Sensors

Industrial control

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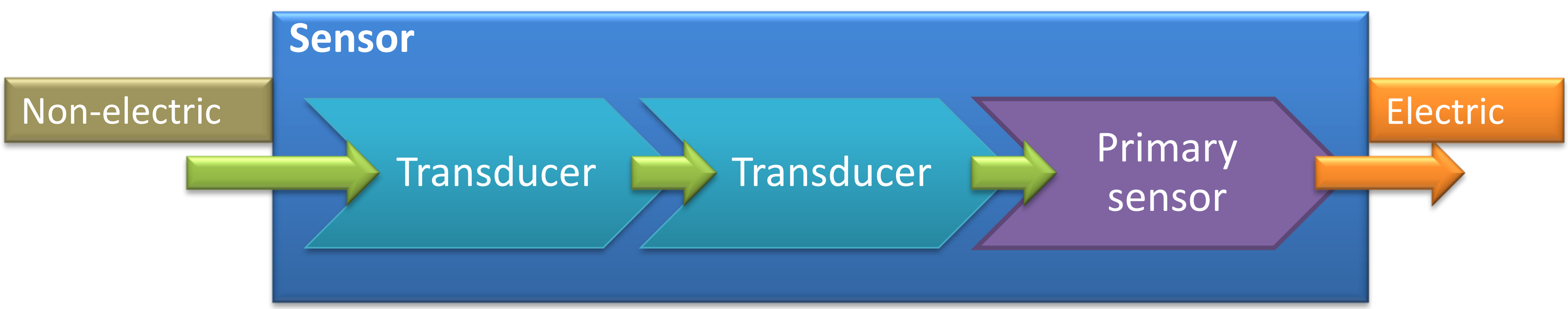
Role of sensors

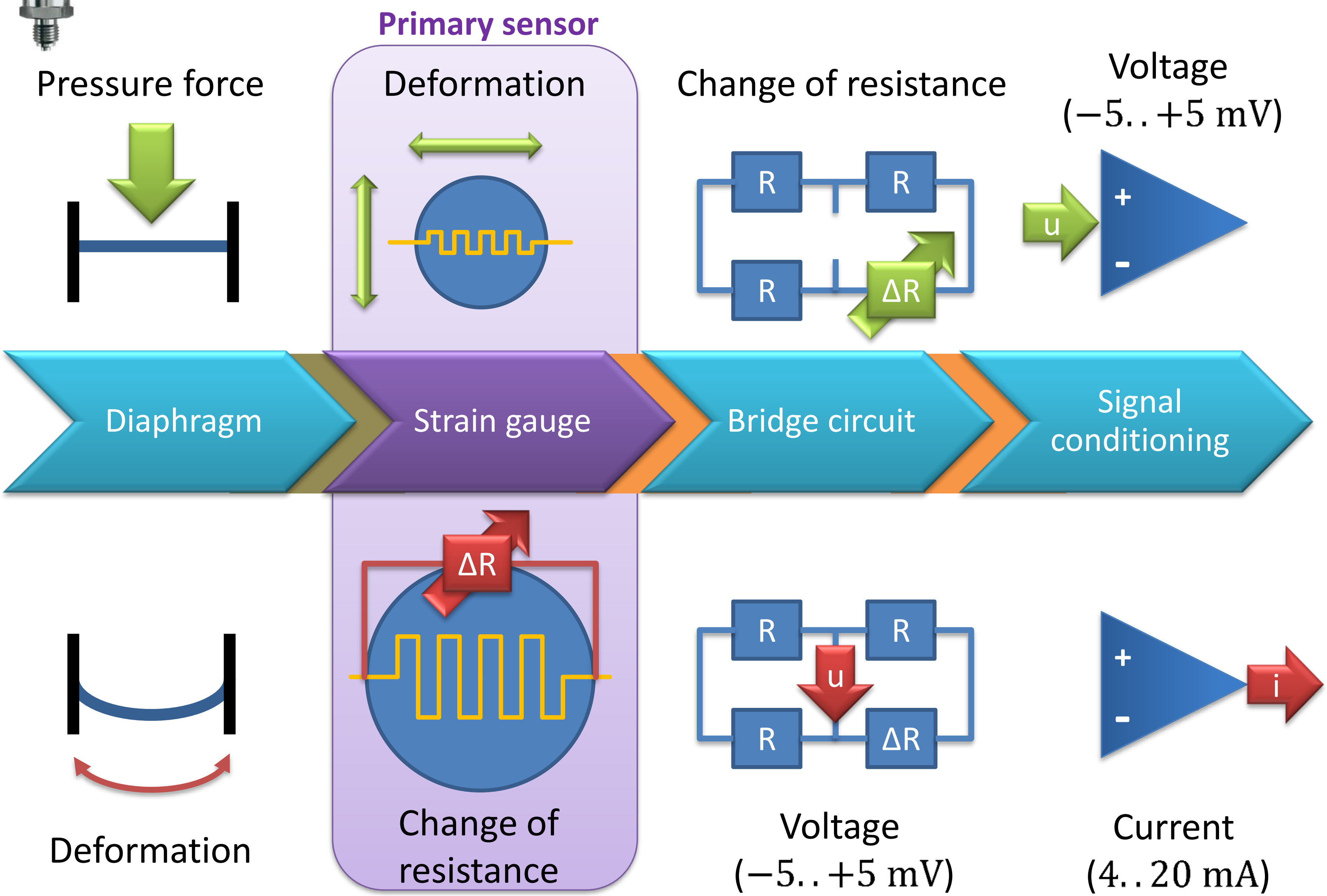
- Role of sensors: provide information about
  - the technology under control
  - the environment of the technology
- Primary requirements against sensor devices
  - accuracy
  - low uncertainty
  - minimal disturbance of the technology

Sensors and transducers

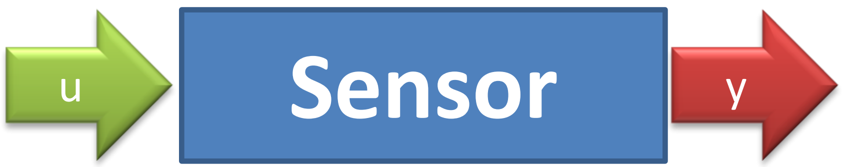
- Transducer: converts some kind of signal (energy) to a different type of signal (energy)
- Sensor: converts some kind of *non-electric* signal to *electric* signal



Typical structure of a sensor - Example



Most important properties of sensors

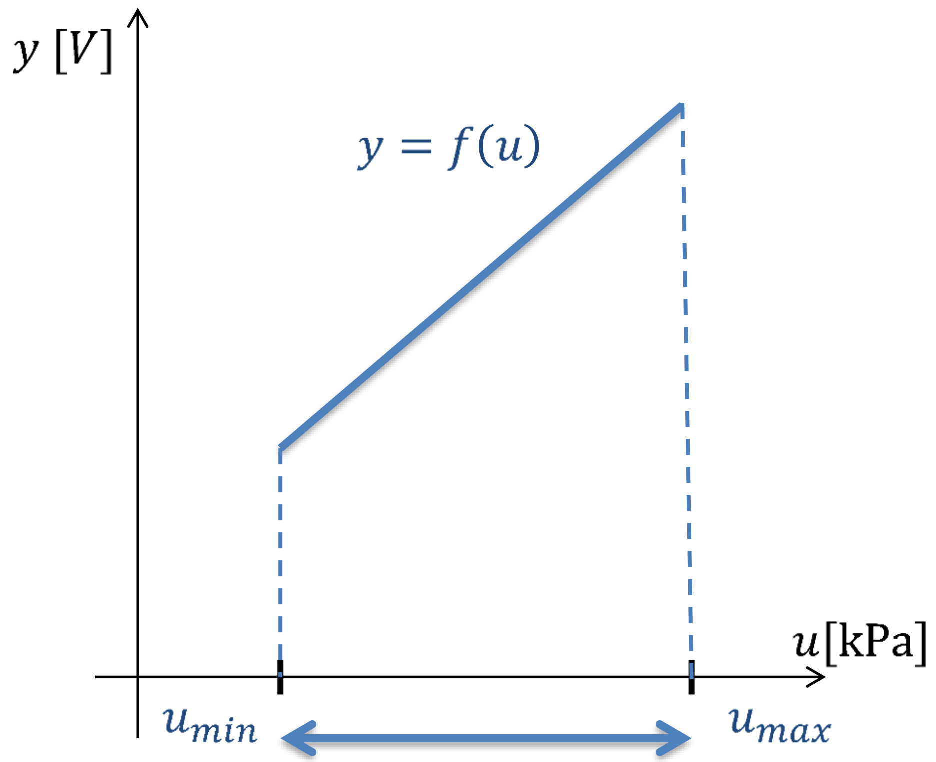


- Input range: $u_{min}, u_{max}, [u]$
- Output range: $y_{min}, y_{max}, [y]$
- Characteristics: $y = f(u)$
  - static characteristics
  - dynamic characteristics
- Resolution
- Errors

Input range

- **Input range**
  - characteristics apply for the whole input range
  - input span:

    $$u_{max} - u_{min}$$

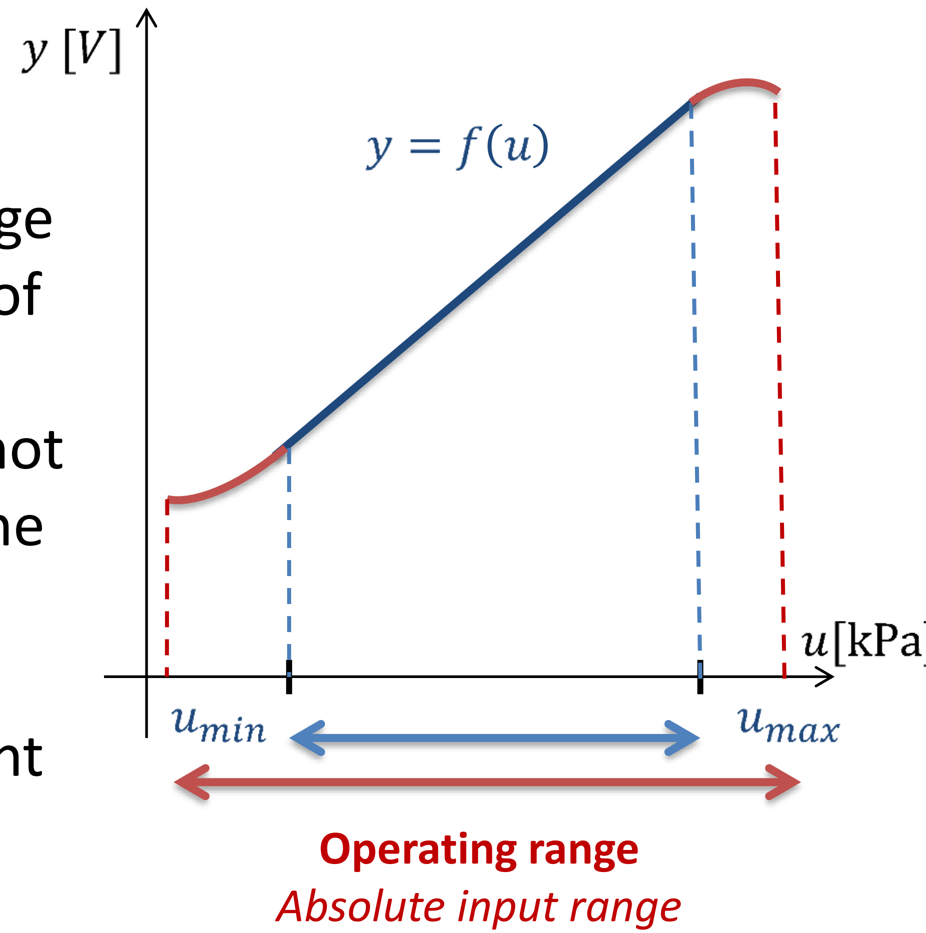


Input range
(Rated) Pressure range

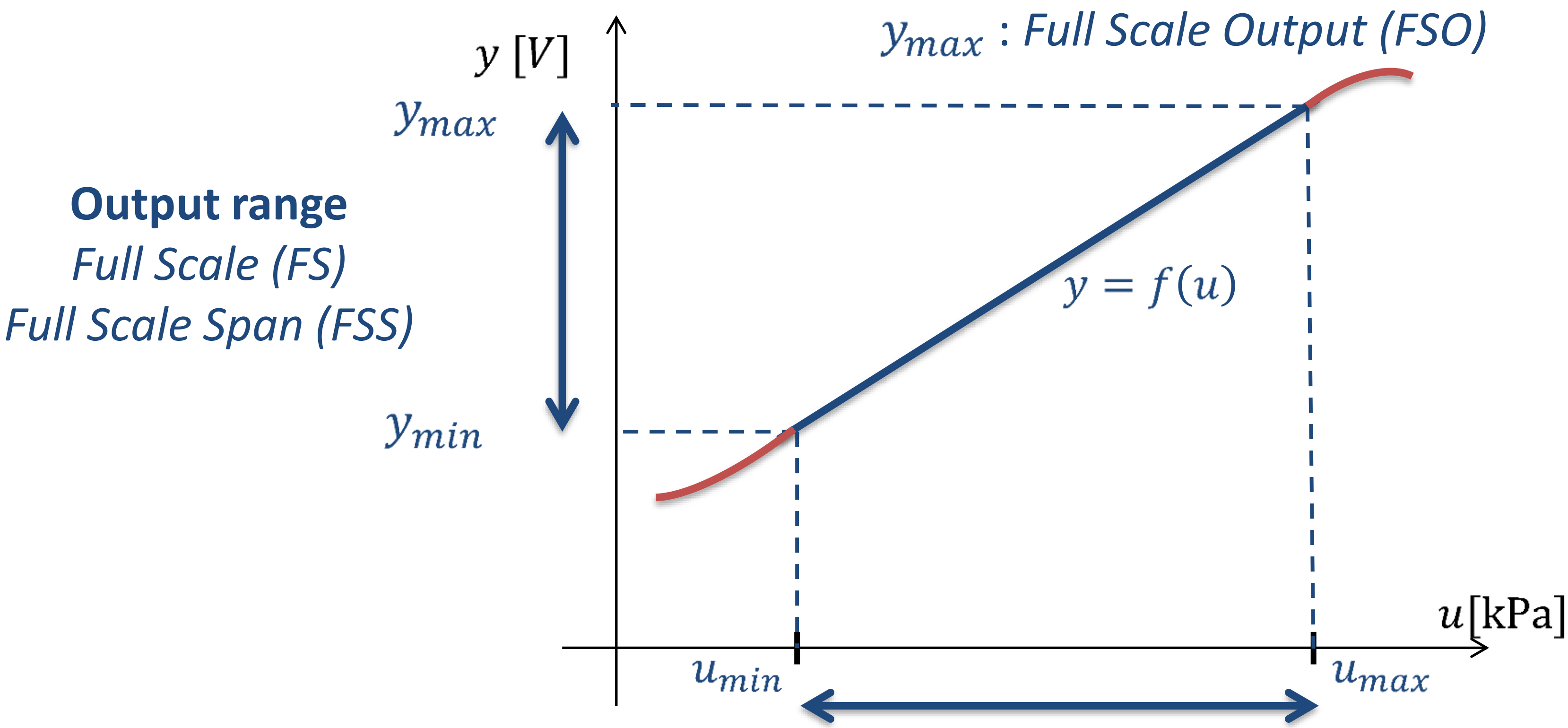
Slides use curves and nomenclature of a voltage-output pressure sensor as an example. For sensors measuring other types of quantities different units and nomenclature are used.

Input range

- **Operating range**
  - working in the operating range does not cause malfunction of the sensor
  - however, characteristics do not apply for the parts outside the input range
  - long-time and short-time (*burst*) operating ranges might be independently defined



Output range



- Range of output corresponding to the **input range**
- $\mathrm{FS} = y_{max} - y_{min}$
- For digital output sensors: *n* bits: $0 \dots 2^n - 1$

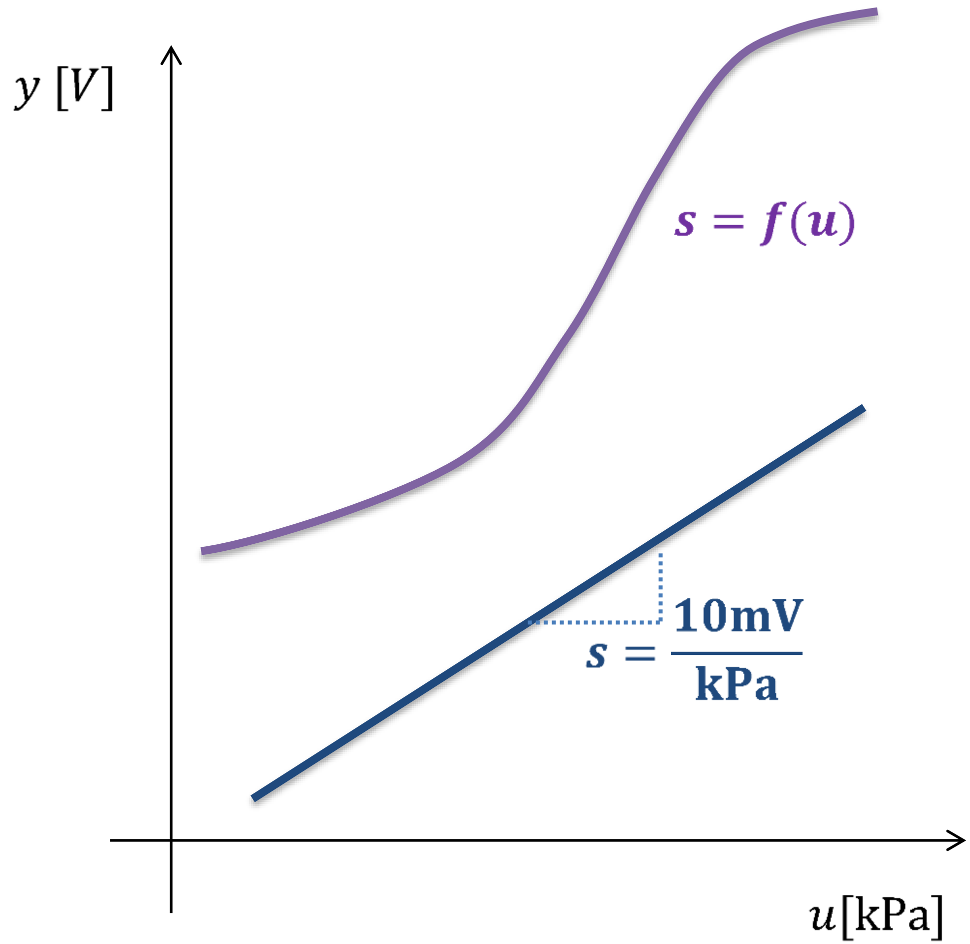
Static characteristics

- Static characteristics are defined for an operating point: $y = f(u)|_{\mathrm{OP}}$
- Environmental parameters are constant:
    - supply (e.g. voltage)
    - circuitry
    - load
    - environmental temperature (except for temperature sensors)
    - …
- Measured quantity has come stay (steady state)

Sensitivity

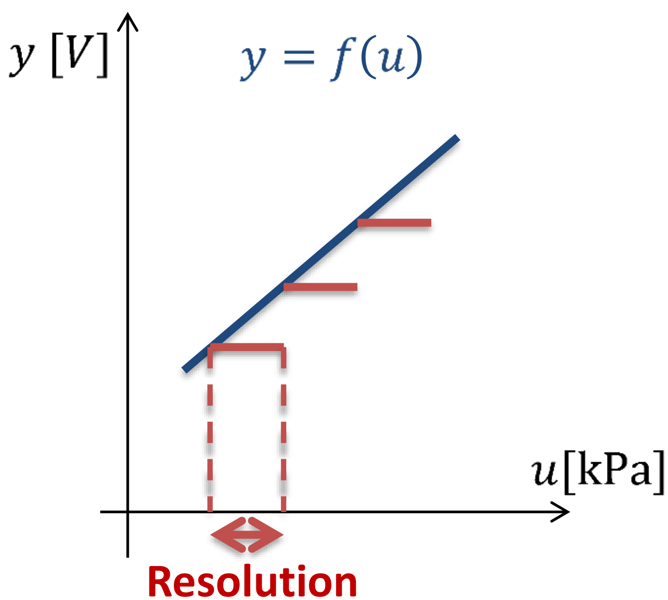
Sensitivity (*scale factor*):

$$s = \frac{df(u)}{du} \approx \frac{\Delta y}{\Delta u}$$



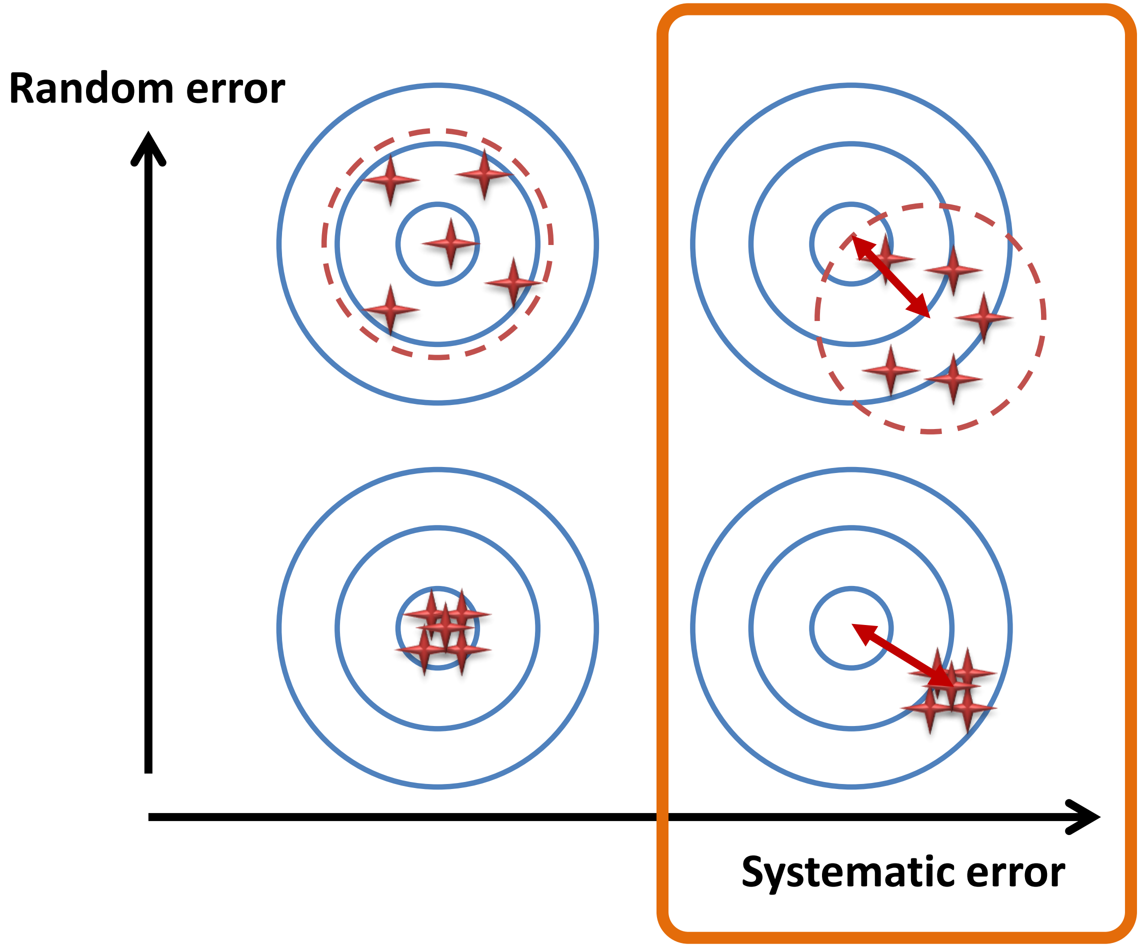
Resolution

- Smallest change of the input, which causes a change of the output
- Digital sensor: value corresponding to ADC LSB $\left(\frac{u_{max}-u_{min}}{2^N-1}\right)$
- Analogue sensor: might originate from physical construction
- Considered infinitesimally small if not indicated



