FIN2010 Financial Management
Lecture 8: Risk and Return

Review—Stock Valuation

- Dividend discount model: one example of cash flow based model

$$P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \frac{Div_3}{(1 + r_E)^3} + \cdots = \sum_{n=1}^{\infty} \frac{Div_n}{(1 + r_E)^n}$$

1) ***Zero growth model**: dividends are constant over time level perpetuity*
2) ***Constant growth model**: dividends grow at a constant rate growth perpetuity*
3) ***Variable growth model***: dividends change for a number of years and then stabilize to a sustainable growth rate irregular cash flows

- Which model to choose? It depends on the maturity stage of the firm!
- Implications:
  - Prices increases with firms' growth prospect, profitability.
  - Price decreases with firms' risk level.
  - When firms cut dividends, prices will increase (decrease) if firms use the retained earnings to invest in efficient (inefficient) projects.

Review—Stock Pricing

- Method of comparable
  - Estimate the value of the firm based on the value of other comparable firms
  - Stock's fair value =EPS * appropriate P/E ratio (from other firms)
- Discounted free cash flow model
  - Another cash flow based model
  - Will come back to it after we learn how to calculate free cash flows.
- All methods have their pros and cons. In reality, analysts tend to use a combination of different methods.

Agenda

- Motivation
- Definition and Measurement of Risk and Return
  - Return
  - Risk
- Empirical Facts on Historical Returns
  - US and Chinese Asset Returns
  - Sharpe Ratio
  - Ponzi Scheme

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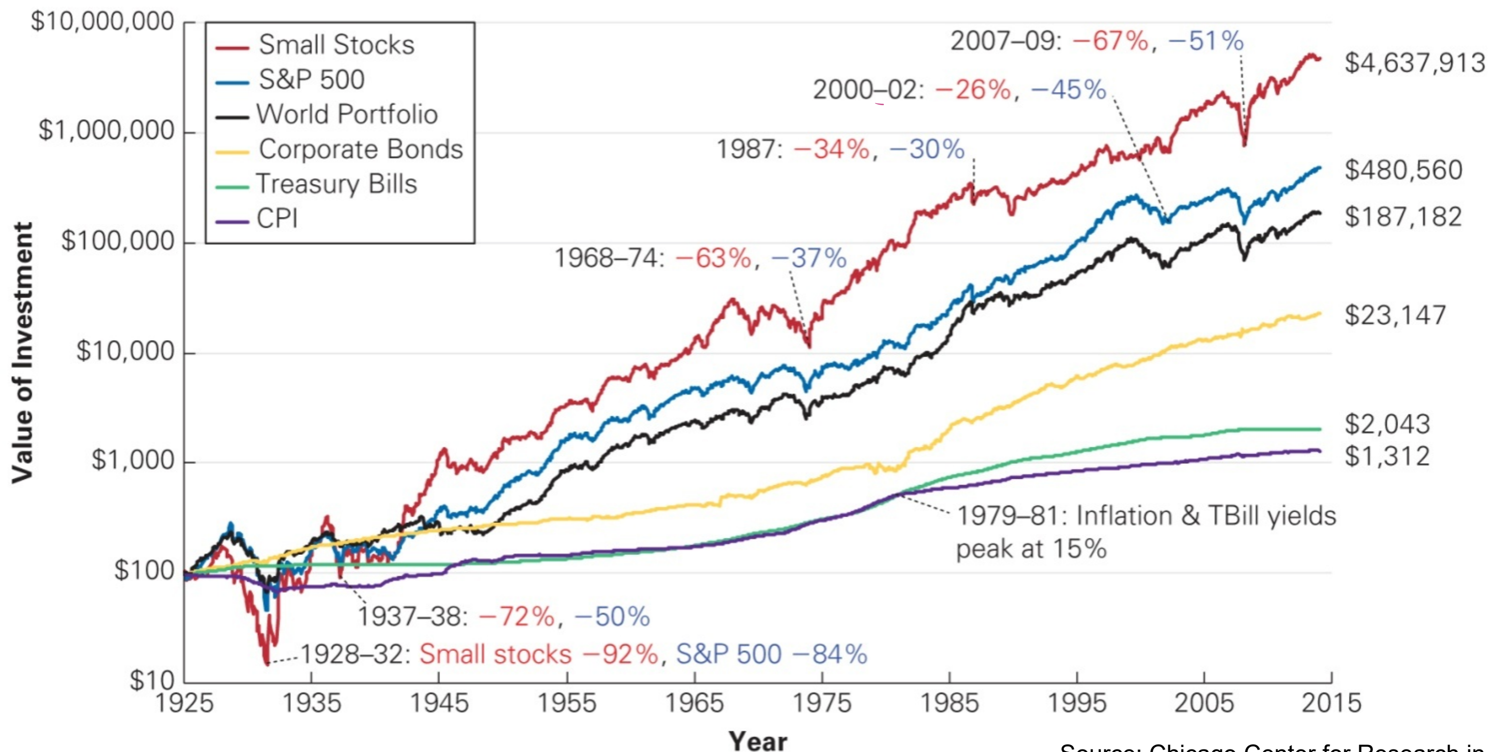
Motivation

(上证指数)

- How would $100 have grown from 1925 to 2015



Source: Chicago Center for Research in Security Prices, Standard and Poor's, MSCI, and Global Financial Data.

- Lessons from the history:
  1. There is positive return on any broad-based index in the long run. 宽基指数
  2. There is a risk and return tradeoff, i.e. higher risks ↔ higher returns.

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Return

- Holding period return (HPR): the rate of return over an investment period one is trying to evaluate (e.g. 4 months, 1 year, 3 years…). Backward looking.

- Average return: average return per year/month/… over an investment period. Backward looking.

- Expected return: the return an investor expects to earn on an investment in the future. Forward looking.

Holding Period Return (HPR)

- Definition: the percent change in wealth over an investment period (e.g. 4 months, 1 year, 3 years…).
  - How much you end up with vs. how much you put in over an investment period

$$R = \frac{Wealth_T}{Wealth_0} - 1$$

- Return on one asset: $R = \frac{P_T + CF}{P_0} - 1$
  - Measures how much money generated per unit of initial investment in this period
    - $\boldsymbol{P_0}$: price of the asset at the beginning of this period
    - $\boldsymbol{P_T}$: price of the asset at the end of this period
    - $\boldsymbol{CF}$: all cash flows generated by this asset in this period
      - For stocks, CF includes dividends
      - For bonds, CF includes coupon payments

Example: Holding Period Return

Yahoo Finance

$P_0 = 170.57$

$CF = 0.63 + 0.73 + 0.73 + 0.73 = 2.82$

$P_T = 150.73$

Return between 12/26/2017 and 12/21/2018:

$$R = \frac{P_T + CF}{P_0} - 1 = \frac{150.73 + 2.82}{170.57} - 1 = -9.98\%$$

Example: Holding Period Return with Dividend Reinvestment

Date	Close
2018-2-28	178.12
… …	
2018-2-12	162.71
2018-2-9	156.41
2018-2-9	0.63 Dividend
2018-2-8	155.15
… …	
2017-12-26	170.57

Yahoo Finance: AAPL

What is your HPR between 12/26/2017 and 2/28/2018?

