RETIREMENT SOLUTION PROPOSAL

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Background

Due to the weakening state of the pension systems, the shift from defined-benefit to defined-contribution schemes in the corporate world and the emergence of individual retirement accounts, individual investors need to pay more and more attention to the retirement risks.

Objective

Not only satisfying the kind of minimum replacement income in retirement needs, we are aiming at a relatively high probability for the investors to achieve target levels of replacement income.

Solution construction

GOAL-BASED-INVESTING Strategy

A combination of a well-diversified **performance-seeking portfolio (PSP)**, which targets to secure the essential goal, and **goal-hedging portfolio (GHP)**, which takes care of the aspirational goal.

Benchmark benefits

Less maximum drawdown and more excess return over a standard cap-weighted index

Improved Goal-Based Retirement Solution

Introduction

As the weakening state of the pension systems, the shift from defined-benefit to defined-contribution schemes in the corporate world and the emergence of individual retirement accounts, individual investors need to pay more and more attention to the retirement risks.

In current practice, our department, the solution development department, has proposed the Goal-Based-Investing strategy for clients. Three risk management techniques that have been introduced in constructing our retirement solution notably as hedging, diversification and insurance.

Target investors

Our target investors are non-high net worth individuals with a financing consumption in retirement.

The initial wealth is how much the investors are supposed to save for the retirement, which is $100000 in our simulation. The income frequency is annually. Then, the duration of liability portfolio is 18.93 years, which is between 10 years and 30 years. And the Maximum Affordable Income is 5945, which means the investor is supposed to get $5945 per year after retirement. Thus, the essential goal EG (protection of say 80% of the initial funding ratio) is $4756 here, and aspirational goal AG (say 130% or 150% of the initial funding ratio) are $7729 and $8918 accordingly. We set cost of live adjusted as 0, which ignores the wealth impact of other factors. Hereafter is the table of details.

Type of goal	**Income Based**
Accumulation Period	10 years
Decumulation Period	20 years
Initial Wealth	100,000
Horizon Date	21/03/30
Cost of Live Adjusted	0
PSP Stochastic Volatility	Yes
PSP Stochastic Sharpe Ratio	Yes
LHP Strategy	Perfect GHP

Figure 1: Initial Factors of target investors

Goals classification

Our goal is not only to secure the minimum replacement income needed to meet our target customers' essential consumption goals in retirement, but also generating a relatively high probability to achieve target levels of replacement income that would allow them to reach their aspirational consumption goals.

The essential goal is typically to protect some fraction (e.g., 80%) of the replacement income that is affordable at date 0, while the aspirational goal is to reach some multiple (e.g., 130% or 150%) of this initial value.

The funding ratio is defined as the currently affordable replacement income divided by the initially affordable one (so it is by definition 100% at date 0). Hence in the example above the essential goal can be equivalently viewed as the objective to keep the funding ratio above 80%, and the aspirational goal is to reach a funding level of 130%.

For the given level of EG and AG, the optimal choice for the multiplier can be get by Monte-Carlo scenarios. The optimal multiplier is defined as a multiplier that could 100% protect the Essential Goal (EG) and achieve the Aspirational Goal (AG) as much as possible.

Solution architecture

- **Goal based investing (GBI) strategy construction**

Investor's welfare can be measured through a quadratic utility function of the final funding ratio. This measure aggregates the preference for high funding levels and aversion for uncertainty over the funding ratio.

Supported by portfolio theory, the GBI strategy recommends that investors should in general hold a combination of a well-diversified performance-seeking portfolio (PSP), which targets diversification, and goal-hedging portfolio (GHP), which takes care of hedging.

- **Goal hedging portfolio design**

In GBI strategy, goal-hedging portfolio (GHP) is defined as the cheapest portfolio that secures the goal, which requires the lowest amount of initial wealth. The perfect GHP strategy can be replaced by a Fixed Mixed Bond Portfolio, a Dynamic Duration Matching method or a Dynamic Duration/Convexity Matching method. In the base case simulations, we chose perfect GHP, which is 50% cash and 50% 10-year ZC Bond. And, in Perfect GHP strategy, the funding ratio volatility, which is the volatility of the ratio of asset value divided by the liability value, is fixed to 0.