Errors of sensors

- Error: difference between the measured value and the actual value

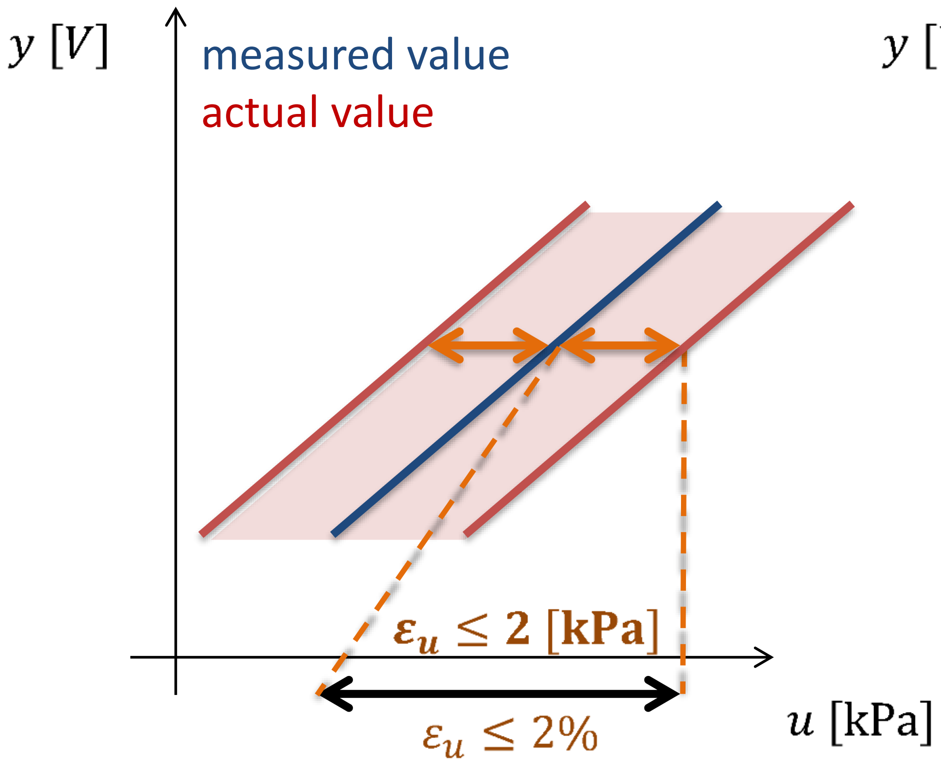


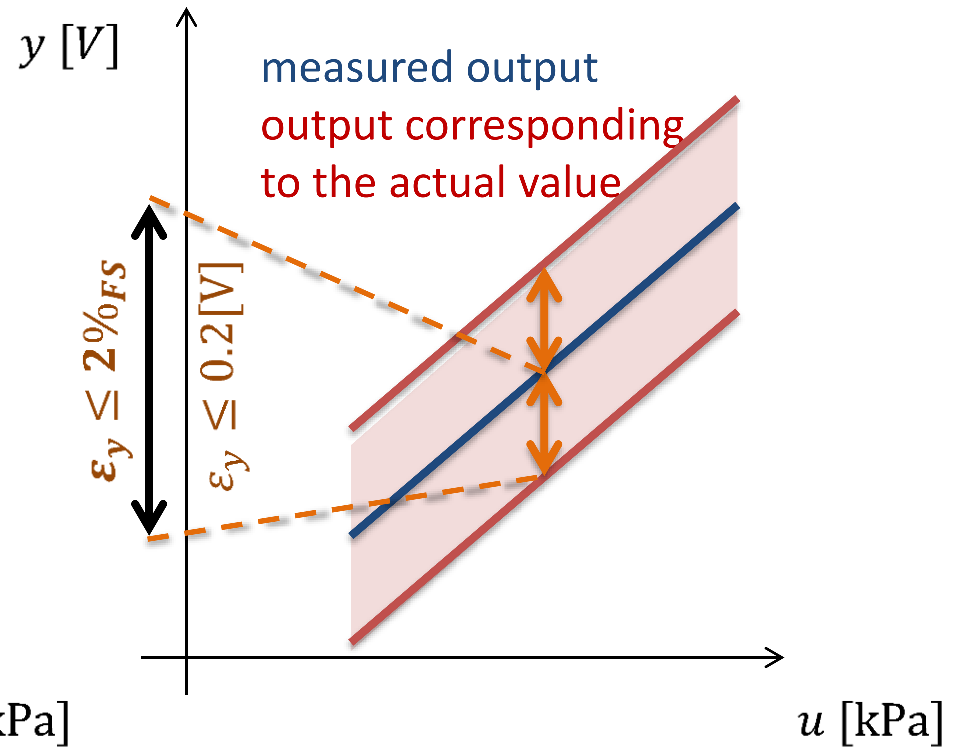
Systematic error can be minimized (nulled ideally) by calibration

Absolute error

- **as measured unit - *kPa***
- as percentage of the input range - %

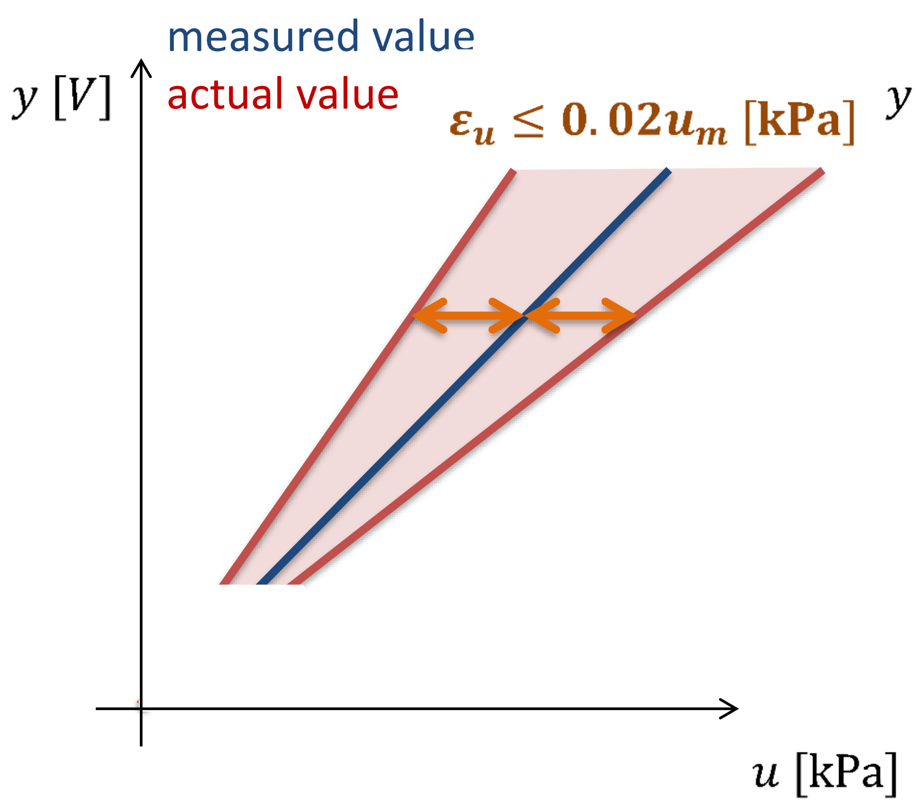
- as output unit – *V*
- **as percentage of the output span - %FS, %VFS**

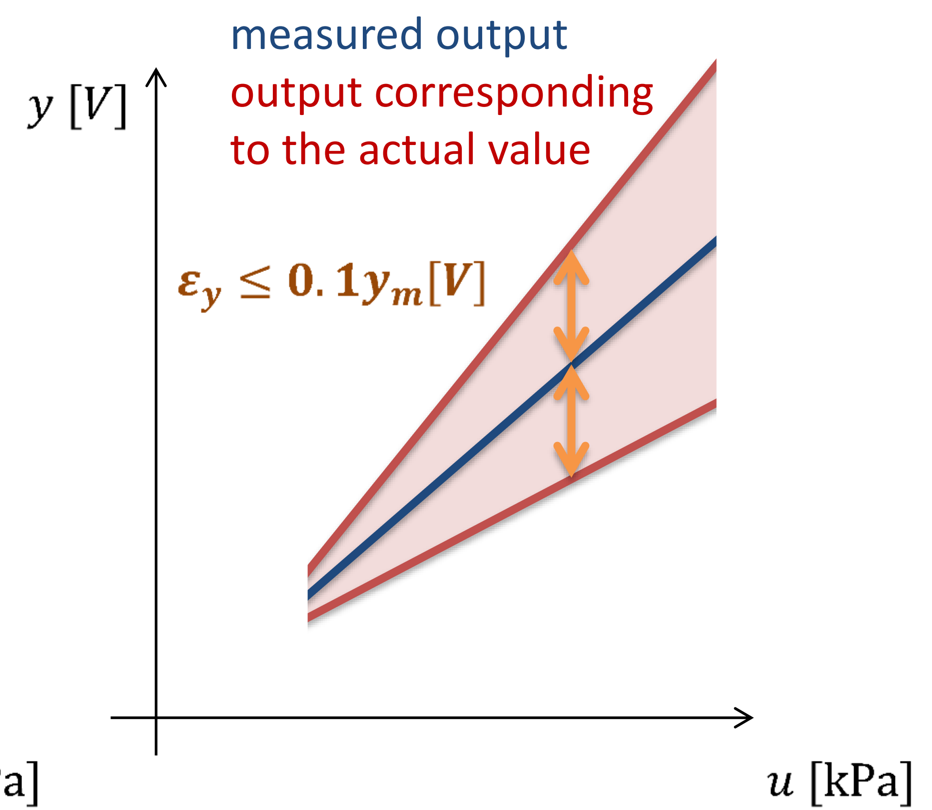




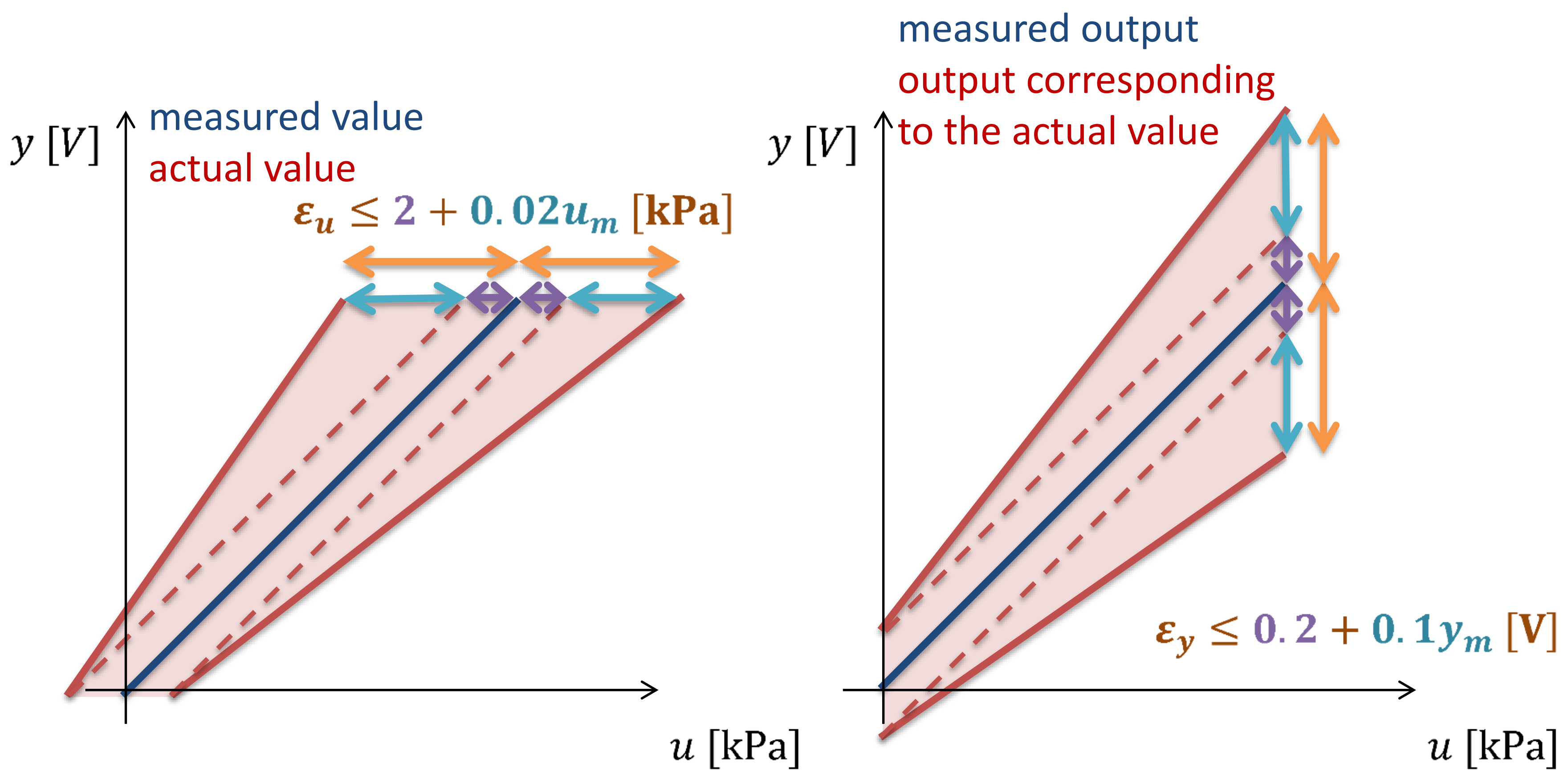
Relative error

- **as percentage of the measured value - %**
- **as percentage of sensor output - %**



