CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

1. domain $= (-\infty, \infty)$; range $= [1, \infty)$

2. domain $= [0, \infty)$; range $= (-\infty, 1]$

3. domain $= [-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow y$ can be any nonnegative real number $\Rightarrow$ range $= [0, \infty)$.

4. domain $= (-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$ can be any nonnegative real number $\Rightarrow$ range $= [0, \infty)$.

5. domain $= (-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3 - t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow$ $3 - t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$ can be any nonzero real number $\Rightarrow$ range $= (-\infty, 0) \cup (0, \infty)$.

6. domain $= (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2 - 16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$ can be any nonzero real number $\Rightarrow$ range $= (-\infty, -\frac{1}{8}] \cup (0, \infty)$.

7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

9. base $= x$; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow$ height $= \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}$ (base)(height) $= \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

10. $s =$ side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

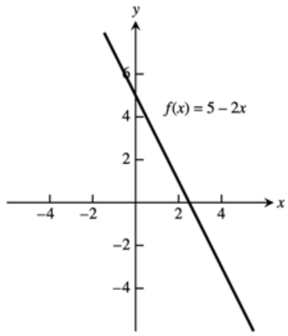
11. Let $D =$ diagonal length of a face of the cube and $\ell =$ the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d^2}{3}\right)^{3/2} = \frac{d^3}{3\sqrt{3}}$.

12. The coordinates of P are $\left(x, \sqrt{x}\right)$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}\ (x > 0)$. Thus, $\left(x, \sqrt{x}\right) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.

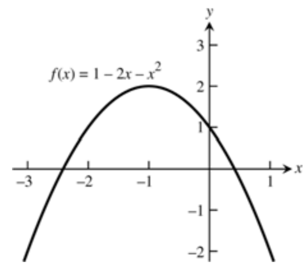
13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$; $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (-\frac{1}{2}x + \frac{5}{4})^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
$= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$

14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$; $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
$= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

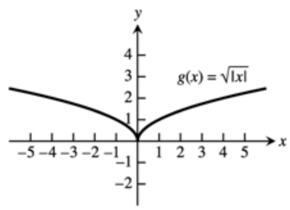
15. The domain is $(-\infty, \infty)$.



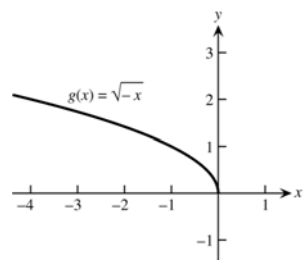
16. The domain is $(-\infty, \infty)$.



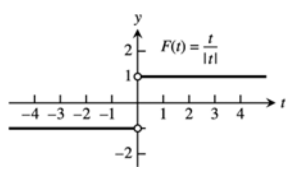
17. The domain is $(-\infty, \infty)$.



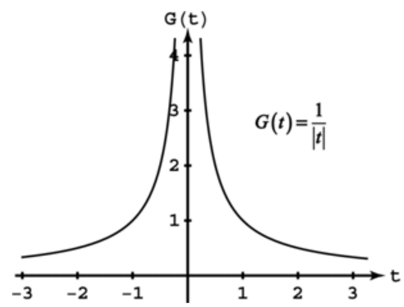
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



20. The domain is $(-\infty, 0) \cup (0, \infty)$.

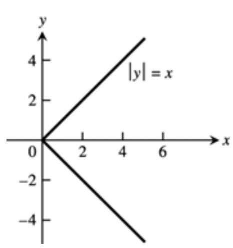


21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

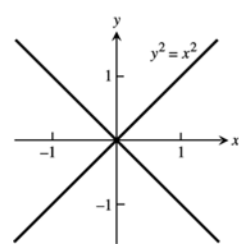
22. The range is $[2, 3)$.

