



香港中文大學(深圳)  
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# Introduction to Computer Science: Programming Methodology

## Lecture 8 Data Structure and Algorithm - Intro

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# Data structure and algorithm

- A **data structure** is a systematic way of organizing and accessing data
- An **algorithm** is a step-by-step procedure for performing some task in a finite amount of time.

# Why study data structure and algorithm?

- Important for **all other branches** of computer science
- Plays a **key role** in modern technological innovation
- **Moore's law** predicts that the density of transistors in integrated circuits would continue to double every 1 to 2 years
- However, in many areas, performance gains due to the **improvements in algorithms** have **greatly exceeded** even the dramatic performance gains due to increased processor speed

# Why study data structure and algorithm?

- Provide novel “lens” on processes outside of computer science and technology, such as physics, economic markets, evolution
- Challenging (good for your brain!!) and funny

# Example: Integer Multiplication

- **Inputs:** two n-digits number  $x$  and  $y$
- **Output:** the product of  $x$  and  $y$
- **Primitive operations:** add or multiply 2 single digit numbers

# The algorithm designer' s mantra

- “Perhaps the most important principle for the good algorithm designer is to refuse to be content”

Aho, Hopcroft, and Ullman, *The Design and Analysis of Computer Algorithms*, 1974

# How do we define a “good” algorithm?

- The primary analysis of algorithms involves characterizing the **running times** and **space usage** of algorithms and data structure operations
- **Running time** is a natural measure of “goodness,” since time is a precious resource—computer solutions should run as fast as possible
- **Space usage** is another major issue to consider when we design an algorithm, since we only have limited storage spaces

# Measuring the running time experimentally

```
from time import time

startTime = time()

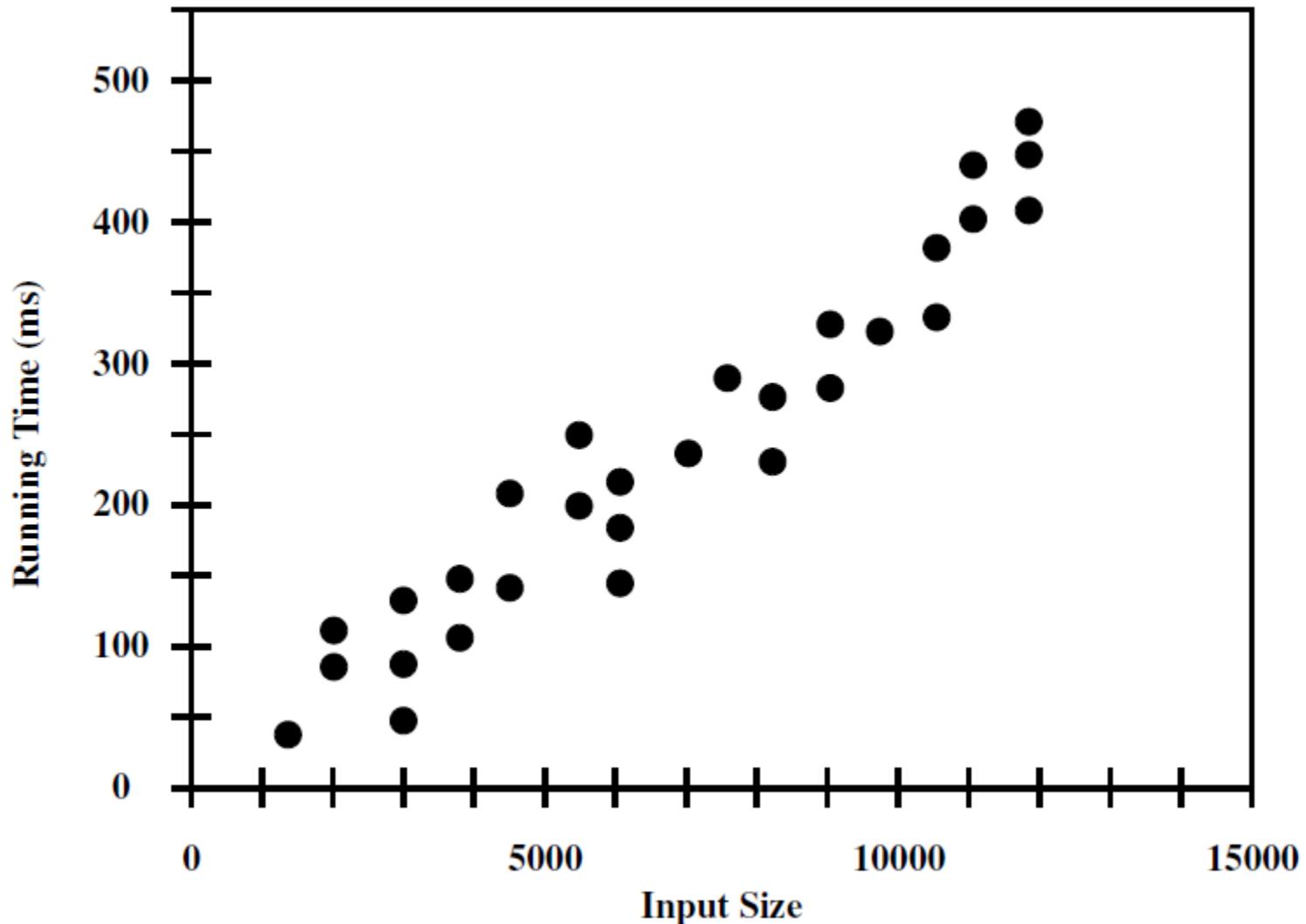
for i in range(1, 20000):
    if i%10 == 0:
        print(i)

endTime = time()

print('The time elapsed is:', endTime - startTime, 'seconds')
```