- Assume the dividend is reinvested at the close price of 2018-2-12
  - Initial investment: 1 share of AAPL, $170.57
  - Final wealth:
    - 1 share of AAPL: $178.12
    - $0.63 dividend obtained on 2018-2-9, purchased $0.63/162.71 = 0.003872$ shares of APPL on 2018-2-12, which is worth $0.003872 * \$178.12 = \$0.69$
  - $R = (178.12 + 0.69)/170.57 - 1 = 4.830\%$
- What if you do not re-invest the dividend?

$$R' = (178.12 + 0.63)/170.57 - 1 = 4.796\%$$

Average Return

- When we look back into the history, returns vary from period to period. One may also be interested in knowing how much return he/she earns per period on average.

- Arithmetic average return

$$\overline{R} = \frac{R_1 + R_2 + \cdots + R_T}{T} = \frac{1}{T}\sum_{i=1}^{T} R_i$$

  – Simple to calculate; often reported by companies/financial institutions; yet can be misleading

- Geometric average return

$$G(R) = \sqrt[T]{(1+R_1)*(1+R_2)*\cdots*(1+R_T)} - 1$$

  – The return really matters – it takes into account the compounding

Example – Average Return

Year	S&P 500
2000	-9.03%
2001	-11.85%
2002	-21.97%
2003	28.36%
2004	10.74%
2005	4.83%
2006	15.61%
2007	5.48%
2008	-36.55%
2009	25.94%
2010	14.82%
2011	2.10%
2012	15.89%
2013	32.15%
2014	13.52%
2015	1.38%
2016	11.77%
2017	21.64%

What is the average return of the S&P 500 stocks between 2000 and 2017?

- Arithmetic average

$$= \frac{-9.03\% + (-11.85\%) + (-21.97\%) + \cdots + 21.64\%}{18}$$

$$= 6.94\%$$

- Geometric average

$$= \sqrt[18]{(1-9.03\%) * (1-11.85\%) * \cdots * (1+21.64\%)} - 1$$

$$= 5.34\%$$

Why Arithmetic Average can be Misleading

- Assume that we have a 6-year sequence of investment returns as follows:
  – {40%   -30%   40%   -30%   40%   -30%}
  – Suppose your initial investment is $100. At the end you have **$94.12**

- Arithmetic $\overline{R} = \frac{40\%+(-30\%)+40\%-30\%+40\%-30\%}{6} = 5\%$
- Geometric $G(R)$
  $$= \sqrt[6]{(1+40\%)*(1-30\%)*\cdots*(1-30\%)} - 1 = -1\%$$

- Theorem: Arithmetic average ≥ geometric average
  Arithmetic average return is not equal to (always higher than) the expected compound return you get when investing
- Proof of the theorem: https://brilliant.org/wiki/arithmetic-mean-geometric-mean/

Expected Return

- When we look forward, we want to know that, on expectation, what the return on an risky investment is. 风险投资
- We model the uncertainty of the future return with a *probability distribution*, which assigns a probability (Pr_i) for each possible return (R_i) that can occur.

- Expected return E(r) = weighted average of the possible returns, where the weights correspond to the probabilities.

$$E[R] = R_1 * Pr_1 + R_2 * Pr_2 + \cdots + R_n * Pr_n = \sum_{i=1}^{n} R_i * Pr_i$$

  – R_i is the return in possible outcome i.

  – Pr_i is the probability of outcome i.

Example- Expected Return

- Example: Assume BFI stock currently trades for $100 per share. In one year, there is a 25% chance the share price will be $140, a 50% chance it will be $110, and a 25% chance it will be $80. What is the expected return in a year?

- Solution:
  - The probability distribution

Current Stock Price ($)	Stock Price in One Year ($)	Probability Distribution: Return, R	Probability Distribution: Probability, P_R
	140	0.40	25%
100	110	0.10	50%
	80	−0.20	25%

  - Expected return: $\mathrm{E}[\mathrm{R}] = 25\%(0.4) + 50\%(0.1) + 25\%(-0.2) = 0.1$

Risk

- Uncertainty of the return
  - Risk a neutral word. A risky asset might have returns lower than your expectation, but might also have returns higher than your expectation
  - E.g. there are 2 investments. Which is risky?

  A. 100% chance lose $100

  B. 50% chance gain $5, 50% chance gain $10

B is risky while A is not!

Risk is the uncertainty of the return, NOT the possibility of losing money.

- Risks come from many different sources. For example, for bonds, there are
  - **Default risk**: borrower might not pay back
  - **Interest rate risk**: change in interest rate will cause change in bond price
  - **Reinvestment risk**: might need to reinvest coupon at a different rate
  - **Inflation risk**: money you get might not be worth as much
  - **Call risk**: principal might be returned earlier than expected

The Most Common Measure of Risks

- Typically measured as ***standard deviation*** (σ) of the return
  - $\sigma(R) = \sqrt{Var(R)} = \sqrt{\mathrm{E}[(R - \mathrm{E}[R])^2]} = \sqrt{\sum_{i=1}^{n} Pr_i (R_i - \mathrm{E}[R])^2}$
  - Interpretation: by how much does return ***typically*** differ from the mean

- Example

Current Stock Price ($)	Stock Price in One Year ($)	Probability Distribution	
		Return, R	Probability, P_R
	140	0.40	25%
100	110	0.10	50%
	80	−0.20	25%

