Efficient Market Portfolio Evaluation

I. Introduction

Maximum Sharpe Ratio (MSR) portfolio, which referred to the market portfolio as a proxy for traditional cap-weighted (CW) indices, may not be mean-variance efficient (MVE) enough due to the inefficient market. The information inefficiency lead to a biased weighting composition. Thus, other weighting portfolios are conducted to be compared with the traditional cap-weighted portfolio, figuring out whether these portfolios are better alternatives as market portfolio in practice.

The paper is organized as follows. The Fama-French 25 size and value portfolios' daily data, downloaded from Kenneth French's website, performed as the passive investment style portfolios. The in-sample period is from July 1st 1963 to July 1st 1993 whereas the out-of-sample period is from July 1st 1993 to July 1st 2019.

Generally, five constant correlation covariance method for risk optimizations, listed as Min-Variance (MV), Max-Diversified (MD), Risk Parity (RP), Equally-Weighted (EW) and Max Sharpe Ratio (MSR), are conducted to estimate the weight matrix. For each portfolio, we use 60 lagged daily data with a one-month (20 trading days) rebalancing frequency to get the changes of weights of the in-sample period. Then, the cumulative performance of different strategies is tested by various risk measurements, decomposed as mean-variance framework and High order moments, are compared against the traditional cap-weighted portfolio.

In the final part, a new portfolio by adding some amount of new asset class is composed to test where this particular strategy, noted as Momentum, makes sense for the high order moments. Finally, the out-of-sample stress test is implemented to determine the robustness of the risk measurements during economic crisis.

II. Market Portfolio Analysis

2.1 Changes of weight in different strategies

This section shows the estimation of the in-sample covariance matrix. Specifically, the changes of weight in each strategy are calculated by using five risk-optimization methods respectively with 60 lagged daily data rebalancing on a monthly basis.

Figure 1 shows that, apart from weights of RP and EW portfolios remain stable during the in-sample period, other three portfolio strategies (called MV, MD, and MSR) show dramatic fluctuation in the weights during the same sample period. Therefore, due to frequent rebalancing, these three strategies generate high transaction costs because of high turnover.

Besides, the weights in portfolio such as MV, MD, and MSR are highly sensitive to the change of portfolio return at each time period, and this high sensitivity leads to both drastic fluctuation in the weight size and relative high trading costs which may reduce the net return of the portfolio.

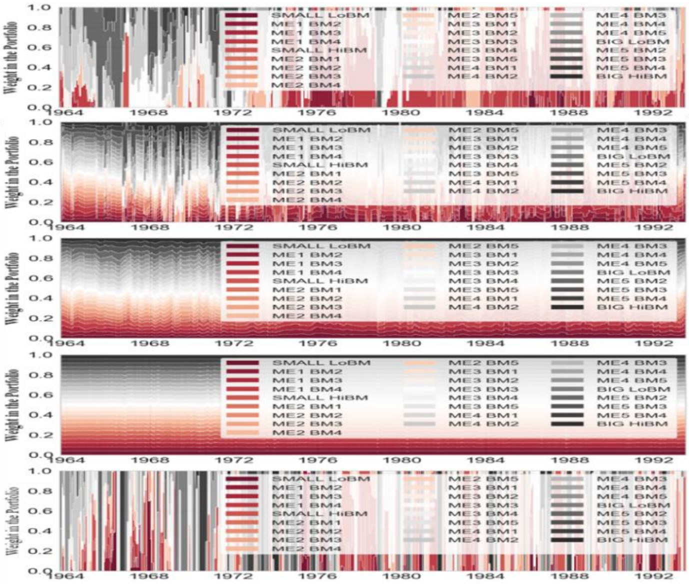

Figure 1: Changes in Weight for different strategies (in-sample period). The figures from top to bottom are in-sample period weight in Min-Variance (MV), Max-Diversified (MD), Risk Parity (RP), Equally-weighted(EW), Max Sharpe Ratio (MSR) portfolio respectively.

2.2 Cumulative Performance Analysis

Then, the gross and the net cumulative performance of the strategies are compared with the traditional cap-weighted portfolio in order to find mean-variance efficiency of traditional cap-weighted portfolio. Besides, the impact of trading costs is also considered in estimating the portfolio performance.

Illustrated by Figure 2, without considering transaction costs, the MSR portfolio has the highest total cumulative performance over the sample period. Its gross return is at least three times as many as second and third place, respectively for RP and EW portfolio, while the cap-weighted portfolio (Market Portfolio in Figure 2) performs worst in the same period. Therefore, after analyzing the total return, the cap-weighted portfolio is not mean-variance efficient.

However, when considering the trading costs, EW and RP portfolios dramatically outperform other strategies, and portfolios with large fluctuations in weight (MV, MD, and MSR) perform relatively poorly, even worse than traditional market portfolios. This phenomenon is consistent with previous conclusion that the more unstable the weight in portfolio, the more frequent the rebalance, and thus the higher the trading costs and lower net return.

