Ex 11.10

Stability of switched system, common Lyapunov function

P19 of L11

Let $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ where $P_1, P_2 > 0$

for system 1

$$PA_1 + A_1^T P = \begin{bmatrix} -2P_1 & 0 \\ 0 & -4P_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$ namely $Q_1 = I$

if we choose $2P_1 = 1$, $4P_2 = 1 \Rightarrow P_1 = 1/2$, $P_2 = 1/4$

$\Rightarrow V(x) = x^T P x$ is a Lyapunov function for system 1

how about system 2?

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} = x^T (A_2 P + P A_2) x$$
$$= x^T \begin{bmatrix} -3 & 0 \\ 0 & -\frac{5}{2} \end{bmatrix} x \leq 0, \forall x$$

with equality only for $x = 0$. $\Rightarrow V(x)$ is also a Lyapunov function for system 2

Thus $V(x)$ is a Common Lyapunov function for system 1 and 2.

Ex 11.11

we consider a positive definite matrix P in the form

$$P = \begin{pmatrix} 1 & s \\ s & r \end{pmatrix}.$$

For system A_1, we require

$$A_1^T P + P A_1 < 0$$

$$\Rightarrow \begin{pmatrix} 2-2q & 2q+1-r \\ 2q+1-r & 2q+2r \end{pmatrix} > 0$$

$\Rightarrow$ ~~$q<1$~~

for 2x2 matrix

$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$, eigenvalue $(\lambda - a)(\lambda - c) - b^2 = 0$

$\lambda^2 - (a+c)\lambda + ac - b^2 = 0$

both positive if $a + c > 0$, $ac - b^2 > 0$

$D(q_1) = \{x \in \mathbb{R}^2 \mid x_1 \geq 0, x_2 \geq 0\}$

$D(q_2) = \{x \in \mathbb{R}^2 \mid x_1 < 0, x_2 \geq 0\}$

$D(q_3) = \{x \in \mathbb{R}^2 \mid x_1 < 0, x_2 < 0\}$

$D(q_4) = \{x \in \mathbb{R}^2 \mid x_1 \geq 0, x_2 < 0\}$

$R(q_i, q_j, x) = \{x\}$, put at last , $\forall (q_i, q_j) \in E$

$E = \{(q_1, q_4), (q_4, q_3), (q_3, q_2), (q_2, q_1)\}$

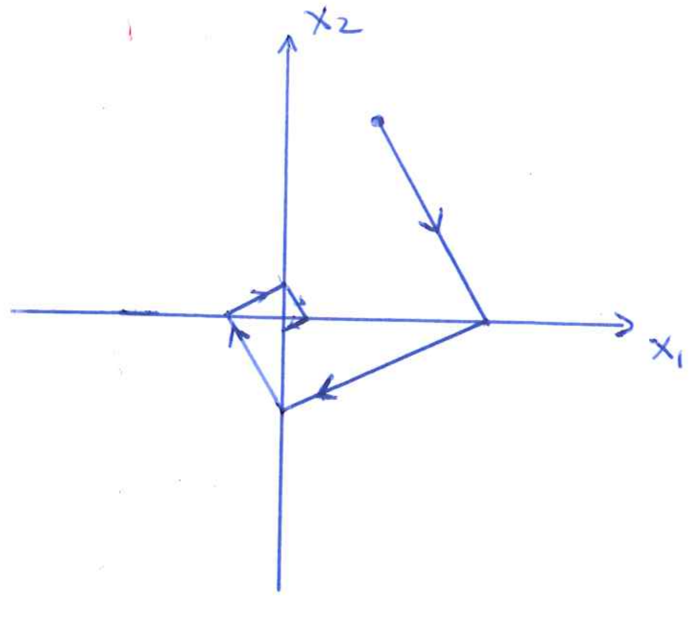
$G(q_1, q_4) = \{x \in \mathbb{R}^2 \mid x_2 < 0\}$

$G(q_4, q_3) = \{x \in \mathbb{R}^2 \mid x_1 < 0\}$

$G(q_3, q_2) = \{x \in \mathbb{R}^2 \mid x_2 \geq 0\}$

$G(q_2, q_1) = \{x \in \mathbb{R}^2 \mid x_1 \geq 0\}$

(b) looks like a "spiral" which moves towards the origin. The number of switches increases as we approach the origin.
$\Rightarrow$ infinite number of switches within finite time $\Rightarrow$ zeno behaviors



Ex. 11

11.4 , 11.5 , 11.10 , 11.11

* analyze stability of hybrid systems
* common Lyapunov function

Ex 11.4

Definition of stability   P8 of L11

Lyapunov's second method   P9 of L11

Consider the following Lyapunov function

$$V(x) = \frac{1}{2}x^T x = \frac{1}{2}(x_1^2 + x_2^2)$$

where $x = [x_1\ x_2]^T$.

