E.5

5.1, 5.2, 5.3

* key points from Lecture
  - modes for delay, jitter and loss
  - compensation for known and unknown delays
  - identify and select appropriate controller realization
  - quantization

* extra notes

James Taylor
jamest@kth.se
laperss@kth.se

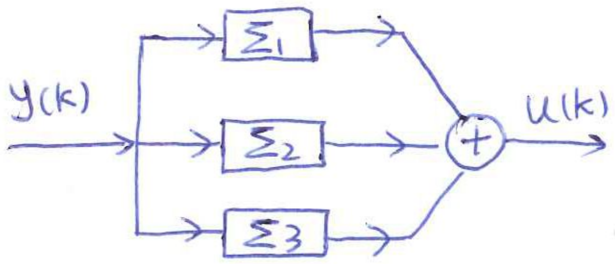
E 5.1

* parallel form

motive: control functions that can only handle $\boxed{\text{lower-order}}$ difference equations

* Robust to pertubation in parameter and states
* parallel form implementation, as a cascade of first and second-order filters

* extra notes



implementation by

- direct Form
- canonical form $\begin{cases} \text{controllable} \\ \text{observable} \end{cases}$

Both sensitive

P25-29, L5

$$H(z) = \frac{1}{(z-1)(z-1/2)(z^2+1/2z+1/4)} \qquad (z+\tfrac{1}{4})^2+\tfrac{3}{16}$$

$$= \underbrace{\frac{A}{z-1}}_{\triangleq u_1(z)} + \underbrace{\frac{B}{z-1/2}}_{\triangleq u_2(z)} + \underbrace{\frac{Cz+D}{z^2+1/2z+1/4}}_{\triangleq u_3(z)}$$

$\Rightarrow A = 1.14,\ B = -2.67,\ C = 1.52,\ D = 0.95$

Implementation of three parts (see E4.1 (b) for canonical representation)

$$\frac{A}{z-1}: \quad \boxed{\begin{aligned} x_1(k+1) &= x_1(k) + y(k) \\ u_1(k) &= 1.14\, x_1(k) \end{aligned}} \qquad \left(\text{Same for } \frac{B}{z-1/2}\right)$$

$$\frac{cz+D}{z^2+1/2z+1/4}: \quad \boxed{\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= -\tfrac{1}{4}x_1(k) - \tfrac{1}{2}x_2(k) + y(k) \\ u_3(k) &= C\,x_2(k) + D\,x_1(k) \end{aligned}}$$

$u(k) = u_1(k) + u_2(k) + u_3(k)$

nsider • first order term

$$u(z) = \frac{\beta_i}{z-\lambda_i} Y(z)$$ distinct poles

$$\Rightarrow \quad u(k+1) = \lambda_i u(k) + \beta_i y(k)$$

• Second order term complex pairs

$$u(z) = \frac{a_1 z + a_2}{\underbrace{z^2+\beta_1 z+\beta_2}_{=(z-\sigma_1)^2+w_1^2}} Y(z)$$

$$= \frac{a_1 z + a_2}{[z-(\sigma_1+w_1 j)][z-(\sigma_1 - w_1 j)]} Y(z)$$

$$\begin{bmatrix} v_1(k+1) \\ v_2(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_1 & w_1 \\ -w_1 & \sigma_1 \end{bmatrix}}_{=\Phi} \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} + \underbrace{\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}}_{=\Gamma} y(k)$$

$$u(k) = \underbrace{[\, c_1 \quad c_2 \,]}_{=C} \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}$$

$$\Rightarrow \quad u(z) = [\, c_1 \quad c_2 \,] \left[z I_2 - \Phi \right]^{-1} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} y(z)$$

$$= [\, c_1 \quad c_2 \,] \frac{1}{(z-\sigma_1)^2+w_1^2} \begin{bmatrix} z-\sigma_1 & w_1 \\ -w_1 & z-\sigma_1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} y(z)$$

Set

$$[\, c_1 \quad c_2 \,] \begin{bmatrix} \dfrac{r_1 z - r_1\sigma_1 + w_1 r_2}{(z-\sigma_1)^2+w_1^2} \\ \dfrac{r_2 z - r_2\sigma_1 - r_1 w_1}{(z-\sigma_1)^2+w_1^2} \end{bmatrix} y(z)$$

if $c_2 = 0$

$$\begin{cases} c_1 r_1 = a_1 \\ (w_1 r_2 - r_1\sigma_1) c_1 = a_2 \end{cases}$$

$$// \; a_1 z + a_2$$

$$= \frac{(c_1 r_1 + r_2 c_2) z + (w_1 r_2 - r_1 \sigma_1) c_1 - (r_2\sigma_1 + r_1 w_1) c_2}{(z-\sigma_1)^2 + w_1^2}$$