Overall, we summarize the results derived from both gross and net performance of different strategies and conclude that, in both cases (with or without considering trading costs), the cap-weighted portfolio is not mean-variance efficient. Besides, the analysis of net return show that instead of using cap-weighted indices, it is much better to use RP or EW portfolios as alternative proxy for MSR portfolio.

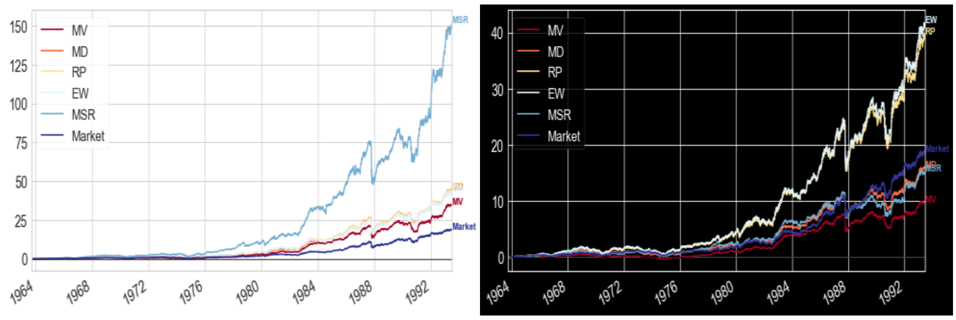

Figure 2&3: Gross & Net Cumulative Performance of strategies (in-sample period)

2.3 Risk Measurements of strategies' performance

Furthermore, different risk measurements have been taken to explore the performance for each strategy.

2.3.1 mean-variance framework

Introduced by Huberman and Kandel (1987), the mean-variance framework is widely used by most practitioner to measure investment performance. This framework assumes either that all asset returns are normally distributed or that investors have mean-variance preferences (and thus ignore skewness).

Table 1 shows the benchmark approach of the mean-variance framework. The RP and EW portfolio displayed attractive Sharpe Ratio and information ratio, which implies a better level of skill to generate higher excess returns. Besides, based on the theoretical performance index instead of the market index, Jensen alpha is also conducted to estimate the strategy outperformance, which displays an extension to multifactor models.

Mean-variance Framework					
	MV	**MD**	**RP**	**EW**	**MSR**
Mean	0.0852	0.1022	0.1318	0.1340	0.1029
Std	0.1000	0.1162	0.1184	0.1225	0.1320
Sharpe Ratio	0.2038	0.3220	0.5660	0.5650	0.2882
Information Ratio	-0.0809	-0.0239	0.0750	0.0825	-0.0218

Table 1: benchmark approach of the mean-variance framework

2.3.2 High order moments

Assumed by the mean-variance framework, the asset return follows random walk, which means the portfolio return is normally distributed. However, displayed by Table 2, examined by Jarque-Bera test, all strategies present abnormality distribution, which are leptokurtic (Kurtosis above 3) and left-skewed (skewness below 0), meaning that the market portfolio is mean-variance inefficient and has higher probability in extreme losses. Thus, superior performance measurements are required.

The Calmar ratio is a comparison of the average annual compounded rate of return and the maximum drawdown risk, which assesses the relative riskiness of one strategy versus another. The lower the Calmar ratio, the worse the investment performed on a risk-adjusted basis within the specified time period. And like the Sharpe Ratio, the Sortino ratio replaces the standard deviation with a downside deviation, which is more acceptable to different investors with various levels of minimum acceptable return. Table 2 show that the RP and EW portfolio capture relatively highly risk, shown as -0.108 and -0.111 for Calmar ratio, and better risk-adjusted performance, displayed as 0.685 and 0.68 for Sortino ratio.

In addition, extreme risk analysis (VaR and Cornish-Fisher VaR) as well as Modified SR and Modified VaR (taking into account the skewness and kurtosis of the return distribution), indicate that all strategies have similar such risk.

High Order Moments								
	Jarque-Bera	Sharpe Modified	Max Drawdown	Calmar Ratio	Downside Deviation	Sortino Ratio	VaR Historaical	VaR Modified
MV	271157	0.2032	-0.7342	-0.0281	0.0860	0.2370	-0.0107	-0.0036
MD	167859	0.3200	-0.6922	-0.0551	0.0948	0.3949	-0.0124	-0.0063
RP	150699	0.5477	-0.6442	-0.1076	0.0978	0.6849	-0.0128	-0.0067
EW	152444	0.5471	-0.6477	-0.1107	0.1004	0.6896	-0.0132	-0.0070
MSR	47573	0.2877	-0.6606	-0.0587	0.1084	0.3512	-0.0141	-0.0097

Table 2: Performance measurement with High order moments

2.4 Portfolio Improvement

More rigorous risk measurement methods are implemented by the decomposing the portfolio risk from mean-variance framework into high order moments. And identified that adding new assets is a commonly used technique to diversify the portfolio, we examine whether implement a 5% long/short stock momentum strategy to the initial capital-weighted portfolio can benefit the higher moments.

