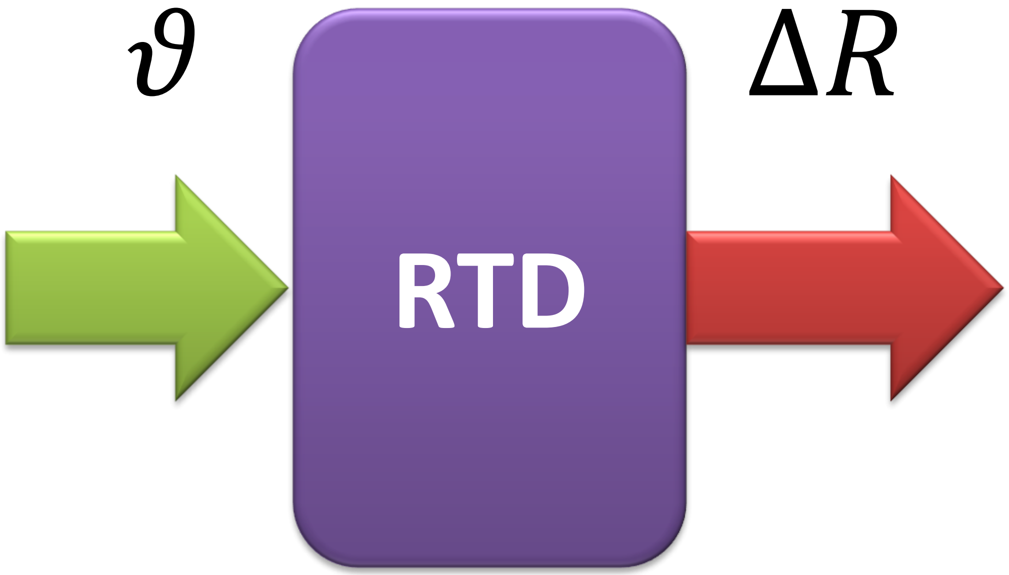
- Celsius scale (ϑ)
  - $0°\mathrm{C}$ - freezing point of water
  - $100°\mathrm{C}$ - boiling point of water
- Kelvin scale (T)
  - $0\mathrm{K}$ – absolute zero temperature
  - unit equals to Celsius unit
  - $0°\mathrm{C} = 273{,}15\mathrm{K}$
- Fahrenheit scale
  - $32°\mathrm{F}$ - freezing point of water
  - $212\ °\mathrm{F}$ - boiling point of water
  - $\vartheta_{°\mathrm{C}} = \frac{5}{9}(\vartheta_{°\mathrm{F}} - 32)$



Temperature sensors

- Resistance Temperature Detectors (RTD)
  - metal
  - semiconductor
- Integrated circuit (IC) thermometers
- Thermocouples
- Other devices

Resistance temperature detectors



Metal RTDs

- Nuclei and ions oscillate with a higher magnitude due to increasing temperature
- Flowing electrons collide more frequently and therefore slow down
- Less electrons leave when the same electric field is applied, i.e. resistance of the material increases

Metal RTDs

- **Copper**
  - linear characteristics
  - low price
  - low resistance
  - low stability, susceptible to oxidation
  - measurement range: $-100℃ \ldots + 180℃$

- **Nickel**
  - high sensitivity
  - highly nonlinear characteristics
  - measurement range: $-100℃ \ldots 180℃$

Metal RTDs

- **Molybdenum**
  - simple to manufacture as thin film resistor
  - stable
  - measurement range: $-50℃ \dots + 200℃$

- **Special RTDs**
  - *Germanium*: below 100K
  - *Carbon-Glass*: below 10K
  - *Rhodium-Iron*: down to 0.5K

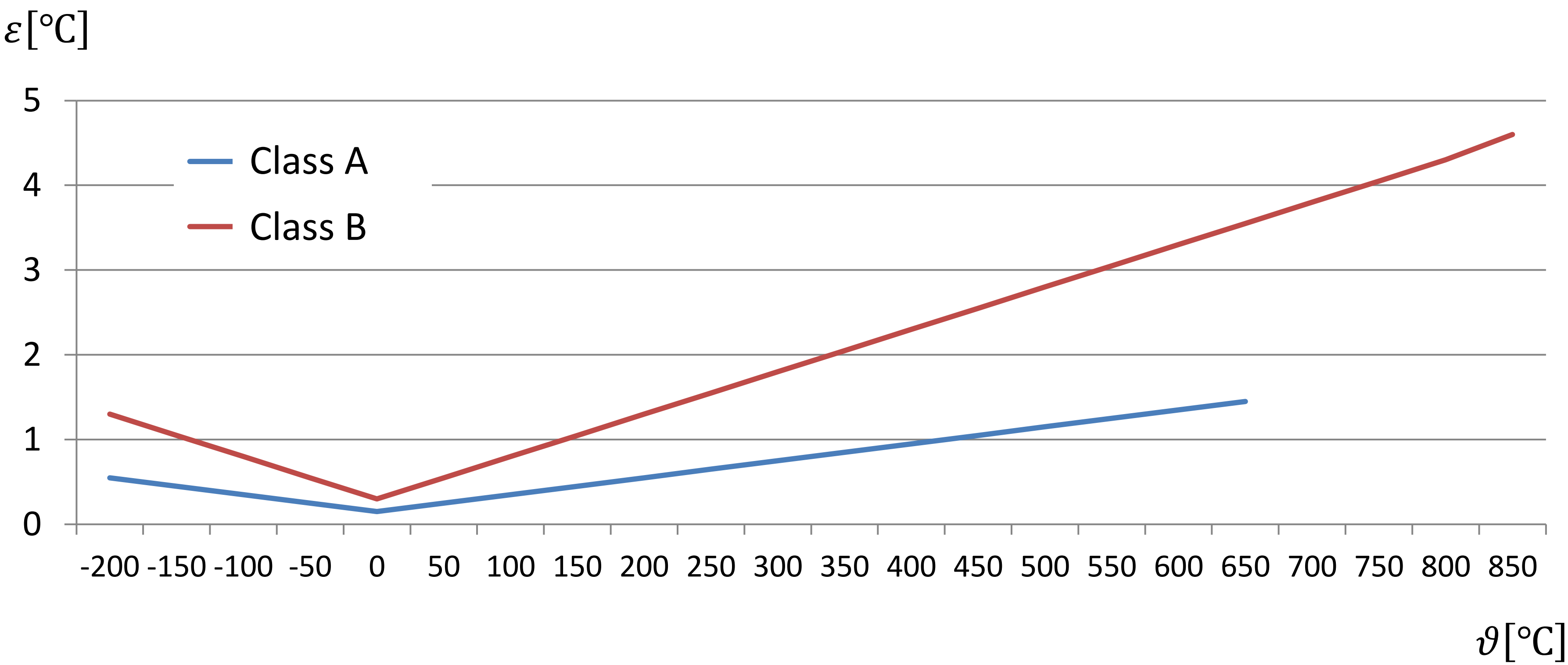
RF-800

Metal RTDs

- **Platinum**
  - relatively expensive
  - nearly linear characteristics in an important range
  - can be manufactured with high purity
  - exceptionally stable
  - high accuracy
  - wide measurement range
  - standardized types
    - Pt100: 100Ω resistance at $0°\text{C}$
    - Pt1000: $1\text{k}\Omega$ resistance at $0°\text{C}$

Tolerance classes of Platinum RTDs

- Class A: $-200°\mathrm{C} \ldots +650°\mathrm{C}$, $\varepsilon \leq 1.45°\mathrm{C}$
- Class B: $-200°\mathrm{C} \ldots +850°\mathrm{C}$, $\varepsilon \leq 4.6°\mathrm{C}$



Static characteristics of Platinum RTDs

- ITS-90 standard:
  - below 0 °C: characteristics defined by a 12th order polynomial
  - above 0 °C: characteristics defined by a 9th order polynomial

- Callendar - Van Dusen equation (IEC 60751):
  - $R_\vartheta = R_0(1 + A\vartheta + B\vartheta^2 + (\vartheta - 100)C\vartheta^3), \vartheta < 0\ °\mathrm{C}$
  - $R_\vartheta = R_0(1 + A\vartheta + B\vartheta^2), 0\ °\mathrm{C} \leq \vartheta \leq 850 °\mathrm{C}$
    - ϑ: temperature [°C]
    - R_ϑ: resistance at ϑ °C temperature
    - R_0: resistance at $\vartheta_0 = 0 °\mathrm{C}$ temperature

  - $A = 3.90803 \cdot 10^{-3}$
  - $B = -5.775 \cdot 10^{-7}$
  - $C = -4.183 \cdot 10^{-12}$

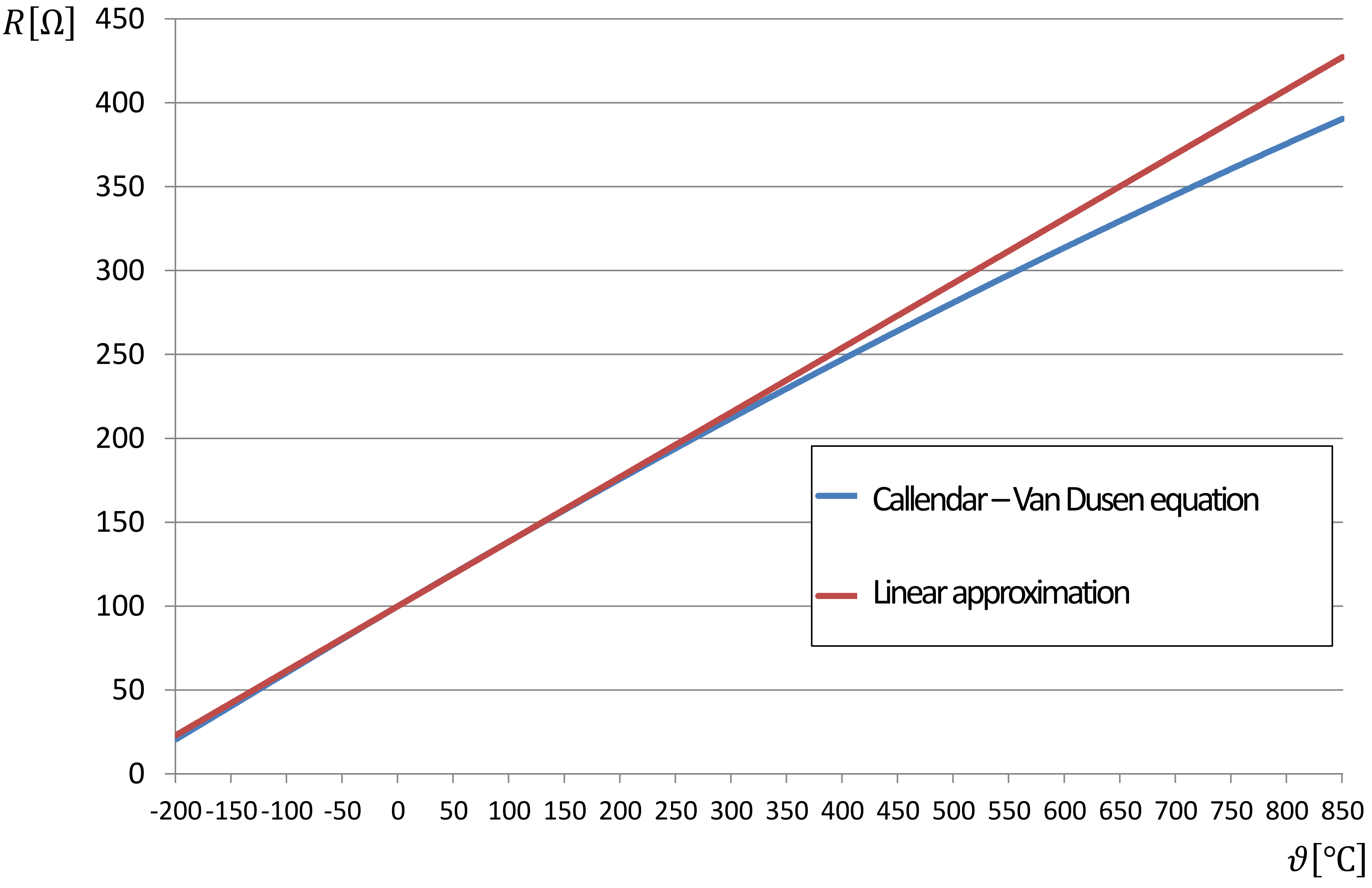
Static characteristics of platinum RTDs

- Linear approximation:

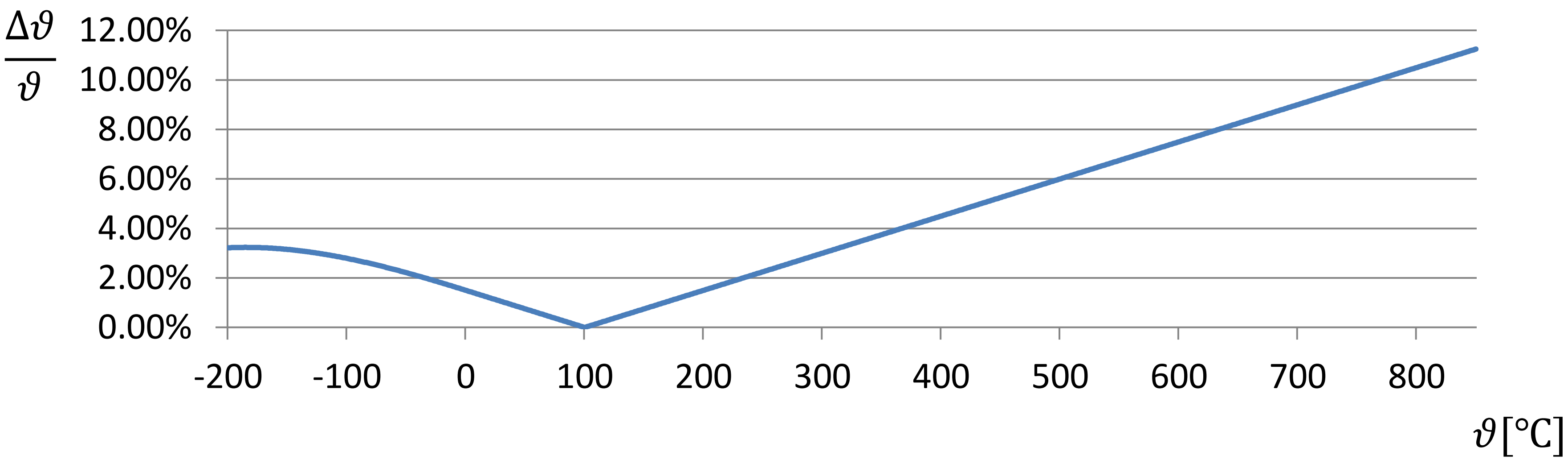
$$R_\vartheta = R_0(1 + A\vartheta + B\vartheta^2) \approx R_0(1 + \alpha\vartheta) = R_0 + \Delta R$$

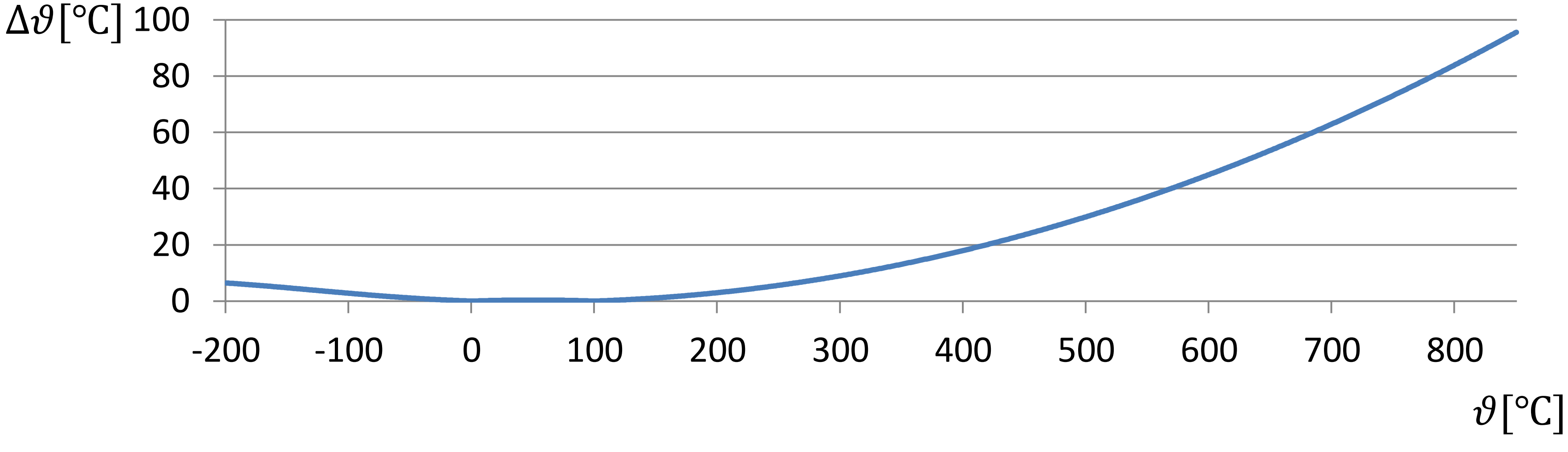
  - $\alpha = \frac{R_\vartheta - R_0}{\vartheta R_0} := \frac{R_{100} - R_0}{100 R_0} \approx 0.00385 \frac{1}{°\mathrm{C}}$
  - $\Delta R = R_0 \alpha \vartheta$
  - $\boldsymbol{R_\vartheta} = R_0 + \Delta R = \boldsymbol{R_0(1 + \alpha\vartheta)}$
  - $\boldsymbol{\vartheta} = \frac{R_\vartheta - R_0}{R_0}\frac{1}{\alpha} = \boldsymbol{\frac{\Delta R}{R_0}\frac{1}{\alpha}}$
  - For Pt100 ($R_0 = 100\Omega$) : $\vartheta = \frac{\Delta R}{100\alpha} = \frac{\Delta R}{0.385}$

Linear approximation



Error of linear approximation





Self-heating

- A resistor dissipates power if current flows through it:

$$P = UI = I^2 R$$

- Dissipated power increases the temperature of the resistor:

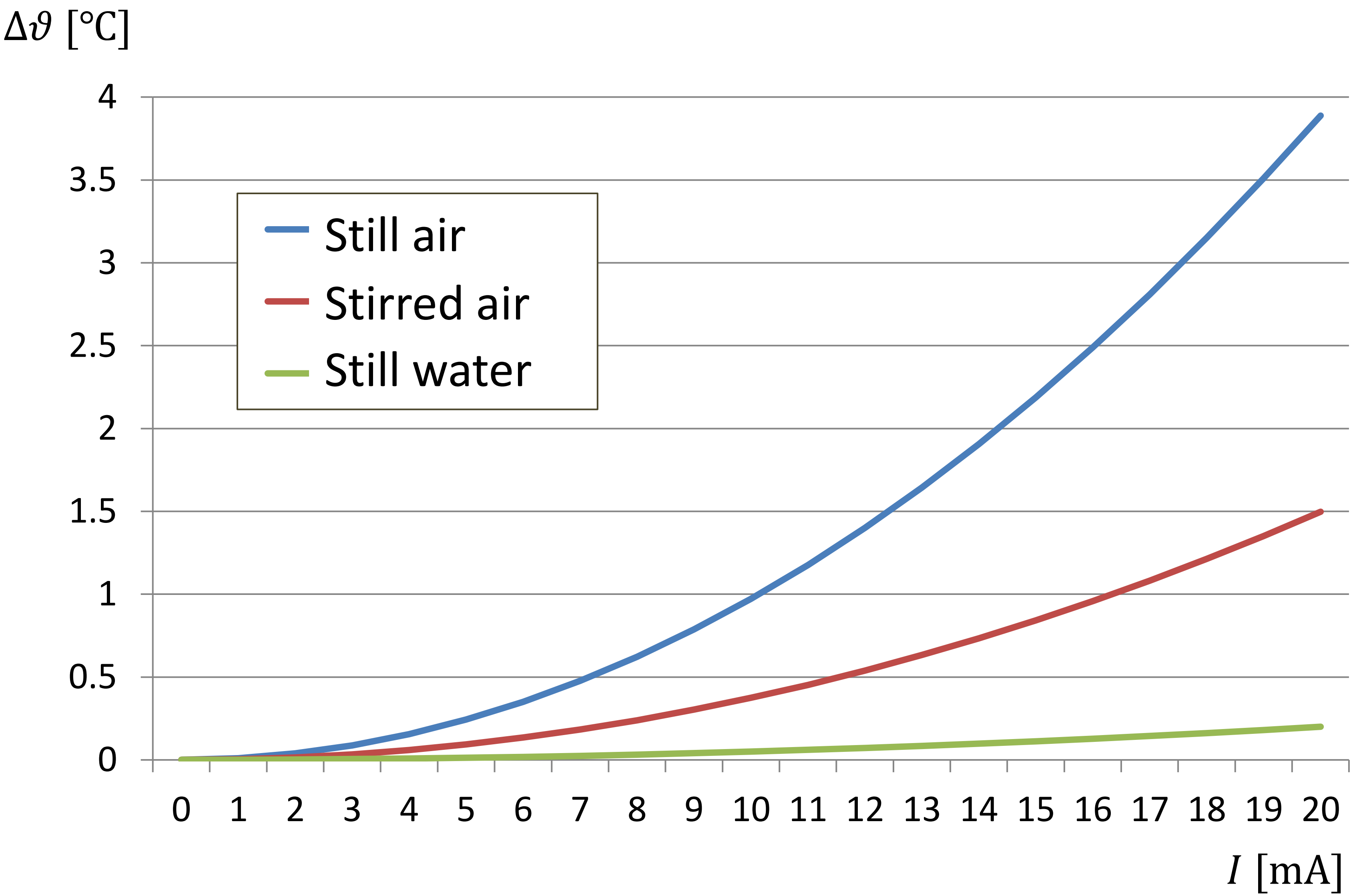
$$\boldsymbol{\Delta\vartheta = P/C = I^2 R_\vartheta / C}$$

  - C: Dissipation constant – power increasing the temperature of the resistor by $1°\mathrm{C}$ in steady state $\left[\frac{W}{°\mathrm{C}}\right]$