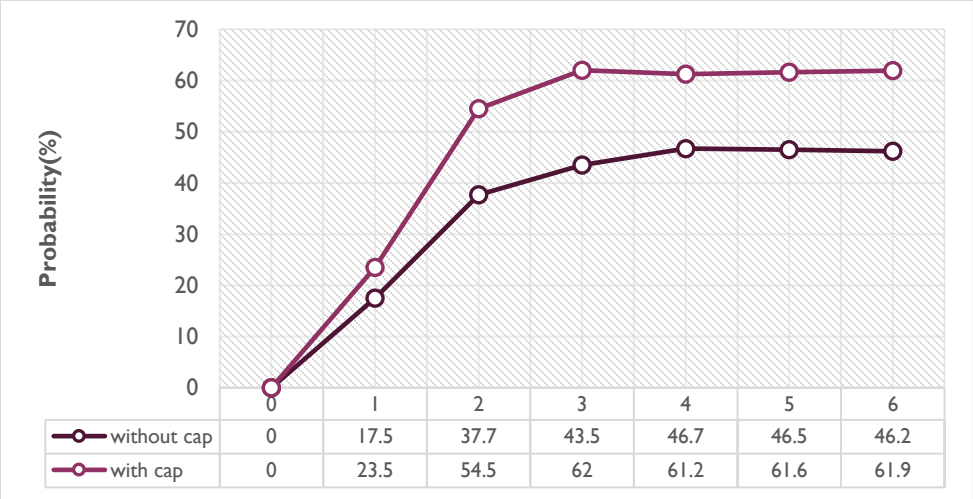
- **Performance seeking portfolio design**

For the portfolio to have potential for performance, it must not be invested only in hedging assets dedicated to the protection of essential goals, and some fraction must be allocated to a performance-seeking portfolio (PSP). PSPs are included for their upside potential, which allows the investor to reach aspirational goals in "good" scenarios. To achieve these potentially high returns, the portfolio must efficiently collect the risk premia of available risky assets. Here, in this case, cap-weighted equity portfolio is intended to be the PSP tool to capture the equity factor premia.

Optimal multiplier selection

The optimal choice for the multiplier can be calculated by using Monte-Carlo scenarios. Also, to reduce the opportunity cost of insurance against downside risk, one can give up of the upside premia above a given threshold by imposing a cap in addition to the floor.

Then, we run the Monte-Carlo Simulation in two Scenarios under a 130% funding ratio level, with or without cap, to find the optimal multiplier. We set Sharpe Ratio at 0.4 and volatility at 0.16 here.



	0	1	2	3	4	5	6
without cap	0	17.5	37.7	43.5	46.7	46.5	46.2
with cap	0	23.5	54.5	62	61.2	61.6	61.9

Chart 1: Probability (%) to Reach Aspirational Goal 130% with Different Multiplier

From the Chart 1 above, we figured out that the optimal multipliers are 3 and 4, with and without cap respectively, and the probabilities of reaching certain level of Funding Ratio are 78.30% and 56.90% accordingly. Apparently, introduced the cap into the strategy, the probability of reaching AG is larger. By forgoing performance beyond a certain threshold, where the Aspirational Goal is reached, investors benefit from a decrease in the cost of the downside protection. In the without cap scenario, though multiplier 4 achieves higher probability of AG, the probability of EG (as indicator of GBI shows) is not 100%, thus we choose multiplier of 3 instead of 4 for without cap scenario as well. **In short, we choose the multiplier as 3** to maximize our probability no matter with or without cap.

- ### Robustness of the multiplier

Now we have the optimal multiplier and we still need to test the robustness of it. In this test, we changed the risk and return parameters for performance-seeking portfolio, and changes in EG/AG levels.

1. Sharpe Ratio change

If we change the Sharpe Ratio from 0.4 to 0.5, we have the chart as below. The probability of reaching AG with cap (72.8%) and without cap (58.6%) both reach maximum when the multiplier

is 4. However, at that time, the GBI is not 100% for both with cap and without cap. Therefore, we can only pick 3 as our optimal multiplier as before, which means the multiplier is robust. If we change the Sharpe Ratio from 0.4 to 0.6, we have the optimal multiplier at 4 but same problem with GBI, so we can only choose 3 as our optimal multiplier, which means the the multiplier is robust.

