Force, torque and pressure sensors

Industrial Control

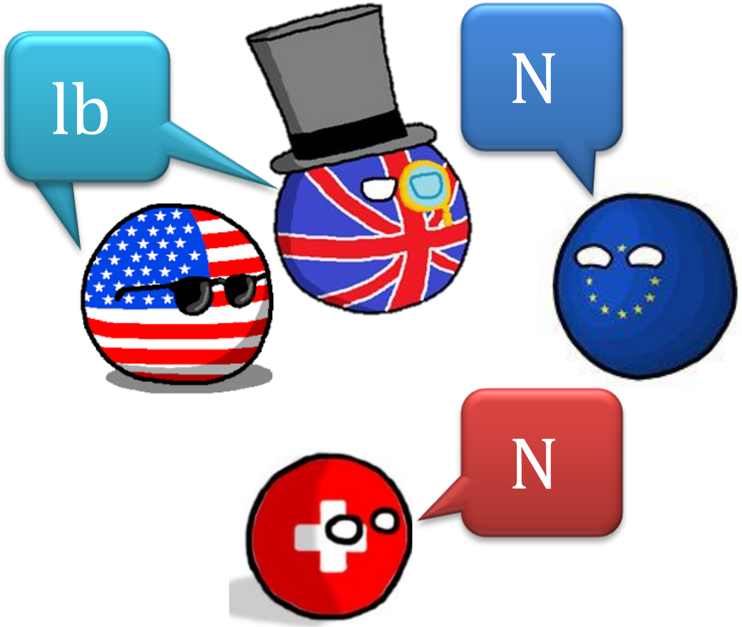
KOVÁCS Gábor
gkovacs@iit.bme.hu

Force

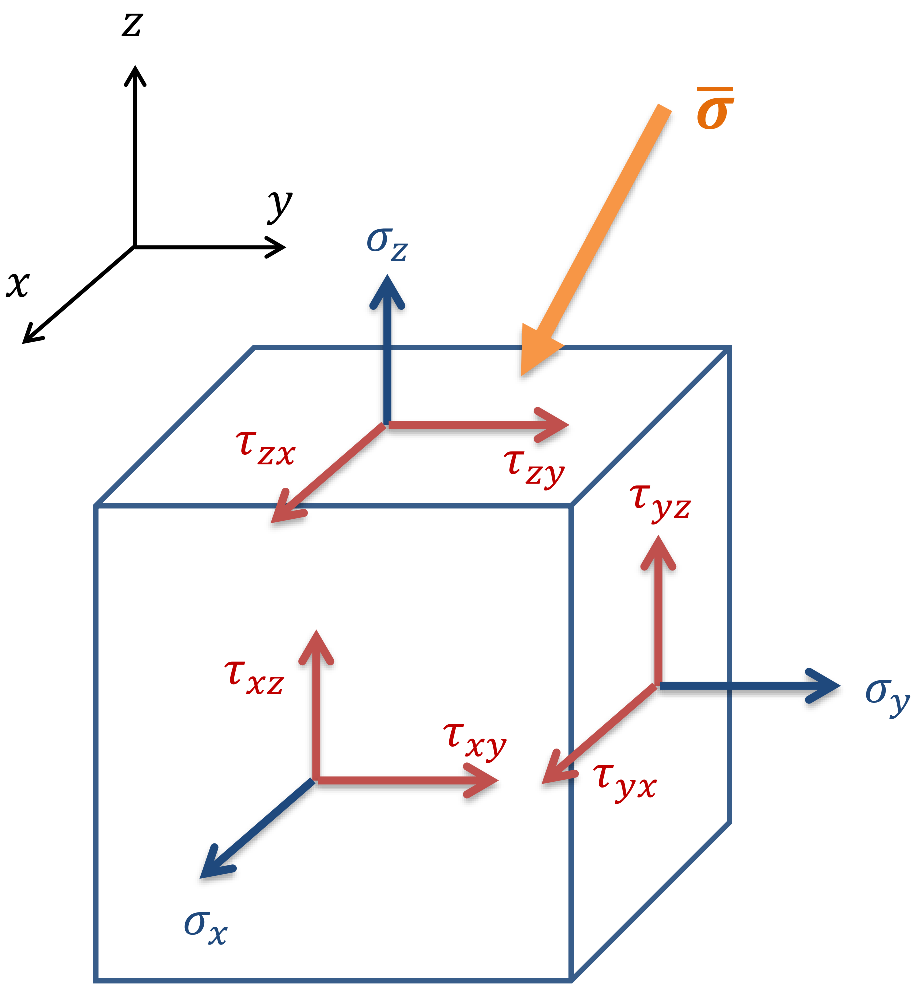
- What do we know about force?
  - a force is an interaction which causes an object with mass to change its velocity
  - SI base unit
  - vector quantity

Units

- Newton - N
  - 1N is the force which accelerates a mass of 1kg by $1\ m/s^2$
- Pound – lb, lbf
  - 1 lb is the force which accelerates a mass of 1 slug by $1\ \ ft/s^2$
  - 1 lbf is the force exerted by standard gravity on a mass of 1 lbm
  - $1\ \mathrm{lb} = 4.44822\mathrm{N}$
- Kilogramm - kg
  - 1 kg is the force exerted by standard gravity on a mass of 1 kg



Mechanical stress



Stress:

$$\bar{\sigma} = \frac{\bar{F}}{A} \left[\frac{\mathrm{N}}{\mathrm{m}^2}, \mathrm{Pa}\right]$$

- Normal stress:

$$\sigma_x, \sigma_y, \sigma_z$$

- Shear stress:

$$\tau_{ij} = \tau_{ji}$$

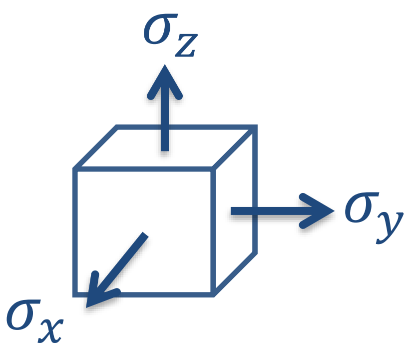
Elastic strain caused by normal stress

Up to a limit of stress, a mass is exposed to elastic strain (Hooke's law):

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{bmatrix} = \begin{bmatrix}1/E & -\nu/E & -\nu/E\\ -\nu/E & 1/E & -\nu/E\\ -\nu/E & -\nu/E & 1/E\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{bmatrix}$$

where

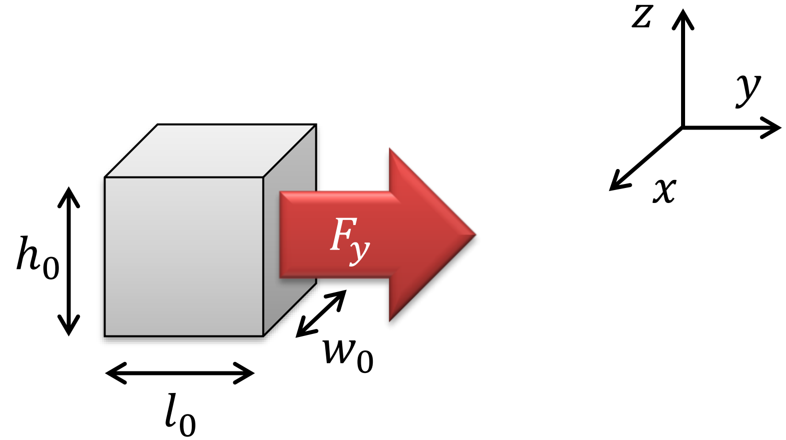
- ε_x is the relative strain in direction x
- E is Young's modulus
  (silicon: 117GPa, steel: 200GPa)
- ν is Poisson's ratio
  ($\nu \leq 0.5$, silicon: 0.22, steel: 0.3)



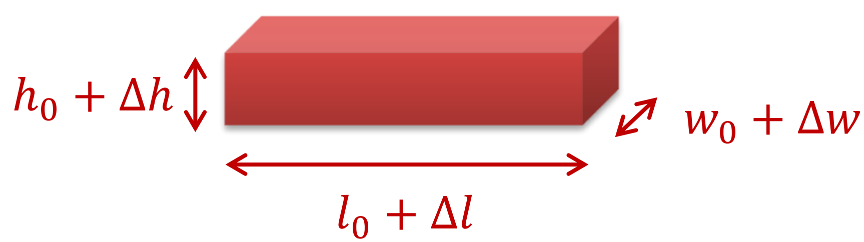
Elastic strain

- Only normal stress is exerted:

$$\sigma_y = \frac{F_y}{A}$$



- Relative strain:
  - $\varepsilon_y = \dfrac{\sigma_y}{E}$
  - $\varepsilon_x = \varepsilon_z = -\dfrac{\nu\sigma_y}{E}$



- Absolute strain:
  - $\Delta l = l_0\varepsilon_y = l_0\dfrac{\sigma_y}{E} > 0$
  - $\Delta w = w_0\varepsilon_x = w_0\dfrac{-\nu\sigma_y}{E} < 0$
  - $\Delta h = h_0\varepsilon_z = h_0\dfrac{-\nu\sigma_y}{E} < 0$

Problem

Give the change of the height and base area of a steel cube with 10 cm edge length if a mass of 100 kg is placed on it. Young's modulus and Poisson-ratio for steel are $E = 200\text{GPa}$ and $\nu = 0.3$, respectively.

- $\varepsilon_z = \frac{\sigma_z}{E} = \frac{-mg}{A}/E \approx \frac{-1000}{0.1 \cdot 0.1}/(200 \cdot 10^9) = -\frac{10^5}{2 \cdot 10^{11}} = -5 \cdot 10^{-7}$

$$\Delta h = \varepsilon_z h_0 = -5 \cdot 10^{-7} \cdot 0.1\text{ m} = -0.05\mu\text{m}$$

- $\varepsilon_x = \varepsilon_y = -\frac{\nu \sigma_z}{E} = 1.5 \cdot 10^{-7}$

$\Delta A = (l_0 + \varepsilon_x l_0) \cdot \left(w_0 + \varepsilon_y w_0\right) - l_0 w_0 = (0.1 + 0.1 \cdot 1.5 \cdot 10^{-7})^2 - 0.1^2 =$
$= 2 \cdot 0.1 \cdot 0.1 \cdot 1.5 \cdot 10^{-7} + 2.25 \cdot 10^{-16} \approx 0.003\text{mm}^2$

Effect of strain on electrical properties

- Resistance – resistance of a conductor depends on its resistivity, length and diameter

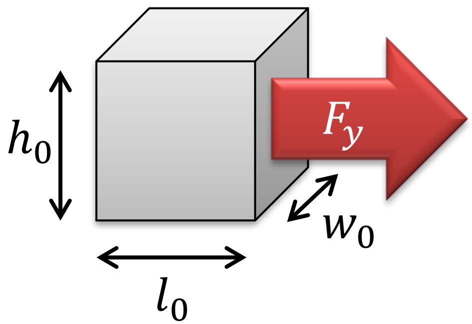
- $R = \rho \frac{l}{A}$, where

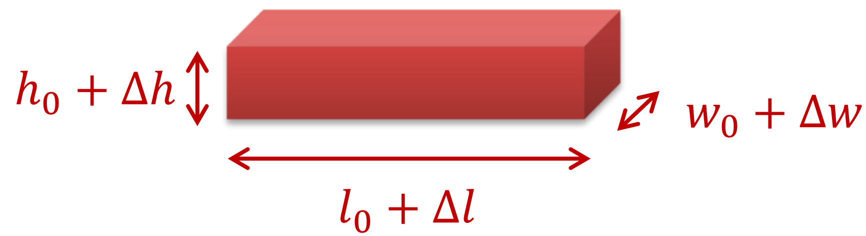
  $\rho = 1/\sigma$ is the resistivity $[\Omega\mathrm{m}]$,

  l is the length of the conductor $[\mathrm{m}]$,

  A is the cross section of the conductor $[\mathrm{m}^2]$

Geometrical effect of normal strain





$$R' = \rho \frac{l'}{A'} = \rho \frac{l_0 + \Delta l}{(w_0 + \Delta w)(h_0 + \Delta h)} = \rho \frac{l_0(1 + \varepsilon_y)}{w_0(1 + \varepsilon_x)h_0(1 + \varepsilon_z)} = \rho \frac{l_0}{A_0} \frac{1 + \varepsilon_y}{(1 + \varepsilon_x)(1 + \varepsilon_z)}$$

- $\varepsilon_x = \varepsilon_z \Rightarrow (1 + \varepsilon_x)(1 + \varepsilon_z) = 1 + \varepsilon_x + \varepsilon_z + \varepsilon_x \varepsilon_z = 1 + 2\varepsilon_x + \varepsilon_x^2 = 1 - 2\nu\varepsilon_y + \nu^2\varepsilon_y^2$
- $\varepsilon_y \ll 1, \nu < 1 \Rightarrow 1 - 2\nu\varepsilon_y + \nu^2\varepsilon_y^2 \approx 1 - 2\nu\varepsilon_y$
- $\frac{1}{1-2\nu\varepsilon_y} = \frac{1+2\nu\varepsilon_y}{1-(2\nu\varepsilon_y)^2} \approx 1 + 2\nu\varepsilon_y$

$$\Rightarrow \quad R' \approx \rho \frac{l_0}{A_0}(1 + \varepsilon_y)(1 + 2\nu\varepsilon_y) \approx \rho \frac{l_0}{A_0}(1 + \varepsilon_y(1 + 2\nu))$$

Piezoresistive effect

- Mechanical stress modifies inter-atomic spacing which results in change of the resistivity
- For isotropic effects, the change reads

$$\frac{\Delta\rho}{\rho} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & & & \\ \pi_{12} & \pi_{11} & \pi_{12} & & & \\ \pi_{12} & \pi_{12} & \pi_{11} & & & \\ & & & \pi_{44} & & \\ & & & & \pi_{44} & \\ & & & & & \pi_{44} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

π_{ij} piezoresistive coefficients, for metals $\pi_{44} = 0$

Piezoresistive effect

- Simplified formula:

$$\frac{\Delta\rho}{\rho} = \pi_l\sigma_l + \pi_t\sigma_t,$$

where π_l adn π_t are the longitudinal and transversal (relative to the direction of current flow) piezoresistive coefficients:

$$\pi_{l,NiCr} \approx 10^{-3}\,\frac{1}{\mathrm{GPa}},\ \pi_{l,Si} \approx 10^{-1}\,\frac{1}{\mathrm{GPa}}$$

- Omitting the transversal effect: $\frac{\Delta\rho}{\rho} = \pi_l\varepsilon_l E$

Joint effect of mechanical stress

$$R' = (\rho_0 + \Delta\rho)\frac{l_0}{A_0}\left(1 + \varepsilon_y(1 + 2\nu)\right) =$$

$$\rho_0\left(1 + \frac{\Delta\rho}{\rho_0}\right)\frac{l_0}{A_0}\left(1 + \varepsilon_y(1 + 2\nu)\right) =$$

$$\rho_0\frac{l_0}{A_0}\left(1 + \varepsilon_y\pi_l E\right)\left(1 + \varepsilon_y(1 + 2\nu)\right)$$

$$\varepsilon_y^2 \approx 0 \Rightarrow \boldsymbol{R' \approx R_0(1 + \varepsilon_y(E\pi_l + 1 + 2\nu))}$$

$$\boldsymbol{\Delta R = R' - R_0 = R_0\varepsilon(E\pi_l + 1 + 2\nu)}$$

ε denotes longitudinal strain (ε_y) in the followings

Gauge factor

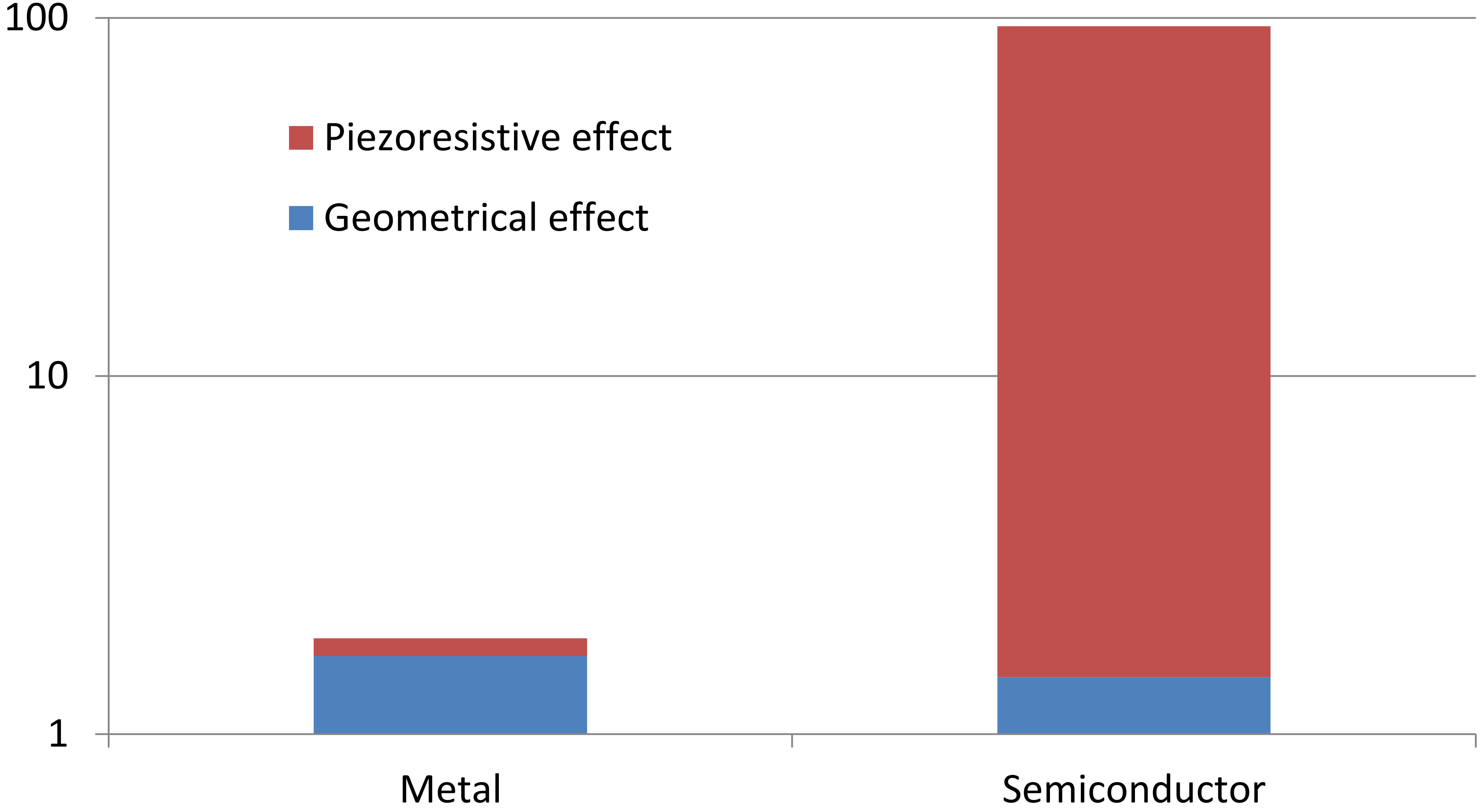
- $\Delta R = R_0 \varepsilon (E\pi_l + 1 + 2\nu)$
- $\boldsymbol{g = GF = \frac{\Delta R/R}{\Delta l/l} = \frac{\Delta R/R}{\varepsilon} = E\pi_l + 1 + 2\nu}$ **[1]**