- Value of the dissipation constant depends on
  - type of medium (water/air/… , still/stirred)
  - construction of the sensor

- Typical range: $1 - 100\ \mathrm{mW}/°\mathrm{C}$

Effect of current on self-heating



Problem

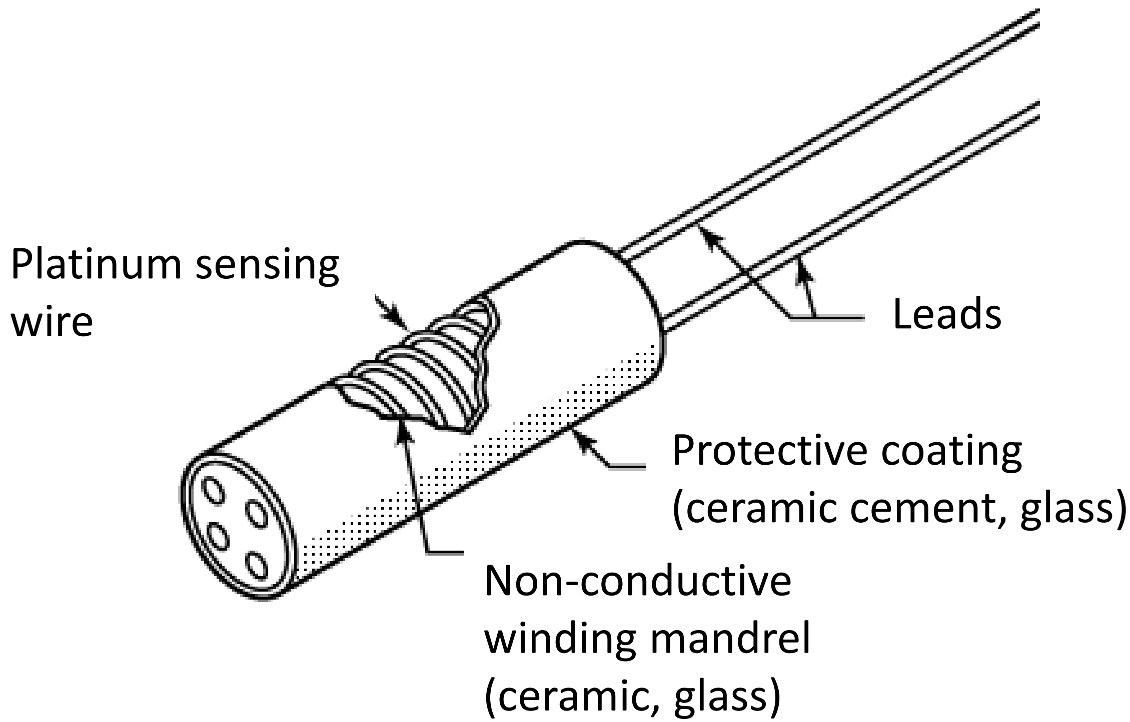
Dissipation constant of a Pt100 RTD is $12\ \mathrm{mW/°C}$ for steady air. Give the maximal current applicable if self-heating of the sensor should be at most $1\mathrm{°C}$ at $20\mathrm{°C}$ ambient temperature!

$$\Delta\vartheta_{max} = \frac{I_{max}^2 R_\vartheta}{C} \Rightarrow I_{max} = \sqrt{\frac{C\Delta\vartheta_{max}}{R_\vartheta}}$$

$$I_{max} = \sqrt{\frac{12 \cdot 0.001 \cdot 1}{100 + 20 \cdot 0.385}} = 0.0105\mathrm{A} = 10.5\mathrm{mA}$$

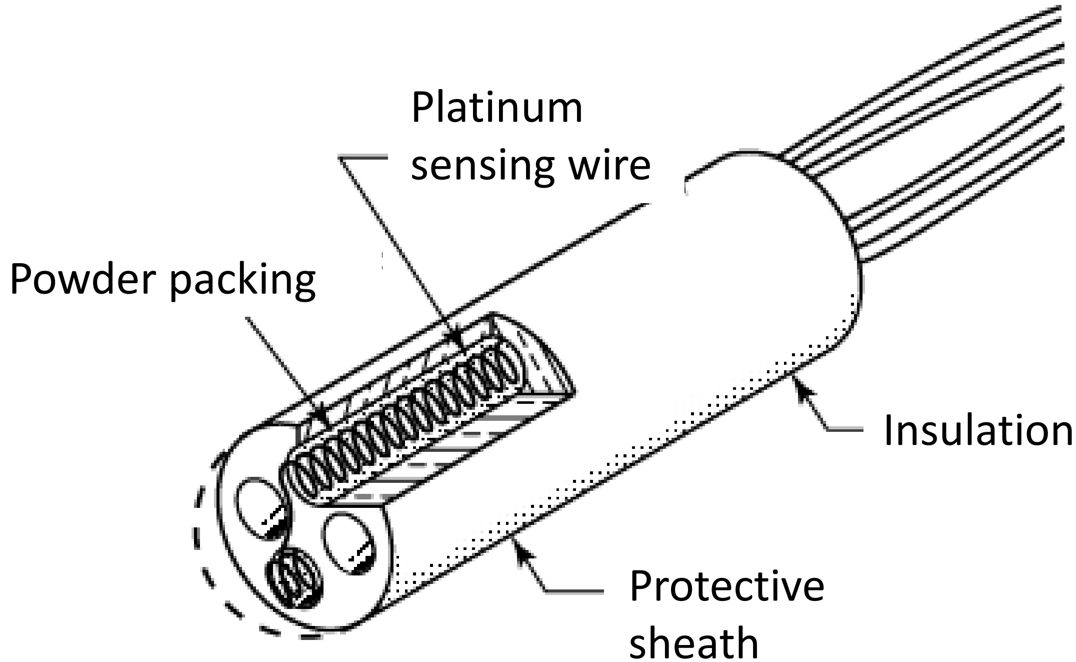
Construction types of Platinum RTDs

Wire-wound RTDs



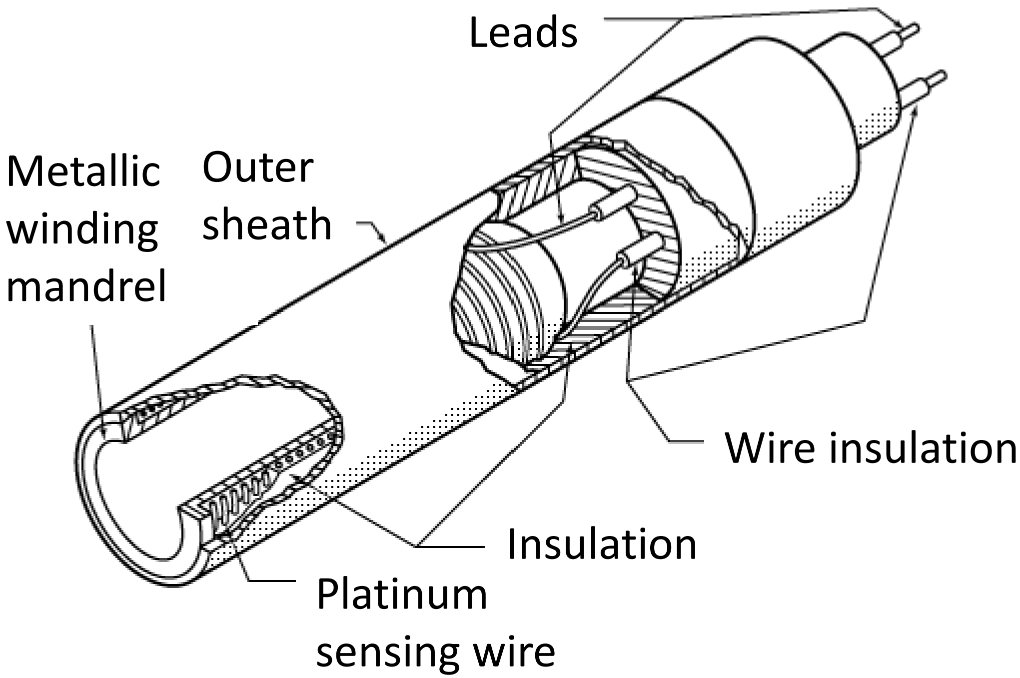
- Thermal expansion of the elements are similar
- Reduced tolerance for cyclic change of temperature
- Significant hysteresis
- Measurement range up to $500°C$

Coiled element RTDs



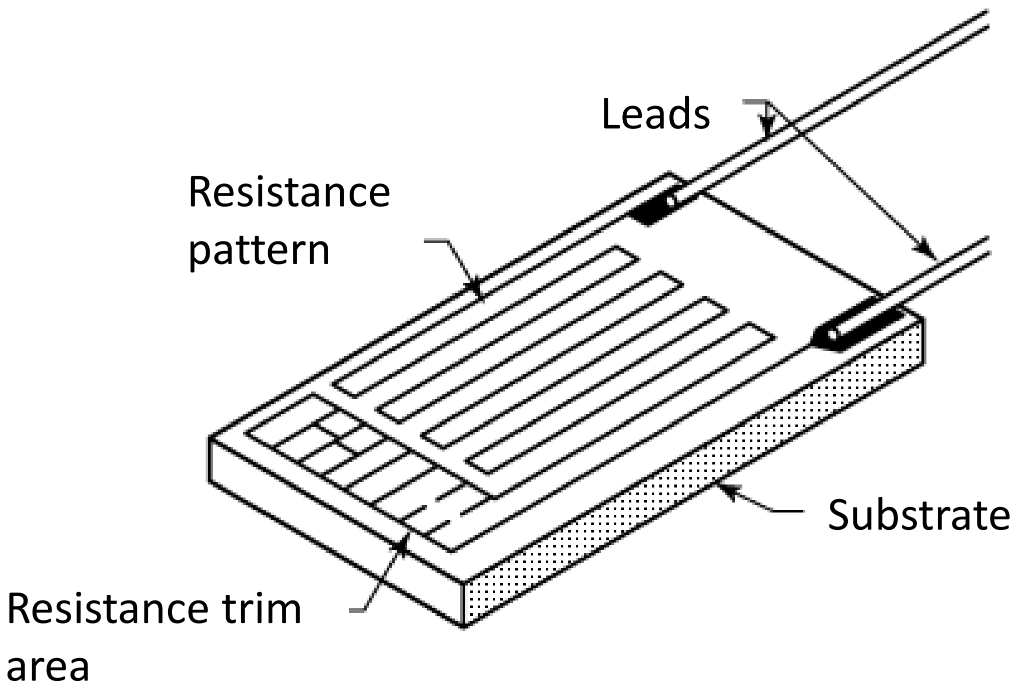
- Free of mechanical stress
- If hermetic sealing is not required, air might flow around helical sensing wires
- Measurement range: $-200℃ \dots +850℃$

Hollow annulus type RTD-s



- Fully insulated
- Low time constant
- Long winding length, i.e. long platinum sensing wire with high resistance can be used
- High cost

Film-type RTDs



- Possible to trim with high accuracy
- Large area, low time constant
- Insensitive to resonance
- Measurement range: $-50℃ \ldots 500℃$

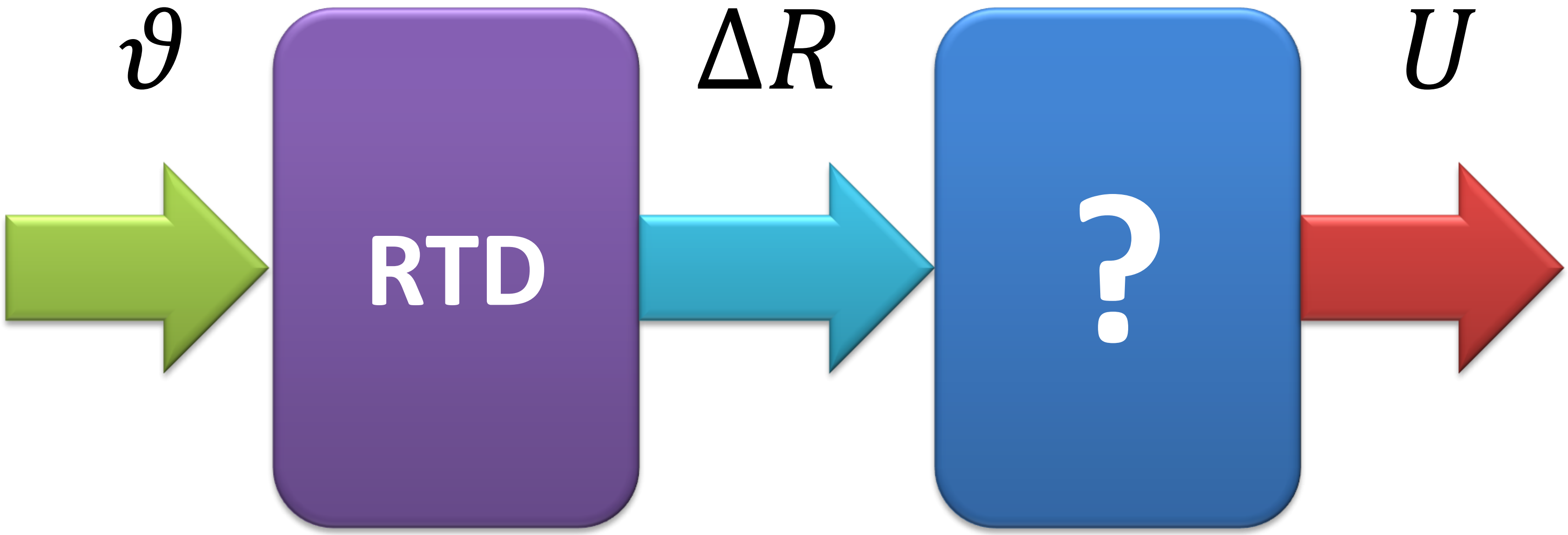
RTD probe assemblies

- Probe
  - copper (rare)
  - stainless steel
  - inconel
  - ceramics

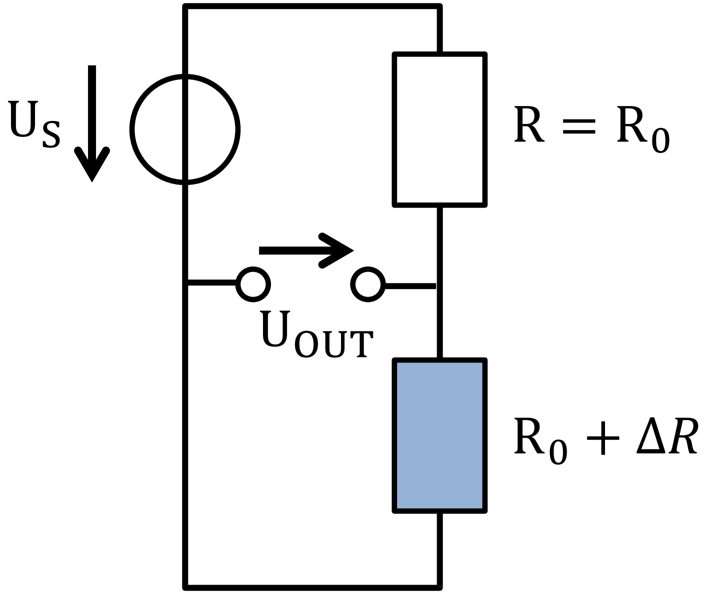
- Head
  - wiring
  - transmitter device
  - optional display

- Mounting elements

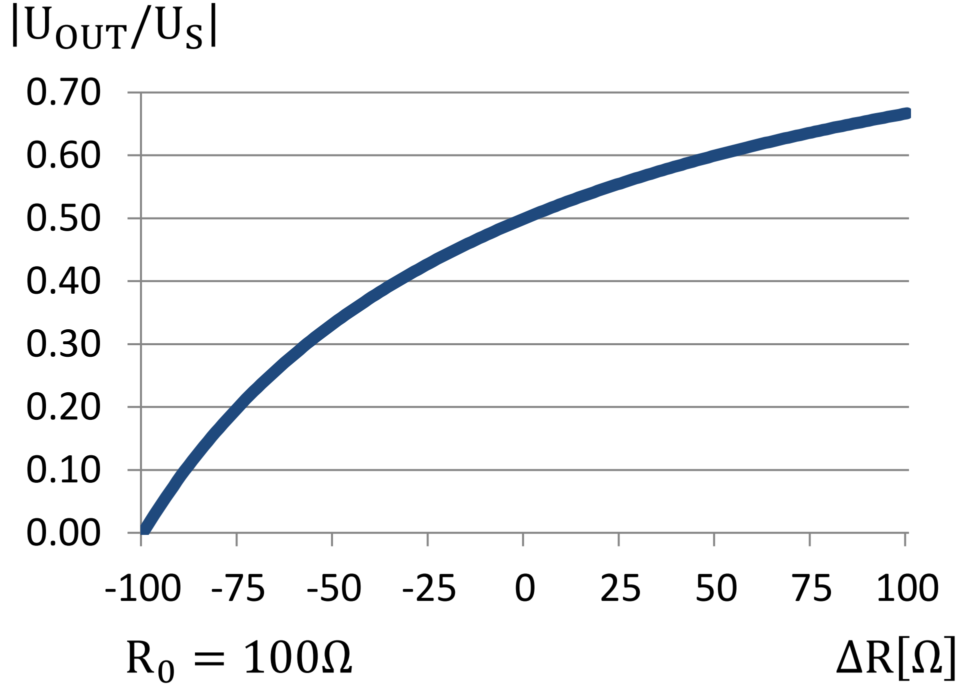
Measurement circuits for RTDs



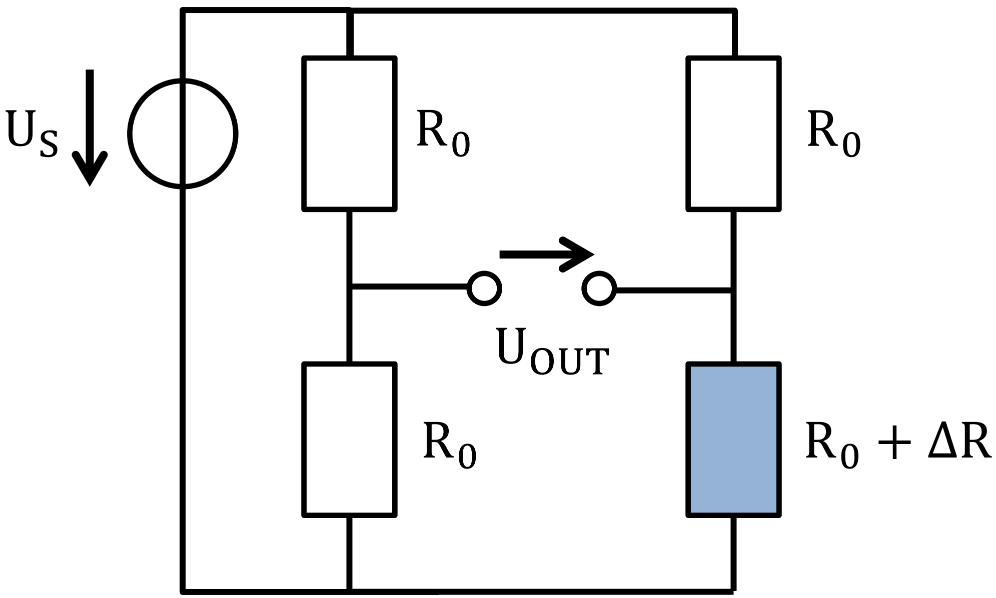
Voltage divider

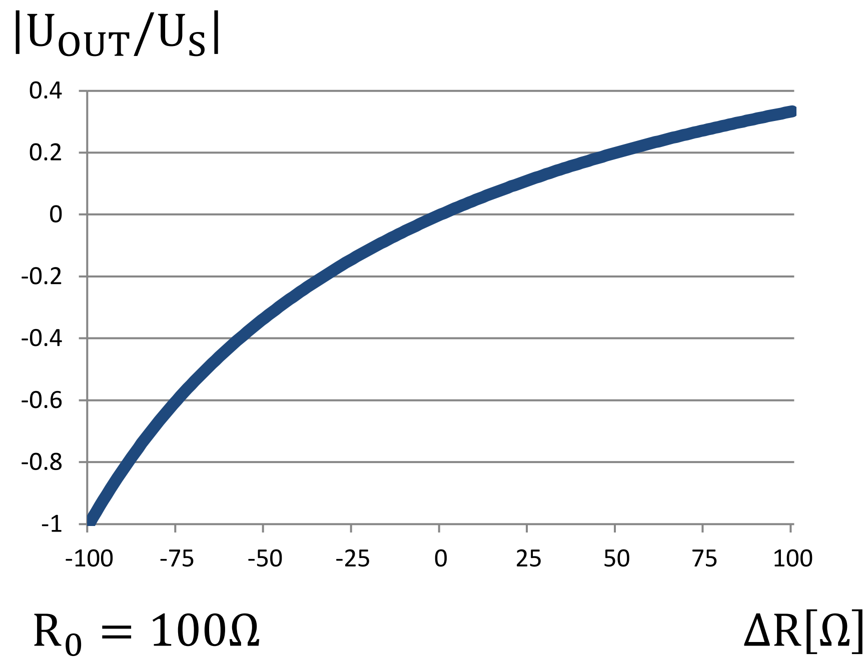


$$U_{OUT} = -U_S \frac{R_0 + \Delta R}{2R_0 + \Delta R}$$



Bridge circuit

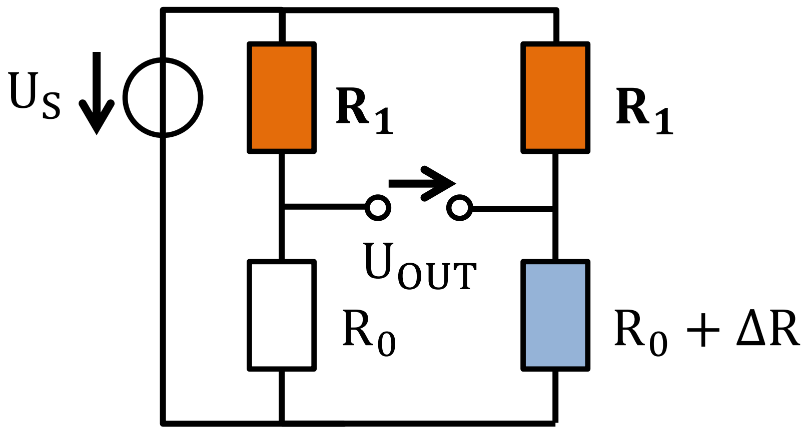




$$U_{OUT} = \frac{U_S}{2} - U_S\frac{R_0 + \Delta R}{2R_0 + \Delta R} = \frac{U_S}{2}\left(\frac{2R_0 + \Delta R - 2R_0 - 2\Delta R}{2R_0 + \Delta R}\right) =$$

