FIN2010 Financial Management
Lecture 3: Time value of money II

Review—Time Value of Money

- For a single cash flow: $FV = PV * \left(1 + \frac{APR}{m}\right)^{m*n}$
  - APR: how interest rate is quoted. Equals to per period interest times m (#periods per year)
  - EAR: actual interest accrued in a year

- For cash flow streams: FV (or PV) is the sum of FV (or PV) of individual cash flows
  - Ordinary annuity: $PV = PMT\frac{1-(1+r)^{-n}}{r}$
  - Annuities due: $PV = PMT\left(1 + \frac{1-(1+r)^{-n+1}}{r}\right) = PMT(1+r)\frac{1-(1+r)^{n}}{r}$
  - Level perpetuity: $PV = PMT/r$
  - Growth perpetuity: $PV = \frac{PMT}{r-g}$

Agenda

- Examples questions of annuities
  - Demonstration of the financial calculator
- Amortized loan
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Example- PV of Annuities

Suppose you win a lottery of $10 million. The money is paid in equal annual installments of $500,000 over 20 years. The discount rate is 5% (EAR) per year.

- If the first payment will be paid 1 year from today, how much is the lottery actually worth today?
  - $PV = 500{,}000(1 - 1.05^{-20})/0.05 = 6{,}231{,}105.17$

- If the first payment is paid today, what is the present value?
  - $PV = 500{,}000(1.05)(1 - 1.05^{-20})/0.05 = 6{,}542{,}660.43$

Example-FV of Annuities

- Suppose you begin saving for your retirement by depositing $2,000 per year at the end of each year in an bank account. The interest rate is 7.5% per year.
  - How much will you have in 40 years?
    - $FV = 2{,}000(1.075^{40} - 1)/0.075 = 454{,}513.04$
  - Now suppose instead of annual deposits of $2,000, you deposit $4,000 every two years. The first deposit is made at the end of year two. How much will you have at the end of year 40?
    - $FV = 4{,}000[(1.075^{2})^{20} - 1)/(1.075^{2}-1) = 438{,}084.86$

Example-Level Perpetuity

- Suppose the Fellini Co. wants to sell a level perpetuity at $100m.
  - Dividends from preferred stock are level perpetuities.
  - Assume the perpetuities are paid every quarter. If Fellini expects that investors will require a return of 10% (APR), what dividend will Fellini have to offer if the level perpetuity is going to sell?

- Solution
  - Perpetuity formula: PV = PMT / r
  - Required return per quarter=10%/4=2.5%
  - Amount for new payment:
    - 100 = PMT / 2.5%
    - PMT = 2.50 per quarter

Example: Growth Perpetuity

- The expected payment next year is $1.30, and payments are expected to grow at 5% forever. If the discount rate is 10%, what is the value of this promised cash flow stream?

- Solution:
  - $PV=\frac{\$1.30}{10\%-5\%}=\$26$

Example: Irregular Starting Time

Suppose you just won $10 million in a lottery. You have 3 choices:

1) Get the $10 million now
2) Get $1 million per year, starting from now, for 12 years
3) Get $2 million per year from the end of year 5 to year 11

Suppose you think the fair interest rate is 7% per year. Which should you prefer?

	PV	FV (at the end of year 11)
1	10M	$10\text{M} * 1.07^{11} = 21.049\text{M}$
2	$1\text{M}\left(1 + \frac{1 - 1.07^{-11}}{0.07}\right) = 8.499\text{M}$	$1\text{M}\frac{1.07^{12} - 1}{0.07}1.07 = 17.889\text{M}$
3	$2\text{M}(1.07)^{-4}\frac{1 - 1.07^{-7}}{0.07} = 8.223\text{M}$	$2\text{M}\frac{1.07^{7} - 1}{0.07} = 17.308\text{M}$

Example: Non-integer Time Interval

Compare the following 4 schemes of salary, assuming 5% EAR:

1) $1000 per month, paid at beginning of each month
2) $1004 per month, paid at the end of each month
3) $501 paid at the beginning and the middle of each month
4) $502 paid at the middle and the end of each month

Since these four cash flow streams offer the same EAR and covers the same # of months, we can simply compare the month-end value for each payments:

1) $1000 * 1.05^{1/12} \approx 1004.07$
2) 1004
3) $501 * 1.05^{1/12} + 501 * 1.05^{1/24} \approx 1005.06$
4) $502 * 1.05^{1/24} + 502 \approx 1005.02$

Note that interest rate/month = $(1 + 5\%)^{1/12} - 1$

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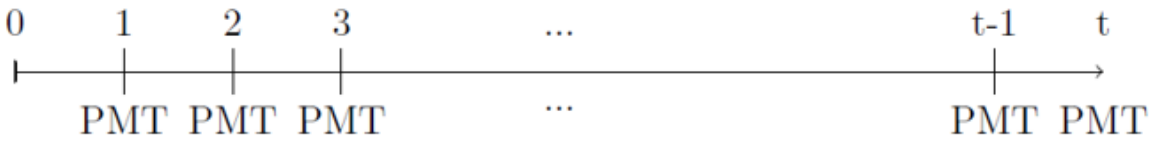
What is amortized loan?

