Homework

$$\begin{cases} x^{'}(t) = 2x(t-1) - x^{'}(t-1), & t \in [0,3], \\ x(t) = t, & t \in [-1,0], \end{cases}$$

{# } 1. $t \in [0,1]$ $x(t-1) = t-1$

$$\begin{cases} x^{'}(t) = 2t-3, & t \in [0,1], \\ x(0) = 0. \end{cases}$$

$x(t) = t^2 - 3t, x \in [0,1]$ 2. $t \in [1,2]$ $x(t-1) = (t-1)^2 - 3(t-1)$

$$\begin{cases} x^{'}(t) = 2(t-1)^2 - 8(t-1) + 3, & x \in [1,2], \\ x(1) = -2. \end{cases}$$

$x(t) = (t-1)^3 - 4(t-1)^2 + 3(t-1) - 2, x \in [1,2]$ 3. $t \in [2,3]$ $x(t-1) = (t-2)^3 - 4(t-2)^2 + 3(t-2) - 2$

$$\begin{cases} x^{'}(t) = 2(t-2)^3 - 10(t-2)^2 + 14(t-2) - 7, & x \in [2,3], \\ x(2) = -2. \end{cases}$$

$x(t) = \frac{1}{2}(t-2)^4 - \frac{10}{3}(t-2)^3 + 7(t-2)^2 - t(t-2) - 2, x \in [2,3]$ 4.

$|b| < -\mathfrak{Re}(a)$

$$x^{'}(t) = ax(t) + bx(t-\tau), a, b \in \mathbb{C}, \tau > 0,$$

$x(t)$ ### {# } $x = e^{\lambda t}$

$$\lambda = a + be^{-\lambda\tau}.$$

$\mathfrak{Re}(\lambda) < 0$ $|b| < -\mathfrak{Re}(a)$ $\mathfrak{Re}(\lambda) < 0$
$|b| < -\mathfrak{Re}(a)$ $\mathfrak{Re}(\lambda) \geq 0$
$0 \leq |b| < -\mathfrak{Re}(a)$ $\mathfrak{Re}(a) < 0$

$$\begin{aligned} \mathfrak{Re}(\lambda) =& \mathfrak{Re}(a + be^{-\lambda\tau}) \\ =& \mathfrak{Re}(a) + \mathfrak{Re}(be^{-\lambda\tau}) \\ =& \mathfrak{Re}(a) + \mathfrak{Re}(|b|e^{iarg(b) - \tau\mathfrak{Re}(\lambda) - i\tau\mathfrak{Im}(\lambda)}) \\ =& \mathfrak{Re}(a) + |b|e^{-\tau\mathfrak{Re}(\lambda)}\mathfrak{Re}(e^{i\theta}) \end{aligned}$$

$\theta = arg(b) - \tau \mathfrak{Im}(\lambda)$

$$
\begin{aligned}
&|b| \geq 0,\\
&0 < e^{-\tau \mathfrak{Re}(\lambda)} \leq 1,\\
&-1 \leq \mathfrak{Re}(e^{i\theta}) \leq 1,
\end{aligned}
$$

$$-|b| \leq |b| e^{-\tau \mathfrak{Re}(\lambda)} \mathfrak{Re}(e^{i\theta}) \leq |b|,$$

$$\mathfrak{Re}(\lambda) = \mathfrak{Re}(a) + |b| e^{-\tau \mathfrak{Re}(\lambda)} \mathfrak{Re}(e^{i\theta}) \leq \mathfrak{Re}(a) + |b| < 0,$$

$\mathfrak{Re}(\lambda) \geq 0$