

LEARNING OBJECTIVES

CHAPTER 1. Functions

Section 1. Functions and Their Graphs

1. Find the domain and range of a function.
2. Determine if a graph is a function.
3. Find a formula for a function.
4. Graph a function and determine its domain.
5. Graph piecewise-defined functions.
6. Find a formula for a piecewise-defined function from its graph.
7. Understand the greatest and least integer functions.
8. Determine where a graph is increasing or decreasing.
9. Identify even and odd functions.
10. Graph linear and power functions.
11. Solve applied problems using functions.

Section 2. Combining Functions; Shifting and Scaling Graphs

1. Find the domain and range of algebraic combinations of functions.
2. Find the domain and range of and sketch the graph of composites of functions.
3. Write the equation for and sketch the graph of a function that has been shifted vertically or horizontally.
4. Write the equation for and sketch the graph of a function that has been transformed by a vertical or horizontal scaling.
5. Write the equation for and sketch the graph of a function that has been transformed by a vertical or horizontal reflection.
6. Write the equation for and sketch the graph of a function that has been transformed by a combination of shifts, scalings, and reflections.

Section 3. Trigonometric Functions

1. Use the formula to find arc length.
2. Evaluate the six trigonometric functions of an angle.
3. Find the value of trigonometric functions given the value of one of them.
4. Graph trigonometric functions.
5. Derive trigonometric identities.
6. Use the angle addition formula.
7. Use the double angle and half-angle formulas.
8. Solve trigonometric equations.
9. Use the law of cosines and the law of sines.
10. Identify the characteristics of the general sine function.

Section 4. Graphing with Software

1. Determine the most appropriate viewing window.
2. Graph functions using the most appropriate window.
3. Make a scatterplot of data, and find the regression line or quadratic curve fit.

CHAPTER 2. Limits and Continuity

Section 1. Rates of Change and Tangents to Curves

1. Find the average rate of change of a function over an interval.
2. Find the slope of the tangent line at a given point.
3. Solve applied problems using rates of change.

Section 2. Limit of a Function and Limit Laws

1. Find the limit from graphs of a function.
2. Find the limit of algebraic functions.
3. Find the limit of trigonometric functions.
4. Recognize the rules for limits.
5. Find the limit using the rules for limits.
6. Evaluate the limit of average rates of change.
7. Use the sandwich theorem.
8. Estimate limits using tables.
9. Find the limit of $f(x)$ given information about f .

Section 3. The Precise Definition of a Limit

1. Center intervals about a point.
2. Find delta graphically.
3. Find delta algebraically.
4. Use the formal definition to find limits.
5. Prove limit statements.
6. Solve applied problems involving limits.
7. Prove that a limit does not exist.

Section 4. One-Sided Limits

1. Find one-sided limits graphically.
2. Find one-sided limits algebraically.
3. Find the limit of trigonometric functions using $\lim (\sin x)/x = 1$.
4. Use the formal definition to find one-sided limits.

Section 5. Continuity

1. Determine where a function is continuous or discontinuous.
2. Use the algebraic properties of continuous functions to prove continuity.
3. Determine where a composite function is continuous.
4. Find limits involving continuous functions.
5. Fill in values to make a function continuous.
6. Use continuity and the Intermediate Value Theorem to solve problems.
7. Solve equations graphically.

Section 6. Limits Involving Infinity; Asymptotes of Graphs

1. Find limits graphically.
2. Find limits as x approaches infinity or negative infinity.
3. Find horizontal or oblique asymptotes.
4. Find limits where $f(x)$ approaches infinity or negative infinity.
5. Find vertical asymptotes.

6. Graph rational functions.
7. Understand the formal definition of a limits involving infinity.
8. Find and graph a function that satisfies given conditions.

CHAPTER 3. Derivatives

Section 1. Tangents and the Derivative at a Point

1. Find the slope of the tangent line at a given point.
2. Find the equation of the tangent line at a given point.
3. Find the point where the graph has a vertical or horizontal tangent.
4. Determine if a function has a tangent at a given point.
5. Find instantaneous rates of change and solve applications.

Section 2. The Derivative as a Function

1. Calculate a derivative from the definition.
2. Use the alternate formula for the derivative.
3. Compute one-sided derivatives.
4. Determine where a function does not have a derivative.
5. Match a graph with the graph of its derivative.
6. Graph the derivative given the graph of a function.
7. Compute derivatives as limits to determine differentiability.
8. Determine differentiability and continuity from a graph.
9. Graph and analyze a function and its derivative.