# Visualize the running time

- Running time and space usage are dependent on the **size of the input**
- Perform independent experiments on many different **test inputs of various sizes**
- Visualize the results by plotting the performance of each run of the algorithm as a point



# Challenges of experimental analysis

- Experimental running times of two algorithms are difficult to directly compare unless the experiments are performed in the same hardware and software environments
- Experiments can be done only on a limited set of test inputs; hence, they leave out the running times of inputs not included in the experiment (and these inputs may be important)
- An algorithm must be fully implemented in order to execute it to study its running time experimentally

算法分析三原则

# Principle of algorithm analysis 1: Counting primitive operations

- To analyse the running time of an algorithm without performing experiments, we perform an analysis directly on a high-level description of the algorithm
- We define a set of primitive operations such as the following:
  - ✓ Assigning a variable to an object  $a = 10$
  - ✓ Determining the object associated with a variable  $a = b + 10$
  - ✓ Performing an arithmetic operation (for example, adding two numbers)  $a = b + 10$
  - ✓ Comparing two numbers  $a >= b$
  - ✓ Accessing a single element of a Python list by index  $b = a[10]$
  - ✓ Calling a function (excluding operations executed within the function)  $\text{print}['...']$
  - ✓ Returning from a function.

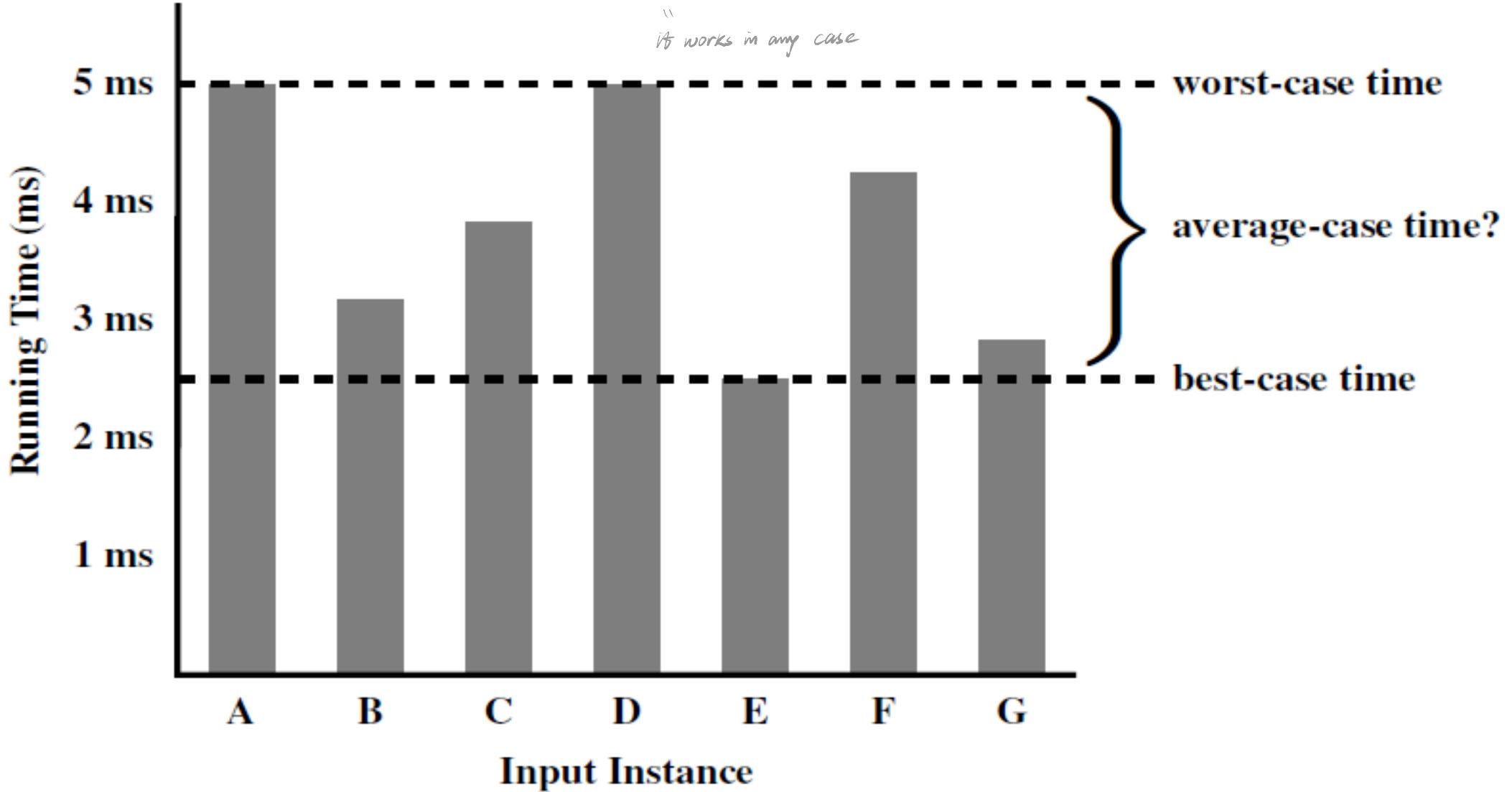
# Principle of algorithm analysis 2: Measuring Operations as a Function of Input Size

