FIN2010 Financial Management
Capital Asset Pricing Model

Agenda

- Efficient Frontier
- Capital Asset Pricing Model
  - Beta (β): a Measure of Systematic Risks
  - Estimating Beta of a Single Stock
  - Portfolio Beta
  - CAPM and the Security Market Line
  - Does it work?
  - Alpha (α)

The mathematical derivation is not required for our exams. You will learn more in FIN2020

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Efficient Frontier

What is it: a curve that answer the following question: suppose we want to obtain a given level of expected return (R_p). What is the minimum risk that we can achieve?

$$\min_{w_i} Var(R_P) \quad = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i\neq j} w_i w_j \rho_{ij}\sigma_i\sigma_j$$

$$\text{subject to } \sum_i w_i E[R_i] = R_p \;\; and \;\; w_1 + w_2 + \cdots + w_n = 1$$

How do we find the curve: a quadratic optimization problem with analytical solution (not required for this course)

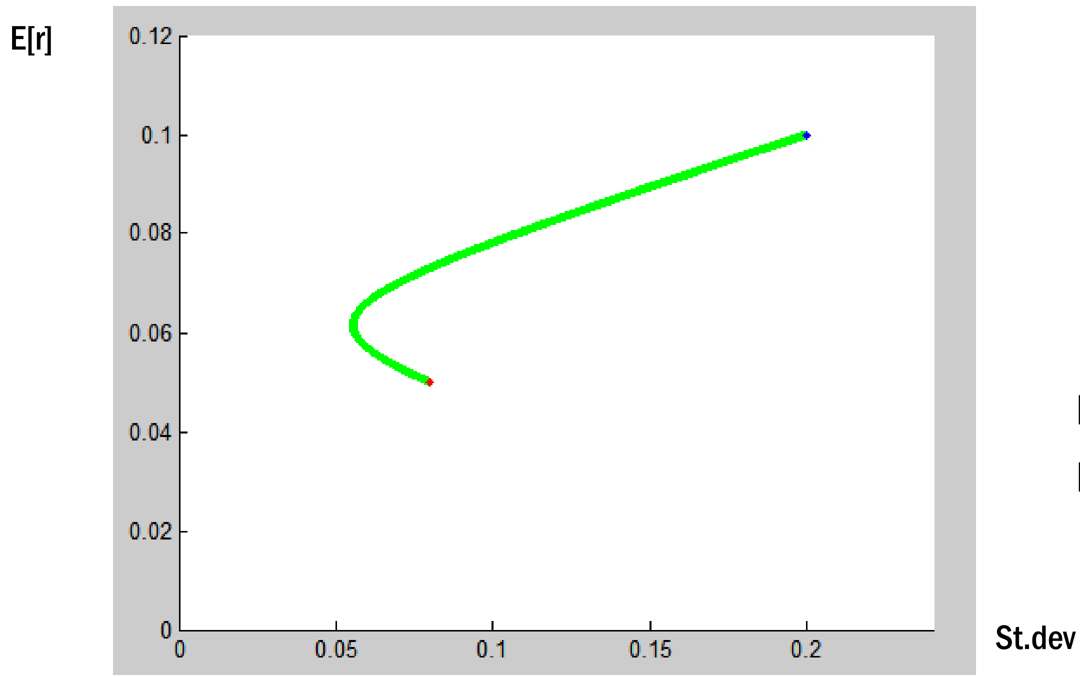
Why do we care about it: (1) it gives a graphical illustration of diversification benefit; (2) quantifies the diversification benefit; (3) answers the asset allocation problems

Example—Two Assets

- 2 assets with mean return μ_1, μ_2, correlation ρ and standard deviation σ_1, σ_2. To find the efficient frontier,

$$\text{Minimize } \sigma_p^2 = w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2\rho w(1-w)\sigma_1\sigma_2$$

$$\text{s.t. } E[R_P] = w\mu_1 + (1-w)\mu_2$$



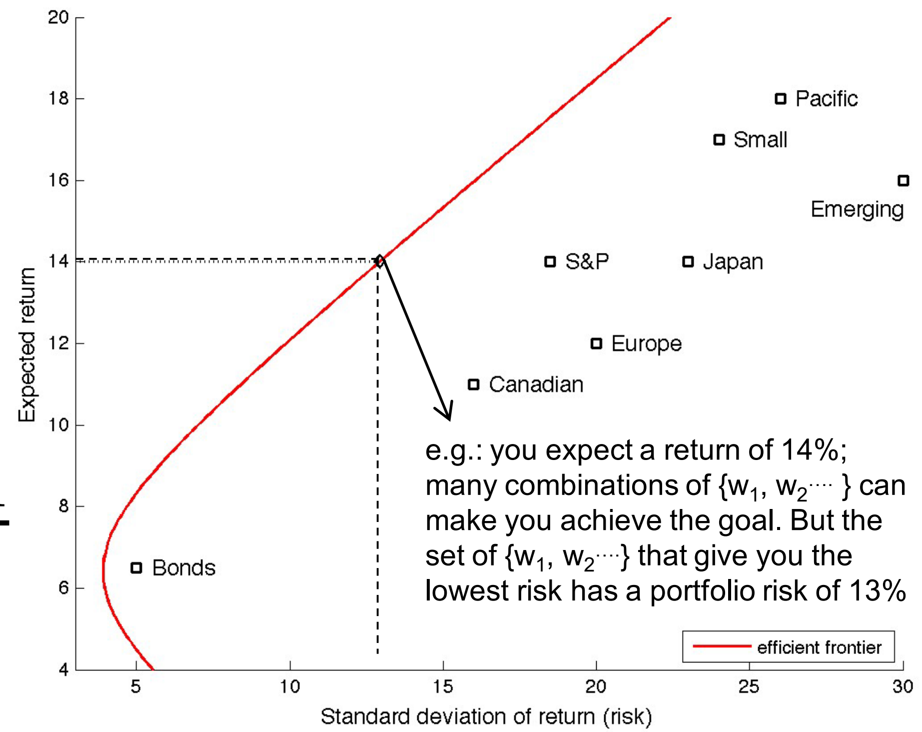
The shape of the efficient frontier is a hyperbola

Interpretation: given a level of return, the minimum risk you need to bear

General Case with n Assets

- n assets with mean return μ (n x 1 vector) and covariance matrix Σ (n x n matrix). We want to minimize the risks for a required level of return c

$$\min_{w} w^T \Sigma w \qquad s.t.\, \mu^T w = c, 1^T w = 1$$



Analytical solution:

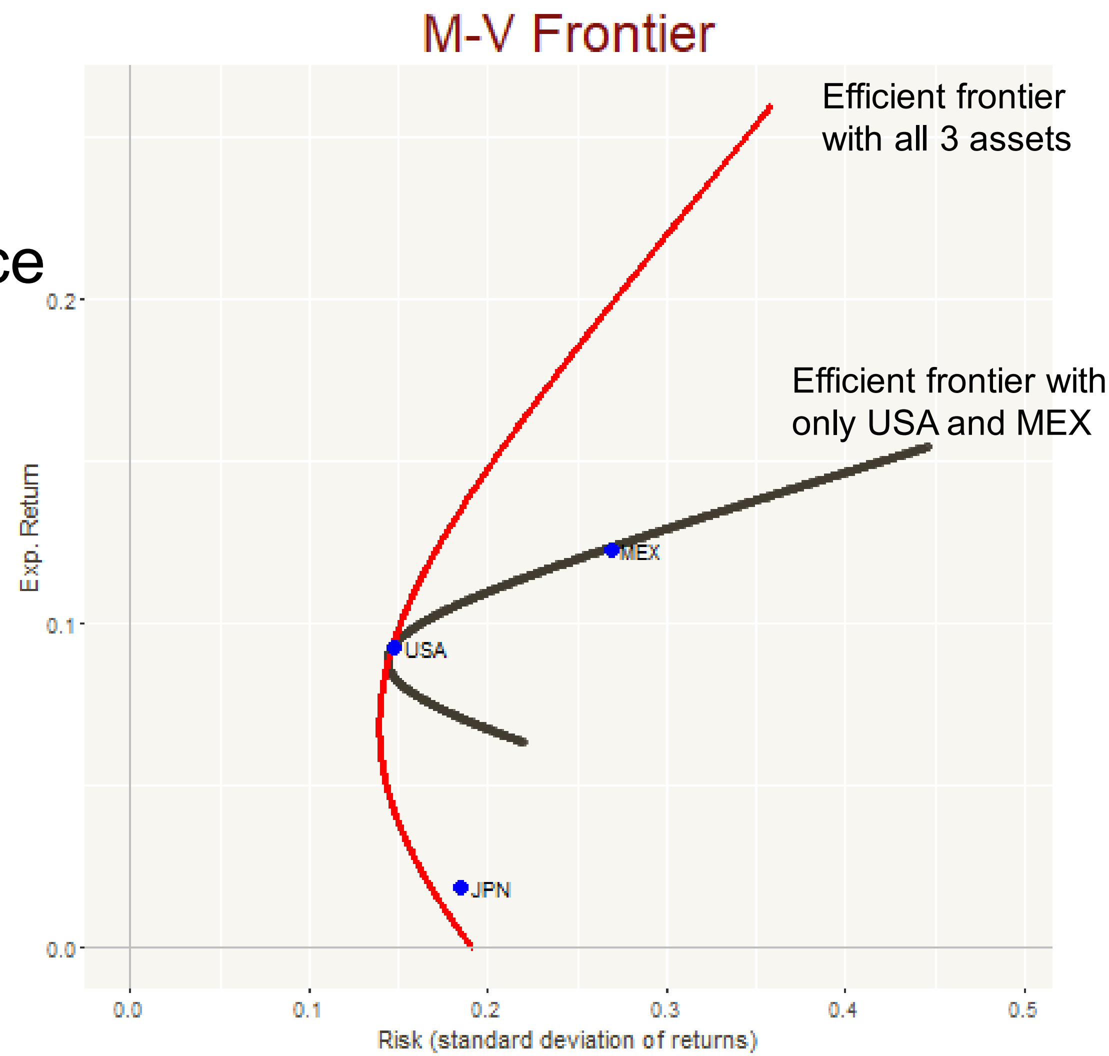
$$\text{Let } w_\mu = \frac{\Sigma^{-1}\mu}{1^T\Sigma^{-1}\mu}, w_1 = \frac{\Sigma^{-1}1}{1^T\Sigma^{-1}1}$$

$$w^* = \frac{c(w_\mu - w_1) - \mu^T w_1 w_\mu + \mu^T w_\mu w_1}{\mu^T w_\mu - \mu^T w_1}$$

Example: USA, MEX and JPN

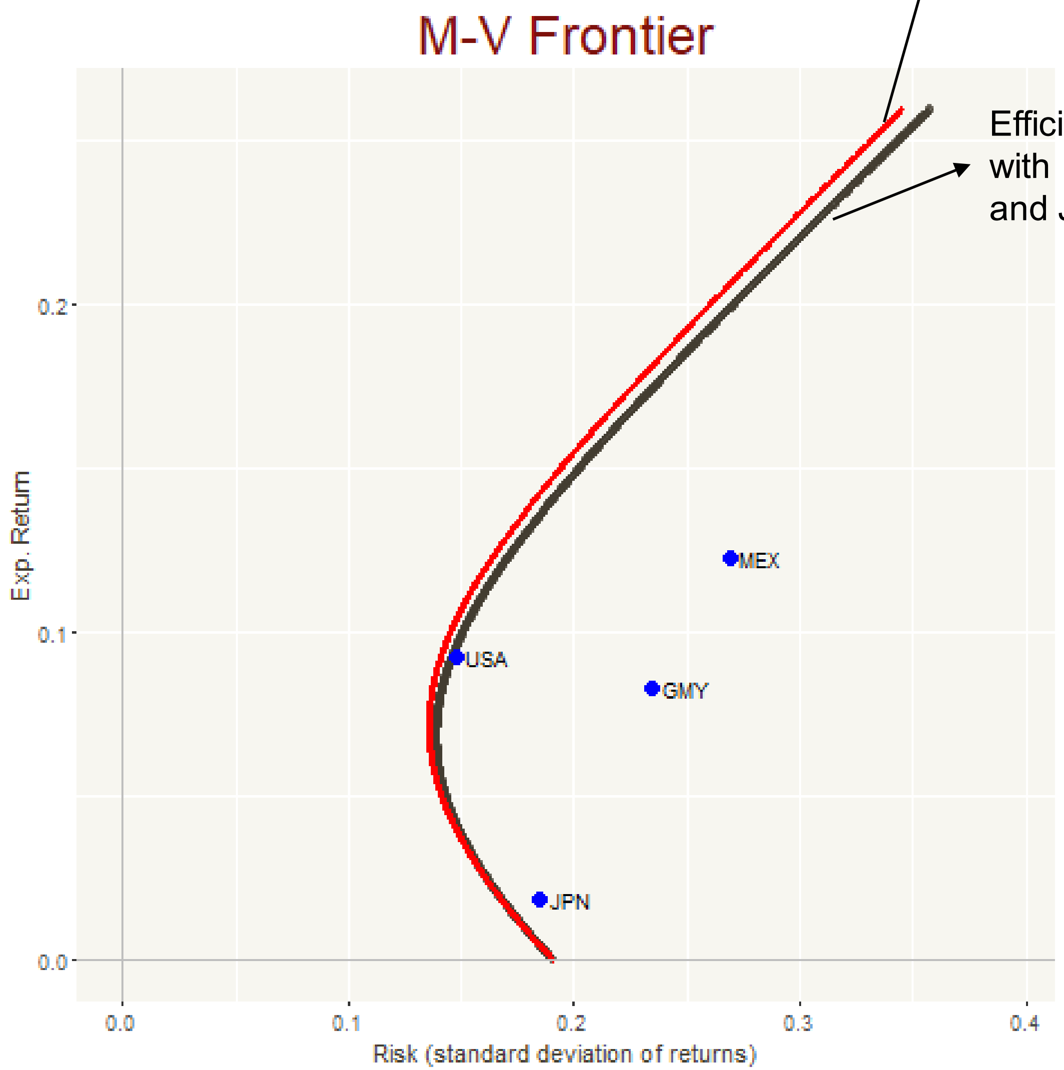
Suppose we have 3 portfolios: USA stock, Mexico (MEX) stock, Japanese (JPN) stock

