Problem Set 2

Tips

Step 1: Read the question roughly (Determine Topic): Bond/Stock/debt/Mortgage/ /Portfolio/CAPM/Derivatives……?

Step 2: Read the question again to extract and process the information: List the information (PV, r, %, SD, EPS…)

Step 3: Draw the timeline

Step 4: Write down the relevant equations, enter all informed numbers, and calculate the answer

1. (1) Suppose a company just paid a dividend of $4. Its manager promises to pay annual dividend growing at 6% per year. If the required return is 10% (in EAR), what would the current price be?

- EAR=10% g=6% DIV0=$4

$$P_0 = \frac{DIV1}{r-g} = \frac{\$4*(1+6\%)}{10\%-6\%} = \$\ 106$$

(2) Now assume the manager has decided to pay quarterly dividends instead of annual dividends. It has just paid a dividend of $1. The next dividend will be 3 months from now, with value equal to $\$1*1.06^{1/4}$, and will grow at an annual rate of 6%. That is, each dividend payment will be $1.06^{1/4}$ times the previous one. What is the share price now?

- Div 0 = $1; EAR=(1+r)^4-1= 10% → Quarterly rate= 2.41%;
- Quarterly g= (1+6%)^1/4 -1=1.47%

$$P_0 = \frac{DIV1}{r-g} = \frac{\$1*(1+6\%)^{0.25}}{2.41\%-.1.47\%} = \$\ 107.49$$

(3) Now assume the dividend will grow more slowly at a rate of 1% per year from the 40th quarterly dividend. That is, the 41st dividend will be equal to the 40th dividend multiplied by $1.01^{1/4}$. What is the share price now?

- CF 1: From year 1 to year 40: $Div_1 = 1 * 1.06^{0.25}$; g=1.06^(1/4) -1; r = 1.1^(1/4) -1; n=40

Limited Growth: $PV_0 = PMT_1 * \frac{1 - \frac{(1+g)^\wedge t}{(1+r)^\wedge t}}{r-g}$

- CF 2: From year 41 to infinity: Div_{41} = 1*1.06^10 *1.01^0.25 ; g=1.01^(1/4) -1 ; r = 1.1^(1/4) -1

Perpetuity Growth: $PV_0 = \frac{PMT_1}{r-g}$

$$P_0 = [(1 * 1.06^{0.25}) * \frac{1 - (\frac{1.06^{10}}{1.1^{10}})}{1.1^{1/4} - 1.06^{1/4}}] + [\frac{1 * 1.06^{10} * 1.01^{0.25}}{(1.1^{1/4} - 1) - (1.01^{1/4} - 1)}]/(1.1^{1/4})^\wedge 40$$

$= \$ 65.28$

2. The returns of two stocks and one bond in 4 possible states of economies are given below.

	Probability	Stock A	Stock B	Bond C
Recession	30%	-25%	-10%	5%
Normal-bad	10%	-5%	1%	5%
Normal-good	20%	10%	6%	3%
Boom	40%	30%	9%	1%

(1) What is expected return and the standard deviation of the three assets?

E(Ra)= -25%*30%-5%*10%+10%*20%+30%*40%= 6%

$\sigma(Ra) = \sqrt{(-25\%-6\%)^2 * 30\% + (-5\%-6\%)^2 * 10\% + (10\%-6\%)^2 * 20\% + (30\%-6\%)^2 * 40\%} = 23.11\%$

E(Rb)= -10%*30%+1%*10%+6%*20%+9%*40%= 1.9 %

$\sigma(Rb) = \sqrt{(-10\% - 1.9\%)^2 * 30\% + (1\% - 1.9\%)^2 * 10\% + (6\% - 1.9\%)^2 * 20\% + (9\% - 1.9\%)^2 * 40\%} = 8.13\ \%$

E(Rc)=5%*30%+5%*10%+3%*20%+1%*40%=3 %

$\sigma(Rc) = \sqrt{(5\% - 3\%)^2 * 30\% + (5\% - 3\%)^2 * 10\% + (3\% - 3\%)^2 * 20\% + (1\% - 3\%)^2 * 40\%} = 1.79\ \%$

(2) What are the pairwise correlations among the 3 assets (i.e., $\rho_{AB}, \rho_{AC}, \rho_{BC}$)?

$$Corr(R_a, R_b) = \frac{Cov(R_a, R_b)}{\sigma(R_a)\sigma(R_b)} = \frac{E[(R_a - \bar{R}_a)(R_b - \bar{R}_b)]}{\sigma(R_a)\sigma(R_b)} =$$

$$\frac{(-25\% - 6\%)(-10\% - 1.9\%)30\% + (-5\% - 6\%)(1\% - 1.9\%)10\% + (10\% - 6\%)(6\% - 1.9\%)20\% + (30\% - 6\%)(9\% - 1.9\%)40\%}{23.11\% * 8.13\%}$$

= 0.9747

$$Corr(R_a, R_c) = \frac{Cov(R_a, R_c)}{\sigma(R_a)\sigma(R_c)}$$

$$= \frac{(-25\% - 6\%)(5\% - 3\%)30\% + (-5\% - 6\%)(5\% - 3\%)10\% + (10\% - 6\%)(3\% - 3\%)20\% + (30\% - 6\%)(1\% - 3\%)\,40\%}{23.11\% * 1.79\%}$$