*Find out f(n) represents the # of ops. n  $\Rightarrow$  input size*

- To capture the order of growth of an algorithm's running time, we will associate, with each algorithm, a function  $f(n)$  that characterizes the number of primitive operations that are performed as a function of the problem size  $n$

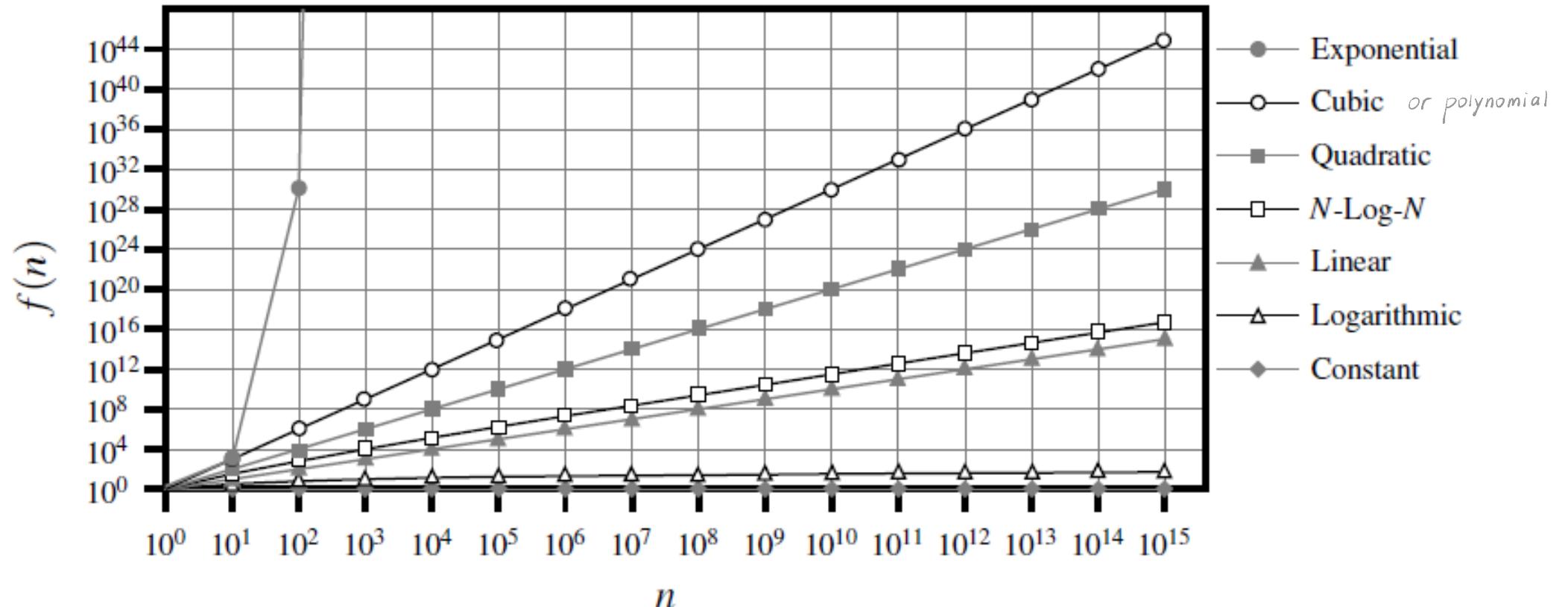
# Principle of algorithm analysis 3: Focusing on the Worst-Case Input

*in most cases*



# The 7 functions used in algorithm analysis

- We may use the following 7 functions to measure the time complexity of an algorithm: **constant, logarithm, linear, N-log-N, quadratic, cubic and other polynomials, exponential**



