Ex 13 Hybrid system bisimulations

13.1 13.2

13.3
M33

key points
* bisimulations for timed automata
* bisimulation for multi-rate automata
* bisimulation for rectangular automata

extra notes

13.1

∘ Def: pre and post for hybrid automata P_{21} of L_{12}

$Pre(P) = \{(q_p, x_p) \mid \exists (q,x) \in P, (q_p, x_p) \xrightarrow{t} (q,x)$ or
Predecessor
$(q_p, x_p) \xrightarrow{e} (q,x)\}$

$Post(P) = \{(q_p, x_p) \mid \exists (q,x) \in P, (q,x) \xrightarrow{t} (q_p, x_p)$ or
successor
$(q,x) \xrightarrow{e} (q_p, x_p)\}$

(a) $Post(q_{off,off}, x=175, y \in [100,200])$
only time-driven transitions are available $(t=0)$
$= \{(q_{off,off}, x'_p, y'_p) \mid x'_p = 175 + 5t,\ y'_p = y - 18t,\ t \in [0,5]\}$

(b) $Post(q_{off,off}, x=200, y=200)$
at time $t=0$, only time-driven enabled $(q_{off,off}, x=200, y=200)$ why?
afterwards, an event-driven transition $(q_{off,on}, x=200, y=200)$
$= \{(q_{off,off}, x=200, y=200), (q_{off,on}, x=200, y=200)\}$

(c) $Pre(q_{on,on}, x=150, y=200)$

at $t=0$, the time-driven transition $(q_{on,on}, x=150, y=200)$

the event-driven transition $(q_{off,on}, x=150, y=200)$ error in the solution?

$= \{(q_{on,on}, x=150, y=200), (q_{off,on}, x=150, y=200)\}$

(d) reachable set from an initial state

definiton P22 of L12

$Post(q_{off,off}, x=190, y=200) = \{(q_{off,off}, 190+5t, 200-18t), t \in [0,2]\}$

where $y \in [164, 200]$, $x \in [190, 200]$

$Post(q_{off,off}, x=200, y=164) = \{(q_{off,on}, x=200, y=164), (q_{off,off} \cdots)\}$

$Post(q_{off,on}, x=200, y=164) = \{(q_{off,on}, 200-25t, 164+12t), t \in [0,2]\}$

where $y \in [164, 188]$, $x \in [150, 200]$

$Post(q_{off,on}, x=150, y=188) = \{(q_{off,on}, x=150, y=188), (q_{on,on}, \cdots)\}$

$Post(q_{on,on}, x=150, y=188) = \{(q_{on,on}, 150-5t, 188+12t), t \in [0,10]\}$

where $y \in [188, 308]$, $x \in [100, 150]$

$Post(q_{on,on}, x=100, y=308) = \{(q_{on,on}, 100, 308), (q_{on,off}, \cdots)\}$

$Post(q_{on,off}, x=100, y=308) = \{(q_{on,off}, 100+25t, 308-18t), t \in [0,3]\}$

where $y \in [254, 308]$, $x \in [100, 175]$

$Post(q_{on,off}, 175, 254) = \{(q_{on,off}, 175, 254), (q_{off,off}, 175, 254)\}$

$Post(q_{off}, 175, 254) = \{(q_{off,off}, 175+5t, 254-18t), t \in [0,5]\}$

$x \in [175, 200]$ $y \in [164, 254]$

$Post(q_{off,off}, \overset{200}{\cancel{190}}, 164) = \{(q_{off,off}, \overset{200}{\cancel{190}}, 164), (q_{off,on}, \overset{200}{\cancel{190}}, 164)\}$

once again reaches state $(q_{off,off}, 200, 164)$

As the system is deterministic, we have found all the reachable states.

(e) it can be solved by setting $190+5t = 200-18t \Rightarrow t = \frac{10}{23}$

error in the solution?

$175+5t = 254-18t \Rightarrow t = \frac{79}{23}$

13.3

multi-rate automata (P16-17 of L13)
rectangular automata (P18-19 of L13)

(a) def. rectangular automaton to be initialized if the rate of a variable changes along an edge

