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1. (a) $P(A\cup B)=0.6$

(b) $P(A\cap B')=0.1$

(c) $P(A'\cup B')=0.7$

2. (a) $P(A_1\cup A_2\cup A_3)=P(A_1)+P(A_2)+P(A_3)-P(A_1\cap A_2)-P(A_1\cap A_3)-P(A_2\cap A_3)+P(A_1\cap A_2\cap A_3)$

$= 1-3\times\frac{1}{9}+\frac{1}{27}=\frac{19}{27}$

(b) $P(A_1\cup A_2\cup A_3) = 1-P(A_1\cup A_2\cup A_3)' = 1-P(A_1'\cap A_2'\cap A_3')$

$i=1,2,3 \quad P(A_i') = 1-P(A_i) = 1-\frac{1}{3}$

$\therefore P(A_1\cup A_2\cup A_3) = 1-(1-\frac{1}{3})^3$

prove:

$\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$

3. $P(A\cup B\cup C)=P((A\cup B)\cup C) = P(A\cup B)+P(C)-P((A\cup B)\cap C)$

$= P(A)+P(B)+P(C)-P(A\cap B)-P((A\cap C)\cup(B\cap C))$

$= P(A)+P(B)+P(C)-P(A\cap B)-(P(A\cap C)+P(B\cap C)-P((A\cap C)\cap(B\cap C)))$

$= P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

4. $P=\frac{2}{3}$

5. (a) $A_9^9 = 9! = 362880$

(b) The numbers of lineups $=\frac{9!}{3!\,6!} = 84$

(c) $2^9=512$

6. (a) $N(S)={}^{52}C_{13}$

$N(X)={}^{13}C_5\times{}^{13}C_4\times{}^{13}C_3\times{}^{13}C_1$

$P=\frac{N(X)}{N(S)}=\frac{\frac{13!}{5!}\times\frac{13!}{4!}\times\frac{13!}{3!}\times\frac{13!}{1!}}{\frac{52!}{13!}}=0.0054$

(b) $P=\frac{N(X)}{N(S)}=\frac{{}^{13}C_5\times{}^{13}C_4\times{}^{13}C_2\times{}^{13}C_2}{{}^{52}C_{13}}=0.0088$

(c) $P=\frac{N(X)}{N(S)}=\frac{{}^{13}C_5\times{}^{13}C_4\times{}^{13}C_1\times{}^{13}C_3}{{}^{52}C_{13}}=0.0054$

(d) split 2,2

$${}^{13}C_2 \times {}^{13}C_2 = 6084$$

split 3,1

$${}^{13}C_1 \times {}^{13}C_3 + {}^{13}C_3 \times {}^{13}C_1 = 7436$$

So that splitting 1,3 is more likely than spliting 2,2

7. $\binom{10-1+36}{36} = \frac{45!}{36!\,9!} = 886163135$

8. (a) $P = \frac{{}^{13}C_1 \times {}^{48}C_1}{{}^{52}C_5} = \frac{524}{{}^{52}C_5} \approx 0.0002401$

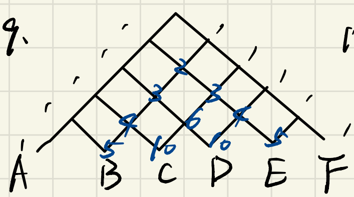
(b) $P = \frac{{}^{13}C_1 \times {}^{4}C_3 \times {}^{12}C_1 \times {}^{4}C_2}{{}^{52}C_5} = \frac{3744}{{}^{52}C_5} \approx 0.001441$

(c) $P = \frac{{}^{13}C_1 \times {}^{4}C_3 \times {}^{12}C_2 \times ({}^{4}C_1)^2}{{}^{52}C_5} \approx 0.02113$

(d) $P = \frac{{}^{13}C_2 \times {}^{4}C_2 \times {}^{4}C_2 \times {}^{11}C_1 \times {}^{4}C_1}{{}^{52}C_5} = \frac{123552}{{}^{52}C_5} \approx 0.04754$

(e) $P = \frac{{}^{13}C_1 \times {}^{4}C_2 \times {}^{4}C_1 \times {}^{12}C_3 \times {}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_5} = \frac{1098240}{2598960} \approx 0.4226$

9.



$N(S) = 1+5+10+10+5+1 = 32$

$P(B) = \frac{5}{32}$

$P(C) = \frac{10}{32} = \frac{5}{16}$

10. $N(S) = 4^k$

① begin with 1/2/3 : $a_k = 3a_{k-1}$

② begin with 0 : $a_k = 4^{k-1} - a_{k-1}$

$a_1 = 3$ $\quad a_1 - \frac{1}{2}\times 4 = 1$

$\therefore a_k = 2a_{k-1} + 4^{k-1}$

$\therefore a_k - \frac{1}{2}4^k = 2(a_{k-1} - \frac{1}{2}4^{k-1})$ $\quad \therefore a_k - \frac{1}{2}4^k = 2^{k-1}$ $\quad \therefore a_k = 2^{k-1} + \frac{1}{2}\cdot 4^k$

$\therefore P = \frac{2^{k-1} + \frac{1}{2}4^k}{4^k} = \frac{1}{2} + \frac{1}{2^{k+1}}$