渐近分析

# Asymptotic Analysis



- In algorithm analysis, we focus on the growth rate of the running time as a function of the input size  $n$ , taking a “big-picture” approach
- Vocabulary for the analysis and design of algorithms
- “Sweet spot” for high-level reasoning about algorithms
- Coarse enough to suppress unnecessary details, e.g. architecture/language/compiler... *fair comparison*
- Sharp enough to make meaningful comparisons between algorithms

# The big Oh notation

物理意义

- Let  $f(n)$  and  $g(n)$  be functions mapping positive integers to positive real numbers.
- We say that  $f(n)$  is  $O(g(n))$  if there is a real constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that

$$f(n) \leq cg(n), \text{ for } n \geq n_0$$

upper bound 上界  $\Rightarrow$  worst-case time

- This definition is often referred to as the “big-Oh” notation
- Example: The function  $8n+5$  is  $O(n)$ .

(eg.  $f(n) = n$   $c=9$   $n_0=10$ )

# The big Oh notation

- The big-Oh notation allows us to say that a function  $f(n)$  is “less than or equal to” another function  $g(n)$  up to a constant factor and **in the asymptotic sense** as  $n$  grows toward infinity
- The big-Oh notation is used widely to characterize **running times** and **space bounds** in terms of some parameter  **$n$** , which varies from problem to problem, but is always defined as a chosen measure of the “**size**” of the problem

# Some Properties of the Big-Oh Notation

- The big-Oh notation allows us to ignore constant factors and lower-order terms and focus on the main components of a function that affect its growth
- Example:  $5n^4 + 3n^3 + 2n^2 + 4n + 1$  is  $O(n^4)$
- Example:  $2^{n+2}$  is  $O(2^n)$
- Example:  $2n + 100\log n$  is  $O(n)$
- In general, we should use the big-Oh notation to characterize a function as closely as possible

# Comparative analysis

**Question:** Suppose two algorithms solving the same problem are available: an algorithm A, which has a running time of  $O(n)$ , and an algorithm B, which has a running time of  $O(n^2)$ . Which algorithm is better?