  Gauge factor is a unitless quantity describing the ratio of relative elastic strain and relative change of resistance

- $\boldsymbol{R = R_0(1 + g\varepsilon)}$

- Value of gauge factor
  - for metals: $2 \dots 5$
  - for semiconductors: $125 \dots 200$ (depends on doping)

Components of the gauge factor

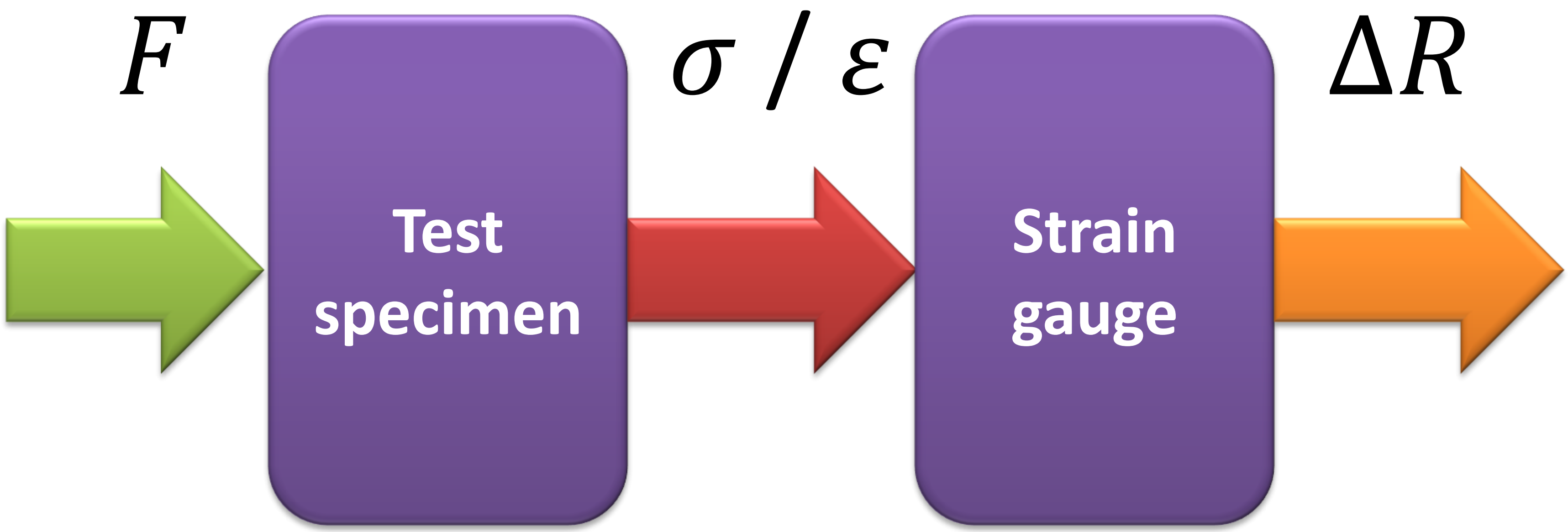


Effect of shear strain

- Piezoresistive effect might change the resistivity if shear strain is exerted
- No effect of shear strain for metals ($\pi_{44} = 0$)
- For semiconductors the effect of shear strain can be significant

Strain gauges

(strain gages)



Dynamical transducer up to a few kHz frequency

Magnitude of change in resistance

- Metal strain gauges
  - maximal strain: $\varepsilon_{max} \approx \pm 0.2\% \ldots \pm 2\%$
  - gauge factor: $g \approx 2 - 5$
  - $\Delta R = g\varepsilon < 10\%$, typically $\boldsymbol{\Delta R \approx 1 \ldots 5}\%$
- Semiconductor strain gauges
  - maximal strain: $\varepsilon_{max} \approx \pm 0.5\%$
  - gauge factor: $g \approx 125 - 200$
  - $\boldsymbol{\Delta R = g\varepsilon > 50}\%$, up to 100%
- Change of resistance limited by the maximal strain of the test specimen!
  - steel: ca. 0.12%
  - aluminum: ca. 0.35%

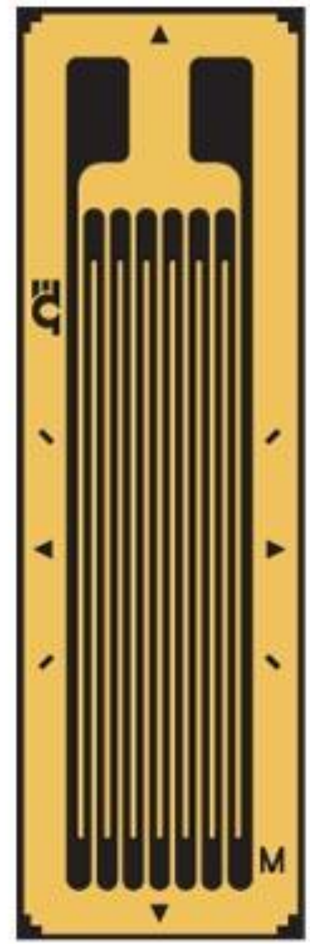
Attachment of strain gauges

- Bond between the strain gauge and the test specimen is of paramount importance: strains of the gauge and the test specimen should be the same
- Strain gauge: resistance layer (metal or semiconductor) on insulated backing
- Attachment by gluing
- Principle cause of error: crawling due to different thermal expansion properties of the test specimen , the strain gauge and the glue
- Use of a special glue is strongly recommended

Metal foil strain gauges

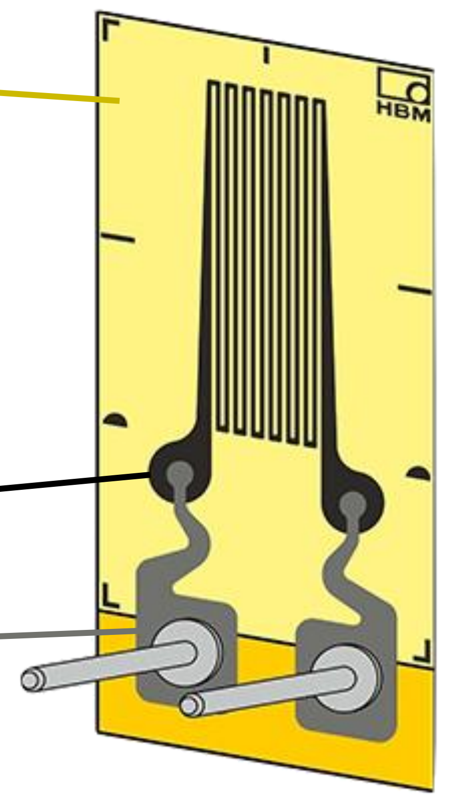
- Goal: high resistance with small footprint
- Solution: zig-zag pattern
- Great sensitivity for strain in the principal direction
- Non-zero transversal sensitivity:

  $g_t = K_t g$, where typically $K_t < 1\%$



Metal foil strain gauges

- Backing
  - insulation
  - attachment (by glue)
  - typical material: epoxy
- Metal foil resistance pattern
- Lead or solder pad
- Optional external enclosure (rare)



Strain gauge alloys

- Constantan (Cu-Ni) - $g \approx 2$
  - low cost
  - thermal expansion can be set by alloying
  - wide linear range
  - ideal for generic use
  - Irreversible change in properties at high temperature
- Karma (K-alloy, Ni-Cr, Nichrome) - $g \approx 2 - 2.4$
  - exceptional linerarity
  - high accuracy and sensitivity
  - high stability
  - wide temperature range: $-269 \ldots + 260\ ℃$

Strain gauge alloys

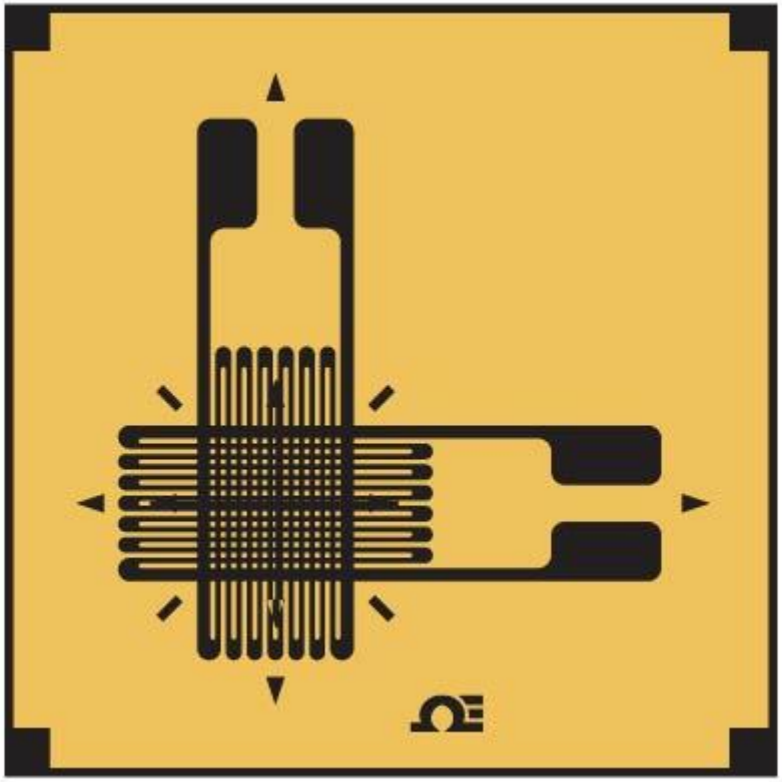
- Platinum alloys - $g \approx 4 - 5$
  - wide temperature range
  - exceptional stability
  - high piezoresistive effect
  - high cost
- Isoelastic - $g \approx 3.5$
  - for measurement of dynamic strain
  - high thermal expansion, sensitive for temperature change

Parameters of metal foil strain gauges

- Base resistance: $90 - 120 - 350 - 600 - 1000\ \Omega$
- Size (length): $1.5 \ldots 50\ \mathrm{mm}$
- Thickness: few microns
- Gauge factor: $2 \ldots 5$
- Elasticity (bending radius): ca. 1/10 of the length
- Accuracy
  - base resistance: $\pm 0.1\ - 1\%$
  - gauge factor: varies with batch

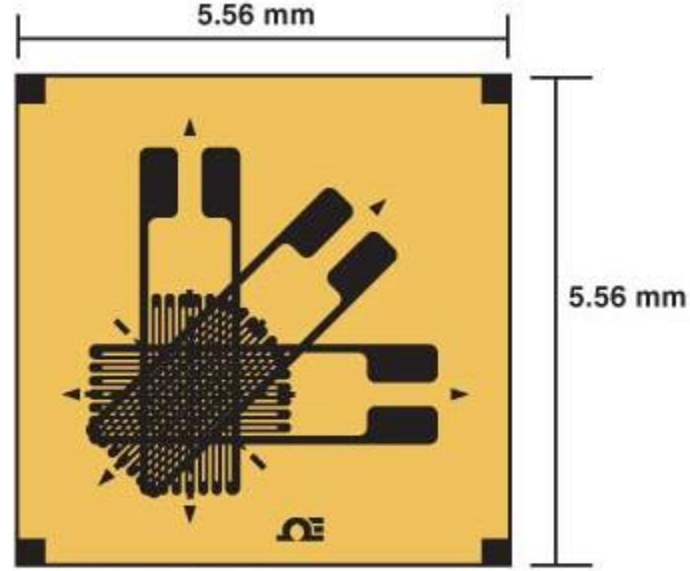
Biaxial (Tee) strain gauges

- Two zig-zag patterns in orthogonal configuration
- For measurement of stress if principal directions are known and no shear stress is present

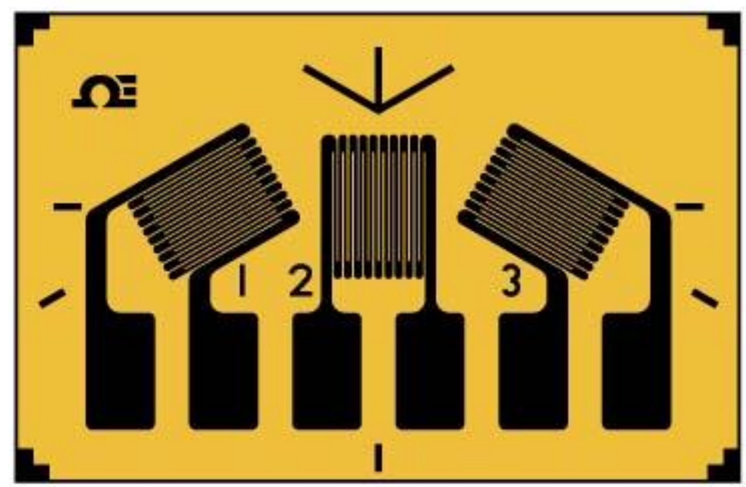


Rosette strain gauges

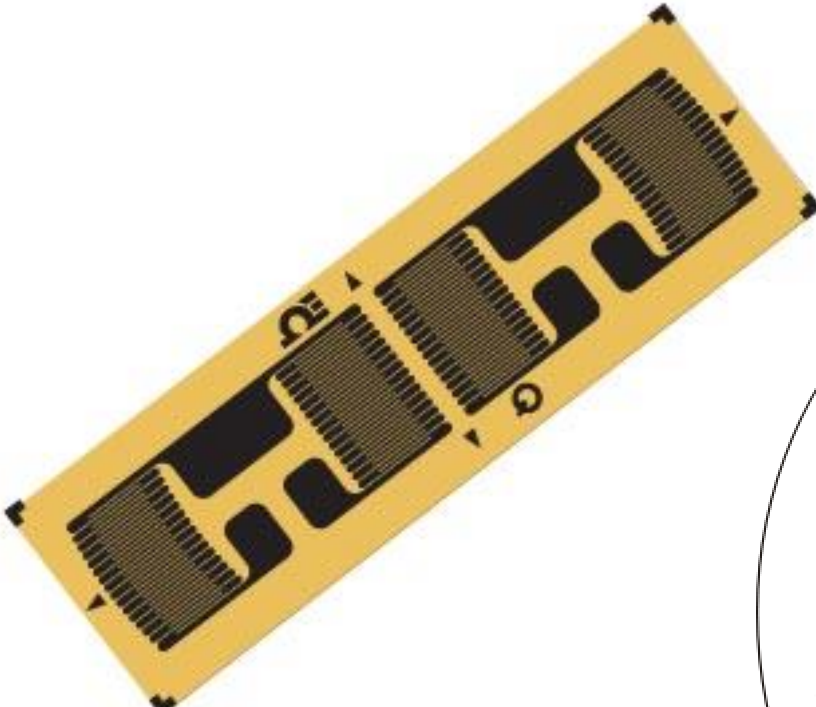
- Suitable for stress measurement on a plane
  - 3 strains
  - 3 components (normal strains and shear strain)

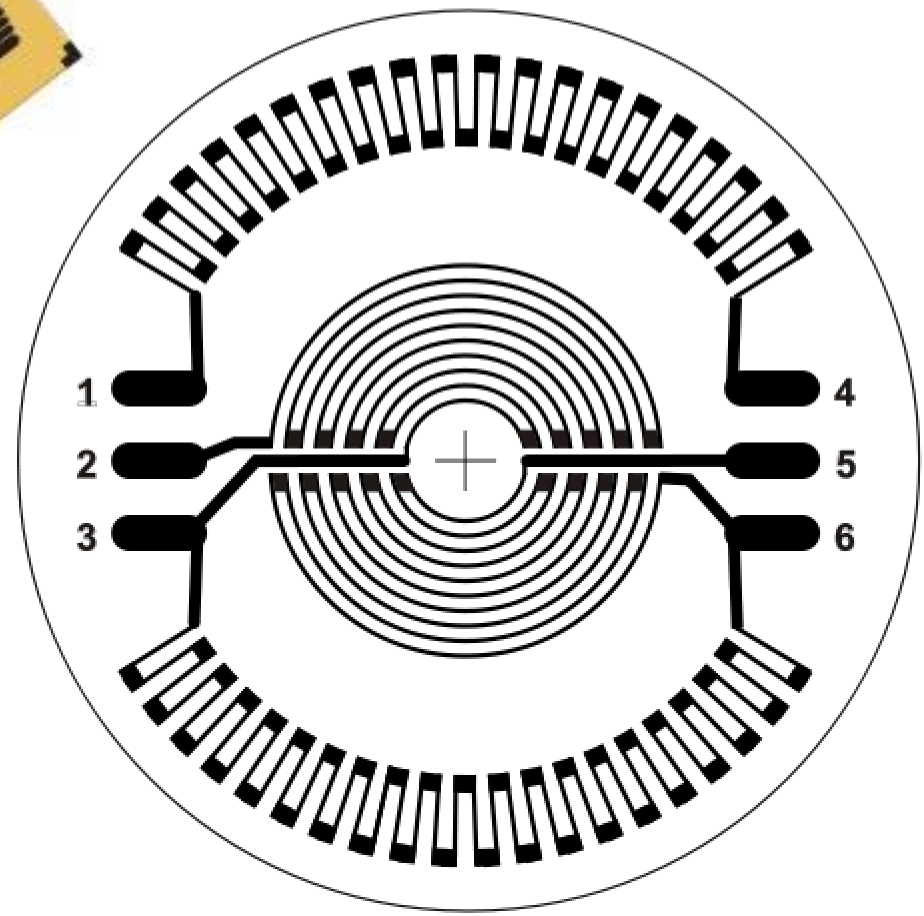


- Zig-zag configurations
  - beside or over each other (in different layers)
  - Angles of 45° or 60°



