FIN2010 Financial Management
Lecture 9: Risk and Return in a Portfolio

Review—Risk and Return

- Return: $R = \frac{Wealth_T}{Wealth_0} - 1 = \frac{P_T + CF}{P_0} - 1$
  - Average return: arithmetic v.s. geometric
  - Expected return $= E[R] = \sum_{i=1}^{n} R_i * Pr_i$
  - Risk premium: excess return (over the risk-free rate) required to compensate investors for holding risky assets
- Risk: standard deviation of returns,

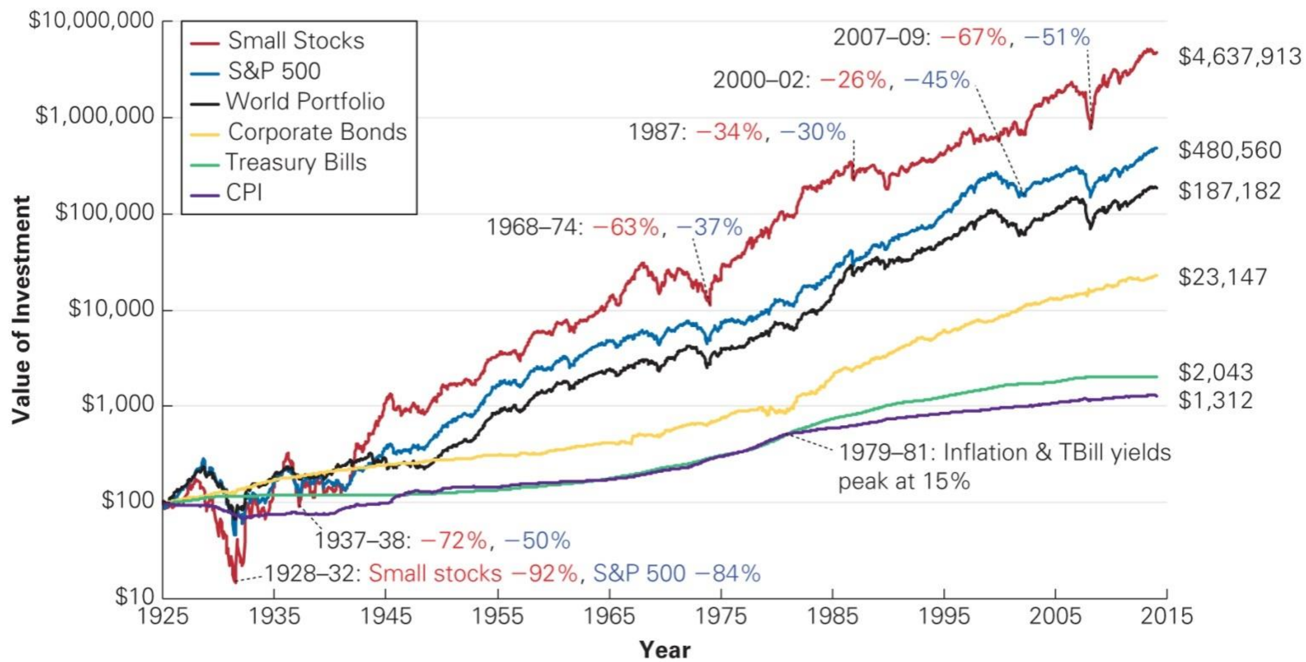
$$\sigma(R) = \sqrt{\mathrm{E}[(R - \mathrm{E}[R])^2]} = \sqrt{\sum_{i=1}^{n} P_i (R_i - \mathrm{E}[R])^2}$$

  - We generally do not know the probability distribution of future returns. Instead, we estimate risks from a <u>sample</u> of historical return

$$\hat{\sigma} = \sqrt{\frac{\sum (R_i - \bar{R})^2}{T - 1}}$$

- Sharpe ratio: $\frac{E[R - R_f]}{\sigma(R - R_f)}$, reward to risk ratio

Lessons from the History



- Average returns are positive
- Higher return requires high risks
- Over a long horizon, risks usually pay off
- Nobody can PROMISE you a high return without any risk

Agenda

- Portfolio
  - Motivation
  - Weights
  - Portfolio returns
  - Portfolio risks
    - Examples and intuitions
    - Math formulas
- Diversification
  - Idiosyncratic and systematic risks

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Motivations

In reality, investors usually hold a portfolio of stocks.

- Portfolio: a combination of different assets, e.g., different stocks, stocks and bonds etc..
  - E.g. Berkshire Hathaway (CEO: Warren Buffet) portfolio as of 12/31/2019

*Shares**	*Company*	*Percentage of Company Owned*	*Cost***	*Market*
			(in millions)	
151,610,700	American Express Company	18.7	$ 1,287	$ 18,874
250,866,566	Apple Inc.	5.7	35,287	73,667
947,760,000	Bank of America Corp.	10.7	12,560	33,380
81,488,751	The Bank of New York Mellon Corp.	9.0	3,696	4,101
5,426,609	Charter Communications, Inc.	2.6	944	2,632
400,000,000	The Coca-Cola Company	9.3	1,299	22,140
70,910,456	Delta Air Lines, Inc.	11.0	3,125	4,147
12,435,814	The Goldman Sachs Group, Inc.	3.5	890	2,859
60,059,932	JPMorgan Chase & Co.	1.9	6,556	8,372
24,669,778	Moody's Corporation	13.1	248	5,857
46,692,713	Southwest Airlines Co.	9.0	1,940	2,520
21,938,642	United Continental Holdings Inc.	8.7	1,195	1,933
149,497,786	U.S. Bancorp	9.7	5,709	8,864
10,239,160	Visa Inc.	0.6	349	1,924
345,688,918	Wells Fargo & Company	8.4	7,040	18,598
	Others***		28,215	38,159
	Total Equity Investments Carried at Market		$110,340	$248,027