$$H = \frac{1.52 z + 0.95}{z^2 + \frac{1}{2} z + \frac{1}{4}}$$

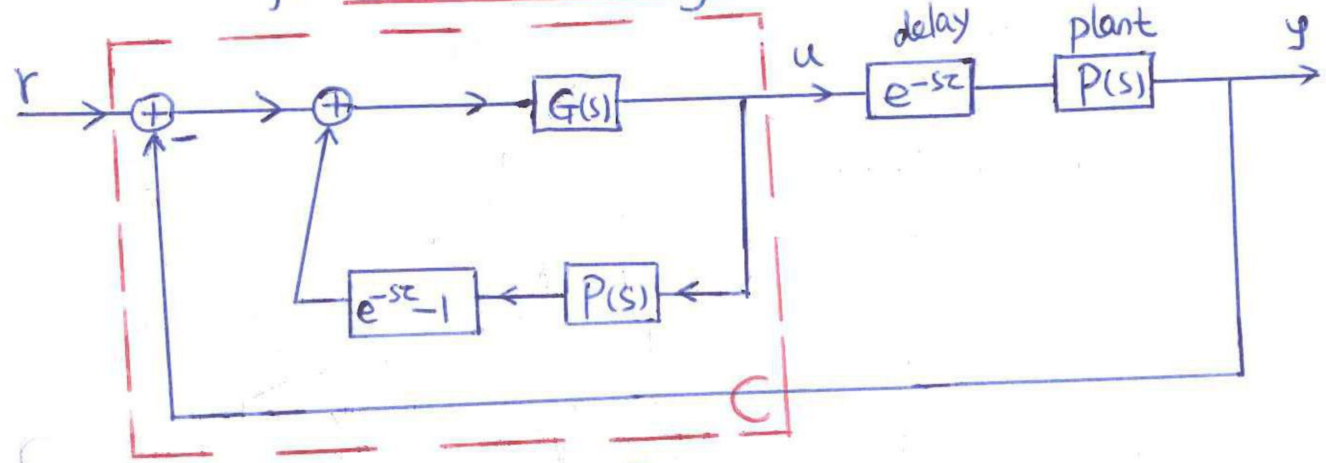
$$\begin{bmatrix} S_3(k+1) \\ S_4(k+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} S_3(k) \\ S_4(k) \end{bmatrix} + \begin{bmatrix} 1.756 \\ 1.521 \end{bmatrix} y(k)$$

$$u(k) = \begin{bmatrix} 0.8677 & 0 \end{bmatrix} \begin{bmatrix} S_3(k) \\ S_4(k) \end{bmatrix}$$

E5.2

* Smith predictor

for known time delays



$$C(s) = \frac{G(s)}{1 - P(s)\,G(s)\,(e^{-s\tau}-1)}$$

* extra notes

(a) $C_s(s) \Rightarrow H_{cl}(s) = \dfrac{C_s(s)P(s)e^{-s\tau}}{1+C_s(s)P(s)e^{-s\tau}} = \dfrac{C(s)P(s)}{1+C(s)P(s)}e^{-s\tau}$

$\Rightarrow C_s(s) = \dfrac{C(s)}{1+C(s)P(s)-C(s)P(s)e^{-s\tau}}$ ← how to derive Smith predicator

diagram see above.

(b) $P(s) = \dfrac{1}{s+1}$, $H_{cl}(s) = \dfrac{8}{s^2+4s+8}e^{-s\tau} = \dfrac{C(s)P(s)}{1+C(s)P(s)}e^{-s\tau}$

$\Rightarrow C(s) = \dfrac{8s+8}{s^2+4s}$

$\Rightarrow C_s(s) = \dfrac{8(s+1)}{s^2+4s+8(1-e^{-s\tau})}$

E5.3

$P_{7-9}, L5$

☆ time delay & stability

① CTS with phase margin φ_m at w_c, then the maximum (fixed) time delay:

$$\tau < \varphi_m / w_c$$

(Nyquist criterion)

② CTS with unknown time-varying delay, $0 < \tau(t) < \tau_{max}$

if

$$\left| \frac{P(jw)\,C(jw)}{1+P(jw)\,C(jw)} \right| < \frac{1}{\tau_{max} w}, \quad \forall w \in [0, \infty)$$

sufficient condition

(small-gain theorem)

③ DTS with unknown time-varying delay, $0 < \tau(t) < N \cdot h$

if

$$\left| \frac{P(e^{jwh})\,C(e^{jwh})}{1+P(e^{jwh})\,C(e^{jwh})} \right| < \frac{1}{N|e^{jwh}-1|}, \quad \forall w \in [0, 2\pi]$$

☆ extra notes

$$\left|\frac{1}{e^{jwh}-0.5}\right| |e^{jw}-1|$$

$$= \frac{\sqrt{(\cos\theta-1)^2+\sin^2\theta}}{\sqrt{(\cos\theta-0.5)^2+\sin^2\theta}} \leq 2 \quad (\theta = 0)$$

h?