23. Neither graph passes the vertical line test

(a)

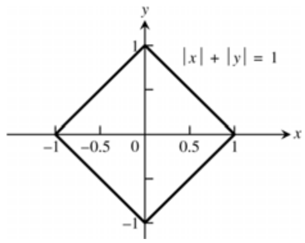


(b)

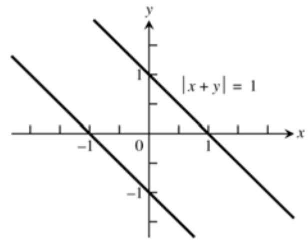


24. Neither graph passes the vertical line test

(a)



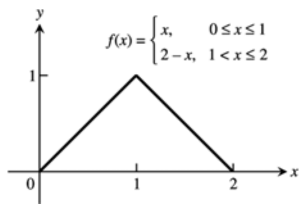
(b)



$$|x+y| = 1 \Leftrightarrow \left\{\begin{matrix} x+y=1 \\ \text{or} \\ x+y=-1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} y=1-x \\ \text{or} \\ y=-1-x \end{matrix}\right\}$$

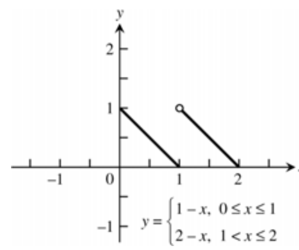
25.

x	0	1	2
y	0	1	0

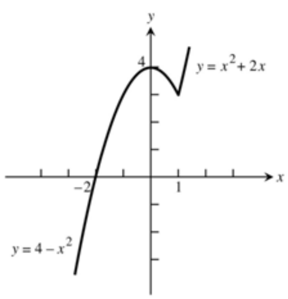


26.

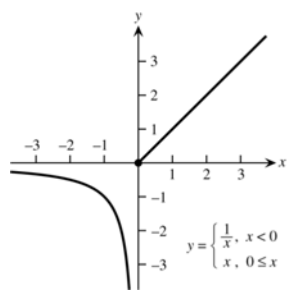
x	0	1	2
y	1	0	0



27. $F(x) = \begin{cases} 4-x^2, & x \le 1 \\ x^2+2x, & x > 1 \end{cases}$



28. $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \le x \end{cases}$



29. (a) Line through (0, 0) and (1, 1): $y = x$; Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

(b) $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2)+1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1)+1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \le x < 0 \\ 1 & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \le x \le 0 \\ -2x+2 & 0 < x \le 1 \\ -1 & 1 < x \le 3 \end{cases}$$

32. (a) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

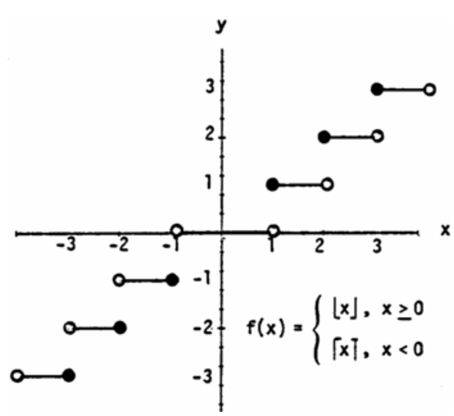
(b) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$ (b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

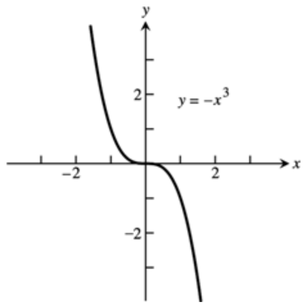
34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x, $n \le x \le n+1$, where n is an integer. Now: $n \le x \le n+1 \Rightarrow -(n+1) \le -x \le -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x.

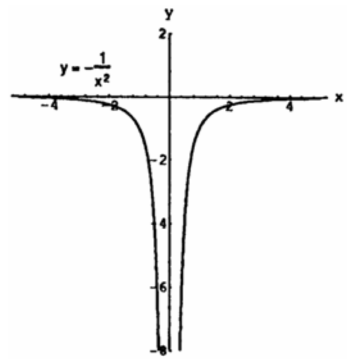
36. To find $f(x)$ you delete the decimal or fractional portion of x, leaving only the integer part.



