STA2001 Tutorial 6

1. 3.2-16. Cars arrive at a tollbooth at a mean rate of 5 cars every 10 minutes according to a Poisson process. Find the probability that the toll collector will have to wait longer than 26.30 minutes before collecting the eighth toll.

1分钟 0.5辆车
2分钟 1辆车
每10分钟5辆车
过26.3分钟第8辆车通过的概率

$\lambda = \frac{1}{2}$

$\cancel{f(x) = \frac{e^{-\frac{1}{2}}}{2^x \cdot x!} = \frac{1}{\sqrt{e} \cdot 2^x \cdot x!}}$

2. 3.2-19. A bakery sells rolls in units of a dozen. The demand X (in 1000 units) for rolls has a gamma distribution with parameters $\alpha = 3$, $\theta = 0.5$, where θ is in units of days per 1000 units of rolls. It costs \$2 to make a unit that sells for \$5 on the first day when the rolls are fresh. Any leftover units are sold on the second day for \$1. How many units should be made to maximize the expected value of the profit?

第一天 +3

第二天 −1

应生产几打？

$\theta = \frac{1}{2}$

$\lambda = 2_{天} \Longrightarrow$ 1000个卷

即1天可以生产500个？

$f(x) = \frac{x^2 e^{-2x}}{(3-1)!\,(\frac{1}{2})^3} = \frac{4x^2}{e^{2x}}$

$= \frac{x^2}{\frac{1}{2}\frac{1}{2}\,e^{2x}}$

3. 3.3-11. A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints is $\mathcal{N}(21.37, 0.16)$.

(a) Let X denote the weight of a single mint selected at random from the production line. Find $P(X > 22.07)$.

(b) Suppose that 15 mints are selected independently and weighed. Let Y equal the number of these mints that weigh less than 20.857 grams. Find $P(Y \leq 2)$.