= -0.9676

$$Corr(R_b, R_c) = \frac{Cov(R_b, R_c)}{\sigma(R_b)\,\sigma(R_c)} = \frac{E[(R_b - \bar{R}_b)(R_c - \bar{R}_c)]}{\sigma(R_b)\,\sigma(R_c)} =$$

$$\frac{(-10\% - 1.9\%)(5\% - 3\%)30\% + (1\% - 1.9\%)(5\% - 3\%)10\% + (6\% - 1.9\%)(3\% - 3\%)20\% + (9\% - 1.9\%)(1\% - 3\%)\,40\%}{8.13\% * 1.79\%}$$

= -0.8939

(3) What is the Sharpe ratio of a 20% stock A, 10% stock B, and 70% bond portfolio (assuming R_f=0)?

E(Rp) = 20%* 6% + 10%* 1.9% + 70% * 3% = 3.49%

$$\sigma_P = \sqrt{w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + w_c^2\sigma_c^2 + 2 * w_a w_b \rho_{ab}\sigma_a\sigma_b + 2 * w_a w_c \rho_{ac}\sigma_a\sigma_c + 2 * w_b w_c \rho_{bc}\sigma_b\sigma_c}$$

$$= \sqrt{= (20\% * 23.11\%)^2 + (10\% * 8.13\%)^2 + (70\% * 1.79\%)^2 + 2 * 20\% * 10\% * 0.9747 * 23.11\% * 8.13\% + 2 * 20\% * 70\% * (-0.676) * 23.11\% * 1.79\% + 2 * 10\% * 70\% * (-0.8939) * 8.13\% * 1.79\%}$$

= 0.0423

$$\text{Sharp ratio} = \frac{E[Rp-Rf]}{\sigma[Rp-Rf]} = \frac{E(Rp)}{\sigma(Rp)} = \frac{3.49\%}{0.0423} = 0.83$$

3. The return profiles of 2 assets are given below. What is the minimum risk, in terms of standard deviation, that can be achieved with a portfolio that holds these two assets? The weight of the 2 assets must be positive and sum up to 1. That is, holding 0 of each and thus having 0 risk is not allowed. (Hint: let weight in asset A be w, and weight in asset B be 1-w. The standard deviation of the portfolio is a quadratic function of w. You only need to find the minimum for the quadratic function)

	E[r]	St.dev	Correlation
Asset A	10%	20%	-0.2
Asset B	5%	10%	

1) Let' s assume the weight of Asset A is w and the weight of Asset B should be (1-w)

σ(p)= $\sqrt{w^2\sigma A^2 + (1-w)^2\sigma B^2 + 2w(1-w)\rho\sigma A\sigma B}$

=$\sqrt{0.002 * (29W^2 - 14W + 5)}$ $(w > 0)$

If w= -$\frac{-14}{2*29}$ = $\frac{7}{29}$, we will get minimum risk which is 24.14%.

Extra credit (10pts. Can be used to make up for points you lost in this problem set. However, the total score of the problem set cannot exceed 100): Assume R_f=0. What is the highest Sharpe ratio that can be achieved with the 2 assets?

Ep= wEa+(1-w)Eb=0.05 (W+1)

Sharpe Ratio = $\sqrt{1.25 * \frac{(1+W)^2}{(29W^2-14W+5)}}$

$$U' = \frac{0.0478588 * \left(-\frac{1}{3}+W\right) * (1+W)}{\left(\frac{5}{29}-\frac{14W}{29}+W^2\right)^2 * \sqrt{\frac{(1+W)^2}{29W^2-14W+5}}} = 0, \quad \text{where } (0<W<1)$$

If W= 1/3, the portfolio will achieve the highest Sharpe ratio, by 0.790569.

4. Your friend Kevin is evaluating the stocks of North Great Timber Company. North Great Timber Company expects to have an EPS of $5/share next year and pay a dividend of $1.50 a share. You expect that the firms' ROE (meaning the return on both old and new investments) will stay at 5% in the future and its payout ratio will remain unchanged. According to Kevin's analysis of the firms' past stock return, the firm's beta is 1.25. He also expects that the market return will be 4% and the T-bill rate will be 2%. Currently the market price of the stock is $50/share. (10pts)

(1) What is the expected growth rate of the firms' dividend?

- When EPS= 5/share and Div_1= 1.5/share, ROE =5%

  Dividend Payout Ratio=1.5/5 = 3%
  g= 5%*(1-3%)=3.5%

(1) According to CAPM, what is the appropriate required rate of return on the stock?

We simply take T-bill rate as the risk –free return which rf=2% , Rm = 4%, Beta = 1.25
E(R)= 2%+1.25*(4%-2%)=4.5%

(3). Please estimate the fair value of the stock using DDM. Use the required return derived from CAPM as the discount rate in the DDM model.

Po=Div1/r-g= 1.5/(4.5%-3.5%)= 150

(4) Can you make an investment recommendation to your friend?

The market value of the stock is lower than its fair value. It's undervalued. I will recommend my friend to invest on this company.