香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

INTRODUCTION TO COMPUTER SCIENCE: PROGRAMMING METHODOLOGY

TUTORIAL 10

ALGORITHM ANALYSIS

Primitive operations

Counting primitive operations:

~ Assigning an identifier to an object
~ Performing an arithmetic operation
~ Comparing two numbers
~ Calling a function
…

Measuring operations as a function of input size.

To capture the order of growth of an algorithm's running time, we will associate, with each algorithm, a function f(n) that characterize the number of primitive operations as a function of input size.

An quadratic-time algorithm

Example:

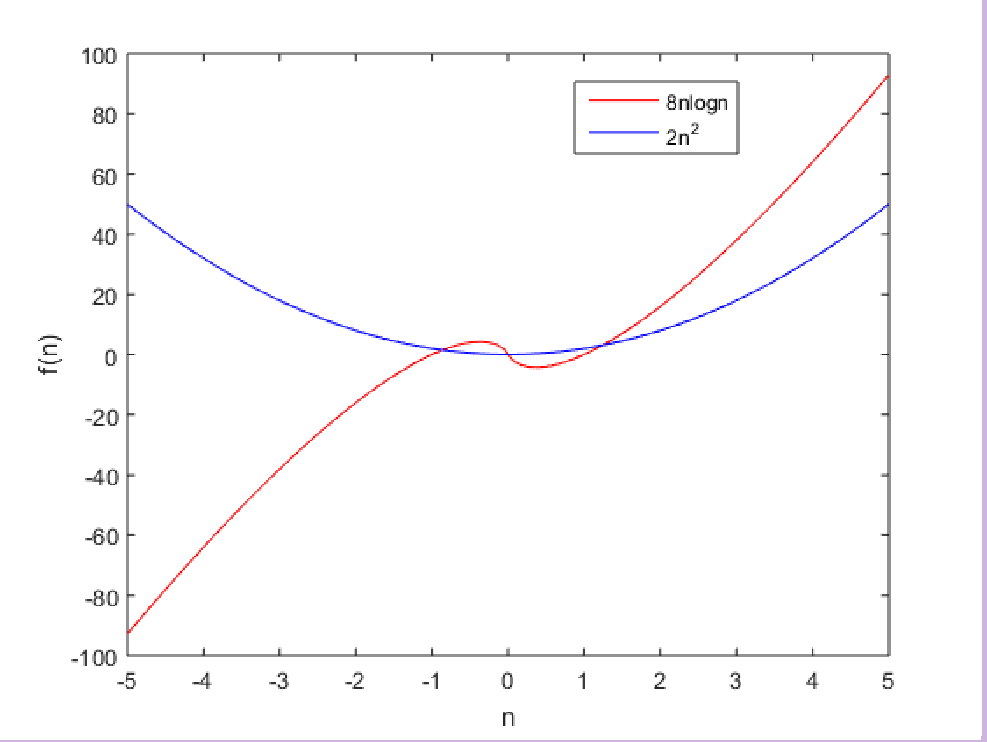
```
def prefix_average1(S):
  """Return list such that, for all j, A[j] equals average of S[0], ..., S[j]."""
  n = len(S) ——→ O(1)
  A = [0] * n ——→ O(n)           # create new list of n zeros
  for j in range(n):
    total = 0 ——→ O(n)           # begin computing S[0] + ... + S[j]
    for i in range(j + 1):
      total += S[i] ——→ n(n+1)/2->O(n^2)
    A[j] = total / (j+1) ——→ O(n) # record the average
  return A
```