⇓

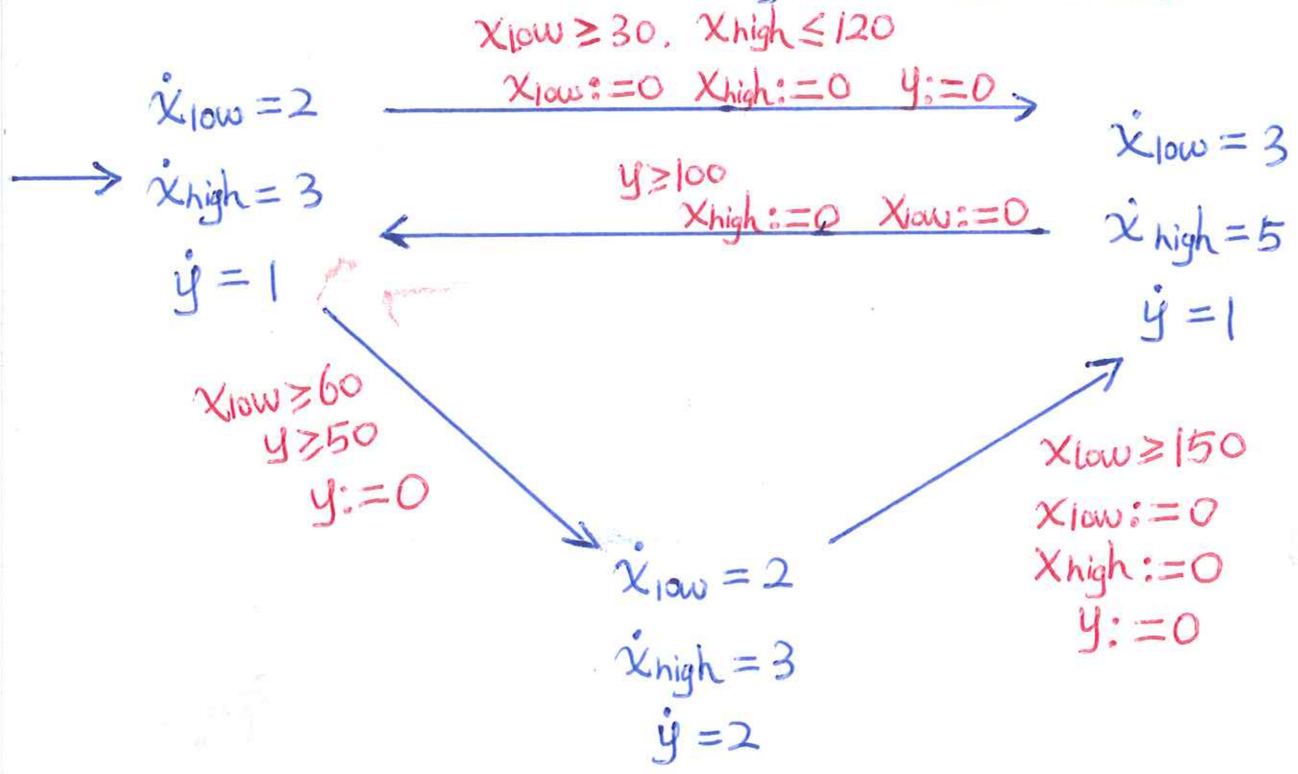
that variable has to be reset along that edge

Yes, for both x, y on all edges

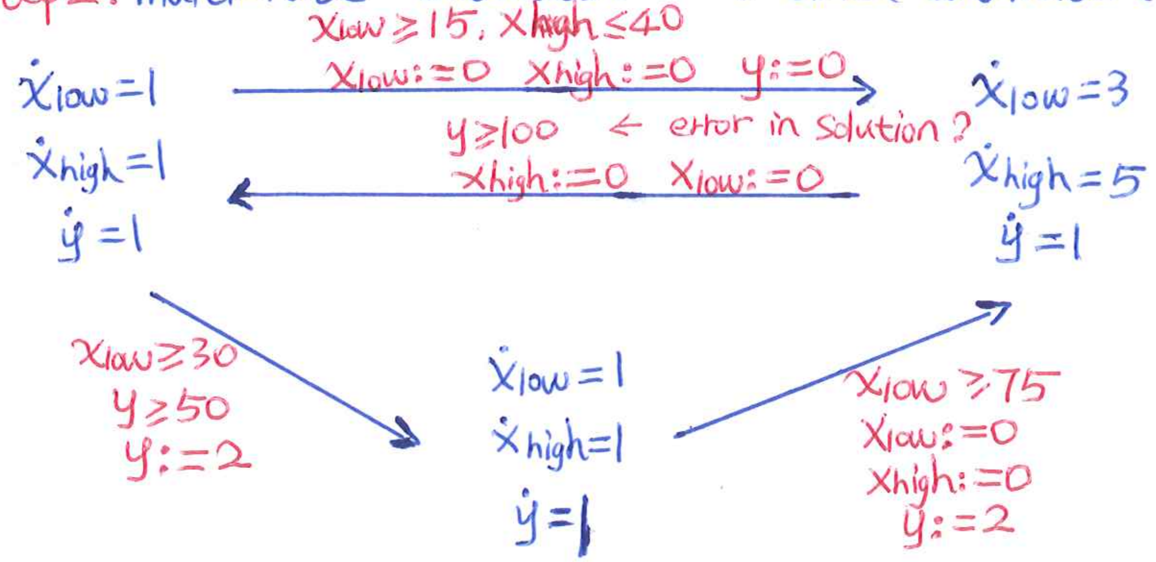
(b) translate it into a timed automaton

Step1. rectangular automaton ⟶ multi-rate automaton

add x_{low} and x_{high} as state variables



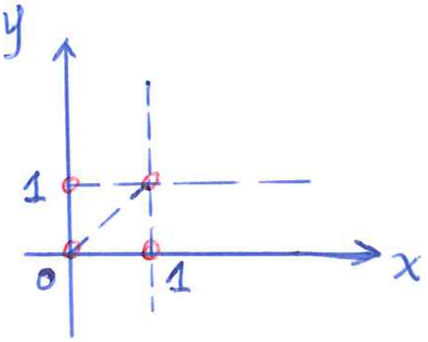
Step2. multi-rate automaton ⟶ timed automaton