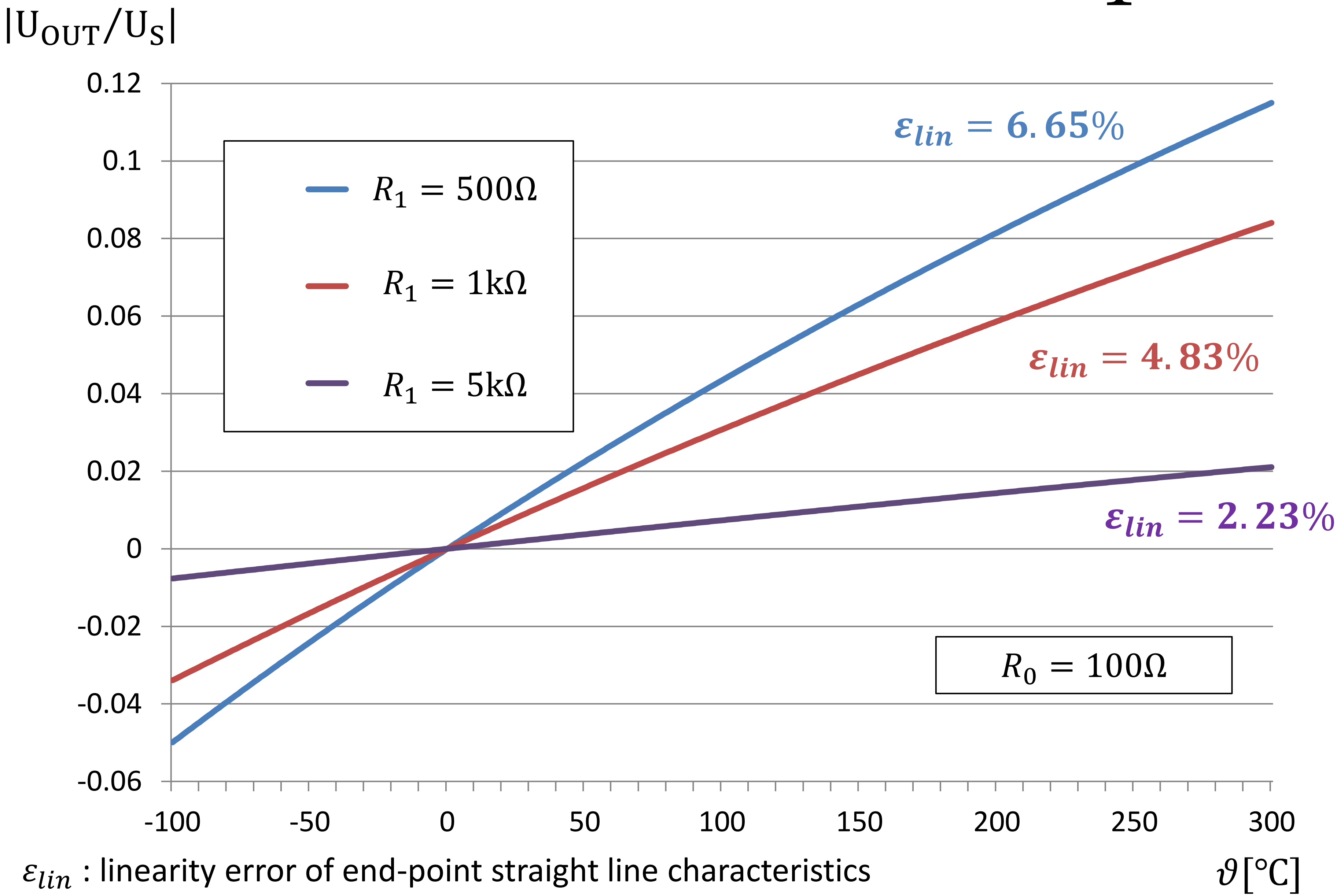
$$= -\frac{U_S}{2}\frac{\Delta R}{2R_0 + \Delta R}$$

Bridge circuit



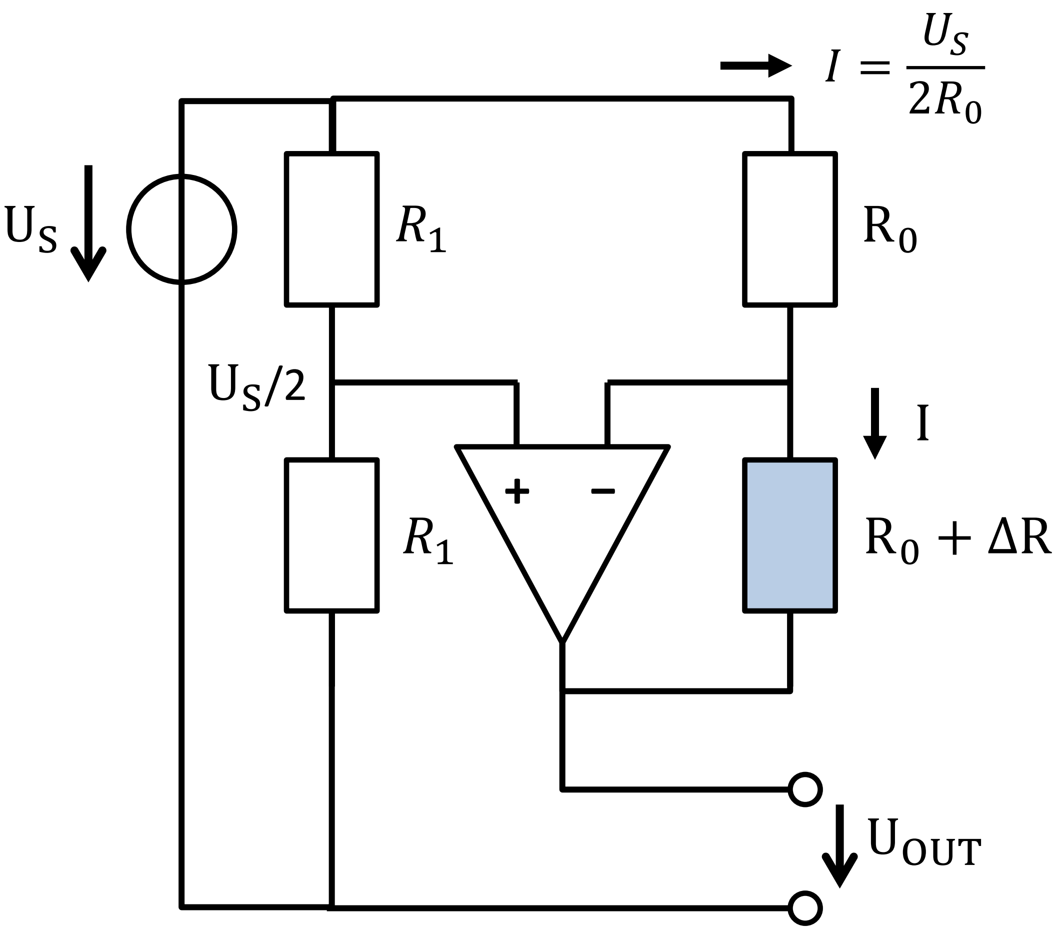
- $U_{OUT} = U_S \frac{R_0}{R_1+R_0} - U_S \frac{R_0+\Delta R}{R_1+R_0+\Delta R}$
- $R_1 \gg R_0, \mathrm{R}_1 \gg \Delta R \Rightarrow U_{OUT} \approx U_S \frac{R_0}{R_1+0} - U_S \frac{R_0+\Delta R}{R_1+0+0} = -U_S \frac{\Delta R}{R_1}$
- By using a resistor $R_1 \gg R_0$, linearity error can be significantly decreased, but then the output voltage is also decreased
- To achieve higher sensitivity the supply voltage needs to be increased

Effect of the resistance R_1



ε_{lin} : linearity error of end-point straight line characteristics

Linearizing the resistance-voltage characteristics with active bridge circuit



$$U_{OUT} = \frac{U_S}{2} - I(R_0 + \Delta R) =$$

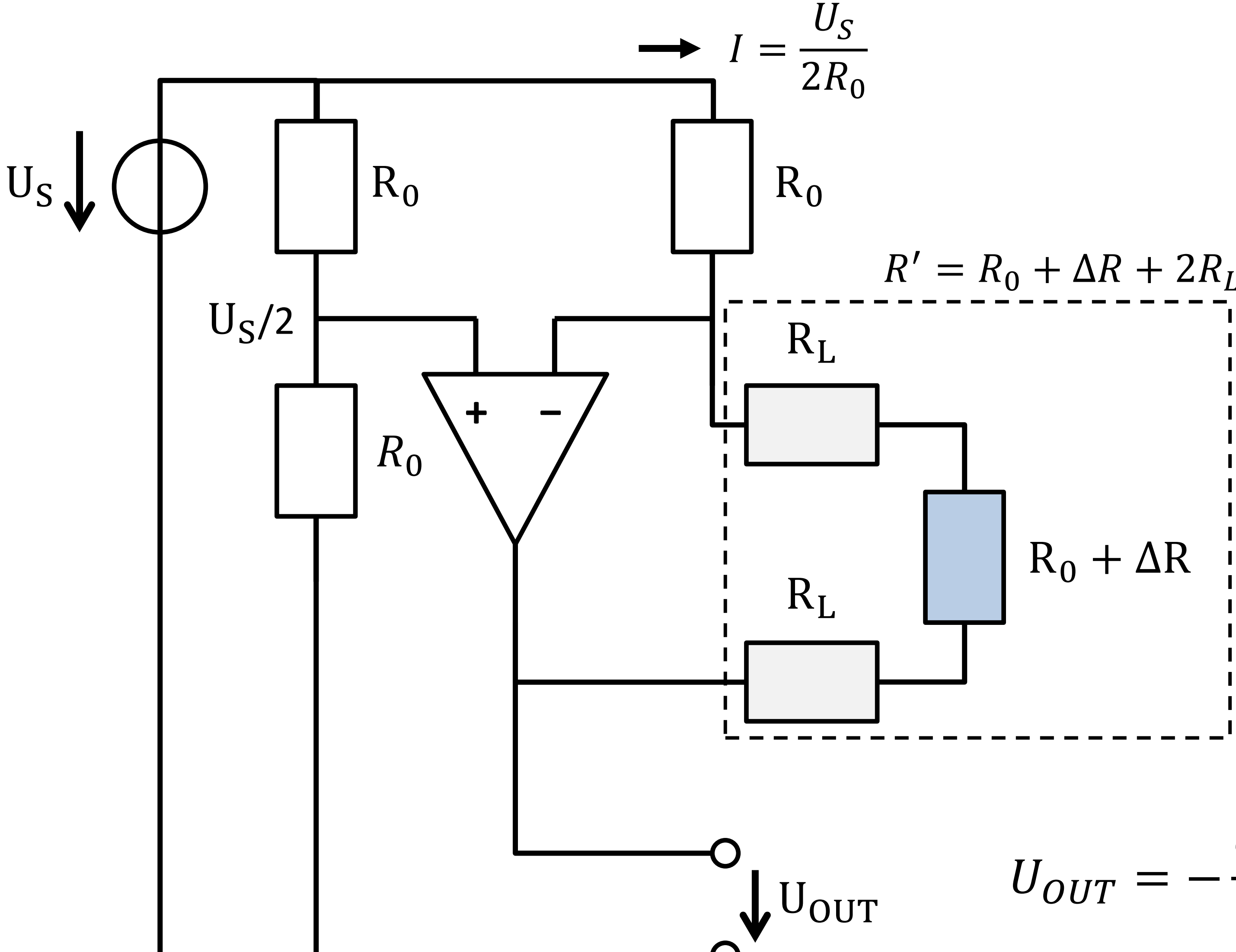
$$= \frac{U_S}{2}\frac{R_0 - (R_0 + \Delta R)}{R_0} =$$

$$= -\frac{U_S}{2}\frac{\Delta R}{R_0} = -\frac{U_S}{2}\alpha\vartheta$$

$$\Rightarrow \quad \boldsymbol{\vartheta = -\frac{2}{\alpha}\frac{U_{OUT}}{U_S}}$$

Output voltage is a linear function of the resistance, sensitivity is not deteriorated

Effect of lead resistance



$$U_{OUT} = -\frac{U_T}{2}\frac{\Delta R + 2R_L}{R_0}$$

Effect of lead resistance

- If the resistance of a lead wire is R_L then the output voltage reads:

$$U_{OUT} = -\frac{U_S}{2}\frac{\Delta R + 2R_L}{R_0}$$

- A temperature-independent term is added to the nominal output voltage:

$$\Delta U_{OUT} = -\frac{U_S}{2}\frac{2R_L}{R_0}$$

- As the temperature-voltage characteristics is linear, an absolute error is also added to the measured temperature

Problem

A Pt100 RTD is installed in a distance of 100m from the bridge circuit. Resistivity of the required AWG 20 lead wire is $0.033\ \Omega/\mathrm{m}$. Give the error of temperature measurement considering linear characteristics ($\alpha = 0.00385\ [1/°\mathrm{C}]$) !

- Lead resistance: $R_L = l \cdot \rho = 100 \cdot 0.033 = 3.3\ \Omega$
- Output voltage with lead resistance:

$$U_{OUT} = -\frac{U_S}{2}\frac{\Delta R + 2R_v}{R_0} = -\frac{U_S}{2}\alpha\vartheta - \frac{U_S}{2}\frac{2R_v}{R_0}$$

- Calculation of the temperature based on the output voltage:

$$\vartheta_m = -\frac{2}{\alpha}\frac{U_{OUT}}{U_S} = -\frac{2}{\alpha}\,\frac{-\frac{U_S}{2}\alpha\vartheta - \frac{U_S}{2}\frac{2R_L}{R_0}}{U_S} = \vartheta + \frac{2R_L}{R_0\alpha}$$

- Error of temperature measurement:

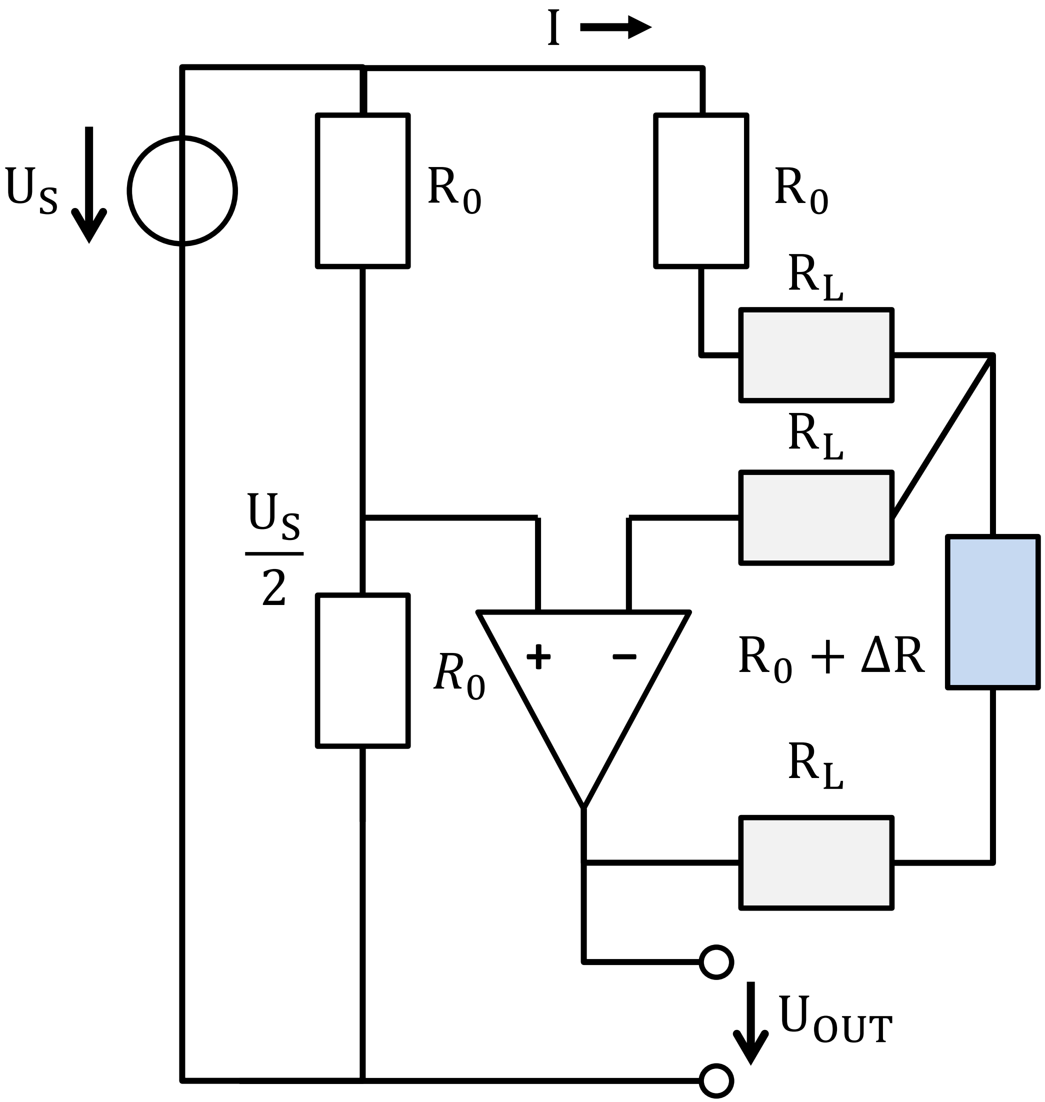
$$e_\vartheta = \vartheta_m - \vartheta = \frac{2R_L}{R_0\alpha} = \frac{2 \cdot 3.3}{100 \cdot 0.0385} \approx 17.15°\mathrm{C}$$

Problem – Alternative solution

A Pt100 RTD is installed in a distance of 100m from the bridge circuit. Resistivity of the required AWG 20 lead wire is $0.033\ \Omega/\mathrm{m}$. Give the error of temperature measurement considering linear characteristics ($\alpha = 0.00385\ [1/°\mathrm{C}]$) !

- Resistance of 100m lead wire: 3.3Ω, i.e. temperature-independent error of resistance is $2 \cdot 3.3\Omega = 6.6\Omega$
- Error of temperature measurement: $\frac{6.6}{0.385} \approx 17.15°\mathrm{C}$

Three-wire configuration



$$I = \frac{U_S}{2(R_0 + R_L)}$$

$$U_{OUT} =$$

$$= \frac{U_S}{2} - I(R_0 + \Delta R + R_L) =$$

$$= -\boldsymbol{\frac{U_S}{2} \frac{\Delta R}{R_0 + R_L}}$$

Effect of lead resistance for three-wire configuration

- Nominal output voltage without lead resistance:

$$U_{OUT,n} = -\frac{U_S}{2}\frac{\Delta R}{R_0}$$

- Measured output voltage (with lead resistance):

$$U_{OUT} = -\frac{U_S}{2}\frac{\Delta R}{R_0 + R_L} = -\frac{U_S}{2}\frac{\Delta R}{R_0}\frac{R_0}{R_0 + R_L} = U_{OUT,n}\frac{R_0}{R_0 + R_L}$$

- Lead resistance causes a relative error in output voltage:

$$\varepsilon_U = \frac{U_{OUT} - U_{OUT,n}}{U_{OUT}} = \frac{R_0}{R_0 + R_L} - 1 = -\frac{R_L}{R_0 + R_L}$$

- As temperature-voltage characteristics is linear, relative error of temperature measurement is the same, but with different sign:

$$\boldsymbol{\varepsilon_{\vartheta} = \frac{R_L}{R_0 + R_L}}$$

Problem

Give the error of temperature measurement caused by 100 m long leads with resistivity of $0.033\Omega/\mathrm{m}$ if three-wire configuration is used! Use linear approximation of resistance characteristics ($\alpha = 0.00385$).