Note: R codes that produce the graphs can be found on Blackboard



Add Germany

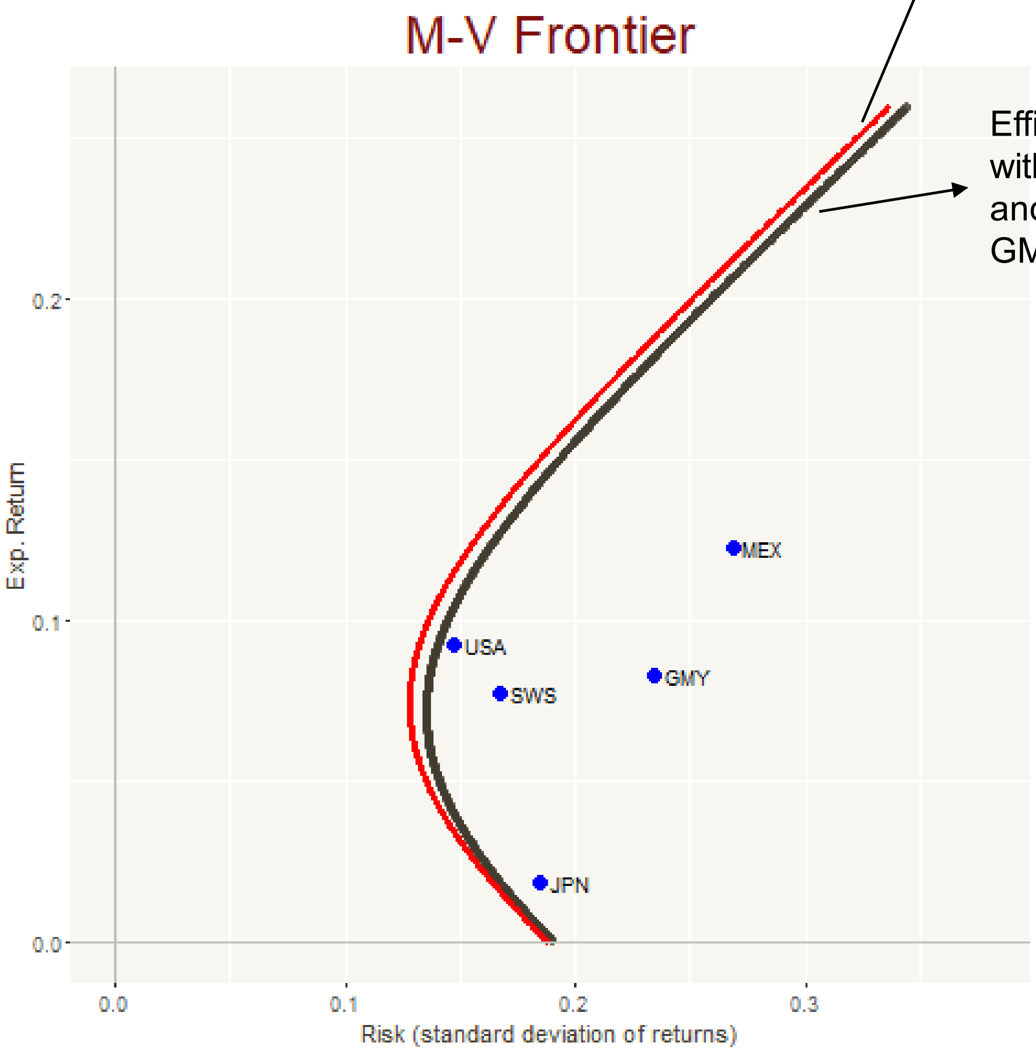
Efficient frontier after adding Germany (GMY)