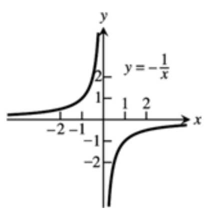
37. Symmetric about the origin
Dec: $-\infty < x < \infty$
Inc: nowhere



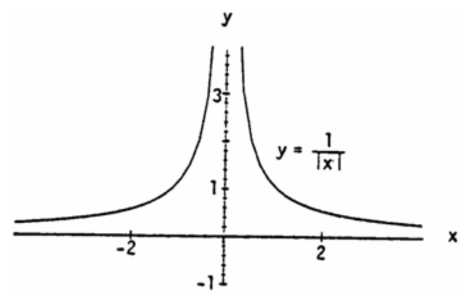
38. Symmetric about the y-axis
Dec: $-\infty < x < 0$
Inc: $0 < x < \infty$



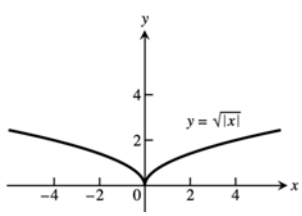
39. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < 0$
$0 < x < \infty$



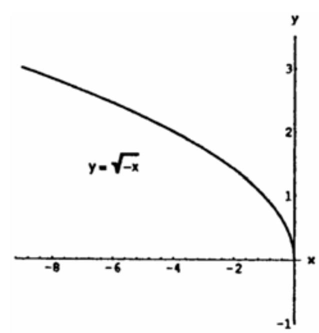
40. Symmetric about the y-axis
Dec: $0 < x < \infty$
Inc: $-\infty < x < 0$



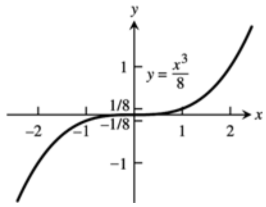
41. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



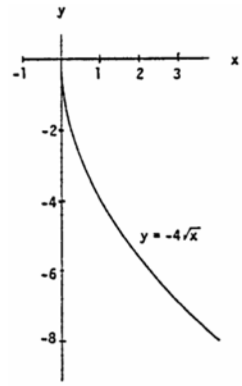
42. No symmetry
Dec: $-\infty < x \le 0$
Inc: nowhere



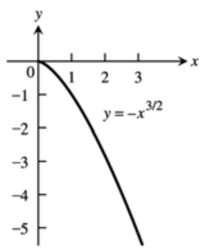
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



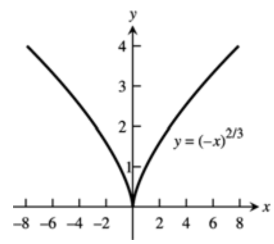
44. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$. The function is even.

50. Since $[f(x) = x^2 + x] \ne [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \ne [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t - 1}$; $h(-t) = \frac{1}{-t - 1}$; $-h(t) = \frac{1}{1 - t}$. Since $h(t) \ne -h(t)$ and $h(t) \ne h(-t)$, the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|, h(t) = h(-t)$ and the function is even.

57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.

58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.

59. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$

60. $K = c\,v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules

61. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$

62. $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$; $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2\ \text{cm}^3$

63. $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$.

64. (a) Let h = height of the triangle. Since the triangle is isosceles, $\left(\overline{AB}\right)^2 + \left(\overline{AB}\right)^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So, $h^2 + 1^2 = \left(\sqrt{2}\right)^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is $y = f(x) = -x + 1$; $x \in [0, 1]$.
(b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$.

65. (a) Graph h because it is an even function and rises less rapidly than does Graph g.
(b) Graph f because it is an odd function.
(c) Graph g because it is an even function and rises more rapidly than does Graph h.

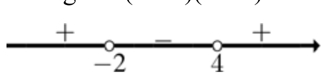
66. (a) Graph f because it is linear.
(b) Graph g because it contains (0, 1).
(c) Graph h because it is a nonlinear odd function.

67. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$
(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

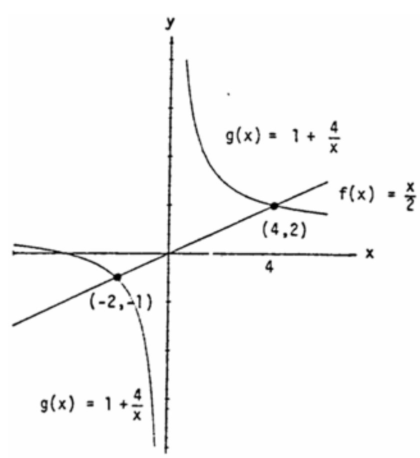
$x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$
$\Rightarrow x > 4$ since x is positive;

$x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$
$\Rightarrow x < -2$ since x is negative;

sign of $(x - 4)(x + 2)$



Solution interval: $(-2, 0) \cup (4, \infty)$



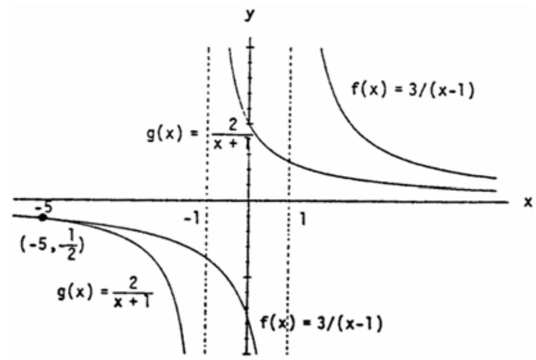
68. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$
$\Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5$.
Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$
$\Rightarrow 3x+3 > 2x-2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5$
which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.



69. A curve symmetric about the x-axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x-axis, a horizontal line for which there is a single y-value, 0, for any x.

70. price $= 40 + 5x$, quantity $= 300 - 25x \Rightarrow R(x) = (40+5x)(300-25x)$

71. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost $= 5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h\left(\sqrt{2}+2\right)$

72. (a) Note that $2 \text{ km} = 2{,}000 \text{ m}$, so there are $\sqrt{250^2 + x^2}$ meters of river cable at \$180 per meter and $(2{,}000 - x)$ meters of land cable at \$100 per meter. The cost is $C(x) = 180\sqrt{250^2 + x^2} + 100(2{,}000 - x)$.

(b) $C(0) = \$245{,}000$
$C(100) \approx \$238{,}466$
$C(200) \approx \$237{,}628$
$C(300) \approx \$240{,}292$
$C(400) \approx \$244{,}906$
$C(500) \approx \$250{,}623$
$C(600) \approx \$257{,}000$
Values beyond this are all larger. It would appear that the least expensive location is less than 300 m from the point P.

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \ge 1 \Rightarrow D_{f+g} = D_{fg}: x \ge 1.$ $R_f: -\infty < y < \infty, R_g: y \ge 0, R_{f+g}: y \ge 1, R_{fg}: y \ge 0$

2. $D_f: x+1 \ge 0 \Rightarrow x \ge -1, D_g: x-1 \ge 0 \Rightarrow x \ge 1.$ Therefore $D_{f+g} = D_{fg}: x \ge 1.$
$R_f = R_g: y \ge 0, R_{f+g}: y \ge \sqrt{2}, R_{fg}: y \ge 0$

3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty, R_f: y = 2, R_g: y \ge 1, R_{f/g}: 0 < y \le 2,$
$R_{g/f}: \frac{1}{2} \le y < \infty$

4. $D_f: -\infty < x < \infty$, $D_g: x \ge 0$, $D_{f/g}: x \ge 0$, $D_{g/f}: x \ge 0$; $R_f: y = 1$, $R_g: y \ge 1$, $R_{f/g}: 0 < y \le 1$, $R_{g/f}: 1 \le y < \infty$

5. (a) 2 (b) 22 (c) $x^2 + 2$
(d) $(x+5)^2 - 3 = x^2 + 10x + 22$ (e) 5 (f) -2
(g) $x + 10$ (h) $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$

6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1} - 1 = \frac{-x}{x+1}$
(d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
(g) $x - 2$ (h) $\frac{1}{\frac{1}{x+1}+1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$

7. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x)+1 = 13-3x$

8. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1)+4 = 6x^2+1$

9. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x}+4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x}+1} = \sqrt{\frac{5x+1}{1+4x}}$

10. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{\left(\sqrt{2-x}\right)^2}{\left(\sqrt{2-x}\right)^2+1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$

11. (a) $(f \circ g)(x)$ (b) $(j \circ g)(x)$ (c) $(g \circ g)(x)$
(d) $(j \circ j)(x)$ (e) $(g \circ h \circ f)(x)$ (f) $(h \circ j \circ f)(x)$

12. (a) $(f \circ j)(x)$ (b) $(g \circ h)(x)$ (c) $(h \circ h)(x)$
(d) $(f \circ f)(x)$ (e) $(j \circ g \circ f)(x)$ (f) $(g \circ f \circ h)(x)$

13.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	$x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x+2$	$3x$	$3(x+2) = 3x+6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$
(e)	$\frac{1}{x-1}$	$1+\frac{1}{x}$	x
(f)	$\frac{1}{x}$	$\frac{1}{x}$	x

14. (a) $(f \circ g)(x) = |g(x)| = \frac{1}{|x-1|}$.
(b) $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$, so $g(x) = x+1$.
(c) Since $(f \circ g)(x) = \sqrt{g(x)} = |x|$, $g(x) = x^2$.
(d) Since $(f \circ g)(x) = f\left(\sqrt{x}\right) = |x|$, $f(x) = x^2$. (Note that the domain of the composite is $[0, \infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f\circ g)(x)$
$\frac{1}{x-1}$	$\lvert x\rvert$	$\frac{1}{\lvert x-1\rvert}$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x\rvert$
$\sqrt{x}$	x^2	$\lvert x\rvert$

15. (a) $f(g(-1)) = f(1) = 1$ (b) $g(f(0)) = g(-2) = 2$ (c) $f(f(-1)) = f(0) = -2$
 (d) $g(g(2)) = g(0) = 0$ (e) $g(f(-2)) = g(1) = -1$ (f) $f(g(1)) = f(-1) = 0$

16. (a) $f(g(0)) = f(-1) = 2-(-1) = 3$, where $g(0) = 0-1 = -1$
 (b) $g(f(3)) = g(-1) = -(-1) = 1$, where $f(3) = 2-3 = -1$
 (c) $g(g(-1)) = g(1) = 1-1 = 0$, where $g(-1) = -(-1) = 1$
 (d) $f(f(2)) = f(0) = 2-0 = 2$, where $f(2) = 2-2 = 0$
 (e) $g(f(0)) = g(2) = 2-1 = 1$, where $f(0) = 2-0 = 2$
 (f) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2-\left(-\frac{1}{2}\right) = \frac{5}{2}$, where $g\left(\frac{1}{2}\right) = \frac{1}{2}-1 = -\frac{1}{2}$

17. (a) $(f\circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x}+1} = \sqrt{\frac{1+x}{x}}$
 $(g\circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$
 (b) Domain $(f\circ g)$: $(-\infty, -1]\cup(0,\infty)$, domain $(g\circ f)$: $(-1,\infty)$
 (c) Range $(f\circ g)$: $(1,\infty)$, range $(g\circ f)$: $(0,\infty)$

18. (a) $(f\circ g)(x) = f(g(x)) = 1-2\sqrt{x}+x$
 $(g\circ f)(x) = g(f(x)) = 1-\lvert x\rvert$
 (b) Domain $(f\circ g)$: $[0,\infty)$, domain $(g\circ f)$: $(-\infty,\infty)$
 (c) Range $(f\circ g)$: $(0,\infty)$, range $(g\circ f)$: $(-\infty, 1]$

19. $(f\circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = (g(x)-2)x = x\cdot g(x) - 2x$
 $\Rightarrow g(x) - x\cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$

20. $(f\circ g)(x) = x+2 \Rightarrow f(g(x)) = x+2 \Rightarrow 2(g(x))^3 - 4 = x+2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$

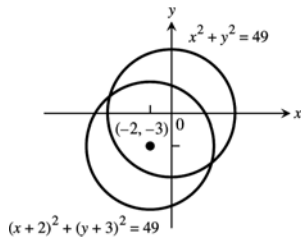
21. (a) $y = -(x+7)^2$ (b) $y = -(x-4)^2$

