Ex. 12 simulation and bisimulation

12.1 12.2
12.5
Q33

key points from Lecture

* simulation and bismulation
* quotient
* simulation relation verification, reachability

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extra notes

transition system with initial and final states

Ex 12.1 Given T and T', find the bisimulation relation

Def. relation (P_7 of L_{12})

→ relation R from A to B is a subset of A×B.

$$\boxed{aRb \text{ if } (a,b)\in R \quad , a\in A, b\in B}$$

Simulation relation (P_8 of L_{12})

→ given T and T', a relation $\sim \in S\times S'$ is a simulat ion relation, if

- $\forall s\in S_0 \Rightarrow \exists s'\in S_0'$ that $s\sim s'$
- $s\sim s' \wedge s\in S_F \Rightarrow s'\in S_F'$
- $s\sim s' \wedge s\xrightarrow{\sigma} r \Rightarrow \exists r'\in S'$. s.t. $s'\xrightarrow{\sigma} r'$ and $r\sim r'$

Bisimulation relation (P_9 of L_{12})

→ if $\sim$ is a simulation relation from T to T', and $\sim' = \{(s',s) \mid (s,s')\in\sim\}$ is a simulation from T' to T then $\sim$ is a bisimulation relation

step 1. there is only one initial state in both T and T'
$s_1 \sim s'_1$

step 2. $s_1 \sim s'_1$, $s_1 \xrightarrow{a} s_4$, $s'_1 \xrightarrow{a} s'_2 \Rightarrow s_4 \sim s'_2$

$s_1 \sim s'_1$, $s_1 \xrightarrow{b} s_2$, $s'_1 \xrightarrow{b} s'_3 \Rightarrow s_2 \sim s'_3$

$s_4 \sim s'_2$, $s_4 \xrightarrow{a} s_6$, $s'_2 \xrightarrow{a} s'_4 \Rightarrow s_6 \sim s'_4$

$s_4 \sim s'_2$, $s_4 \xrightarrow{b} s_5$, $s'_2 \xrightarrow{b} s'_5 \Rightarrow s_5 \sim s'_5$

$s_2 \sim s'_3$, $s_2 \xrightarrow{a} s_5$, $s'_3 \xrightarrow{a} s'_6 \Rightarrow s_5 \sim s'_6$

$s_2 \sim s'_3$, $s_2 \xrightarrow{b} s_3$, $s'_3 \xrightarrow{b} s'_7 \Rightarrow s_3 \sim s'_7$

$s_6 \sim s'_4$, $s_6 \xrightarrow{c} s_7$, $s'_4 \xrightarrow{c} s'_2 \Rightarrow s_7 \sim s'_2$

$s_5 \sim s'_5$, $s_5 \xrightarrow{c} s_7$, $s'_5 \xrightarrow{c} s'_3 \Rightarrow s_7 \sim s'_3$

$s_5 \sim s'_6$, $s_5 \xrightarrow{c} s_7$, $s'_6 \xrightarrow{c} s'_2 \Rightarrow s_7 \sim s'_2$

$s_3 \sim s'_7$, $s_3 \xrightarrow{c} s_7$, $s'_7 \xrightarrow{c} s'_3 \Rightarrow s_7 \sim s'_3$

$s_7 \sim s'_2$, $s_7 \xrightarrow{a} s_5$, $s'_2 \xrightarrow{a} s'_4 \Rightarrow s_5 \sim s'_4$

$s_7 \sim s'_2$, $s_7 \xrightarrow{b} s_5$, $s'_2 \xrightarrow{b} s'_5 \Rightarrow s_5 \sim s'_5$

$s_7 \sim s'_3$, $s_7 \xrightarrow{a} s_5$, $s'_3 \xrightarrow{a} s'_6 \Rightarrow s_5 \sim s'_6$

$s_7 \sim s'_3$, $s_7 \xrightarrow{b} s_5$, $s'_3 \xrightarrow{b} s'_7 \Rightarrow s_5 \sim s'_7$

$s_5 \sim s'_4$, $s_5 \xrightarrow{c} s_7$, $s'_4 \xrightarrow{c} s'_2 \Rightarrow s_7 \sim s'_2$

$s_5 \sim s'_7$, $s_5 \xrightarrow{c} s_7$, $s'_7 \xrightarrow{c} s'_3 \Rightarrow s_7 \sim s'_3$

find non-repetitive ones

$\Rightarrow \mathcal{R} = \{ (s_1, s'_1), (s_4, s'_2), (s_2, s'_3), (s_6, s'_4), (s_5, s'_5)$
$(s_5, s'_6), (s_3, s'_7) (s_7, s'_2), (s_7, s'_3), (s_5, s'_4), (s_5, s'_7) \}$

EX 12.2

Quotient transition system

the equivalent class, (the partition given S and an equivalence relation)

Given T and a partition of S, the quotient transition system P14 of L12 ←

↙ write down

* Bisimulation quotient Algorithm (P16 of L12)

step 1. initialize $S/\sim = \{ \{ S \setminus S_F \}, S_F \}$ typo in P16?

step 2. loop. while $\exists P, P' \in S/\sim$, $\sigma \in \Sigma$ s.t. $\emptyset = P \cap Pre_\sigma(P') \neq P$

find the partition!