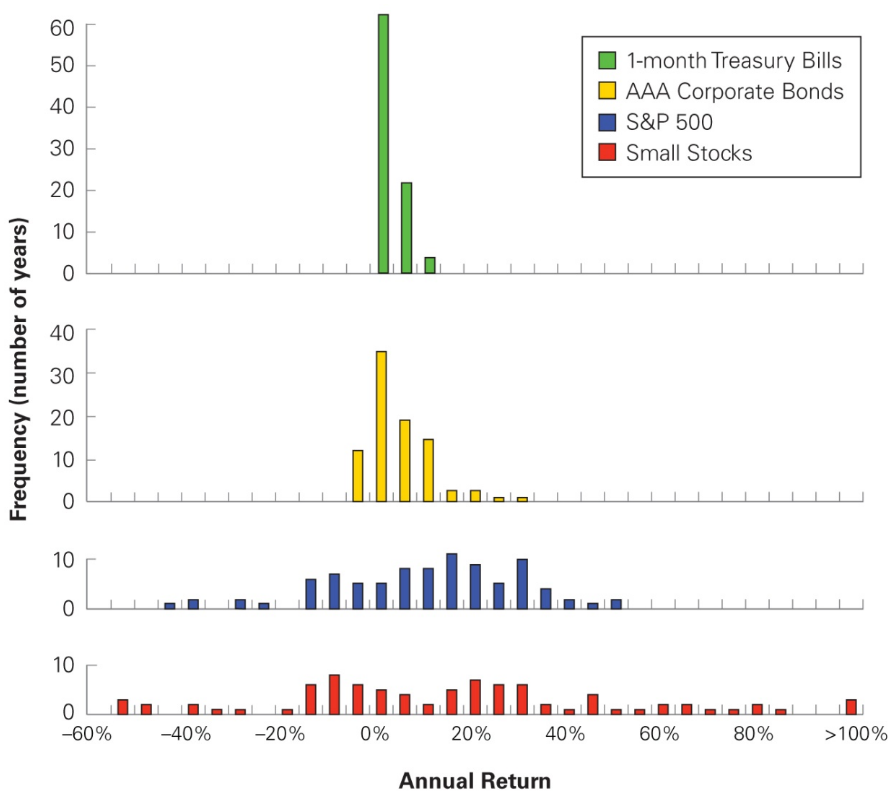
  - Expected return: $\mathrm{E}(R) = 25\%(0.4) + 50\%(0.1) + 25\%(-0.2) = 0.1$
  - Variance: $Var(R) = 25\%(0.4 - 0.1)^2 + 50\%(0.1 - 0.1)^2 + 25\%(-0.2 - 0.1)^2$
    $= 0.045$
  - Standard deviation: $\sigma(R) = \sqrt{Var(R)} = 0.212$

Use Historical Data to Estimate Expected Return and Risk

- Investor's goal: identify the probability distributions of future returns
- But this is almost impossible. In practice, we often rely on past experiences to forecast the future.
  - By counting the number of times a realized return falls within a particular range, we can estimate the underlying probability distribution.
- Assumption: the past realization has an equal chance of repeating itself in the future

Example – Empirical Distributions of Annual Returns of Different Assets

Figure: The Empirical Distribution of Annual
Returns for U.S. Large Stocks (S&P 500), Small Stocks, Corporate Bonds, and Treasury Bills, 1926–2014



Use Historical Data to Estimate Expected Return and Risk

- Expected return estimated using the historical returns:

$$E(R) = \frac{1}{T}(R_1 + R_2 + \cdots + R_T) = \frac{1}{T}\sum_{t=1}^{T} R_t$$

- Risks estimated using historical returns

$$\delta(R) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - E(R)\right)^2}$$

?

* 1. [Why is the s.d. of a sample different from the s.d. of the population?]
2. [Derivation of the s.d. of a sample]

Theorem 5 (6.5). Suppose $\mathbf{X}^n = (X_1, \cdots, X_n)$ is an IID random sample from a population with (μ, σ^2). Then for all $n > 1$,

$$E(S_n^2) = \sigma^2.$$

Proof: Using the formula $(a-b)^2 = a^2 - 2ab + b^2$, we have

$$\begin{aligned}
\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 &= \sum_{i=1}^{n}[(X_i - \mu) - (\bar{X}_n - \mu)]^2 \\
&= \sum_{i=1}^{n}(X_i - \mu)^2 - 2\sum_{i=1}^{n}(X_i - \mu)(\bar{X}_n - \mu) + \sum_{i=1}^{n}(\bar{X}_n - \mu)^2 \\
&= \sum_{i=1}^{n}(X_i - \mu)^2 - 2(\bar{X}_n - \mu)\sum_{i=1}^{n}(X_i - \mu) + n(\bar{X}_n - \mu)^2 \\
&= \sum_{i=1}^{n}(X_i - \mu)^2 - 2n(\bar{X}_n - \mu)^2 + n(\bar{X}_n - \mu)^2 \\
&= \sum_{i=1}^{n}(X_i - \mu)^2 - n(\bar{X}_n - \mu)^2,
\end{aligned}$$

where we have used the fact

$$\sum_{i=1}^{n}(X_i - \mu) = n(\bar{X}_n - \mu).$$

Taking the expectations for both sides, we have

$$\begin{aligned}
E\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 &= \sum_{i=1}^{n} E(X_i - \mu)^2 - nE[(\bar{X}_n - \mu)^2] \\
&= n\sigma^2 - n\cdot\frac{\sigma^2}{n} = (n-1)\sigma^2,
\end{aligned}$$

where we have used the fact that $E(\bar{X}_n - \mu)^2 = \frac{\sigma^2}{n}$ from Section 6.2. It follows that

$$E(S_n^2) = E\left[\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2\right] = \sigma^2.$$

Example

Year	S&P 500
2000	-9.03%
2001	-11.85%
2002	-21.97%
2003	28.36%
2004	10.74%
2005	4.83%
2006	15.61%
2007	5.48%
2008	-36.55%
2009	25.94%
2010	14.82%
2011	2.10%
2012	15.89%
2013	32.15%
2014	13.52%
2015	1.38%
2016	11.77%
2017	21.64%

What is the expected return and risk of S&P 500 in 2019?

- Estimating expected returns from historical data:

$$E[R] = \frac{1}{T}(R_1 + R_2 + \cdots + R_T) = 6.94\%$$

- Estimating risks from historical data (standard deviation of a sample):

$$\sigma = \sqrt{\frac{\sum(R_t - \bar{R})^2}{T - 1}} = 17.77\%$$

Other Measures of Risks (Not Required)