22. (a) $y = x^2+3$ (b) $y = x^2-5$

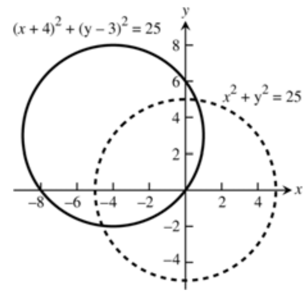
23. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

24. (a) $y = -(x-1)^2+4$ (b) $y = -(x+2)^2+3$ (c) $y = -(x+4)^2-1$ (d) $y = -(x-2)^2$

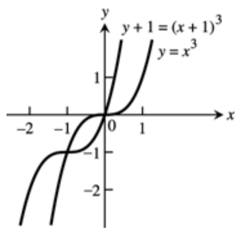
25.



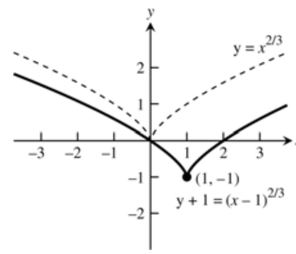
26.



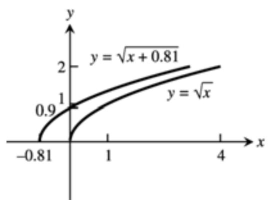
27.



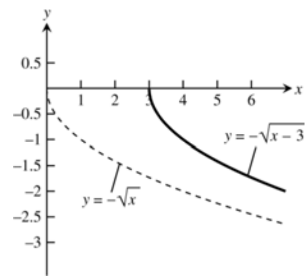
28.



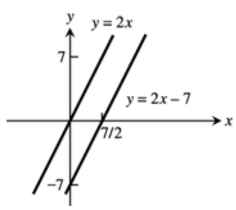
29.



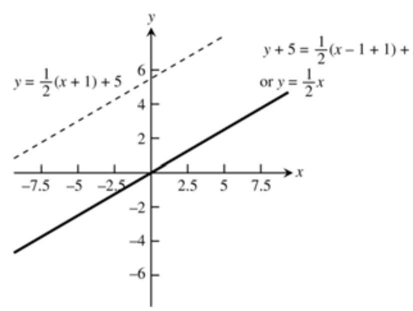
30.



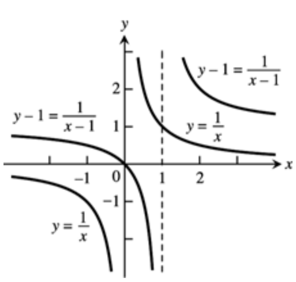
31.



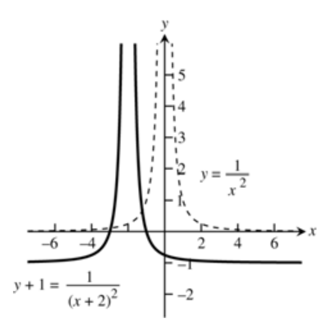
32.



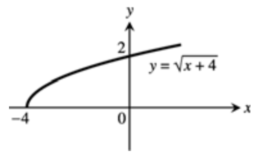
33.



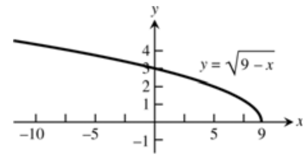
34.



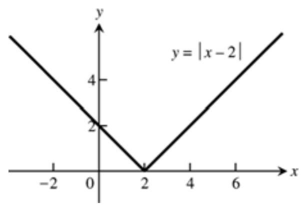
35.



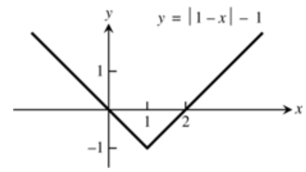
36.



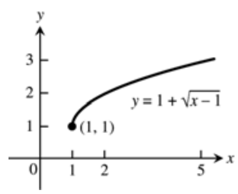
37.



