E9.

9.1 9.2
9.3 9.5
Q 31

* keys points from the lecture
  - transition system
  - reachability

* extra notes

E. 9.1

Consider a Discrete Event system described by an automaton and model it formally as a transition system

Def.

- automaton model: $A = (Q, E, \delta, q_0, Q_m)$ P6 of L8
- transition system: $T = (S, \Sigma, \rightarrow)$ P5 of L9

  states S. generators Σ (a special one time t)

  transition relation $\rightarrow \subset S \times \Sigma \times S$

  $s \xrightarrow{\sigma} s'$ or $(s, \sigma, s') \in \rightarrow$

essentially, they are equivalent if we add the starting states S_S and marked states S_F

"a transition system is just another formal way to describe a discrete event system"

E. 9.2

Model the basic functionalities of a keypad of a mobile phone including mainmenu, contacts and lock

Def. $T = (S, \Sigma, \longrightarrow)$

need to determine these elements

states $S = \{$ mainmenu, contacts, lockscreen $\}$ (display on the screen)

generator $\Sigma = \{$ unlock, choose_contact, exit, lock $\}$

transitions relation

lockscreen $\xrightarrow{\text{unlock}}$ mainmenu

lockscreen $\xrightarrow{\text{choose\_contact}}$ lockscreen (same for exit, lock)

mainmenu $\xrightarrow{\text{choose\_contact}}$ contacts

mainmenu $\xrightarrow{\text{lock}}$ lockscreen

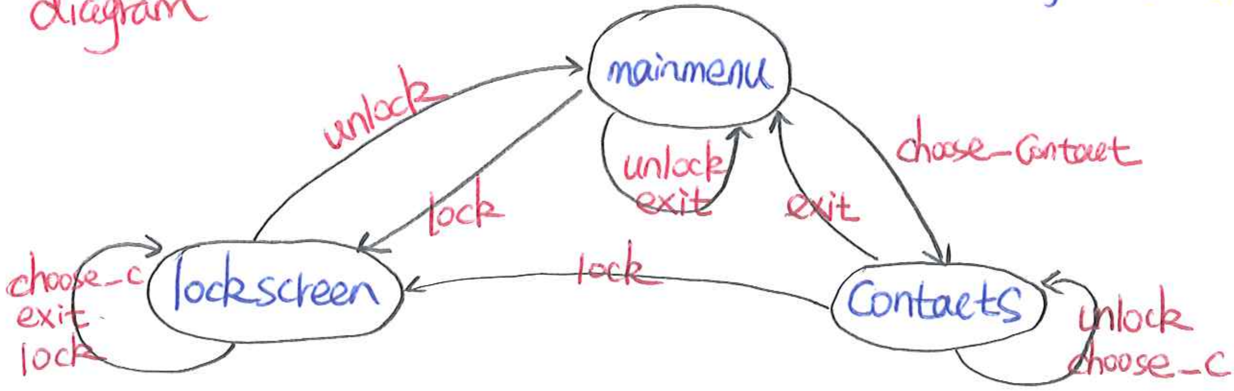
mainmenu $\xrightarrow{\text{unlock}}$ mainmenu (same for exit)

contacts $\xrightarrow{\text{lock}}$ lockscreen

contacts $\xrightarrow{\text{exit}}$ mainmenu

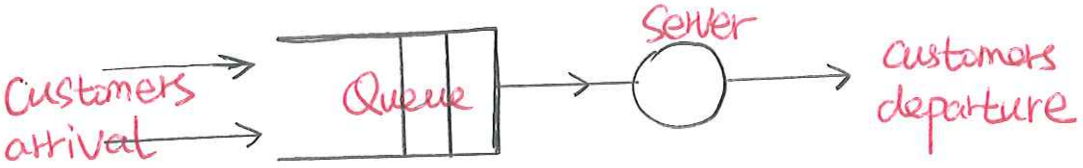
contacts $\xrightarrow{\text{unlock}}$ contacts (same for choose_contact)

diagram



E 9.3

Queuing system



model the queue system by a transition system

Def: $T = (S, \Sigma, \rightarrow)$

states $S = \{0, 1, 2 \cdots\}$, number of customers in the queue

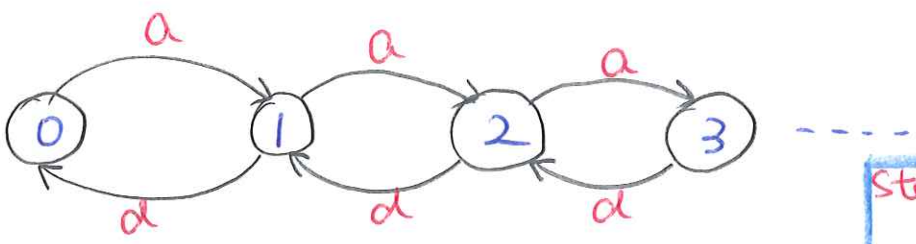
generators $\Sigma = \{\text{arrivial}, \text{departure}\}$

transition relation

arrival: $x \rightarrow x+1$, $x \xrightarrow{\text{arrival}} x+1$, $\forall x \in S$

departure: $x \rightarrow x-1$, $x \xrightarrow{\text{departure}} x-1$, $\forall x \in S \setminus \{0\}$

diagram:



$\boxed{\text{States: infinite but countable}}$

E 9.5

* reachable set. $T = (S, \Sigma, \rightarrow)$
  - Given a subset $P \subset S$, ~~then~~

Predecessor $Pre(P) = \{ s \in S : \exists \sigma \in \Sigma, \exists p \in P, s \xrightarrow{\sigma} p \}$

$Pre_\sigma(P) = \{ s \in S : \exists p \in P, s \xrightarrow{\sigma} p \}$

Successor $Post(P) = \{ s \in S : \exists \sigma \in \Sigma, \exists p \in P, p \xrightarrow{\sigma} s \}$

$Post_\sigma(P) = \{ s \in S : \exists p \in P, p \xrightarrow{\sigma} s \}$

recursively $Post^k(P) = Post(Post^{k-1}(P))$, $Post^0(P) = P$

$Pre^k(p) = Pre(Pre^{k-1}(p))$, $Pre^0(p) = p$

- reach set

P11 of L9

$Reach(S_0)$: set of states that can be reached from S_0.

$$Reach(S_0) = \bigcup_{k=0,1\ldots} Post^k(S_0)$$

- reachability algorithm   P14 of L9

$Reach_{-1} = \phi$, $Reach_0 = S_0$, $i=0$

while $Reach_i \neq Reach_{i-1}$ **do**

  $Reach_{i+1} = Reach_i \cup Post(Reach_i)$

  $i = i+1$

end

after termination $\Rightarrow Reach(S_0) = Reach_i$

for finite transition system, algorithm terminates in a finite number of steps.

extra notes

Given the transition system in Fig 9.5

① $S_S = \{3\}$

run the algorithm virtually

$Reach_{-1} = \phi$, $Reach_0 = \{3\}$, $i=0$

iter. 1. $Reach_1 = Reach_0 \cup Post(Reach_0)$

$= \{3\} \cup \underbrace{Post(\{3\})}_{=\{1,5,6\}}$

$= \{1,3,5,6\}$

Since $Reach_1 \neq Reach_0$

iter. 2 $Reach_2 = Reach_1 \cup Post(Reach_1)$

$= \{1,3,5,6\} \cup Post(\{1,3,5,6\})$

$Post(\{1,3,5,6\}) = \{2,3\} \cup \{1,5,6\} \cup \{\phi\} \cup \{\phi\}$

$= \{1,2,3,5,6\}$

$Reach_2 = \{1,2,3,5,6\}$

since $Reach_2 \neq Reach_1$

iter 3. $Reach_3 = Reach_2 \cup Post(Reach_2)$

$= \{1,2,3,5,6\} \cup Post(\{1,2,3,5,6\})$

$Post(\{1,2,3,5,6\}) = \{2,3\} \cup \{4,5\} \cup \{5,6\} \cup \{\phi\} \cup \{\phi\}$

$= \{2,3,4,5,6\}$

$Reach_3 = \{1,2,3,4,5,6\}$

since $Reach_3 \neq Reach_2$

iter. 4. $Reach_4 = Reach_3 \cup Post(Reach_3)$

$= \{1,2,3,4,5,6\} \cup \{1,2,3,4,5,6\}$

$= \{1,2,3,4,5,6\}$

since $Reach_4 = Reach_3$ terminates $\Rightarrow Reach(\{3\}) = \{1,2,3,4,5,6\}$

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② $S_s = \{2\}$

run the algorithm virtually

$Reach_{-1} = \phi$, $Reach_0 = \{2\}$, $i=0$

iter. 1. $Reach_1 = Reach_0 \cup Post(Reach_0)$

$= \{2\} \cup \{4,5\} = \{2,4,5\}$

since $Reach_1 \neq Reach_0$

iter. 2. $Reach_2 = Reach_1 \cup Post(Reach_1)$

$= \{2,4,5\} \cup \{4,5\}$

$= \{2,4,5\}$

since $Reach_2 = Reach_1$ terminates $\Rightarrow Reach(\{2\}) = \{2,4,5\}$