- Output voltage without lead resistance:

$$U_{OUT,n} = -\frac{U_S}{2}\frac{\Delta R}{R_0} = -\frac{U_S}{2}\frac{R_0(\alpha\vartheta)}{R_0} = -\frac{U_S}{2}\alpha\vartheta$$

- Output voltage with lead resistance:

$$U_{OUT} = -\frac{U_S}{2}\frac{\Delta R}{R_0 + R_L} = -\frac{U_S}{2}\frac{R_0\alpha\vartheta}{R_0 + R_L} = -\frac{U_S}{2}\frac{R_0}{R_0 + R_L}\alpha\vartheta = \frac{R_0}{R_0 + R_L}U_{OUT,n}$$

- Relative error of temperature measurement: $-\frac{U_{OUT}-U_{OUT,n}}{U_{OUT,n}} = 1 - \frac{R_0}{R_0+R_L} =$
$= \frac{R_L}{R_0+R_L} = \frac{3.3}{103.3} \approx \ 3.2\%$

- Absolute error: 0.032ϑ

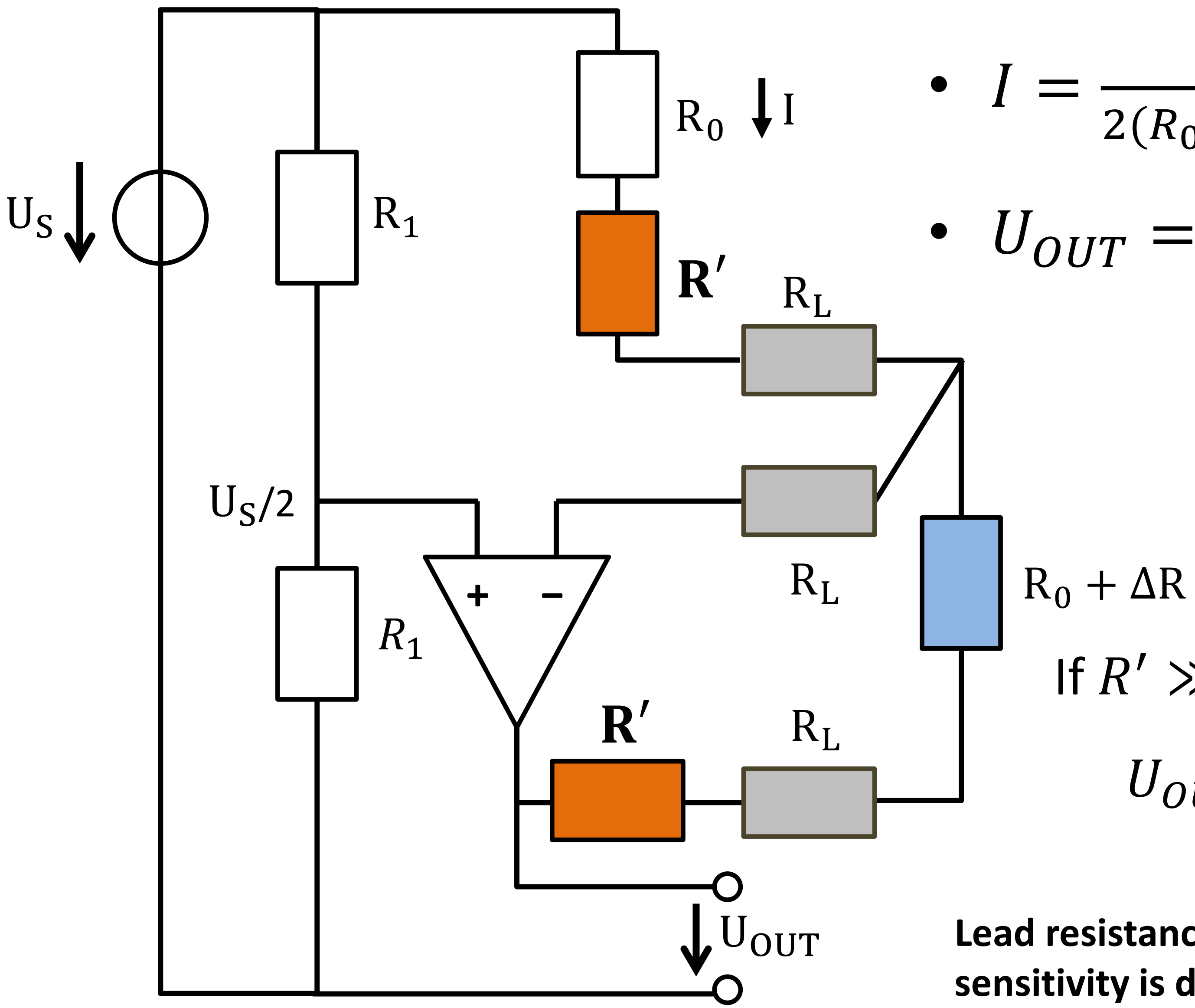
Two-wire vs. three-wire configuration

- The measurement using three-wire configuration is more accurate if the relative error is smaller than the absolute error for two-wire configuration :

$$|0.032\vartheta| < 17.15°\mathrm{C} \Rightarrow |\vartheta| < 535.94°\mathrm{C}$$

- Three-wire configuration provides more accurate results in a wide temperature range (especially around $0°\mathrm{C}$)

Three-wire configuration



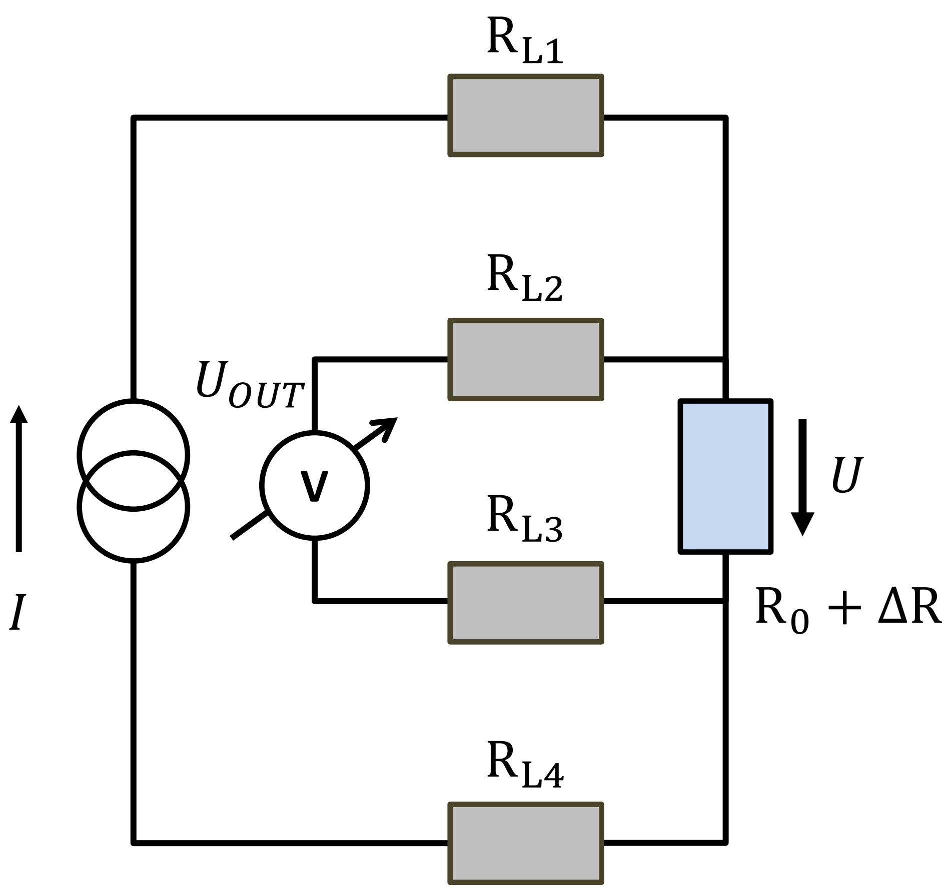
- $I = \dfrac{U_S}{2(R_0 + R' + R_L)}$
- $U_{OUT} = -\dfrac{U_S}{2}\dfrac{\Delta R}{R' + R_0 + R_L}$

If $R' \gg R_L$, $R' \gg R_0$:

$$U_{OUT} \approx -\frac{U_T}{2}\frac{\Delta R}{R'}$$

Lead resistance can be neglected but sensitivity is decreased significantly.

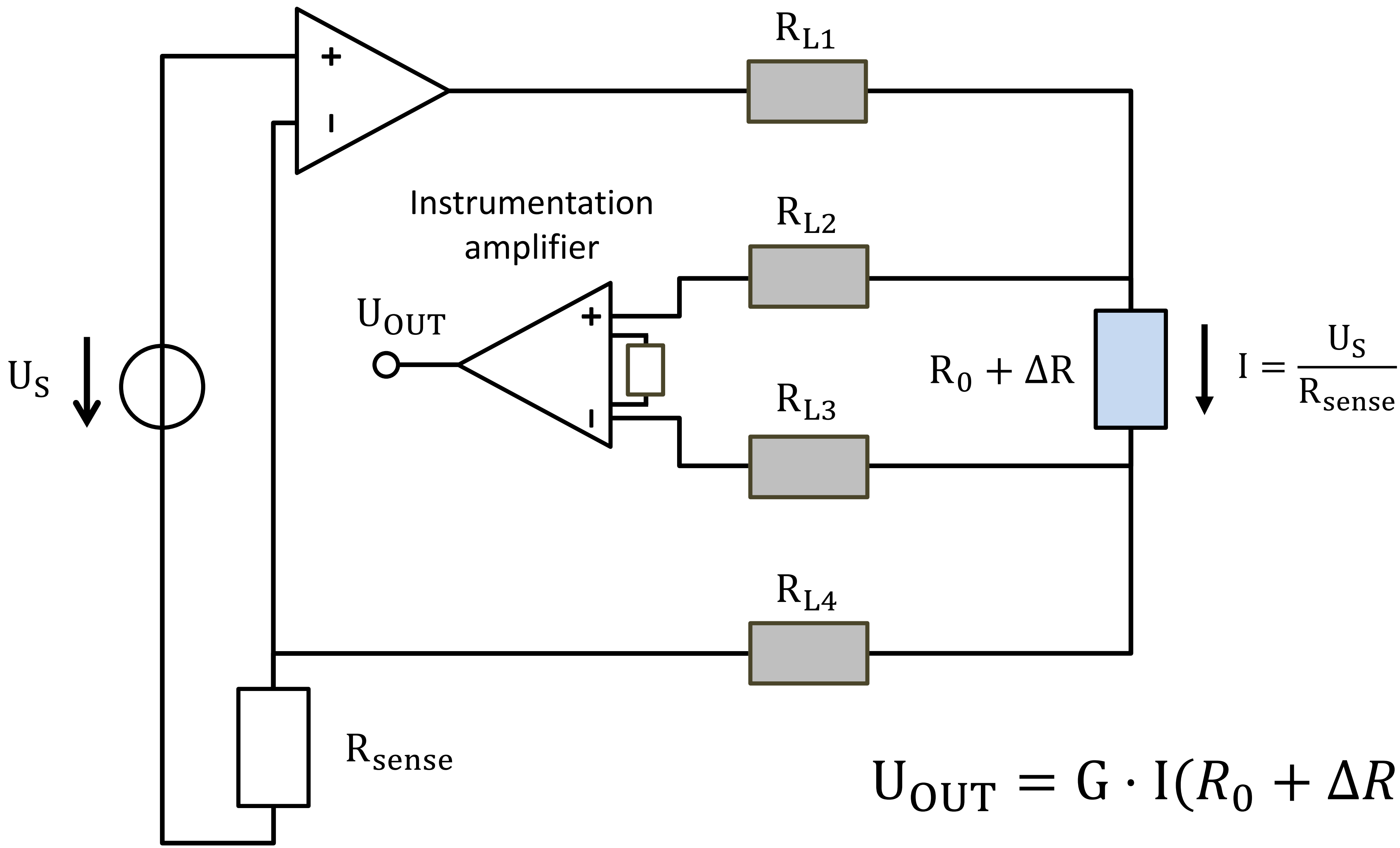
Four-wire configuration



$$U_{OUT} = (R_0 + \Delta R) I$$

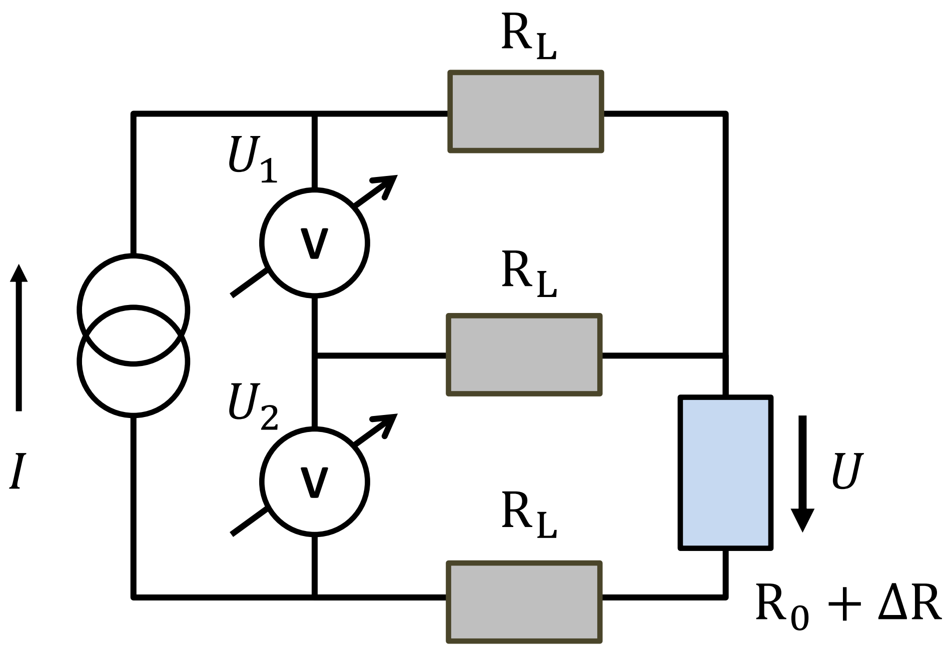
- A current generator can provide a load-independent constant current flowing through the RTD eliminating the effect of lead resistances R_{L1} and R_{L4}
- The voltage drop U is directly proportional to $R_0 + \Delta R$
- By using a device with high input impedance, effects of lead resistances R_{L2} and R_{L3} on voltage measurement can be also eliminated

Four-wire configuration



$$U_{OUT} = G \cdot I(R_0 + \Delta R)$$

Three-wire configuration with lead compensation



- Decrease of wiring cost
- Need for extra circuitry
- Accuracy identical to four-wire configuration if lead resistances are identical
- Compensation might be analog or digital

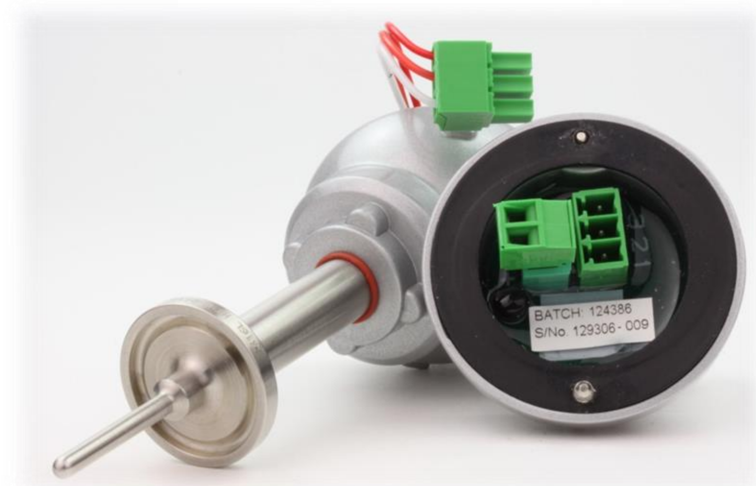
$$U_1 = R_L I$$

$$U_2 = (R_L + R_0 + \Delta R) I$$

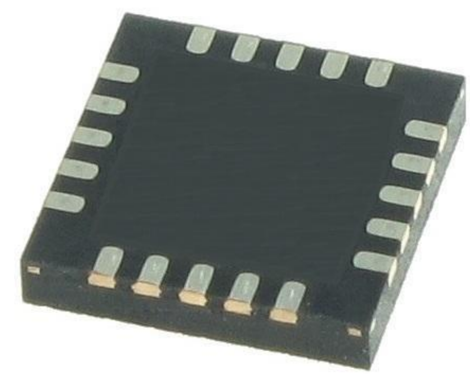
$$U_{OUT} = U_2 - U_1 = (R_0 + \Delta R) I$$

Off-the-shelf RTD circuits

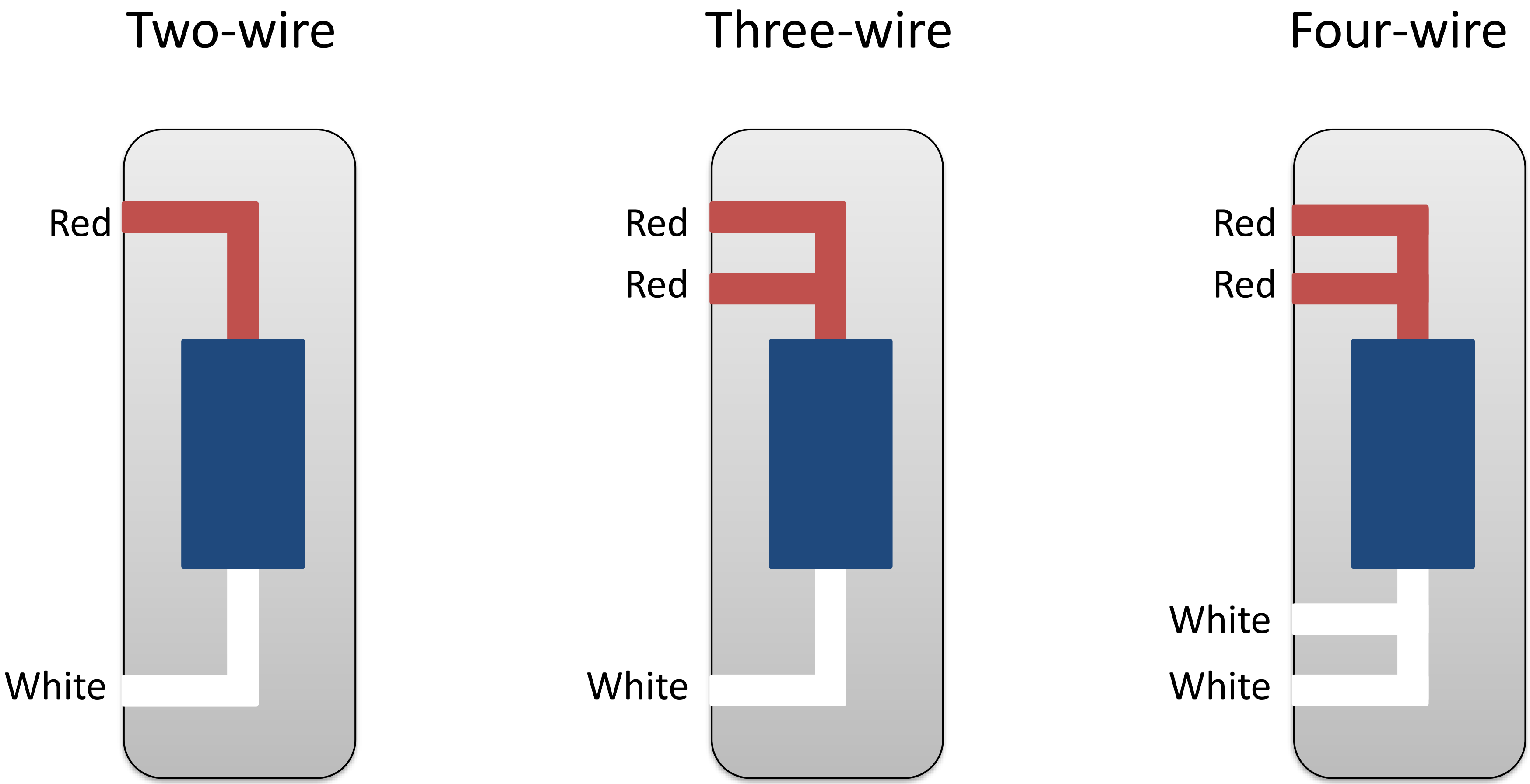
- Transmitters
  - 4-20mA current output
  - industrial grade circuitry
  - might be installed inside the RTD head



- Integrated circuits
  - voltage output
  - integrated ADC, digital output (SPI, I2C)
  - a single IC can be connected to multiple RTDs



Metal RTD leads



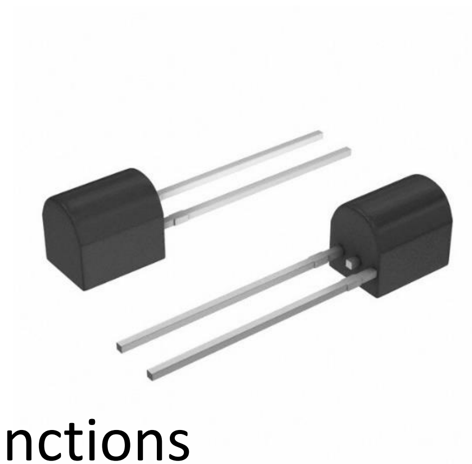
Metal RTDs

+ outstanding stability
+ high accuracy
+ low nonlinearity
+ standardized

− low resistance
− low sensitivity
− need of current excitation
− self-heating
− relatively high price

Semiconductor temperature sensors

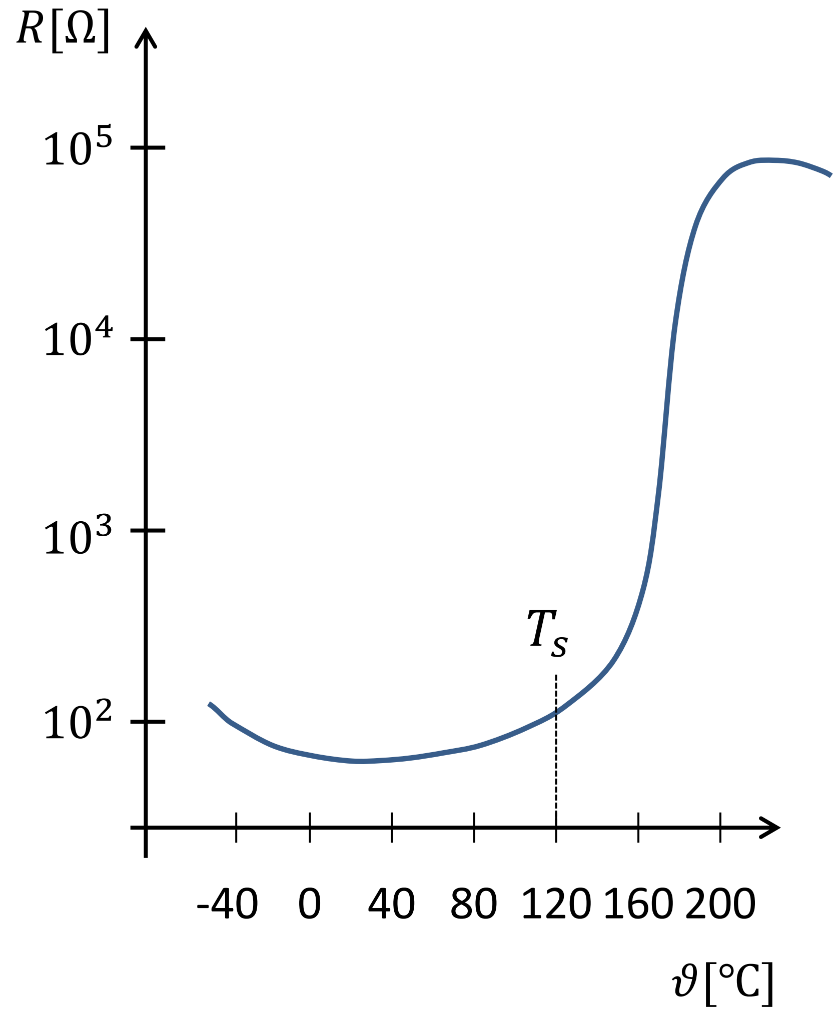
- Thermistors
  - semiconductor RTD
  - no p-n junction inside
  - measurement is based on properties of semiconductors as materials