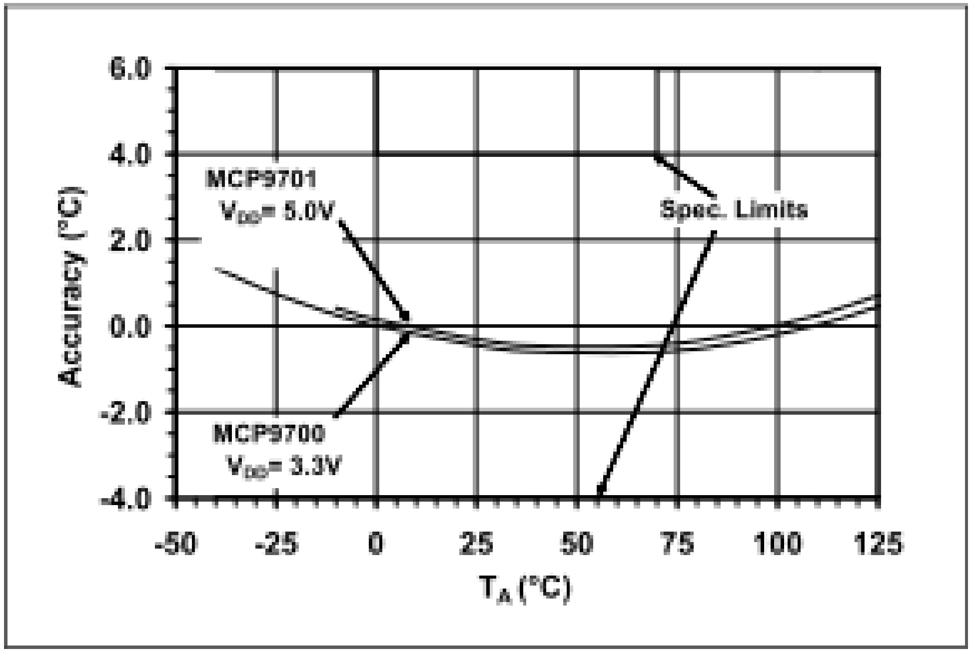


Combined relative and absolute error



Validity of error characteristis

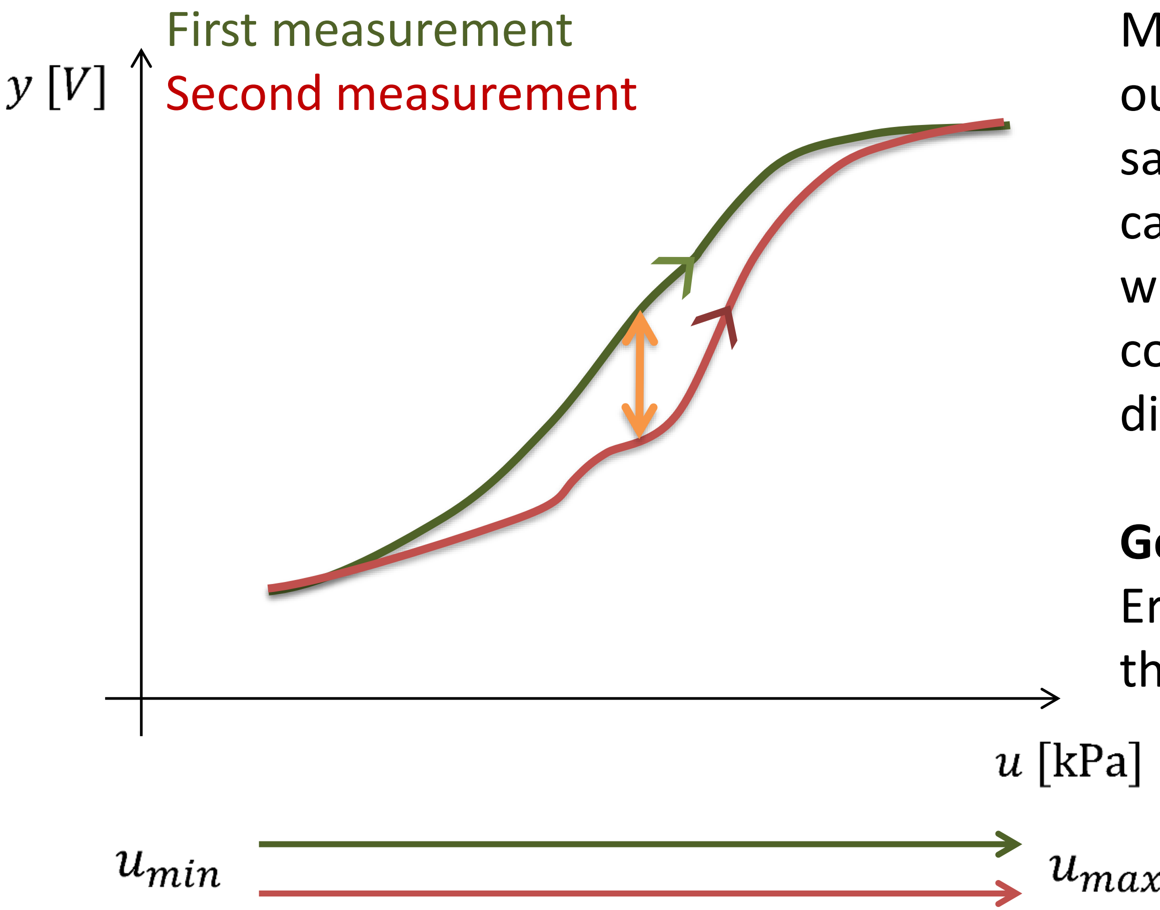
- On the whole input range
- On a subrange of the input range
- At a given operating point



Sensor Accuracy (Notes 1, 2)					
$T_A = +25°C$	T_{ACY}	—	±1	—	°C
$T_A = 0°C$ to $+70°C$	T_{ACY}	-2.0	±1	+2.0	°C
$T_A = -40°C$ to $+125°C$	T_{ACY}	-2.0	±1	+4.0	°C

Source: Microchip MPC970X series datasheet

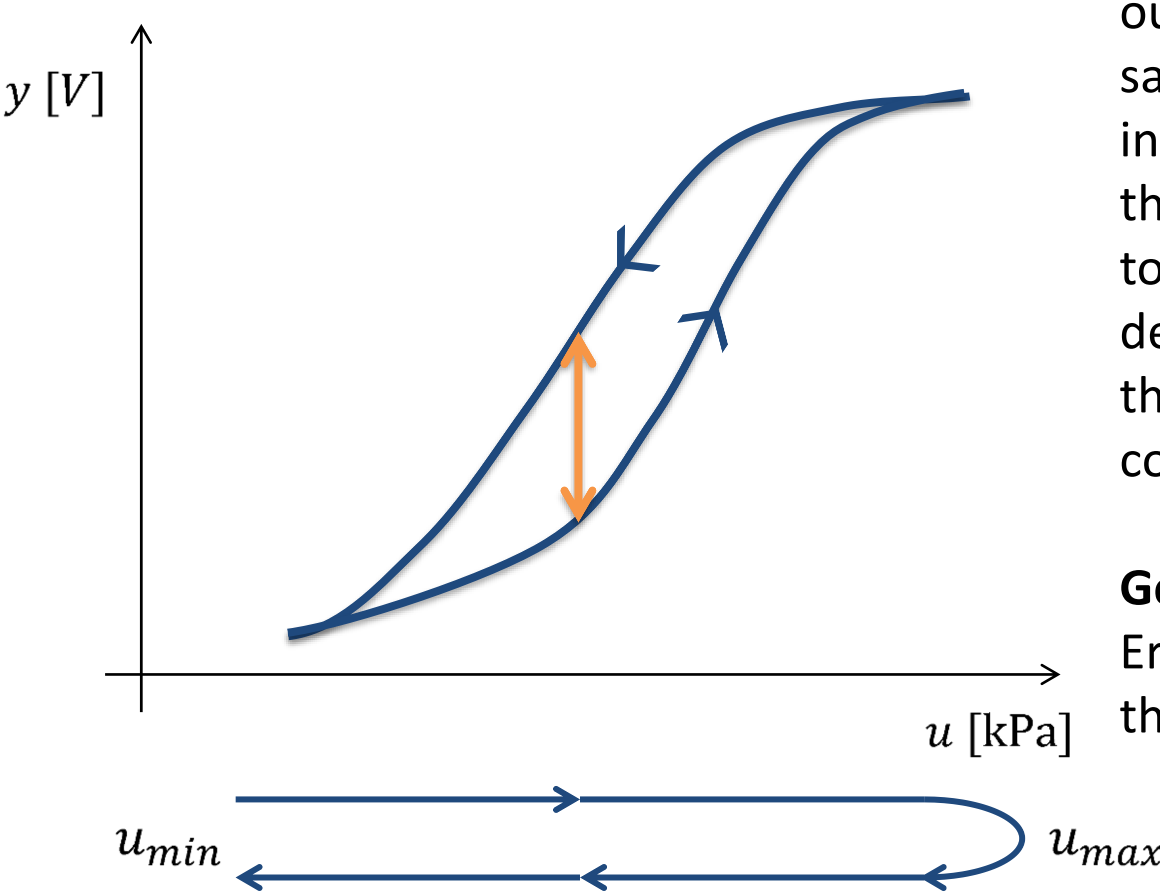
Repeatability error



Definition:
Maximal difference between the outputs corresponding to the same actual value of two calibration cycles carried out with the same environmental conditions and to the same direction.

General description:
Error band around the theoretical characteristics.

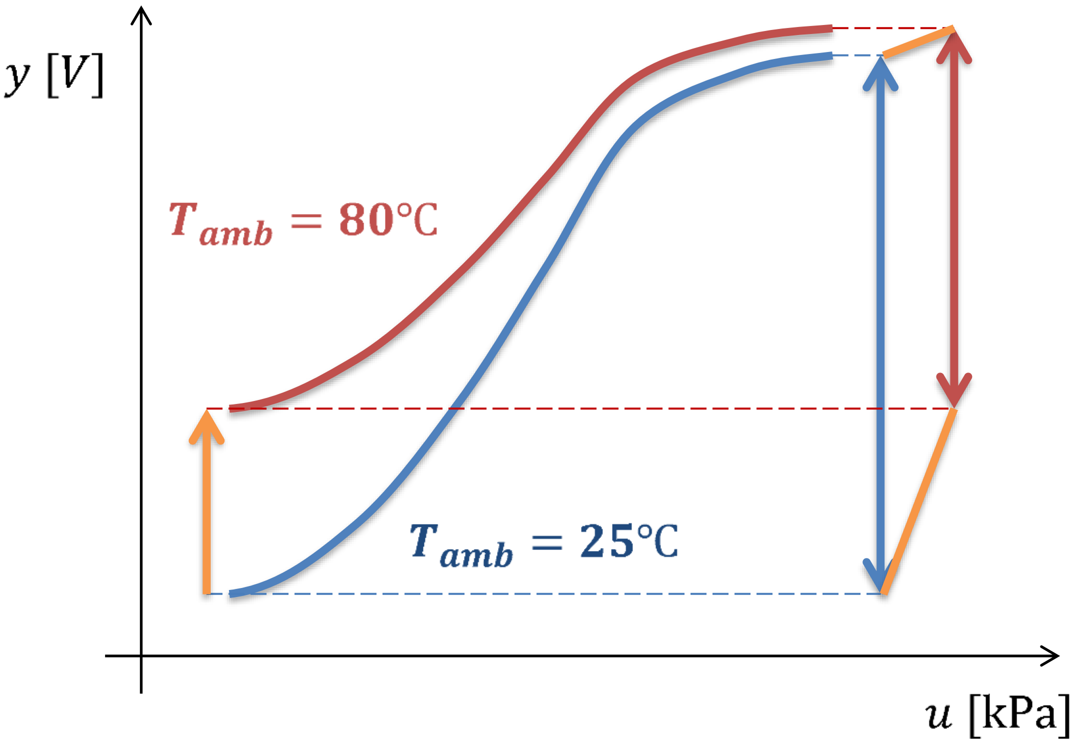
Hysteresis error



Definition:
Maximal difference between the outputs corresponding to the same actual value when the input is at first increased from the minimum of the input range to the maximum and then decreased from the maximum to the minimum (environmental conditions are the same).

General description:
Error band around the theoretical characteristics.

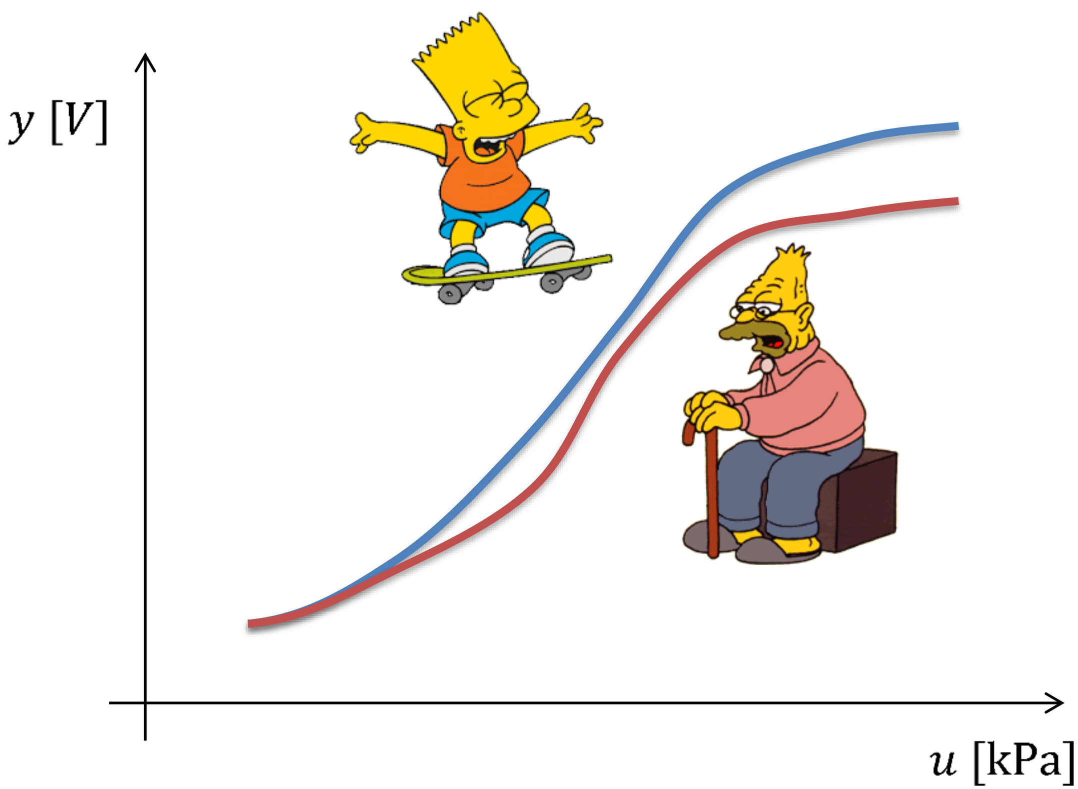
Thermal effect



Definition:
Difference between static characteristics corresponding to the same actual value measured by different environmental conditions.

