Problem 4 Ex10.6

Problem

Ex.7

Three periodic control tasks A, B and C are executed on a CPU. The tasks have the following characteristics:

Task	T_i	D_i	C_i
A	4	4	1
B	5	5	2
C	x	x	3

(a) Suppose that all of the CPU capability is available for A, B and C. What is the minimum task period x for the task C to guarantee that all tasks A, B and C are schedulable under earliest deadline first (EDF) scheduling? **[2p]**

Ex7.3

(b) Suppose $x = 10$. Try to schedule the above tasks using earliest deadline first (EDF) scheduling algorithm, and verify your result from (a). What is the worst response time for control task A, B and C ? **[3 p]**

(c) Suppose $x = 10$, again. Are the tasks A, B and C schedulable under rate monotonic (RM) scheduling? Try to answer by computing the worst response time for control task A, B and C. **[3 p]** Ex 7.4

~~(d)~~ Comment on your results from (b) and (c). **[2 p]**

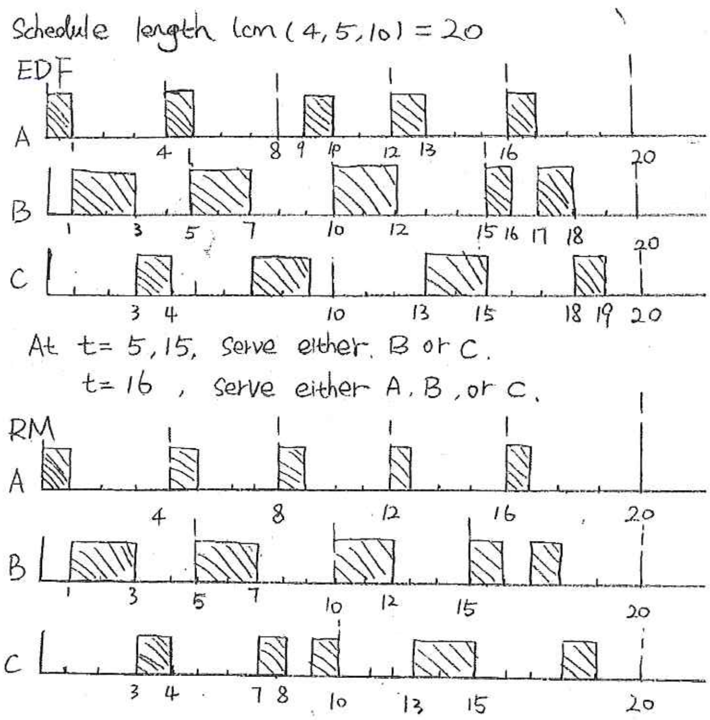
Solution

(a) In the case where EDF is used, the schedulability bound is one (sufficient and necessary). So

$$U = \sum_{i=1}^{3} \frac{C_i}{T_i} = \frac{1}{4} + \frac{2}{5} + \frac{3}{x} \leq 1 \Rightarrow x \geq \frac{60}{7},$$

which gives the minimal task period for for task C is $x = \frac{60}{7} \approx 8.57$.

(b) Tasks A, B and C should be schedulable by EDF as $10 > \frac{60}{7}$ from (a). The schedule length is given by $lcm(T_A, T_B, T_C) = lcm(4, 5, 10) = 20$. Below is the corresponding schedule:



Problem 1

V22

Consider the model of normalized DC motor

$$G(s)=\frac{1}{s(s+1)}$$

(a) find the state-space representation of $G(s)$

(b) Sample $G(s)$ with sampling period $h=\ln 2 \approx 0.7$

(c) Determine the linear state-feedback controller

$$u(k)=[-l_1 \quad -l_2]\,x(k)$$

such that the characteristic polynomial of the closed-loop system is $z^2-0.6z+0.3=0$

(d) Design a full-state observer for the state based on the output.

Solution

(a) $G(s)=\frac{1}{s^2+s}$, the ~~observable~~ canonical form E4.1

$$\dot{x}_1=\underbrace{\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}}_{\triangleq A}x+\underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{\triangleq B}u$$

$$y=\underbrace{[1 \quad 0]}_{\triangleq C}x$$

(b) Sample the system by $h=\ln 2$ E 2.1

$$x(k)=\underbrace{e^{Ah}}_{\triangleq\Phi}x(k-1)+\underbrace{\int_0^h e^{As}ds\cdot B}_{\triangleq\Gamma}\,u(k-1)$$

$$y(k)=C\,x(k)$$

where E 1.2

$$e^{Ah}=\mathcal{L}^{-1}\left\{\begin{bmatrix}s & -1\\ 0 & s+1\end{bmatrix}\right\}\Bigg|_{t=h}=\frac{1}{s(s+1)}\begin{bmatrix}s+1 & 1\\ 0 & s\end{bmatrix}$$

$\frac{1}{s}-\frac{1}{s+1}$

$$=\begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+1)}\\ 0 & \frac{1}{s+1}\end{bmatrix}\ \overset{\mathcal{L}^{-1}}{\frown}\ \begin{bmatrix}1 & 1-e^{-t}\\ 0 & e^{-t}\end{bmatrix}$$

$$\left(1 \overset{\mathcal{L}}{\frown} \frac{1}{s},\quad e^{at}\overset{\mathcal{L}}{\frown}\frac{1}{s-a}\right)$$

