Welcome to STA2001!

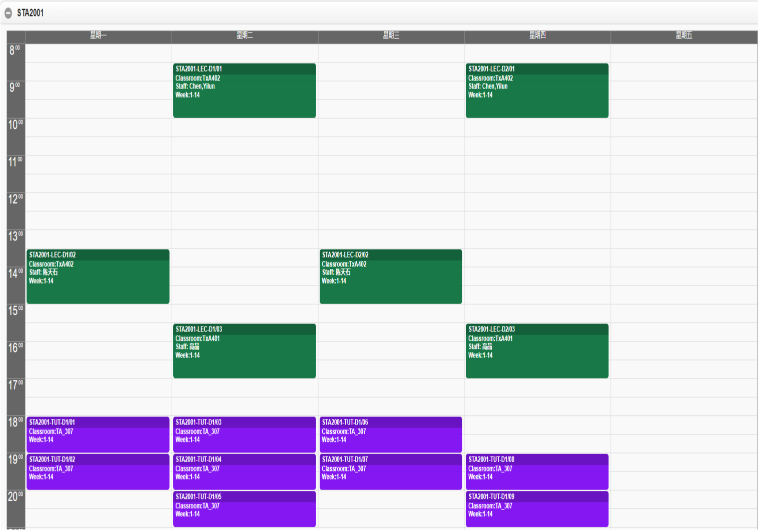
Teaching Staff

- Instructors
  - CHEN, Tianshi (leading instructor)
  - CHEN, Yilun
  - GAO, Pin
- Teaching Assistants
  - XU, Yu (leading TA)
  - FANG, Xiaozhu
  - YOU, Runze
  - ZHANG, Meng
  - CHEN, Zhiyu
  - QU, Wei
- USTF (Requested 8; Obtained 5 so far):
  - TANG, Yanchuan
  - CHEN, Jiaming
  - ZHAO, Xueling
  - LEI, Mingcong
  - TONG, Zhen

Course Components

- 2 lectures/week and each lecture is repeated 3 times
- 1 tutorial/week and each tutorial is repeated 9 times
- 17 + 3 (?) office hours/week
- 11 assignments and each contains 10 questions, plus 1 optional computer-based question
- 1 mid-term exam, March 16 (SAT), 9:00am – 12:00am, Indoor Sports Hall.
  (around 24 multiple choice problems and 2 regular problems, no make-up exam). **Mark your calender now!**
- 1 final term exam (around 10 regular problems, 50% from assignments, if necessary, there would be 1 make-up exam.)

Teaching Schedule



Tutorials will start from the second week, i.e., January 15, 2024!

Assessment Scheme and Textbooks

- Assessment scheme

Component	Weight
Assignments	20%
Mid-Term Exam	30%
Final Term Exam	50%

- Textbook
  - Required:
    Hogg, R. V., Tanis, E. A. Zimmerman, D. L. (2015) Probability and Statistical Inference, 9th edition, Pearson.
  - Recommended:
    Hogg, McKean and Craig (2005) Introduction to mathematical statistics, 6th edition, Prentice Hall.

Teaching Plan

Week	Content
1	Sections 1.1-1.2
2	Sections 1.3-1.5
3	Sections 2.1-2.3
4	Sections 2.3-2.6
5	Sections 3.1-3.2
6	Sections 3.2-3.3
7	Half-time Review and Mid-term exam
8	Sections 4.1-4.2
9	Sections 4.3-4.5
10	Sections 5.1-5.3
11	Sections 5.3-5.5
12	Sections 5.5-5.7
13	Sections 5.7-5.9
14	Full-time Review

Sections 3.4 and 5.2 are not included in teaching and exam!

