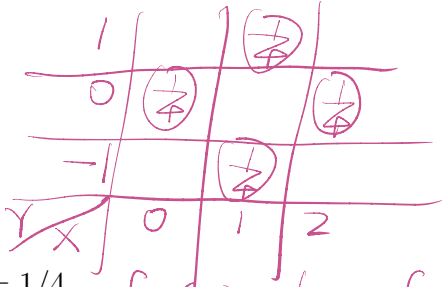
STA2001 Tutorial 8

1. 4.1-6. The torque required to remove bolts in a steel plate is rated as very high, high, average, and low, and these occur about 25%, 35%, 20%, and 20% of the time, respectively. Suppose $n = 31$ bolts are rated; what is the probability of rating 9 very high, 10 high, 7 average, and 5 low? Assume independence of the 31 trials.

VH 25% 9
H 35% 10
A 20% 7
L 20% 5

$$\frac{31!}{9!\,10!\,7!\,5!}\left(\frac{25}{100}\right)^9\left(\frac{35}{100}\right)^{10}\left(\frac{20}{100}\right)^7\left(\frac{20}{100}\right)^5$$



2. 4.2-7 Let the joint pmf of X and Y be

$$f(x, y) = 1/4$$

where $(x, y) \in S = \{(0, 0), (1, 1), (1, -1), (2, 0)\}$.

$f_X(0) = \frac{1}{4}$ $f_Y(-1) = \frac{1}{4}$

$f_X(1) = \frac{1}{2}$ $f_Y(0) = \frac{1}{2}$

$f_X(2) = \frac{1}{4}$ $f_Y(1) = \frac{1}{4}$

(a) Are X and Y independent? dependent

(b) Calculate $\text{cov}(X, Y)$ and ρ .

This exercise also illustrates the fact that dependent random variables can have a correlation coefficient of zero.

$E[X] = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 = 1$

$E[Y] = 0$

$E[XY] = 0$

$0 - 0 = 0 = \text{cov}(X, Y)$

$\rho = 0$

3. 4.2-8. A certain raw material is classified as to moisture content X (in percent) and impurity Y (in percent). Let X and Y have the joint pmf given by

y\x	1	2	3	4
1	0.05	0.05	0.15	0.1
2	0.1	0.2	0.3	0.05

(a) Find the marginal pmfs, the means, and the variances of X and Y, respectively.

(b) Find the covariance and the correlation coefficient of X and Y.

(c) If additional heating is needed with high moisture content and additional filtering with high impurity such that the additional cost is given by the function $C = 2X + 10Y^2$ in dollars, find $E(C)$.