- Semiconductor (IC) temperature sensors
  - voltage or current output
  - has p-n junction(s)
  - measurement based on properties of p-n junctions

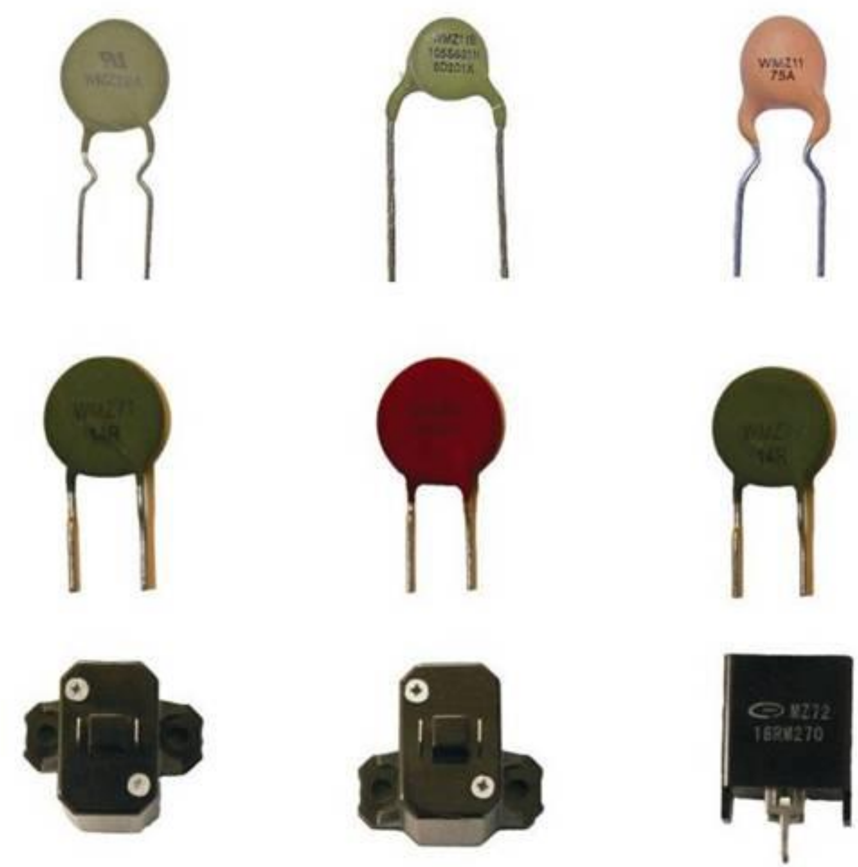
PTC thermistors

- Barium-Titanite doped ceramic
- Below the Curie point (T_s) high dielectric constant prevents formation of potential barriers, i.e. resistance is low
- Above the Curie point the dielectric constant decreases, potential gates are formed so resistance increases abruptly
- Curie point can be set by doping



PTC thermistors

- Due to their steep and nonlinear characteristics, PTC thermistors can not be used for temperature measurement even in a narrow range
- However, they serve well as temperature switches or resettable fuses
- Most common application: protection circuits



NTC thermistores

- Sintered metal oxide (n- or p-type)
- Increasing heat increases the number of charge carriers in the conduction bend
- Conductivity increased by heat
- Resistance decreased by heat

Static characteristics of NTC thermistors

- Steinhart-Hart equation:

$$\frac{1}{T} = a + b\ln(R) + c(\ln(R))^3$$

Magnitude of parameters ($R = 3k\Omega$ @ $25°\mathrm{C}$)

  - $a = 1.4 \cdot 10^{-3}$
  - $b = 2.37 \cdot 10^{-4}$
  - $c = 9.9 \cdot 10^{-8}$

Approximation error $< 0.02°\mathrm{C}$

- T $[\mathrm{K}]$ – absolute temperature (in Kelvin)

Transformation of static characteristics

- Steinhart-Hart equation: $\frac{1}{T} = a + b\ln(R) + c(\ln(R))^3$
- Denote the resistance of the thermistor at temperature T_0 by R_0 (e.g. $3\mathrm{k}\Omega$ @ $298{,}15\mathrm{K}$)
- Third-order term might be neglected: $c = 0$
- $\frac{1}{T_0} = a + b\ln(R_0) \Rightarrow a = \frac{1}{T_0} - b\ln(R_0)$
- $\frac{1}{T} = a + b\ln(R_0) = \frac{1}{T_0} - b\ln(R_0) + b\ln(R)$
- $\ln(R) - \ln(\mathrm{R}_0) = \frac{1}{b}\left(\frac{1}{\mathrm{T}} - \frac{1}{\mathrm{T}_0}\right) \Rightarrow \frac{R}{R_0} = \mathrm{e}^{\frac{1}{\mathrm{b}}\left(\frac{1}{\mathrm{T}} - \frac{1}{\mathrm{T}_0}\right)}$
- $B := \frac{1}{b} \Rightarrow \boldsymbol{R} = \boldsymbol{R_0}\boldsymbol{e}^{\mathbf{B}\left(\frac{\mathbf{1}}{\boldsymbol{T}} - \frac{\mathbf{1}}{\boldsymbol{T_0}}\right)}$

Static characteristics of NTC thermistors

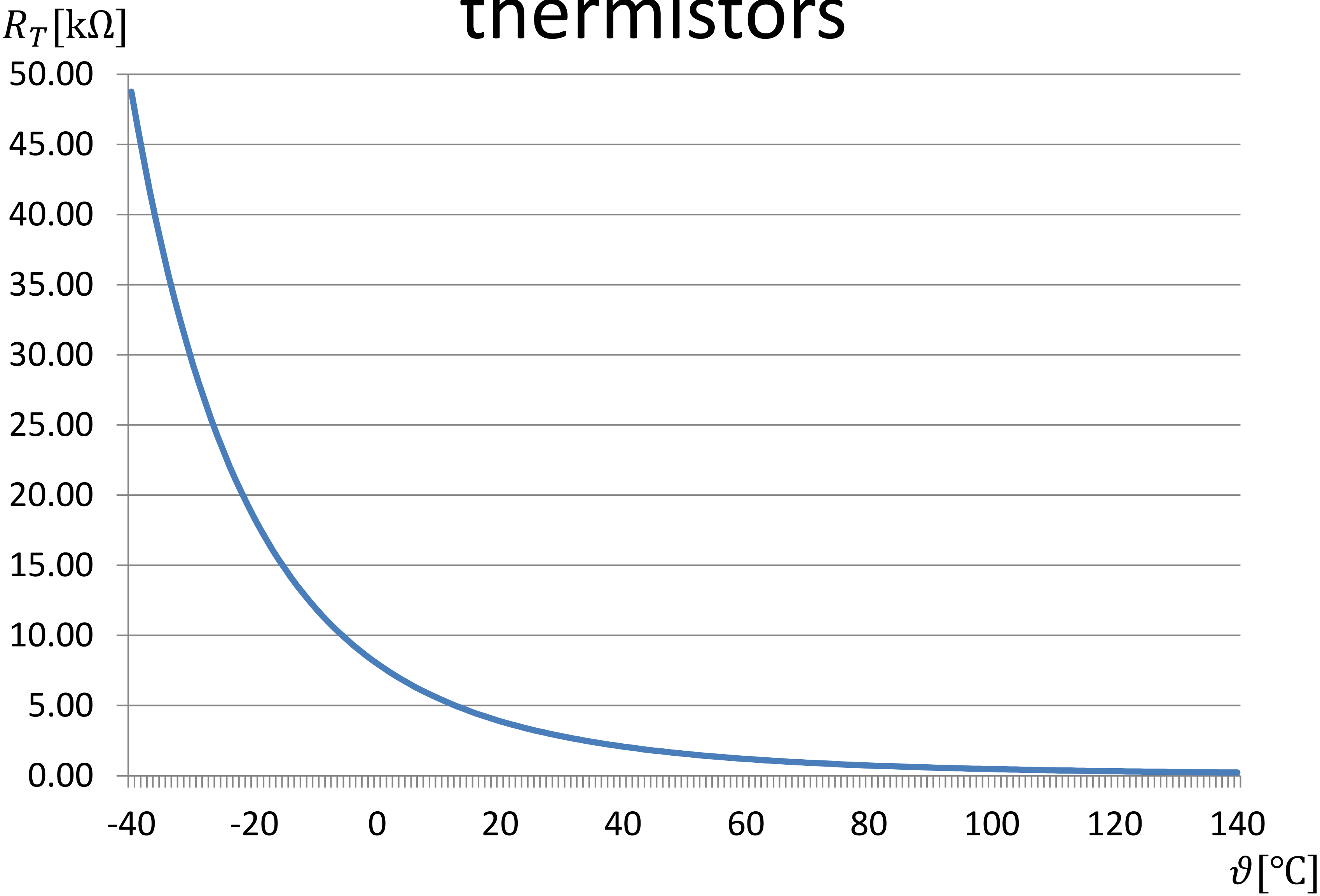
- Characteristics used in practice:

$$\boldsymbol{R_T = R_0 e^{B\left(\frac{1}{T}-\frac{1}{T_0}\right)}}$$

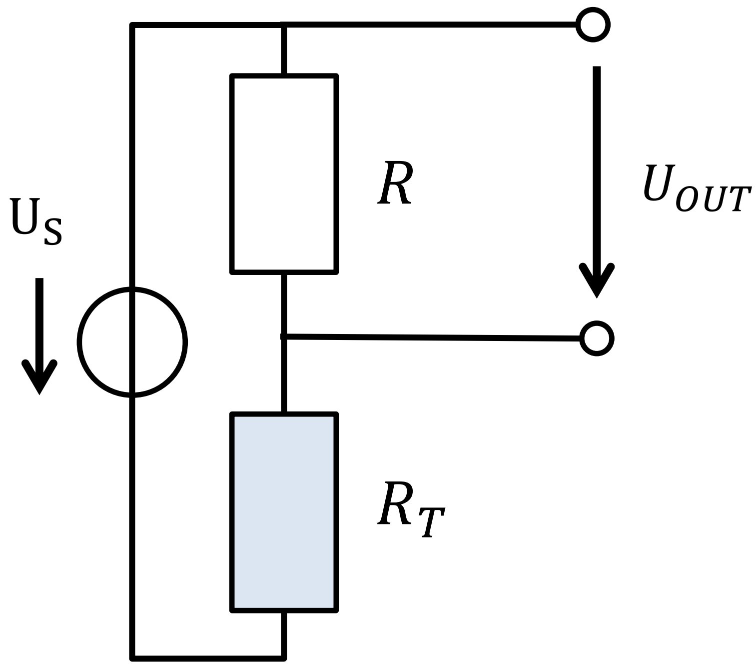
  - T_0: reference temperature $[\mathrm{K}]$
    (commonly $T_0 = 298.15\mathrm{K} = 25℃$)
  - R_0: resistance at reference temperature T_0 $[\Omega]$ (kΩ)
  - B: device-specific constant $[\mathrm{K}]$ (3000-4500)

- More useful form for calculations:
  - $R_\infty = R_0 e^{-\frac{B}{T_0}}$ (constant) $\Rightarrow R_T = R_\infty e^{\frac{B}{T}}$
  - $T = \dfrac{B}{\ln(R_T/R_\infty)}$

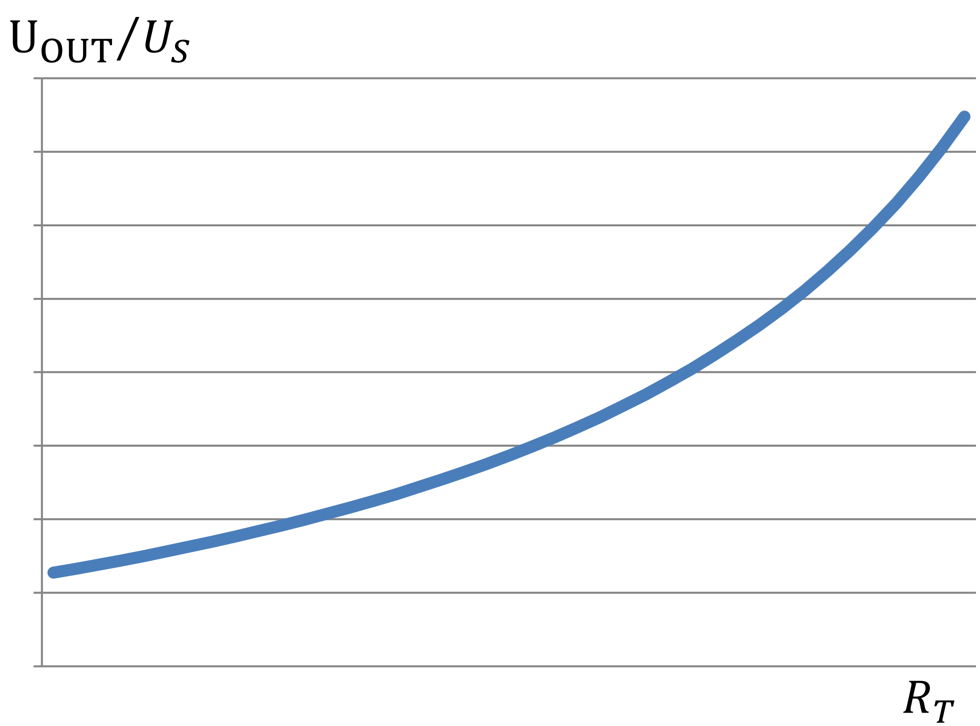
Static characteristics of NTC thermistors



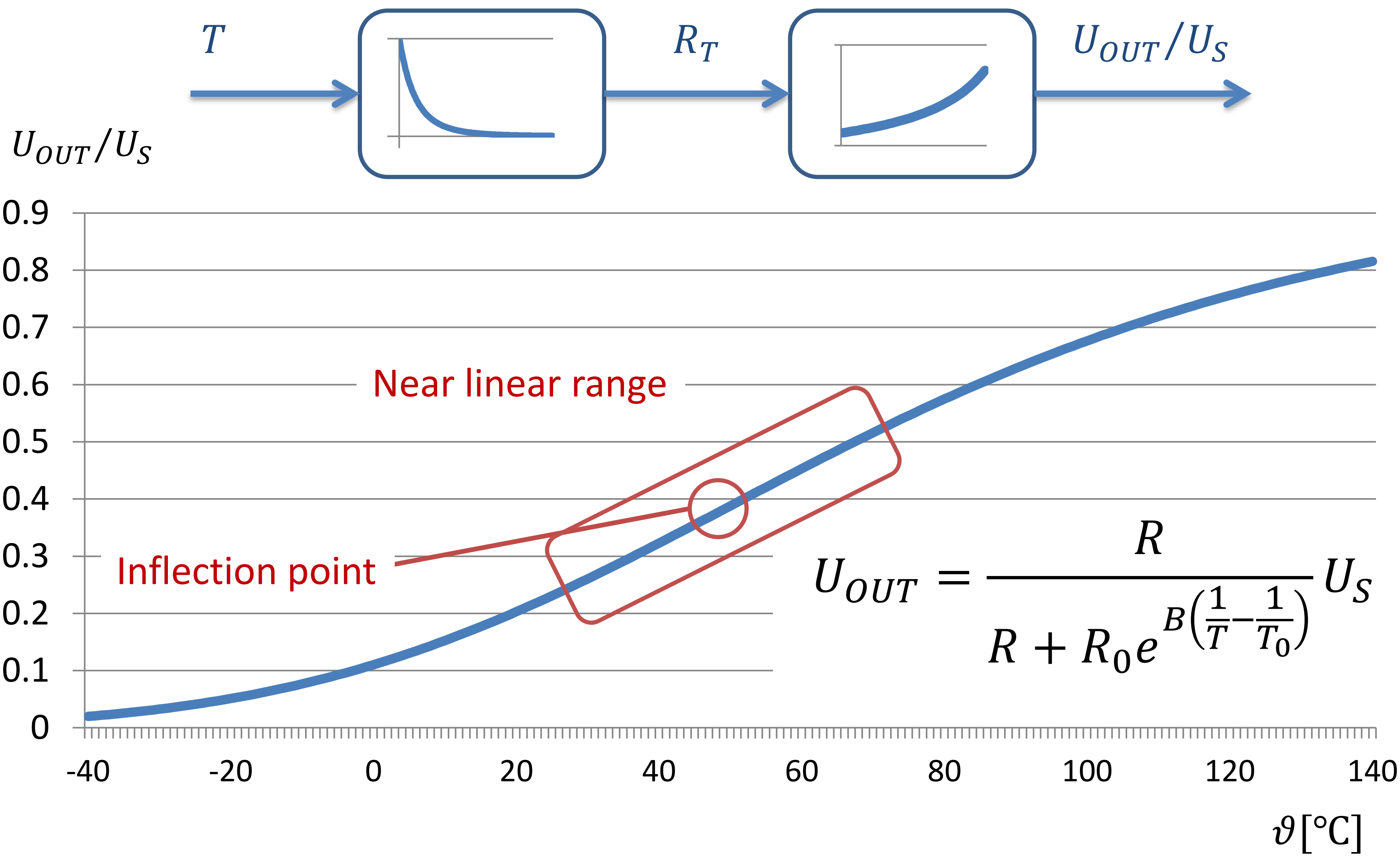
Linear temperature-voltage characteristics for NTC thermistors



$$U_{OUT} = \frac{R}{R + R_T} U_S$$



Linear temperature-voltage characteristics for NTC thermistors



Linear temperature-voltage characteristics for NTC thermistors

- The temperature-voltage characteristics is nearly linear in the proximity of its inflection point

- Inflection point:

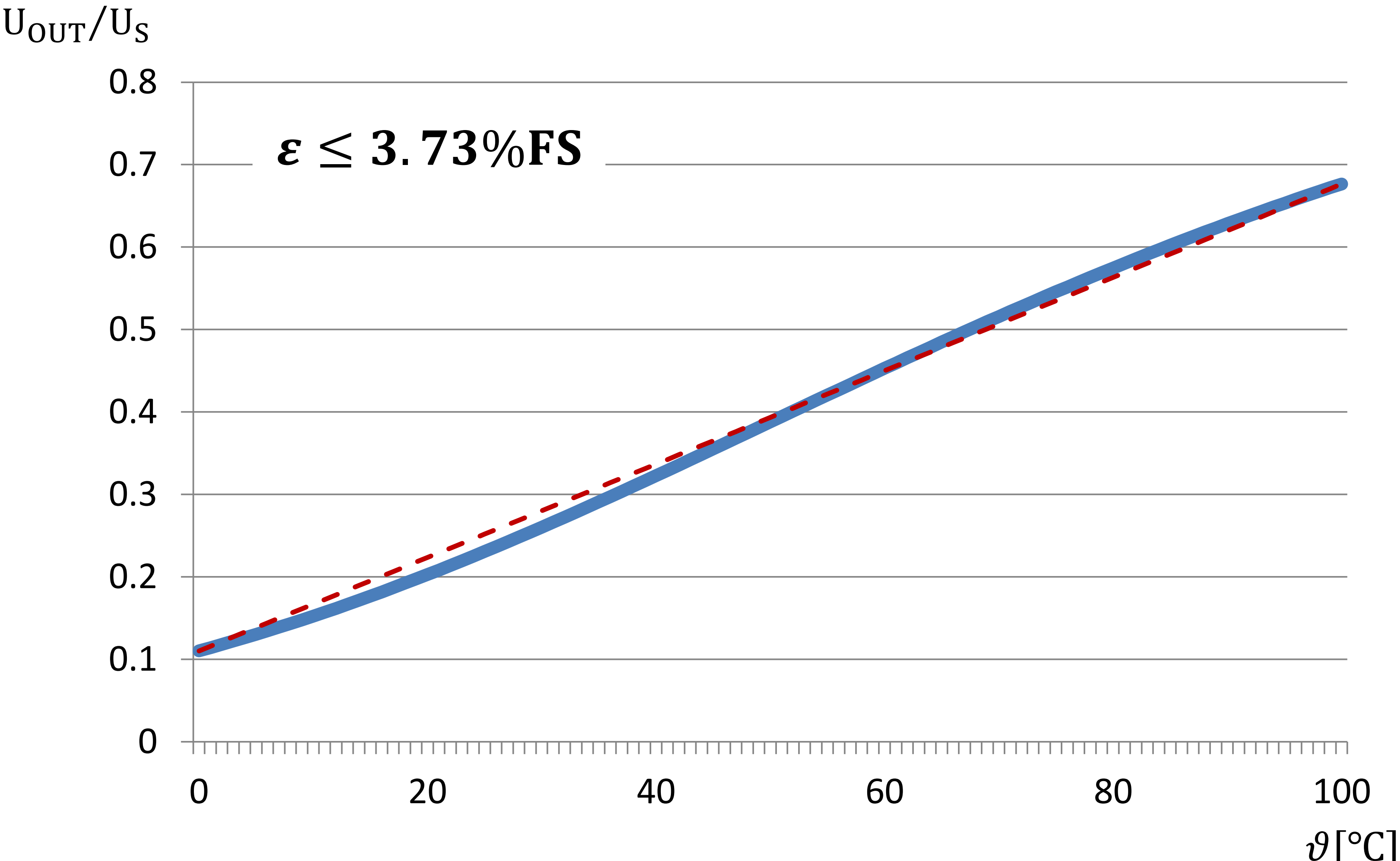
$$\frac{\partial^2}{\partial T^2} \frac{R}{R+R_0 e^{B\left(\frac{1}{T}-\frac{1}{T_0}\right)}} = 0 \Rightarrow R \text{ can be calculated}$$

- $R = R_{TM} \frac{B-2T_M}{B+2T_M}$

    - T_M: midpoint of the temperature range of linearization [K]

    - R_{TM}: resistance of the thermistor at temperature T_M

Linear temperature-voltage characteristics for NTC thermistors



Linear temperature-voltage characteristics for NTC thermistors

- Acceptable accuracy for a narrow range only (at most $100°\mathrm{C}$)
- Decreased sensitivity
- Output voltage range can be set by changing U_{S}
- More accurate (although more complex) circuits are also available

Problem

Given an NTC thermistor with B-constant $B = 3977\mathrm{K}$ and base resistance of $10\mathrm{k}\Omega$ at $25℃$. In order to use the thermistor in a household heating control system, its linear temperature-voltage characteristics needs to be established for the range $0℃ - 40℃$. Sample points of the nonlinear characteristics are given in the table below.