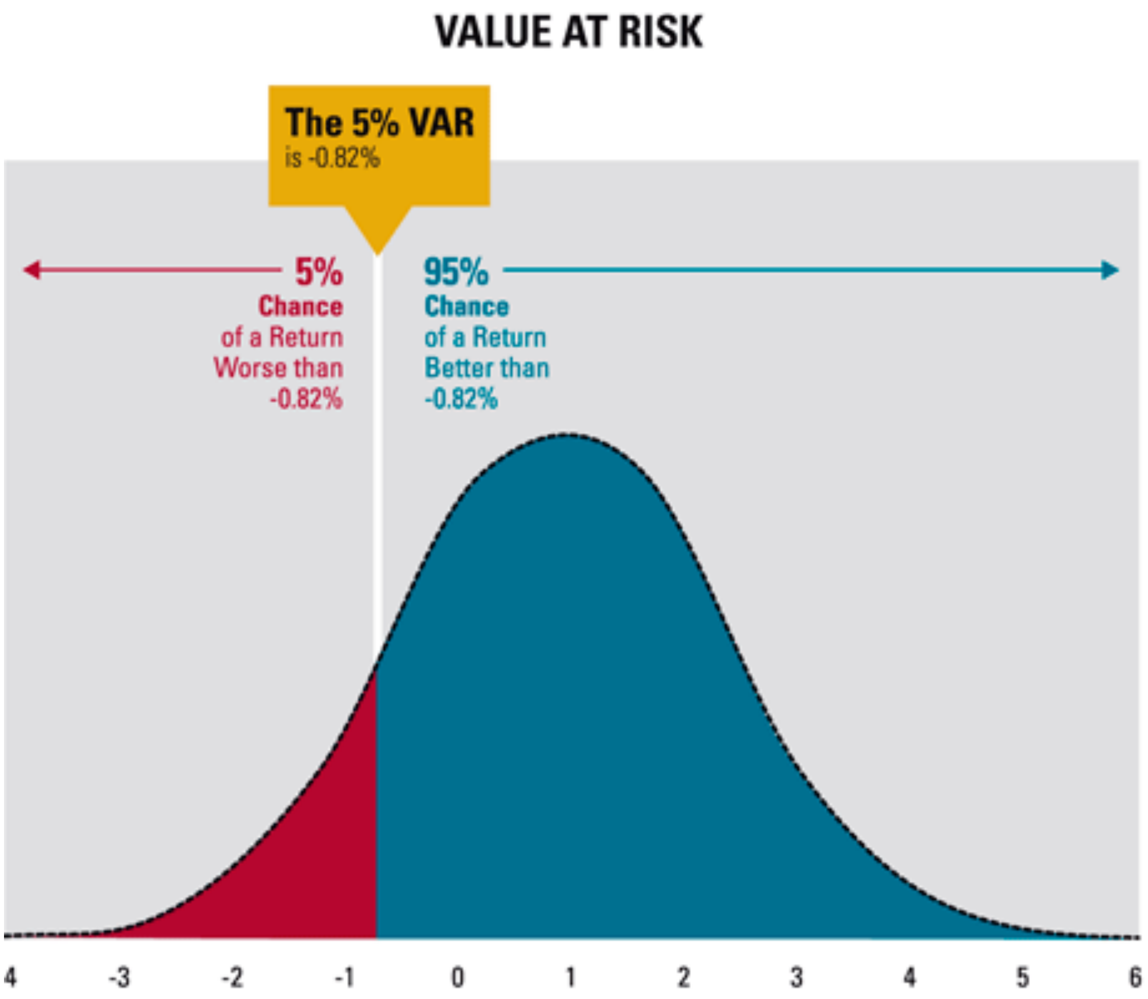
- Value at risk (VAR)

  X% of the time (e.g. 95%) return will be higher than this

- Expected Shortfall

  Expected loss if the most unfortunate X% event happens

Focus on extreme situations, often used in banks



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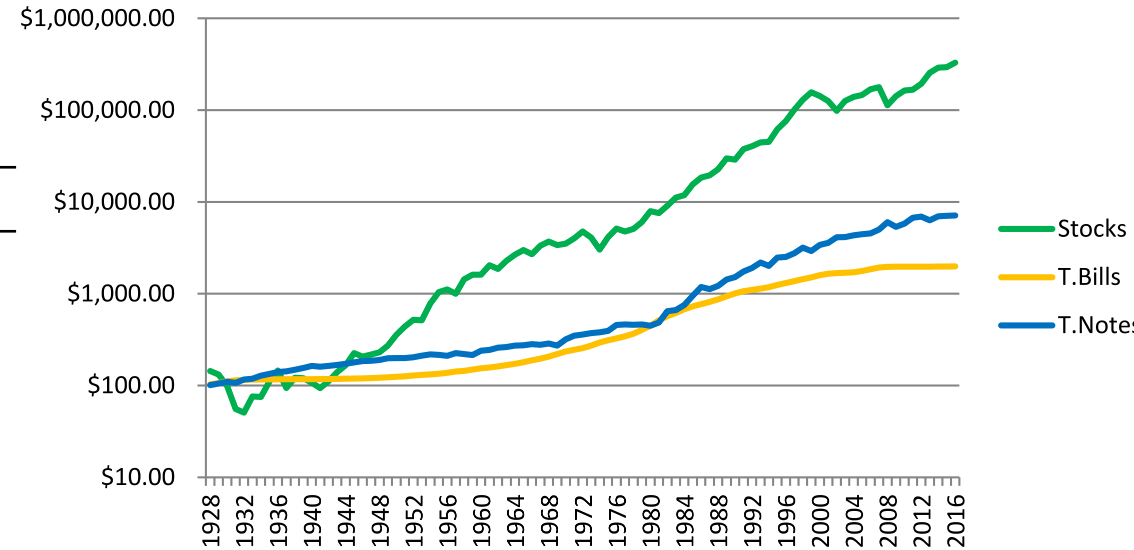
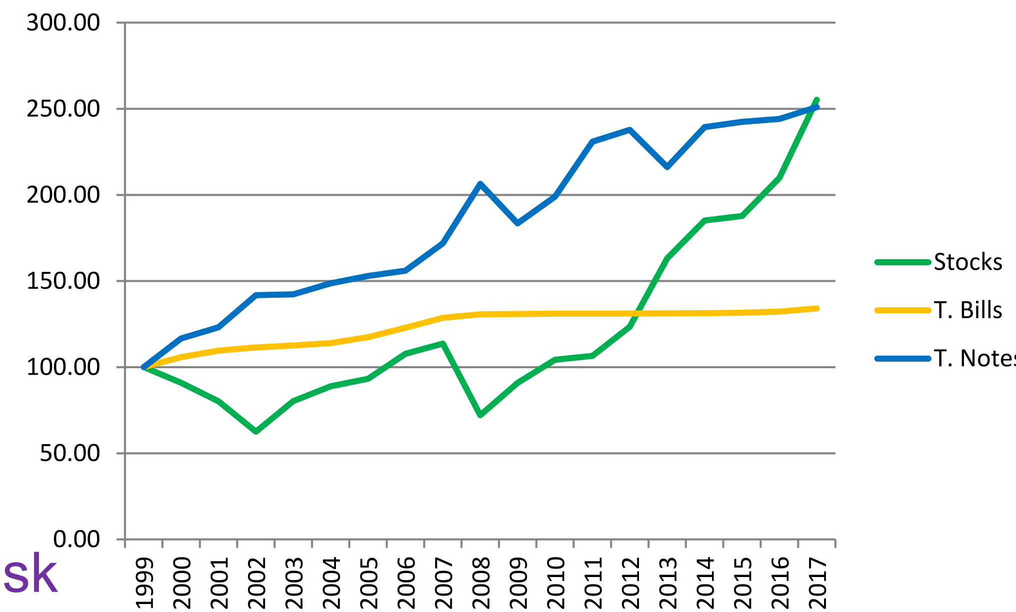
Historical Return of US Assets

	S&P 500	T-Bill	T-Note
	1928-2017		
Total Ret (HPR)	3999.27	20.17	73.07
Arithmetic avg. of annual ret	$\boxed{11.53\%}$	$\boxed{3.44\%}$	5.15%
Geometric avg. of annual ret	9.65%	3.39%	4.88%
St. dev of annual ret (δ)	19.62%	3.05%	7.72%
	2000-2017		
Total Ret(HPR)	2.55	1.34	2.51
Arith avg	$\boxed{6.94\%}$	$\boxed{1.66\%}$	5.56%
Geo avg	5.90%	1.33%	4.35%
St.dev (δ)	17.77%	1.88%	8.37%