Proof see Paper 3 of P.h.D thesis [L]

Very interesting to construct the transformed system

☆ apply the criterion ③,

closed-loop system without delay

$$H_{cl}(z) = \frac{P(z)\,C(z)}{1+P(z)\,C(z)} = \frac{z\,[(k_p+k_i)z - k_p]}{(z-0.5)(z-1) + (k_p+k_i)z^2 - k_p z}$$

$$= \frac{0.3z^2 - 0.2z}{1.3z^2 - 1.7z + 0.5}$$

(pole 0.86, 0.44) stable

$\Rightarrow$ plot $|H_{cl}(e^{jwh})| \cdot |e^{jwh}-1| < \frac{1}{N}$

explicit calculation hard even for low-order system

$e^{jwh} = \cos\theta + \sin\theta j$

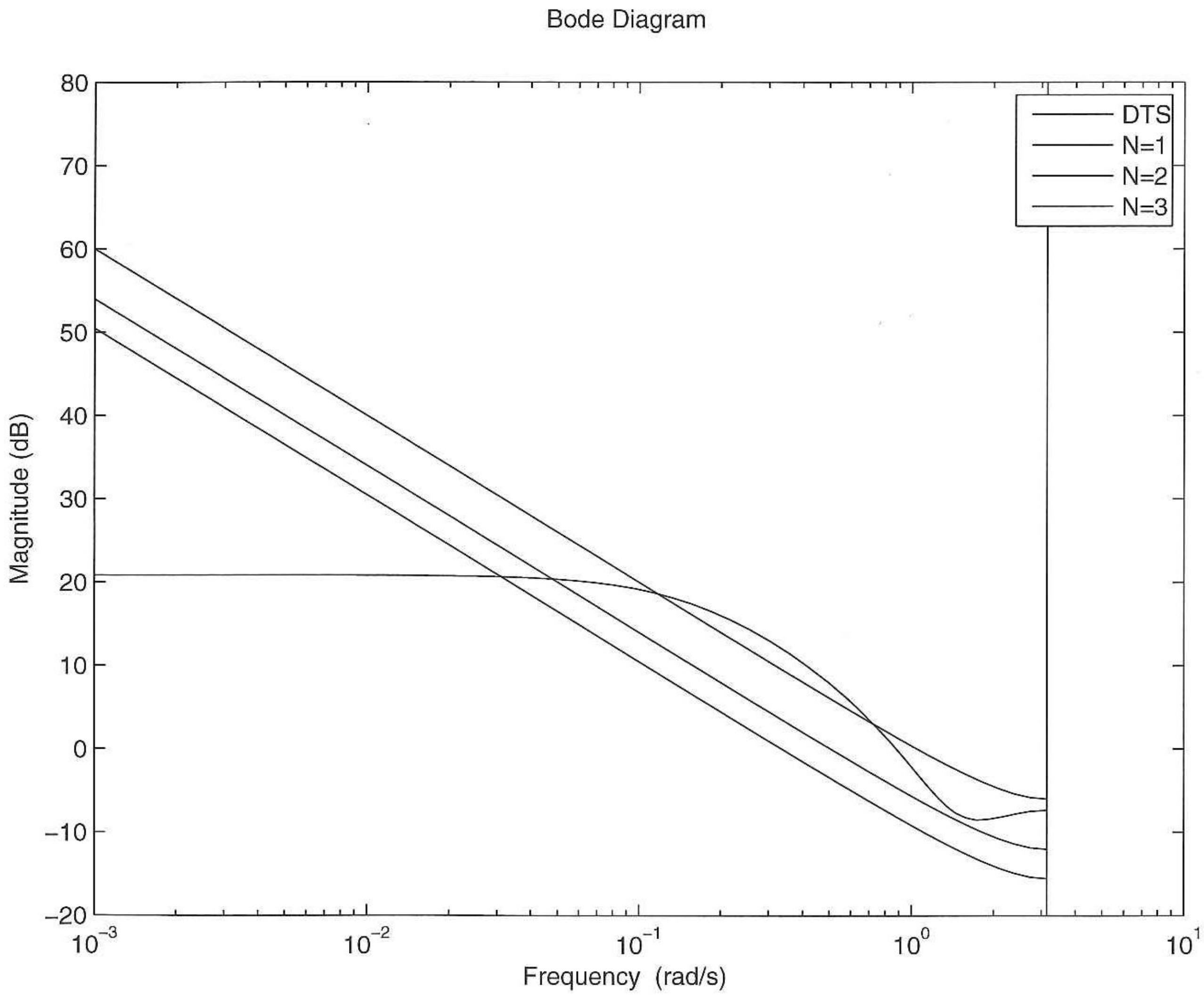
find upper-band

h=1 ?

$\boxed{\cancel{0.7832}}$ ✓ ≈ 1.003, N=1

(matlab function bode)

```
% E 5.3
ts=1; %100
DTS = tf([0.3 -0.2 1], [1.3 -1.7 0.5], ts);
delay1 = tf([1], 1*[1 -1], ts);
delay2 = tf([1], 2*[1 -1], ts);
delay3 = tf([1], 3*[1 -1], ts);
figure(1)
bodemag(DTS,'b',delay1,'r',delay2,'k',delay3,'c')
legend('DTS','N=1','N=2','N=3')
```



Bode Diagram
80
70
60
50
40
30
20
10
0
−10
−20
Magnitude (dB)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)
DTS
N=1
N=2
N=3