$P_1 = P \cap Pre_\sigma(P')$

$P_2 = P \setminus Pre_\sigma(P')$

$S/\sim = (S/\sim \setminus \{P\}) \cup \{P_1, P_2\}$

apply the algorithm / P_{8-9}, L9 remind

Step 1. initially $S/\sim = \{\{s_1, s_2, s_3, s_4, s_5, s_6\}, \{s_7, s_8\}\}$

Step 2. loop $\quad$ $Pre_\sigma(P)$ not properly defined?

iter1. choose $P = \{s_1, s_2, s_3, s_4, s_5, s_6\}$, $P' = \{s_7, s_8\}$

then $Pre_b(P') = \{s_3, s_5\}$, $P \cap Pre_b(P') = \{s_3, s_5\}$

thus $\phi \neq P \cap Pre_b(P') \neq P$

$\Rightarrow P_1 = \{s_3, s_5\}$, $P_2 = \{s_1, s_2, s_4, s_6\}$

$S/\sim = \{\{s_1, s_2, s_4, s_6\}, \{s_3, s_5\}, \{s_7, s_8\}\}$

iter2. choose $P = \{s_1, s_2, s_4, s_6\}$, $P' = \{s_3, s_5\}$

then $Pre_a(P') = \{s_1, s_2\}$, $P \cap Pre_a(P') = \{s_1, s_2\}$

thus $\phi \neq P \cap Pre_a(P') \neq P$

$\Rightarrow P_1 = \{s_1, s_2\}$, $P_2 = \{s_4, s_6\}$

$S/\sim = \{\{s_1, s_2\}, \{s_4, s_6\}, \{s_3, s_5\}, \{s_7, s_8\}\}$

iter3. choose $P = \{s_1, s_2\}$, $P' = \{s_3, s_5\}$

then $\underline{Pre_b(P') = \{s_1\}}$, $P \cap Pre_b(P') = \{s_1\}$

thus $\phi \neq P \cap Pre_b(P') \neq P$

$\Rightarrow P_1 = \{s_1\}$, $P_2 = \{s_2\}$

$S/\sim = \{\{s_1\}, \{s_2\}, \{s_4, s_6\}, \{s_3, s_5\}, \{s_7, s_8\}\}$

this result can be verified.

(draw the diagram)

EX 12.5

Given the hybrid automaton H, is it bisimilar to T_1, T_2, T_3?

- Hybrid automaton as transition systems $\quad$ P_{18} of L12
- time as the generator
- verification use reability algorithm of bisimilar quotient

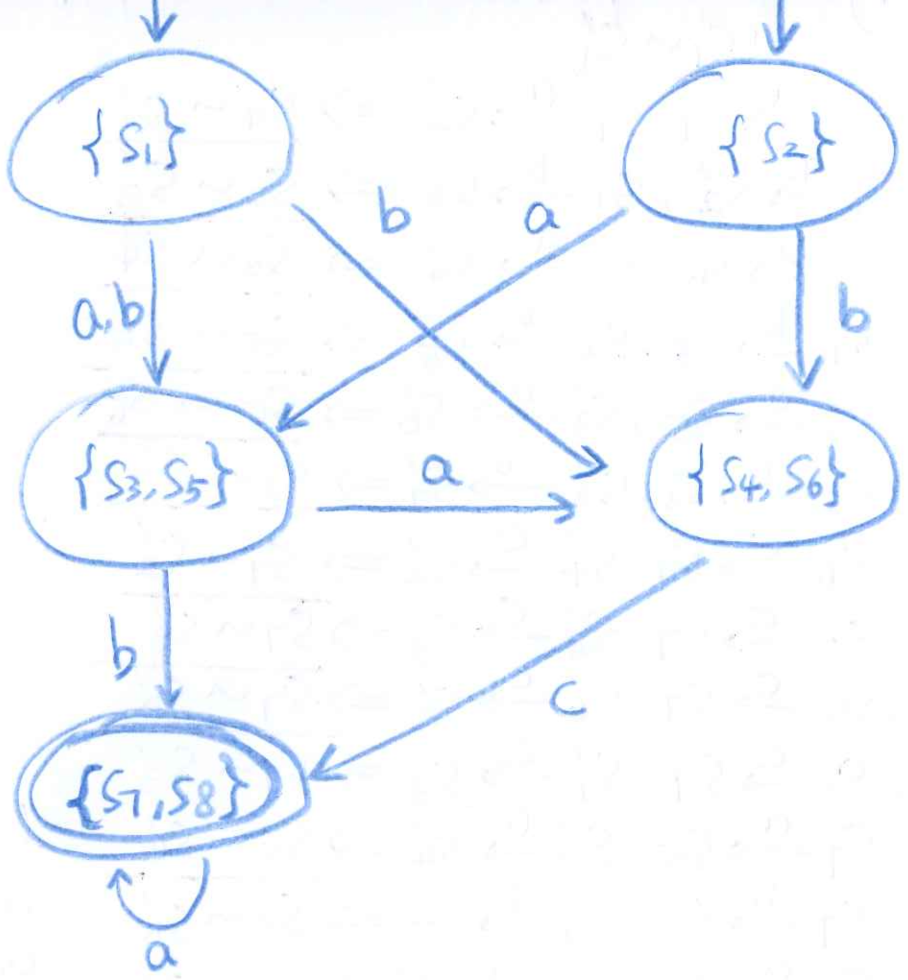
T_1 : not bisimilar, as "turn-right" and "turn-left" can't be immediate

T_2 : not, as the system can't follow "time + ~~driven~~ ~~forever~~ turn right + time + turn right".

T_3 : yes

T_4 : not, it only allows "two" time transition. (not properly defined? t is virtual / continuous anyway?)

(P_{22}, transition / reachability exists some "t")


{S1}
{S2}
a,b
b
a
b
{S3,S5}
a
{S4,S6}
b
c
{S7,S8}
a