$$V(0) = 0, \quad V(x) > 0, \ \forall x \in \mathbb{R}^2 \setminus \{0\}$$

$$\begin{aligned}
\dot{V}(x) &= x_1 \cdot \dot{x}_1 + x_2 \cdot \dot{x}_2 \\
&= x_1(-x_1 + g(x_2)) + x_2(-x_2 + h(x_1)) \\
&= -x_1^2 - x_2^2 + x_1 g(x_2) + x_2 h(x_1) \\
&\le -x_1^2 - x_2^2 + |x_1 g(x_2)| + |x_2 h(x_1)| \\
&= -x_1^2 - x_2^2 + |x_1||g(x_2)| + |x_2||h(x_1)| \\
&\le -x_1^2 - x_2^2 + \frac{1}{2}|x_1||x_2| + \frac{1}{2}|x_2||x_1| \\
&= -x_1^2 - x_2^2 + |x_1||x_2| \\
&\le -x_1^2 - x_2^2 + \frac{1}{2}(x_1^2 + x_2^2) = -\frac{1}{2}(x_1^2 + x_2^2)
\end{aligned}$$

which means $\dot{V}(x) \le 0, \ \forall x \in \mathbb{R}^2$

By Lyapunov's second method, $[0, 0]$ is stable

Ex 11.5   P3-4, L11

(a) Formally $H = (Q, X, Init, f, D, E, G, R)$ where

$Q = \{q_1, q_2, q_3, q_4\}$

$X = \mathbb{R}^2$

$Init = (q_0, x_{10}, x_{20})$. to be given

$f(q_1, x) = (1, -3)^T$

$f(q_2, x) = (3, 1)^T$

$f(q_3, x) = (-1, 3)^T$ $f(q_4, x) = (-3, -1)^T$

$$\begin{cases} 2+2r > 0 \\ (2q+1-r)^2 - (2-2q)(2q+2r) < 0 \end{cases}$$

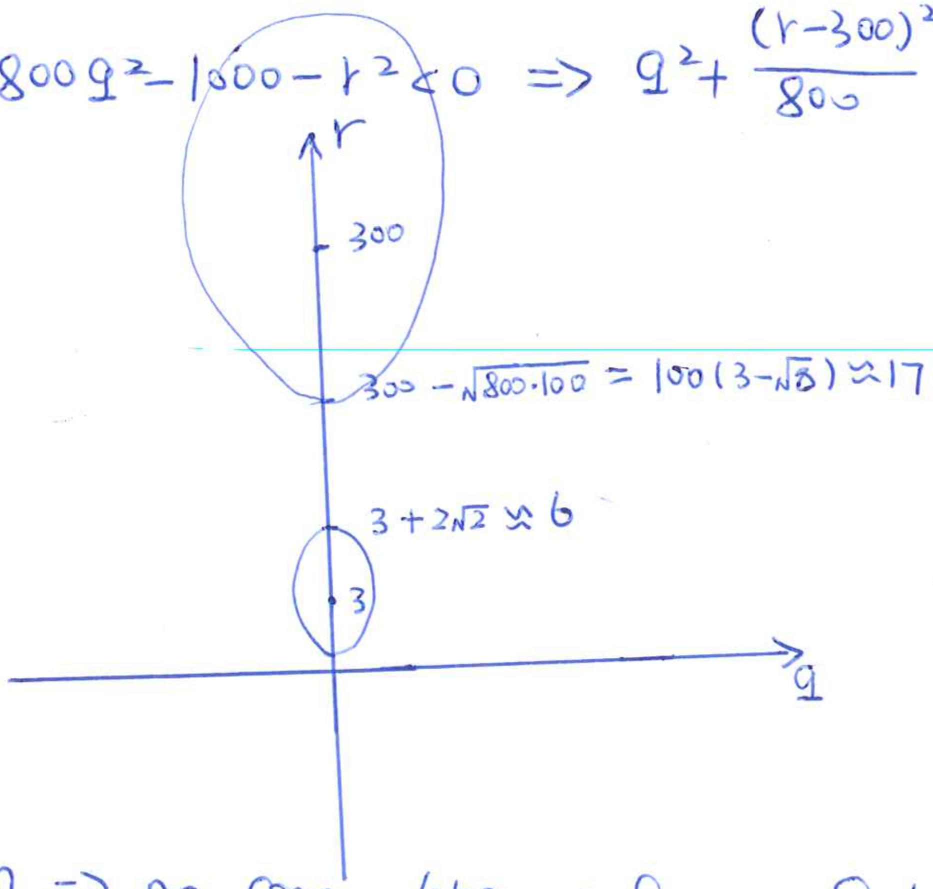
$$\Rightarrow \begin{cases} r > -1 \\ 8q^2+r^2-6r+1 < 0 \Rightarrow q^2 + \frac{(r-3)^2}{8} < 1 \end{cases}$$

For system 2, A_2. we require

$$A_2^T P + P A_2 < 0$$

$$\Rightarrow \begin{pmatrix} 2-\frac{1}{5}q & 2q+10-\frac{1}{10}r \\ 2q+10-\frac{r}{10} & 20q+2r \end{pmatrix} > 0$$

$$\Rightarrow \begin{cases} 10+99q+10r > 0 \\ 600r - 800q^2 - 1000 - r^2 < 0 \Rightarrow q^2 + \frac{(r-300)^2}{800} < 100 \end{cases}$$



NO intersection $\Rightarrow$ no common Lyapunov function for both systems