Section 3. Differentiation Rules

1. Find derivatives using differentiation rules, including the product and quotient rules.
2. Compute second-order and higher-order derivatives.
2. Work with tangent or normal lines.
3. Solve theory and application problems for derivatives.

Section 4. The Derivative as a Rate of Change

1. Understand the relationship between derivatives and instantaneous rates of change.
2. Calculate quantities related to motion, including position, velocity, and acceleration.
3. Solve applications involving motion of a object under gravity.
4. Analyze motion from graphs.
5. Solve other applications of rate of change.

Section 5. Derivatives of Trigonometric Functions

1. Find derivatives of trigonometric functions.
2. Work with tangent lines of trigonometric functions.
3. Find limits that involve trigonometric functions.
4. Solve applications that involve trigonometric functions.

Section 6. The Chain Rule

1. Find derivatives of composite functions using the chain rule.
2. Find derivatives of powers of a function.
3. Find second derivatives using the chain rule.
4. Find the tangent to a curve at a given value.

5. Solve problems involving theory and applications related to the chain rule.

Section 7. Implicit Differentiation

1. Understand implicitly defined functions.
2. Use implicit differentiation to find derivatives.
3. Use implicit differentiation to find second derivatives.
4. Find the slope, tangent line, or normal line at a given point by using implicit differentiation.
5. Solve problems involving theory and applications related to implicit differentiation.

Section 8. Related Rates

1. Solve related rates problems.

Section 9. Linearization and Differentials

1. Find the linearization of a function at a given point.
2. Use the approximation $(1 + x)^k = 1 + kx$.
3. Find the derivative in differential form.
4. Use differentials to estimate the value of a function.
5. Use differentials to find the change in a function f , the value of the estimate df , and the approximation error.
6. Find differential formulas that estimate changes in volume or surface area.
7. Solve applications involving differentials.

CHAPTER 4. Applications of Derivatives

Section 1. Extreme Values of Functions

1. Find extrema from graphs.
2. Find the absolute extrema on finite closed intervals.
3. Find all critical points and local extrema of a function.
4. Find the extreme values and where they occur.
5. Solve applications involving extreme values.

Section 2. The Mean Value Theorem

1. Find the values that satisfy the conclusion of the mean value theorem.
2. Show that a function has exactly one zero in a given interval.
3. Find all possible functions that have a given derivative.
4. Find a function from a given derivative and a given point.
5. Find position from velocity or acceleration.
6. Solve theory and application problems involving the mean value theorem.

Section 3. Monotonic Functions and the First Derivative Test

1. Find intervals on which a function is increasing and decreasing.
2. Use the first derivative test to find local extrema.
3. Find local and absolute extrema in a given domain.
4. Graph a function given its first derivative.
5. Solve theory and application problems by using the first derivative test.

Section 4. Concavity and Curve Sketching

1. Identify inflection points, local extrema, and concavity from a graph.

2. Use the second derivative test to find intervals on which a function is concave up or down.
3. Use the second derivative test for local extrema.
4. Graph equations, find intervals on which the function is increasing/decreasing or concave up/down, find any local extrema, find inflection points, and find any asymptotes.
5. Graph $f(x)$ given the graphs of $f'(x)$ and $f''(x)$.
6. Graph rational functions.
7. Solve applications involving concavity.

Section 5. Applied Optimization

1. Solve mathematical and geometric problems involving optimization.
2. Solve physical problems involving optimization.
3. Solve business and economics problems involving optimization.

Section 6. Newton's Method

1. Use Newton's method to estimate solutions to problems.
2. Solve theory problems related to root-finding.

Section 7. Antiderivatives

1. Find an antiderivative or indefinite integral.
2. Check an antiderivative formula by computing a derivative.
3. Solve initial value problems.
4. Solve applications involving antiderivatives.

CHAPTER 5. Integrals

Section 1. Area and Estimating with Finite Sums

1. Estimate the area under the graph of a function by using upper and lower sums.
2. Solve applications involving estimating an integral.
3. Estimate the average value of a function.

Section 2. Sigma Notation and Limits of Finite Sums

1. Write a sum without sigma notation and evaluate.
2. Write a sum in sigma notation.
3. Find the value of a finite sum.
4. Apply the algebra rules for finite sums.
5. Graph a function and rectangles for a corresponding Reimann sum approximation.
6. Find the norm of a partition.
7. Find a formula for a Reimann sum and calculate the corresponding area.

Section 3. The Definite Integral

1. Express a definite integral as a limit, and a limit as a definite integral.
2. Use the definite integral rules.
3. Evaluate definite integrals by using known formulas and areas.
4. Find an area by evaluating a definite integral.
5. Find the average value of a function.
6. Solve theory problems involving definite integrals.