- **Amortized loan**: a loan where the principal of the loan is paid down over the loan according to an amortized loan schedule, typically through equal payments.
  - One typical example is mortgage (home loan).
  - In contrast, bullet loan is one where principal is paid at one shot at the maturity date.
- Each payment consists of two parts
  1. Principal repayment
  2. Interest payment

Two questions:

- What is the amount of each payments?
- Out of each payment, how much of it goes towards repaying the principal and how much of it goes towards paying the interest?

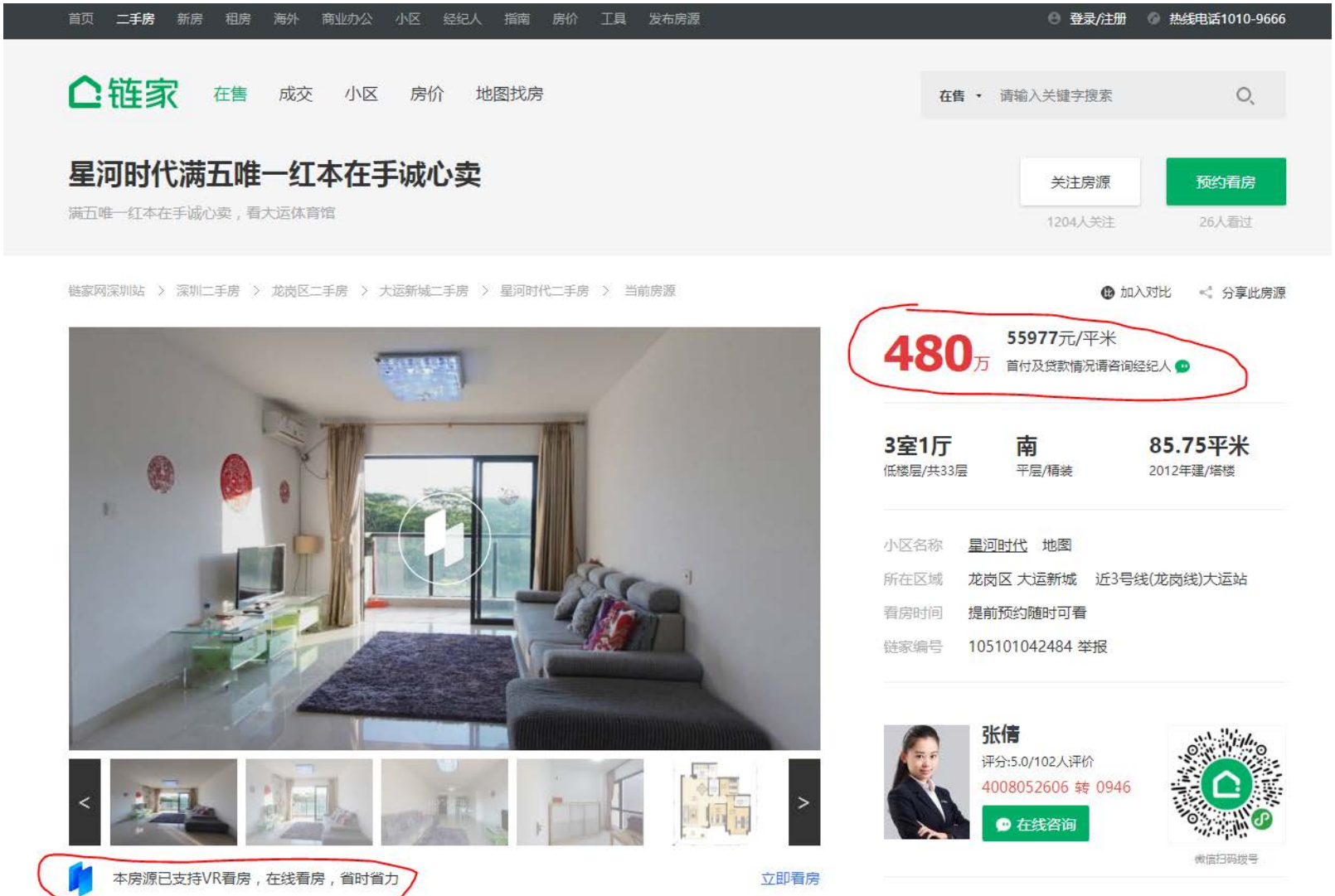
How do we solve for the PMT?