Efficient frontier with USA, MEX, and JAP

Further decrease in risks after adding Germany

Add Switzerland

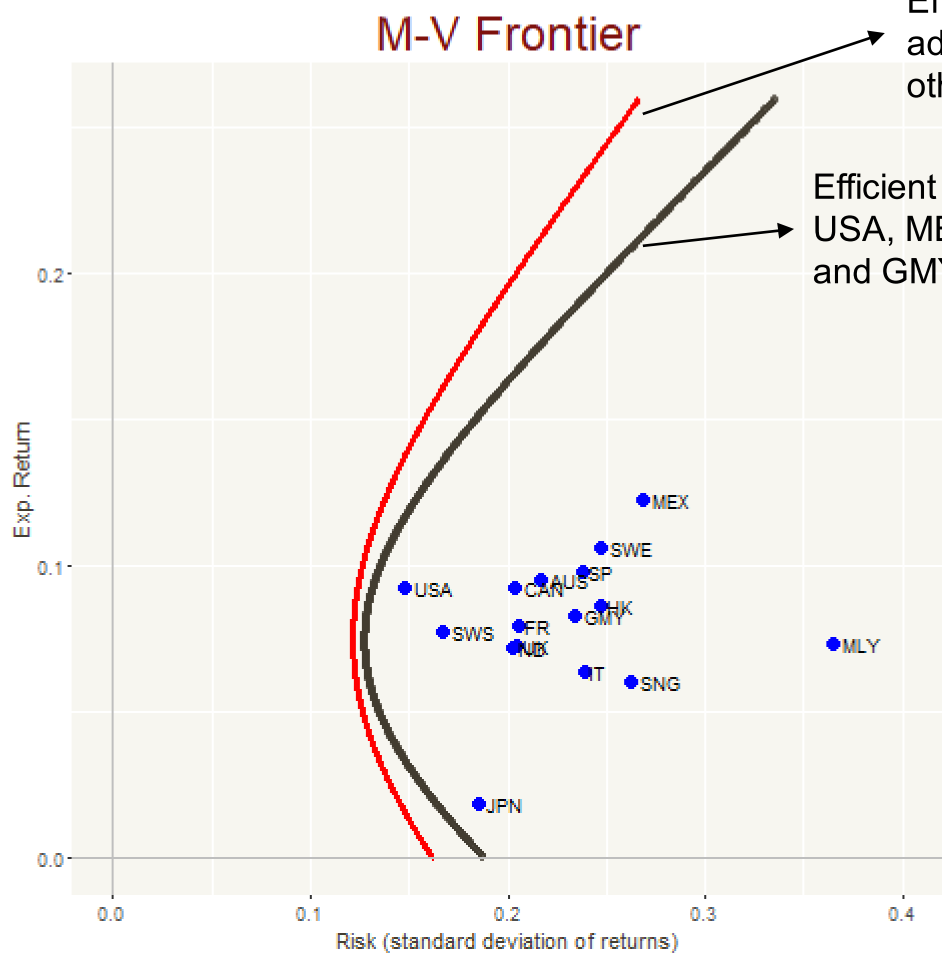


Efficient frontier after adding Switzerland (SWS)

Efficient frontier with USA, MEX, and JAP, and GMY

Add a whole bunch

Australia, Canada, UK, Italy, Singapore, France, Hong Kong, Spain, Sweden, Malaysia and the Netherlands

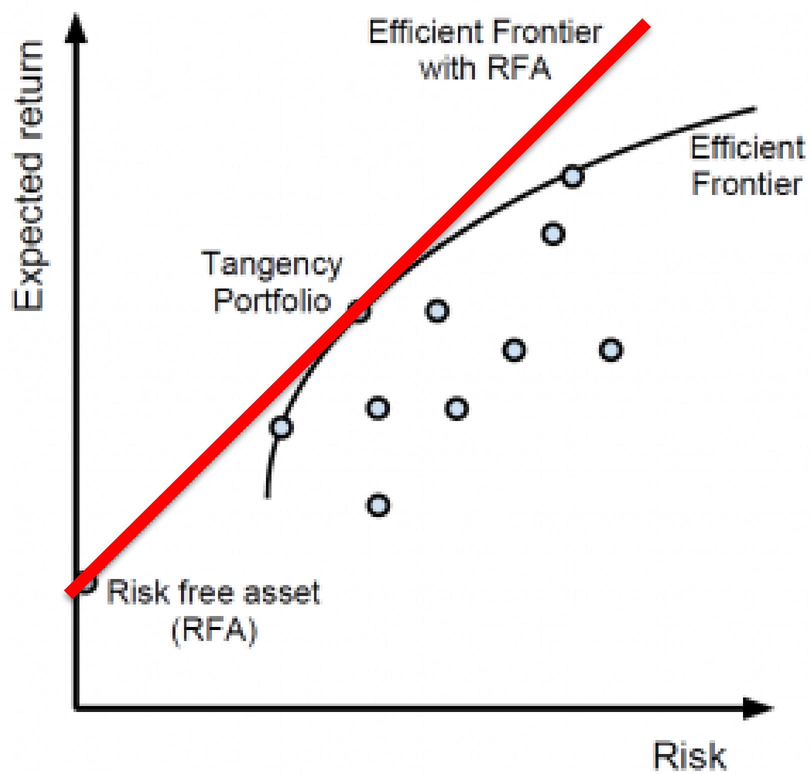


Efficient frontier after adding a whole bunch of other stock markets

Efficient frontier with USA, MEX, and JAP, and GMY and SWS

Efficient Frontier with a Risk-free Asset

- If we add a risk-free assets to the portfolio, the efficient frontier becomes a straight line that connects the risk-free asset and the tangent portfolio
  - The tangency portfolio: the portfolio of risky assets with the highest Sharpe ratio



Separation Theorem

- "*"Every portfolio on the mean-variance frontier can be replicated by a combination of any two frontier portfolios*"
- Thus, no matter what return investors wants, he/she can achieve the goal by combining two assets: risk free asset and tangency portfolio
- E.g. if an investor wants to earn an return of r, he can buy both the risk-free assets and the tangency portfolio so that

$$w_{R_f} \cdot R_f + w_{tan} \cdot E(R_{tan}) = r$$

Lessons from the Efficient Frontier

- Diversification can reduce risk
  - A well-constructed portfolio has risks that are much lower than those of the individual assets
  - By diversifying internationally, we can further reduce risks
    - However, there are systematic risks that still cannot be eliminated by diversifying internationally
  - The lower the correlation, the more the improvement in the efficient frontier
- Efficient frontier
  - If you expect a certain return, many portfolios can get you there. However, the ones on the efficient frontier are the ones with the lowest risk.
- You can achieve any portfolio on the efficient frontier by combining two portfolios: risk-free asset and the tangency portfolio

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Determinants of Required Return

- Required rate of return = Risk free rate of return + risk premium

  = Real risk free return + expected inflation + risk premium

- Risk-free rate: the return that can be earned on a risk-free investment.
- Risk premium: the excess return an investor requires over risk-free rate of return to compensate for the risk that he/she assumes.
- Expected inflation: represents the rate of inflation expected over an investment's life.
- The real risk-free return: the increase in purchasing power that an risk free investment provides.
- Required return is also called expected return.

What is the Capital Asset Pricing Model (CAPM)?

- In general higher risk ↔ higher return
- But how exactly are they related? That is, what is the functional form of $E[R] = f(risk)$?