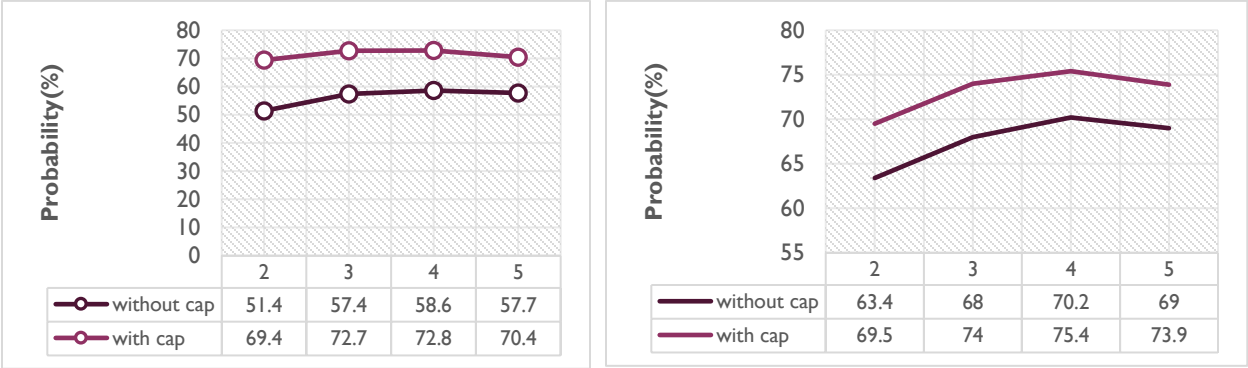


Chart 2: Probability (%) to Reach Aspirational Goal 130% with SR=0.5 and 0.6

2. Volatility change

If we change the volatility from 0.16 to 0.26, we have the chart as below. The probability of reaching AG with cap (58.5%) and without cap (45.5%) both reach maximum when multiplier is 2, which means the multiplier is not robust, so we decide to change the volatility from 0.16 to 0.21, we have the chart as below. The probability of reaching AG with cap (72.8%) and without cap (58.6%) both reach maximum when multiplier is 4. However, at that time, the GBI is not 100% for both with cap and without cap. Therefore, we can only pick 3 as our optimal multiplier as before, which means the multiplier is robust.

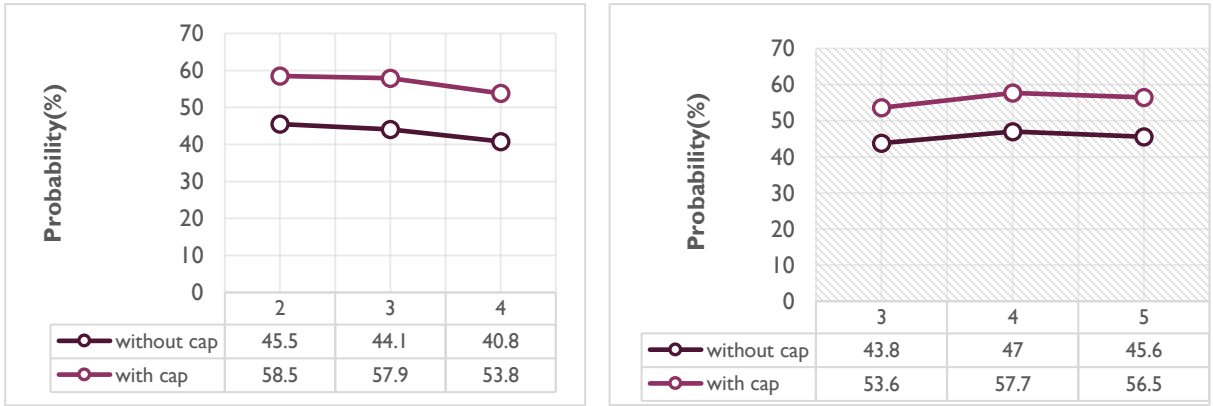
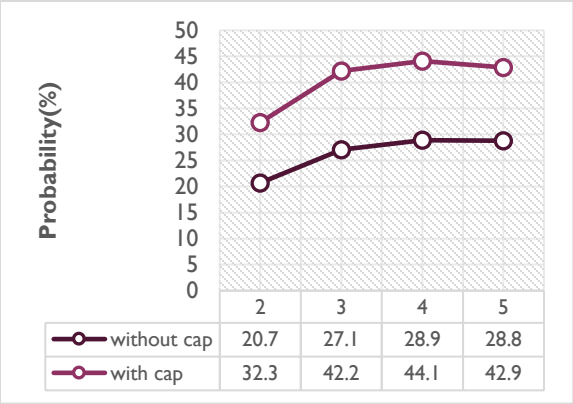


Chart 3: Probability (%) to Reach Aspirational Goal 130% with volatility=0.26 and 0.21

3. EG level change

As the chart below shows, we change the EG level from 80 to 90, we find the optimal multiplier should be 4 for both with cap (28.9%) and without cap (44.1%). However, at that time, the GBI is not 100% for both with cap and without cap. Therefore, we can only pick 3 as our optimal multiplier as before, which means the multiplier is robust. And if we change the EG level from 80 to 70, we find the optimal multiplier should be 5 for both with cap (60.5%) and without cap (75.8%). However, the GBI is not 100% for both with cap and without cap when multiplier is 5 or even 4. Therefore, we can only pick 3 as our optimal multiplier as before, which means the multiplier is robust.