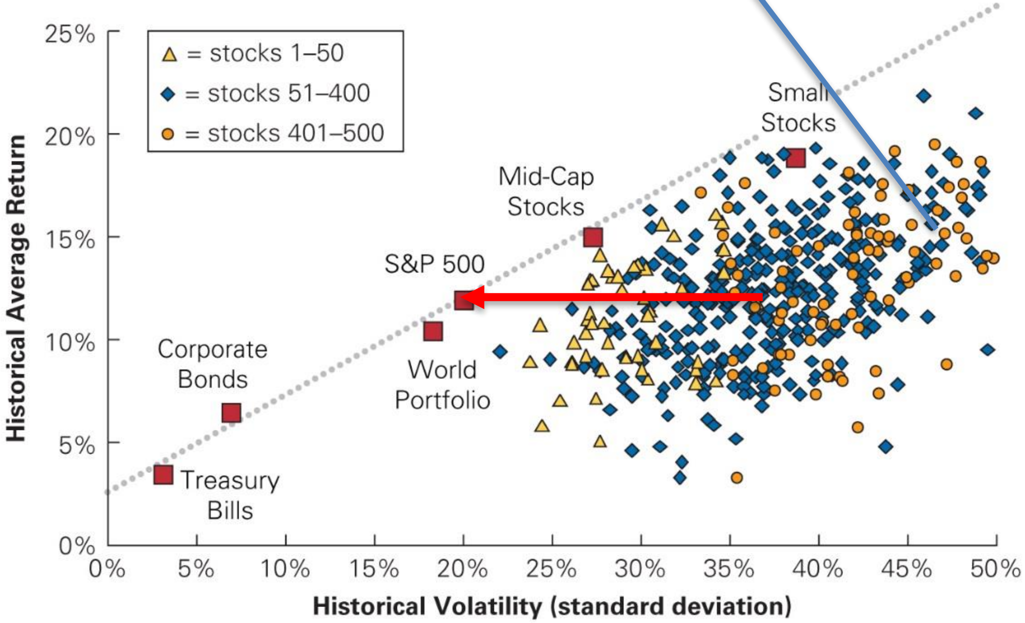
Source: https://www.berkshirehathaway.com/2019ar/2019ar.pdf

- Question: (1) Why do they hold so many stocks?
  (2) What is the return and risk of a portfolio?

Preview of the Results

- Why should an investor hold multiple assets?
  - Diversification reduces risks, but not returns
  - Will discuss "why" in the rest of the lecture

Ideally, we want an asset with high return and low risk



Conventional wisdom:
Don't put all your eggs in one basket!

The best we can do is to reduce some risks while maintaining the same return

投资组合权重

Portfolio Weights

- Weight of asset i: $w_i = \frac{\text{value of asset i}}{\text{Total assets value}}$

Commonly denoted as TNA (total net assets)

- Berkshire Hathaway's portfolio as of 2019/12/31
  – Note: the weight of an asset changes over time

Shares*	Company	Cost**	Cost/share ($)	Price on 2018/12/31	Price on 2019/12/31	Market value on 2018/12/31 (in million)	Market value on 2019/12/31 (in million)	Weight on 2018/12/31	Weight on 2019/12/31
		(in millions)							
151,610,700	American Express Company	$1,287	$8	95.3	124.5	14451.5	18874.0	10.4%	9.3%
250,866,566	Apple Inc	35,287	$141	157.7	293.6	39571.7	73667.0	28.4%	36.3%
947,760,000	Bank of America Corp	12,560	$13	24.6	35.2	23352.8	33380.1	16.8%	16.4%
81,488,751	The Bank of New York Mellon Corp	3,696	$45	47.1	50.3	3835.7	4101.3	2.8%	2.0%
5,426,609	Charter Communications Inc	944	$174	285.0	485.1	1546.4	2632.3	1.1%	1.3%
400,000,000	The Coca-Cola Company	1,299	$3	47.3	55.3	18940.0	22140.0	13.6%	10.9%
70,910,456	Delta Air Lines, Inc	3,125	$44	49.9	58.5	3538.4	4146.8	2.5%	2.0%
12,435,814	The Goldman Sachs Group Inc	890	$72	167.1	229.9	2077.4	2859.4	1.5%	1.4%
60,059,932	JPMorgan Chase & Co	6,556	$109	97.6	139.4	5863.1	8372.4	4.2%	4.1%
24,669,778	Moody's Corporation	248	$10	140.0	237.4	3454.8	5856.9	2.5%	2.9%
46,692,713	Southwest Airlines Co	1,940	$42	46.5	54.0	2170.3	2520.5	1.6%	1.2%
21,938,642	United Continental Holdings Inc	1,195	$54	83.7	88.1	1836.9	1932.6	1.3%	1.0%
149,497,786	US Bancorp	5,709	$38	8.0	13.0	1190.0	1939.0	0.9%	1.0%
10,239,160	Visa Inc	349	$34	131.9	187.9	1351.0	1923.9	1.0%	0.9%
345,688,918	Wells Fargo & Company	7,040	$20	46.1	53.8	15929.3	18598.1	11.5%	9.2%
Total						139109.3	202944.2	100%	100%