- Give the value of the serial linearizing resistor!
- Give the voltage-temperature characteristics (formula to calculate the temperature based on the output voltage) if the supply voltage is $\mathrm{U_S} = 5\mathrm{V}$!

ϑ [℃]	R_T [$k\Omega$]
0	32.56
5	25.34
10	19.87
15	15.70
20	12.49
25	10.00
30	8.059
35	6.535
40	5.33

Serial linearizing resistor

$$R = R_{TM}\frac{B - 2T_M}{B + 2T_M}$$

$$R = 12.49\text{k}\frac{3977 - 2 \cdot 293.15}{3977 + 2 \cdot 293.15}$$

$$= \mathbf{9.28\ k\Omega}$$

Absolute temperature [K]

ϑ [°C]	R_T [$k\Omega$]
0	32.56
5	25.34
10	19.87
15	15.70
20	12.49
25	10.00
30	8.059
35	6.535
40	5.33

Temperature-voltage characteristics
(end-point straight line)

- $U = \frac{R}{R+R_T} U_S$
- $U_1 = U_{0°C} = \frac{9.28k}{9.28k+32.56k} \cdot 5 = 1.11V$
- $U_2 = U_{40°C} = \frac{9.28k}{9.28k+5.33k} \cdot 5 = \frac{1910}{316} = 3.18V$

- $U_2 - U_1 = 1.07V$
- $\vartheta_2 - \vartheta_1 = 40°C$

$$\Rightarrow S = \frac{2.07}{40} = 0.052 \frac{V}{°C}$$

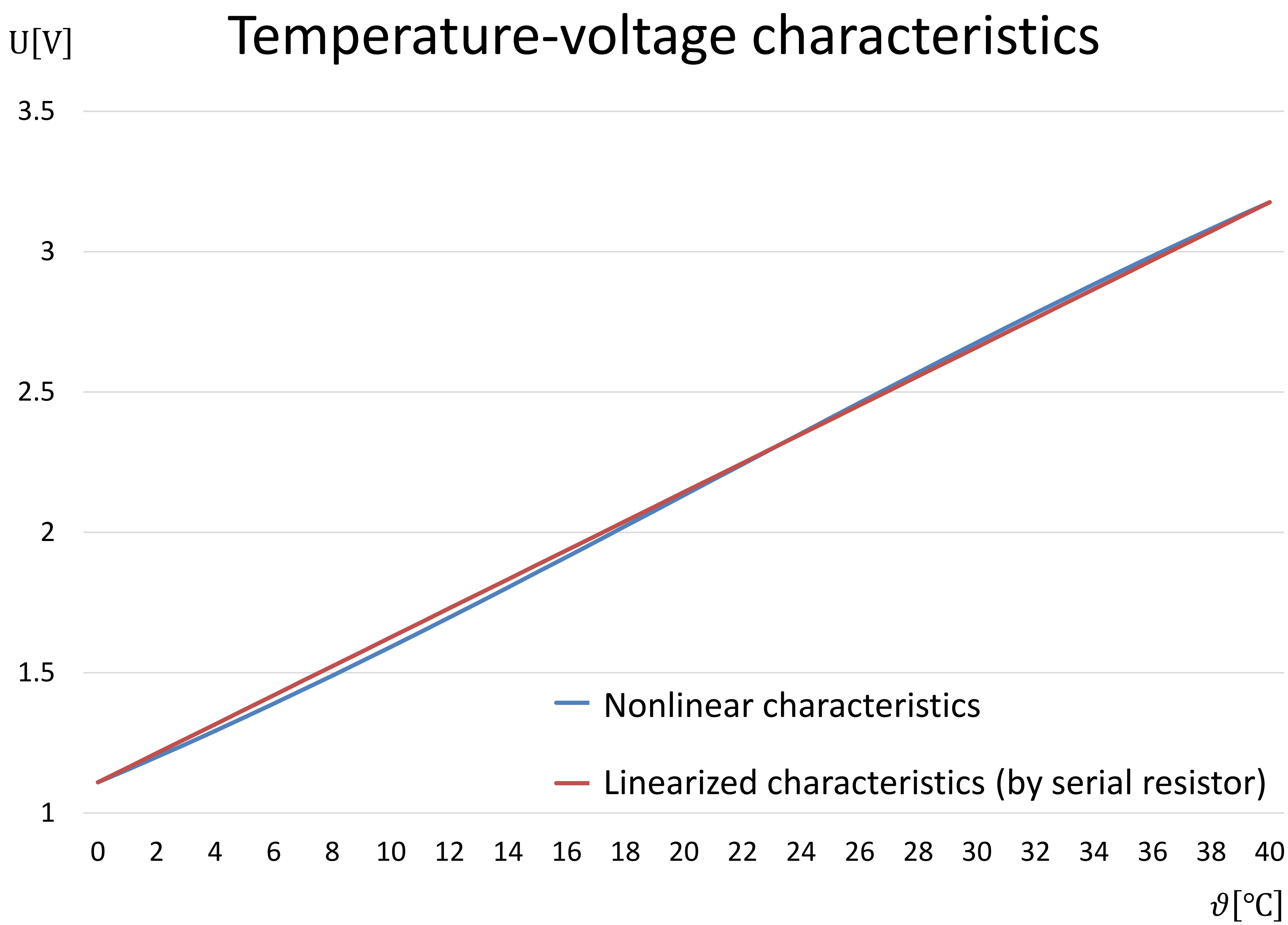
ϑ [°C]	R_T [$k\Omega$]
0	32.56
5	25.34
10	19.87
15	15.70
20	12.49
25	10.00
30	8.059
35	6.535
40	5.33

Temperature-voltage characteristics
(end-point straight line)

- $U = U_1 + (\vartheta - \vartheta_1)S \Rightarrow \vartheta = \vartheta_1 + \frac{U - U_1}{S}$

- $\vartheta = 0°\mathrm{C} + \frac{U - 1.11}{0.052} = 19.23(U - 1.11) =$

$$= \mathbf{19.23}\boldsymbol{U} - \mathbf{21.15}\ [°\mathrm{C}]$$


Temperature-voltage characteristics
U[V]
3.5
3
2.5
2
1.5
1
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
ϑ[°C]
Nonlinear characteristics
Linearized characteristics (by serial resistor)

Self-heating

- As for metal RTDs, self-heating must be considered
- $\Delta T = \frac{I_T^2 R_T}{C}$, where C is the dissipation constant
- Maximal current allowed for a given temperature change ΔT_{max}:

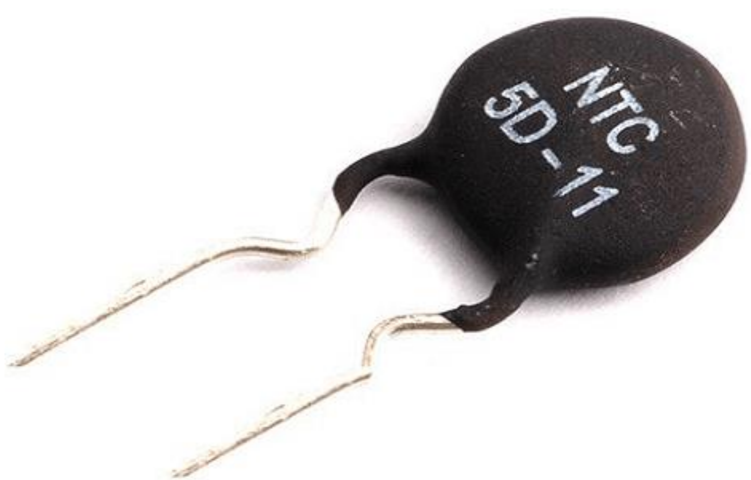
$$I_{T,max} = \sqrt{\frac{\Delta T_{max} C}{R_T}}$$

- Self-heating phenomenon can be exploited in protection circuits

NTC thermistors

+ small size
+ low cost
+ low time constant
+ high output range
+ high sensitivity

– low accuracy ($\pm 5\%$)
– nonlinear static characteristics
– narrow input range
– sensitivity to environmental effects
– need for current excitation
– self-heating



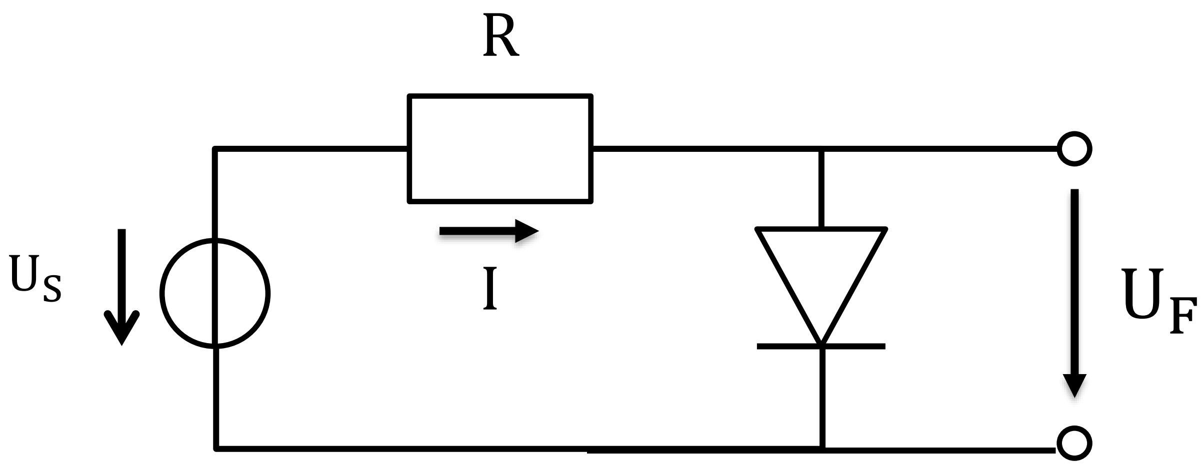
Diodes as temperature sensors

- Forward voltage at $25°\mathrm{C}$: $\approx 600\mathrm{mV}$
- Temperature of dependence of forward voltage: $\mathrm{C} \approx 0.2\mathrm{mV/K}$
- Linear static characteristics:

$$U_F = U_{G0} - CT \Rightarrow T = \frac{U_{G0} - U_F}{C}$$

- Band gap voltage: $U_{G0} \approx 1.205\mathrm{V}$

Diodes as temperature sensors



Constant current is required!

Recommended: use lowest current possible

Diodes as temperature sensors

- Low accuracy
  - drift of sensitivity
  - manufacturing variance – junctions of transistors have better properties
  - total accuracy: $\approx \pm 4°\mathrm{C}$ for input range $-50 \ldots 150°\mathrm{C}$

- Special diodes calibrated for temperature measurement (Si, GaAs)
  - high accuracy
  - input range: $4 - 400\mathrm{K}$

Base – emitter junctions of bipolar transistors

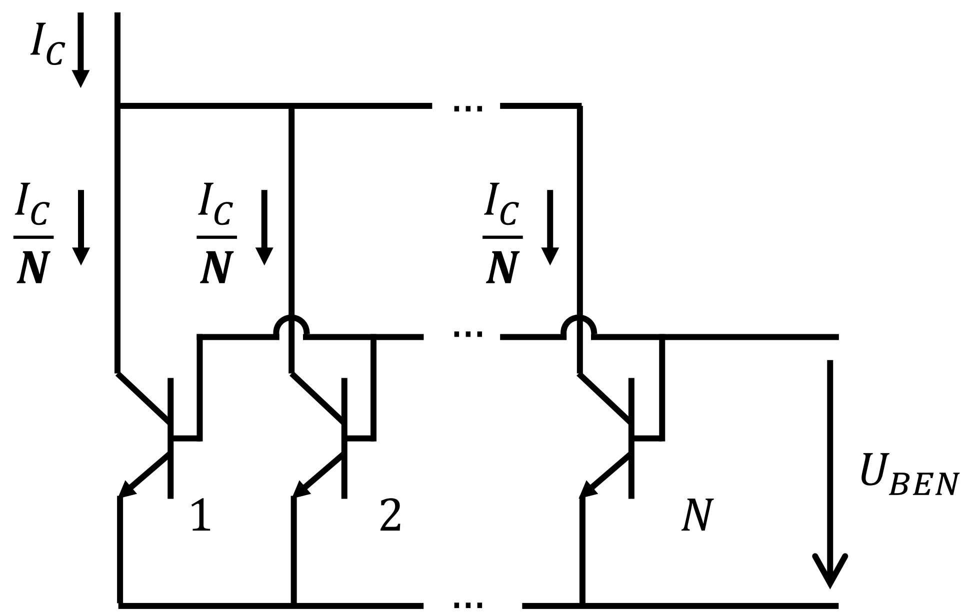
Temperature dependence of base-emitter voltage:

$$U_{BE} = U_{G0}\left(1-\frac{T}{T_0}\right) + U_{BE0}\left(\frac{T}{T_0}\right) + \left(\frac{nKT}{q}\right)\ln\left(\frac{T_0}{T}\right) + \frac{KT}{q}\,ln\left(\frac{I_c}{I_{c0}}\right)$$

- U_{G0}: band gap voltage at $0\mathrm{K}$
- U_{BE0}: band gap voltage at operating point
- T_0: temperature at operating point
- n: device-specific constant
- K: Boltzmann-constant
- q: electron charge
- I_{c0}: parameter depending on silicon geometry and temperature

Base – emitter junctions of bipolar transistors

Connect N transistors in parallel and apply the current I_C to the assembly!



$$U_{BEN} = U_{G0}\left(1 - \frac{T}{T_0}\right) + U_{BE0}\left(\frac{T}{T_0}\right) + \left(\frac{nKT}{q}\right)\ln\left(\frac{T_0}{T}\right) + \frac{KT}{q}\,ln\left(\frac{I_c}{\boldsymbol{N}I_{c0}}\right)$$

Base – emitter junctions of bipolar transistors

- Single transistor:

$$U_{BE} = U_{G0}\left(1-\frac{T}{T_0}\right)+U_{BE0}\left(\frac{T}{T_0}\right)+\left(\frac{nKT}{q}\right)\ln\left(\frac{T_0}{T}\right)+\frac{KT}{q}ln\left(\frac{I_c}{I_{c0}}\right)$$

- N transistors connected in parallel:

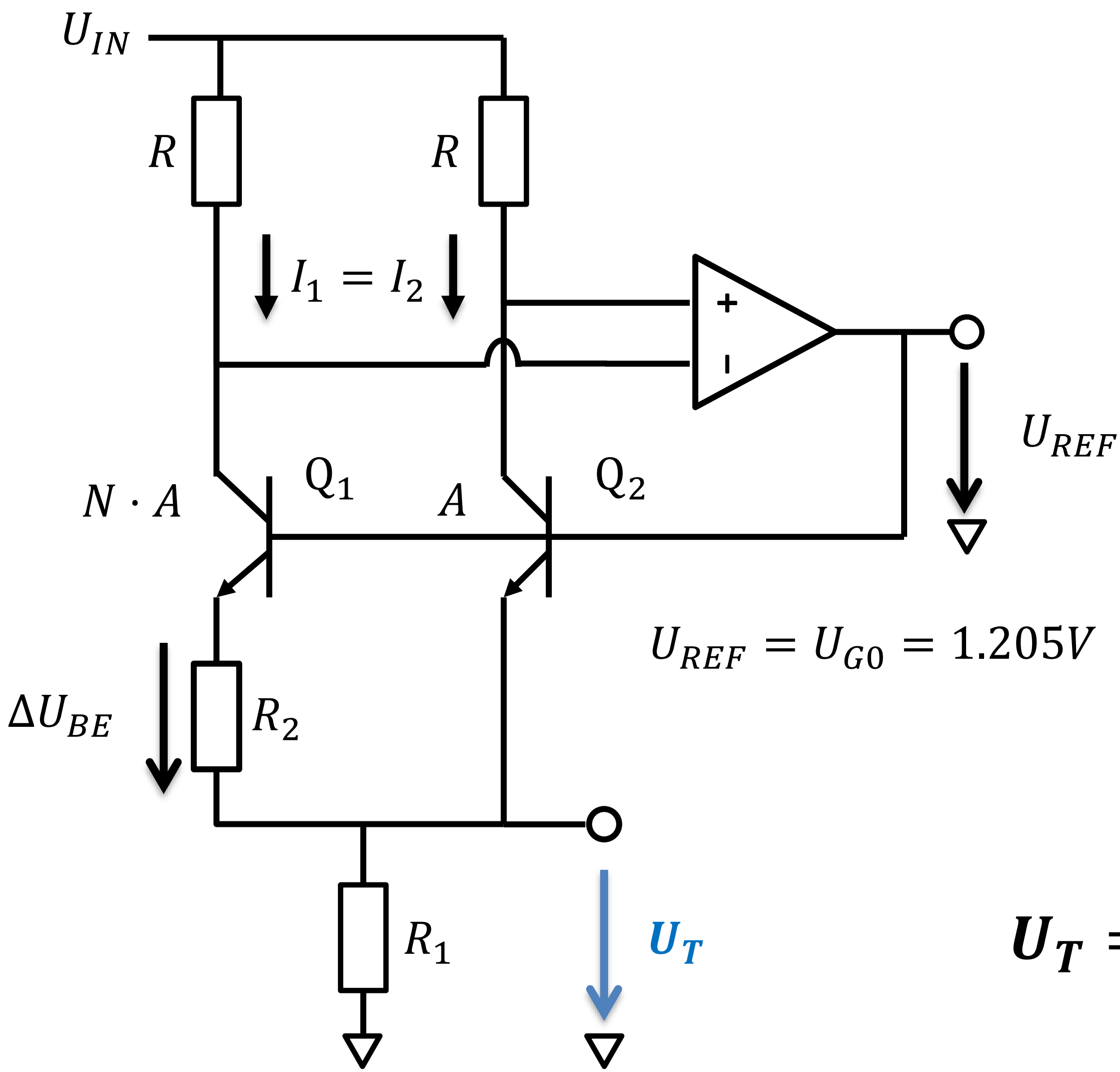
$$U_{BEN} = U_{G0}\left(1-\frac{T}{T_0}\right)+U_{BE0}\left(\frac{T}{T_0}\right)+\left(\frac{nKT}{q}\right)\ln\left(\frac{T_0}{T}\right)+\frac{KT}{q}ln\left(\frac{I_c}{\boldsymbol{N}I_{c0}}\right)$$

- Difference of Base-Emitter voltages:

$$\Delta U_{BE} = U_{BE} - U_{BEN} = \frac{KT}{q}\ln\left(\frac{I_c}{I_{c0}}\right)-\frac{KT}{q}\ln\left(\frac{I_c}{NI_{c0}}\right)=\frac{K}{q}\ln(N)\boldsymbol{T}$$

- Is it necessary to use N base-emitter junctions? No, one junction with a surface N times larger is sufficient.