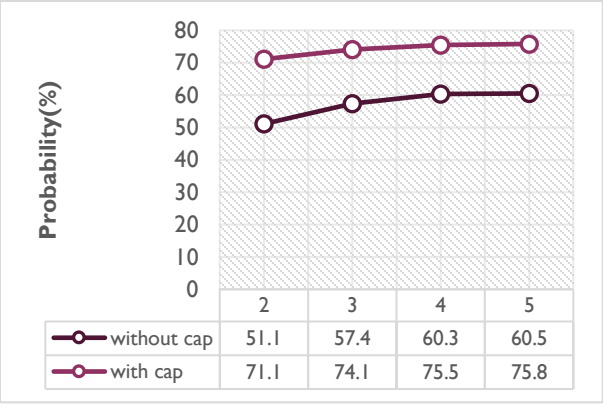
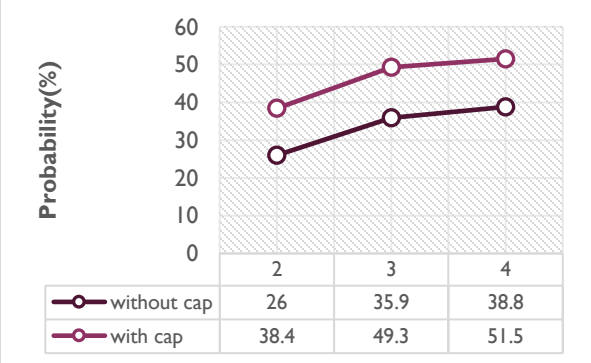


Chart 4: Probability (%) to Reach Aspirational Goal 130% with EG=90 and 70

4. AG level change

As the chart below shows, we change the AG level from 130 to 150, though the probability goes up when we choose high multiplier, we find the GBI is not 100% for both with cap and without cap since multiplier is 4. Therefore, we can only pick 3 as our optimal multiplier as before, which means the multiplier is robust. And we also change the AG level from 130 to 110, but in that case, we have optimal multiplier of 2.



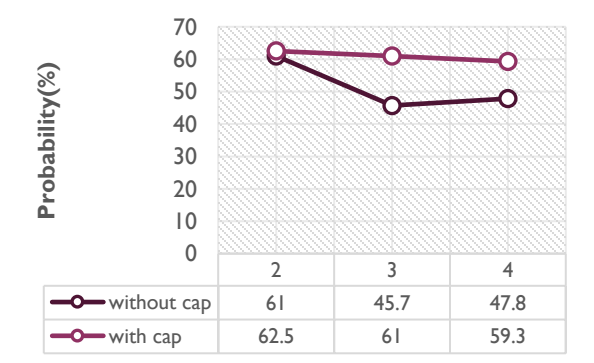


Chart 5: Probability (%) to Reach Aspirational Goal 130% with AG=150 and 110

In conclusion, all the robustness tests show that the multiplier of 3 is robust with change of the risk and return parameters for performance-seeking portfolio, and changes in EG or AG levels as long as the parameters do not change so dramatically in one direction.

Evaluation against other strategies

Compared with other strategies, such as balanced funds and target date funds, the static GBI (when multiplier equals to 1) and GBI got the largest probability (100%) to seize the Essential Goal (80% Funding Ratio). In the view of aspirational goal, the balanced funds and target date funds strategies perform better than GBI. From the perspective of upside potential gains such as 200%, balanced funds capture the biggest probability. And when we take a deep look into the statistics on funding ratio, the balanced funds and target date funds both stand a chance to miss the essential goal. Although GBI strategy have the biggest standard deviation, it has 100% guarantee to seize the floor.

Funding Ratio Level	Balanced Fund	Target-Date Fund	Static GBI	GBI
80%	97.20	96.70	100.00	100.00
110%	79.50	74.30	34.30	32.30
130%	59.20	51.50	7.60	22.80
150%	40.10	30.40	0.90	17.90
200%	12.40	5.80	0.00	9.00