	Initial Portfolio(P)	New Asset(A)	New portfolio(Pa)	Beta	Benefits higher moment diversification or not
Mean	0.0532%	0.0373%	0.0524%		
Variance	0.0060%	0.0023%	0.0054%	0.0856	**Benefits**
μ^3	-5.53493E-07	-8.15934E-08	-5.08221E-07	0.4463	**Benefits**
Skewness	-1.2038	-0.7185	-1.2700	1.0588	**Not Benefits**
μ^4	8.86074E-08	5.80892E-09	7.66615E-08	0.2829	**Benefits**
Kurtosis	21.9698	7.5623	22.9957	1.0377	**Not Benefits**

Table 3: risk measures of high moments

Shown by the results, allocating 5% of the portfolio to a momentum strategy does benefit the portfolio third and fourth moment (mu^3 and mu^4) diversification, when asset A has a beta less than 1 respect to the existing portfolio. While due to the normalization, estimation errors, and the degree of freedom problems, the allocating does not benefit the normalized cumulants (Skewness, Kurtosis) by decreasing the skewness and increasing the kurtosis of the portfolio, with the betas higher than 1.

III. Stress Test Analysis

To further proof the conclusions above, Stress test strategies over out-of-sample period from the first of July 1993 until the first of July 2019 are conducted to compare the in-sample results with out-of-sample results for gross and net cumulative performance of the strategies against the traditional market portfolio.

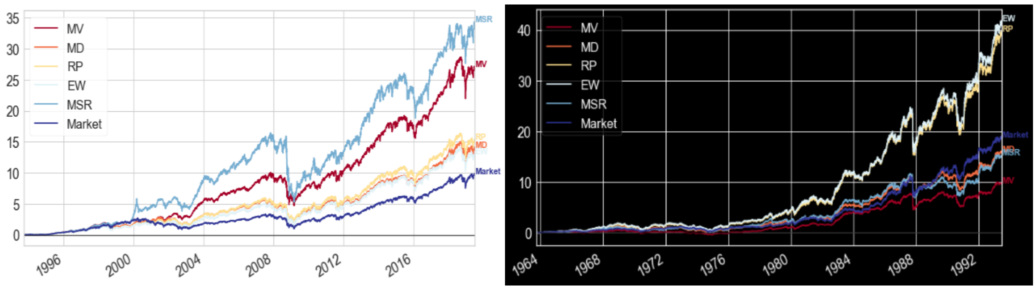

Figure 4&5: Gross & Net Cumulative Performance of strategies (out-of-sample period)

Figure 4&5 shows the same conclusion for in-sample and out-of-sample period. With the impact of trading cost, rather than use CW indices, RP and EW portfolio are better proxy for MSR portfolio because of mean-variance efficient advantages. Then, different risk measurements, noted as mean-variance framework and high order moments, are taken to reach the same conclusion, that the RP and EW portfolio displayed better risk-adjusted performance compared with the traditional cap-weighted benchmark.

	Mean-variance Framework				High Order Moments					
	Mean	Std	Sharpe Ratio	Information Ratio	Jarque-Bera	Sharpe Modified	Calmar Ratio	Sortino Ratio	VaR Historaical	VaR Modified
MV	0.1050	0.1500	0.5427	-0.0151	71321	0.5391	-0.1607	0.6460	-0.0141	-0.0118
MD	0.0863	0.1842	0.3404	-0.0853	20415	0.3402	-0.0984	0.4230	-0.0175	-0.0160
RP	0.1220	0.1838	0.5347	0.0483	16012	0.5335	-0.1701	0.6773	-0.0175	-0.0166
EW	0.1197	0.1900	0.5055	0.0398	14106	0.5046	-0.1650	0.6466	-0.0182	-0.0173
MSR	0.0809	0.1941	0.2948	-0.1059	25879	0.2946	-0.0856	0.3595	-0.0184	-0.0161

Table 4: Performance measurement of out-of-sample test

Conclusions

Compared against the traditional cap-weighted portfolio, we implement the Risk Parity portfolio and Equally Weighted portfolio to better replace the market portfolio. According to the theorical analysis and empirical tests, the RP portfolio and EW portfolio outperform the traditional cap-weighted portfolio in terms of mean-variance efficient advantages and higher moments benefits. Taking the transaction costs generated by high turnover into consideration, the stress test of this proposal is realizable.

Presented by group 4

Ying SUN (56868)   Yu JIN (56887)   Mengdi MA (67596)

Individual Contribution:

- **Coding completing**
  In class Lab2, we **three** completed the blanking part of all coding together.
- **Debugging**
  Ying SUN & Yu JIN
- **Report Writing**
  Ying SUN & Mengdi MA