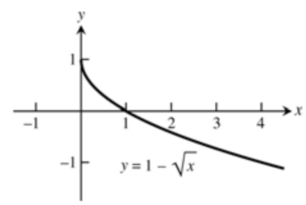
38.



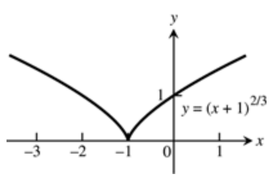
39.



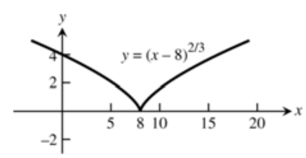
40.



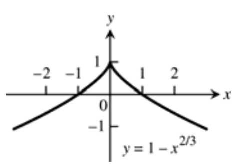
41.



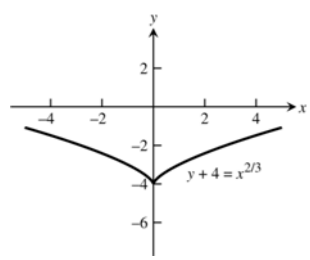
42.



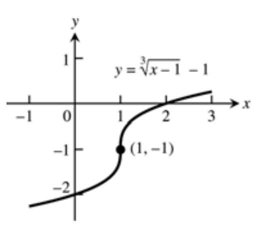
43.



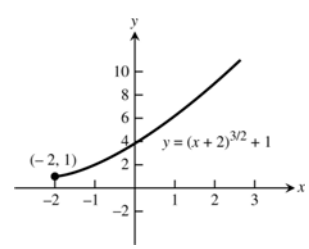
44.



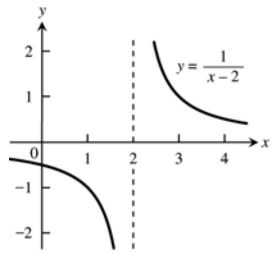
45.



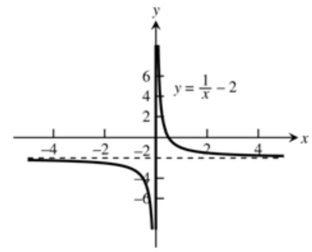
46.



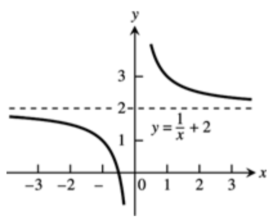
47.



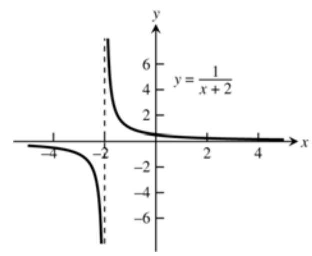
48.



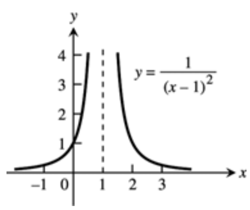
49.



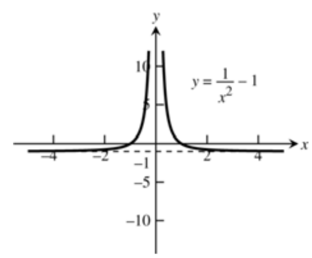
50.



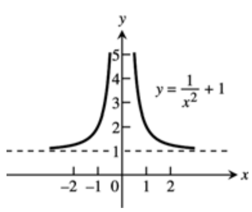
51.



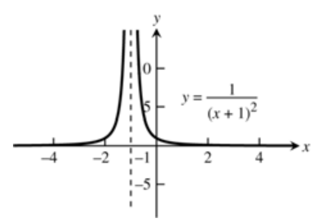
52.



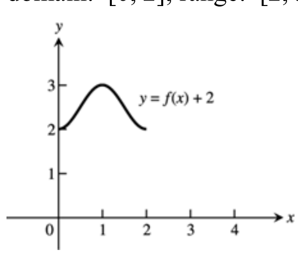
53.