- CAPM provides a way to determine
  - the expected return
  - given the systematic risks of an asset.

CAPM - Assumptions

1. Investors are rational and have mean-variance preference
   - Investors choose portfolios based <u>only</u> on portfolio risk and variance
   - They like higher return and lower risks
2. There is no friction in the market
   - No trading costs and taxations
   - Can borrow and lend unlimited amount at the risk-free rate
   - Stocks are infinitely divisible
3. Investors cannot influence stock prices
4. All information is available to investors, and investors have homogeneous beliefs (no disagreement)
   - They all invest in the risk-free asset and the tangent portfolio as shown in the previous section.
   - Tangent portfolio = market portfolio (a portfolio of all risky assets)

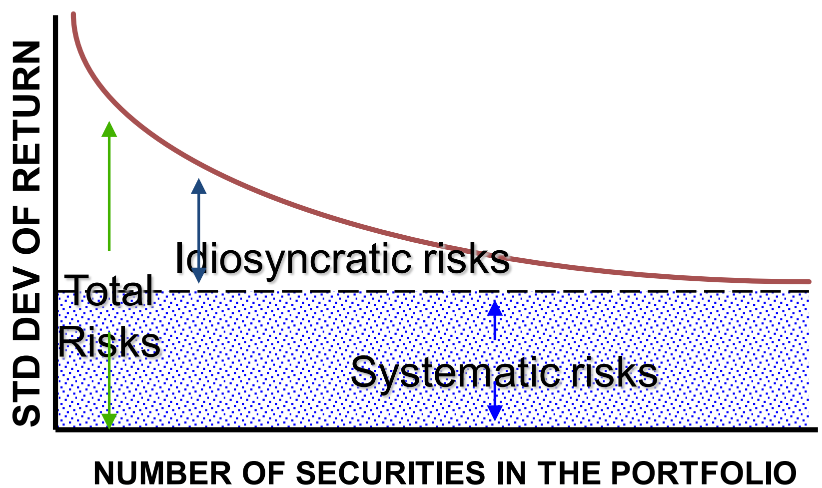
Implications of CAPM Assumptions

→ All investors hold a combination of the risk-free asset and the market portfolio

→ The only risk they will bear is the systematic risk

- For a given risky asset, total risks = systematic risk + idiosyncratic risk
  – Investors demand a reward for bearing the unavoidable (systematic) risks
  – In equilibrium, there is NO reward for bearing the avoidable (idiosyncratic) risk

- **Thus, the expected return on a risky asset depends only on the asset's systematic risk**
- $E(R) = f(\text{systematic risk})$
- $E[R_i] = R_f + \frac{Cov(R_m, R_m)}{Var(R_m)}(E[R_m] - R_f)$



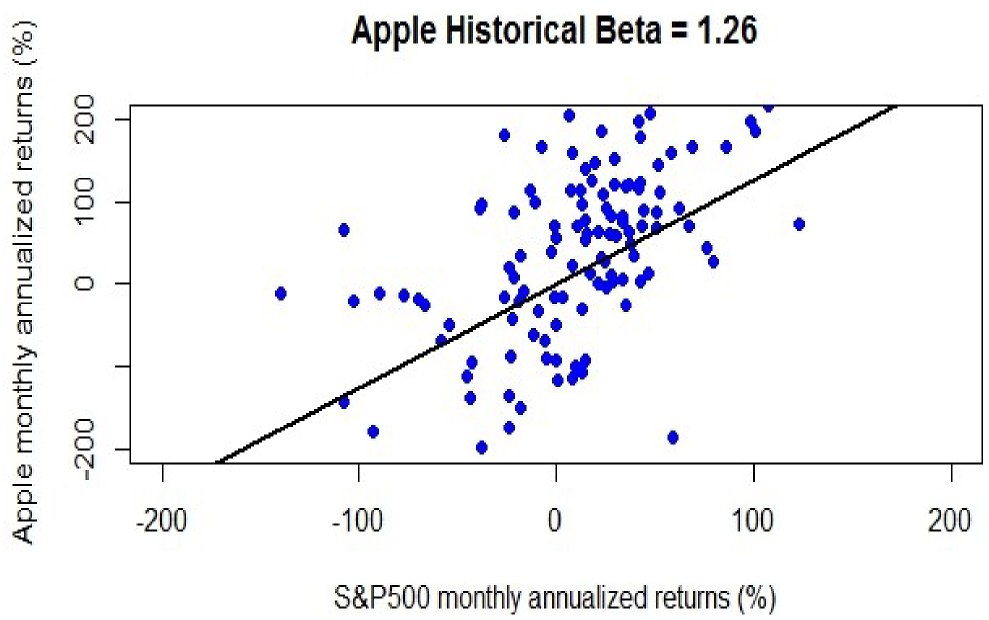
Measuring Systematic Risks

- What are the systematic risks for a risky asset?
  - The risk that makes a risky asset's return co-move with the market portfolio.

- Thus, systematic risk can be measured as the sensitivity of an asset's return to changes in the return of the market portfolio. Such a measure is called beta (β)

Estimating β

- Mathematical definition: $\beta_i = \frac{Cov(R_i,R_m)}{Var(R_m)} = \frac{\rho_{i,M} \cdot \sigma_i}{\sigma_M}$
- Interpretation: on average, if market is up 1%, stock i goes up β%
- Estimation: regress $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$
  - Use historical data to estimate the regression
  - R_i: historical returns of asset i
  - R_m: historical returns of market portfolio. In reality, no one can hold all risky assets (including stocks, bonds, real estate, etc.) in the market. We typically use stock index as the proxy for the market portfolio
  - β_i: systematic risk of asset i



One Caveat about Estimating β

MarketWatch

KEY DATA

OPEN $171.25	P/E RATIO 14.01
DAY RANGE 169.75 - 171.70	EPS $12.16
52 WEEK RANGE 142.00 - 233.47	YIELD 1.71%
MARKET CAP $803.58B	DIVIDEND $0.73
SHARES OUTSTANDING 4.73B	EX-DIVIDEND DATE Feb 8, 2019
PUBLIC FLOAT 4.72B	SHORT INTEREST 40.36M 01/31/19
BETA 1.18	% OF FLOAT SHORTED 0.85%
REV. PER EMPLOYEE $1.98M	AVERAGE VOLUME 40.67M

Different sources give different estimation of Apple's β. Why?