Section 4. The Fundamental Theorem of Calculus

1. Use the FTC to evaluate definite integrals.
2. Use the FTC to find derivatives of integrals.
3. Express the area of a region as a definite integral and evaluate.
4. Express the solution of an initial value problem as an integral.
5. Solve theory and application problems by using the FTC.

Section 5. Indefinite Integrals and the Substitution Method

1. Use the substitution method to evaluate indefinite integrals.
2. Solve initial value problems.
3. Solve applications involving integrals.

Section 6. Definite Integral Substitution and the Area Between Curves

1. Evaluate definite integrals by using the substitution formula.
2. Find the total area of a region.
3. Find the area of a region enclosed by lines and curves.
4. Solve theory and application problems involving integrals.

CHAPTER 6. Applications of Definite Integrals

Section 1. Volumes Using Cross-Sections

1. Find the volume of a solid by slicing.
2. Find the volume of a solid by the disk method.
3. Find the volume of a solid by the washer method.
4. Find the volume of a solid of revolution.
5. Solve theory and application problems involving volumes.

Section 2. Volumes Using Cylindrical Shells

1. Use the shell method to find the volume of a solid obtained by revolving a region about the y -axis.
2. Use the shell method to find the volume of a solid obtained by revolving a region about the x -axis.
3. Use the shell method to find the volume of a solid obtained by revolving the region about given lines.
4. Determine the appropriate method to use to find the volume of a solid of revolution.
5. Solve theory and application problems involving volumes.

Section 3. Arc Length

1. Find the arc length of a curve.
2. Solve theory and application problems involving arc length.

Section 4. Areas of Surfaces of Revolution

1. Find the area of a surface that has been revolved around the x -axis.
2. Find the area of a surface that has been revolved around the y -axis.
3. Solve theory and application problems involving area of surfaces.

Section 5. Work and Fluid Forces

1. Find the work done by a variable force moving an object.

2. Solve applications involving springs.
3. Find the work required to pump a liquid from a container.
4. Solve applications involving work and kinetic energy.
5. Find fluid forces.

Section 6. Moments and Centers of Mass

1. Find the center of mass of a thin plate with constant density.
2. Find the center of mass of a thin plate with varying density.
3. Find the moment or the center of mass of a thin wire.
4. Find the centroid of a thin plate bounded by curves.
5. Solve problems involving the theorems of Pappus.

CHAPTER 7. Transcendental Functions

Section 1. Inverse Functions and Their Derivatives

1. Identify one-to-one functions graphically by using the horizontal line test.
2. Graph the inverse of a one-to-one function by using symmetry with respect to the line $y = x$.
3. Find the formula for the inverse of a one-to-one function.
4. Find the derivative of the inverse of a function.
5. Solve theory and application problems involving inverse functions.

Section 2. Natural Logarithms

1. Understand the definition and properties of $\ln x$.
2. Use properties of logarithms to simplify expressions.
3. Find derivatives of logarithmic functions.
4. Use logarithmic differentiation.
5. Evaluate integrals involving logarithms.
6. Solve theory and application problems involving logarithms.

Section 3. Exponential Functions

1. Understand the definition and properties of e^x .
2. Find derivatives involving exponential functions.
3. Evaluate integrals involving exponential functions.
4. Solve initial value problems.
6. Solve theory and application problems involving exponential functions.

Section 4. Exponential Change and Separable Differential Equations

1. Understand the definition of exponential change.
2. Solve initial value problems and separable differential equations.
3. Solve application problems involving exponential change.

Section 5. Indeterminate Forms and L'Hôpital's Rule

1. Recognize indeterminate forms where L'Hôpital's Rule is applicable.
2. Use L'Hôpital's Rule to find limits.
3. Use logarithms and L'Hôpital's rule to find limits involving indeterminate powers.
4. Solve theory and application problems involving limits.

Section 6. Inverse Trigonometric Functions

1. Find common values of inverse trigonometric functions.
2. Evaluate expressions involving inverse trigonometric functions.
3. Find limits involving inverse trigonometric functions.
4. Find derivatives of inverse trigonometric functions.
5. Evaluate integrals involving inverse trigonometric functions.
5. Solve theory and application problems related to inverse trigonometric problems.

Section 7. Hyperbolic Functions

1. Find the values of hyperbolic functions.
2. Rewrite expressions in terms of exponentials.
3. Find derivatives of hyperbolic functions.
4. Evaluate integrals of hyperbolic functions.
5. Rewrite expressions in terms of natural logarithms.
6. Evaluate integrals in terms of inverse hyperbolic functions and natural logarithms.
7. Solve theory and application problems related to hyperbolic functions.