Current balance (at any time) = PV of all future payments

- You can think of the future payments as the benefits banks receive and the loan amount they lend out as the costs. The benefits have to equal to the costs after adjusting for the interest rate they charge.
- Or we can prove this using backward induction:
  - On the last payment date t, how much do I owe? Does the relationship hold?
  - On date $t-1$, how much do I owe after making the payment? How much do I owe before the payment? Does the relationship hold?
  - In general, how much do I owe on date s for some $s < t$ after making the payment due on date s?

Example—Home Mortgage



Loan Amount and Loan Rate

深圳市最全类别限购限贷政策一览(更新于2020年5月13日)

家庭户籍情况	婚姻情况	可在深圳购买住宅套数	房贷情况	最低首付成数	需符合条件
深户（夫妻一方为深户即算深户）	未婚、离异丧偶	1	无房贷记录	3成	因离异时间长短会影响贷款情况，此处婚姻情况中的离异指的是离异2年后
			房贷已结清 有一笔房贷未结清	5成	
			有2笔及以上房贷未结清	停贷	
	已婚	2	无房贷记录	3成	在深圳无房
			房贷已结清 有一笔房贷未结清	5成	
			无房贷记录/贷款已结清 有一笔贷款未结清	7成	在深圳有1套房
			有2笔及以上房贷未结清	停贷	/

深圳商业银行房贷利率情况（2020.5.13）

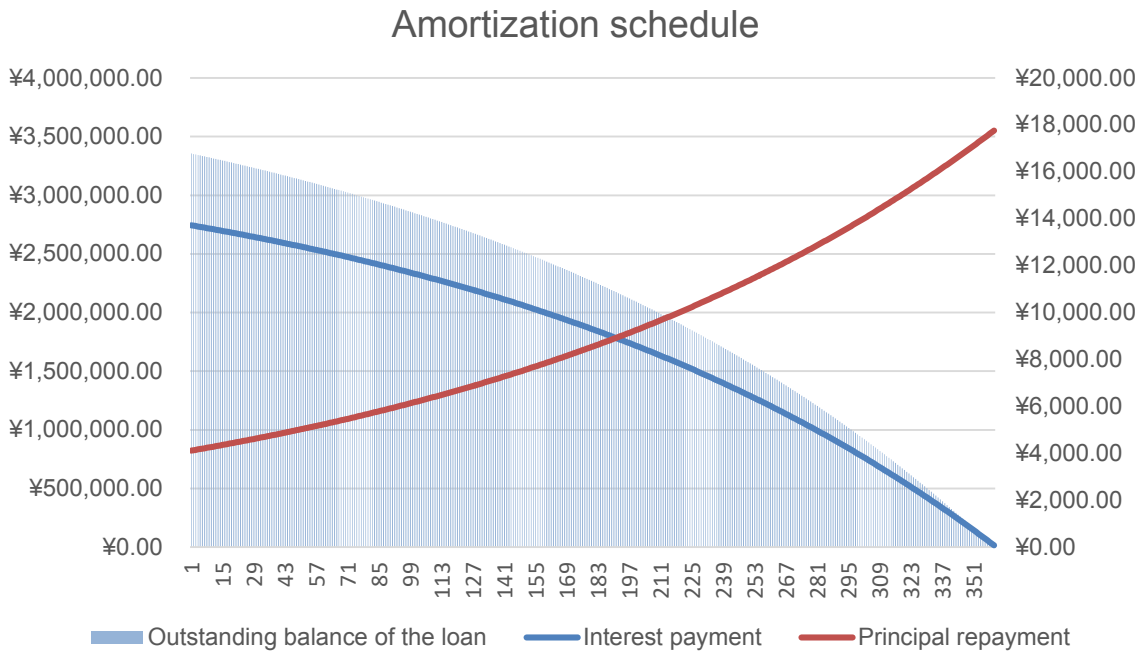
银行	首套住宅		二套住宅	
	利率	首付	利率	首付
中国银行	LPR+30BP（4.95%）	三成	LPR+60BP（5.25%）	七成
农业银行	LPR+30BP（4.95%）	三成	LPR+60BP（5.25%）	七成
工商银行	LPR+30BP（4.95%）	三成	LPR+60BP（5.25%）	七成
建设银行	LPR+30BP（4.95%）	三成	LPR+60BP（5.25%）	七成
交通银行	LPR+30BP（4.95%）	三成	LPR+60BP（5.25%）	七成

Assume you are buying your first apartment.