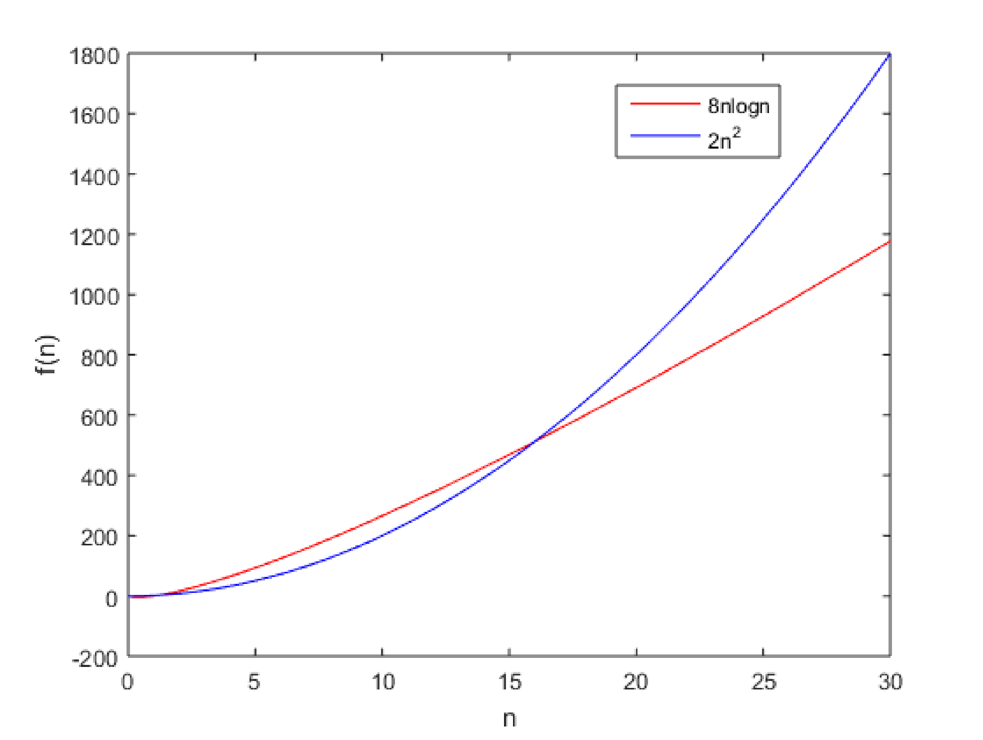
The running time of this program is $O(n^2)$: quadratic-time algorithm

Q1:Verify 8nlogn better than $2n^2$

The number of operations executed by algorithms A and B is $8n\log n$ and $2n^2$, respectively. Determine n_0 such that A is better than B for $n \geq n_0$.

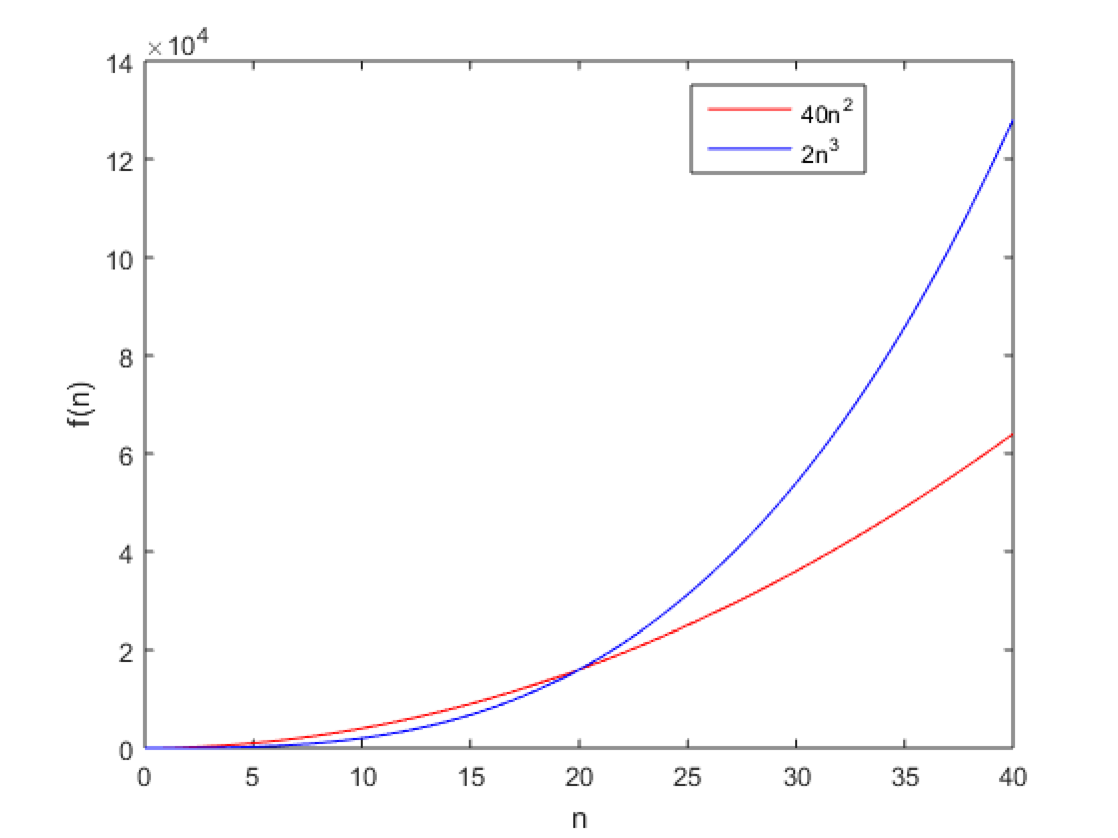
In this tutorial, logn means $\log_2 n$.