Let $t=\ln 2 \implies e^{Ah}=\begin{bmatrix}1 & 0.5\\ 0 & 0.5\end{bmatrix}$

$$\int_0^h e^{As}ds = \int_0^h \begin{bmatrix} 1 & 1-e^{-\tau} \\ 0 & e^{-\tau} \end{bmatrix} d\tau = \begin{bmatrix} h & h-1+e^{-h} \\ 0 & 1-e^{-h} \end{bmatrix}$$

$\int_0^h 1\,d\tau = h$, $\int_0^h (1-e^{-\tau})d\tau = h - \int_0^h e^{-\tau}d\tau$

$= h-(-e^{-\tau})\big|_0^h = h-(1-e^{-h})$

Let $h = \ln 2 \approx 0.7 \Rightarrow \begin{bmatrix} 0.7 & 0.2 \\ 0 & 0.5 \end{bmatrix}$

$\Rightarrow$ state-space model

$$x(k) = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix} x(k-1) + \begin{bmatrix} 0.7 & 0.2 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k-1)$$

$$= \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix} x(k-1) + \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} u(k-1)$$

$$y(k) = [1 \quad 0]\, x(k)$$

(c) the closed-loop system is obtained by

$$u(k-1) = [-l_1 \quad -l_2]\, x(k-1)$$

$$\Rightarrow x(k) = \underbrace{\begin{bmatrix} 1-0.2l_1 & 0.5-0.2l_2 \\ -0.5l_1 & 0.5-0.5l_2 \end{bmatrix}}_{\triangleq G} x(k-1)$$

Thus the characteteristic polynomial is given by

$$\underbrace{z^2 + (\underbrace{\tfrac{l_1}{2} + \tfrac{l_2}{5} - \tfrac{3}{2}}_{=0.6})z + (\underbrace{\tfrac{3l_2}{20} - \tfrac{l_1}{2} + \tfrac{1}{2}}_{=0.3})}_{=\det(zI-G)} = 0$$

implies $l_1 = 1$, $l_2 = 2$

(d) Full-order observer: E 3.11 (b)

The estimation error

$$\tilde{x}(k+1|k) = (\Phi - KC)\,\hat{x}(k|k-1)$$

Let $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$

$$\Phi - KC = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-k_1 & 0.5 \\ -k_2 & 0.5 \end{bmatrix}$$

the characteristic equation is given by

$$z^2 + (k_1 - 1.5)z + 0.5 - 0.5k_1 + 0.5k_2 = 0$$

If we set $p_1 = p_2 = 0$ (deadbeat observer)

$$\begin{matrix} k_1 - 1.5 = 0 \\ 0.5 - 0.5k_1 + 0.5k_2 = 0 \end{matrix} \Rightarrow \begin{matrix} k_1 = 1.5 \\ k_2 = 0.5 \end{matrix}$$

Thus the observer is given by

$$\hat{x}(k+1|k) = \Phi\, x(k|k-1) + \Gamma u(k) + K[y(k) - C\hat{x}(k|k-1)]$$

As a result, V can serve as a common Lyapunov function by choosing $d>0$. Thus the system is globally asympotically stable

(b)

$$A_3 = \begin{bmatrix} -3 & 1 \\ k_1-1 & 3+k_2 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -1 & 3 \\ k_1-3 & k_2-5 \end{bmatrix}$$

$$V = x^T \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} x$$

$$\Rightarrow A_3^T P + P A_3 = \begin{bmatrix} -6d_1 & d_1+(k_1-1)d_2 \\ d_1+(k_1-1)d_2 & 2(3+k_2)d_2 \end{bmatrix}$$

$$A_4^T P + P A_4 = \begin{bmatrix} -2d_1 & 3d_1+(k_1-3)d_2 \\ 3d_1+(k_1-3)d_2 & 2(k_2-5)d_2 \end{bmatrix}$$

If $d_1=d_2=d>0$, $k_1=0$, $k_2<-3$, then it serves as a global Lyapunov function.

Problem 2

EX 11

Consider the following switched system with $i \in \{1,2\}$

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ -2 & -1 \end{bmatrix} x(t) + \lambda_i \begin{bmatrix} 1 & 1 \\ -1 & -4 \end{bmatrix} x(t) + b\, u(t)$$

a) show that the system is asymptotically stable when $\lambda_1 = 0$, $\lambda_2 = 1$ and $b = 0$

~~b)~~ design feedback controller (no time)

$$u(t) = [k_1, \ k_2]\, x(t)$$

which stabilizes the system when $\lambda_1 = -1$, $\lambda_2 = 1$, $b = [0 \ 1]^T$

Solution

(a) when $b = 0$ and $\lambda_1 = 0$

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ -2 & -1 \end{bmatrix} x = A_0 x$$

when $\lambda_i = \lambda_2 = 1$

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ -1 & -4 \end{bmatrix} x$$

$$= \begin{bmatrix} -1 & 3 \\ -3 & -5 \end{bmatrix} x = A_1 x$$

In order to identify a global Lyapunov function, we assume a quadratic form

$$V = x^T \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} x = x^T P x$$

By requiring $A_0^T P + P A_0$ be negative definite,

$$A_0^T P + P A_0 = \begin{bmatrix} -4d & 0 \\ 0 & -2d \end{bmatrix} < 0 \Rightarrow d > 0$$

By requiring $A_1^T P + P A_1$ be negative definite,

$$A_1^T P + P A_1 = \begin{bmatrix} -2d & 0 \\ 0 & -10d \end{bmatrix} < 0 \Rightarrow d > 0$$