Yahoo Finance

Apple Inc. (AAPL)
NasdaqGS - NasdaqGS Real Time Price. Currency in USD

Add to watchlis

170.42 -0.38 (-0.22%)
At close: February 15 4:00PM EST

Buy

Summary | Chart | Conversations | Statistics | Historical Data | P

Previous Close	170.80	Market Cap	803.578B
Open	171.25	Beta (3Y Monthly)	0.99
Bid	170.43 x 900	PE Ratio (TTM)	14.06
Ask	170.50 x 1200	EPS (TTM)	12.12
Day's Range	169.76 - 171.70	Earnings Date	Apr 29, 2019 - May 3, 2019
52 Week Range	142.00 - 233.47	Forward Dividend & Yield	2.92 (1.71%)
Volume	24,626,814	Ex-Dividend Date	2019-02-08
Avg. Volume	39,365,424	1y Target Est	178.49

Many decisions to make when estimating β:

- Horizon
  - How many years of data should I use? Past 1yr, 3yr, or 5yr?
- Data Frequency
  - Should I use the daily returns or the monthly returns?
- Benchmark
  - Which is proxy of the market portfolio? S&P index? A portfolio of all stocks?

Example: Estimating β of APPL

- Download return data from Yahoo Finance
  - Apple Inc.
  - Market portfolio choice 1: S&P 500
  - Market portfolio choice 2: Total Market
- Beta can vary when we change
  1. Horizon
  2. Data Frequency
  3. Benchmark

Example: Apple's beta using different horizon/frequency/benchmark

Benchmark		S&P	Total Market	S&P	Total Market
Horizon	1yr	1.3937	1.3911	1.0373	0.9960
	3yr	1.2436	1.2138	0.9452	0.8746
	5yr	1.1825	1.1561	1.0798	1.0359
Data Frequency		Daily		Monthly	

As of 2019-2-15. See attached Excel.

Beta of a Portfolio

- Equal to the weighted average of individual stock's beta

$$\beta_P = w_1\beta_1 + w_2\beta_2 + \cdots + w_n\beta_n$$

*Proof can be found on the next page

- Example: suppose I hold a portfolio as follows

Ticker	Company	Weight %	Beta
AAPL	Apple Inc	40.00%	1.18
BAC	Bank of America Corporation	18.13%	1.32
WFC	Wells Fargo & Co	16.32%	1.05
KO	Coca-Cola Co	12.97%	0.50
KHC	The Kraft Heinz Co	12.59%	0.81

– The beta of this portfolio is

$$\beta_P = 40\% * 1.18 + 18.13\% * 1.32 + 16.32\% * 1.05 + 12.97\% * 0.50 + 12.59\% * 0.81$$
$$= 1.05$$

Beta of a Portfolio (Not Required)

- I prove the results for the simple case where the portfolio is made up of 2 assets. The proof is similar for the case of n assets.
- $\beta_P = \frac{Cov(R_P, R_m)}{Var(R_m)} = \frac{1}{Var(R_m)} E[(R_P - \bar{R}_P)(R_m - \bar{R}_m)]$

$$= \frac{1}{Var(R_m)} E[(w_1 R_1 + w_2 R_2 - w_1 \bar{R}_1 - w_2 \bar{R}_2)(R_m - \bar{R}_m)]$$

$$= \frac{1}{Var(R_m)} E[((w_1 R_1 - w_1 \bar{R}_1) + (w_2 R_2 - w_2 \bar{R}_2))(R_m - \bar{R}_m)]$$

$$= \frac{1}{Var(R_m)} E[(w_1 R_1 - w_1 \bar{R}_1)(R_m - \bar{R}_m) + (w_2 R_2 - w_2 \bar{R}_2)(R_m - \bar{R}_m)]$$

$$= \frac{1}{Var(R_m)} [Cov(w_1 R_1, R_m) + Cov(w_2 R_2, R_m)]$$

$$= \frac{w_1 Cov(R_1, R_m) + w_2 Cov(R_2, R_m)}{Var(R_m)}$$

$$= w_1 \beta_1 + w_2 \beta_2$$

Relationship between β and Expected Return

- CAPM states that: $\frac{E[R_i]-R_f}{\beta_i} = \frac{E[R_m]-R_f}{\beta_m}$ = reward to risk ratio
  - Required risk premium on stock i is proportional to its systematic risk (β)
  - Note that market portfolio has a beta of 1: $\beta_m = \frac{Cov(R_m,R_m)}{Var(R_m)} = 1$
  - Thus, the expected return of risky asset i:

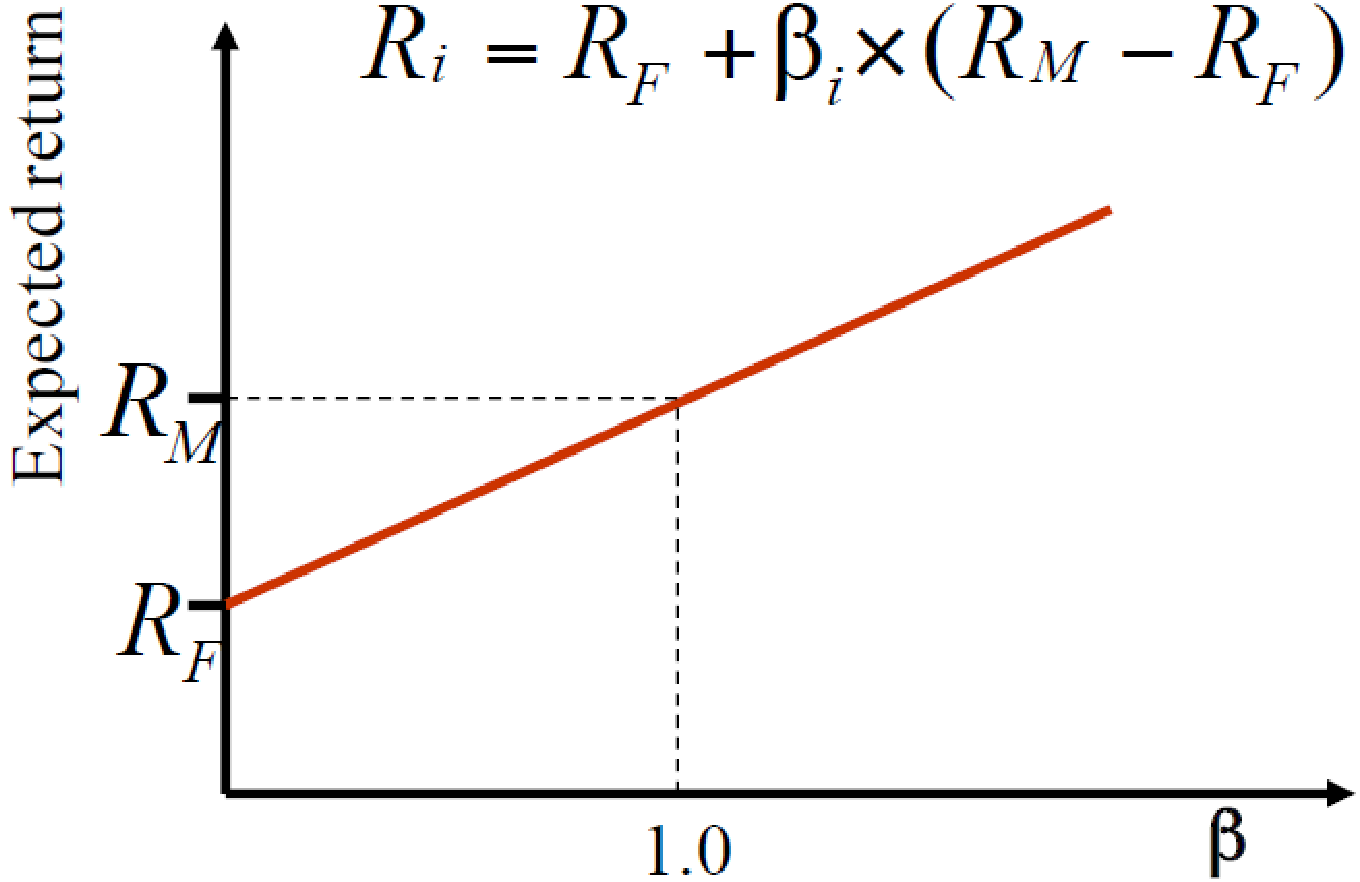
$$E[R_i] = R_f + \beta_i(E[R_m] - R_f)$$

- R_i : return on stock i
- R_f : the risk free rate
- β_i: exposure (loading) to the market risks
- R_m : return on the market
- $R_m - R_f$: market risk premium

Security Market Line – a Graphical Representation of the CAPM

$$\overline{R}_i = R_F + \beta_i \times (\overline{R}_M - R_F)$$

$R_M - R_F =$ Market risk premium



Note: x-axis is β (systematic risks), not σ (total risks)

Insights from the CAPM

$$E[R_i] = R_f + \beta(E[R_m] - R_f)$$

- Factors that affect the expected return of a stock:
  - β: level of systematic risks of the stock
  - R_f: risk-free rate
  - $R_m - R_f$: market risk premium
- Only systematic risks matter for the expected return. Idiosyncratic risks do not because they can be diversified away.
- Required risk premium is linear in β

CAPM FAQ

1. Can beta be 0?
2. What is the expected return for a stock with 0 beta?
3. Can beta be negative?
4. Can the expected return of a risky asset be below R_f?
5. Why would someone want an asset with expected return lower then R_f?
6. Stock A has higher correlation with the market than stock B. Does it mean that stock A has higher beta?
7. Stock A has higher beta than stock B. Does it mean that stock A has higher volatility?

CAPM FAQ

1. Can beta be 0? Yes
2. What is the expected return for a stock with 0 beta? Rf
3. Can beta be negative? Yes
4. Can the expected return of a risky asset be below R_f? Yes, when beta<0
5. Why would someone want such an asset? To hedge against market risks
6. Stock A has higher correlation with the market than stock B. Does it mean that stock A has higher beta? No. Also depends on the volatilities of stock A and B
7. Stock A has higher beta than stock B. Does it mean that stock A has higher volatility? No. Also depends on the correlation between stock A and the market

Application of CAPM - Example

Ticker	Company	Weight %	Beta	E[R]
AAPL	Apple Inc	40.00%	1.18	7.58%
BAC	Bank of America Corporation	18.13%	1.32	8.42%
WFC	Wells Fargo & Co	16.32%	1.05	6.80%
KO	Coca-Cola Co	12.97%	0.50	3.50%
KHC	The Kraft Heinz Co	12.59%	0.81	5.36%
Total		100%	1.05	6.80%

- Suppose $R_f = 0.5\%$ and $\mathrm{E}[R_m] - R_f = 6\%$
- What is the expected return of Apple?

$$E\left[R_{Apple}\right] = 0.5\% + 1.18(6\%) = 7.58\%$$

Note: Expected Return ≠ Realized Return

- Actual return in the past year

Ticker	Company	Weight %	E[R]	R_{2018}
AAPL	Apple Inc.	40.00%	7.58%	-1.88%
BAC	Bank of America Corporation	18.13%	8.42%	-8.64%
WFC	Wells Fargo & Co	16.32%	6.80%	-15.61%
KO	Coca-Cola Co	12.97%	3.50%	6.39%
KHC	The Kraft Heinz Co	12.59%	5.36%	-47.71%