风险溢价

Risk premium: difference in returns between risky and risk-free assets (e.g. 11.53%-3.44%=8.09% is called equity risk premium)

Long Run Performance

- In the long run, stocks > long-term bonds > short-term bonds
  - There are many periods that stocks lose money, but…
  - Stocks outperform bonds in 92% of times in 10-yr periods
  - Stocks outperform bonds in any 20-yr period after 1929

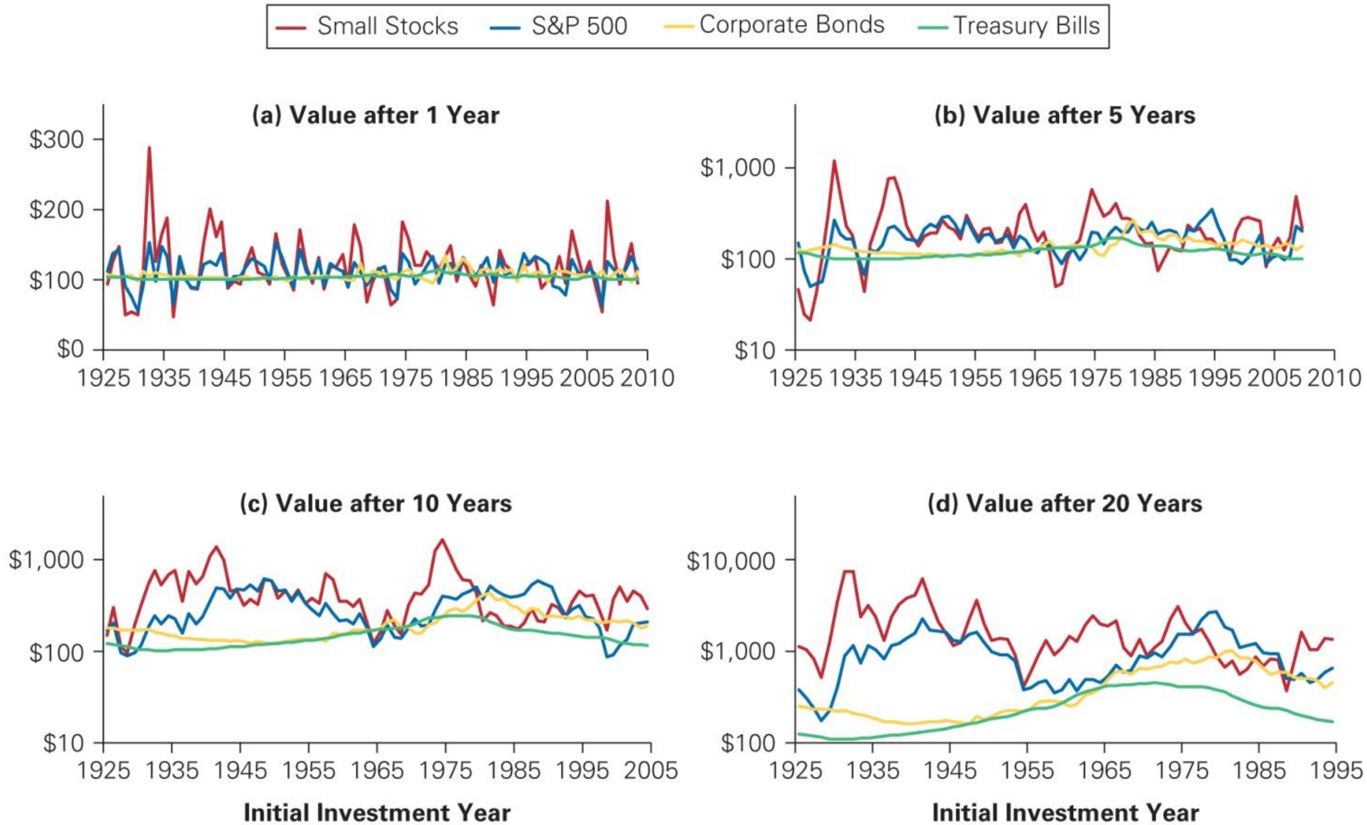
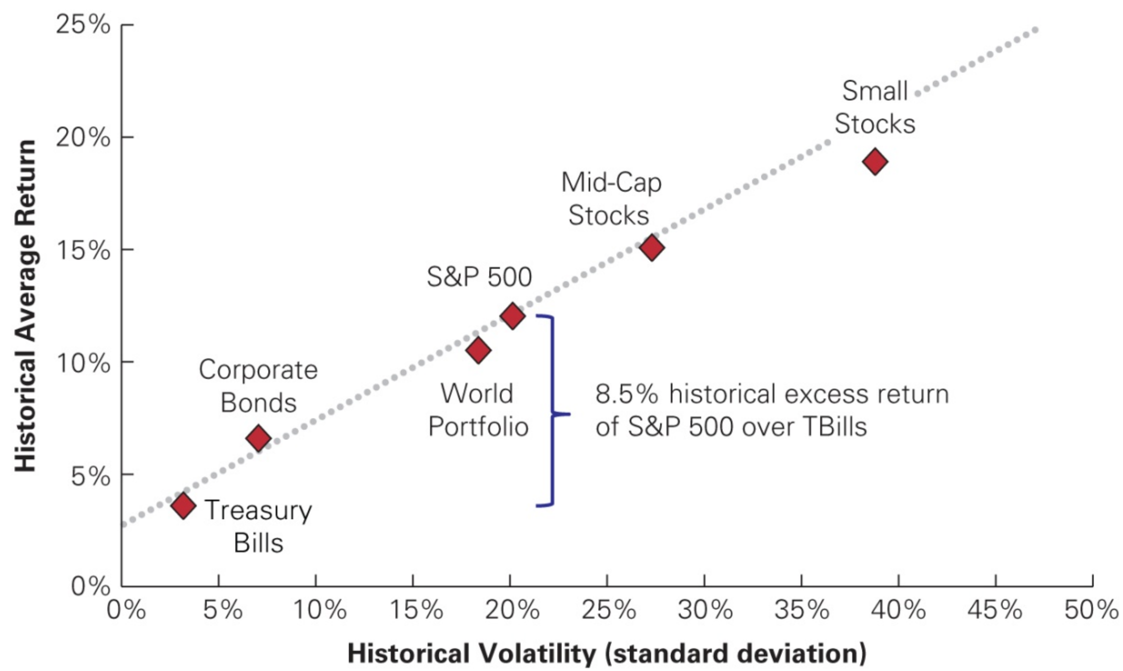


Figure: How much will you have if you invest $100 in asset x in year t for n years?

E.g. Red line in figure (a) tells you how much you will have in one year if you invest $100 in a portfolio of small stocks in any year between 1925 and 2010.