Special strain gauges

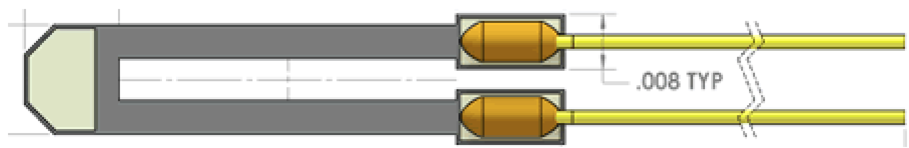


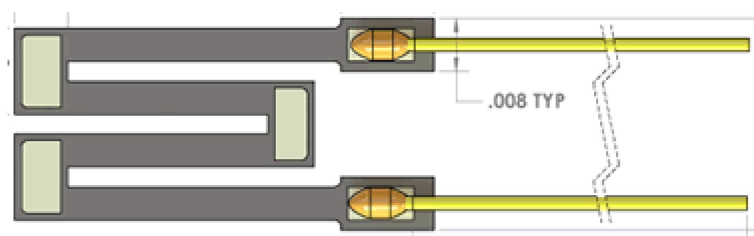


Semiconductor strain gauges

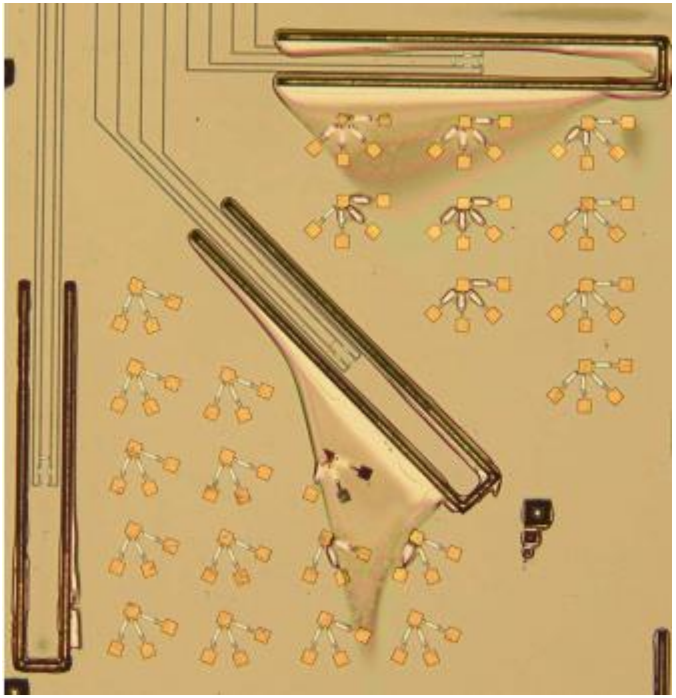
- Doped single crystal or polycrystalline (Si, Ge)
- Base resistance can be set by doping
- No need for complex zig-zag patterns
- High gauge factor
  - single crystal: $g \approx -100\ -\ +200$
  - polycrystalline: $g \approx 30 - 45$
- No need for backing
- Sensitive for multiple normal strains and shear strain
- High temperature sensitivity
- Rigid
- High cost

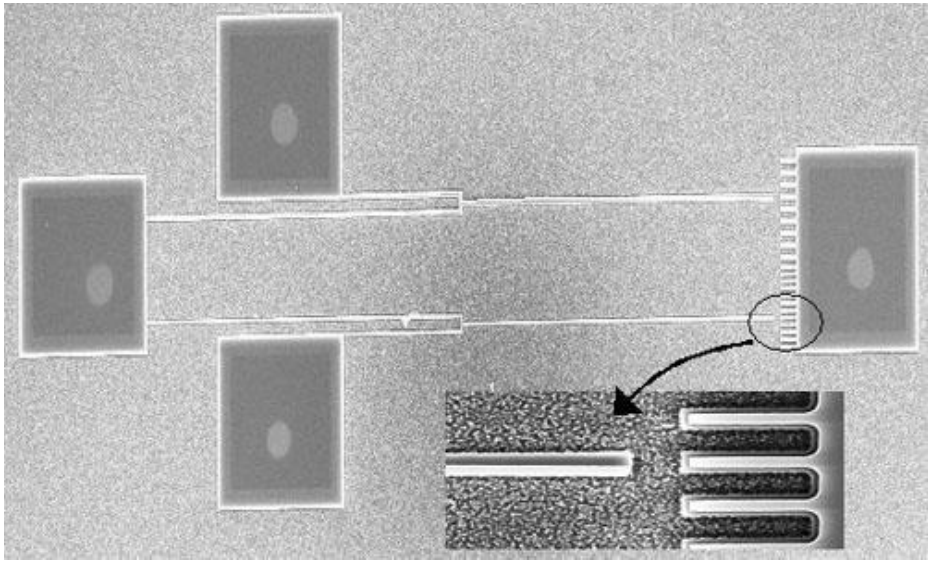
Semiconductor strain gauges



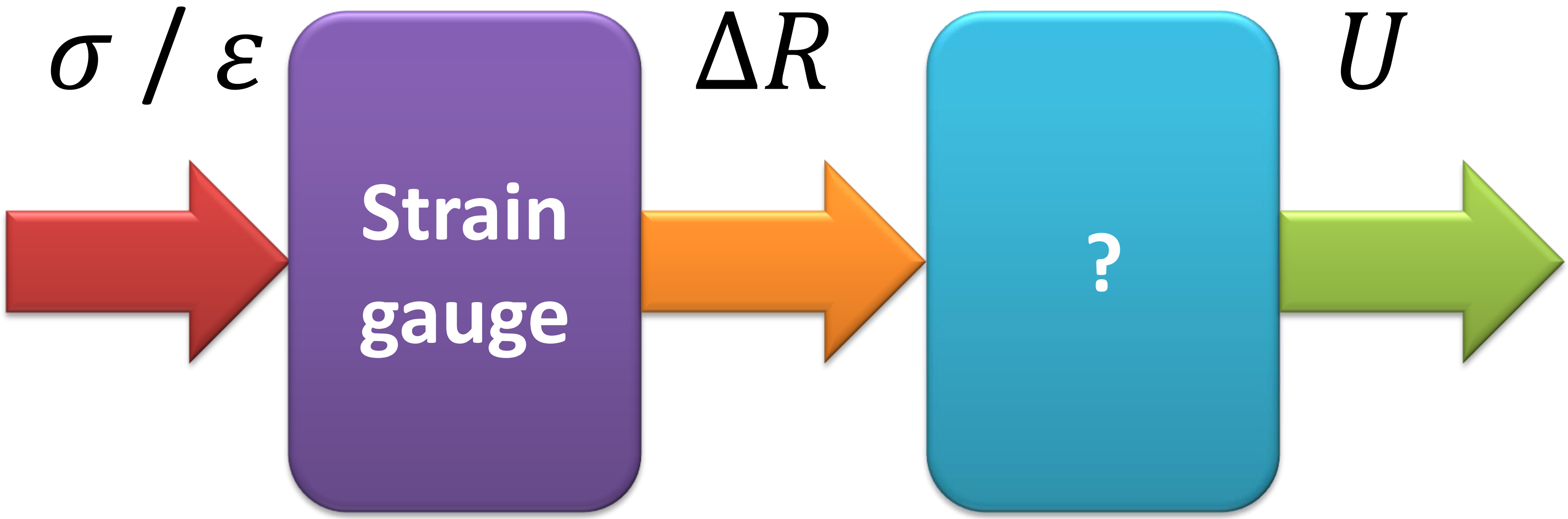


Semiconductor strain gauges

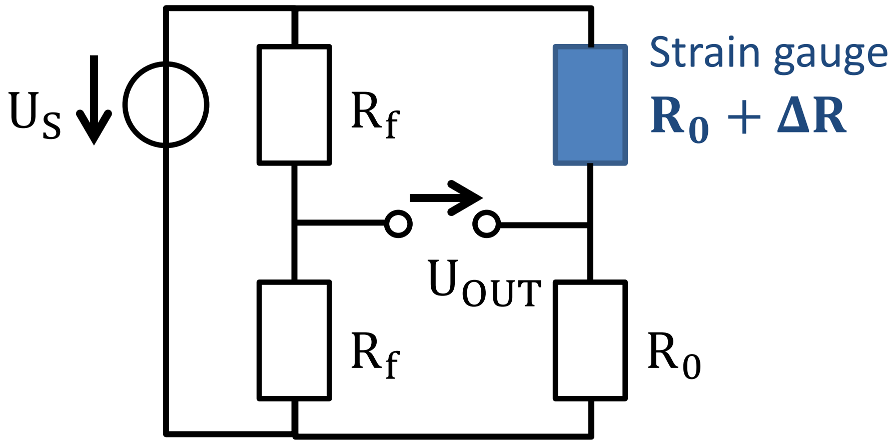




Measurement circuitry for strain gauges



Simple bridge circuit



$$\boldsymbol{U_{OUT}} = \frac{U_S}{2} - U_S\frac{R_0}{R_0 + (R_0 + \Delta R)} = U_S\frac{(2R_0 + \Delta R) - 2R_0}{2(2R_0 + \Delta R)} = U_S\frac{\Delta R}{2(2R_0 + \Delta R)} \approx$$

$$\approx U_S\frac{\Delta R}{4R_0} = \boldsymbol{\frac{U_S}{4} g\varepsilon}$$

Thermal effect

- As for metal RTDs, resistance depends on the temperature by $R \approx R_0(1 + \alpha(\vartheta - \vartheta_0))$, where R_0 is the base resistance at temperature ϑ_0, α is the temperature coefficient of resistance, ϑ is the actual temperature:

$$\frac{\Delta R}{R_0} = \alpha(\vartheta - \vartheta_0) = \alpha\Delta\vartheta$$

- Thermal effect is considered as a virtual strain:

$$\boldsymbol{\varepsilon = \frac{1}{g}\frac{\Delta R}{R_0} = \frac{\alpha}{g}(\vartheta - \vartheta_0) = \frac{\alpha}{g}\Delta\vartheta}$$

Thermal effect - problem

Temperature of a constantan strain gauge is increased by $10°C$. How much is the virtual strain caused by the thermal effect?

For a constantan strain gauge $\alpha = 3 \cdot 10^{-5}\ 1/°C, g = 2$

Virtual strain:

$$\varepsilon = \frac{\alpha}{g}(\vartheta - \vartheta_0) = \frac{3 \cdot 10^{-5}}{2} \cdot 10 = 0.015\%$$

Full scale error if the maximal strain is $\varepsilon_{max} = 1\% = 10^{-2}$:

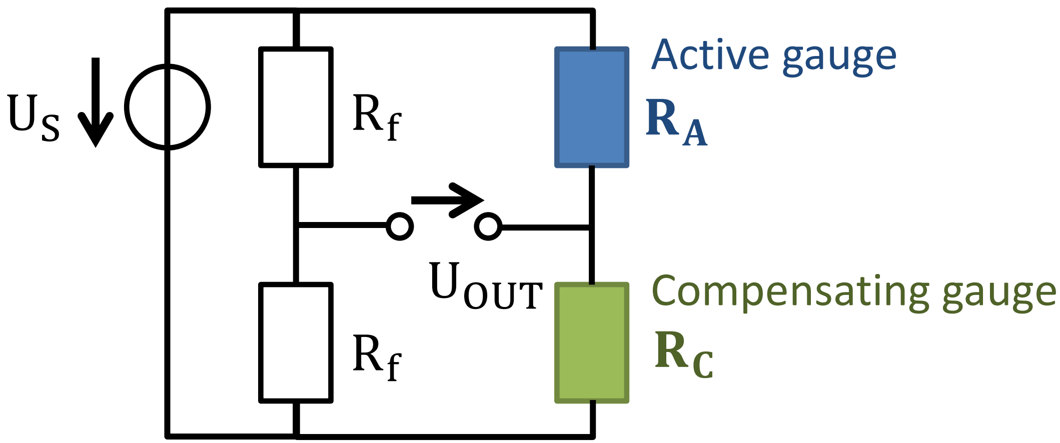
$$\frac{0.015}{1} = 0.015 = \mathbf{1.5\%_{FS}}$$

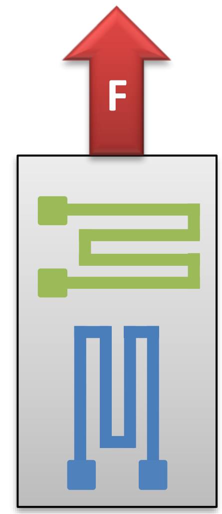
Compensation of thermal effect

- What kind of device shall we use for compensation?
  - a resistor with same base resistance
  - a resistor with same temperature coefficient

- Use an identical strain gauge!
  - no stress should be exerted
  - if not applicable, the stress should be transversal

Compensation of thermal effect





- If only transversal effect is exerted on the compensating gauge: $R_C = R_0(1 + gK_t\varepsilon)(1 + \alpha\Delta\vartheta)$

- $U_{OUT} = \frac{U_T}{2}\frac{R_A - R_C}{R_A + R_C} = \frac{U_T}{2}\frac{R_0(1+g\varepsilon)(1+\alpha\Delta\vartheta) - R_0(1+gK_t\varepsilon)(1+\alpha\Delta\vartheta)}{R_0(1+g\varepsilon)(1+\alpha\Delta\vartheta) + R_0(1+gK_t\varepsilon)(1+\alpha\Delta\vartheta)}$

Compensation of thermal effect

- $U_{OUT} = \frac{U_S}{2} \frac{R_0(1+g\varepsilon)(1+\alpha\Delta\vartheta) - R_0(1+gK_t\varepsilon)(1+\alpha\Delta\vartheta)}{R_0(1+g\varepsilon)(1+\alpha\Delta\vartheta) + R_0(1+gK_t\varepsilon)(1+\alpha\Delta\vartheta)} = \frac{U_S}{2} \frac{(1+g\varepsilon)-(1+gK_t\varepsilon)}{(1+g\varepsilon)+(1+gK_t\varepsilon)} = \frac{U_S}{2} \frac{g\varepsilon - gK_t\varepsilon}{2+g\varepsilon+gK_t\varepsilon} = \frac{U_S}{2} \frac{g\varepsilon(1-K_t)}{2+g\varepsilon(1+K_t)}$

- Simplification: $0 \le K_t \le 0.1$, hence

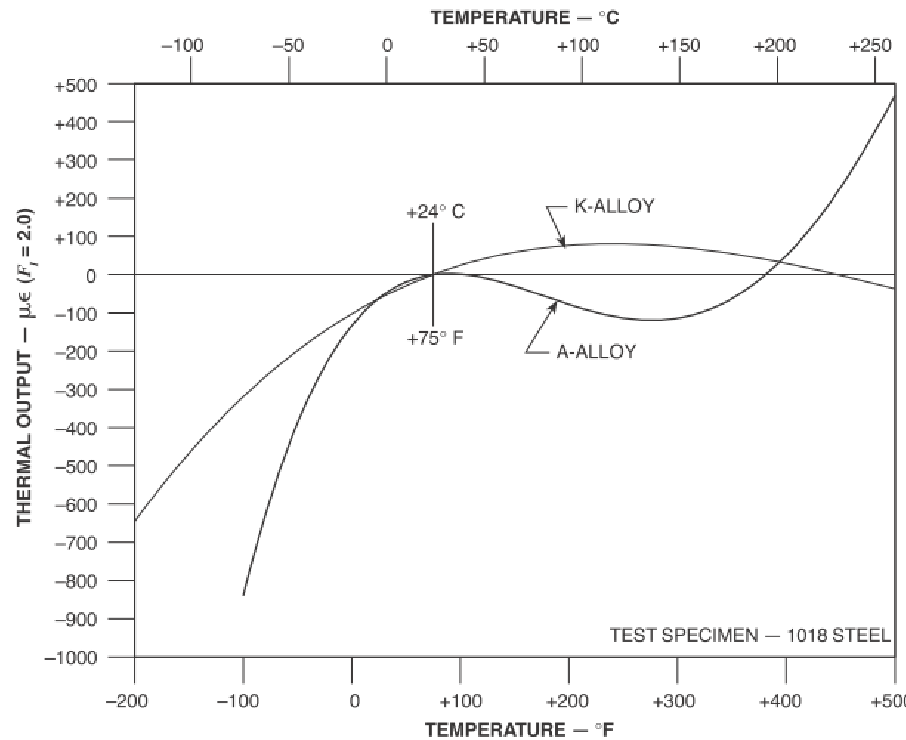
$$(1+K_t) \approx (1-K_t) \approx 1, g\varepsilon \ll 1$$

$$\Rightarrow \boldsymbol{U_{OUT}} = \frac{U_S}{2} \frac{g\varepsilon}{2+g\varepsilon} \approx \frac{U_S}{2} \frac{g\varepsilon}{2} = \boldsymbol{\frac{U_S}{4} g\varepsilon}$$

- Output voltage is
  - temperature-independent
  - nearly linear function of the strain