13.2

region automaton / region graph P_{15} of L_{13}

region equivalence P_{13} of L_{13}



we have two state variables

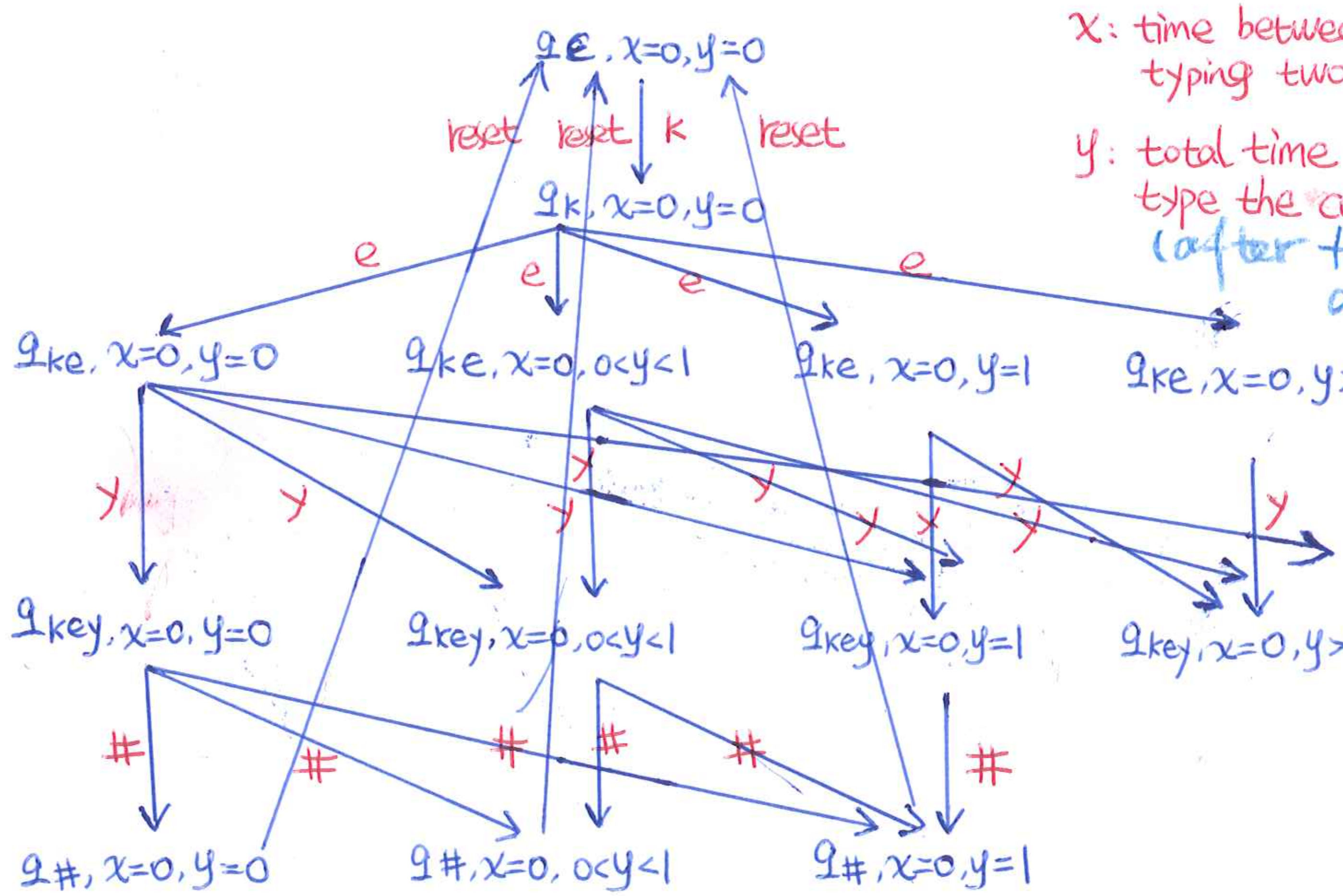
Note:

Timed automata, the clock dynamics $f(q, x) = (1, \cdots, 1)^T$ is often omited in the diagram

Equivalent classes/regions (P_{14} of L_{13}) ✓ no need to write the rest?

$\{x=0, y=0\}, \{x=0, y=1\}, \{x=0, 0<y<1\}$ $\{x=0, y>1\}$

(Timed automata $\xrightarrow{TA}$ labeled timed automata $\xrightarrow{TA}$ transition system T_{TA} of timed automata $\longrightarrow$ untimed transition system $[[TA]]$ $\longrightarrow$ bisimulation quotient for $[[TA]]$, $[[TA]]/\sim$, region graph



x: time between typing two alphabets

y: total time to type the code (after first code)