Lessons from US Historical Returns

- There is positive return on any broad-based index in the long run
  - Do not let cash sit idly!
- Higher risk → higher return (Risk- return tradeoff)
  - If you investment horizon is long, you are better off holding risky assets
- The range of risk premium
  - 5-8% for stocks
  - 1-2% for long-term government bonds



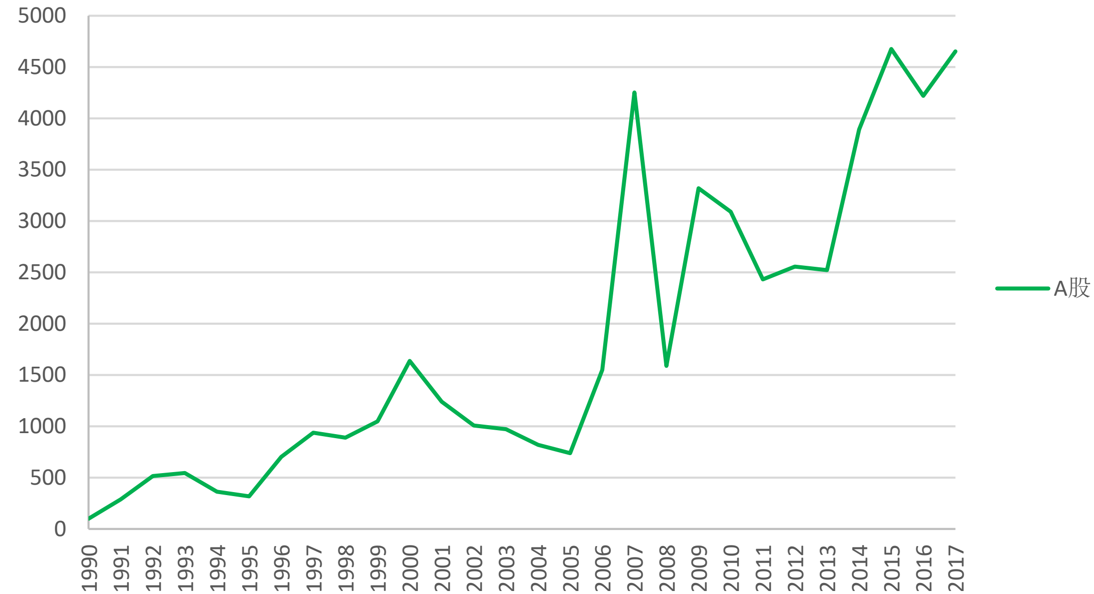
Historical Return of Chinese Stocks

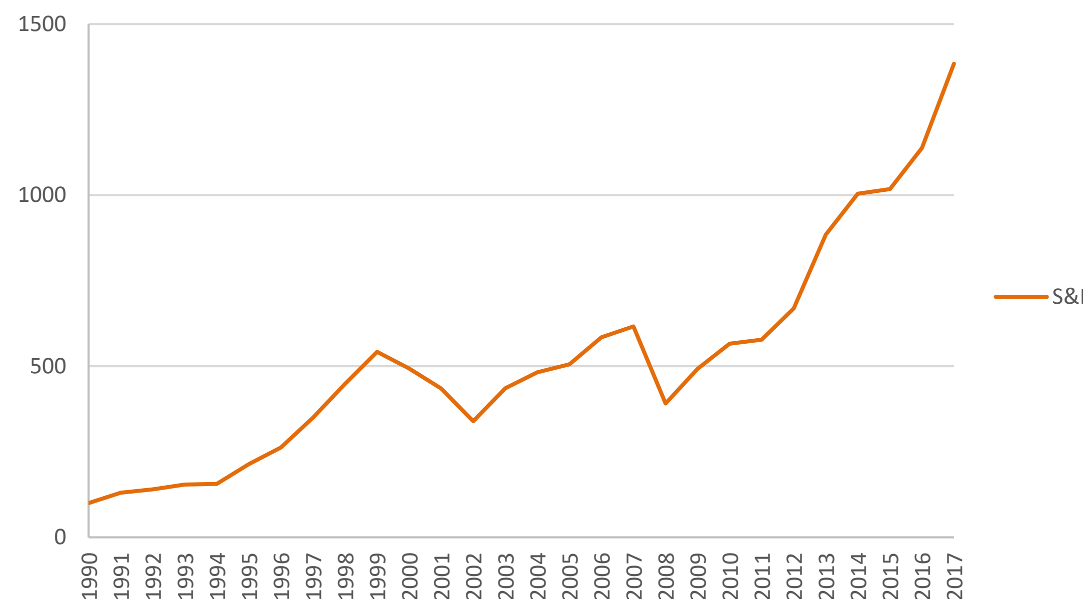
A Shares

	From 1991	From 2000
Total Ret	46.52	4.42
Arithmetic avg.	28.12%	20.30%
Geometric avg.	15.28%	8.64%
St.dev (δ)	63.38%	59.15%

S&P 500

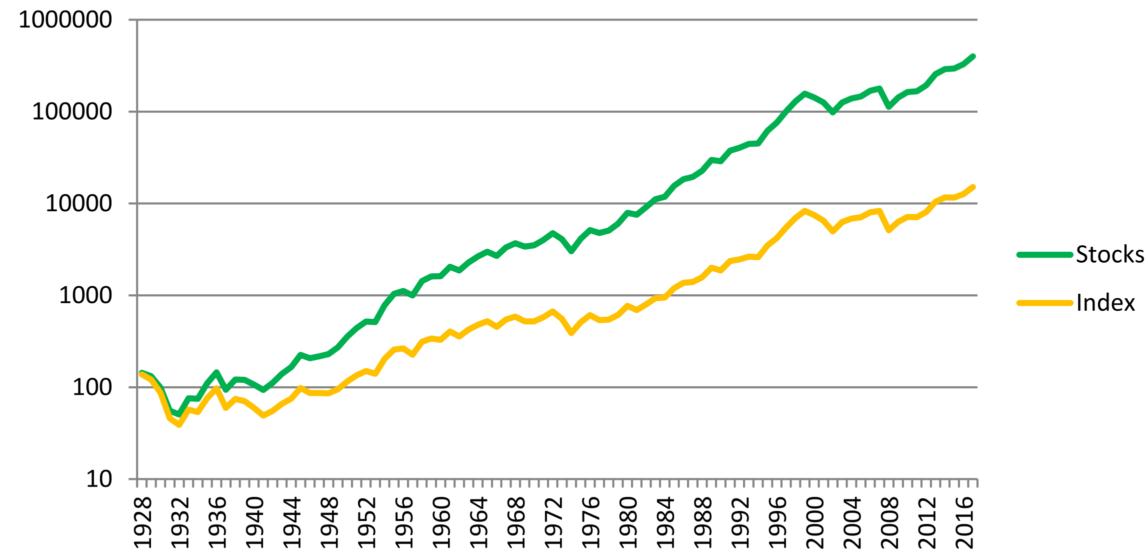
	From 1991	From 2000
Total Ret	13.84	2.55
Arithmetic avg.	11.71%	6.92%
Geometric avg.	9.15%	5.90%
St.dev (δ)	17.36%	17.77%