Q2: Verify $40n^2$ better than $2n^3$

The number of operations executed by algorithms A and B is $40n^2$ and $2n^3$, respectively. Determine n_0 such that A is better than B for $n \geq n_0$.



Q3: Ordering functions asymptotically

Order the following functions by asymptotic growth rate.

$$\begin{array}{ccc} 4n\log n + 2n & 2^{10} & 2^{\log n} \\ 3n + 100\log n & 4n & 2^n \\ n^2 + 10n & n^3 & n\log n \end{array}$$

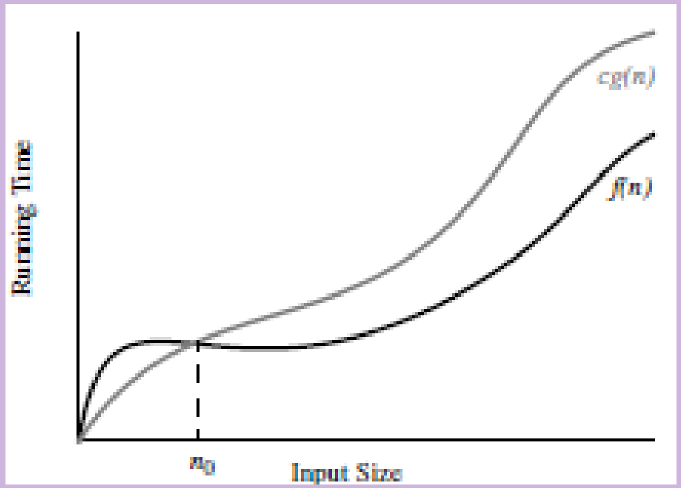
Hints: Using Matlab, Python(download matplotlib module) or Excel to plot the graph of each function for n in the range [0,10], [0,100], [0,1000] respectively.

Definition of Big-Oh Notation

Definition: Let f(n) and g(n) be functions mapping positive integers to positive real numbers. We say that f(n) is O(g(n)) if there is a real constant c>0 and an integer constant $n_0 \geq 1$ such that

$$f(n) \leq cg(n), \text{ for } n \geq n_0$$

This definition is often referred to as the 'big-Oh' notation, for it is sometimes pronounced as 'f(n) is big-Oh of g(n).' Or you can say 'f(n) is order of g(n)'.



Q4: Prove Big Oh of a function

Use the definition of Big Oh notation shown before to
i) Show that $8n+5$ is $O(n)$.
ii) Show that $5n^4+3n^3+2n^2+4n+1$ is $O(n^4)$.
iii) Show that $5n^2+3n\log n+2n+5$ is $O(n^2)$.
iv) Show that $16n\log n+n$ is $O(n\log n)$.

Rules: Characterizing functions in simplest terms.
Use the 7 functions. Our 7 functions are ordered by increasing growth rate in the following sequence.

constant	logarithm	linear	n-log-n	quadratic	cubic	exponential
1	$\log n$	n	$n \log n$	n^2	n^3	a^n

better → **worse**

Q5: Time Complexity

13) What is time complexity of fun(n)?

```
def fun(n):
    count = 0
    m = n//2
    for i in range(n, 0, -m):
        m = m//2
        for j in range(0, i, 1):
            count += 1
    return count
```

A. $O(n^2)$
B. $O(n * \log n)$
C. $O(n)$
D. $O(n * \log n * \log n)$

Q6:Time complexity

i)What is the functionality of the function product()?
ii)What is the time complexity of this function?

```python
def product(n, m):
    if n==0:
        return 0
    elif n==1:
        return m
    else:
        if n%2==1:
            return product(n//2, m)*2+m
        else:
            return product(n//2, m)*2
```