**Answer:** Algorithm A is asymptotically better than algorithm B

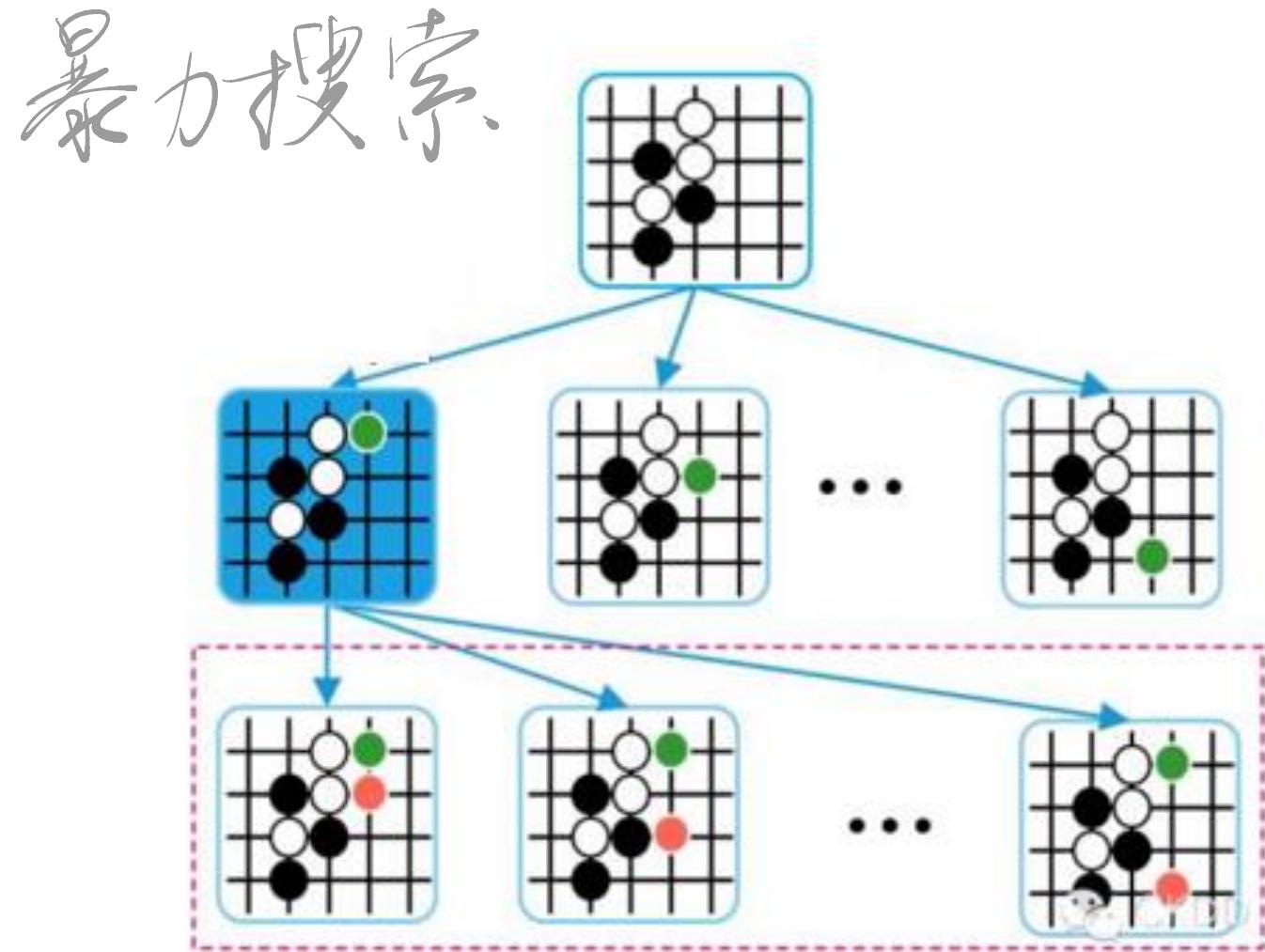
# Comparative analysis

- We can use the big-Oh notation to order classes of functions by asymptotic growth rate
- Our seven functions are ordered by increasing growth rate in the following sequence

$n$	$\log n$	$n$	$n \log n$	$n^2$	$n^3$	$2^n$
8	3	8	24	64	512	256
16	4	16	64	256	4,096	65,536
32	5	32	160	1,024	32,768	4,294,967,296
64	6	64	384	4,096	262,144	$1.84 \times 10^{19}$
128	7	128	896	16,384	2,097,152	$3.40 \times 10^{38}$
256	8	256	2,048	65,536	16,777,216	$1.15 \times 10^{77}$
512	9	512	4,608	262,144	134,217,728	$1.34 \times 10^{154}$

# Why AlghaGo is a remarkable achievement?

- If we use brutal-force to search the best move in Go, the time complexity is at the order of  $O(10^n)$



# The line of tractability

可行性

- To differentiate **efficient** and **inefficient** algorithms, the general line is between **polynomial time algorithms** and **exponential time algorithms**
- The distinction between polynomial-time and exponential-time algorithms is considered a robust measure of **tractability**

# Example: Finding the smallest number in a list

1. primitive op: line
2. Input Size: size of list

3.  $f(n) = \begin{cases} 4 & g(n) = n \\ 1 & \times n+3 = 5n+3 \\ 4 & \end{cases}$

$$f(n) \sim O(n)$$

linear

```
smallest_so_far = None
print('Before', smallest_so_far)

for num in [9, 39, 21, 98, 4, 5, 100, 65]:
    if smallest_so_far == None:
        smallest_so_far = num
    elif num < smallest_so_far:
        smallest_so_far = num
    print(smallest_so_far, num)

print('After', smallest_so_far)
```

first ignore

- What is the time complexity of this algorithm?

# Recursion

递归

a class of 方法

- Recursion is a technique by which a function makes one or more calls to itself during execution
- Recursion provides an elegant and powerful alternative for performing **repetitive tasks**
- Recursion is an important technique in the study of data structures and algorithms

# Inception





## Example: The factorial function

- The **factorial** of a positive integer  $n$ , denoted  $n!$ , is defined as follows:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1 & \text{if } n \geq 1. \end{cases}$$

- The factorial function is important because it is known to equal the number of ways in which  $n$  distinct items can be arranged into a sequence, that is, the number of permutations of  $n$  items

用递归解决问题，首先要给出递归的定义

# The recursive definition

- First, a recursive definition contains one or more **base cases**, which are defined **non-recursively** in terms of fixed quantities
- Second, it also contains one or more **recursive cases**, which are defined by appealing to the definition of the function being defined

# The recursive definition of factorial function

- The factorial function can be naturally defined in a recursive way, for example,  $5! = 5 \cdot (4 \cdot 3 \cdot 2 \cdot 1) = 5 \cdot 4!$
- More generally, for a positive integer  $n$ , we can define  $n!$  to be  $n \cdot (n-1)!$
- Therefore, the recursive definition of factorial function is:

$$\underline{n!} = \begin{cases} 1 & \text{if } n = 0 \\ \underline{n \cdot (n-1)!} & \text{if } n \geq 1. \end{cases}$$

用自己定义自己：递归定义

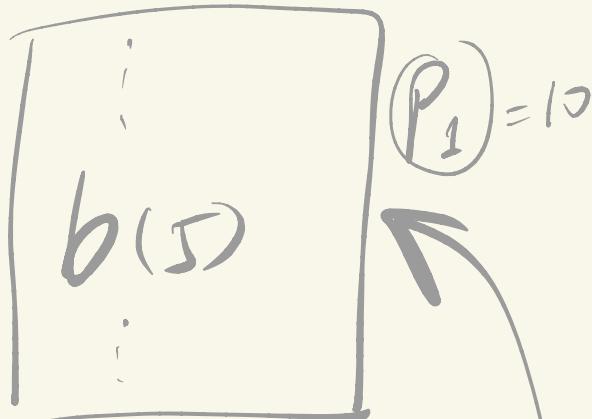
# Solution

```
def facFunc(n):  
    if n<0:  
        print(' Invalid input. ')  
        return None  
    elif n == 0:  base case  
        return 1  
    else:  
        return n*facFunc(n-1)  
 recursive case
```

