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16.3

② $\frac{\partial P}{\partial y} = x\cos z = \frac{\partial N}{\partial z}$, $\frac{\partial M}{\partial z} = y\cos z = \frac{\partial P}{\partial x}$, $\frac{\partial N}{\partial x} = \sin z = \frac{\partial M}{\partial y}$ ∴ conservative

⑥ $\frac{\partial P}{\partial y} = 0 = \frac{\partial N}{\partial z}$, $\frac{\partial M}{\partial z} = 0 = \frac{\partial P}{\partial x}$, $\frac{\partial N}{\partial x} = -e^x \sin y = \frac{\partial M}{\partial y}$ ∴ conservative

⑩ $\frac{\partial f}{\partial x} = y\sin z$ ∴ $f(x,y,z) = xy\sin z + g(y,z)$

$\frac{\partial f}{\partial y} = x\sin z + \frac{\partial g}{\partial y} = x\sin z \Rightarrow \frac{\partial g}{\partial y} = 0$ ∴ $g(y,z) = h(z)$ $f(x,y,z) = xy\sin z + h(z)$

$\frac{\partial f}{\partial z} = xy\cos z + h'(z) = xy\cos z$

∴ $h'(z) = 0$ $h(z) = C$ $f(x,y,z) = xy\sin z + C$

⑰ Let $F(x,y,z) = (\sin y\cos x)\vec{i} + (\cos y\sin x)\vec{j} + \vec{k}$

$\int_{(1,0,0)}^{(0,1,1)} \sin y\cos x\,dx + \cos y\sin x\,dy + dz = f(0,1,1) - f(1,0,0) = 1$

⑳ Let $F(x,y,z) = (2x\ln y - yz)\vec{i} + \left(\frac{x^2}{y} - xz\right)\vec{j} - (xy)\vec{k}$

$\frac{\partial P}{\partial y} = -x = \frac{\partial N}{\partial z}$ $\frac{\partial M}{\partial z} = -y = \frac{\partial P}{\partial x}$, $\frac{\partial N}{\partial x} = \frac{2x}{y} - z = \frac{\partial M}{\partial y}$

∴ $f(x,y,z) = x^2\ln y - xyz + C$

$\int_{(1,2,1)}^{(2,1,1)} (2x\ln y - yz)dx + \left(\frac{x^2}{y} - xz\right)dy - xy\,dz = -\ln 2$

㉚ $\frac{\partial P}{\partial y} = xe^{yz} + xyze^{yz} + \cos y = \frac{\partial N}{\partial z}$ $\frac{\partial M}{\partial z} = ye^{yz} = \frac{\partial P}{\partial x}$ $\frac{\partial N}{\partial x} = ze^{yz} = \frac{\partial M}{\partial y}$

∴ F is conservative

(a) $\text{work} = \int_A^B \vec{F}\cdot dr = [xe^{yz} + z\sin y]_{(1,0,1)}^{(1,\frac{\pi}{2},0)} = 0$

(b) $\text{work} = \int_A^B \vec{F}\cdot dr = [xe^{yz} + z\sin y]_{(1,0,1)}^{(1,\frac{\pi}{2},0)} = 0$

(c) $\text{work} = \int_A^B \vec{F}\cdot dr = [xe^{yz} + z\sin y]_{(1,0,1)}^{(1,\frac{\pi}{2},0)} = 0$

㉝ (a) $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z} \Rightarrow 2ay = cy$ ∴ $2a = c$

$\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \Rightarrow$ ∴ $2cx = 2cx$

$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y} \Rightarrow by = 2ay \Rightarrow b = 2a$ ∴ $2a = b = c$

(b) $\vec{F} = \nabla f$ $a = 1, b = 2, c = 2$

㉞ $\vec{F} = \nabla f$ $g(x,y,z) = \int_{(0,0,0)}^{(x,y,z)} \vec{F}\cdot dr = \int_{(0,0,0)}^{(x,y,z)} \nabla f\, d\vec{r} = f(x,y,z) - f(0,0,0)$