Important Notes

- The content of lectures and tutorials each week are synchronized. **You are free to attend any lecture and tutorial that best suits your time.**
- Avoid asking questions through emails. **You are suggested to visit the teaching stafff during the 17 office hours (3.4 hours for each week day)!**
- **Add/drop requests should be made by submitting your applications to the School Office of SDS.**

Important Notes

- The bivariate continuous random variable part in STA2001 is based on the **Multivariable Calculus** and therefore, the students are suggested to have some fundamental knowledge of Multivariable Calculus, which can be learned from MAT1002 Calculus II or MAT 1012 Calculus (Extended) II.
- Late submission of assignments: **1/7 of 100 points will be deducted every day, and 0 points for the assignment at the 8th day after the deadline, except that you are sick or have some emergency, and unable to do the assignment.**

STA2001 Probability and Statistics (I)

Lecture 1

Tianshi Chen

The Chinese University of Hong Kong, Shenzhen

Preamble

Question

What is probability theory and statistics?

Preamble

Question

What is probability theory and statistics?

- Probability theory: a branch of mathematics concerned with the analysis of random phenomena, cf. Encyclopedia Britannica.
  - Random phenomena: flipping a coin, rolling a die, winning a lottery, stock index
  - Probability: the tool we use to analyze the random phenomena

Preamble

Question

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  - Random phenomena: flipping a coin, rolling a die, winning a lottery, stock index
  - Probability: the tool we use to analyze the random phenomena
- Statistics: the theory for the analysis of the data, how to extract information from data is the core of statistics, information can be used for making decisions and predictions
  - Data: observation/measurements of random phenomena
  - Information: data becomes information once it has been analyzed in some fashion, cf. Wikipedia.

Preamble

Question

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- Probability theory – the mathematical foundation of statistics
- Statistics – application of probability theory

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- Probability theory – the mathematical foundation of statistics
- Statistics – application of probability theory

Question

What is the importance of probability theory and statistics?

- fundamental for many disciplines in science and engineering such as biology, machine learning, big data, artificial intelligence, signal processing, and many others!

A Question Throughout This Course

Question

Facing these random phenomena in our daily life, how would you build a mathematical framework to study them in a rigorous way?

In this course, we will review how mathematicians build probability theory to study random phenomena.

Section 1.1 Properties of Probability

Fundamental Concepts

Definition[Experiment]

Any procedure that can be infinitely repeated and has a well-defined set of possible outcomes.

Definition[Random Experiment]

An experiment is said to be random if it has more than one possible outcomes.

Definition[Sample Space]

Given a random experiment, the collection of all possible outcomes is called the sample space, denoted by S.

Fundamental Concepts

Definition[Event]

Given a sample space S, an event A is a set that contains part of outcomes in S; that is, $A \subseteq S$.

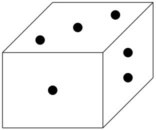
Definition[An event A has occurred]

When a random experiment is performed, if the outcome of the experiment is in A, then we say that the event A has occurred.

Example 1

Throwing a fair 6-sided die

1. This is a random experiment
2. Sample space S=$\{1,2,3,4,5,6\}$
3. Event A=$\{1,2\}$
4. Throw the die, if the outcome is either 1 or 2, then A has occurred.



Algebra of Sets (Set theory)

Set theory: fundamental role in probability theory

- Algebra [Reunion of broken parts]: the study of mathematical symbols and the rules for manipulating these symbols.
- Set: a collection of distinct elements
- $\emptyset$: the null or empty set

Algebra of Sets (Set theory)

In the following, let A and B be two sets.

- $A \subseteq B$: A is a subset of B (every element of A is also an element of B).

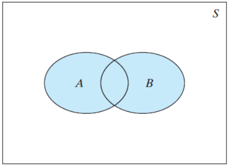
- $A \cup B$: the union of A and B (set of elements that belong to either A or B).

- $A \cap B$: the intersection of A and B.

- A': the complement of A in S is the set of all elements in S that are not in A.