# How Python implements recursion

- In Python, each time a function (recursive or otherwise) is called, a structure known as an **activation record** or **frame** is created to store information about the progress of that invocation of the function
- This activation record stores the function call's **parameters** and **local variables**
- When the execution of a function leads to a nested function call, the execution of the former call is suspended and its activation record stores the place in the source code at which the **flow of control should continue** upon return of the nested call

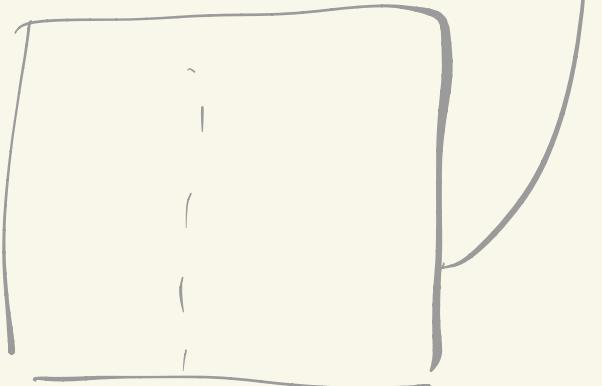
def a( $P_1$ )



(Suppose)

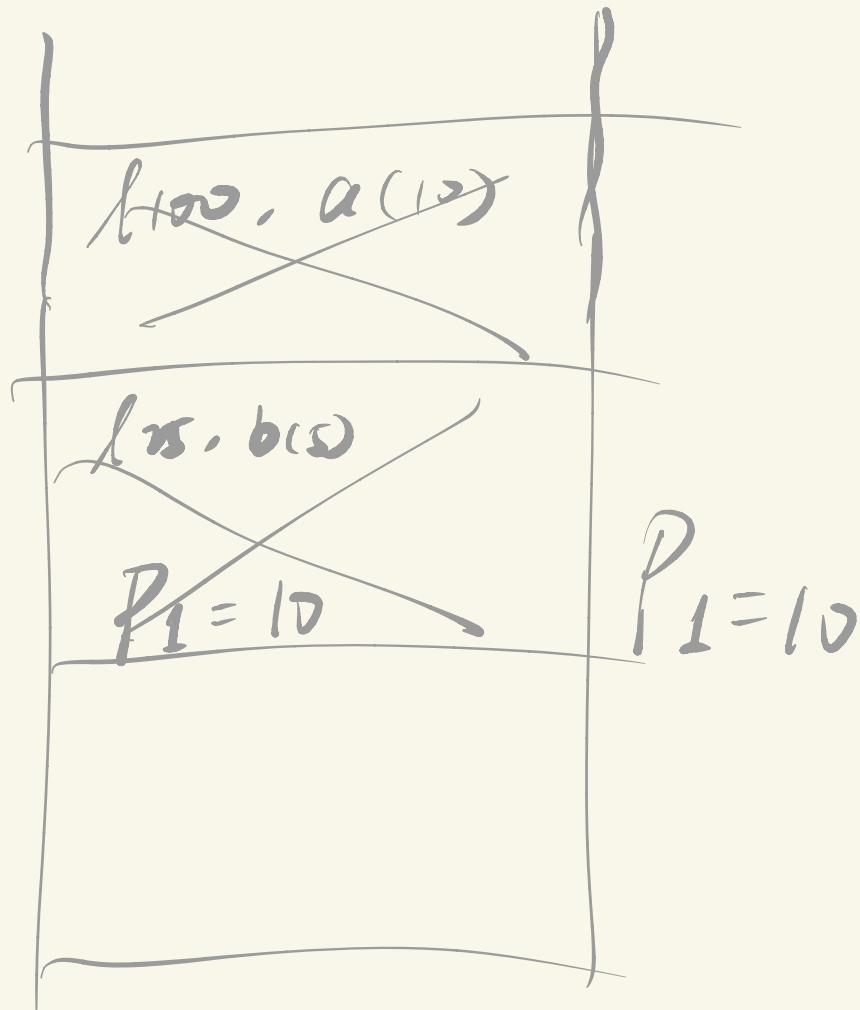
$l \in$

def b( $P_2$ )