Self-compensating gauges

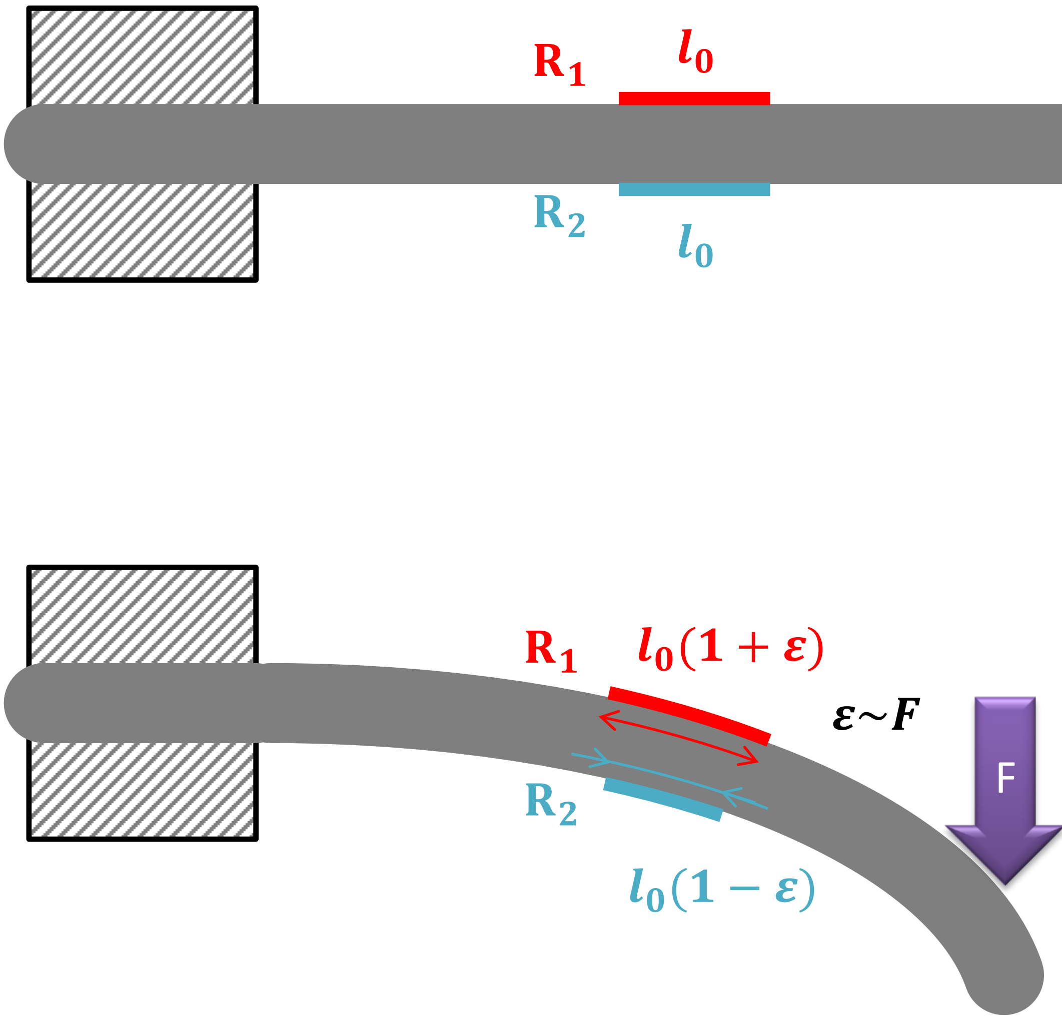
- Problem: thermal extension of the test specimen causes strain, which is considered as effect of an external force
- Self temperature-compensation: temperature dependent resistivity of specific alloys compensate thermal expansion in the most frequently used temperature range
- Various alloys for test specimens of different materials



Self-heating

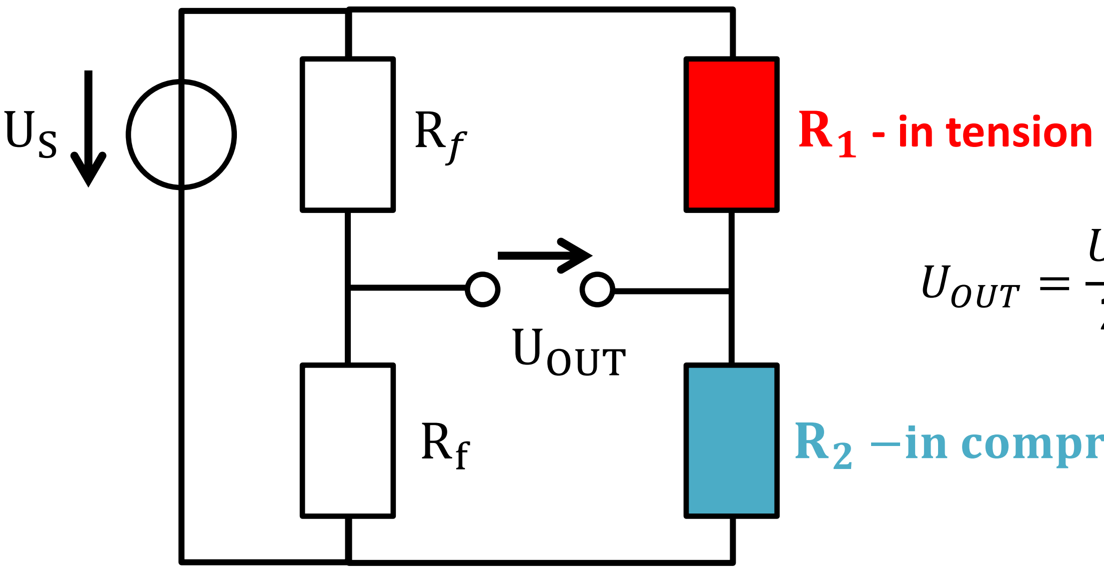
- Strain gauges are resistors, hence self-heating effect is present
- Like metal RTDs the current used for measurement shall be the lowest possible (magnitude: mA)

Two active gauge configuration (half bridge)



- Two identical strain gauges
- One is in compression, other is in tension
- Their relative strains are identical

Two active gauge configuration (half bridge)

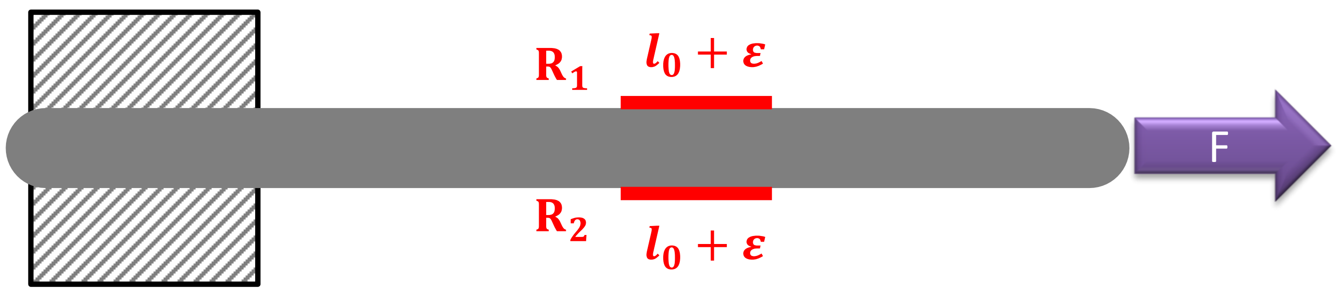


$$U_{OUT} = \frac{U_S}{2} - U_S \frac{R_2}{R_1 + R_2} = U_S \frac{R_1 - R_2}{R_1 + R_2}$$

$$\boldsymbol{U_{OUT}} = \frac{U_S}{2} \frac{R_1 - R_2}{R_1 + R_2} = \frac{U_S}{2} \frac{R_0(1 + \alpha\Delta\vartheta)(1 + g\varepsilon) - R_0(1 + \alpha\Delta\vartheta)(1 - g\varepsilon)}{R_0(1 + \alpha\Delta\vartheta)(1 + g\varepsilon) + R_0(1 + \alpha\Delta\vartheta)(1 - g\varepsilon)} = \frac{U_S}{2} \frac{2g\varepsilon}{2} = \boldsymbol{\frac{U_S}{2} g\varepsilon}$$

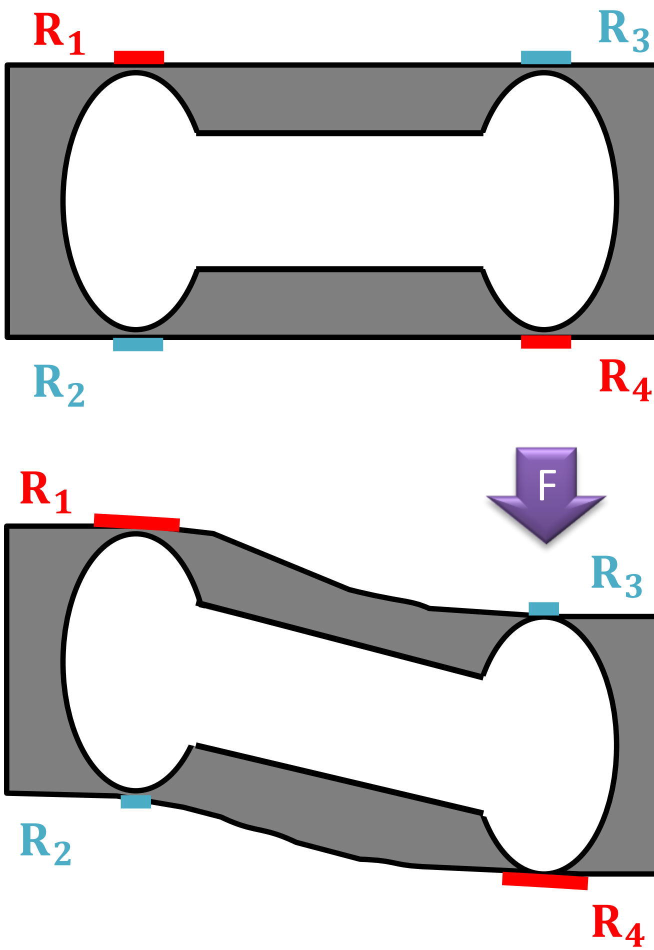
By using two active gauges the sensitivity is doubled, output voltage is temperature-independent and is a linear function of the strain.

Two active gauge configuration (half bridge)



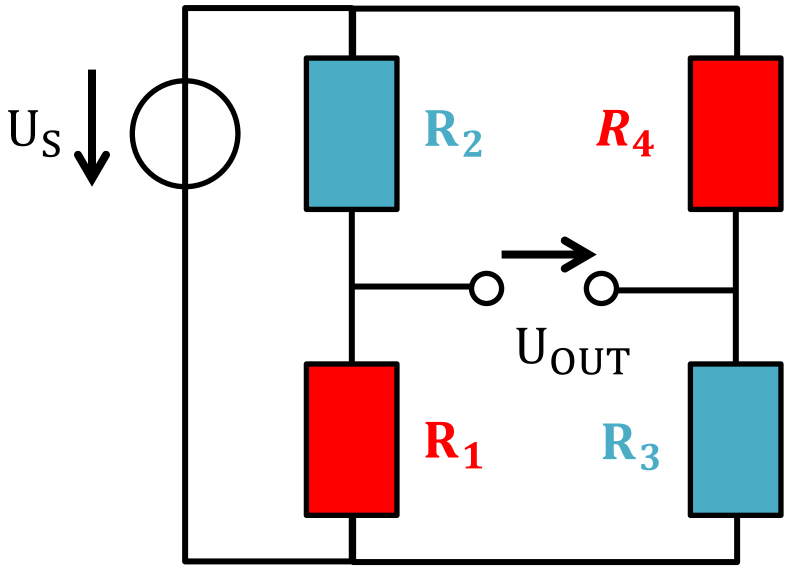
- $U_{OUT} = \frac{U_S}{2} \frac{R_1 - R_2}{R_1 + R_2} = 0$
- Configuration is sensitive to bending force only (not to tension force)
- Tension force can be measured by using one of the gauges

Four active gauge configuration (full bridge)



- Four identical strain gauges
- R_2 and R_3 in compression, R_1 and R_4 in tension
- Relative strains of the four gauges are identical

Four active gauge configuration (full bridge)



R_1, R_4 : in tension

R_2, R_3 : in compression

$$\boldsymbol{U_{OUT}} = U_S \frac{R_1}{R_1 + R_2} - U_S \frac{R_3}{R_3 + R_4} = U_S \frac{R_1 - R_2}{R_1 + R_2} =$$

$$= U_S \frac{R_0(1 + \alpha\Delta\vartheta)(1 + g\varepsilon) - R_0(1 + \alpha\Delta\vartheta)(1 - g\varepsilon)}{R_0(1 + \alpha\Delta\vartheta)(1 + g\varepsilon) + R_0(1 + \alpha\Delta\vartheta)(1 - g\varepsilon)} = \boldsymbol{U_S g \varepsilon}$$

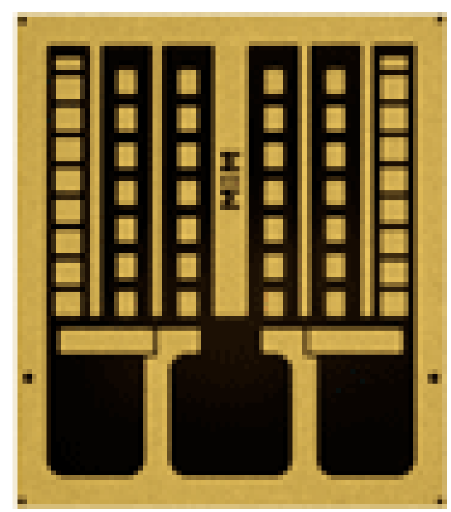
By using four active gauges the sensitivity is increased by a factor or four, output voltage is temperature-independent and is a linear function of the strain.

Fine tuning of bridge cicruits

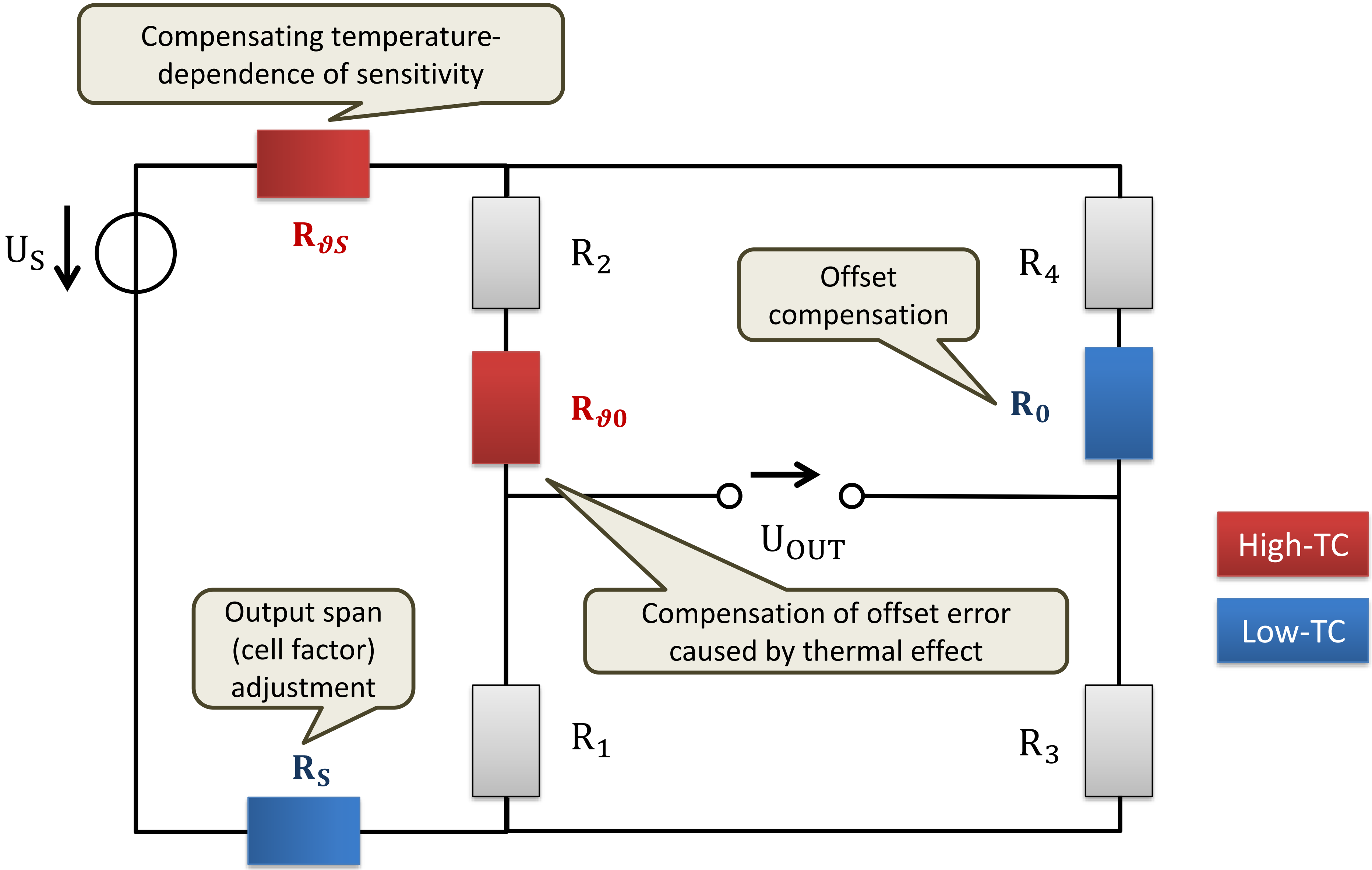
- Bridge circuits suffer from various problems:
  - thermal offset error caused by thermal expansion and manufacturing variance of gauges
  - unbalanced bridge due to manufacturing variance of gauges
  - thermal effect on sensitivity caused by temperature-dependence of Young’s modulus and gauge-factor
- Output span (cell factor) should be set to a prescribed value

Fine tuning of bridge circuits

- Bridges are tuned by resistors with high and low temperature coefficient („high-TC” and „low-TC” resistors)
- Special resistors similar to strain gauges
- Resistor value can be trimmed by the user (trimmable ladder pattern)