- Total price: ￥4,800,000
- Down payment: 4,800,000*30%
  =1,440,000
- Loan amount (PV):
  4,800,000*70%
  = 3,360,000
  - What is LPR? [1] [2]
- Interest rate per month (r)
  =4.95%/12
- Monthly payment= ￥17,935

Amortization Schedule

- Definition: a table that shows the breakdown of each payment
  - Interest payment: goes down over time
  - Principal repayment: goes up over time
- For each payment, interest = APR/m * outstanding balance
- Outstanding balance = PV of all remaining payments.



Example-Amortized Schedule

- In the previous example, 1) What is your amortization schedule? 2) how much do you owe after the 358th payment? 3) what is the total interest payment over the 30-year period?
  - 1)

Month	Remaining balance of principal owed (1)	Monthly payment (2)	Interest payment (3)=(1)*r	Repayment of principal (4)=(2)-(3)	Outstanding balance of the loan (5)=(1)-(4)
1	¥3,360,000.00	¥17,832	¥13,720.00	¥4,112.42	¥3,355,887.58
2	¥3,355,887.58	¥17,832	¥13,703.21	¥4,129.21	¥3,351,758.37
3	¥3,351,758.37	¥17,832	¥13,686.35	¥4,146.07	¥3,347,612.30
…	…	…	…	…	…

  - 2) Two payments left.
    - Outstanding balance=$\frac{17,832}{(1+0.408\%)^1}+\frac{17,832}{(1+0.408\%)^2}=35,446.92$
  - 3) Total interest payment:
    - Total payments – principal $=17,832*360-3,360,000=3,059,520$

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Net Present Value

- Definition: PV of cash **in**flows– PV of cash **out**flows
- You can think of it as benefits-costs
- Calculating the NPV of future cash flows allows us to evaluate an investment decision.
  - If the benefit ≥ cost: accept the investment
  - If the benefit< cost: decline the investment

Example – NPV

- You are considering an investment that will pay you $1,000 in one year, $2,000 in two years and $3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

- According to calculation, PV of the investment=$4,815.93

- Will you be willing to pay $5,000 for the investment?
  - No
- What is the NPV?
  - 4,815.93 - 5,000 = -184.07

Example – NPV

- Would you be willing to pay $6,000 for the following stream of cash flows if the discount rate is 7%?
  - Year 1: 3000 Year 2: 2000 Year 3: 2000

- Solution
  - The present value of the benefits is:

    $3000 / (1.07) + 2000 / (1.07)^2 + 2000 / (1.07)^3 = 6183.21$

  - The present value of the cost is $6,000, because it occurs now.
  - The *NPV* = *PV*(benefits) – *PV*(cost)

    = 6183.21 – 6000 = 183.21

The Internal Rate of Return (IRR)

- Definition: the return that makes the NPV = 0.
- Interpretation: it is the return that makes the benefits of an investment equal to its costs.
- Calculation: it is the same as calculating r in any time value of money question.

Example - IRR in perpetuity

- Jessica has just graduated with her MBA. Rather than take the job she was offered at a prestigious investment bank – Baker, Bellingham, and Botts – she has decided to go into business for herself. She believed that her business will require an initial investment of $1million. After that, it will generate a cash flow of $100,000 at the end of one year, and this amount will grow by 4% per year thereafter forever. What is the IRR of this investment opportunity?

- Solution
  - $1{,}000{,}000 = \frac{100{,}000}{r-0.04}$
  - r = 0.14

Example - IRR in annuity

- Baker was so impressed with Jessica that it has decided to fund her business. In return for providing the initial capital of $1million, Jessica has agreed to pay them $125,000 at the end of each year for the next 30 years. What is the internal rate of return for Baker's investment on Jessica's company, assuming she fulfills her commitment?

- **Solution**
  - Annuity problem
  - $1{,}000{,}000 = 125{,}000 * \frac{1}{r}\left(1 - \frac{1}{(1+r)^{30}}\right)$
  - r = 12%

Example - IRR in non-standardized cash flows

- You are considering an investment that will pay you $1,000 in one year, $2,000 in two years and $3,000 in three years. If you pay $5,000 for this investment, what is your IRR?