Return of a Portfolio

= weighted average of the return of each individual investment

– Historical return:

$$R_p = w_1 R_1 + w_2 R_2 + \cdots + w_n R_n = \sum_{i=1}^{n} w_i R_i$$

- Why? Recall the definition of return: $R = \frac{Wealth_T}{Wealth_0} - 1 = \frac{Profit}{P_{i,0}}$

  For each individual asset, $R_i = \frac{Profit_i}{P_{i,0}}$

  Portfolio return $R_p = \frac{Profit_1 + Profit_2 + \cdots}{P_{1,0} + P_{2,0} + \cdots}$

$$= \frac{Profit_1}{TNA_0} + \frac{Profit_2}{TNA_0} + \cdots$$

$$= \frac{Profit_1}{P_{1,0}} w_1 + \frac{Profit_2}{P_{2,0}} w_2 + \cdots$$

$$= w_1 R_1 + w_2 R_2 + \cdots$$

$$w_1 = \frac{P_1}{TNA} \Rightarrow \frac{1}{TNA} = w_1 \frac{1}{P_1}$$

– Expected return:

$$\mathrm{E}[R_p] = w_1 \mathrm{E}[R_1] + w_2 \mathrm{E}[R_2] + \cdots + w_n \mathrm{E}[R_n] = \sum_{i=1}^{n} w_i \mathrm{E}[R_i]$$

Berkshire Hathaway's Actual Return in 2019

Berkshire Hathaway's portfolio return in 2019			
Shares*	**Company**	**Weight on 2018/12/31**	**Annual return in 2019**
1,516,107	American Express Company	10.4%	0.32
250,866,566	Apple Inc	28.4%	0.88
947,760,000	Bank of America Corp	16.8%	0.46
81,488,751	The Bank of New York Mellon Corp	2.8%	0.09
5,426,609	Charter Communications Inc	1.1%	0.70
400,000,000	The Coca-Cola Company	13.6%	0.20
70,910,456	Delta Air Lines, Inc	2.5%	0.20
12,435,814	The Goldman Sachs Group Inc	1.5%	0.40
60,059,932	JPMorgan Chase & Co	4.2%	0.46
24,669,778	Moody's Corporation	2.5%	0.71
46,692,713	Southwest Airlines Co	1.6%	0.18
21,938,642	United Continental Holdings Inc	1.3%	0.05
149,497,786	US Bancorp	0.9%	0.63
10,239,160	Visa Inc	1.0%	0.43
345,688,918	Wells Fargo & Company	11.5%	0.21
	Portfolio return in 2019		48.4%

$$R_p = \sum_{i=1}^{n} w_i R_i$$

To calculate portfolio return over a period, use the most recent prices to calculate the weights at the beginning of the investment period (instead of purchase price).

Berkshire Hathaway's Expected Return in 2020

Berkshire Hathaway's portfolio EXPECTED return in 2020		
Company	Weight on 2019/12/31	Expected return in 2020
American Express Company	9.3%	12.9%
Apple Inc	36.3%	48.8%
Bank of America Corp	16.4%	12.3%
The Bank of New York Mellon Corp	2.0%	4.8%
Charter Communications Inc	1.3%	33.9%
The Coca-Cola Company	10.9%	7.1%
Delta Air Lines, Inc	2.0%	20.0%
The Goldman Sachs Group Inc	1.4%	12.5%
JPMorgan Chase & Co	4.1%	11.9%
Moody's Corporation	2.9%	23.1%
Southwest Airlines Co	1.2%	10.8%
United Continental Holdings Inc	1.0%	15.1%
US Bancorp	1.0%	12.2%
Visa Inc	0.9%	30.7%
Wells Fargo & Company	9.2%	7.6%
Expected return in 2020		25%

$$E[R_p] = \sum_{i=1}^{n} w_i E[R_i]$$

- **Challenging to forecast the next year's return.**
- **As one commonly used method, let's use each stock's historical average annual return (2001-2019) as the expected annual return for 2020.**
- **We will discuss another method, the CAPM, next time.**

To calculate the expected return of a portfolio, use the most recent prices to calculate the weights (instead of purchase price).

Risk of a portfolio

The *standard deviation* of a <u>portfolio's</u> return	$\leq$	The <u>weighted average</u> of the *standard deviations of returns of the individual investments* in the portfolio

- That is, $\sigma_P \leq \sum w_i \sigma_i$
- Such benefit is a result of simple statistical property.
- Diversification: combining investments in a portfolio ***almost always*** leads to a reduction in risks.