Fine tuning of bridge circuits

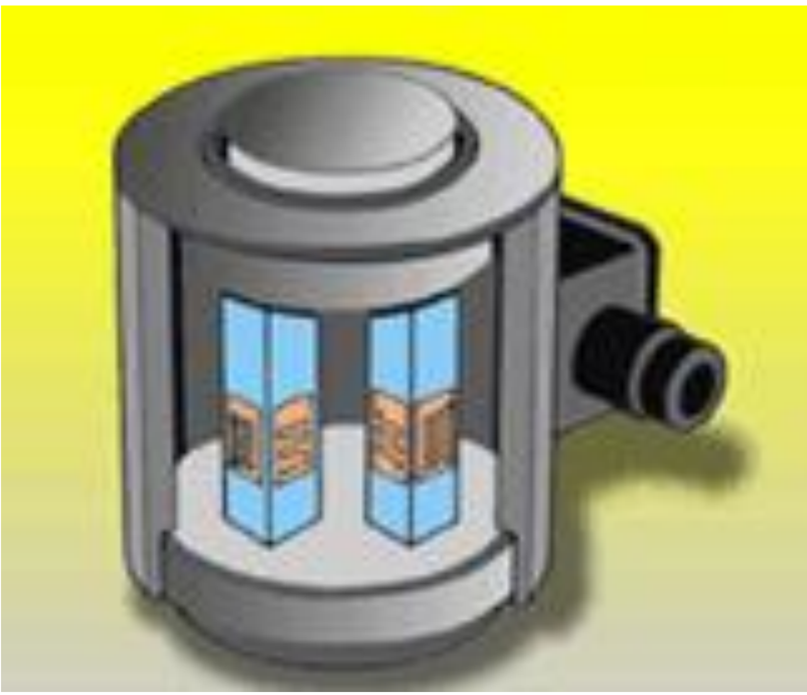


Load cells

- Test specimen, strain gauges and bridge circuit integrated to a single device

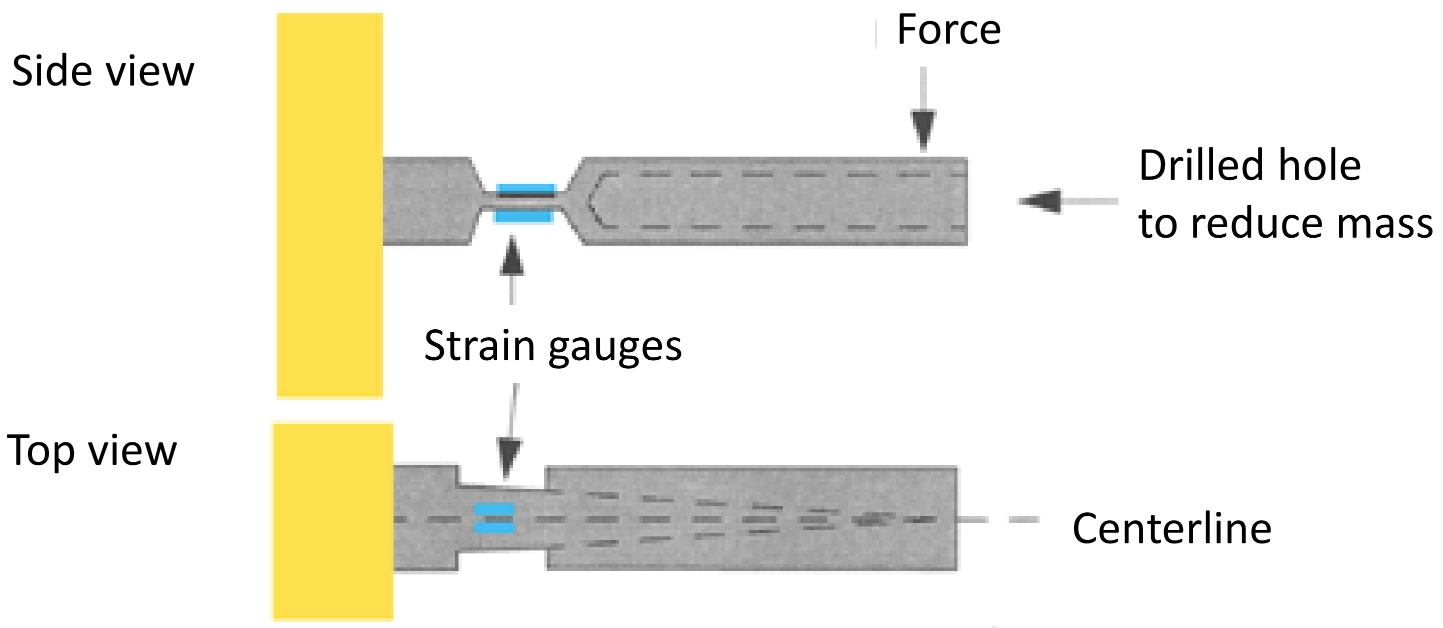
- Most important features of the test specimen
  - strain can be related unambiguously (preferably in a linear way) to mechanical stress
  - strain is elastic in the desired measurement range
  - high stability

Load cells




Serialno
Capacity
Part no.
Class
Model
270540-98
40 t
899531-32
C4
CSPM
Transducers Europe

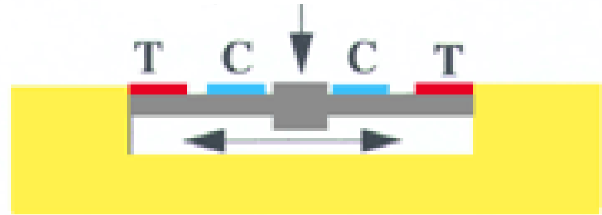
Load cells



Side view
Force
Drilled hole
to reduce mass
Strain gauges
Top view
Centerline

Load cell

Force

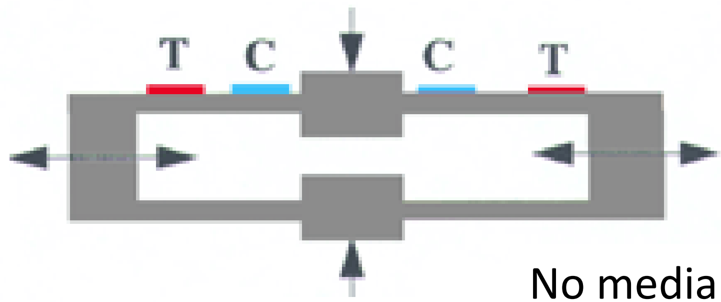


Rigid clamp

C – in compression
T – in tension

Median tensile loads are present

Force



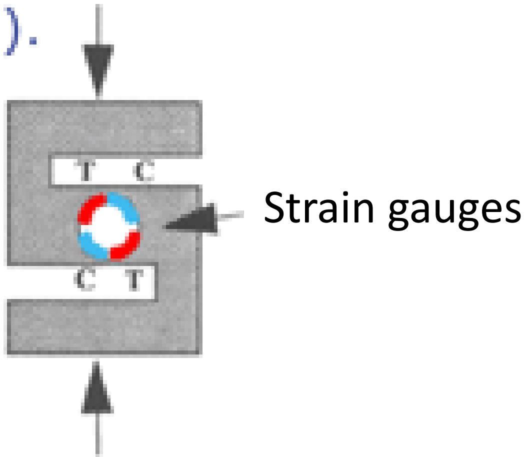
Mobile clamp

No median tensile loads

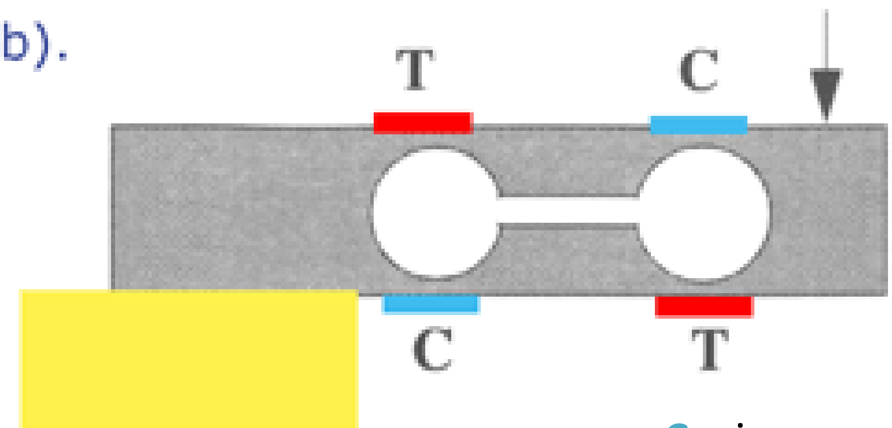
Force

Load cells

a).

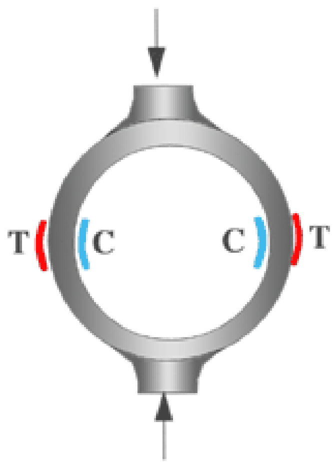


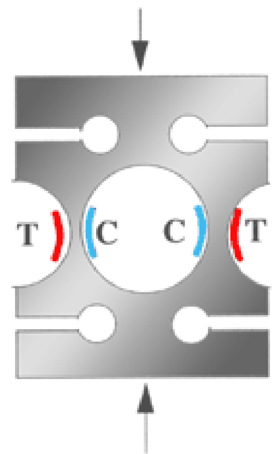
b).

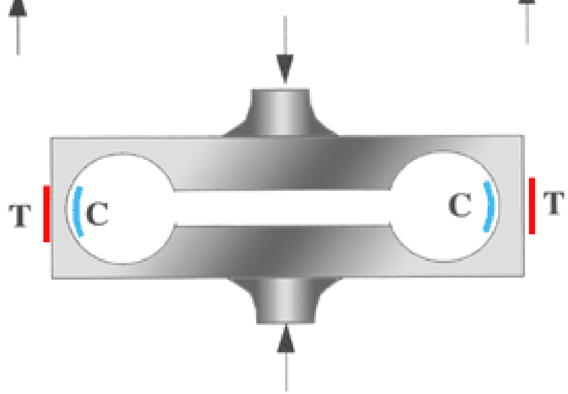


C – in compression
T – in tension

Load cells

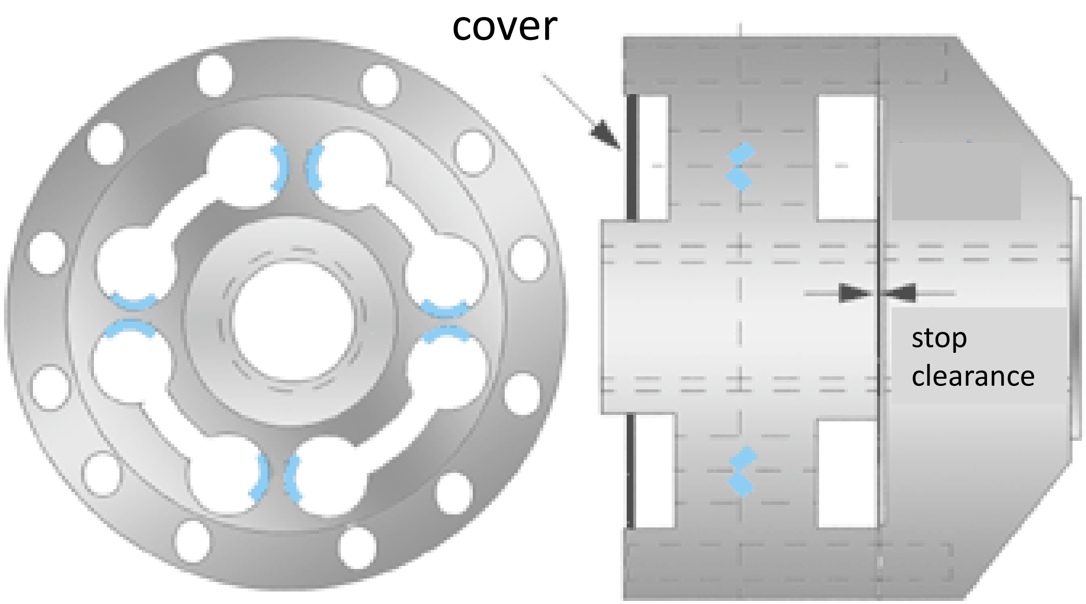




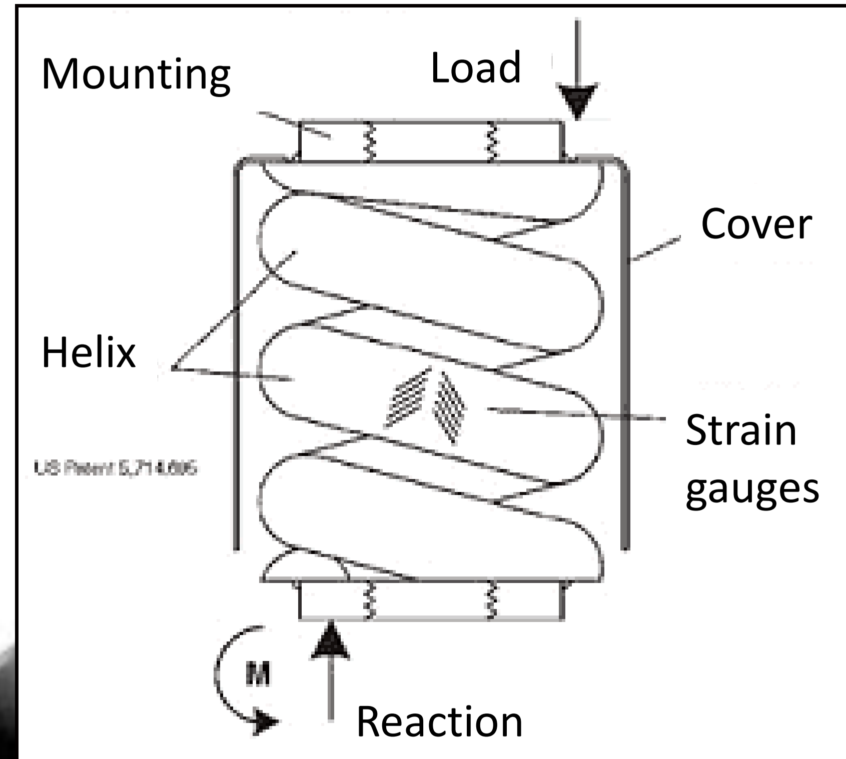


C – in compression
T – in tension

Load cells

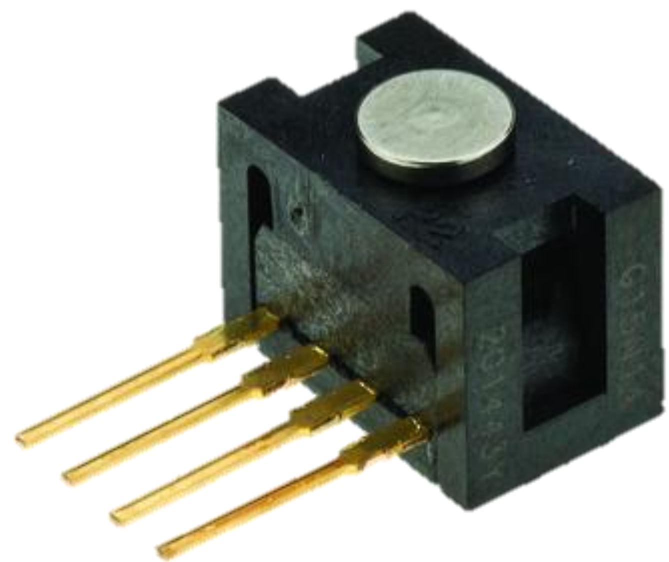


Load cells



MEMS load cells

- In silico micro-manufactured load cell
- Small size
- Narrow measurement range
- Sensitive to overload



Cell factor

- Consider linear output voltage, i.e. $U_{OUT} = U_S a F$, where a is a device-specific constant

- Cell factor:

$$C = \left.\frac{U_{OUT}}{U_S}\right|_{F=F_{max}} = \frac{U_{FS}}{U_S}\ [\mathrm{mV/V}]$$

- Output voltage corresponding to a force F:

$$U_{OUT} = C \cdot U_S \cdot \frac{F}{F_{max}}$$

- Force corresponding to output voltage U_{OUT} :

$$F = F_{max} \frac{1}{C} \frac{U_{OUT}}{U_S}$$

Cell factor - problem

A load cell contains four constantan strain gauges ($g = 2$), its maximal load is 10kg, causing a strain of $\varepsilon_{max} = 0.1\%$ for each gauge. Give the cell factor of the load cell. Give the output voltage for a load of 2.5kg if the supply voltage is $U_S = 24\text{V}$.

- Output voltage of the bridge: $U_{OUT} = U_S g \varepsilon$
- Output voltage corresponding to maximal load:

$$U_{FS} = U_S g \varepsilon_{max} = 0.002 U_S$$

- Cell factor:

$$C = \frac{U_{FS}}{U_S} = 2\text{mV/V}$$

- Output voltage for 2.5 kg load with 24V supply voltage:

$$U_{OUT} = C \cdot U_S \cdot \frac{F}{F_{max}} = 0.002 \cdot 24 \cdot 0.25 = 12\text{mV}$$

Cell factor - problem

Maximal load of a load cell is $5\mathrm{kg}$, its cell factor is $C = 1\ \mathrm{mV/V}$, the supply voltage is $U_S = 2.5\mathrm{V}$. Give the actual force applied if the output voltage of the cell is $U_{OUT} = 0.25\mathrm{mV}$?