General description:
Error band around the theoretical characteristics: absolute offset error and/or relative error.

Long-term stability / drift



Definition
Error caused by aging of the sensor.

General description:
Error band around the theoretical characteristics.

Errors indicated on datasheets

- Repeatability error
- Hysteresis error
- Temperature effects
- Stability

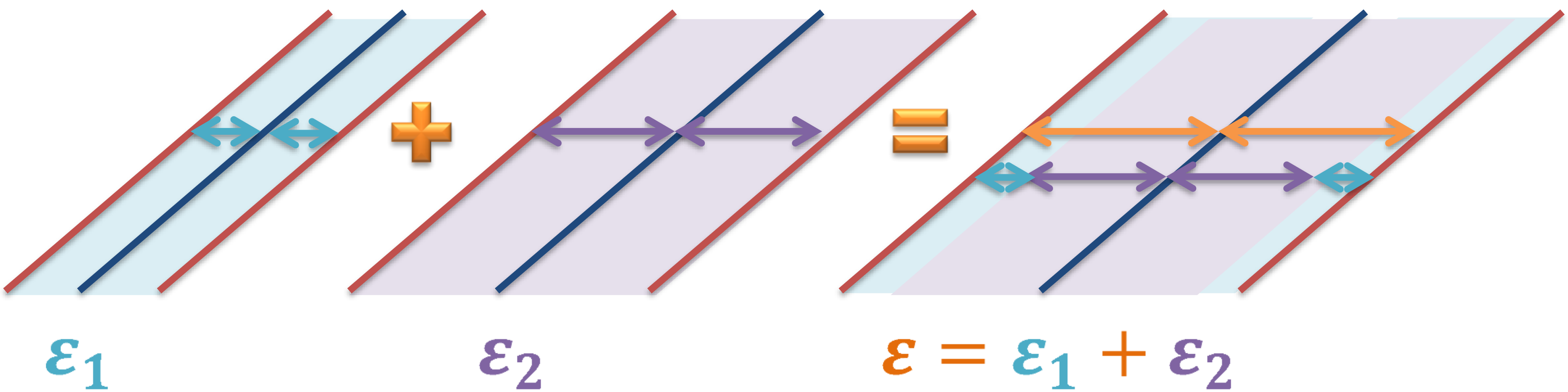
Accuracy

Accuracy

Accuracy

No standard convention exists about which errors are indicated individually on datasheets.

Error summation: Worst Case sum



Problem

Operating Characteristics

Table 1. Operating Characteristics (V_S = 10 V_{DC}, T_A = 25°C unless otherwise noted, P1 > P2)

Characteristic	Symbol	Min	Typ	Max	Units
Pressure Range[1]	P_{OP}	0	—	50	kPa
Supply Voltage[2]	V_S	—	10	16	V_{DC}
Supply Current	I_O	—	6.0	—	mAdc
Full Scale Span[3]	V_{FS}	38.5	40	41.5	mV
Offset[4]	—	–1.0	—	1.0	mV
Pressure Hysteresis (0 to 50 kPa)	—	—	±0.1	—	$\%V_{FS}$
Temperature Hysteresis (-40° to 125°C)	—	—	±0.5	—	$\%V_{FS}$
Temperature Coefficient of Full Scale	TCV_{FS}	–1.0	—	1.0	$\%V_{FS}$
Temperature Coefficient of Offset	TCV_{OFF}	–1.0	—	1.0	mV
Offset Stability[6]	—	—	±0.5	—	$\%V_{FS}$

Give the absolute output error of pressure measurement!

Source: Freescale semiconductor MPXV2050 series datasheet

Absolute output error

1. Full scale span (worst case): $V_{FS} = 41.5\text{mV}$
2. Errors relative to full scale span:

Pressure Hysteresis	$0.1\%V_{FS} =$	0.0415 mV
Temperature Hysteresis	$0.5\%V_{FS} =$	0.2075 mV
Temperature Coeff. FS	$1\%V_{FS} =$	0.4150 mV
Offset Stability	$0.5\%V_{FS} =$	0.2075 mV
Total		$\mathbf{0.8715\ mV}$

3. Absolute error:

Offset	1 mV
Temperature Coeff. Offset	1 mV
Total	$\mathbf{2\ mV}$

4. Total error:

2.8715mV ≈ 7% V_{FS}

Problem

- Data sheet of a pressure sensor provides the following information about errors:

Accuracy	$0 \leq P \leq 40$ kPA	2%
	$0 \leq P \leq 120$ kPA	5%
Hysteresis error		± 0.4kPa
Temperature effect on offset		± 1kPa
Drift (1)		± 0.2kPa

- Give the **relative** error of **pressure measurment** if the actual value is 20kPa!

Solution

- Accuracy: $0 \leq 20 \text{ kPA} \leq 40 \text{ kPA} \Rightarrow 2\% = 0.4 \text{ kPA}$
- Hysteresis error: 0.4 kPa
- Temperature effect on offset: 1 kPa
- Drift: 0.2 kPa
- Total absolute error: 2 kPa

- Relative error: $\frac{2 \text{ kPa}}{20 \text{ kPa}} = \mathbf{10}\%$

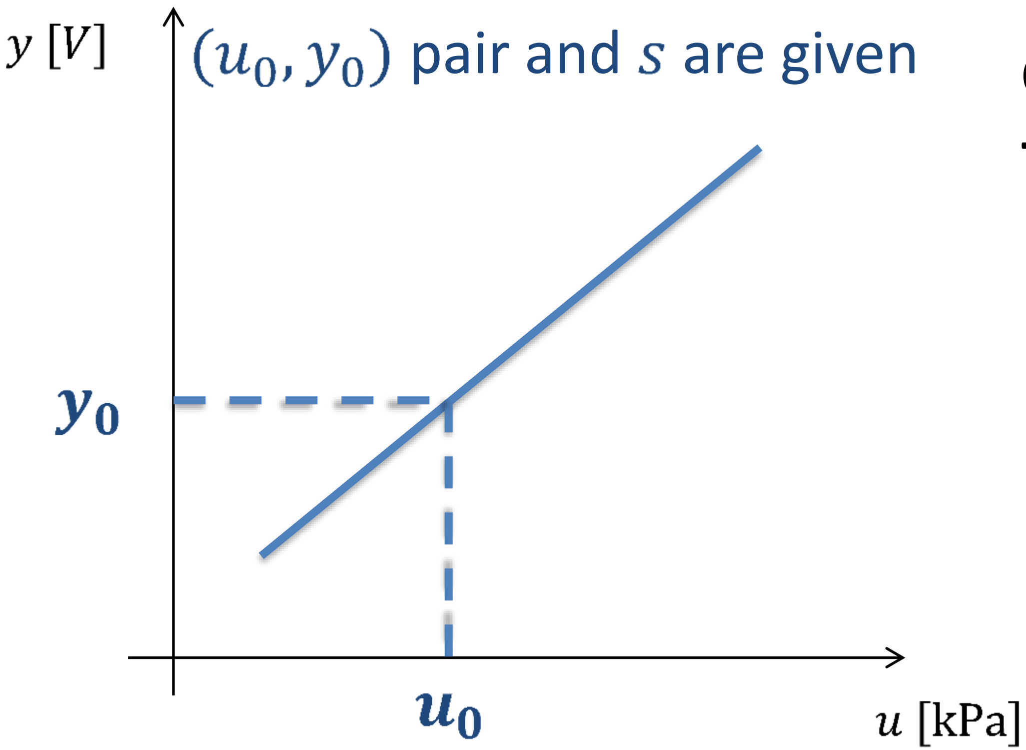
Linear static characteristics

Linear characteristics:

$$y = f(u) = a + bu$$

Sensitivity:

$$s = \frac{df(u)}{du} = b$$



Calculating output based on the measured value:

$$y = y_0 + s(u - u_0)$$

$$u = u_0 + \frac{(y - y_0)}{s}$$

Problem

Operating Characteristics

Table 1. Operating Characteristics (V_S = 10 V_{DC}, T_A = 25°C unless otherwise noted, P1 > P2)

Characteristic	Symbol	Min	Typ	Max	Units
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Temperature Hysteresis (-40° to 125°C)	—	—	±0.5	—	$\%V_{FS}$
Temperature Coefficient of Full Scale	TCV_{FS}	–1.0	—	1.0	$\%V_{FS}$
Temperature Coefficient of Offset	TCV_{OFF}	–1.0	—	1.0	mV
Offset Stability(6)	—	—	±0.5	—	$\%V_{FS}$

Give the relative error of pressure measurement at 30 kPa!

We have already calculated that the absolute output error is 2.8715 mV

Source: Freescale semiconductor MPXV2050 series datasheet

Error of pressure measurement

- Sensitivity:

$$s = \frac{40\text{mV}}{50\text{kPA}} = 0.8\,\frac{\text{mV}}{\text{kPA}}$$

- Absolute error of pressure measurement:

$$\varepsilon_u = \frac{\varepsilon_y}{s} = 2.875\text{mV} \cdot \frac{1}{0.8}\frac{\text{kPa}}{\text{mV}} = 3.59\text{kPa}$$

- Relative error of pressure measurement:

$$\frac{3.59}{30} = 11.96\%$$

Establishing linear characteristics

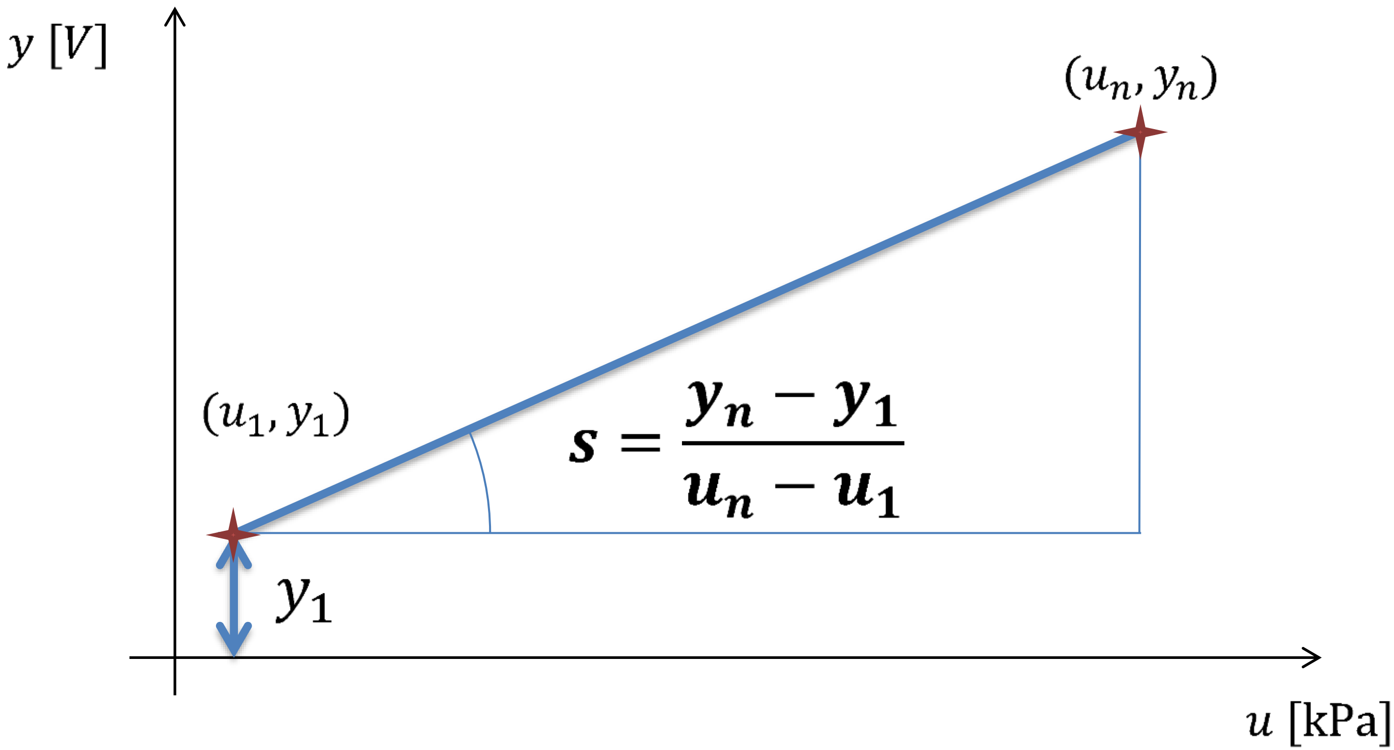
- Given $n \geq 2$ input-output pairs:

$$(u_i, y_i), i = 1 \dots n, u_i < u_{i+1}$$

- Goal: establish linear characteristics in form $y = a + bu$

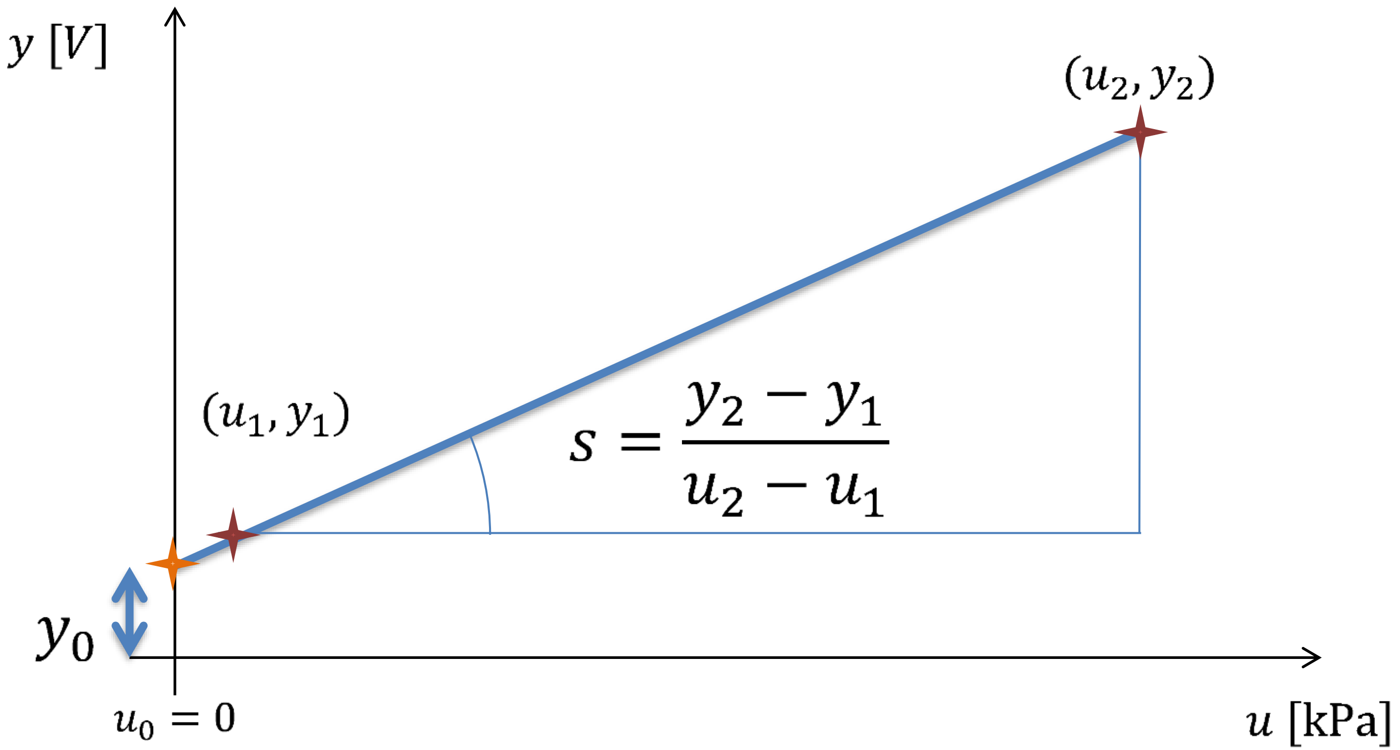
- How parameters of the linear characteristics can be found?

End-Point Straigth Line



$$y = y_1 + s(u - u_1) \qquad u = \frac{y - y_1}{s} + u_1$$

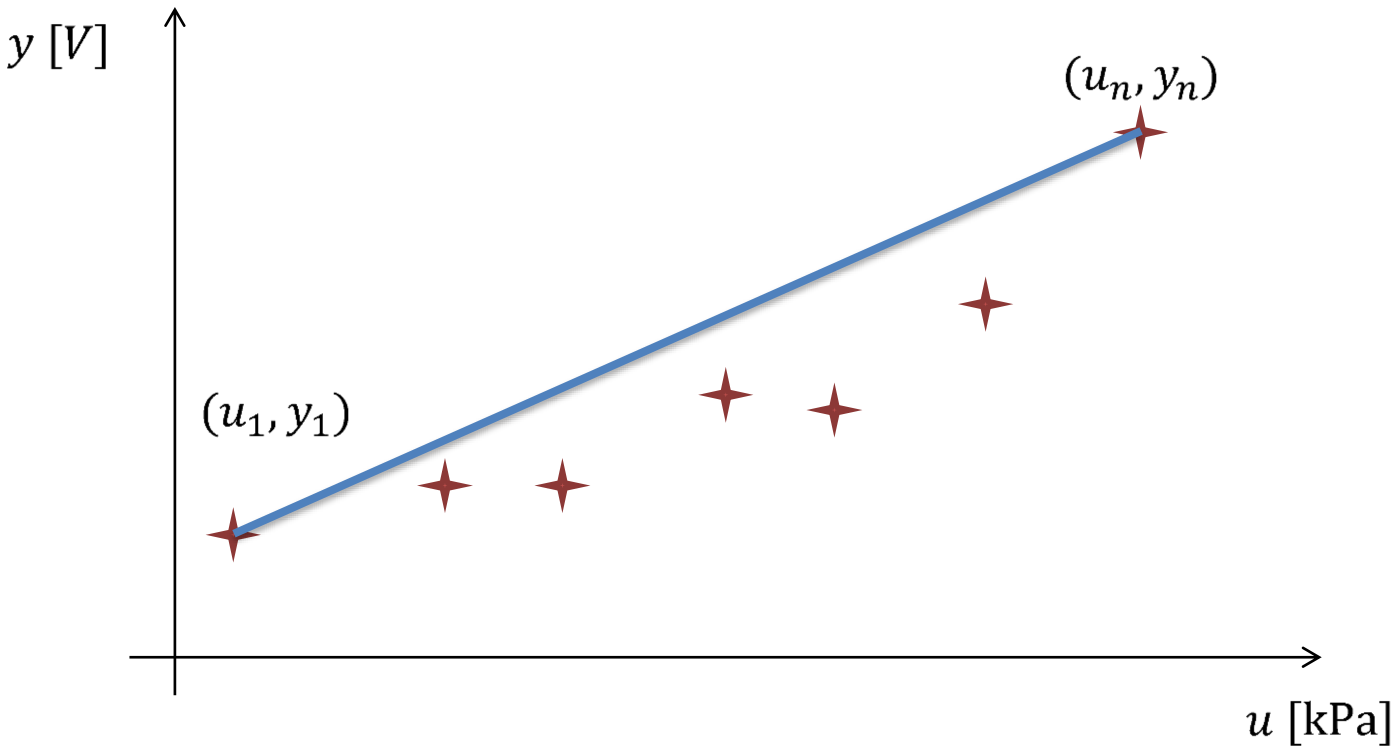
Linear extrapolation



Output value corresponding to the input value $u = 0$: $y_0 = y_1 - su_1$

$$\boldsymbol{y = y_0 + su} \qquad \boldsymbol{u = \frac{y - y_0}{s}}$$

End-Point Straight Line

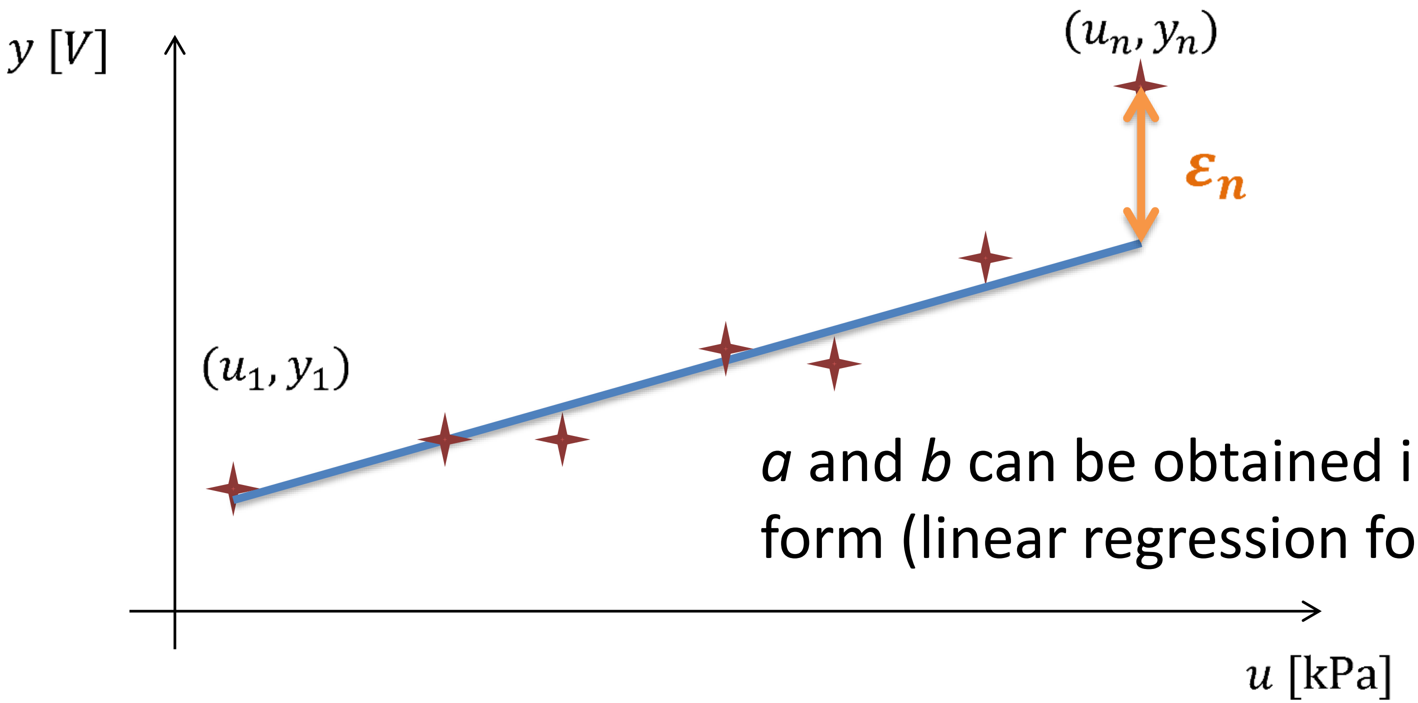


Linear characteristics is exact at end points, but depending on the nonlinearity of the actual characteristics, error might be significant for values between the end points.

Best Fit Straight Line

$$V(a,b) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - (a + bu_i))^2$$

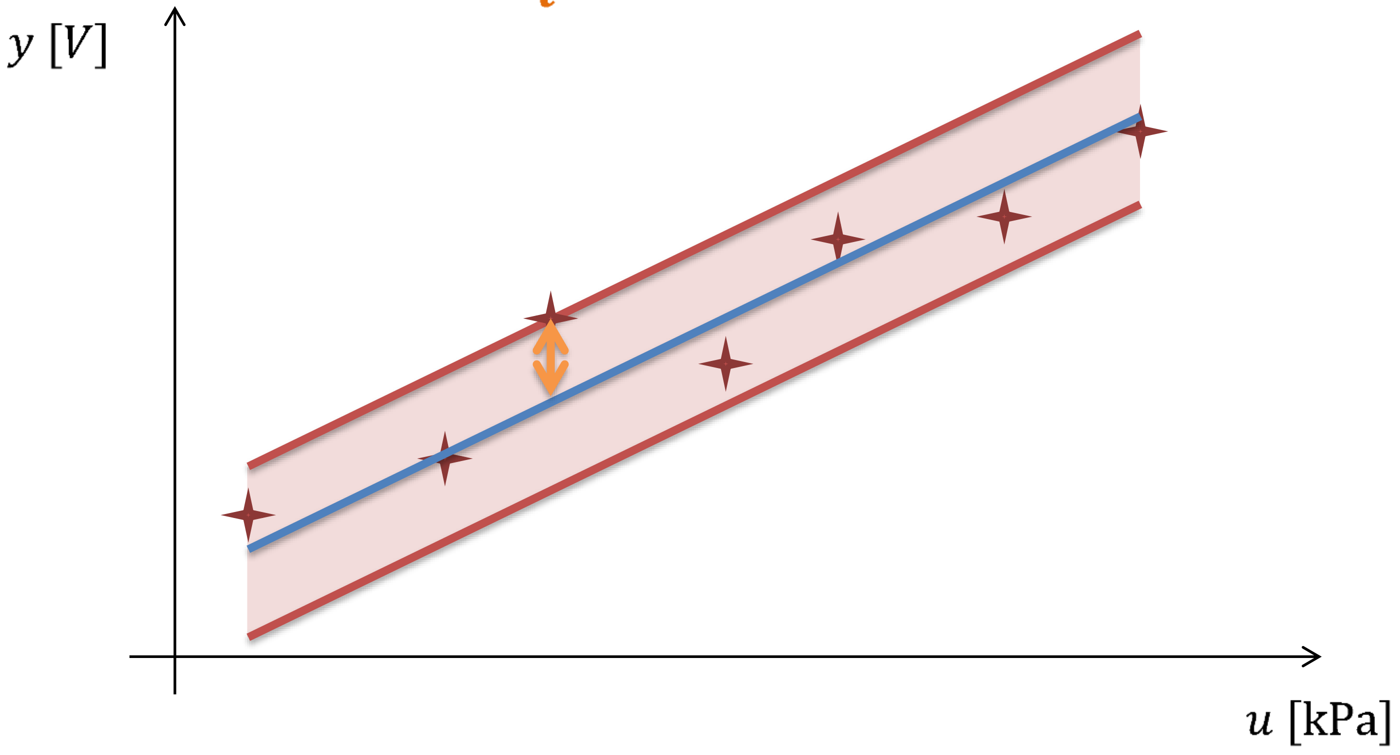
$$(a,b) = \arg\min V(a,b)$$



a and *b* can be obtained in closed form (linear regression formula)