Brokaw bandgap reference



- Difference of base-emitter voltages:
$$\Delta U_{BE} = \frac{KT}{q}\ln(N)$$
- Current flowing through resistor R_2:
$$I_1 = \frac{\Delta U_{BE}}{R_2}$$
- The operational amplifier keeps collector currents at the same value: $I_1 = I_2$
- Current flowing through resistor R_1: $I_1 + I_2 = 2I_1$

$$\boldsymbol{U_T} = 2I_1R_1 = \boldsymbol{2\frac{R_1}{R_2}\frac{KT}{q}\ln(N)}$$

Brokaw bandgap reference

- $U_T = 2\frac{R_1}{R_2}\boldsymbol{T}\frac{K}{q}\ln(N)$

  Output voltage is directly proportional to the temperature, sensitivity can be set by the ratio of resistor values

- „Side effect”: accurate, temperature-independent reference voltage (U_{ref}) equal to bang gap voltage (hence the name *Brokaw bandgap reference*)

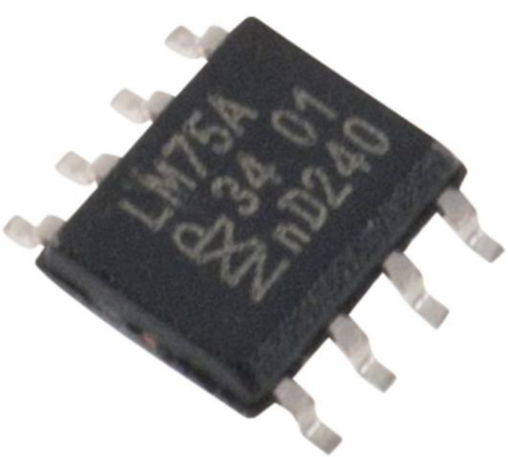
IC temperature sensors

- Off-the-shelf integrated circuits
- Analog ICs
  - typical sensitivity: $10\mathrm{mV/K}$
  - direct Celsius- or Fahrenheit-output with minimal external circuitry
  - other types of ICs (e.g. current sources) might also be used for temperature measurement
- Digital ICs
  - integrated ADC
  - bus output (I2C / SPI)
  - additional functions (e.g. alarms, storage of extreme temperature)

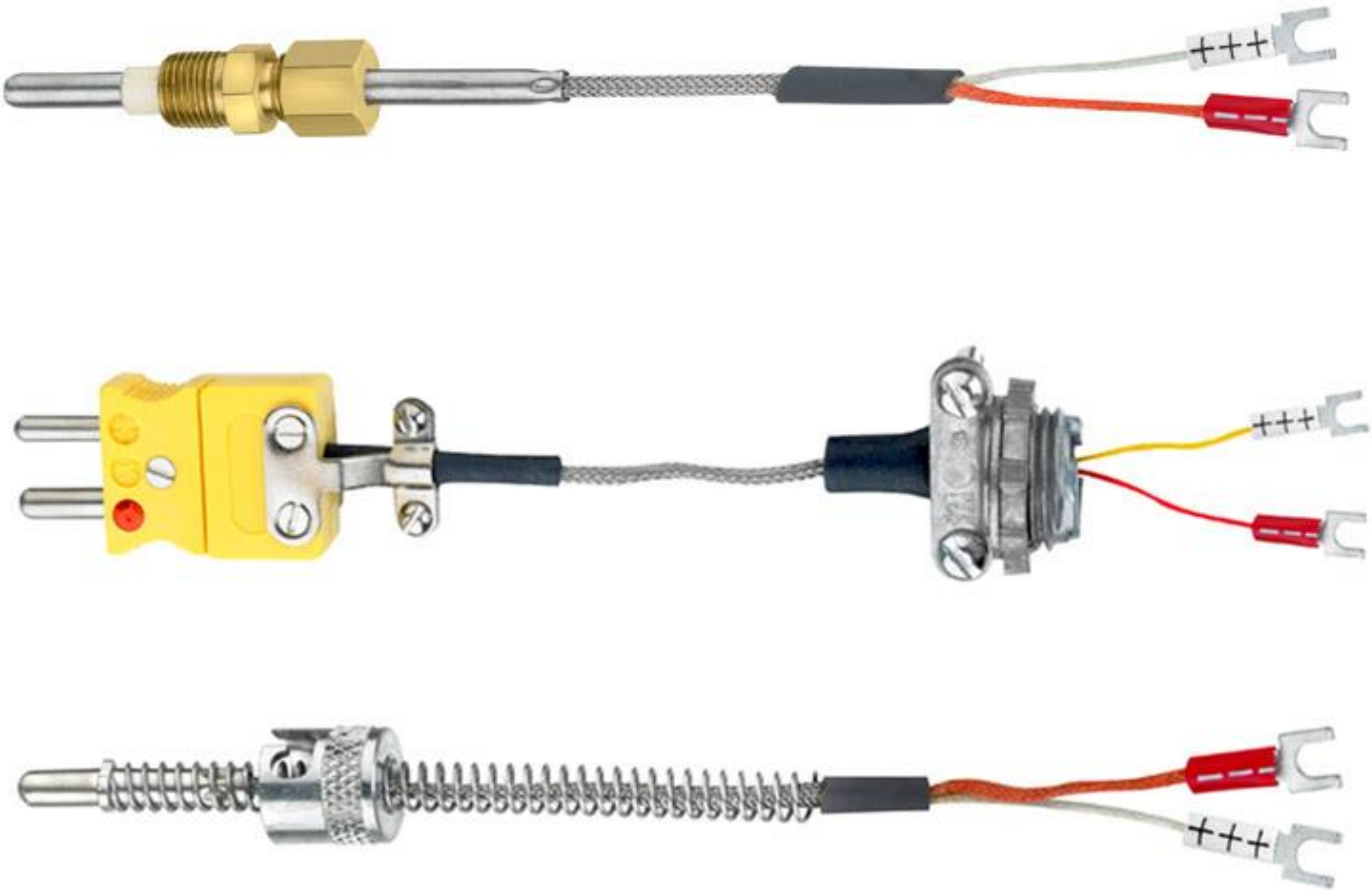
IC temperature sensors

+ linear characteristics
+ high sensitivity
+ low cost
+ can be integrated to devices at wafer level
+ accuracy up to $\pm 0.5°C$

− narrow input range ($-50 \ldots 150°C$)
− needs stable voltage supply
− high time constant
− self-heating
− error up to $\pm 10°C$



Thermocouples

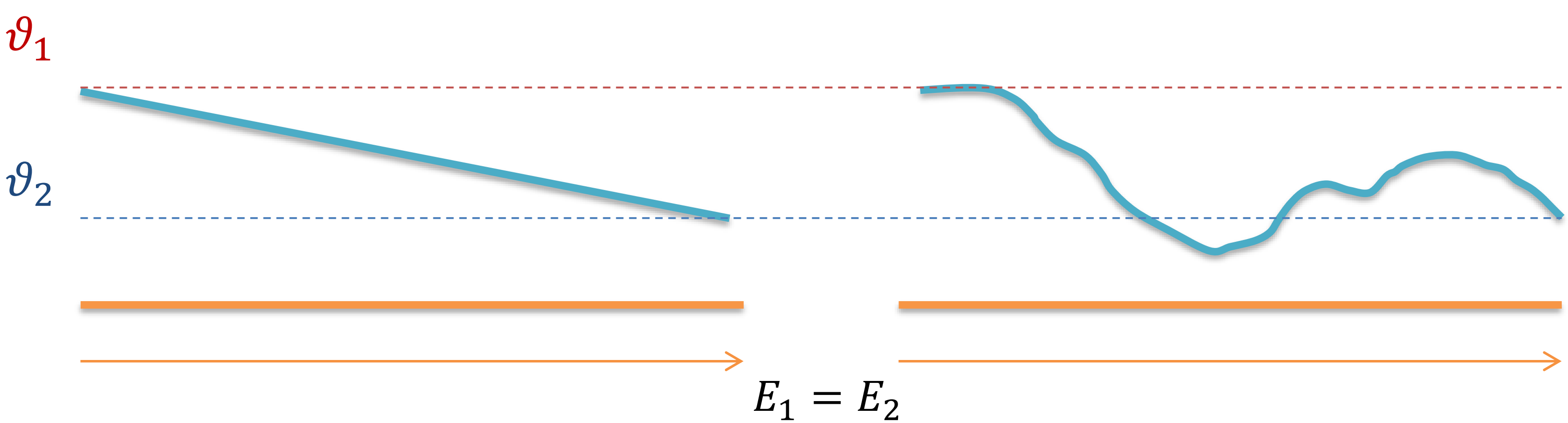


Seebeck effect

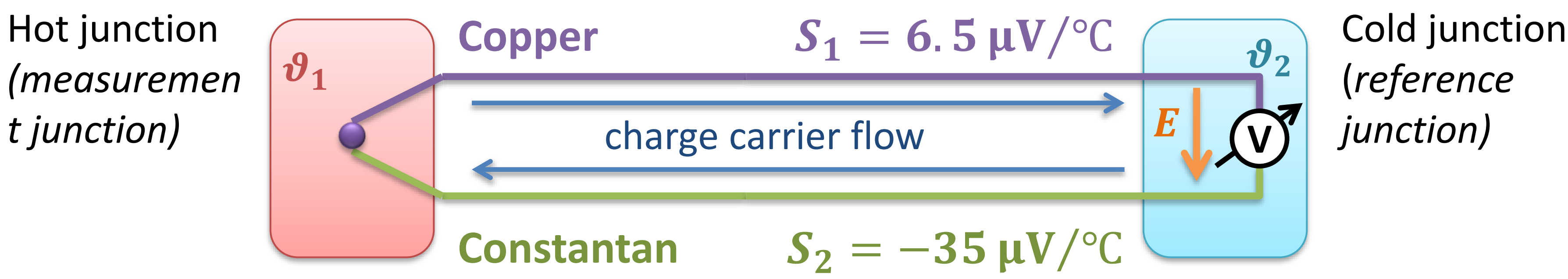
- Voltage is induced in a conductor having temperature gradient
- The induced voltage is independent from the gradient, depends only the temperatures at the endpoints and the properties of the material:

$$E = S(\vartheta_1 - \vartheta_2)[\mathrm{V}]$$

- S $[\mu\mathrm{V/K}]$: Seebeck-coefficient
    - Platinum: $-5\mu\mathrm{V/K}$
    - Copper: $6.5\mu\mathrm{V/K}$



Seebeck effect

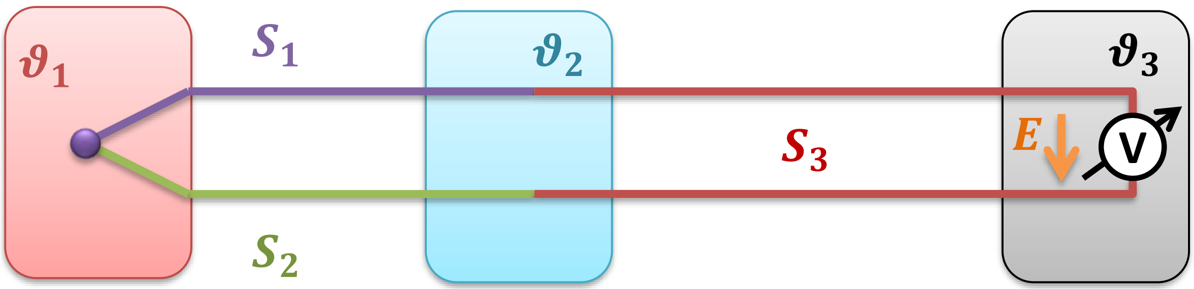


$$E = S_1(\vartheta_1 - \vartheta_2) - S_2(\vartheta_1 - \vartheta_2) = (S_1 - S_2)(\vartheta_1 - \vartheta_2)$$

$$E_{Cu-CuNi} = 41.5(\vartheta_1 - \vartheta_2)\ [\mu\mathrm{V}]$$

Sensitivity depends on the Seebeck-constants of materials only

Seebeck effect



$$E = [S_1(\vartheta_1 - \vartheta_2) + S_3(\vartheta_2 - \vartheta_3)] - [S_2(\vartheta_1 - \vartheta_2) + S_3(\vartheta_2 - \vartheta_3)] =$$
$$= (S_1 - S_2)(\vartheta_1 - \vartheta_2)$$

Applies only if connections are kept at the same temperature (ϑ_2)*!*

Metal thermocouples

Type	Material		Class	Input range	Error band
	+ leg	- leg			
J	Iron (Fe)	Constantan (Cu - Ni)	1	$-40 \ldots + 750°C$	$\pm 1.5°C$ or 0.4%
			2	$-40 \ldots + 750°C$	$\pm 2.5°C$ or 0.75%
K	Chromel (Ni - Cr)	Alumel (Ni - Al)	1	$-40 \ldots + 1000°C$	$\pm 1.5°C$ or 0.4%
			2	$-40 \ldots + 1200°C$	$\pm 2.5°C$ or 0.75%
N	Nicrosil (Ni- Cr - Si)	Nisil (Ni - Si)	1	$-40 \ldots + 1000°C$	$\pm 1.5°C$ or 0.4%
			2	$-40 \ldots + 1200°C$	$\pm 2.5°C$ or 0.75%
E	Chromel (Ni - Cr)	Constantan (Cu - Ni)	1	$-40 \ldots + 800°C$	$\pm 1.5°C$ or 0.4%
			2	$-40 \ldots + 900°C$	$\pm 2.5°C$ or 0.75%
T	Copper (Cu)	Constantan (Cu - Ni)	1	$-40 \ldots + 350°C$	$\pm 0.5°C$ or 0.4%
			2	$-40 \ldots + 350°C$	$\pm 1°C$ or 0.75%
			3	$-200 \ldots + 40°C$	$\pm 1°C$ or 1.5%

According to IEC 60584 (European standard) – color, input range and error band values of the US standard are different

Noble metal thermocouples

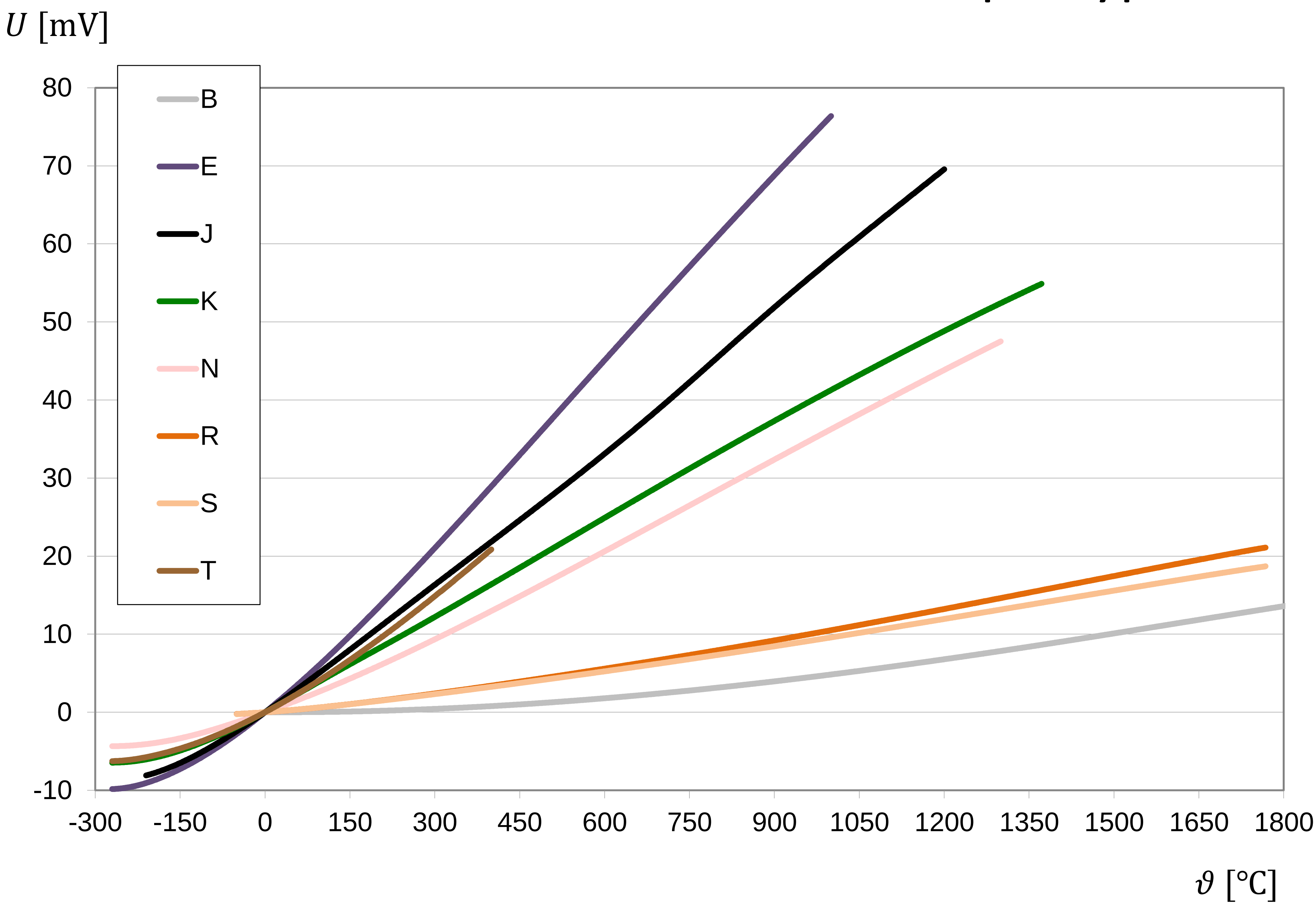
Type	Material		Class	Input range	Error band
	+ leg	- leg			
R	Platinum - Rhodium (Pt - 13% Rh)	Platinum (Pt)	1	$0 \ldots + 1600°C$	$\pm 1°C$ or $\pm[1 + 0.03(\vartheta - 1100)]°C$
			2	$0 \ldots + 1600°C$	$\pm 1.5°C$ or 0.25%
S	Platinum - Rhodium (Pt - 10% Rh)	Platinum (Pt)	1	$0 \ldots + 1600°C$	$\pm 1°C$ or $\pm[1 + 0.03(\vartheta - 1100)]°C$
			2	$0 \ldots + 1600°C$	$\pm 1.5°C$ or 0.25%
B	Platinum - Rhodium (Pt - 30% Rh)	Platinum - Rhodium (Pt - 6% Rh)	2	$+600 \ldots + 1700°C$	0.25%
			3	$+600 \ldots + 1700°C$	$\pm 4°C$ or 0.5%

According to IEC 60584 (European standard) – color, input range and error band values of the US standard are different

Non-standard thermocouples

Type	Material		Class	Input range	Error band
	+ leg	- leg			
G (W)	Wolfram (W)	Wolfram – Rhenium (W – 26% Re)	2	$0 \ldots + 2320°C$	$\underline{+}4.5°C$ or 1%
C (W5)	Wolfram – Rhenium (W – 5% Re)	Wolfram – Rhenium (W – 26% Re)	2	$0 \ldots + 2320°C$	$\underline{+}4.4°C$ or 1%
D (W3)	Wolfram – Rhenium (W – 5% Re)	Wolfram – Rhenium (W – 26% Re)	2	$0 \ldots + 2320°C$	$\underline{+}4.5°C$ or 0.4%

Static characteristics of thermocouple types



Cause of nonlinearity

- Temperature-dependence of Seebeck-coefficient

- Other effects
  - Peltier effect
  - Thomson effect
  - contact potential

Description of nonlinear characteristics

- Polynomial approximation
  - various coefficients for different temperature ranges ($4-10$ coefficients, 10^0-10^{-14} magnitude)
  - direct characteristics: $E=\sum_{i=0}^{n} c_i(t_{90})^i$ [mV]
  - inverse characteristics: $t_{90}=\sum_{i=0}^{n} d_i E^i$ [°C]
  - error of approximation: $\varepsilon \approx 0.05$°C

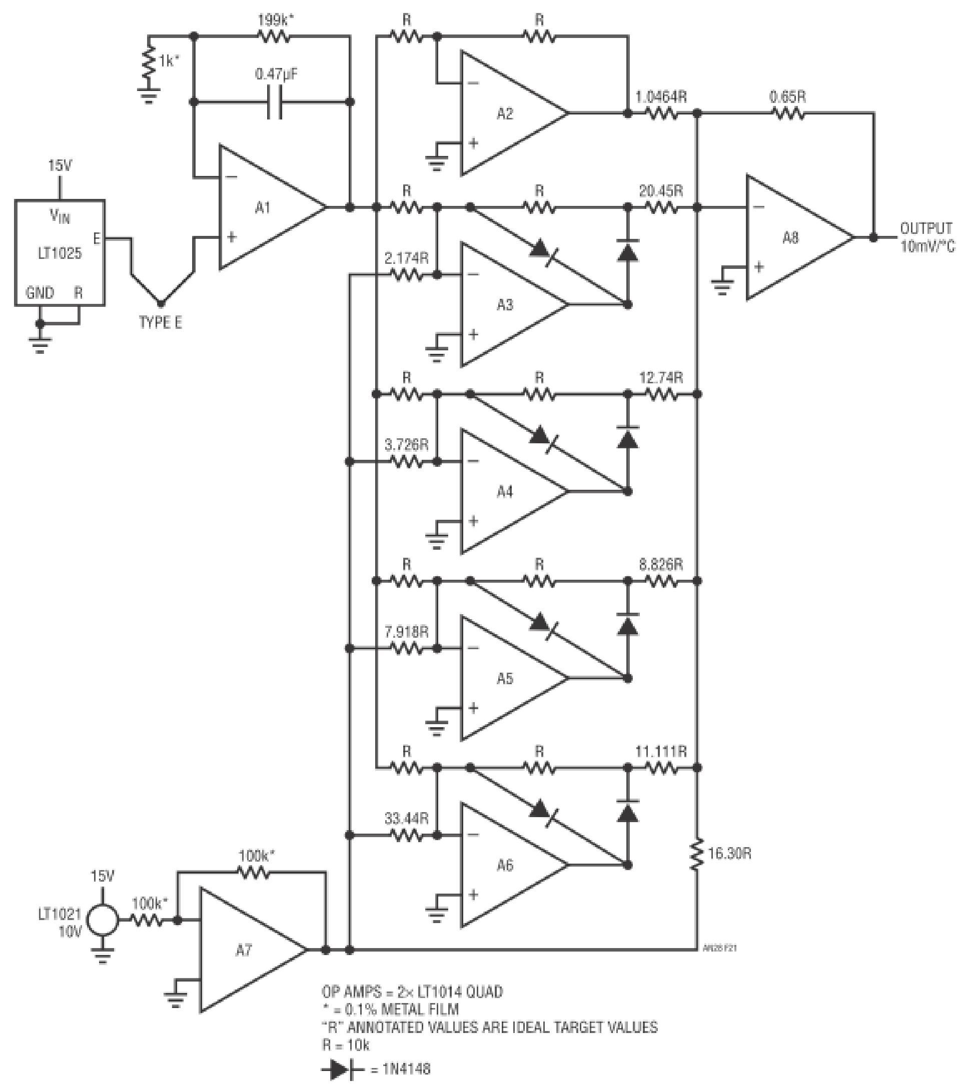
- Linear approximation
  - standard tables
  - error of approximation: $\varepsilon \approx 0.1$°C

Megjegyzés: t_{90} az ITS-90 szabvány jelölése, nem 90°C-t jelent!