Figure 2: Probability (%) to reach certain level of Funding Ratio

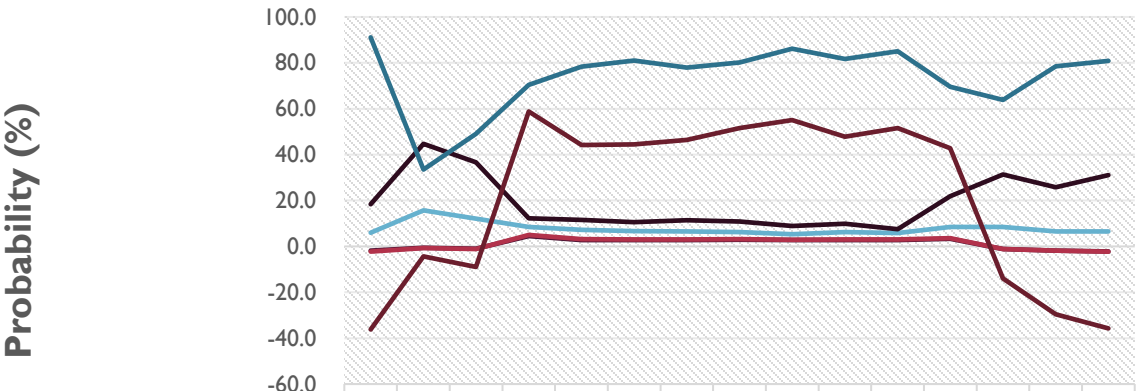
Funding Ratio	Balanced Fund	Target-Date Fund	Static GBI	GBI
Median	146.05	135.52	116.39	146.77
Worst Case (5%)	86.49	85.25	96.66	83.23
Standard Deviation	43.61	35.62	16.29	72.07
Probability of Annual Loss > 20%	15.30	8.20	0.00	21.60
Miss the Essential Goal	2.80	3.30	0.00	0.00
Exp. Shortfall	9.76	7.53	NA	NA

Figure 3: Risk & Performance of Funding Ratio with Different Strategies

Opposed to using the proper GHP, a 5Y constant maturity bond index can be used as a safe asset. However, the duration of asset cannot match duration of liability. Therefore, the performance of the strategy will be influenced more by market interest rate because of the duration gap.

Back-testing of GBI strategy

Now we take the back-test of the GBI strategy with on historical scenarios based on 15-year data from 1985 to 1999. Apparently, the annualized funding ratio return, annualized excess return to liability portfolio, funding ratio volatility and funding ratio maximum drawdown keep stable in while sample periods change. But the annualized excess return to liability portfolio/funding ratio volatility experience a significant fluctuation around 1987. At the same time, the correlation to liability portfolio suffer a big drop.



	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Annualized Funding Ratio Return	-1.9	-0.6	-1	4.5	2.8	2.7	2.7	2.9	2.7	2.7	2.7	3.3	-1.1	-1.8	-2.2
Annualized Excess Return to Liability Portfolio	-2.2	-0.7	-1.1	5	3.2	3	3	3.2	2.9	3	3	3.6	-1.2	-1.9	-2.3
Funding Ratio Volatility	6.0	15.7	12.1	8.5	7.2	6.6	6.5	6.3	5.3	6.2	5.8	8.5	8.5	6.5	6.5
Funding Ratio Maximum Drawdown	18.4	44.7	36.6	12.2	11.5	10.6	11.4	10.9	8.9	9.8	7.5	21.8	31.4	25.8	31.1
Annualized Excess Return to Liability Portfolio/Funding Ratio Volatility	-36.2	-4.4	-9	58.8	44.2	44.5	46.4	51.6	55.1	47.8	51.6	42.8	-14	-29.6	-35.8
Correlation to Liability Portfolio	91.1	33.5	49	70.4	78.4	81	78	80.2	86.2	81.7	85.1	69.6	63.9	78.5	80.8

Chart 6: Back-tests on Historical Scenarios

Impact of changes of the EG level

Now that we have the optimal level of multiplier 3, we design four scenarios to test the **impact the changes of the EG level** (from 0 to 100%, each increasing by 10%) on the probability to achieve the AG target level (130% and 150% in our scenarios case) of replacement income. Here are the scenarios graph below.