Section 8. Relative Rates of Growth

1. Compare the growth rate of functions.
2. Characterize the growth of functions using big-oh and little-oh notation.
3. Solve comparison and other application problems involving rates of growth.

CHAPTER 8. Techniques of Integration

Section 1. Using Basic Integration Formulas

1. Evaluate integrals by using basic integration formulas and algebraic techniques.
2. Solve theory problems related to integrals.

Section 2. Integration by Parts

1. Evaluate indefinite and definite integrals by using integration by parts.
2. Find good choices for u and dv .
3. Evaluate integrals using a substitution prior to integration by parts.
4. Evaluate integrals that require two applications of integration by parts.
5. Evaluate integrals by using reduction formulas.
6. Solve theory and application problems involving integrals.

Section 3. Trigonometric Integrals

1. Evaluate integrals involving powers of sines and cosines.
2. Evaluate integrals involving square roots.
3. Evaluate integrals involving powers of tangents and secants.
4. Evaluate integrals involving products of sines and cosines.
5. Solve application problems involving trigonometric integrals.

Section 4. Trigonometric Substitutions

1. Recognize when a trigonometric substitution will be useful.
2. Evaluate integrals by using trigonometric substitutions.
3. Solve initial value problems.
4. Solve applications problems by using trigonometric substitutions.

Section 5. Integration of Rational Functions by Partial Fractions

1. Expand quotients by partial fractions.
2. Express integrands as a sum of partial fractions and evaluate the integrals.
3. Perform long division on the integrand and evaluate the integral.
4. Solve application problems by using partial fractions.

Section 6. Integral Tables and Computer Algebra Systems

1. Use tables of integrals to evaluate integrals.
2. Use substitution to change an integral into one found in a table of integrals.
3. Use a reduction formula to evaluate integrals.
4. Solve application problems.

Section 7. Numerical Integration

1. Estimate integrals using the trapezoidal rule and Simpson's rule.
2. Estimate the minimum number of subintervals needed to approximate integrals to within a specified error.
3. Estimate the error of an integral evaluated using the trapezoid rule or Simpson's rule.
4. Solve theory and application problems involving numerical integration.

Section 8. Improper Integrals

1. Write an improper integral with infinite limits of integration as a limit.
2. Evaluate improper integrals with infinite limits of integration.
3. Write an improper integral whose integrand has a singularity as a limit.
4. Evaluate improper integrals whose integrand has a singularity.
5. Test improper integrals for convergence or divergence.
6. Solve theory and application problems involving improper integrals.

Section 9. Probability

1. Show that a given function is a probability density function.
2. Find the probability that a random variable has values in a given interval.
3. Find the expected value, mean, and median of a random variable.
4. Find the variance and standard deviation of a random variable.
5. Solve application problems involving exponential or normal distributions.

CHAPTER 9. First-Order Differential Equations**Section 1. Solutions, Slope Fields, and Euler's Method**

1. Recognize first-order differential equations.
2. Verify that a given function is a solution to a first-order initial value problem.
3. Given an equation involving integrals, write an equivalent first-order differential equation with initial condition.
4. Find the slope field for a differential equation.
5. Use Euler's Method to approximate solutions to an initial value problem.

Section 2. First-Order Linear Equations

1. Find the general solution to a first-order linear differential equations.

2. Find the particular solution to a first-order linear differential equation that satisfies a given initial condition.
3. Solve theory problems involving first-order differential equations.

Section 3. Applications

1. Solve motion with resistance applications.
2. Find orthogonal trajectories.
3. Solve mixture problems.

Section 4. Graphical Solutions of Autonomous Equations

1. Determine stable and unstable equilibria.
2. Solve application problems involving autonomous differential equations.

Section 5. Systems of Equations and Phase Planes

1. Use phase-plane analysis to solve applications.

CHAPTER 10. Infinite Sequences and Series

Section 1. Sequences

1. Find terms of a sequence.
2. Find the formula for the n th term of a sequence.
3. Determine if a sequence is monotonic and bounded.
4. Determine if a sequence converges or diverges.
5. Find the limit of a sequence, if one exists.
6. Find the limit of a recursively defined sequence.
7. Solve theory and application problems involving sequences.

Section 2. Infinite Series

1. Find the formula for the n th partial sum of a series.
2. Find the sum of a series, if it converges.
3. Express repeating decimals as the ratio of two integers.
4. Use the n th-term test for divergence.
5. Find the sum of a geometric series and the values for which it converges.
6. Solve theory and application problems involving series.

Section 3. The Integral Test

1. Use the integral test to determine if a series converges or diverges.
2. Estimate bounds for the remainder when using the integral test.
3. Use the integral test to solve theory and application problems involving series.