Linearization of characteristics

- Analog linear interpolation
- Digital linear approximation
- Approximation by continuous function

- Off-the-shelf hardware components and software libraries



Measurement of absolute temperature

- Problem: voltage output of the thermocouple is proportional to the temperature difference between the hot and cold junction:

$$\mathrm{E} = (S_1 - S_2)(\vartheta_1 - \vartheta_2)$$

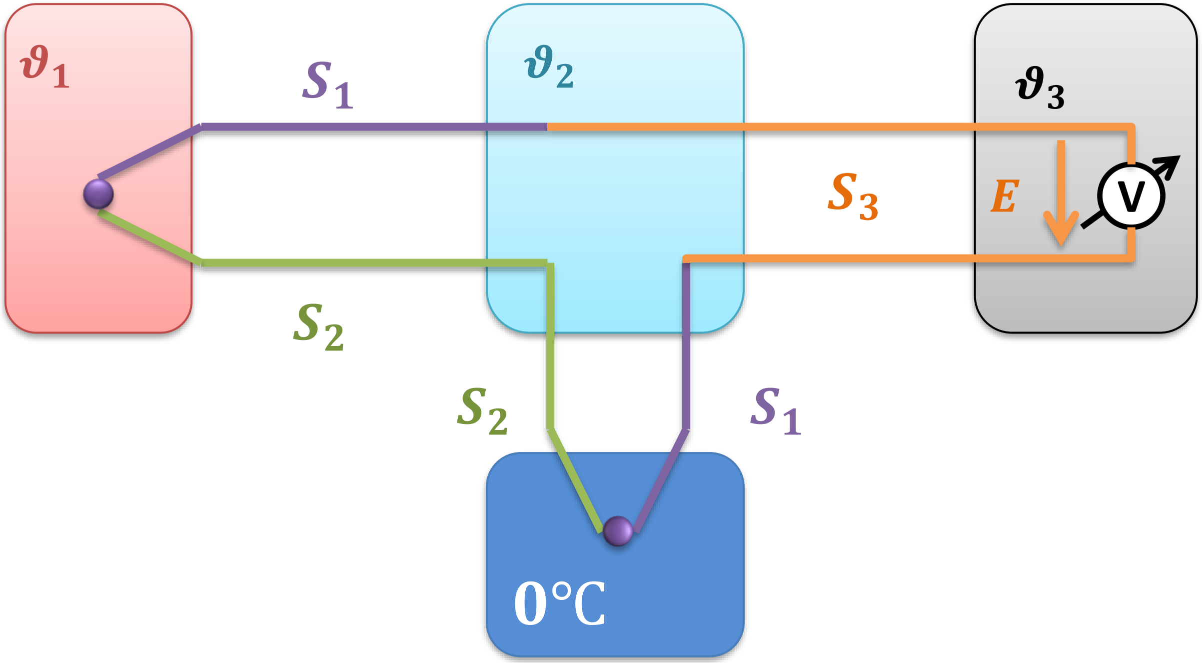
- Goal: output independent from the temperature of the cold junction:

$$E = (S_1 - S_2)\vartheta_1$$

Setting the temperature of the cold junction to $0°C$

- $\vartheta_2 = 0°C \Rightarrow E = (S_1 - S_2)\vartheta_1$
- Exact, no need for compensation
- The standard defines characteristics with cold junction kept temperature of $0°C$
- Melting ice or cooled reference chamber
- Cold junctions of multiple thermocouples might be stabilized simultaneously

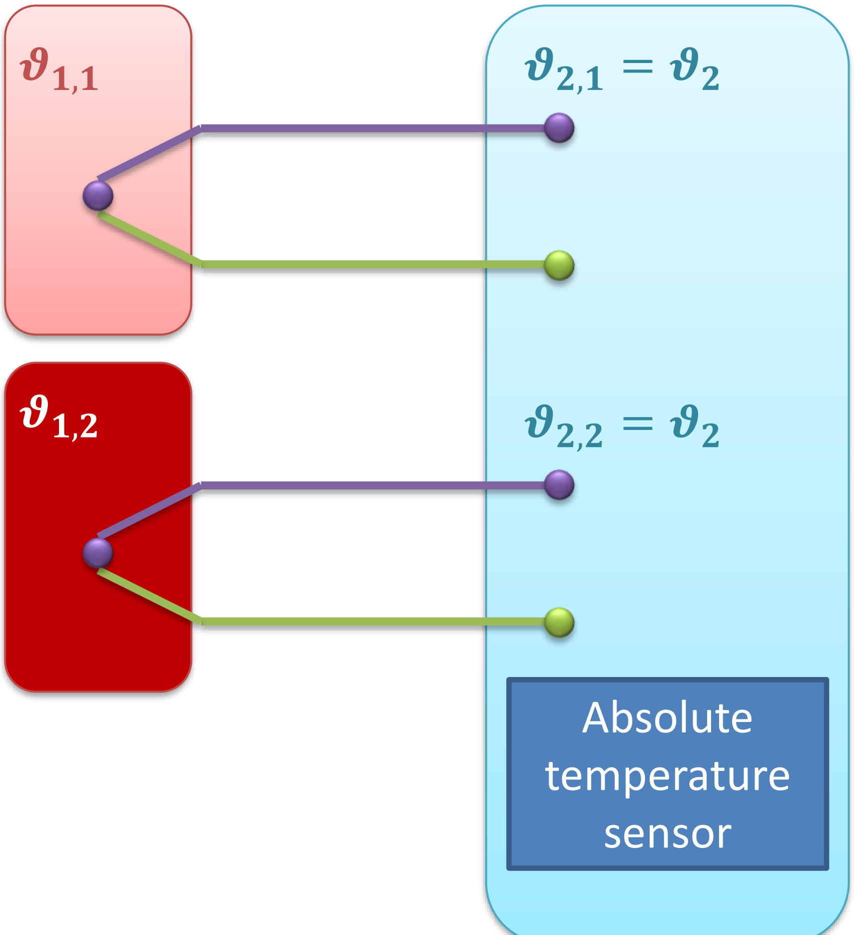
Reference thermocouple



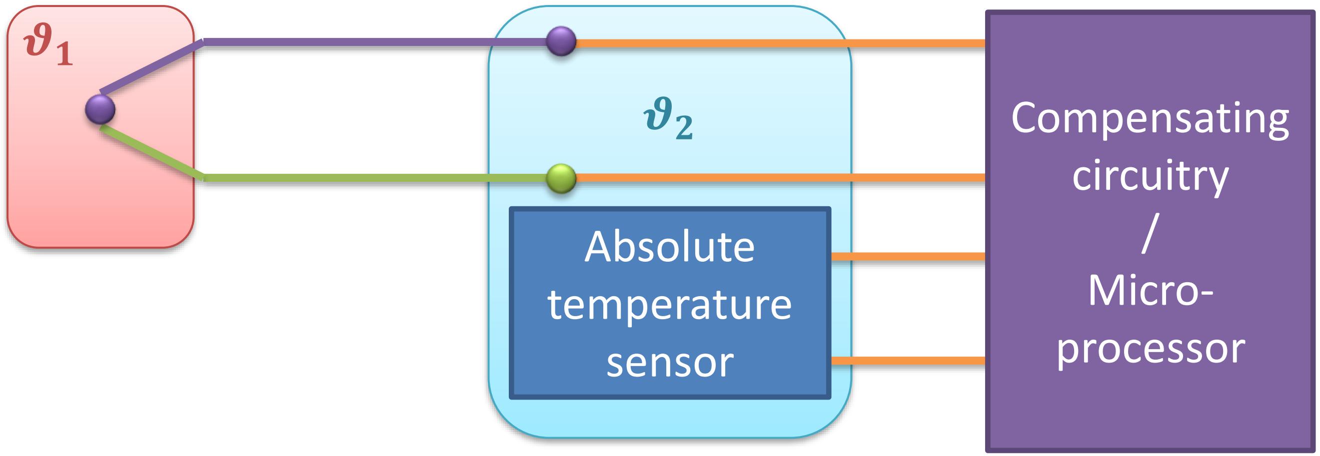
$$E = S_1(\vartheta_1 - \vartheta_2) + S_3(\vartheta_2 - \vartheta_3) - S_2(\vartheta_1 - \vartheta_2) -$$
$$-S_2(\vartheta_2 - 0) - S_1(0 - \vartheta_2) - S_3(\vartheta_2 - \vartheta_3) =$$
$$= (S_1 - S_2)(\vartheta_1 - \vartheta_2) + (S_1 - S_2)(\vartheta_2 - 0) =$$
$$= (S_1 - S_2)(\vartheta_1)$$

Cold junction compensation

- Compensation by measurement of the cold junction temperature
  - measurement by an absolute temperature sensor (IC thermometer)
  - keeping cold junctions of multiple thermometers at an isothermal allows compensation using one single absolute sensor



Cold junction compensation

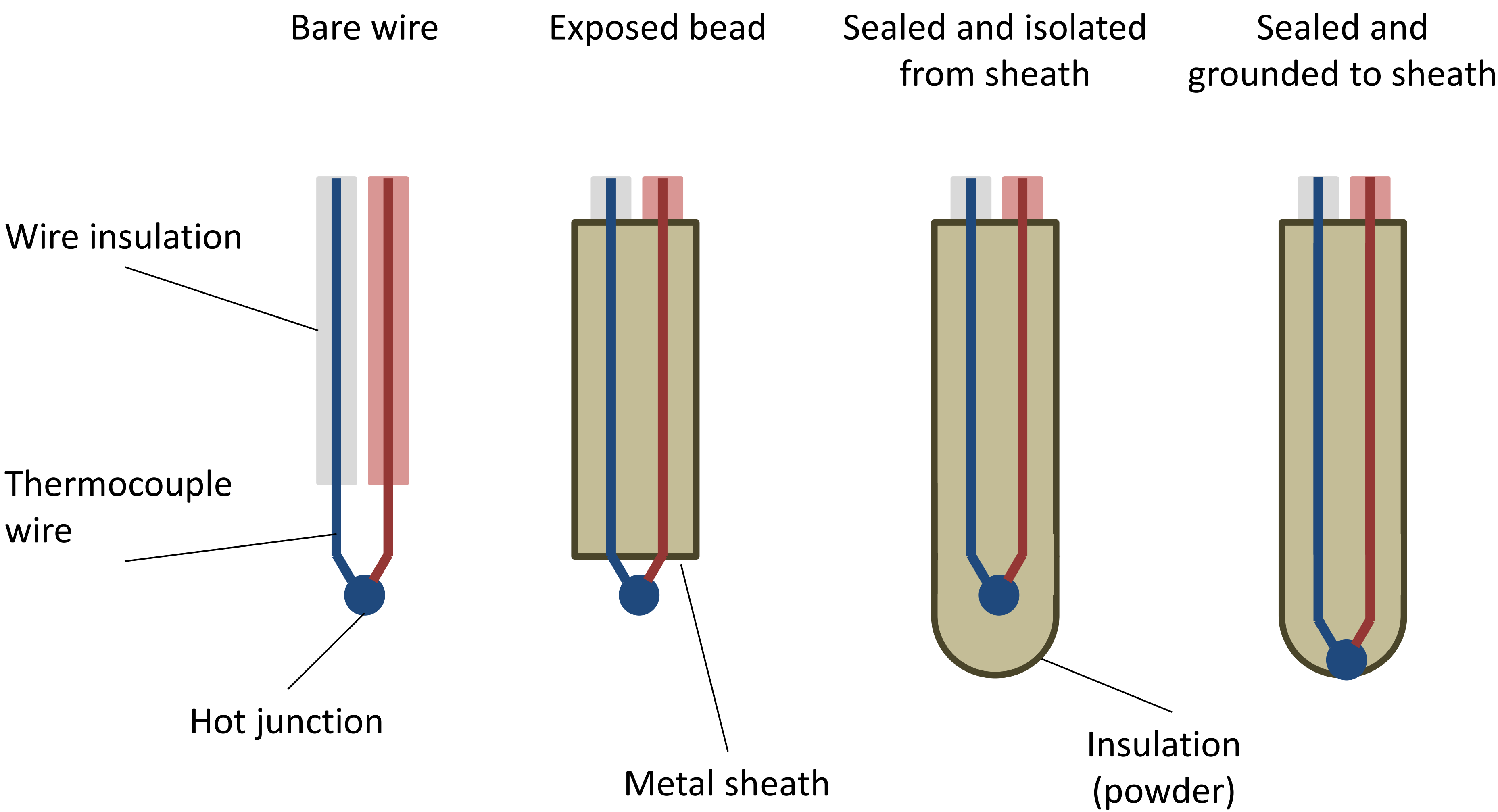


- Compensation according to the temperature of the cold junction
  - by hardware: adding the voltage $E_{comp} = (S_1 - S_2)\vartheta_2$ to the measured output voltage

    $$E = (S_1 - S_2)(\vartheta_1 - \vartheta_2) + E_{comp} = (S_1 - S_2)\vartheta_1$$

  - by software: adding the value ϑ_2 to the temperature value $\vartheta_2 - \vartheta_1$ measured by the thermocouple

Thermocouple sheath types

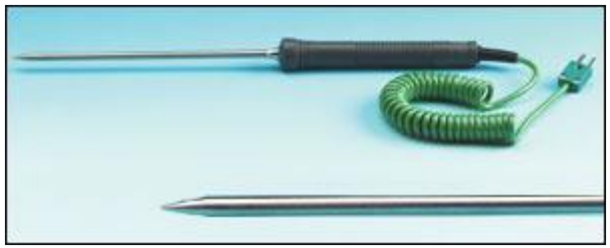


Thermocouple insulation

- Standard insulation material
  - PVC: $-30 \ldots 150°C$
  - Teflon: $-273 \ldots 250°C$
  - glass fiber: $-50 \ldots 800°C$
- Ceramic
- Mineral salt insulation
  - filling of closed metal sheath
  - common use of magnesium-oxide insulation powder
  - $-200 \ldots + 1250°C$

Thermocouple assemblies

- Bare wire with welded hot junction
- Autoclave probe
- Handle probe
- Armoured thermocouple assemblies



Thermocouple wiring

- Color codes
  - positive wire: color of the thermocouple
  - negatív wire: white
  - oversheat: color of the thermometer (blue for explosion proof thermocouples)

- Standardized connectors
  - soldering is prohibited – copper and solder act as a thermocouple ($3\mu V/°C$)

Thermocouple wiring

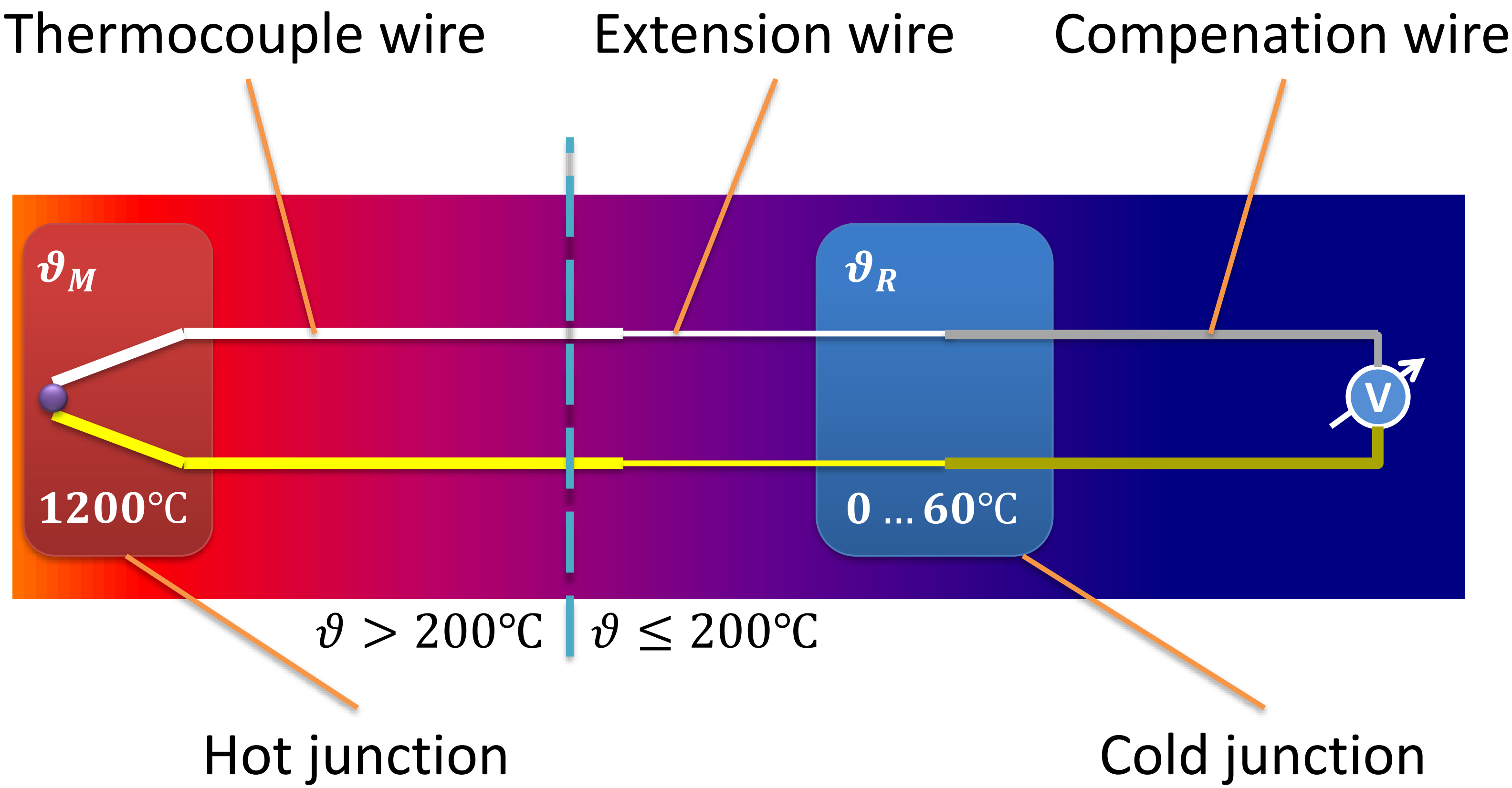
- Thermocouple wire
  - sensing wire without sheath

- E**X**tension wire
  - material identical to the thermocouple wire
  - lower temperature range then the thermocouple
  - lower cost (but still high)
  - notation: X postfix (e.g. JX for J-type thermocouples)
  - Applied where significant temperature gradient is present, i.e. between the end of the thermocouple wire and the cold junction

Thermocouple wiring

- **C**ompensation wire
  - material similar to thermocouple wire but with lower resistance
  - narrow temperature range
  - lower cost than extension wire
  - Notation: C postfix (e.g. JC02 or JCA for J-type thermocouples)
  - applied where no significant temperature gradient is present: between the cold junction and location of voltage measurement

Wiring



Thermocouple wire
Extension wire
Compenation wire
ϑ_M
1200°C
ϑ_R
0 ... 60°C
V
$\vartheta > 200°C$
$\vartheta \leq 200°C$
Hot junction
Cold junction

Thermocouples

+ wide input range
+ robust
+ standardized, interchangable
+ needs no power supply
+ low cost

− low output voltage
− low sensitivity
− low stability
− relative measurement
− high cost of wiring
− nonlinear characteristics

Input range of temperature sensors

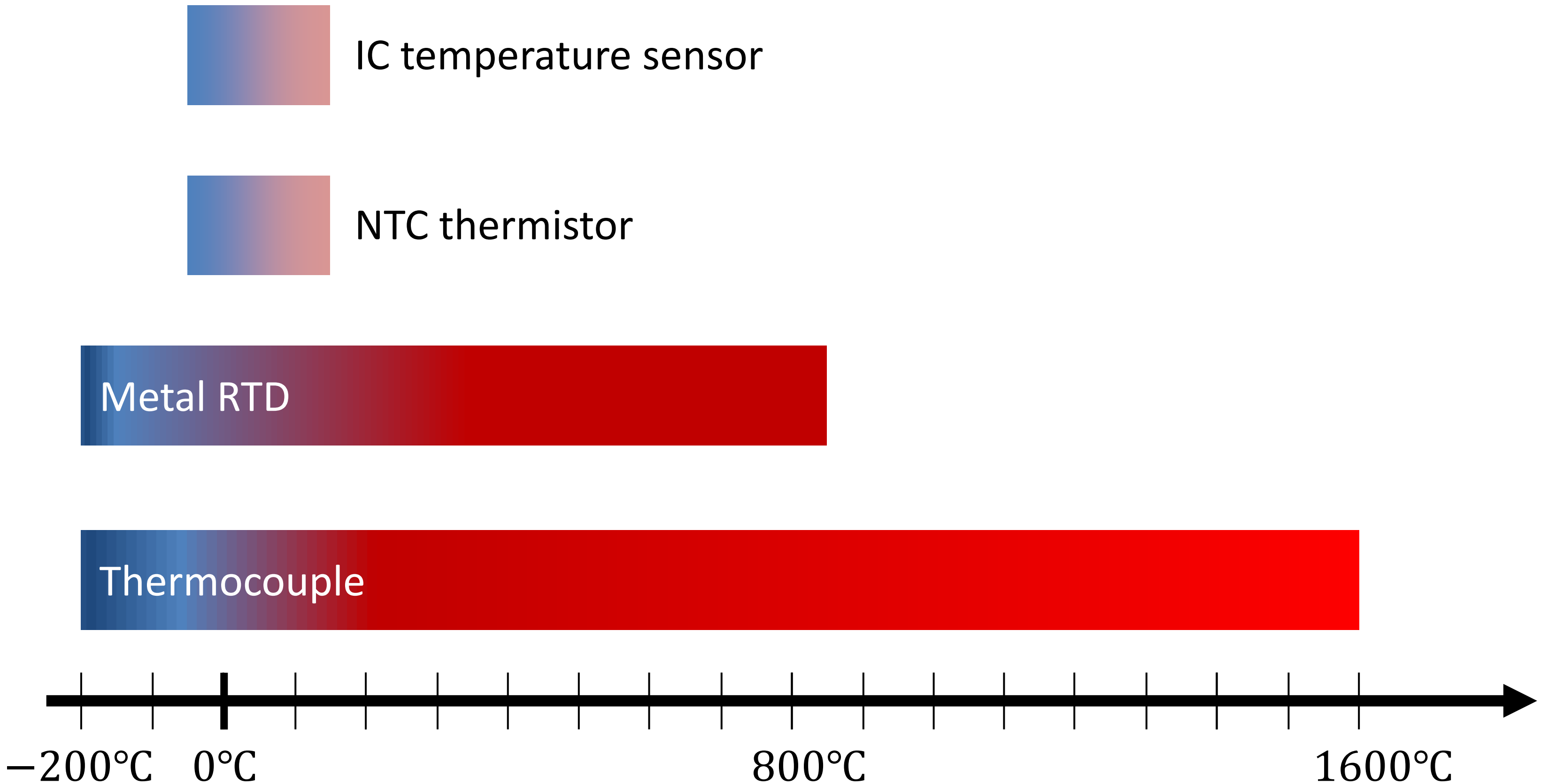


Figure illustrates common input ranges, specific sensors might have wider input range.

Property	Platinum RTD	Thermocouple	NTC thermistor	IC thermometer
Input range	$-200 \ldots + 850°C$	$-200 \ldots + 1600°C$ (varies by type)	$-50 \ldots 150°C$	$-50 \ldots 150°C$
Accuracy (for the input range)	$\pm 1.5°C \approx 0.2\%$	$0.75\% \approx 4.8°C$	$\pm 5\% \approx 10°C$	$\pm 0.5 - 5\%$ $\approx 1 - 10°C$
Linearity	nearly linear	nonlinear	nonlinear	linear
Excitation	current	none	current	supply voltage
Output	resistance	voltage	resistance	voltage / digital
Standardized	yes	yes	no	no
Self-heating	yes	no	yes	yes
Stability	outstanding	average	low	low
Cost	high	average	very low	low
Other features		high wiring cost, relative measurement	low time constant	can be integrated at wafer-level, high sensitivity

Table shows average properties, specific models might have different parameters.