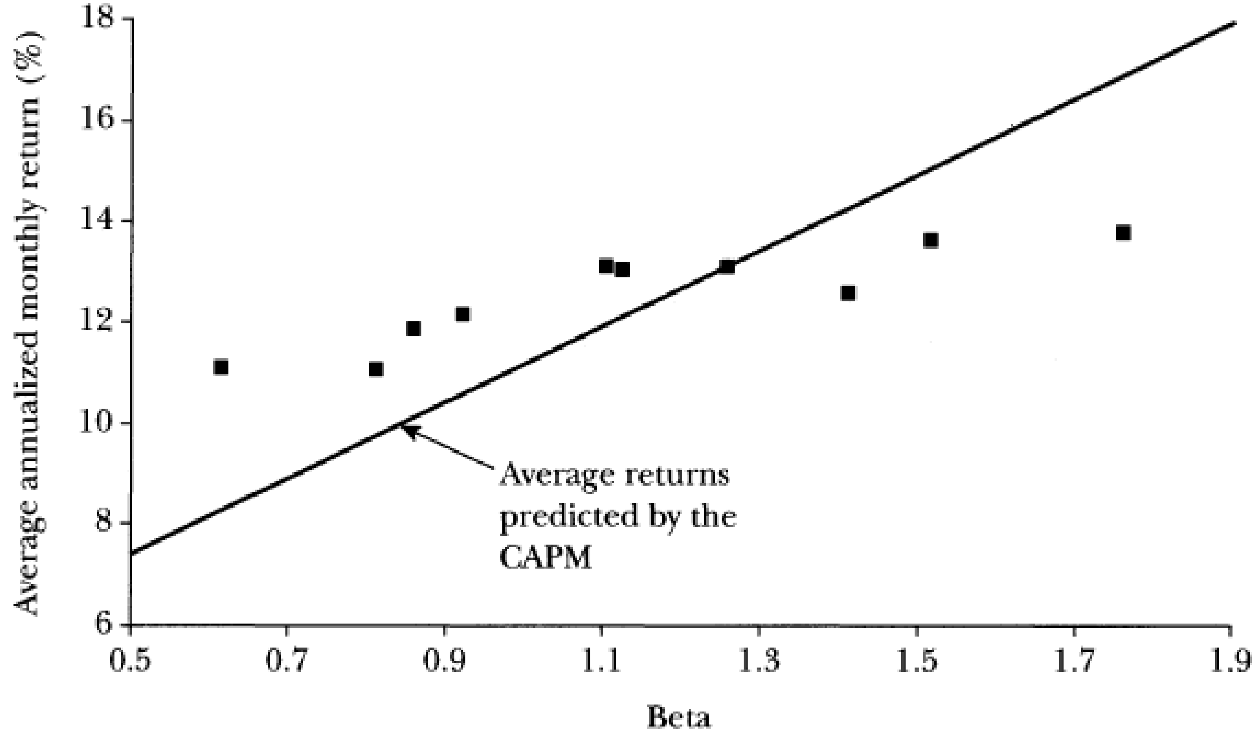
- **E[R]**: expected return in 2018 of a stock according to CAPM
- **R_{2018}**: actual return of stocks in 2018

- Actual returns may not equal to the expected return in reality.

Does CAPM hold empirically?

- Not very well! The average realized return and the predicted return can be quite different.

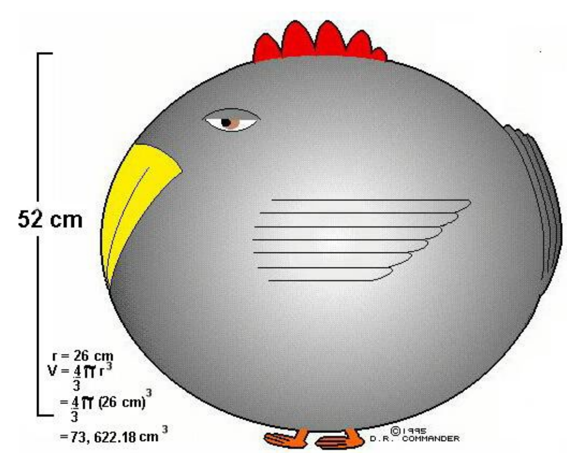
Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



Source: Fama, Eugene F., and Kenneth R. French. "The capital asset pricing model: Theory and evidence." Journal of economic perspectives 18.3 (2004): 25-46.

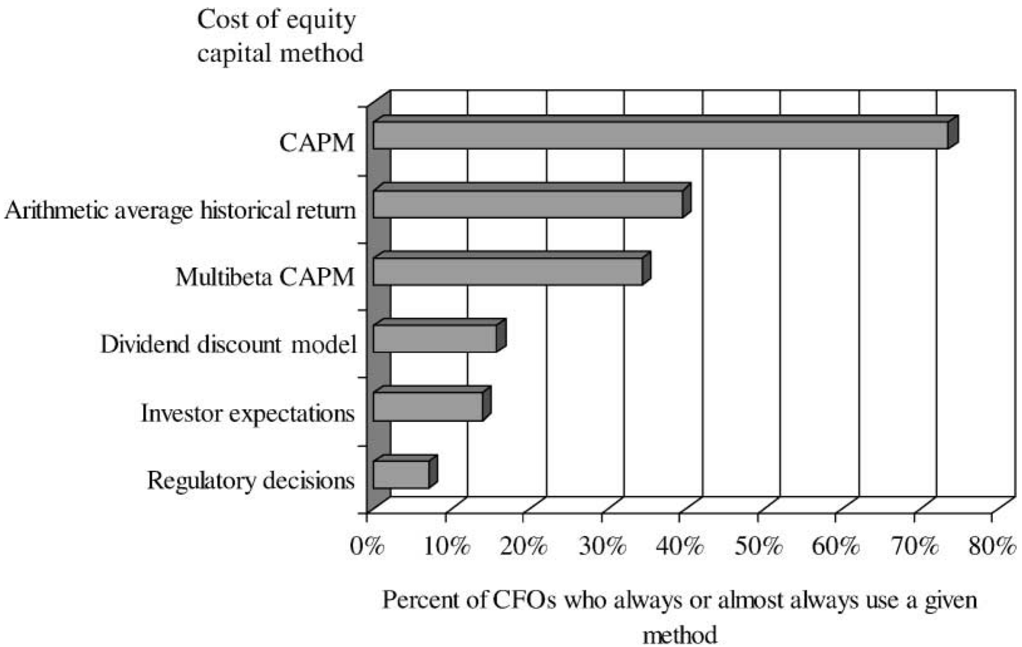
Why does the CAPM not work very well?

- CAPM is based upon many strong assumptions; real people may not act this way!
  - Investors are rational and have mean-variance preference
  - No friction in the market
  - No disagreement among investors
- Spherical chicken in a vacuum:
  - *“There's this farmer, and he has these chickens, but they won't lay any egg. So, he calls a physicist to help. The physicist then does some calculations, and he says, um, I have a solution, but it only works with spherical chickens in a vacuum.”*



Why do We Still Care about CAPM?

- It provides a starting point for many other models that try to explain the relationship between risk and return
- Many firms use CAPM to estimate the cost of capital for equity



J.R. Graham and C.R. Harvey, "The theory and practice of corporate finance: evidence from the field," Journal of Financial Economics 60 (2001)

Summary

- Efficient frontier: the curve (or line) that shows the minimum risk that can be achieved for each level of required return
- CAPM: $\frac{E[R_i]-R_f}{E[R_m]-R_f} = \beta$, or $E[R_i] - R_f = \beta_i\left(E[R_m] - R_f\right)$
  - The required risk premium is proportional to the exposure to the market risk (β)