Linearity error

$$\boldsymbol{\varepsilon = \max_i |y_i - (a + b u_i)|}$$



Problem

Calibration values below have been measured using a temperature sensor. Give the end-point straight line characteristics of the sensor and the formula for calculating the temperature based on the output value! Give the relative linearity error!

u [°C]	30	47	93	100
y $[V]$	2.4	3.1	5.2	5.9

Solution

u [°C]	30	47	93	100
y [V]	2.4	3.1	5.2	5.9

- Input range: $100 - 30 = 70°\mathrm{C}$
- Output range: $5.9 - 2.4 = 3.5\mathrm{V}$
- Sensitivity: $s = 3.5/70 = 1/20\,\mathrm{V}/°\mathrm{C} = 50\,\mathrm{mV}/°\mathrm{C}$
- Linear characteristics:

$$y\ [\mathrm{V}] = 2.4 + 0.05[u[°\mathrm{C}] - 30]$$

- Formula for calculating the temperature:

$$u\ [°\mathrm{C}] = 30 + \frac{y\ [\mathrm{V}] - 2.4}{0.05}$$

Solution

u [°C]	30	47	93	100
y [V]	2.4	3.1	5.2	5.9

- Output at $0°\mathrm{C}$: $2.4 - 0.05 \cdot 30 = 0.9\mathrm{V}$
- Linear characteristics:

$$y\ [\mathrm{V}] = 0.9 + 0.05u\ [°\mathrm{C}]$$

- Formula for calculating the temperature:

$$u\ [°\mathrm{C}] = \frac{y\ [\mathrm{V}] - 0.9}{0.05} = 20y[\mathrm{V}] - 18$$

Solution

u [°C]	30	47	93	100
y [V]	2.4	3.1	5.2	5.9

- Characteristics: $y = 0.9 + 0.05u$
- Linearity error at temperature 47°C:

$$|3.1 - (0.9 + 0.05 \cdot 47)| = 0.15\text{V}$$

- Linearity error at temperature 93°C:

$$|5.2 - (0.9 + 0.05 \cdot 93)| = 0.35\text{V}$$

- Linearity error for the whole input range:

$$\frac{\mathbf{0.35}}{\mathbf{3.5}} = \mathbf{10\%\ FS}$$

Description of nonlinear characteristics

- Static characteristics defined by a nonlinear function, e.g.

$$y = y_0 \mathrm{e}^{B\left(\frac{1}{u} - \frac{1}{u_0}\right)}$$

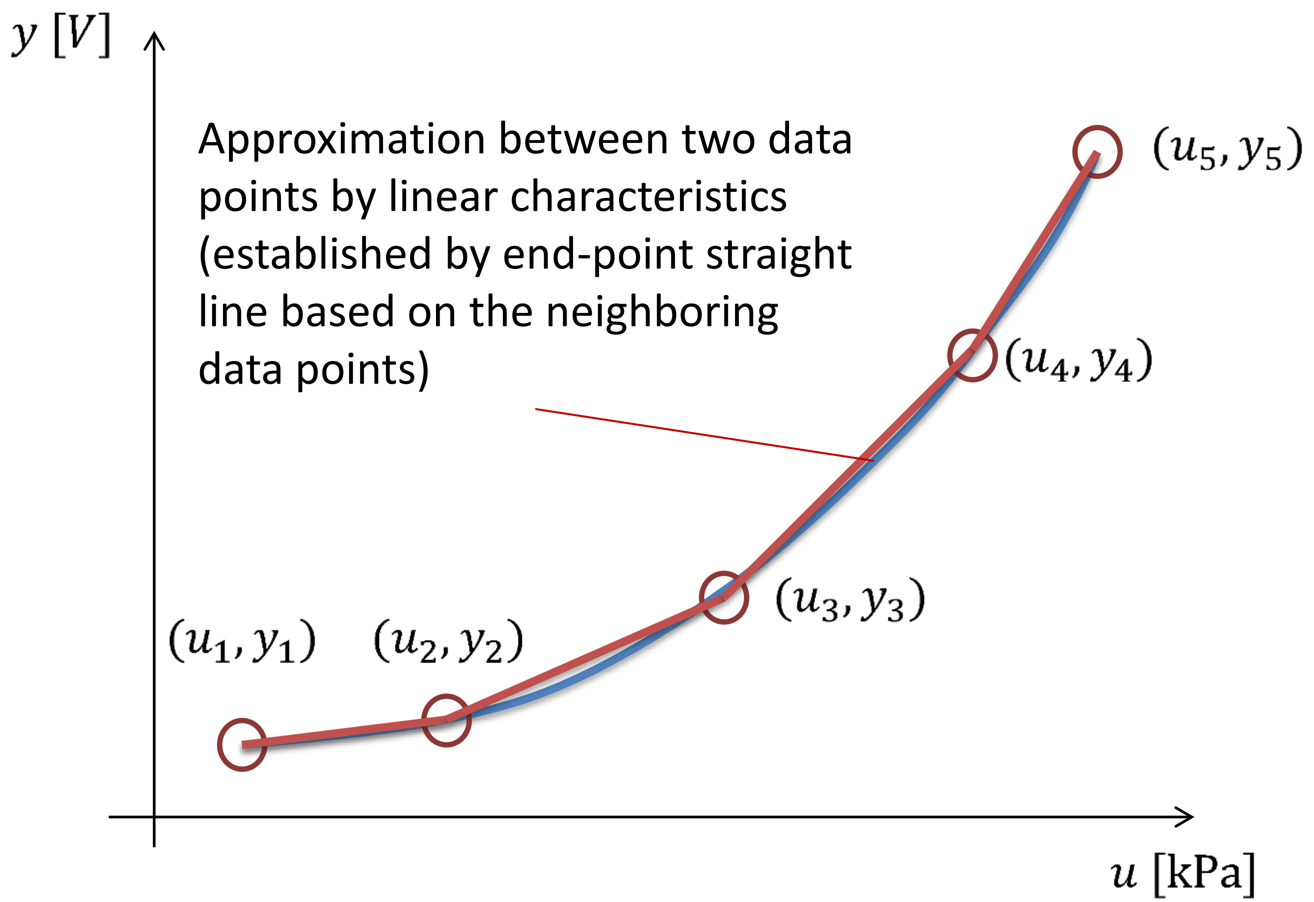
- Approximation by higher order polynomial, e.g.

$$y = a + bu + cu^2$$

- Tabular definition (quantization)

$\boldsymbol{y_1}$	$\boldsymbol{y_2}$	…	$\boldsymbol{y_n}$
$\boldsymbol{u_1}$	$\boldsymbol{u_2}$	…	$\boldsymbol{u_n}$

Linear interpolation



y_1	y_2	y_3	y_4	y_5
u_1	u_2	u_3	u_4	u_5

Problem

The table gives the static characteristics of an NTC thermistor. Give the linearity error of the temperature measurement originating from linear interpolation if the output values and the actual values of temperature are

(a) 821.74 Ω at 12 °C

(b) 31.09 Ω at 107 °C

u [°C]	y [Ω]
5	1110.220
10	887.257
15	713.463
20	577.375
25	470.000
30	384.800
35	316.757
40	262.177
45	218.069
50	182.297
55	153.150
60	129.249
65	109.551
70	93.281
75	79.750
80	68.446
85	58.996
90	51.036
95	44.332
100	38.640
105	33.790
110	29.664
115	26.123
120	23.091

Source: muRata NTC thermistor catalogue

Solution

(a) Since the output is between 887.26Ω and 713.46Ω,

$$u = 10 + \frac{15 - 10}{713.46 - 887.26}(821.74 - 887.26) = 11.88°\text{C}$$

Relative error: $\varepsilon = \frac{11.88-12}{12} = -\mathbf{1}\%$

(b) Since the output is between 29.67Ω and 33.79Ω,

$$u = 105 + \frac{110 - 105}{29.67 - 33.79}(31.09 - 33.79) = 108.28°\text{C}$$

Relative error: $\varepsilon = \frac{108.81-107}{107} = \mathbf{1.19}\%$

Dynamic characteristics

- The measured value is not stationary – how does its change is followed by the output of the sensor?

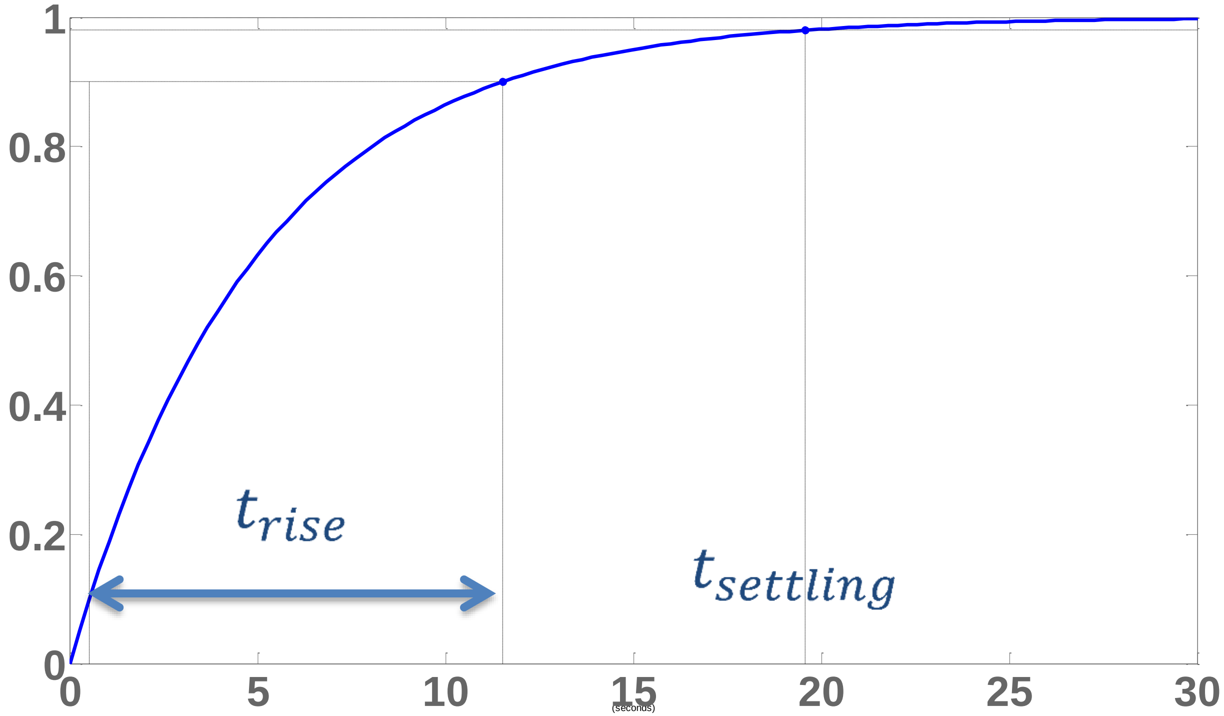
- Dynamic characteristics of sensors is approximated by well-known simple terms:
  - first-order term: $W(s) = \frac{A}{1+sT}$
  - second-order term: $W(s) = \frac{A}{s^2T^2+2\xi Ts+1}$

Description in time domain

- How does output of the sensor follow the change of the input?
- Warm-up time: duration between powering up the sensor and availability the first valid measurement
- Parameters of the step response
  - time constant
  - settling time: 2%, 5%
  - rise time (response time): 10% → 90%
  - overshoot (for second-order terms)