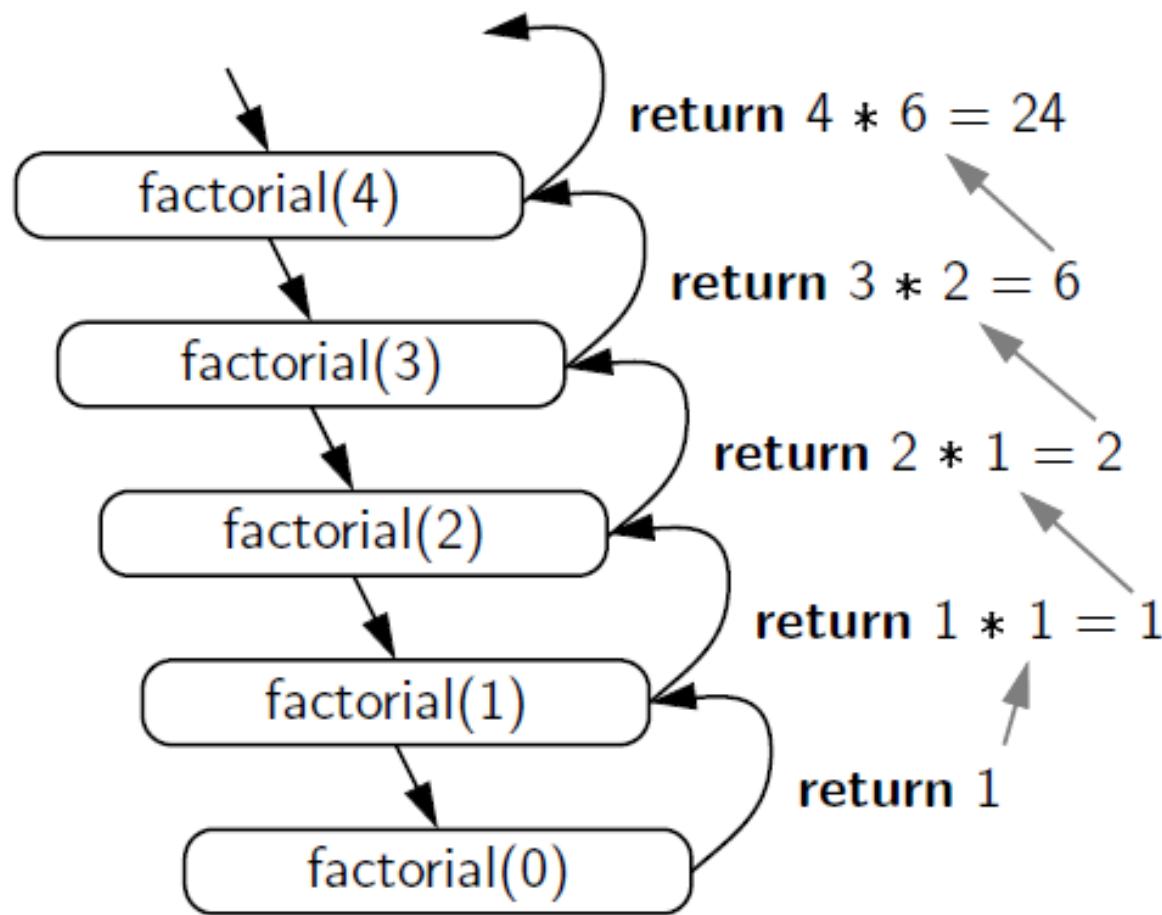
$a(10)$

1. location to call a func
2. Values of local, global variables



$l_{100}$

# The recursive trace



# Example: Binary search

二分查找  
搜索

- A classic and very useful recursive algorithm, **binary search**, can be used to efficiently locate a target value within a **sorted** sequence of **n** elements
- When the sequence is **unsorted**, the standard approach to search for a target value is to use a loop to examine every element, until either finding the target or exhausting the data set; This is known as the **sequential search** algorithm

Much more 效率

顺序查找  
搜索  
(遍历一遍)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	5	7	8	9	12	14	17	19	22	25	27	28	33	37

$O(n)$

# Binary search

- When the sequence is sorted and indexable, binary search is a much more efficient algorithm
- For any index  $j$ , we know that all the values stored at indices  $0, \dots, j-1$  are less than or equal to the value at index  $j$ , and all the values stored at indices  $j+1, \dots, n-1$  are greater than or equal to that at index  $j$

Information is power.

# The strategy of binary search

- We call an element of the sequence a **candidate** if, at the current stage of the search, we cannot rule out that this item matches the target  
*标记 candidate to be checked*
- The algorithm maintains two parameters, low and high, such that all the candidate entries have index at least **low** and at most **high**
- Initially, **low = 0** and **high = n-1**. We then compare the target value to the median candidate, that is, the item **data[mid]** with index  
$$\text{mid} = \lfloor (\text{low}+\text{high})/2 \rfloor$$

# The strategy of binary search

- If the target equals `data[mid]`, then we have found the item we are looking for, and the search terminates successfully
- If `target < data[mid]`, then we recur on the first half of the sequence, that is, on the interval of indices from `low` to `mid-1`
- If `target > data[mid]`, then we recur on the second half of the sequence, that is, on the interval of indices from `mid+1` to `high`

# Solution

```
def binarySearch(data, target, low, high):
    if low>high:
        print(' Cannot find the target number! ')
        return False
    else:
        mid = (low+high)//2
        if target==data[mid]:
            print(' The target number is at position ', mid)
            return True
        elif target<data[mid]:
            return binarySearch(data, target, low, mid-1)
        else:
            return binarySearch(data, target, mid+1, high)

def main():
    data = [1, 3, 5, 6, 16, 78, 100, 135, 900]
    target = 16
    binarySearch(data, target, 0, len(data)-1)
```

# Time complexity of binary search

**Proposition:** The binary search algorithm runs in  $O(\log n)$  time for a sorted sequence with  $n$  elements

Why ?