Risk of a Portfolio—An Ideal Example

- Suppose you open a shop on a beautiful Caribbean island. You can sell two products:

	Return	
	Umbrella	Sunglasses
Rainy days(40% chance)	20%	0%
Sunny days(60% chance)	0%	20%

Expected return and risk of umbrella business:

$$E(r_{\text{umbrella only}}) = r_1 * pb_1 + r_2 * pb_2 = 20\% * 40\% + 0\% * 60\% = 8\%$$

$$\begin{aligned}\sigma_{\text{umbrella only}} &= \sqrt{(r_{\text{rainy}} - E(r_{\text{umbrella only}}))^2 * Pb_{\text{rainy}} + (r_{\text{sunny}} - E(r_{\text{umbrella only}}))^2 * Pb_{\text{sunny}}} \\ &= \sqrt{(20\% - 8\%)^2 * 40\% + (0\% - 8\%)^2 * 60\%} \\ &= 9.798\%\end{aligned}$$

Expected return and risk of sunglasses business:

$$E(r_{\text{Sunglasses only}}) = r_1 * pb_1 + r_2 * pb_2 = 0\% * 40\% + 20\% * 60\% = 12\%$$

$$\begin{aligned}\sigma_{\text{sunglasses only}} &= \sqrt{(r_{\text{rainy}} - E(r_{\text{sunglasses only}}))^2 * Pb_{\text{rainy}} + (r_{\text{sunny}} - E(r_{\text{sunglasses only}}))^2 * Pb_{\text{sunny}}} \\ &= \sqrt{(0\% - 12\%)^2 * 40\% + (20\% - 12\%)^2 * 60\%} \\ &= 9.798\%\end{aligned}$$

Risk of a Portfolio—An Ideal Example

- Suppose you dedicate half of the store to sunglasses and half of the store to umbrellas
  - Weight on sunglasses business: 50%
  - Weight on umbrella business: 50%

	Return	
	Umbrella	Sunglasses
Rainy days(40% chance)	20%	0%
Sunny days(60% chance)	0%	20%

$$\begin{aligned} r_{\text{portfolio, rainy}} &= w_1 * r_1 + w_2 * r_2 \\ &= 50\% * 20\% + 50\% * 0\% \\ &= 10\% \end{aligned}$$

$$\begin{aligned} r_{\text{portfolio, sunny}} &= w_1 * r_1 + w_2 * r_2 \\ &= 50\% * 0\% + 50\% * 20\% \\ &= 10\% \end{aligned}$$

  - Therefore, you will earn 10% regardless of the weather. Standard deviation is 0%. You eliminate all risks by investing in a portfolio in this case!

Risk of a Portfolio—A Realistic Example

State	Probability of occurring	Return on stock	Return on bond
Recession	1/3	-7%	3%
Normal	1/3	12%	5%
Boom	1/3	28%	1%

If you hold only stock or only bond:

$E[R_S] = 11\%, \sigma_S = 14.31\%$

$E[R_B] = 3\%, \sigma_B = 1.63\%$

- You have \$100 . You put \$50 in stock and \$50 in bond
  - Recession: return = 50%*(-7%)+$50\% * 3\% = -2\%$
  - Normal: return =50%*12%+$50\% * 5\% = 8.5\%$
  - Boom: return =50%*28%+$50\% * 1\% = 14.5\%$

(另一种做法)

- $\mathrm{E}[R_p] = \frac{1}{3}(-2\% + 8.5\% + 14.5\%) = 7\%$ (each state has a probability of 1/3)

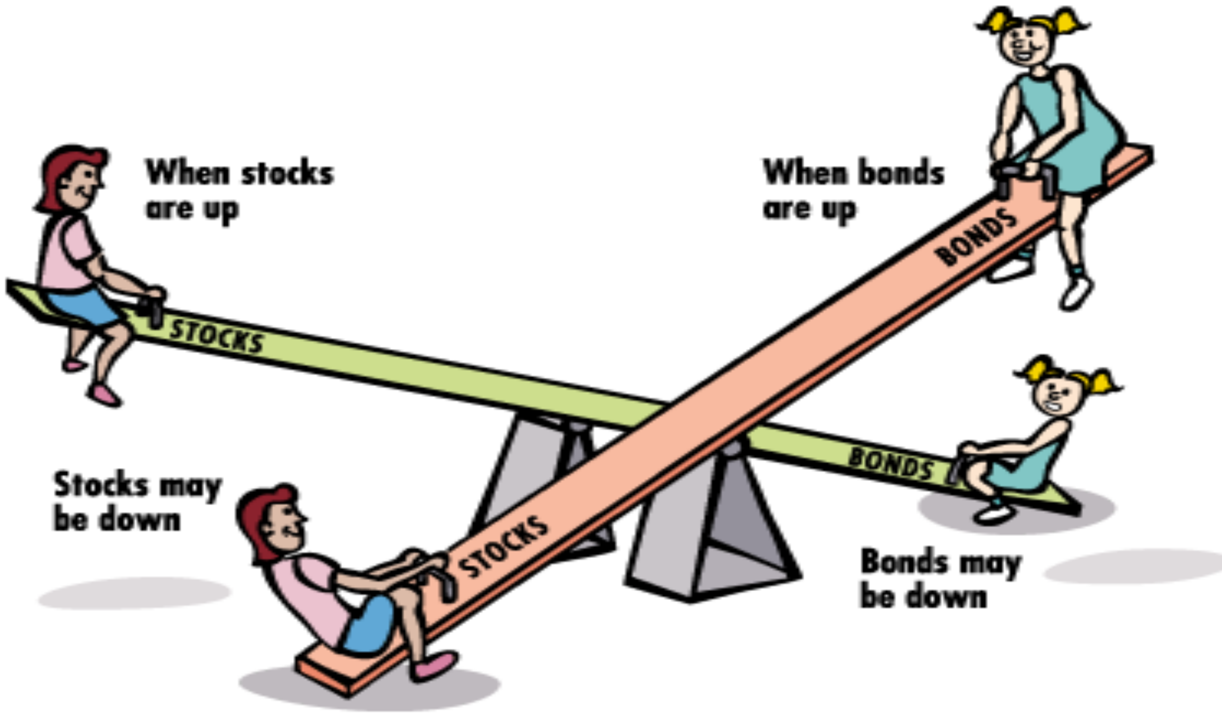
  $= \frac{1}{2}\mathrm{E}[R_S] + \frac{1}{2}\mathrm{E}[R_B]$