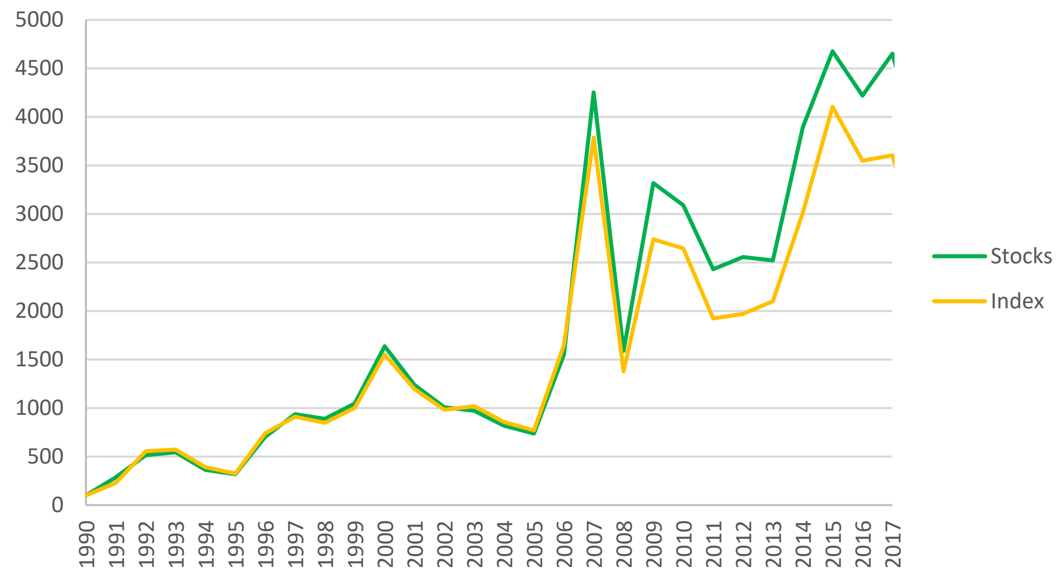


Note: Stock Return ≠ Index Return

S&P 500	Stocks	Index
	1928-2017	
Total Ret	3999.27	151.39
Arithmetic avg. of annual ret	11.53%	7.60%
Geometric avg. of annual ret	9.65%	5.74%
St.dev of annual ret	19.62%	19.18%



A shares	1990-2019	
Total Ret	45.25	33.06
Arith avg	26.33%	24.30%
Geo avg	14.05%	12.82%
St.dev	61.85%	54.35%



Lessons from Chinese Historical Returns

- Despite numerous complaints, the Chinese stock market is actually very profitable
  - A simple buy and hold strategy is very profitable.
  - If someone didn't earn as much as the broad index, he/she should really reflect on his/her trading behavior (more on lecture 23-24).

- Compared to the US market, the volatility in the Chinese stock market is extremely high
  - If you got in at the highest point in 2007, you wouldn't make much money in the past 13 years.
  - Timing is important!
    - Warren Buffet: (be) fearful when others are greedy and greedy when others are fearful

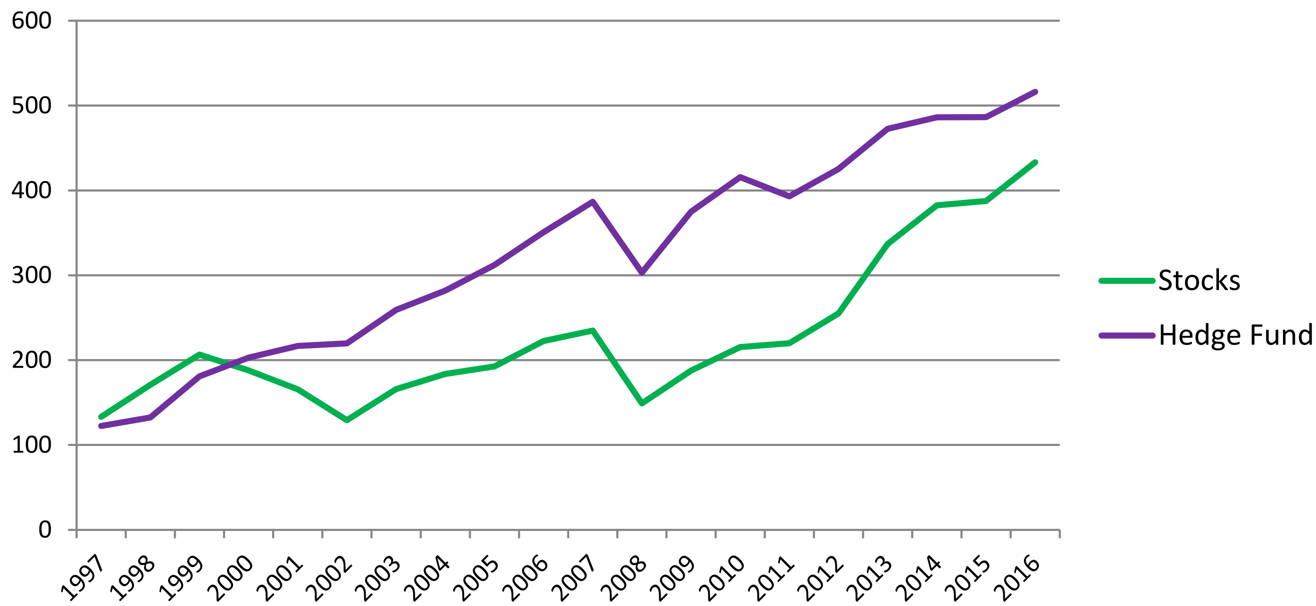
Relative Return Measure —Sharpe Ratio

夏普比率

- Sharpe ratio: $SR = \frac{\mathrm{E}[R - R_f]}{\sigma(R - R_f)}$
  - R_f is the risk-free rate. In practice we usually use the treasury bill rate as a proxy for risk-free rate.
- It measures the return of an investment relative to its risk. Also called reward to risk ratio
  - Assets with higher Sharpe ratio is generally preferred
  - Can be used to compare different investments

	Avg	Avg(R_f)	Avg – Avg(R_f)	St.dev	Sharpe Ratio
			1928-2017		
S&P 500	11.53%	3.44%	8.09%	19.94%	0.4057
Treasury Notes	5.15%	3.44%	1.71%	7.74%	0.2313
			1991-2017		
S&P 500	11.71%	2.00%	9.71%	17.29%	0.5257
T. Notes	4.81%	2.00%	2.81%	8.65%	0.4604
A Shares	28.12%	4.76%	23.36%	63.04%	0.3688

Sharpe Ratio Matters!