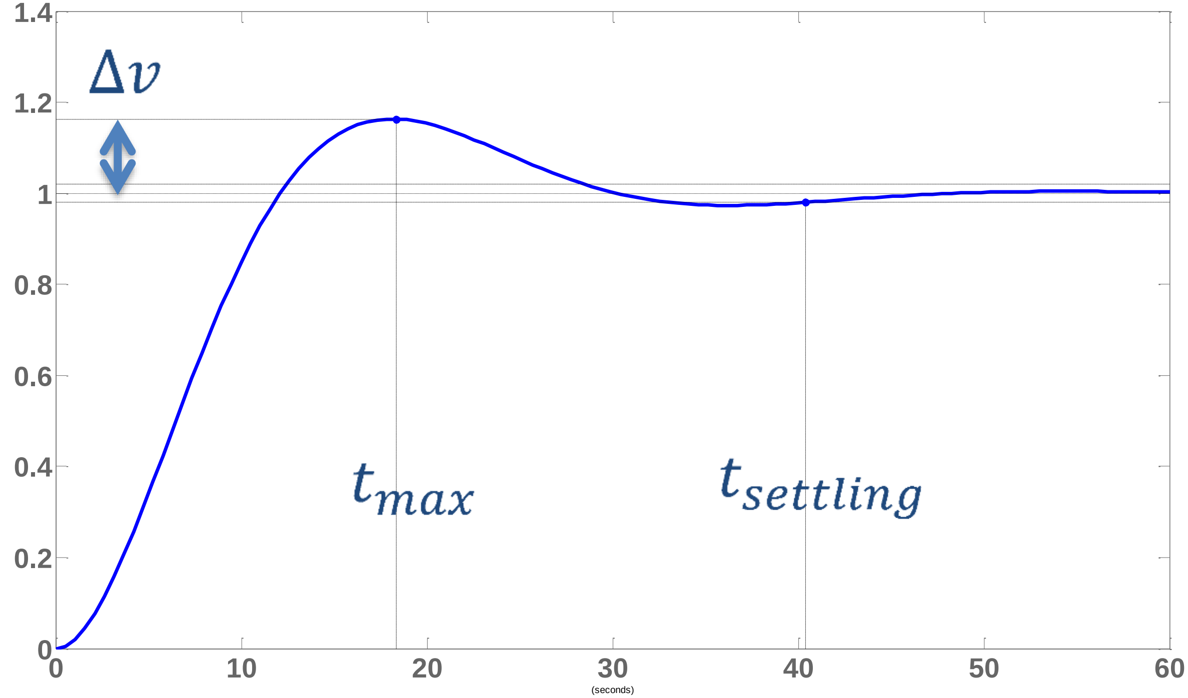
First-order characteristics

- Time constant: T
  - $\boldsymbol{y(T) \approx 0.63 y_\infty}$
  - $y(2T) \approx 0.87 y_\infty$
  - $y(3T) \approx 0.95 y_\infty$
  - $y(5T) \approx y_\infty$



Second-order characteristics

- Undamped natural frequency: ω_0; damping: ξ
- Settling time: $T_{2\%} \approx \frac{4}{\omega_0 \xi}$
- Overshoot: $\Delta V = \exp - \frac{\pi\xi}{\sqrt{1-\xi^2}}$



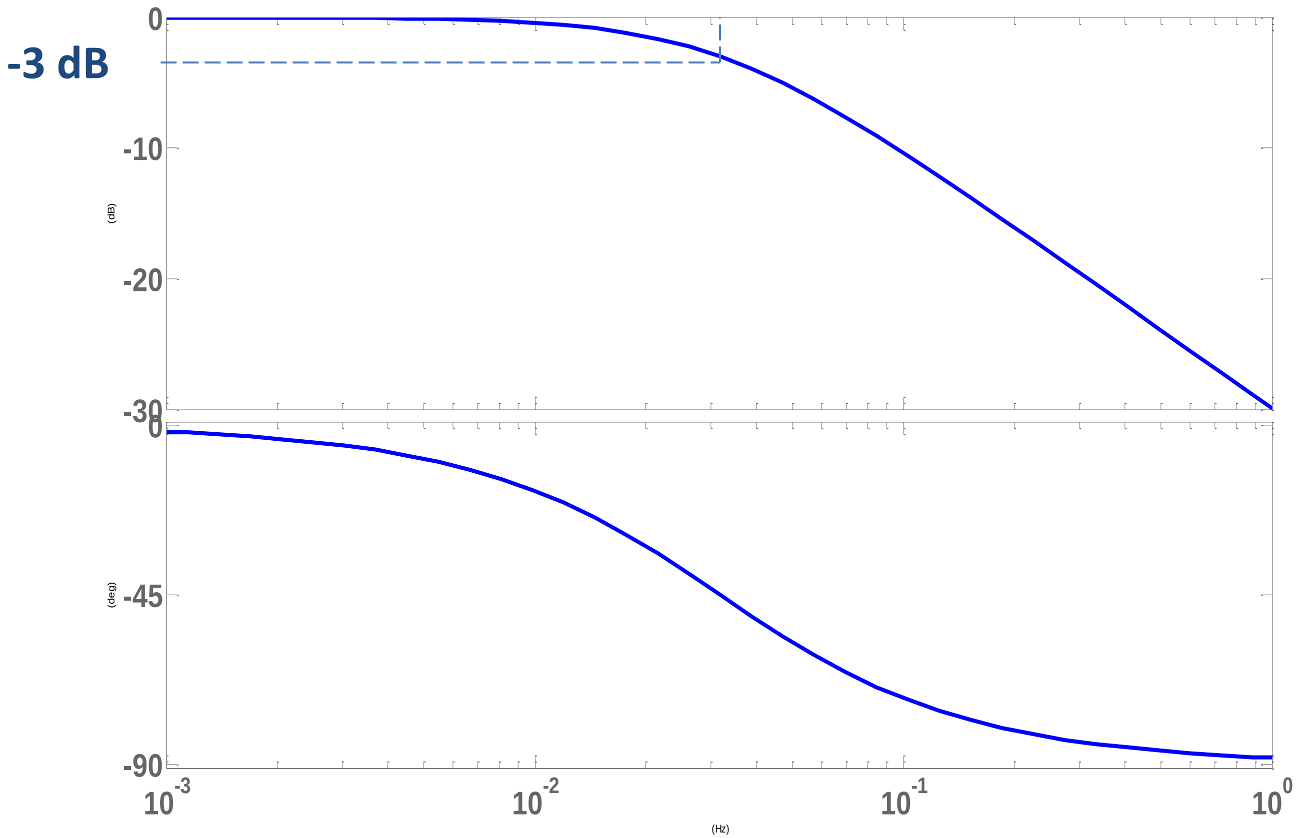
Description in frequency domain

- How fast the input signal of the sensor can change?

- Properties of the Bode diagram:
  - cutoff frequency: generally at $-3\mathrm{dB}$
  - resonant frequency (if exists)

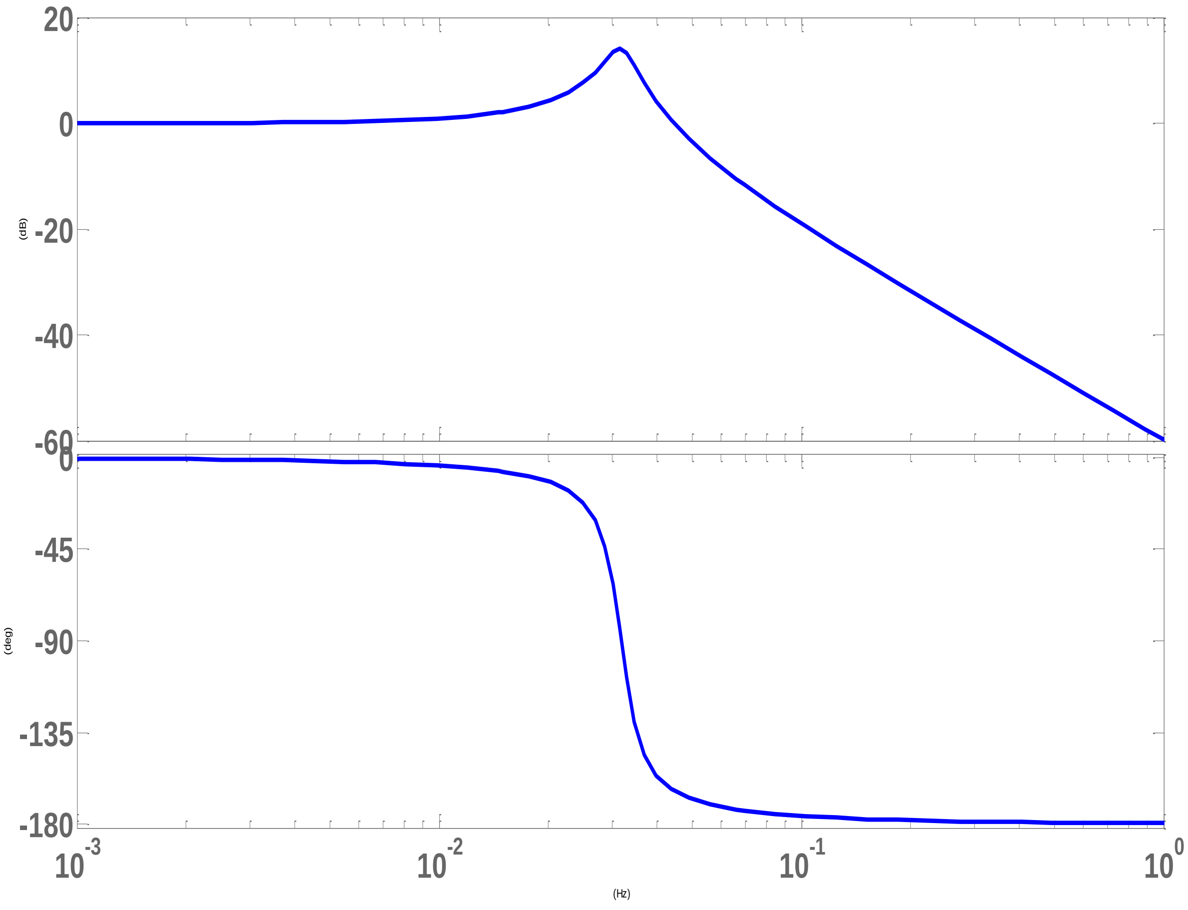
First-order characteristics

$$f_c \approx \frac{0.159}{T}\,[\mathrm{Hz}]$$



Second-order characteristics

- Resonance:
  - $\xi \leq \frac{\sqrt{2}}{2}$
  - $f_{res} = \frac{\sqrt{1-2\xi^2}}{2\pi T}\,[\mathrm{Hz}]$



How to choose the adequate sensor?

You would like to develop a household meteorological station using a microcontroller equipped with an ADC with 10-bit resolution and 5V input range. The desired measurement range is $-10 \ldots + 40\ ^\circ\mathrm{C}$ and you shall aim maximal accuracy. Which sensor you would choose from the options below?

	Anna	**Barbara**	**Cecile**
Operating range	$-100 \ldots + 400\ ^\circ\mathrm{C}$	$-70 \ldots + 130\ ^\circ\mathrm{C}$	$-10 \ldots + 40\ ^\circ\mathrm{C}$
Input range	$-100 \ldots + 400\ ^\circ\mathrm{C}$	$-10 \ldots + 40\ ^\circ\mathrm{C}$	$-10 \ldots + 40\ ^\circ\mathrm{C}$
Accuracy	$\pm 0.02\%\mathrm{FS}$	$\pm 0.1\%\mathrm{FS}$	$\pm 0.02\ ^\circ\mathrm{C}$
Sensitivity	$20\mathrm{mV}/^\circ\mathrm{C}$	$100\mathrm{mV}/^\circ\mathrm{C}$	$100\mathrm{mV}/^\circ\mathrm{C}$

Solution

- Anna
  - input range: $500°\mathrm{C}$
  - output range: $500 \cdot 0.02 = 10\mathrm{V}$
  - output range corresponding to the desired input range $-10 \ldots +40°\mathrm{C}$: $50/500 \cdot 10 = 1\mathrm{V}$

    (using a suitable circuitry the output range can be transformed to the 0-5V range so it can be handled by the ADC of the microcontroller)
  - error: $0.02 \cdot 0.01 \cdot 10\mathrm{V} = 0.002\mathrm{V} = 2\mathrm{mV} \Rightarrow \mathbf{0.1}°\mathrm{C}$

Solution

- Barbara
  - input range: $50°\mathrm{C}$
  - output range: $50 \ \cdot 0.1 = 5\mathrm{V}$
  - error: $0.1 \cdot 0.01 \cdot 5\mathrm{V} = 0.005\mathrm{V} = 5\mathrm{mV} \Rightarrow \mathbf{0.05}°\mathrm{C}$

- Cecile
  - input range: $50°\mathrm{C}$
  - output range: $50 \ \cdot 0.01 = 5\mathrm{V}$
  - error: $\mathbf{0.02}°\mathrm{C}$

Solution

	Anna	Barbara	Cecile
Absolute error	$0.1°C$	$0.05°C$	$0.02°C$

- **But**: as we have an ADC with the resolution of 10 bits, the resolution is $50/1023 \approx 0.05°C$
- There is no point choosing a sensor with higher accuracy
- It is recommended to choose the one with the highest operating range from the sensors with accuracy of $0.05°C$ or better
- The best sensor for the job: Barbara

How to select the adequate sensor?

- The sensor should withstand the environment it is used in
  - operating range
  - environmental temperature, humidity…
  - power supply
- Input range shall be the closest possible to the required range
  - price for a wide input range must be payed: in accuracy, in sensitivity to environmental effects, in cost
- Choose a sensor accurate enough but not too accurate!
  - there is no point using a sensor with $0.01°\mathrm{C}$ accuracy in a room thermostat

How datasheets should be read?

CAREFULLY!

- Double-checks the units!
- Consider the validity range of parameters!
- Never consider typical errors - aim for worst case!
- Always read the footnotes!
- Double-check to which models the given parameters apply!