Algebra of Sets (Set theory)

- $A_1, A_2, \cdots, A_k$ are said to be
  1. mutually exclusive if $A_i \cap A_j = \emptyset, i \neq j$
  2. exhaustive if $A_1 \cup A_2 \cup \cdots \cup A_k = S$
  3. mutually exclusive and exhaustive if 1 & 2 holds.



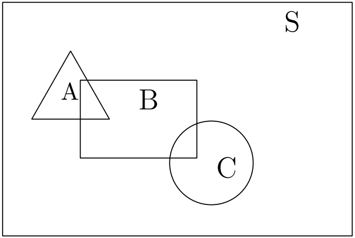
- Commutative laws:

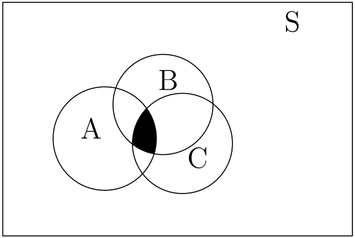
$$A \cup B = B \cup A, A \cap B = B \cap A$$

Algebra of Sets (Set theory)

- Associative Law:

$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$





Algebra of Sets (Set theory)

- Distributive law:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- De Morgan's law

$$(A \cup B)' = A' \cap B', \quad (A \cap B)' = A' \cup B'$$

Example 1 (Continued)

Recall

$$S = \{1, 2, 3, 4, 5, 6\}, \quad A = \{1, 2\}$$

Let

$$B = \{2, 3, 4\}, \quad C = \{5, 6\}$$

What is $A \cap B$, $A \cap (B \cup C)$?

An Intuitive Definition of Probability

Problem

how to define the probability of an event A, (the chance of A occurring)

An intuitive idea:

$$\begin{cases} 1. \quad \text{repeat the experiment a number of times, say } n \text{ times} \\ \\ 2. \quad \text{count the number of times that event } A \text{ actually occurs, } \mathcal{N}(\mathrm{A}) \end{cases}$$

- $\frac{\mathcal{N}(A)}{n}$ is called the relative frequency of event A in n repetitions of the experiment

Example 1 (Continued)

$$S = \{1, 2, 3, 4, 5, 6\}, \quad A = \{1, 2\}$$

Outcome is either 1 or 2 $\Rightarrow A$ occurs

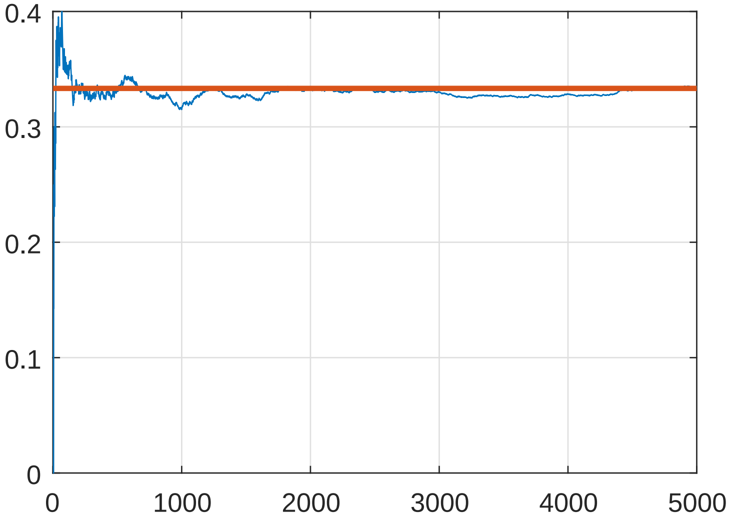
Numerical simulation by computer programs shows

$$\frac{\mathcal{N}(A)}{n} \to \frac{1}{3}, \quad \text{as } n \to \infty$$

The number that $\frac{\mathcal{N}(A)}{n}$ goes to as $n \to \infty$ is called the

probability of event A and is denoted by $P(A) = \lim_{n\to\infty} \frac{\mathcal{N}(A)}{n}$

Example 1 (Continued)



Definition of Probability (Probability Axioms)

Definition[Probability]

A real-valued, set function P that assigns to each event A in the sample space S, a number $P(A)$, called the probability of the event A such that the following properties are satisfied:

1. $P(A) \geq 0$.
2. $P(S) = 1$.
3. if $A_1, A_2, A_3, \cdots$ are countable and mutually exclusive events

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

or equivalently,

$$P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$$

Probability Axioms

The Kolmogorov axioms are the foundations of probability theory introduced by Andrey Kolmogorov in 1933.