Scenarios	With Cap	AG level
1	Yes	130%
2	No	130%
3	Yes	150%
4	No	150%

Figure 4: Scenarios Setting

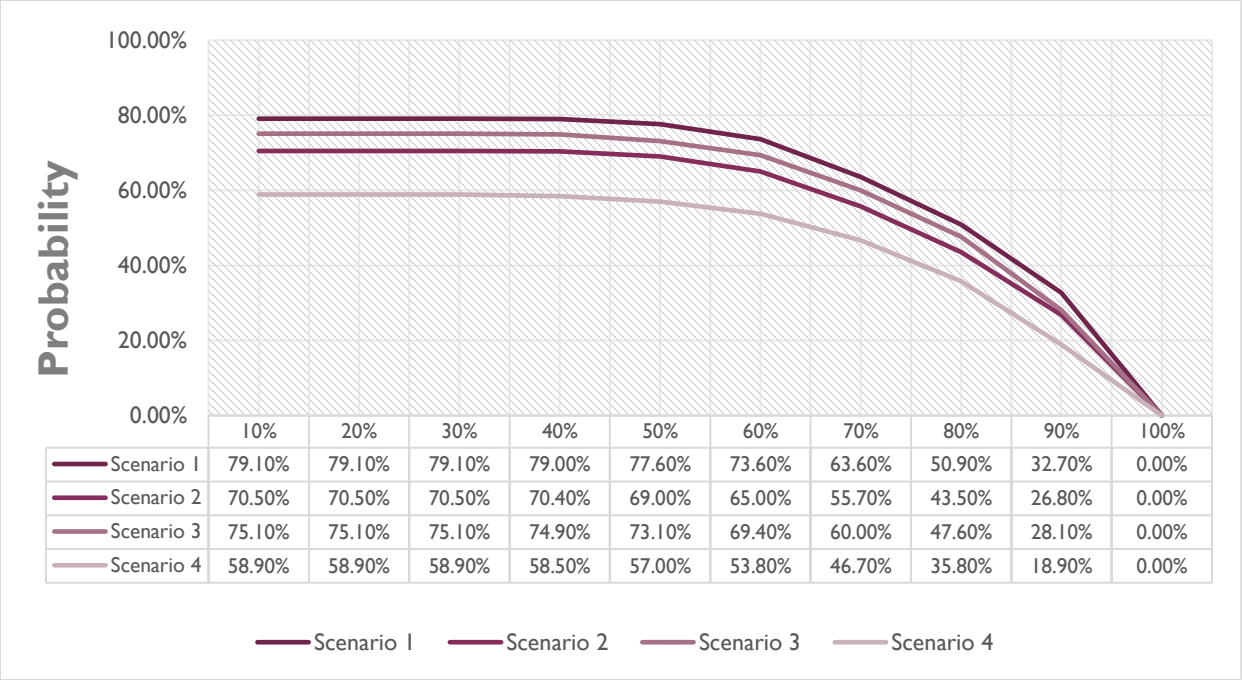
We set all the factors as below. To eliminate the impact of other factors, we take perfect GHP as our LHP Strategy. As explained before, we have 0% as our cost of live adjusted in terms of the US stimulation package announced several days ago. The introduction of caps is an efficient way to pay the cost of downside protection.

Decumulation Period	**20 years**
Initial Wealth	100,000
Aspirational Goal	130%, or 150%
Cost of Live Adjusted	0
PSP Stochastic Volatility	Yes
PSP Stochastic Sharpe Ratio	Yes
LHP Strategy	Perfect GHP

Figure 5: Initial Factors of Monte-Carlo Simulation

We change the EG level (from 0 to 100%, each increasing by 10%) to see how the probability changes to reach the given AG level and hereafter is the result chart. scenario1 (with cap) and scenario 3 (with cap) have the relatively high probability than scenario 2 (without cap) and scenario 4(without cap) to achieve the given AG level, which obviously shows that the **cap could help to reduce the opportunity costs in the transaction.** To be more specifically speaking, a cap would help increase probably 10% higher probability to reach the AG.

With the increases of EG level, all probability of scenarios decreases. On the first increase stage of EG level from 10% to 40%, we see the probability of reaching a certain AG level keeps the same for all scenarios, but after 50% of EG level, the probability decreases dramatically. Take scenario 1 as an example, when EG is from 10% to 20%, the probability remains at 79.1%; however, when EG is from 80% to 90%, the probability drops from 50.9% to 32.7%, which decreases almost 20.22%.



	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Scenario 1	79.10%	79.10%	79.10%	79.00%	77.60%	73.60%	63.60%	50.90%	32.70%	0.00%
Scenario 2	70.50%	70.50%	70.50%	70.40%	69.00%	65.00%	55.70%	43.50%	26.80%	0.00%
Scenario 3	75.10%	75.10%	75.10%	74.90%	73.10%	69.40%	60.00%	47.60%	28.10%	0.00%
Scenario 4	58.90%	58.90%	58.90%	58.50%	57.00%	53.80%	46.70%	35.80%	18.90%	0.00%

Chart 7: Probability (%) to Reach Aspirational Goal on different Scenarios with Different EG Levels

In conclusion, the impact of changes of the EG level on the probability to reach a given AG level is negligible when the EG level is low. However, with a higher EG level, the impact of changes is substantial. Moreover, the opportunity cost, which means the introduction of caps in this case, has a huge impact on the changes of probability. Given a high EG level, the change of probability with caps is less than the one without cap.