# Proof

**Justification:** To prove this claim, a crucial fact is that with each recursive call the number of candidate entries still to be searched is given by the value  $\text{high} - \text{low} + 1$ .

Moreover, the number of remaining candidates is reduced by at least one half with each recursive call. Specifically, from the definition of  $\text{mid}$ , the number of remaining candidates is either

$$(\text{mid} - 1) - \text{low} + 1 = \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor - \text{low} \leq \frac{\text{high} - \text{low} + 1}{2}$$

or

$$\text{high} - (\text{mid} + 1) + 1 = \text{high} - \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor \leq \frac{\text{high} - \text{low} + 1}{2}.$$

Initially, the number of candidates is  $n$ ; after the first call in a binary search, it is at most  $n/2$ ; after the second call, it is at most  $n/4$ ; and so on. In general, after the  $j^{\text{th}}$  call in a binary search, the number of candidate entries remaining is at most  $n/2^j$ . In the worst case (an unsuccessful search), the recursive calls stop when there are no more candidate entries. Hence, the maximum number of recursive calls performed, is the smallest integer  $r$  such that

$$\frac{n}{2^r} < 1.$$

In other words (recalling that we omit a logarithm's base when it is 2),  $r > \log n$ . Thus, we have  $r = \lfloor \log n \rfloor + 1$ ,

which implies that binary search runs in  $O(\log n)$  time. ■

• Binary Search 二分查找 / 二分搜索.

Primitive op.: do recursion / call binarySearch,

$\frac{n}{2 \times 2 \times 2} = 1$

$\frac{n}{2^c} = 1$

$f(n) = \Theta(\log_2 n) \sim O(\log n)$

data = [ ~~1, 3, 5, 6, 7~~ ]  
                ↑  
                mid  
                ↑  
                high  
                ↓  
                low



Practice:

( 92 T 78 )

- How to solve the 8 queens problem using Recursion?

# Solution:

$x[i] = j$ : Queen on row  $i$ , col  $j$   
行 列

```
#Initialization  
N = 8 #8 Queens;  
x = [0]*(N+1) #Save the location of queen at each row;  
total = 0 #Total number of solutions;
```

#Check whether the current placing  
#of queens till row k is valid.

```
def Place(k): #逐行放皇后，k表示放了几行  
    global x, N, total
```

检查前 k 行  
皇后的位置  
是否合法

```
for i in range(1, k):  
    if (x[i] == x[k]) or (abs(i-k) == abs(x[i]-x[k])):  
        return False  
return True
```

#Use recursion to search the valid solutions.

```
def BackTrack(t): #the current row you want to search  
    global x, N, total
```

```
if (t>N): #前8行找完了，找到了合法的解  
    for i in range(1, N+1):  
        print(x[i], end=',')  
    print()  
    total += 1
```

```
else:  
    for i in range(1, N+1):  
        x[t] = i  
        if Place(t):
```

BackTrack(t+1) 有多个 BackTrack 函数同时运行

#Start to find solutions

BackTrack(1)

print('Total number =', total)

# Example: Drawing an English ruler

- We denote the length of the tick designating a whole inch as the ***major tick length***.
- Between the marks for whole inches, the ruler contains a series of ***minor ticks***, placed at intervals of  $1/2$  inch,  $1/4$  inch, and so on.
- As the size of the interval decreases by half, the tick length decreases by one

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(a)

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(b)

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(c)

# Recursive implementation of English ruler

- An interval with a central tick length  $L \geq 1$  is composed of:
  - ✓ An interval with a central tick length  $L-1$
  - ✓ A single tick of length  $L$
  - ✓ An interval with a central tick length  $L-1$

# Solution

```
def draw_line(tickLen, tickLabel=' '):
    line = '-' * tickLen
    if tickLabel:
        line += ' ' + tickLabel
    print(line)

def draw_interval(centerLen):
    if centerLen > 0:
        draw_interval(centerLen - 1)
        draw_line(centerLen)
        draw_interval(centerLen - 1)

def draw_ruler(numInch, majorLen):
    draw_line(majorLen, '0')

    for j in range(1, 1 + numInch):
        draw_interval(majorLen - 1)
        draw_line(majorLen, str(j))
```

# The recursive trace for English ruler