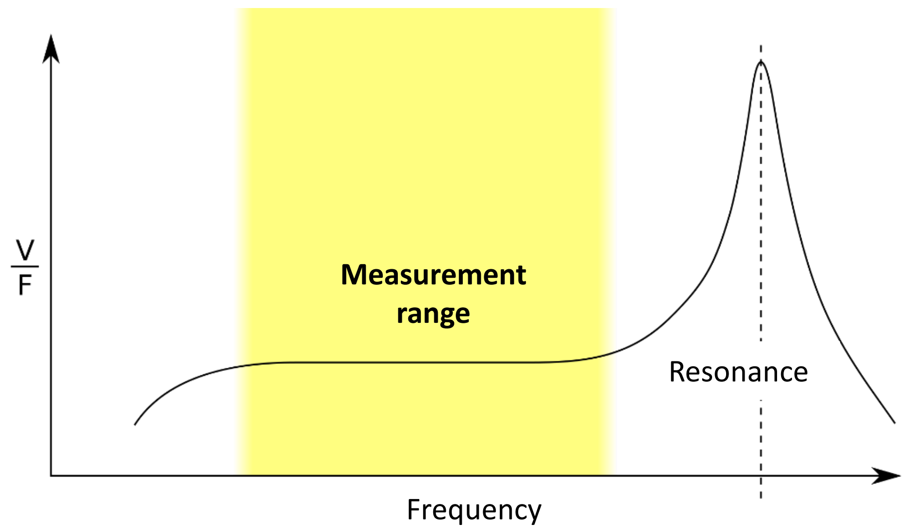
- Output voltage of the cell: $U_{OUT} = U_S C \frac{F}{F_{max}}$
- Actual force:

$$F = F_{max} \frac{1}{C} \frac{U_{OUT}}{U_S} = 5\ [\mathrm{kg}] \cdot \frac{1}{0.001[\mathrm{V/V}]} \cdot \frac{0.00025\ [\mathrm{V}]}{2.5\ [\mathrm{V}]} = 0.5\ \mathrm{kg}$$

Values above are actual parameters of a low-cost load cell with 5 kg maximal load; force exerted corresponds to the weight of an 500 ml plastic bottle filled with water

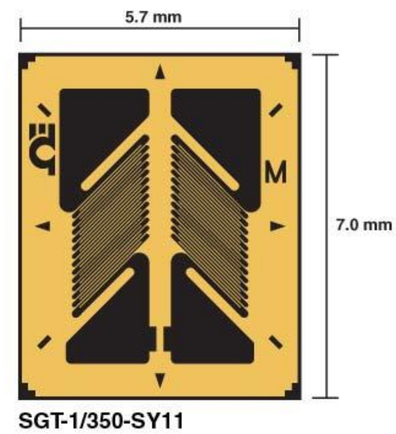
Piezoelectric strain gauges

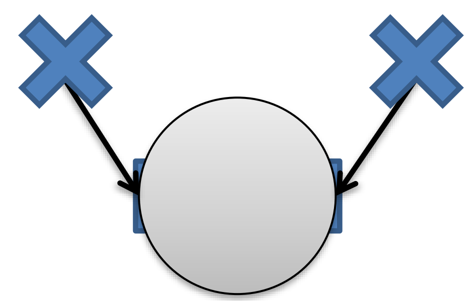
- Piezoelectric materials produce electrical voltage when exposed to mechanical stress (and vice versa)
- Problem: voltage vanishes quickly
- Applicable for measurement of dynamic stress only



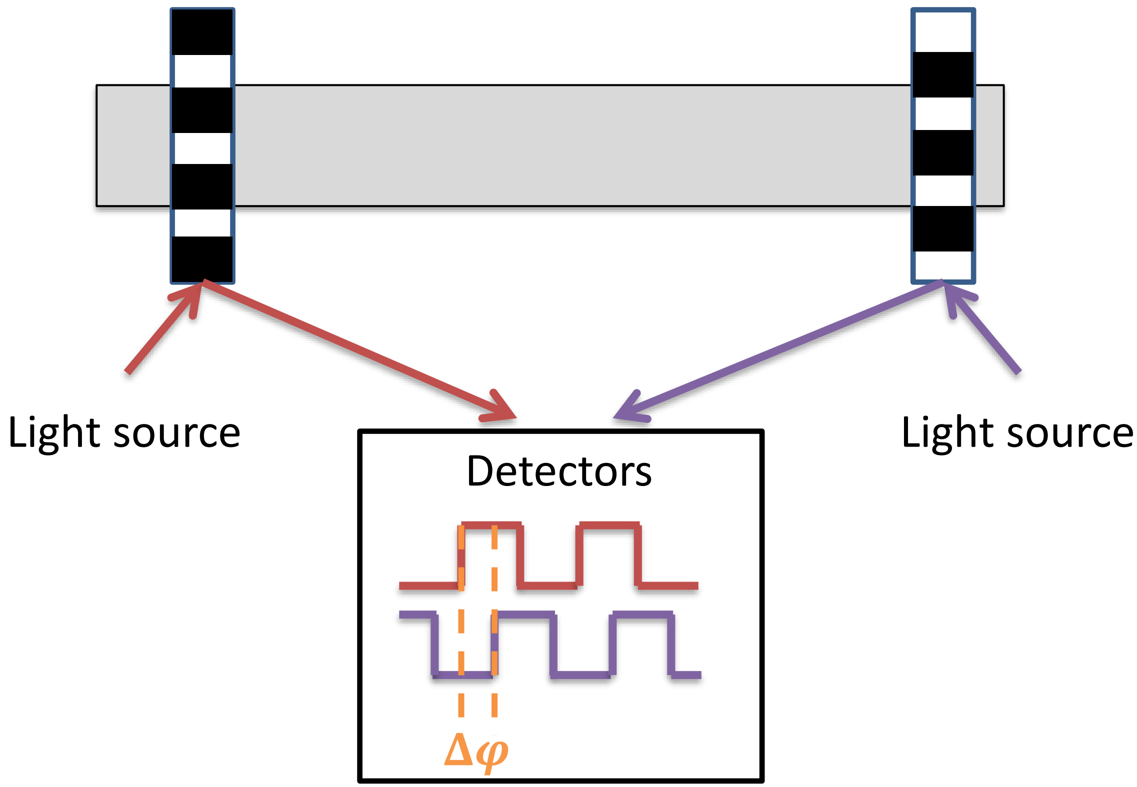
Torque measurement with strain gauges

- Elastic torsion of the shaft exerts shear stress in 45° direction
- Strain due to stress can be measured by 2×2 strain gauges
- Problem: need of slip ring connection to the rotating shaft





Optical torque sensors

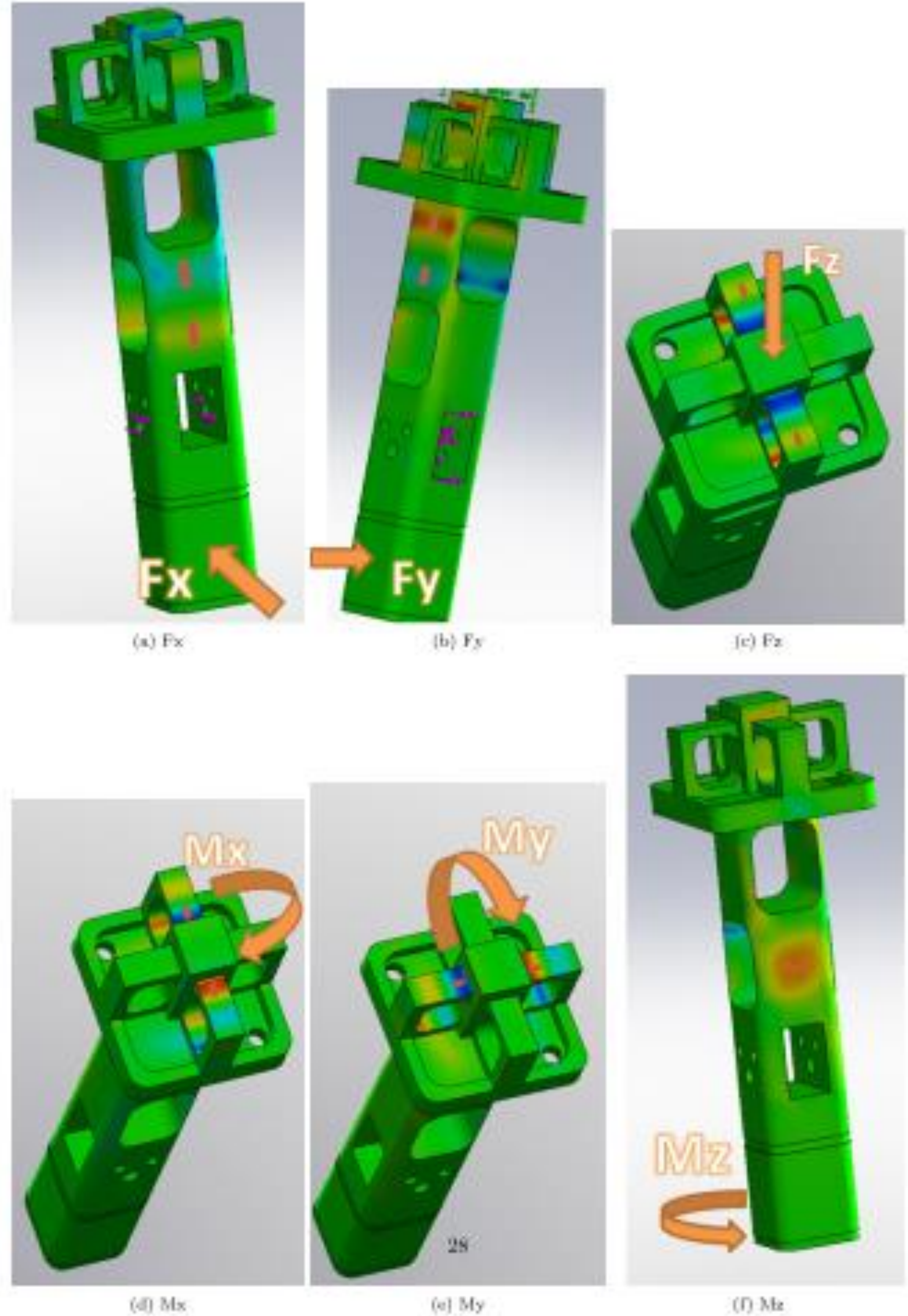


- Torsion of shaft is related directly to the torque
- No active (electrically powered) elements located on the shaft, phase shift can be measured easily

6-DOF force and torque sensors

Quantities measured:

- force along three axis
- torques around three axis



Pressure - Units

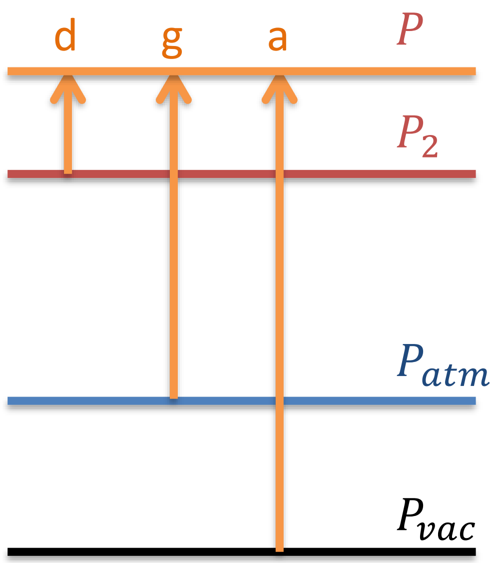
- Pascal - Pa
  - $1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$
- Bar - bar
  - $10^6\ \ \mathrm{dyn/cm^2}$, azaz
    $10^6 \cdot 10^{-5}\mathrm{N}/10^{-4}\mathrm{m}^2$
  - $1\ \mathrm{bar} = 10^5\mathrm{Pa} = 100\mathrm{kPa}$
- Pounds per square inch - psi
  - $1\ \mathrm{psi} = 1\ \mathrm{lb/inch}^2$
  - $1\ \mathrm{bar} \approx 14.5\ \mathrm{psi}$
- Other units
  - $1\ \mathrm{atm} \approx 1.013 \cdot 10^5\mathrm{Pa}$
  - $1\ \mathrm{Hgmm\ (torr)} \approx 133.3224\ \mathrm{Pa}$

Types of pressure scale

- Differential pressure, [psid]
  - referenced to another arbitrary pressure
- Gauge pressure, [psig]
  - referenced to atmospheric pressure
  - atmospheric pressure in Hungary ca.

$$1000\mathrm{hPa} = 10^5\mathrm{Pa} = 1\mathrm{bar}$$

- Absolute pressure, [psia]
  - referenced to full vacuum



Non-electric pressure sensors

- Tube membrane
- Bourdon-tube
  - high accuracy
  - still used as near-field gauge

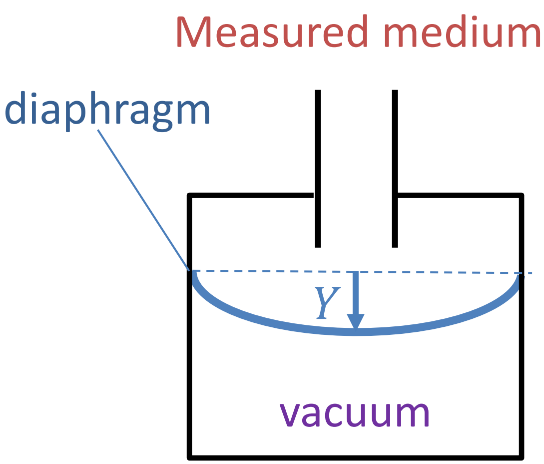
Principle of pressure sensing

- Pressure: force / surface
- Therefore, we shall measure force (if surface is known)
- Atmospheric pressure is ca. $10^5 \mathrm{Pa} = 10 \mathrm{N/cm^2}$
- We shall either use a sensor of large surface or one with elastic sensing element

Principle of pressure sensing

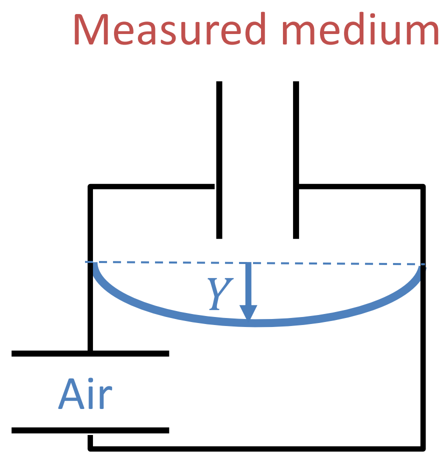
- Test specimen: diaphragm (membrane)
- Circle or square shaped, partially elastic disc clamped at its circumference
- Diaphragm deflects when exposed to pressure

Principle of pressure sensing



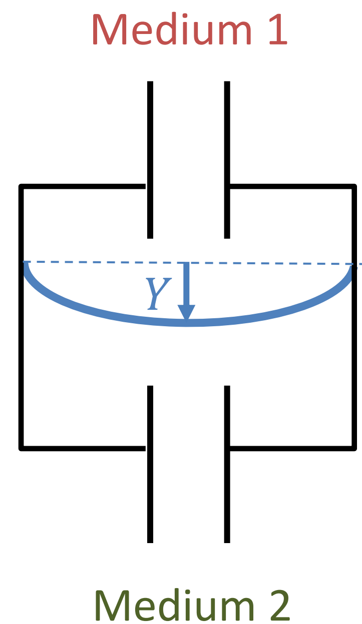
$$Y \propto P_m$$

Absolute pressure
(referenced to vacuum)



$$Y \propto P_m - P_{atm}$$

Gauge pressure
(referenced to atmospheric pressure)

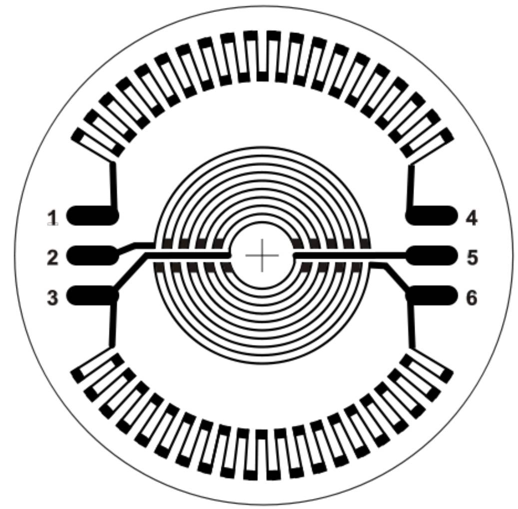


$$Y \propto P_{m1} - P_{m2}$$

Differential pressure
(referenced to medium 2)

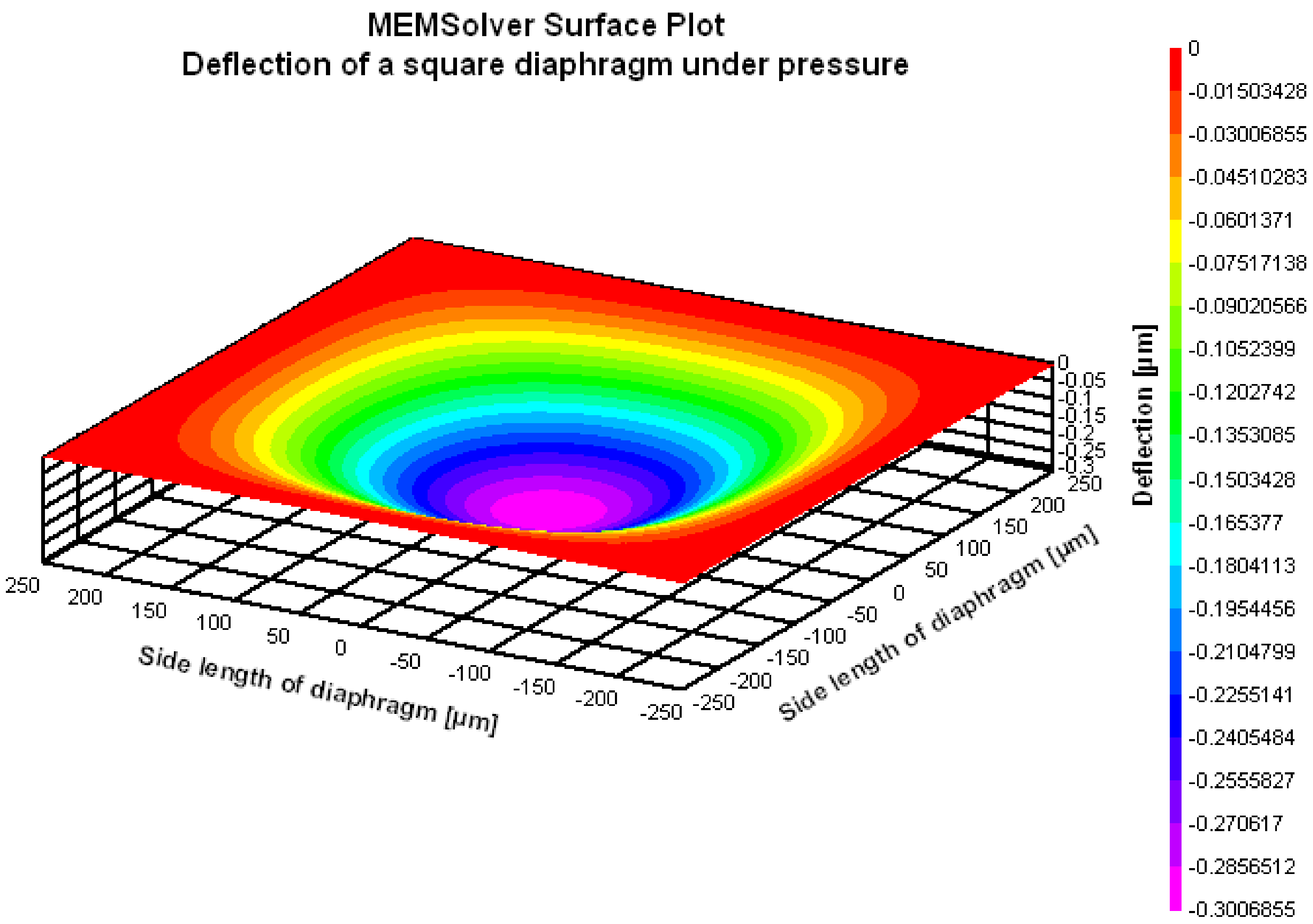
Deflection of the diaphragm

- Radial
  - $\varepsilon_{RC} = \frac{3\boldsymbol{P}R_0^2(1-\nu^2)}{8t^2E}$
  - $\varepsilon_{R0} = \frac{3\boldsymbol{P}R_0^2(1-\nu^2)}{4t^2E}$
- Tangential
  - $\varepsilon_{TC} = \frac{3\boldsymbol{P}R_0^2(1-\nu^2)}{8t^2E}$
  - $\varepsilon_{T0} = 0$
- Displacement of the center
  - $Y_c = \frac{3\boldsymbol{P}R_0^4(1-\nu^2)}{16t^3E^2}$