	Avg	T Bill	Avg - T Bill	St.dev	Sharpe Ratio
			1997-2016		
Stock	9.27%	2.14%	7.13%	18.32%	0.3895
Hedge fund	9.17%	2.14%	7.03%	11.71%	0.6002

Given the same level of arithmetic average return, assets with higher Sharpe ratios are generally preferred

Lessons

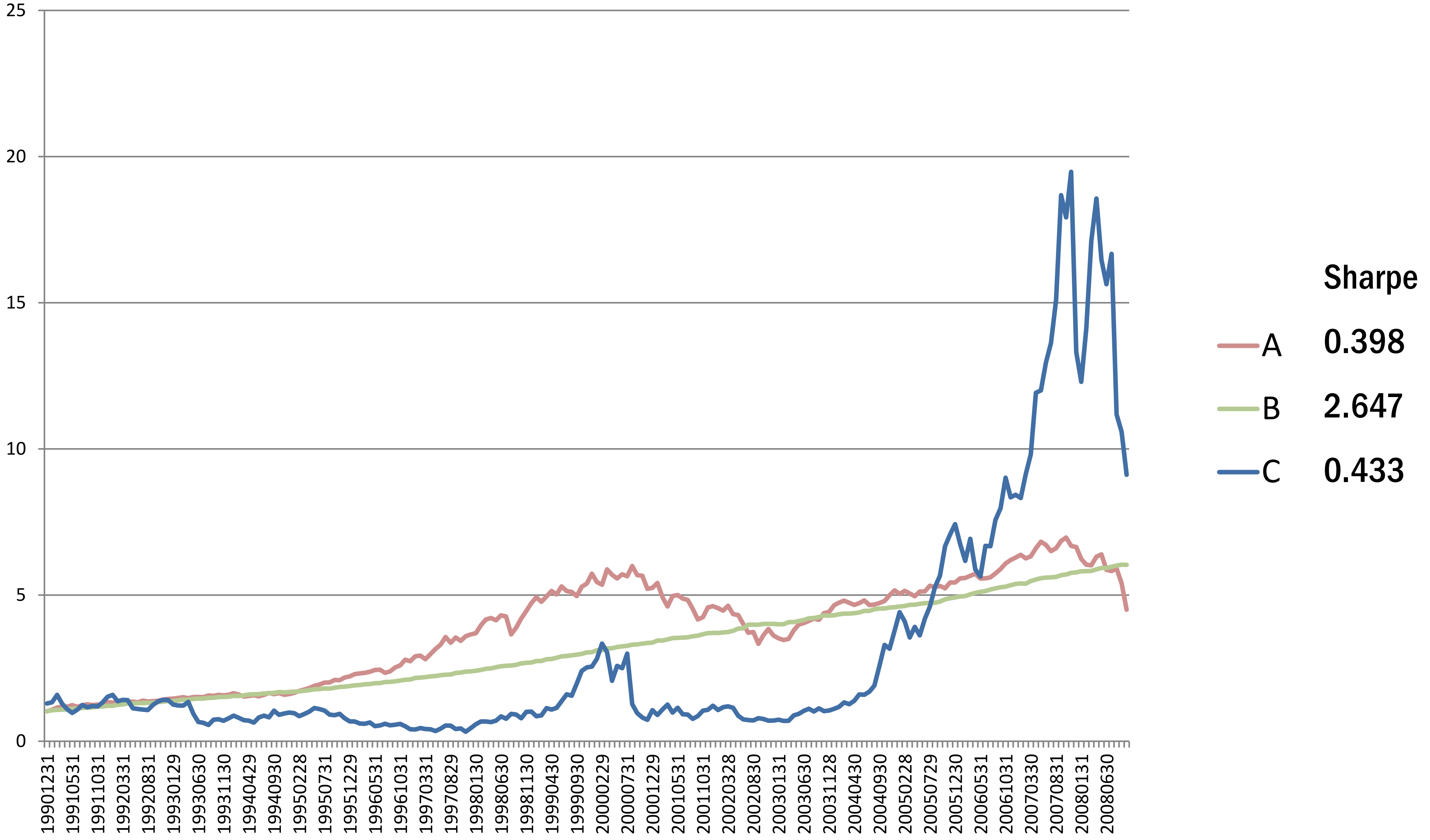
- In general, higher risk → higher return

- Given the same level of average returns, a higher Sharpe ratio usually results in higher compounded return

- On the other hand, if you would like to pursue higher returns, you may have to tolerate lower Sharpe ratios
  - Usually fixed-income assets have very high Sharpe ratios
  - But if you plan to invest for a long term, stocks will outperform despite having a lower Sharpe ratio because they have much higher returns

- The typical range of (annualized) Sharpe ratio is 0.2 to 0.5
  - Only extremely talented money managers achieve Sharpe>1

Which Investment do You Prefer?



What Happened to Investment B?

- Ponzi Scheme
- Origin: In 1919, Charles Ponzi figured that international stamps are priced differently in different countries, so he could buy in one country and sell in another to profit
  - Failed to get a personal loan at banks
  - Promised to pay friends 50% return in 45 days
  - Paid the first group of investors with money obtained from later investors
  - 1920-01: 18 investors, $1,800
  - 1920-03: $25,000
  - 1920-05: $420,000
  - 1920-06: $2,500,000
  - 1920-07: $7,000,000, being suspected by many
  - 1920-08: collapsed

Modern Ponzi Schemes

- Bernard Madoff: largest and longest Ponzi scheme in history
  - Ran a successful stock exchange and a market making business in the 70’s and the 80’s
  - Claimed he found a highly profitable trading strategy and started taking in investors in the late 80’s. His fund yields ~10% every year
  - All the money was actually sitting in a bank
  - Couldn’t make payments to investors in 2008 and collapsed

- P2P
  - Story 1
  - Story 2

Lessons

- <u>If something is too good to be true, it probably is a scam</u>
- Typical range of returns
  - Government bonds:
    risk-free rate + [0%, 2%]
  - Good corporate bonds:
    risk-free rate + [1%, 4%]
  - Stocks:
    risk-free rate + [5%, 10%]

- High return → high risk
  - If someone promise you a return higher than the risk-free rate, there is a chance that he/she cannot keep the promise
- Understanding risk-return tradeoff can help you avoid scams!

Summary

- Return:
  - Holding period return
  - Average return: arithmetic and geometric average
  - Expected return
- Risk
  - Standard deviation (σ)
- History of risk and return:
  - There is positive return on any broad-based index in the long run
  - Return: stocks > long-term bonds > short-term bonds
  - Risk: stocks > long-term bonds > short-term bonds
  - The range of risk premium: 5-8% for stocks; 1-2% for long-term government bonds
- Risk and return tradeoff: return and risk is generally positively related. If someone promises you high returns without risks, it is most likely a scam!

Next Time—Risk and Return of a Portfolio

- Portfolio
  - Motivation
  - Weights
  - Portfolio returns
  - Portfolio risks
    - Examples and intuitions
    - Math formulas
- Diversification
  - Idiosyncratic and systematic risks