- $\sigma_P = \sqrt{\frac{1}{3}(-2\% - 7\%)^2 + \frac{1}{3}(8.5\% - 7\%)^2 + \frac{1}{3}(14.5\% - 7\%)^2} = 6.82\%$

  $< \frac{1}{2}\sigma_S + \frac{1}{2}\sigma_B = 7.97\%$

Portfolio risk is smaller than the weighted average risks of individual assets!

Why are the Portfolio Risks Reduced?



Intuition:

- This reduction in risk arises because bad returns of one asset are offset by good returns of another

Statistics Review—Correlation

- Correlation (ρ)
  - A measure of how much the two assets move together

$$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{\sigma(R_i)\,\sigma(R_j)}$$

$$where: Cov\left(R_i, R_j\right) = E\left[(R_i - \bar{R}_i)\left(R_j - \bar{R}_j\right)\right]$$

  - The correlation between two stocks will always be between –1 and +1.

Formula: Risk of a Portfolio—Two Assets

- In the previous two examples, we show that portfolio risks ≤ weighted average of individual assets' risks. Now let us formally introduce the formula for portfolio risk.

- In a portfolio of two assets with weight w_1 and w_2:

$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2)}$ (See the next page for derivation)

- Theorem: $\sigma_P \leq w_1\sigma_1 + w_2\sigma_2$
  - Why? Because $-1 \leq \rho \leq 1$
  - **Interpretation**: the risk of the portfolio is no higher than the weighted average risks of each individual assets ⇒ holding a p... can lead to diversification benefit
  - Equality holds only when $\rho = 1$
  - If $\rho = -1$, there exist $\{w_1, w_2\}$ such that $\sigma_P = 0$, i.e., that you can eliminate all risks

Derivation (not Required for Exam)

- $\sigma_P^2 = E[(R_P - \bar{R}_P)^2]$
$$= E[(w_1R_1 + w_2R_2 - w_1\bar{R}_1 - w_2\bar{R}_2)^2]$$
$$= E[(w_1(R_1 - \bar{R}_1) + w_2(R_2 - \bar{R}_2))^2]$$
$$= E[w_1^2(R_1 - \bar{R}_1)^2 + w_2^2(R_2 - \bar{R}_2)^2 + 2w_1w_2(R_1 - \bar{R}_1)(R_2 - \bar{R}_2)]$$
$$= w_1^2E[(R_1 - \bar{R}_1)^2] + w_2^2E[(R_2 - \bar{R}_2)^2] + 2w_1w_2E[(R_1 - \bar{R}_1)(R_2 - \bar{R}_2)]$$
$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)$$
$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$$

- In general, for two random variables,
$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$$
$$= \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y$$

Formula: Risk of a Portfolio— n Assets

- 2 assets

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2} \quad \leq w_1\sigma_1 + w_2\sigma_2$$

  – The smaller the correlation, the greater the diversification benefit!

- N assets: suppose the weight on asset i is w_i

$$\sigma_P = \sqrt{\sum w_i^2\sigma_i^2 + \sum_{i\neq j} w_iw_j\rho_{ij}\sigma_i\sigma_j} \quad \leq \sum w_i\sigma_i$$

- An example of a portfolio with 3 assets:

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2 * w_1w_2\rho_{12}\sigma_1\sigma_2 + 2 * w_1w_3\rho_{13}\sigma_1\sigma_3 + 2 * w_2w_3\rho_{23}\sigma_2\sigma_3}$$

Using Covariance to Calculate Portfolio Risks

State	Probability of occurring	Return on stock	Return on bond
Recession	1/3	-7%	3%
Normal	1/3	12%	5%
Boom	1/3	28%	1%

$E[R_S] = 11\%, \sigma_S = 14.31\%$
$E[R_B] = 3\%, \sigma_B = 1.63\%$

- $Cov(R_S, R_B) = E[(R_S - \bar{R}_S)(R_B - \bar{R}_B)]$
  $= \frac{1}{3}(-7\% - 11\%)(3\% - 3\%) + \frac{1}{3}(12\% - 11\%)(5\% - 3\%) + \frac{1}{3}(28\% - 11\%)(1\% - 3\%)$
  $= -0.001067$

  Thus, the correlation coefficient $\rho = \frac{Cov(R_S, R_B)}{\sigma_S \sigma_B} = -0.4566$
- We can plug this information into the formula and calculate portfolio risk in another way (different from page 16):

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2)}$$
$$= \sqrt{\frac{1}{2}^2 14.31\%^2 + \frac{1}{2}^2 1.63\%^2 + 2 \cdot \frac{1}{2} \cdot \frac{1}{2}(-0.4566) \cdot 14.31\% \cdot 1.63\%}$$
$$= 6.82\%$$

When $\rho = 1$, $\sigma_p = w_1\sigma_1 + w_2\sigma_2$

When $\rho = -1$, $\sigma_p = |w_1\sigma_1 - w_2\sigma_2|$

Berkshire Hathaway's Portfolio Risk (not Required for Exam Purpose)

- $\sigma_P = \sqrt{\sum w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \rho_{ij} \sigma_i \sigma_j} = \sqrt{\sum w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \sigma_{ij}}$
  - Where $\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$ is the covariance between asset i and j
- Covariance matrix: a table showing covariance between variables
  - In this example, we use historical data to estimate covariance.