- Solution:
  - $\frac{1,000}{(1+r)^1} + \frac{2,000}{(1+r)^2} + \frac{3,000}{(1+r)^3} = 5,000$
  - To solve r, can use trial-and-error, financial calculator, or excel.
  - r = 8.21%

Interpretations of PV, NPV and IRR

- Assume you are considering a project that are expected to generate certain cash flows.
- PV:
  - What is the future cash flows worth to me **IF** I use a certain discount rate r?
- NPV:
  - We have to pay certain cost if we are to receive the future cash flows.
  - **IF** I use a certain discount rate r, what is the net benefit to me after adjusting for the costs?
  - In other words, benefit-cost=?
- IRR:
  - At what discount rate is the NPV of the project=0?
  - In other words, when do we break even?

Example—Installments

- Financial firms sometimes do not quote an APR—instead, they quote a so called “fee ratio”

各家银行信用卡账单分期手续费率对比

	3期	6期	9期	12期	18期	24期
广发银行		4.20%		8.04%	12.96%	17.28%
兴业银行	2.40%	3.90%		7.80%	11.70%	15.60%
中信银行	2.40%	4.80%	6.84%	8.76%	13.50%	18.00%
光大银行	2.65%	4.65%	6.45%	8.85%		
民生银行	2.46%	4.20%	6.03%	8.04%	12.06%	16.80%
中国银行	1.95%	3.60%	5.40%	7.20%	11.70%	15.00%
工商银行	1.65%	3.60%	5.40%	7.20%	11.70%	15.60%
农业银行	1.80%	3.60%	5.40%	7.20%		14.40%
建设银行	2.25%	4.20%		7.20%	10.80%	14.88%
交通银行	2.16%	4.32%	6.48%	8.64%	12.96%	17.28%
平安银行	2.10%	4.08%	5.94%	14.40%		
招商银行	2.70%	4.50%		7.92%	12.24%	16.32%

What is “fee ratio”?

- Suppose you take a ￥10,000 debt and plan to pay back with 6 monthly installments. The fee ratio is 4.8%.
  - Every month, you need to pay $10{,}000 * (1 + 4.8\%)/6 = 1746.67$
  - What is the EAR?

$$10{,}000 = PV = 1746.67\frac{1-(1+r_m)^{-6}}{r_m}$$

  - Solve for r_m and we get $r_m = 1.356\%$

Equivalent $\text{EAR} = (1 + 1.356\%)^{12} - 1 = 17.55\%$
Equivalent $\text{APR} = 1.356\% \times 12 = 16.28\%$

Beware of the "fee ratio"!

	3 Periods			6 Periods			12 Periods		
Per period fee	Total fee	APR	EAR	Total fee	APR	EAR	Total fee	APR	EAR
0.60%	1.80%	10.77%	11.32%	3.60%	12.24%	12.95%	7.20%	13.03%	13.84%
0.65%	1.95%	11.66%	12.31%	3.90%	13.25%	14.09%	7.80%	14.10%	15.05%
0.70%	2.10%	12.56%	13.30%	4.20%	14.26%	15.23%	8.40%	15.16%	16.26%
0.75%	2.25%	13.45%	14.31%	4.50%	15.27%	16.38%	9.00%	16.22%	17.48%
0.80%	2.40%	14.34%	15.32%	4.80%	16.27%	17.54%	9.60%	17.27%	18.71%
0.85%	2.55%	15.24%	16.35%	5.10%	17.28%	18.72%	10.20%	18.32%	19.94%
0.90%	2.70%	16.13%	17.38%	5.40%	18.28%	19.90%	10.80%	19.37%	21.19%
0.95%	2.85%	17.02%	18.41%	5.70%	19.29%	21.09%	11.40%	20.42%	22.44%

Summary

- Calculate the PV and FV of annuities
  - Make sure you use the appropriate discount factor for each cash flow, and count the number of cash flows correctly
  - Know how to deal with irregular cash flow streams
  - (Know how to use the financial calculator)
- Amortization
  - Know how to calculate monthly payments of a loan
  - Know how to calculate the remaining balance
- NPV and IRR
  - NPV: PV of both investments and future cash flows of a project
  - IRR: implied interest rate that sets NPV=0

Next Lecture—What is a Firm?

- Organizational Forms of a Firm
  - Sole proprietorship
  - Partnership
  - Limited liability company (LLC)
  - Corporation
- Financial Decisions of a Firm
- A Firm's Goals when Making Financial Decisions
- Agency Problems