Therefore, we recommend to have a EG level of 60% to have a relatively high probability of AG level in scenario 1, which is 130% AG level with cap.

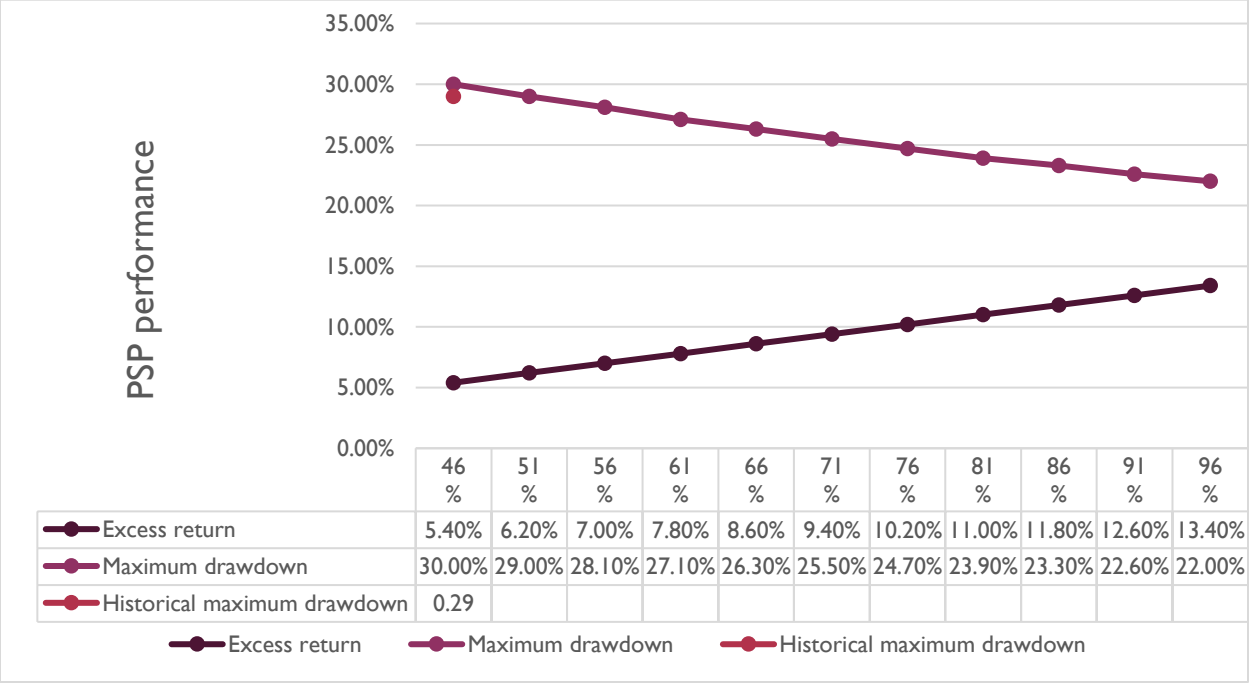
Benefits of using an improved "smart" equity benchmark

Now that we have the strategy, we want to see the benefit of using am improved equity benchmark rather than standard cap-weighted index. Setting the initial factors (as shown in Figure 6 below) in Monte-Carlo Simulation, we would like to change the PSP Sharpe ratio to examine the risk and performance.

Type of goal	Income Based
Accumulation Period	10
Decumulation Period	20
Initial Wealth	100000
Horizon Date	21/03/30
Cost of Living Adjusted (%)	0
PSP Stochastic Volatility	No
PSP Stochastic Sharpe Ratio	No
LHP Strategy	Perfect GHP

Figure 6: Initial Factors of Monte-Carlo Simulation

We have initial PSP Sharpe Ratio of 0.46 and historical maximum drawdown is 0.29. In our simulation, each time the PSP Sharpe ratio increases 5% from 46% to 96% in the X axis of the chart below. And we choose excess return and maximum drawdown as our proxy of PSP performance. Hereafter is the result chart 4.



	46 %	51 %	56 %	61 %	66 %	71 %	76 %	81 %	86 %	91 %	96 %
Excess return	5.40%	6.20%	7.00%	7.80%	8.60%	9.40%	10.20%	11.00%	11.80%	12.60%	13.40%
Maximum drawdown	30.00%	29.00%	28.10%	27.10%	26.30%	25.50%	24.70%	23.90%	23.30%	22.60%	22.00%
Historical maximum drawdown	0.29										

Chart 8: PSP Performance with Different EG Level

As is shown in the Chart 4, we find that when the Sharpe ratio increases from its initial setting of 46% to 100%, the maximum drawdown decreases, while the excess return increases. This indicates a positive correlation between Sharpe Ratio and PSP performance. The grey dot in the Chart 4 is the historical maximum drawdown ---when the Sharpe Ratio is 46%, the maximum drawdown of historical scenario is 0.29 (or 29%), slightly lower than that in the Monte-Carlo simulation results (30%), which proves the robustness of the associated benefits.