	American Express Company	*Apple Inc.*	*Bank of America Corp.*	*The Bank of New York Mellon Corp.*	*Charter Communications Inc.*	*The Coca-Cola Company*	*Delta Air Lines, Inc.*	*The Goldman Sachs Group Inc.*	*JPMorgan Chase & Co.*	*Moodys Corporation*	*Southwest Airlines Co.*	*United Continental Holdings Inc.*	*U.S. Bancorp*	*Visa Inc.*	*Wells Fargo & Company*
American Express Company	0.14														
Apple Inc.	0.10	0.40													
Bank of America Corp.	0.05	0.03	0.14												
The Bank of New York Mellon Corp.	0.04	0.04	0.05	0.06											
Charter Communications Inc.	0.05	0.03	0.03	0.02	0.08										
The Coca-Cola Company	0.04	0.02	0.00	0.02	0.01	0.02									
Delta Air Lines, Inc.	0.06	0.01	0.11	0.08	0.06	0.02	0.20								
The Goldman Sachs Group Inc.	0.11	0.11	0.08	0.06	0.05	0.04	0.08	0.15							
JPMorgan Chase & Co.	0.06	0.04	0.05	0.05	0.03	0.02	0.05	0.08	0.06						
Moodys Corporation	0.06	0.06	0.08	0.02	0.05	0.00	0.07	0.05	0.04	0.11					
Southwest Airlines Co.	0.06	0.02	0.06	0.05	0.04	0.02	0.17	0.07	0.05	0.05	0.15				
United Continental Holdings Inc.	0.07	0.03	0.07	0.05	0.01	0.02	0.12	0.07	0.04	0.05	0.12	0.16			
U.S. Bancorp	0.15	0.08	0.13	0.06	0.08	0.04	0.08	0.16	0.09	0.11	0.06	0.10	0.25		
Visa Inc.	0.06	0.05	0.03	0.00	0.02	0.01	0.02	0.04	0.02	0.05	0.01	-0.04	0.04	0.06	
Wells Fargo & Company	0.02	-0.01	0.03	0.02	0.03	0.01	0.06	0.03	0.02	0.02	0.04	0.03	0.04	0.00	0.03

Berkshire Hathaway's Portfolio Risk (not Required for Exam Purpose)

Risk of a portfolio with n assets: matrix representation

$$\sigma_w^2 = w^T S w$$

$$= \begin{bmatrix} w_1 & w_2 & . & . & w_n \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & & & \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ w_n \end{bmatrix}$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

	American Express Company	*Apple Inc.*	*Bank of America Corp.*	*The Bank of New York Mellon Corp.*	*Charter Communications Inc.*	*The Coca-Cola Company*	*Delta Air Lines, Inc.*	*The Goldman Sachs Group Inc.*	*JPMorgan Chase & Co.*	*Moodys Corporation*	*Southwest Airlines Co.*	*United Continental Holdings Inc.*	*U.S. Bancorp*	*Visa Inc.*	*Wells Fargo & Company*
American Express Company	0.14	0.10	0.05	0.04	0.05	0.04	0.06	0.11	0.06	0.06	0.06	0.07	0.15	0.06	0.02
Apple Inc.	0.10	0.40	0.03	0.04	0.03	0.02	0.01	0.11	0.04	0.06	0.02	0.03	0.08	0.05	-0.01
Bank of America Corp.	0.05	0.03	0.14	0.05	0.03	0.00	0.11	0.08	0.05	0.08	0.06	0.07	0.13	0.03	0.03
The Bank of New York Mellon Corp.	0.04	0.04	0.05	0.06	0.02	0.02	0.08	0.06	0.05	0.02	0.05	0.05	0.06	0.00	0.02
Charter Communications Inc.	0.05	0.03	0.03	0.02	0.08	0.01	0.06	0.05	0.03	0.05	0.04	0.01	0.08	0.02	0.03
The Coca-Cola Company	0.04	0.02	0.00	0.02	0.01	0.02	0.02	0.04	0.02	0.00	0.02	0.02	0.04	0.01	0.01
Delta Air Lines, Inc.	0.06	0.01	0.11	0.08	0.06	0.02	0.20	0.08	0.05	0.07	0.17	0.12	0.08	0.02	0.06
The Goldman Sachs Group Inc.	0.11	0.11	0.08	0.06	0.05	0.04	0.08	0.15	0.08	0.05	0.07	0.07	0.16	0.04	0.03
JPMorgan Chase & Co.	0.06	0.04	0.05	0.05	0.03	0.02	0.05	0.08	0.06	0.04	0.05	0.04	0.09	0.02	0.02
Moodys Corporation	0.06	0.06	0.08	0.02	0.05	0.00	0.07	0.05	0.04	0.11	0.05	0.05	0.11	0.05	0.02
Southwest Airlines Co.	0.06	0.02	0.06	0.05	0.04	0.02	0.17	0.07	0.05	0.05	0.15	0.12	0.06	0.01	0.04
United Continental Holdings Inc.	0.07	0.03	0.07	0.05	0.01	0.02	0.12	0.07	0.04	0.05	0.12	0.16	0.10	-0.04	0.03
U.S. Bancorp	0.15	0.08	0.13	0.06	0.08	0.04	0.08	0.16	0.09	0.11	0.06	0.10	0.25	0.04	0.04
Visa Inc.	0.06	0.05	0.03	0.00	0.02	0.01	0.02	0.04	0.02	0.05	0.01	-0.04	0.04	0.06	0.00
Wells Fargo & Company	0.02	-0.01	0.03	0.02	0.03	0.01	0.06	0.03	0.02	0.02	0.04	0.03	0.04	0.00	0.03
Expected return	12.9%	48.8%	12.3%	4.8%	33.9%	7.1%	20.0%	12.5%	11.9%	23.1%	10.8%	15.1%	12.2%	30.7%	7.6%
Portfolio expected return	25%														
Portfolio variance	9.0%														
Portfolio standard deviation	30.0%														
Sharpe ratio	0.8467														