Section 4. Comparison Tests

1. Use the comparison test to determine if a series converges or diverges.
2. Use the limit comparison test to determine if a series converges or diverges.
3. Use comparison tests to solve theory and application problems involving series.

Section 5. Absolute Convergence; The Ratio and Root Tests

1. Use the Ratio Test to determine whether a series converges absolutely or diverges.
2. Use the Root Test to determine whether a series converges absolutely or diverges.

3. Solve theory problems involving the Root and Ratio Tests.

Section 6. Alternating Series, Absolute and Conditional Convergence

1. Determine if a series converges absolutely, converges conditionally, or diverges.
2. Estimate the error in approximating the sum of an alternating series.
3. Determine the number of terms needed to estimate the sum of an alternating series.
4. Approximate the sum of an alternating series given a specific magnitude of error.
5. Solve theory and application problems involving alternating series.

Section 7. Power Series

1. Find the radius and interval of convergence of a power series.
2. Determine whether a power series diverges, converges conditionally, or converges absolutely at the endpoints of the interval of convergence.
3. Use algebraic operations, term-by-term differentiation, and term-by-term integration to find the sum of a power series.
4. Solve theory and application problems involving power series.

Section 8. Taylor and Maclaurin Series

1. Find the n th Taylor polynomial for a function f at a point $x = a$.
2. Find the Taylor series for a function f at a point $x = a$.
3. Find the Maclaurin series for a function f .
4. Find the values of x for which a Taylor or Maclaurin series converges absolutely.
5. Solve theory problems involving Taylor or Maclaurin series.

Section 9. Convergence of Taylor Series

1. Use substitution and power series operations to find a Taylor series.
2. Show that a Taylor series converges at a given point by estimating the remainder term.
3. Estimate the error when $f(x)$ is approximated by the n th Taylor polynomial $P_n(x)$.
4. Determine how large n must be in order that the Taylor polynomial $P_n(x)$ approximate $f(x)$ to within a given accuracy.
5. Solve theory and application problems involving Taylor series.

Section 10. The Binomial Series and Applications of Taylor Series

1. Find terms of a binomial series.
2. Find a binomial series.
3. Use series to estimate the value of an integral within a specific error.
4. Find a polynomial that will approximate a function given by an integral to a given accuracy.
5. Use series to evaluate limits that involve indeterminate forms.
6. Use algebraic operations and common Taylor series to find the sum of a given series.
7. Solve theory and application problems involving Taylor series.
8. Use Euler's identity.

CHAPTER 11. Parametric Equations and Polar Coordinates

Section 1. Parametrizations of Plane Curves

1. Graph a curve given by a parametric equation.
2. Find and graph a Cartesian equation corresponding to a given parametric equation.
3. Find parametric equations that define a curve or the motion of a particle.

4. Graph parametric curves using a software package.

Section 2. Calculus with Parametric Curves

1. Given a parametric equation, find the parametric formulas for dy/dx and d^2y/dx^2 .
2. Find the tangent to a curve given by a parametric equation.
3. Find the area enclosed by a parametrically defined curve.
4. Find the length of a parametrically defined curve.
5. Find the area of a surface of revolution corresponding to a parametrized curve.
6. Find the coordinates of the centroid of a region defined by a parametrized curve.
7. Solve theory and application problems involving parametric curves.

Section 3. Polar Coordinates

1. Find all of the polar coordinates of a given point.
2. Write Cartesian coordinates for given polar coordinates.
3. Write polar coordinates for given Cartesian coordinates.
4. Graph sets of points whose polar coordinates satisfy a given equation or inequality.
5. Convert polar equations to Cartesian equations.
6. Convert Cartesian equations to polar equations.

Section 4. Graphing in Polar Coordinates

1. Identify the symmetries of a curve and sketch its graph.
2. Find the slope of a curve given in polar coordinates at a given point.
3. Graph curves given in polar coordinates.

Section 5. Areas and Lengths in Polar Coordinates

1. Find the area of a region enclosed by a curve given in polar coordinates.
2. Find the length of a curve given in polar coordinates.

Section 6. Conic Sections

1. Sketch conic section and find quantities related to the conic section, such as vertices, foci, directrix, or asymptotes.
2. Find the standard form of a conic equation.
3. Solve problems involving shifted conic sections.
4. Solve theory and application problems related to conic sections.

Section 7. Conics in Polar Coordinates

1. Find the eccentricity, foci, and directrix of a conic section.
2. Find a standard-form equation in Cartesian coordinates.
3. Find the polar equation for a conic section.
4. Graph a conic section.