∴ $\frac{\partial g}{\partial x} = \frac{\partial f}{\partial x} - 0$ $\frac{\partial g}{\partial y} = \frac{\partial f}{\partial y} - 0$ $\frac{\partial g}{\partial z} = \frac{\partial f}{\partial z} - 0$ ∴ $\nabla g = \nabla f = \vec{F}$

16.4

(8) $M = x+y \quad N = x^2+y^2$

$\text{Flux} = \iint_R (-2x+2y)\,dx\,dy = \int_0^3\int_0^x (-2x+2y)\,dy\,dx = -9$

$\text{Circ} = \iint_R (2x-2y)\,dx\,dy = \int_0^3\int_0^x (2x-2y)\,dy\,dx = \int_0^3 x^2\,dx = 9$

(9). $M = xy+y^2 \quad N = x-y$

$\text{Flux} = \int_0^1\int_{x^2}^{\sqrt{x}} (y-1)\,dy\,dx = \int_0^1 (\frac{1}{2}x - \sqrt{x} - \frac{1}{2}x^4 + x^2)\,dx = -\frac{11}{60}$

$\text{Circ} = \int_0^1\int_{x^2}^{\sqrt{x}} (1-x-2y)\,dy\,dx = \int_0^1 (\sqrt{x} - x^{\frac{3}{2}} - x - x^2 + x^3 + x^4)\,dx = -\frac{7}{60}$

(19) $M = 2xy^3 \quad N = 4x^2y^2$

$\text{work} = \int_0^1\int_0^{x^3} 2xy^2\,dy\,dx = \int_0^1 \frac{2}{3}x^{10}\,dx = \frac{2}{33}$

(23) $M = 6y+x \quad N = y+2x$

$\oint_C (6y+x)\,dx + (y+2x)\,dy = -16\pi$

(26) $M = x = a\cos t \quad N = y = b\sin t$

$dx = -a\sin t\,dt \quad dy = -b\cos t\,dt$

$\text{Area} = \frac{1}{2}\oint_C x\,dy - y\,dx = \frac{1}{2}\int_0^{2\pi} ab\,dt = \pi ab$

(27) $M = x = \cos^3 t \quad N = y = \sin^3 t$

$\text{Area} = \frac{1}{2}\oint_C x\,dy - y\,dx = \frac{1}{2}\int_0^{2\pi} (3\sin^2 t\cos^2 t)\,dt = \frac{3}{8}\int_0^{2\pi} \sin^2 2t\,dt = \frac{3}{16}\left[\frac{u}{2} - \frac{\sin 2u}{4}\right]_0^{4\pi} = \frac{3}{8}\pi$

(30) $M = xy^2 \quad N = x^2y+2x$

$\oint_C xy^2\,dy + (x^2y+2x)\,dy = 2\iint_R dx\,dy = 2$ times the Area of the square

(31) $M = \frac{\partial f}{\partial y} \quad N = -\frac{\partial f}{\partial x} \Rightarrow \frac{\partial M}{\partial y} = \frac{\partial^2 f}{\partial y^2} \quad \frac{\partial N}{\partial x} = -\frac{\partial^2 f}{\partial x^2}$

$\oint_C \frac{\partial f}{\partial y}\,dx - \frac{\partial f}{\partial x}\,dy = \iint_R (-\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2})\,dx\,dy = 0$

(38) $M = \frac{1}{4}x^2y + \frac{1}{3}y^3, \quad N = x$

$\text{Work} = \iint_R (1 - \frac{1}{4}x^2 - y^2)\,dx\,dy$

will be ~~min~~ maximized by $1 = \frac{1}{4}x^2 + y^2$

16.5

(5) $x=r\cos\theta \quad y=r\sin\theta$

$x^2+y^2=r^2 \Rightarrow z^2=9-r^2 \quad z=\sqrt{9-r^2}$

$\vec{r}(r,\theta)=(r\cos\theta)\vec{i}+(r\sin\theta)\vec{j}+\sqrt{9-r^2}\vec{k}$

