STA2001 Tutorial 13

似乎没有过分的难…但是真的太多了!

1. 5.7-13. Let $X_1, X_2, \cdots, X_{36}$ be a random sample of size 36 from the geometric distribution with pmf $f(x) = (1/4)^{x-1}(3/4), x = 1, 2, 3, \cdots$. Approximate

   (a) $P(46 \le \sum_{i=1}^{36} X_i \le 49)$.

   (b) $P(1.25 \le \bar{X} \le 1.50)$.

   Hint: Observe that the distribution of the sum is of the discrete type.

$X_1, \cdots, X_{36} \sim$ Geometric dis $p(\frac{3}{4})$

$$E(X_i) = \frac{1}{p} = \frac{4}{3} \qquad Var(X_i) = \frac{1-p}{p^2} = \frac{4}{9}$$

$\sum_i X_i = Y$: <THM 5.3-2>

$$E(Y) = \frac{4}{3} \cdot 36 = 48 \qquad Var(Y_i) = \frac{4}{9} \cdot 36 = 16$$

(1) $P(46 \le Y \le 49) = P\left(\frac{46-48-0.5}{4} \le Z \le \frac{49-48+0.5}{4}\right)$

$= \Phi(0.375) - \Phi(-0.625) = 0.3802$

(2) $\cdots = P(1.25 \times 36 \le 36 \cdot \bar{X} \le 1.5 \times 36) = P(45 \le Y \le 54)$

$= P\left(\frac{45-48-0.5}{4} \le Z \le \frac{54-48+0.5}{4}\right) = \Phi(1.625) -$

$0.7571 = \Phi(-0.875)$

只知道可能要用 CLT

2. 5.9-3. Let S^2 be the sample variance of a random sample of size n from $N(\mu, \sigma^2)$. Show that the limit, as $n \to \infty$, of the mgf of S^2 is $e^{\sigma^2 t}$.

For $\chi^2(r)$: mgf is: $M(t)=(1-2t)^{-\frac{r}{2}}$, $t<\frac{1}{2}$

THM 5.5-2: $W=\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$M_W(t)=E(e^{tW})=(1-2t)^{-\frac{n-1}{2}}$, $t<\frac{1}{2}$

with $W=\frac{(n-1)S^2}{\sigma^2} \Leftrightarrow S^2=\frac{\sigma^2}{n-1}W$

$M_{S^2}(t)=M_W\left(\frac{\sigma^2}{n-1}t\right)=(1-\frac{\sigma^2 t}{(n-1)}$

...... 真的死定了 看什么字好看呢？

三横是什么意思啊......

他走了我操!!! 问题真不会……

3. Let $X_n \xrightarrow{d} X$ where $X \equiv x$ is a constant random variable. Prove that $X_n \xrightarrow{p} X$.

Note that $\xrightarrow{d}$ is the convergence in distribution and $\xrightarrow{p}$ is the convergence in probability.

Fix $\varepsilon > 0$, then: $P_r(|X_n - X| \le \varepsilon) = P_r(|X_n - x| \le \varepsilon)$

$= P_r(x - \varepsilon \le X_n \le x + \varepsilon) \ge P_r(x - \varepsilon < X_n \le x + \varepsilon)$

$= F_{X_n}(x+\varepsilon) - F_{X_n}(x-\varepsilon) \xrightarrow{n\to\infty} F_X(x+\varepsilon) - F_X(x-\varepsilon)$

$= P_r(x-\varepsilon) < X \le x+\varepsilon) = 1$

$\rightarrow \lim_{n\to\infty} P_r(|X_n - X| \le \varepsilon) = 1$ for any $\varepsilon > 0$