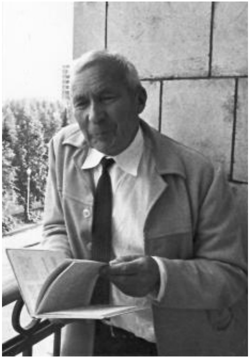


Figure: Andrey Kolmogorov (25 April 1903 – 20 October 1987) was a Soviet mathematician who contributed to the mathematics of probability theory, topology, intuitionistic logic, turbulence, classical mechanics, algorithmic information theory and computational complexity.

Properties of Probability

Property 1: For each event A, $P(A) = 1 - P(A')$.

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$$S = A \cup A', \quad A \cap A' = \emptyset$$

$$1 = P(S) = P(A \cup A') = P(A) + P(A') \Rightarrow P(A) = 1 - P(A')$$

Property 2: $P(\emptyset) = 0$. By property 1 and take $A' = S$.

Properties of Probability

Property 3: If events A and B are such that $A \subseteq B$, then

$$P(A) \leq P(B)$$

Properties of Probability

Property 3: If events A and B are such that $A \subseteq B$, then

$$P(A) \leq P(B)$$

$$B = B \cup A = (B \cup A) \cap S = (B \cup A) \cap (A' \cup A) = (B \cap A') \cup A$$

note that $(B \cap A') \cap A = \emptyset$ and $P(B \cap A') \geq 0$

$$P(B) = P((B \cap A') \cup A) = P(B \cap A') + P(A) \geq P(A)$$

Properties of Probability

Property 4: For each event A, $\quad P(A) \leq 1$.

$$P(S) = 1 = P(A \cup A') = P(A) + P(A') \geq P(A)$$

Property 5: For any two events A and B.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Properties of Probability

Property 4: For each event A, $\quad P(A) \leq 1$.

$$P(S) = 1 = P(A \cup A') = P(A) + P(A') \geq P(A)$$

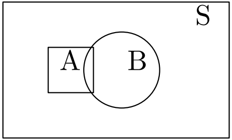
Property 5: For any two events A and B.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$A \cup B = (A \cup B) \cap S = (A \cup B) \cap (A \cup A') = A \cup (A' \cap B)$$

$$A \cup B = A \cup (A' \cap B), \quad \text{where } A \cap (A' \cap B) = \emptyset$$

Properties of Probability



$$P(A \cup B) = P(A) + P(A' \cap B) \qquad (1)$$

$$B = B \cap S = B \cap (A \cup A')$$

$$B = (A \cap B) \cup (A' \cap B), \quad \text{where } (A \cap B) \cap (A' \cap B) = \emptyset$$

$$P(B) = P(A \cap B) + P(A' \cap B) \qquad (2)$$

$$(1) + (2) \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability Space*

A probability space is a triple (S, F, P)

1. S: the sample space

2. F is a σ-algebra on S, a collection of subsets of S, and called the event space

$$\begin{cases} \bullet \quad S \in F \\ \bullet \quad F \text{ is closed under complement} \\ \bullet \quad F \text{ is closed under countable unions} \end{cases}$$

3. $P : F \to [0, 1]$ is the probability measure such that

$$P(A) \geq 0, \forall A \in F, \quad P(S) = 1, \quad P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$$

for countable and mutually exclusive $A_1, A_2, \cdots$

Note: This slide is included here for your possible interest but not included in the exam.