Weights	
American Express Company	9.3%
Apple Inc	36.3%
Bank of America Corp	16.4%
The Bank of New York Mellon Corp	2.0%
Charter Communications Inc	1.3%
The Coca-Cola Company	10.9%
Delta Air Lines, Inc	2.0%
The Goldman Sachs Group Inc	1.4%
JPMorgan Chase & Co	4.1%
Moody's Corporation	2.9%
Southwest Airlines Co	1.2%
United Continental Holdings Inc	1.0%
US Bancorp	1.0%
Visa Inc	0.9%
Wells Fargo & Company	9.2%

Agenda

- Portfolio
  - Motivation
  - Weights
  - Portfolio returns
  - Portfolio risks
    - Examples and intuitions
    - Math formulas
- Diversification
  - Idiosyncratic and systematic risks

Diversification

- Holding multiple asset with the hope of reducing risks
  - $R_P = \sum w_i R_i$
  - $\sigma_P = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \rho_{ij} \sigma_i \sigma_j} \leq \sum w_i \sigma_i$
  - The smaller the correlation (ρ), the larger the risk reduction

- What can and cannot diversification do?
  - Can diversification eliminate all risks?

Can Diversification Eliminate all Risks?

Suppose there are n identical investments with the same risk σ, and the pairwise correlations are ρ.

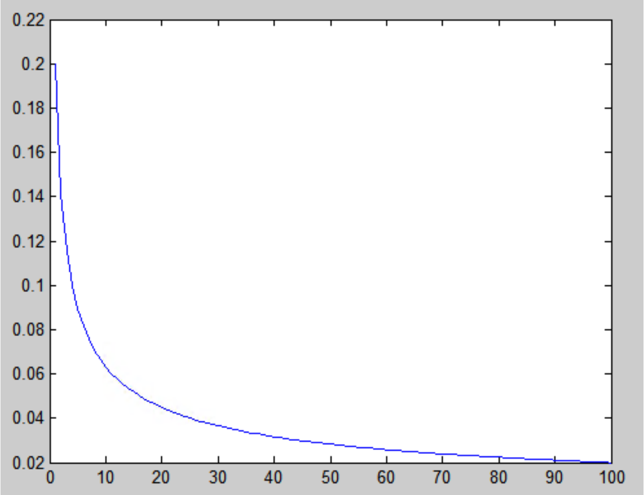
- Pairwise correlation: correlation between any two investments.
- E.g., for three assets A, B, and C, there are three pair-wise correlations: $\rho_{A,B}, \rho_{B,C}, \rho_{A,C}$

- If we invest in *n* of them with equal weight in each, what is the expected return and risk of the portfolio?
- $\sigma_n = \sqrt{(\frac{\sigma}{n})^2 + \cdots + \frac{1}{n}\frac{1}{n}\rho\sigma\sigma + \cdots} = \sqrt{n\frac{\sigma^2}{n^2} + n(n-1)\rho\frac{\sigma^2}{n^2}} = \sigma\sqrt{\frac{1+(n-1)\rho}{n}}$

Can Diversification Eliminate all Risks?

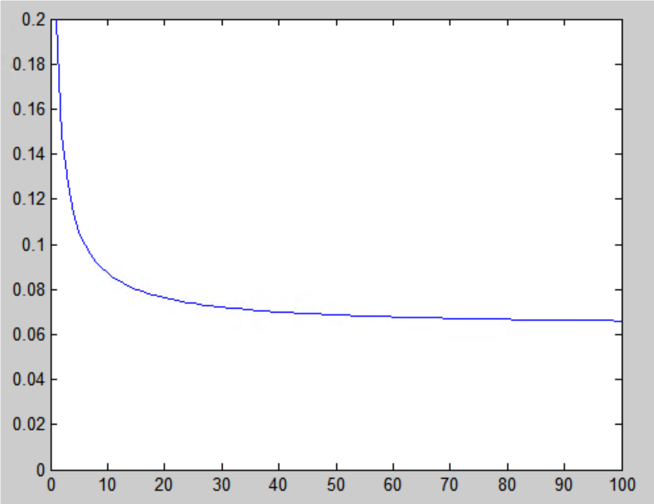
When ρ=0:

- $\sigma_n = \sigma\sqrt{\frac{1+(n-1)\rho}{n}} = \sigma\sqrt{\frac{1}{n}}$
- $\lim_{n\to\infty} \sigma_n$ =0
- If n becomes large, the risk can shrink to 0



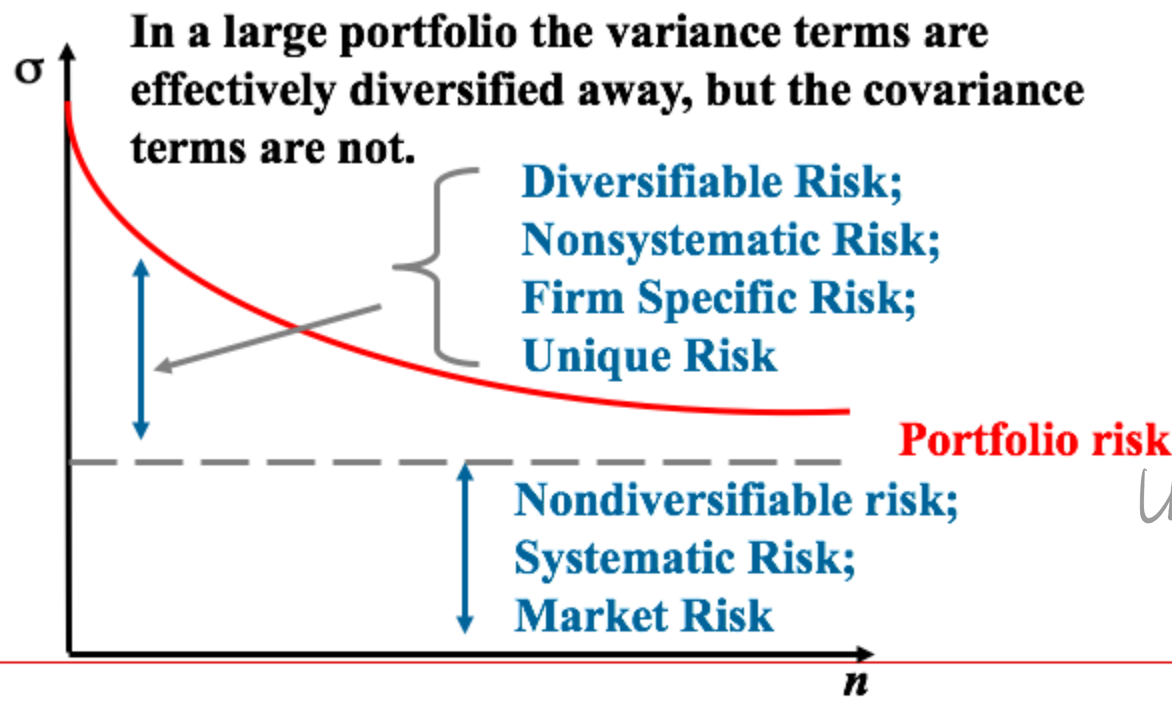
When ρ >0:

- $\sigma_n = \sigma\sqrt{\frac{1+(n-1)\rho}{n}} = \sigma\sqrt{\frac{1-\rho}{n} + \rho}$
- $\lim_{n\to\infty} \sigma_n = \sigma\sqrt{\rho}$ >0
- No matter how many assets you include in the portfolio, the risk will always > 0



Returns of different stocks are usually correlated. Thus, a portfolio of hundreds of stocks still have significant level of risks!

Implication: Systematic Risk



Diversification can eliminate idiosyncratic risks but NOT systematic risks!

Diversifiable

Idiosyncratic risks:

- Fluctuations of a stock's return that are due to firm-specific news, e.g.,
- The CEO suddenly dies
- A factory catches fire
- Consumer don't like the new product

Undiversifiable

Systematic risks:

- Fluctuations of a stock's return that are due to market-wide news, e.g.,
- Economy is in recession
- A large bank fails
- Interest rate changes
- Global warming
- Coronavirus outbreak

What can and can't diversification do?

- Can diversification guarantee an reduction in risk?
  - Only when $\rho \neq 1$, $\because \sigma_P = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \rho_{ij} \sigma_i \sigma_j} \leq \sum w_i \sigma_i$
- Can diversification eliminate all risks?
  - Only under very stringent conditions
    - 2 assets: $\rho = -1$
    - N assets: no systematic risk (ρ=0)
  - In most situations, no.

Summary

- Weights: $w_i = \frac{value\ of\ asset\ i}{Total\ value\ of\ the\ portfolio}$

- Return: $R_P = \sum_{i=1}^{n} w_i R_i$

- Risk: $\sigma_P = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \rho_{ij} \sigma_i \sigma_j} \leq \sum w_i \sigma_i$

- Diversification: hold multiple assets so that we can reduce risks without reducing return
  - Still cannot eliminate systematic risks

Next Time—Capital Asset Pricing Model

- Efficient Frontier
- Capital Asset Pricing Model
  - Beta (β): a Measure of Systematic Risks
  - Estimating Beta of a Single Stock
  - Portfolio Beta
  - CAPM and the Security Market Line
  - Does it work?
  - Alpha (α)