- RC/TC: at center
- $R0/T0$: around circumference
- P: pressure (Pa)
- R_0: diaphragm radius (mm)
- ν: Poisson ration
- t: diaphragm thickness (mm)
- E: Young's modulus (Pa)

Deflection of the diaphragm



Deflection of the diaphragm

- **For small deflections pressure is proportional to the deflection of the elastic diaphragm (deflection is a linear function of pressure)**
- What is considered as small deflection?
  - rule of thumb: not greater than the thickness of the diaphragm
  - to ensure linearity error of 0.3%: quarter of diaphragm thickness
- When the diaphragm can be considered as elastic?
  - rule of thumb: if radius is at least 200 times the thickness
  - elasticity of diaphragm decreases linearity error
- If strain gauges are used for the measurement, maximal strain of the diaphragm must not be greater than the maximal strain of the gauge
- For strain gauge pressure sensors, the magnitude of maximal deflection is $1\mu m$

Strain-based measurement of diaphragm deflection

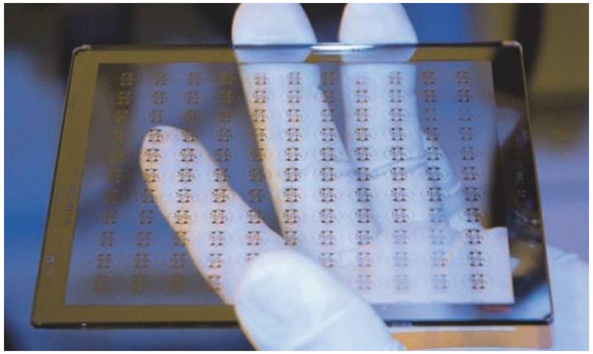
- Bonded strain gauge
- Thin-film strain gauge
- Thick-film strain gauge
- Integrated (piezoresistive, semiconductor) strain gauge

Bonded strain gauge sensors

- Significant drift
- High mechanical wear of strain gauge
- Low reliability

Metal thin-film diaphragms

- Backing
  - stainless steel
  - tantal, special alloys (Hastelloy, Inconel)
  - silver (for chloride and fluoride media)



- Layer deposition
  - insulation layer
  - resistor layer
- Zig-zag pattern on side not exposed to medium formed by photolithography

- Not suitable for small pressures
- Can withstand large pressures
- Stable, insensitive for mechanical vibration

Ceramic thick-film sensors

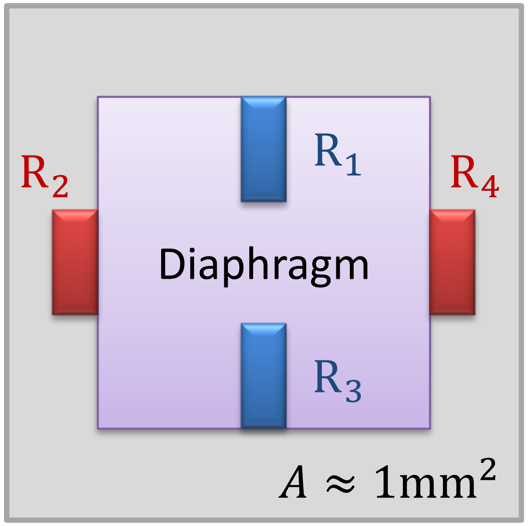
- Backing: Al_2O_3
- Zig-zag pattern applied by screen printing on the side not exposed to the medium
- Corrosion-resistant
- Good long-term stabilty
- Sensitive to burst pressure due to fragility of ceramic

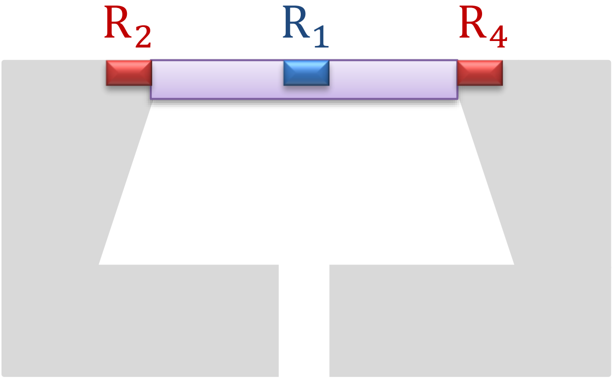
Strain gauge pressure sensors

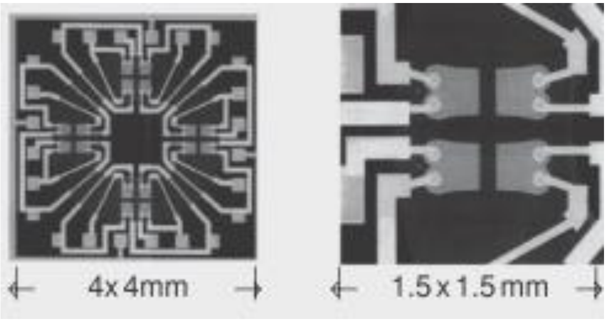
- For measurement of both absolute, gauge and differential pressure
- Measurement range up to 1400 MPa (14 000 bar)
- Accuracy: 0.25%FS
- Long time drift: 0.25%FS / 6 months

Piezoresistive pressure sensors

- In-silico diaphragm and strain gauge
- Base structure manufactured by etching
- Strain gauges manufactured by diffusion
- Longitudinal $(\mathrm{R}_1, \mathrm{R}_3)$ and transversal $(\mathrm{R}_2, \mathrm{R}_4)$ piezoresistive coefficients are of different signs
- In-silico leads and compensation circuitry





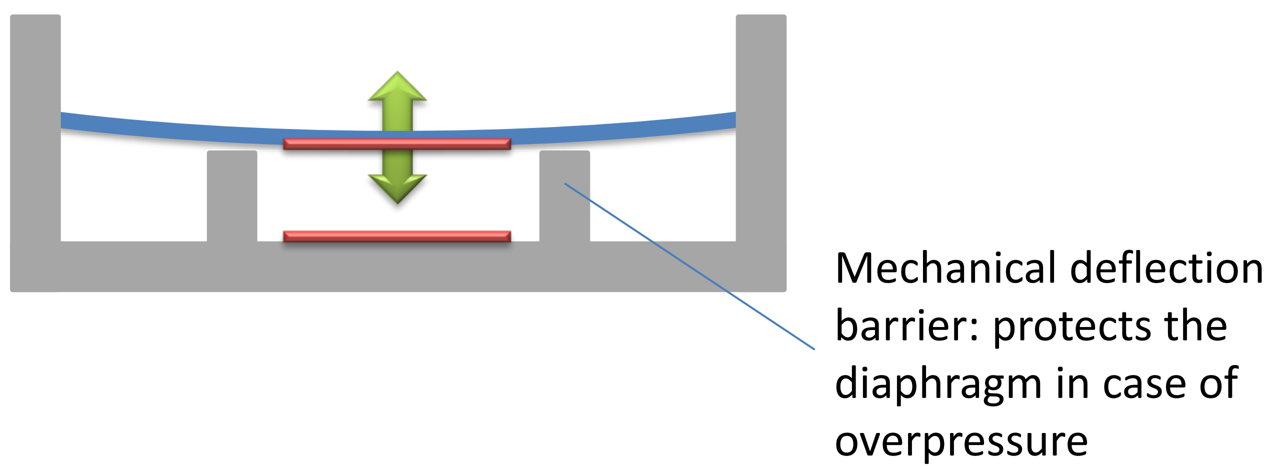


Piezoresistive pressure sensors

- Fragile devices
  - protection by additional diaphragm
  - isolation from environment
  - silica oil pressure transmission
- Significant thermal effects
- In-silico compensating electronics

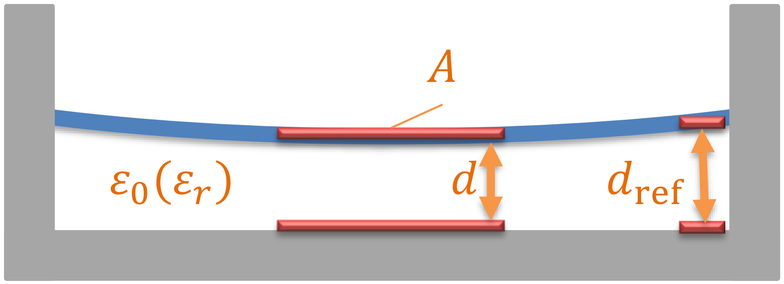
Capacitive pressure sensors

- Membrane and baseplate are plates of a capacitor
- Measurement based not on the deflection of the membrane but on the displacement of its center

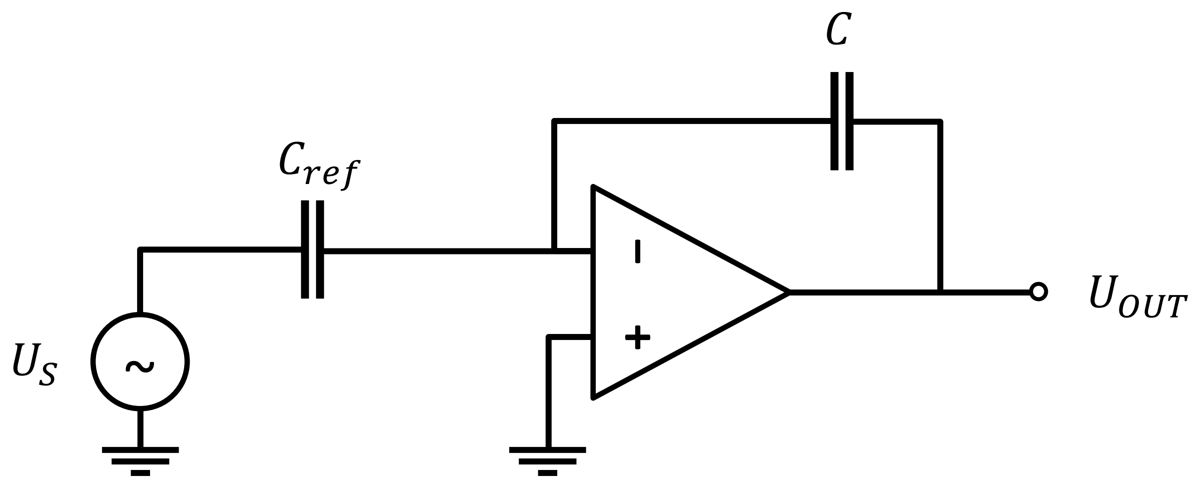


Capacitive pressure sensors

- Capacitance: $C = \varepsilon \frac{A}{d} = \varepsilon_0 \varepsilon_r \frac{A}{d}$
- Displacement is proportional to the pressure
- Measurement range up to 25%
- Reference capacitor
  - at the circumference of the diaphragm (no displacement)
  - distance of plates is constant: $C_{ref} = \frac{\varepsilon\, A_{ref}}{d_{ref}} = \text{constant}$



Capacitance measurement



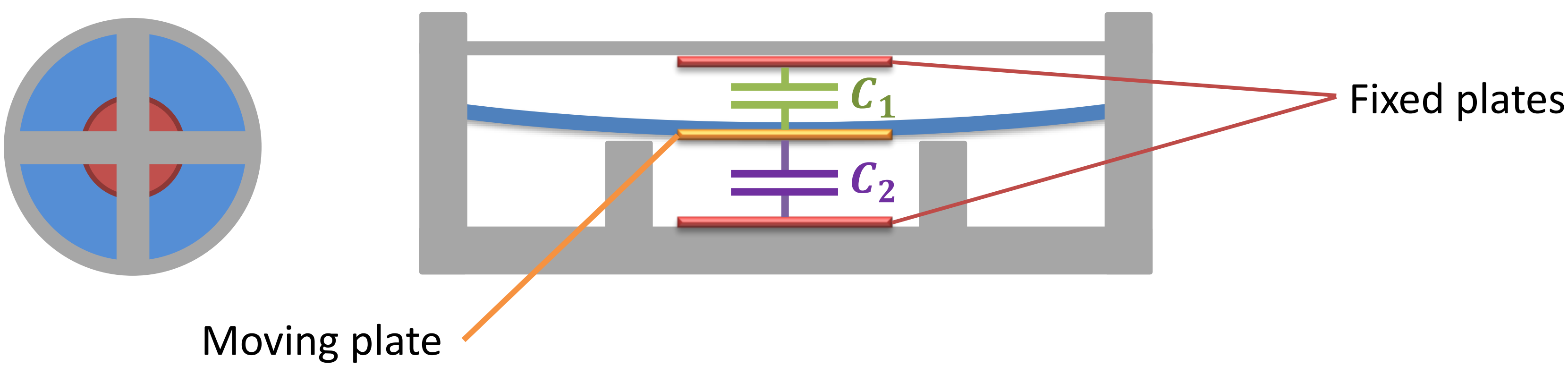
$$U_{OUT} = -U_S \frac{Z}{Z_{ref}} = -U_S \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C_{ref}}} = -U_S \frac{C_{ref}}{C} = -U_S \frac{C_{ref}}{\varepsilon \frac{A}{d}} = -U_S \frac{C_{ref}}{\varepsilon A} \boldsymbol{d}$$

- Output is directly proportional to the displacement of the diaphragm center
- Also directly proportional to pressure if displacement is small
- If area of reference capacitor plates is identical to the area of the moving plates (relative permittivities are the same as the dielectric material is the same):

$$\mathrm{U_{OUT}} = -U_S \frac{C_{ref}}{C} = -U_S \frac{\varepsilon A/d_{ref}}{\varepsilon A/d} = -U_S \frac{d}{d_{ref}}$$

Differential-capacitive pressure sensors

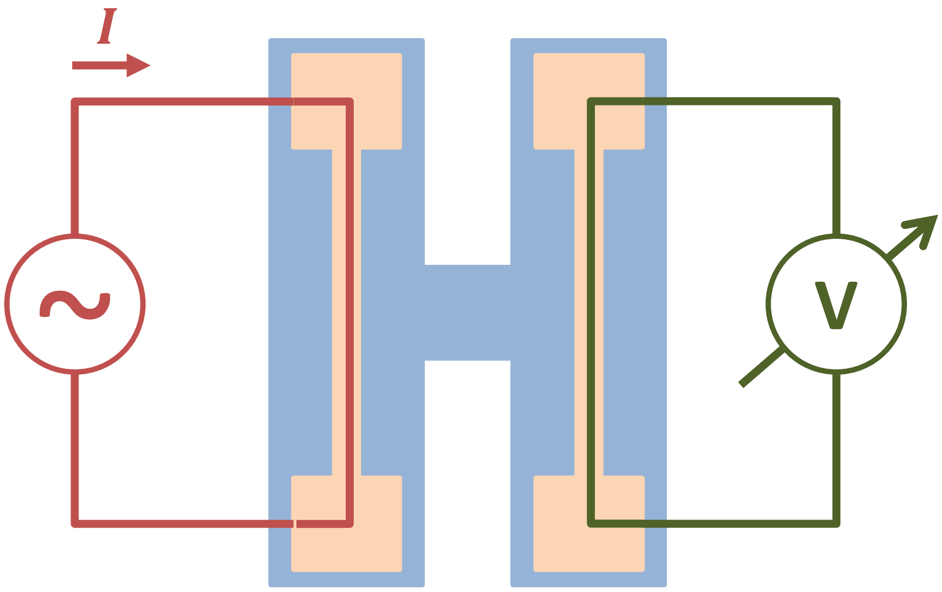
- Membrane is a moving plate between two fixed ones
- Ration C_1/C_2 changes nearly directly proportional to the displacement of diaphragm center
- Displacement can be measured easily (see capacitive displacement sensors)

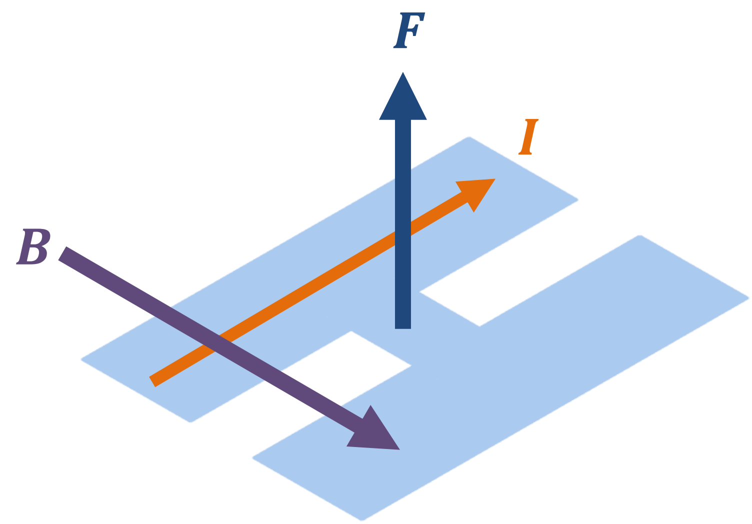


Capacitive pressure sensors

- Larger strain allows wider measurement range
- From vacuum up to 700 bars
- Used especially in low-pressure or vacuum applications
- No need for temperature compensation
- Accuracy up to 0.01%FS

MEMS resonant pressure sensors (Yokogawa DPharp)