$\therefore 0\le r\le\frac{3}{\sqrt{2}}$

(7) $x=\rho\sin\phi\cos\theta \quad y=\rho\sin\phi\sin\theta \quad \rho^2=3 \quad \rho=\sqrt{3}$

$z=\sqrt{3}\cos\phi$

$\vec{r}(\phi,\theta)=(\sqrt{3}\sin\phi\cos\theta)\vec{i}+(\sqrt{3}\sin\phi\sin\theta)\vec{j}+(\sqrt{3}\cos\phi)\vec{k}$

for $\frac{\pi}{3}\le\phi\le\frac{2\pi}{3}$ and $0\le\theta\le 2\pi$

(16) $y=w\cos v \quad z=w\sin v$

$y^2+(z-5)^2=25 \quad y^2+z^2-10z=0 \qquad \therefore w=0$ or $w=10\sin v$

$w=0 \Rightarrow y=0, z=0$ not a cylinder

$w=10\sin v \Rightarrow y=10\sin v\cos v \quad z=10\sin^2 v$

$\vec{r}(u,v)=u\vec{i}+(10\sin v\cos v)\vec{j}+(10\sin^2 v)\vec{k} \qquad 0\le u\le 10,\ 0\le v\le\pi$

(18) let $x=r\cos\theta$, $y=r\sin\theta$, $z=-x=-r\cos\theta$

$\vec{r}_r\times\vec{r}_\theta=r\vec{i}+r\vec{k}$

$|\vec{r}_r\times\vec{r}_\theta|=\sqrt{r^2+r^2}=r\sqrt{2} \Rightarrow A=\int_0^{2\pi}\int_0^2 r\sqrt{2}\,dr\,d\theta=4\pi\sqrt{2}$

(23) $\vec{r}_r=(\cos\theta)\vec{i}+(\sin\theta)\vec{j}-2r\vec{k}$

$\vec{r}_\theta=(-r\sin\theta)\vec{i}+(r\cos\theta)\vec{j}$

$\vec{r}_r\times\vec{r}_\theta=r\sqrt{4r^2+1}$

$A=\int_0^{2\pi}\int_0^1\sqrt{4r^2+1}\,dr\,d\theta=\frac{\pi}{6}(5\sqrt{5}-1)$

(30) $\vec{r}(x,y)=x\vec{i}+y\vec{j}-x^2\vec{k}$ at $P_0(1,2,-1)$

$\vec{r}_x\times\vec{r}_y=\begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & 0\end{vmatrix}=2\vec{i}+\vec{k}$

$\therefore$ the surface is $z=-x^2$

(31) (b) $\vec{r}_u=(-r\sin u\cos v)\vec{i}-(r\sin u\sin v)\vec{j}+(r\cos u)\vec{k}$

$\vec{r}_v=(-(R+r\cos u)\sin v)\vec{i}+((R+r\cos u)\cos v)\vec{j}$

$\vec{r}_u\times\vec{r}_v=-(R+r\cos u)(r\cos v\cos u)\vec{i}-(R+r\cos u)(r\sin v\cos u)\vec{j}+(-r\sin u)(R+r\cos u)\vec{k}$

$A=\int_0^{2\pi}\int_0^{2\pi}(rR+r^2\cos u)\,du\,dv=4\pi^2 rR$

(40) $S=\int_0^{\sqrt{3}}\int_0^x\sqrt{x^2+1}\,dy\,dx=\int_0^{\sqrt{3}}x\sqrt{x^2+1}\,dx=\frac{7}{3}$

(56) (b) $\vec{r}_x(x,\theta)=\vec{i}+f'(x)\cos\theta\vec{j}+f'(x)\sin\theta\vec{k}$, $\vec{r}_\theta(x,\theta)=-f(x)\sin\theta\vec{j}+f(x)\cos\theta\vec{k}$

$\vec{r}_x\times\vec{r}_\theta=f(x)f'(x)\vec{i}-f(x)\cos\theta\vec{j}-f(x)\sin\theta\vec{k}$

$A=\int_a^b\int_0^{2\pi}f(x)\sqrt{1+(f'(x))^2}\,d\theta\,dx=\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$