If we set the EG level at 60%, which we choose in the EG impact analysis, we can have an excess return at around 7.8% and a maximum drawdown at around 27.1%, which is quite better than the cap-weighted portfolio before.

Conclusion

To maximize the probability of achieving AG, we recommend to use **3 as optimal multiplier** with cap and without cap since the robustness test is good for 3, with which GBI would get the largest probability (100%) to seize the Essential Goal (80% Funding Ratio). Therefore, the investors would receive a stable retirement income and an aspirational income at high probability.

Opposed to using the proper GHP, a 5Y constant maturity bond index can be used as a safe asset. However, the duration of asset cannot match duration of liability. Therefore, we should care about the **interest rate** because the performance of the strategy will be influenced more by market interest rate because of the duration gap.

Of course, we should consider how the level of EG influences the AG probablity. Because the impact of changes of the EG level on the probability to reach a given AG level is negligible when the EG level is low but with a higher EG level, the impact of changes is substantial. Moreover, the opportunity cost, which means the introduction of caps in this case, has a huge impact on the changes of probability. Given a high EG level, the change of probability with caps is less than the one without cap. Therefore, we recommend to have a EG level of **60%** to have a relatively high probability of AG level in scenario 1, which is **130% AG level** with cap.

Besides, the improved "smart" equity benchmark with a higher Sharpe ratio has advantages over using a standard cap-weighted index as a benchmark for the PSP. The benefits can be shown on the less maximum drawdown and the more excess return. If we set the EG level at 60%, which we choose in the EG impact analysis, we can have an excess return at around 7.8% and a maximum drawdown at around 27.1%, which is quite better than the cap-weighted portfolio before.

In general, we set the EG level at 60% and AG level at 130%, and a multiplier at 3. We pay attention to the interest rate to reset our parameter moderately and hopefully to achieve around 7.8% excess return, which is obviously better than 5.6% from cap-weighted portfolio and minimize the maximum drawdown at around 27.1%, which is 29% in cap-weighted portfolio.

Appendix

Find optimal multiplier

funding ratio level=130%		SR=0.4
multiplier	without cap	with cap
0	0	0
1	17.5	23.5
2	37.7	54.5
3	43.5	62
4	46.7	61.2
5	46.5	61.6
6	46.2	61.9

Probability to reach AG with different EG levels

	130%		150%	
	with cap	without cap	with cap	without cap
EG	Scenario 1	Scenario 2	Scenario 3	Scenario 4
10%	79.10%	70.50%	75.10%	58.90%
20%	79.10%	70.50%	75.10%	58.90%
30%	79.10%	70.50%	75.10%	58.90%
40%	79.00%	70.40%	74.90%	58.50%
50%	77.60%	69.00%	73.10%	57.00%
60%	73.60%	65.00%	69.40%	53.80%
70%	63.60%	55.70%	60.00%	46.70%
80%	50.90%	43.50%	47.60%	35.80%
90%	32.70%	26.80%	28.10%	18.90%
100%	0.00%	0.00%	0.00%	0.00%

Back-test data

GBI	Annualized Funding Ratio Return	Annualized Excess Return to Liability Portfolio	Funding Ratio Volatility	Funding Ratio Maximum Drawdown	Annualized Excess Return to Liability Portfolio/Funding Ratio Volatility	Correlation to Liability Portfolio
1985	-1.9	-2.2	6.0	18.4	-36.2	91.1
1986	-0.6	-0.7	15.7	44.7	-4.4	33.5
1987	-1	-1.1	12.1	36.6	-9	49
1988	4.5	5	8.5	12.2	58.8	70.4
1989	2.8	3.2	7.2	11.5	44.2	78.4
1990	2.7	3	6.6	10.6	44.5	81
1991	2.7	3	6.5	11.4	46.4	78
1992	2.9	3.2	6.3	10.9	51.6	80.2
1993	2.7	2.9	5.3	8.9	55.1	86.2
1994	2.7	3	6.2	9.8	47.8	81.7
1995	2.7	3	5.8	7.5	51.6	85.1
1996	3.3	3.6	8.5	21.8	42.8	69.6
1997	-1.1	-1.2	8.5	31.4	-14	63.9
1998	-1.8	-1.9	6.5	25.8	-29.6	78.5
1999	-2.2	-2.3	6.5	31.1	-35.8	80.8