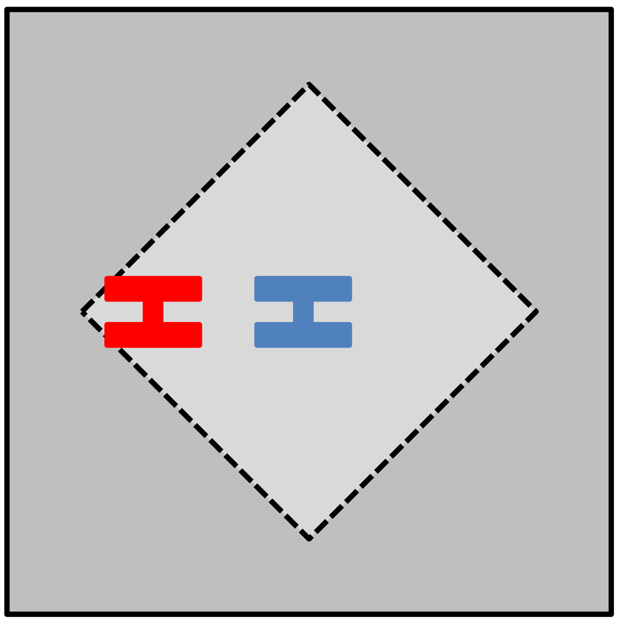
- If excited by alternating current in stationary magnetic field, the H-shaped element resonates (Lorentz force)
- Sinusoidal voltage is induced in the other side of the resonator (Faraday's law)

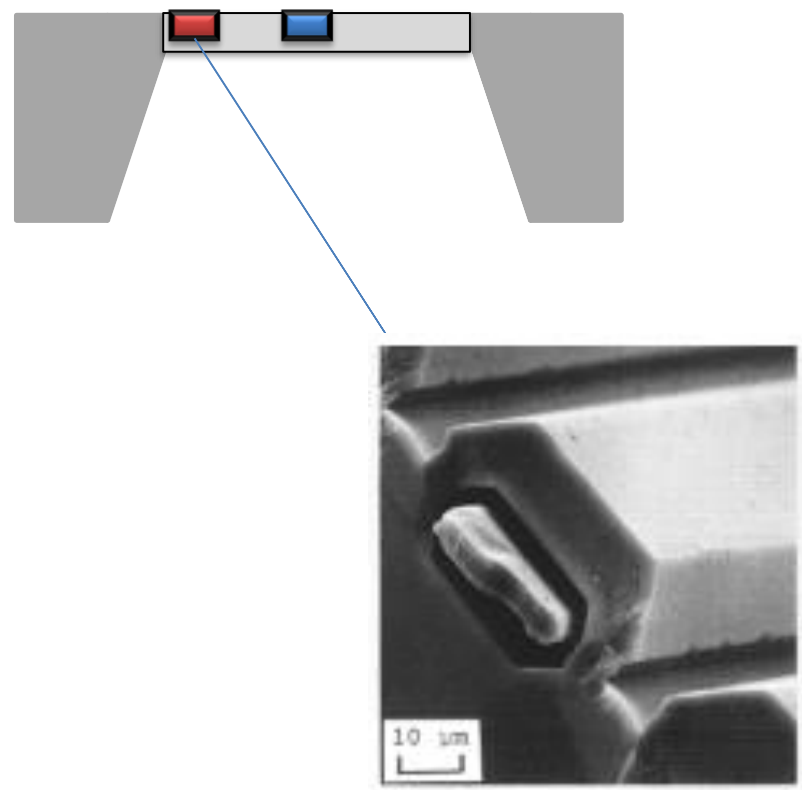
Resonating frequency

- The resonator is resonated at its natural frequency by a PLL
- $f^2 = c\varepsilon,$

  where ε is the tensile force, c is a device-specific constant
- $\varepsilon = \varepsilon_0 + \varepsilon_S \pm \varepsilon_{DP}$
  - ε_0: initial tensile force (originates from the structure of the resonator)
  - ε_S: tensile force due to static pressure (applied to both sides of the resonator)
  - ε_{DP}: tensile force due to differential pressure (difference of pressure between the two sides)

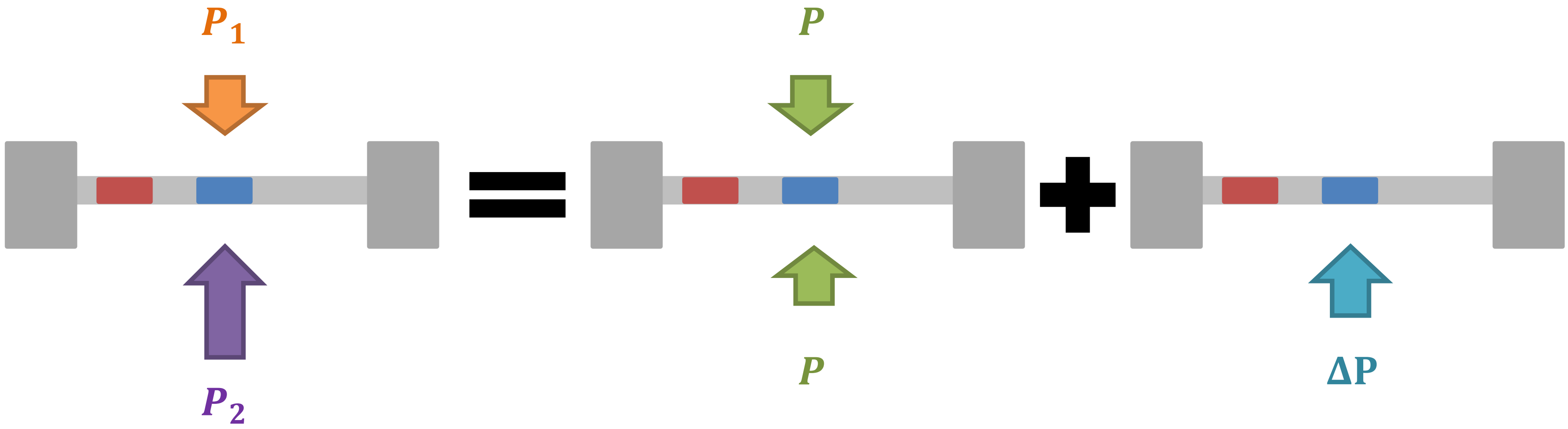
Location of resonators





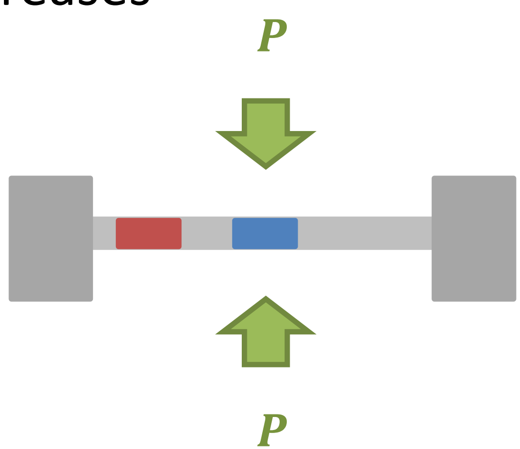
Pressure components

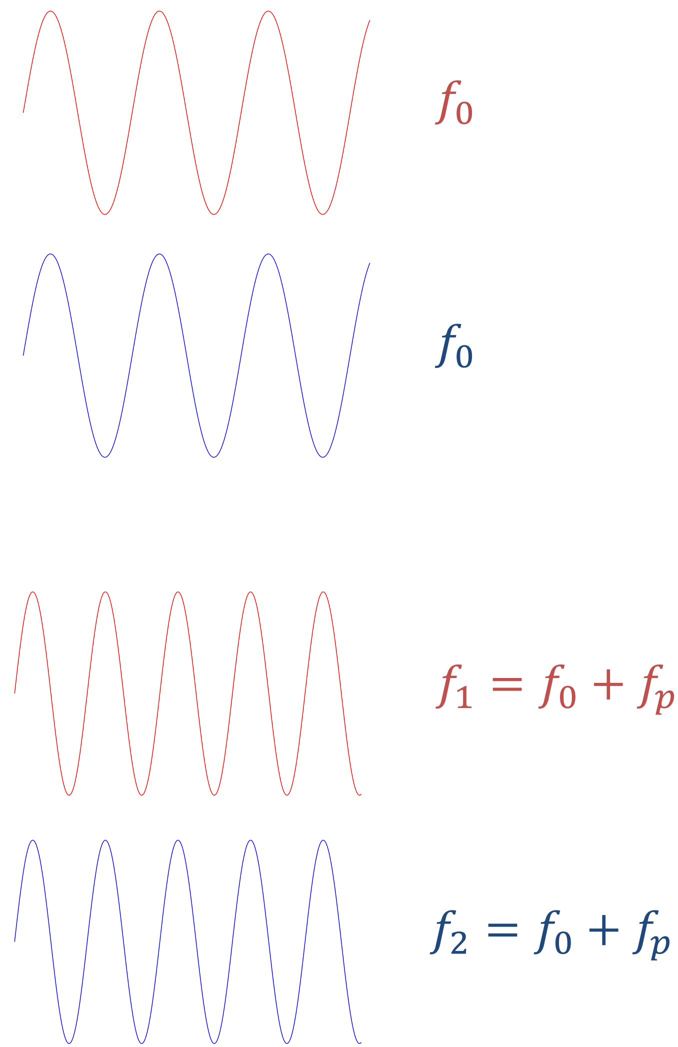
- $P_1 = P$
- $P_2 = P + \Delta P$
- P: static pressure (common component of pressure)
- $\Delta P = P_2 - P_1 = P_2 - P$: differential pressure



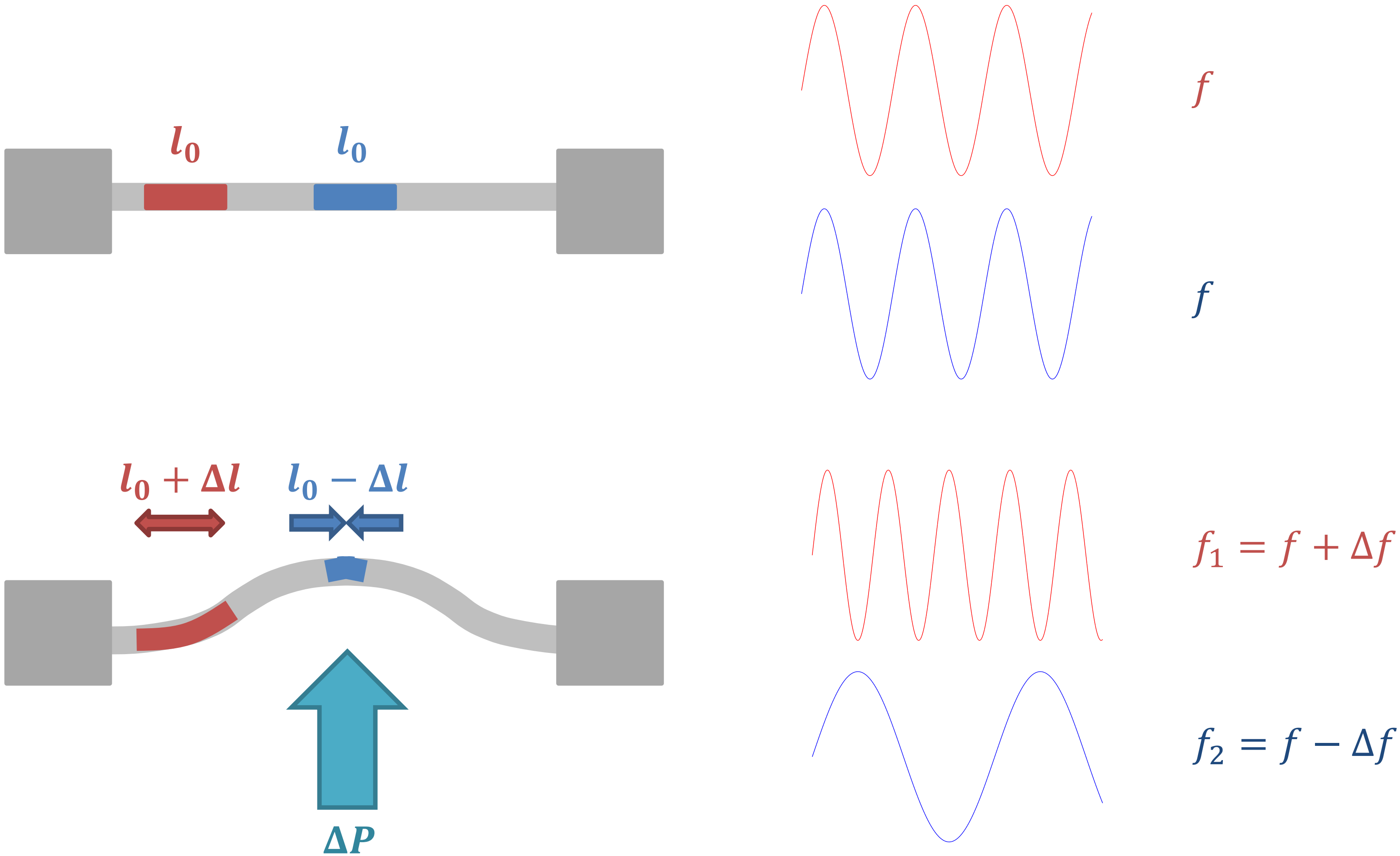
Static pressure

- Pressure increases the tensile force inside the silicon structure
- P increases $\Rightarrow \varepsilon$ increases $\Rightarrow f$ increases





Differential pressure



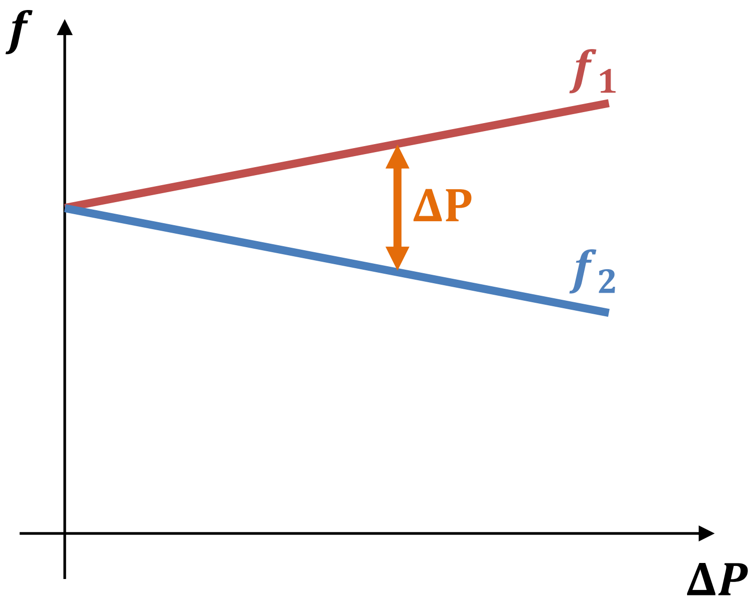
Relationship between pressure and frequency

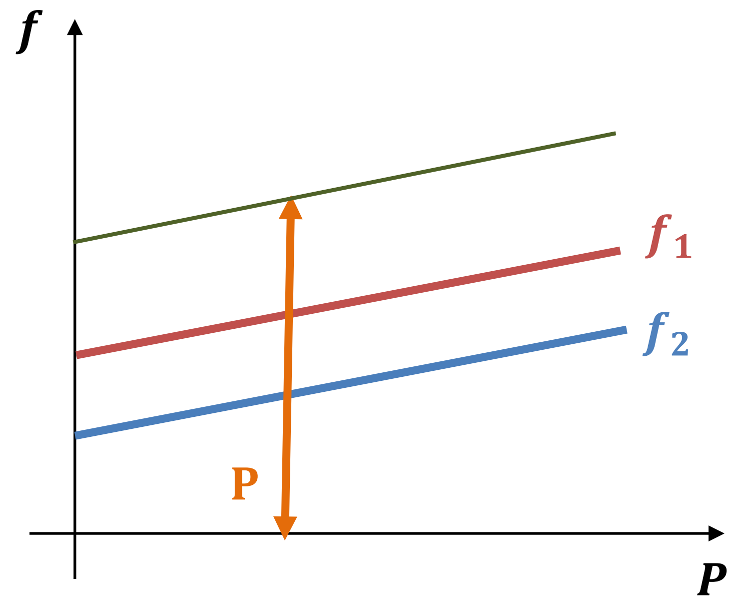
- $f_1 = f_0 + f_P + \Delta f$
- $f_2 = f_0 + f_P - \Delta f$

- $f_P \propto P \Rightarrow f_1 + f_2 = 2f_0 + 2f_P \propto P$

- $\Delta f \propto \Delta P \Rightarrow f_1 - f_2 = 2\Delta f \propto \Delta P$

Relationship between pressure and frequency

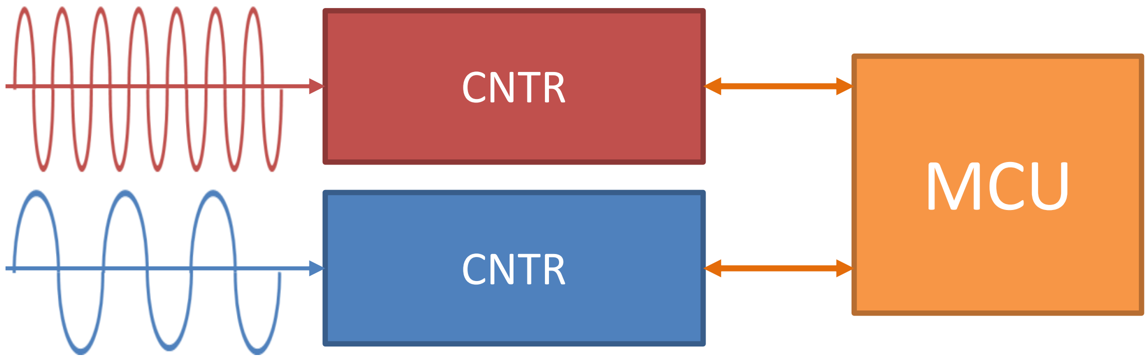




Differential pressure:
$\Delta P \propto f_1 - f_2$

Static pressure:
$P \propto f_1 + f_2$

Frequency measurement



- Frequency can be measured by two counters
- No need for ADC (source of significant error)
- Calculations carried out by the microprocessor based on the two measured frequencies

MEMS resonator pressure sensors

- Measurement range (depending on physical characteristics): 0.1 – 20 000 kPa
- Wide span ($P_{max}/P_{min} \approx 100$)
- Accuracy up to 0.025%
- Long-term stability: 0.1% / 10 év
- Simultaneous measurement of static and differential pressure (necessary for flow and level measurement)
- Direct digital output