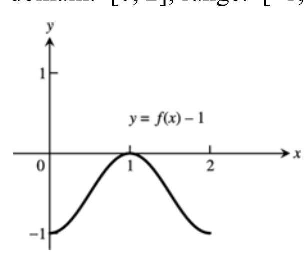
54.



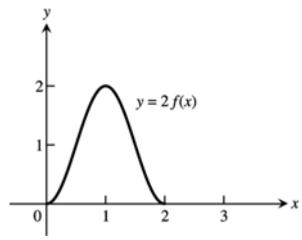
55. (a) domain: [0, 2]; range: [2, 3]



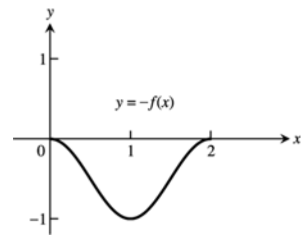
(b) domain: [0, 2]; range: [–1, 0]



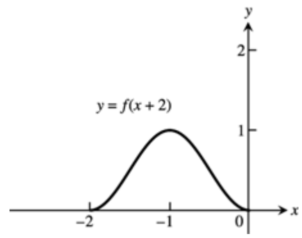
(c) domain: [0, 2]; range: [0, 2]



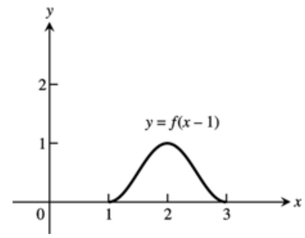
(d) domain: [0, 2]; range: [–1, 0]



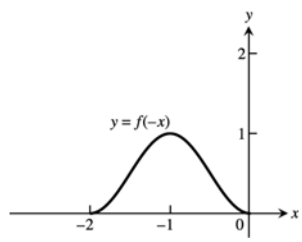
(e) domain: [–2, 0]; range: [0, 1]



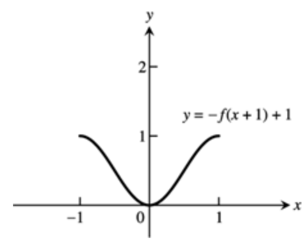
(f) domain: [1, 3]; range: [0,1]



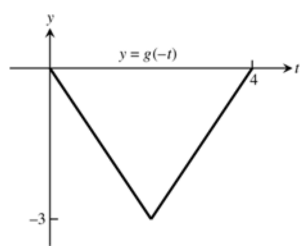
(g) domain: [–2, 0]; range: [0, 1]



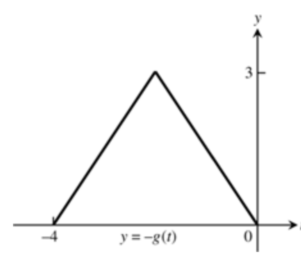
(h) domain: [–1, 1]; range: [0, 1]



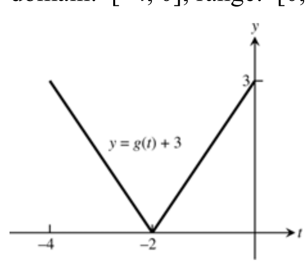
56. (a) domain: [0, 4]; range: [–3, 0]



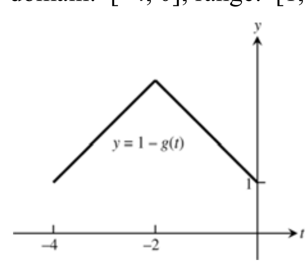
(b) domain: [–4, 0]; range: [0, 3]



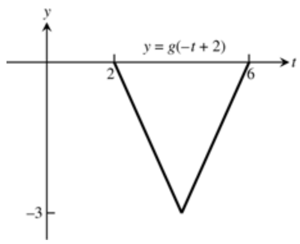
(c) domain: [–4, 0]; range: [0, 3]



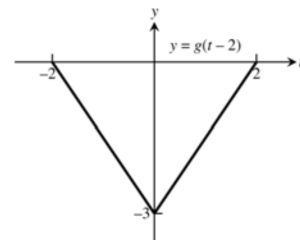
(d) domain: [–4, 0]; range: [1, 4]



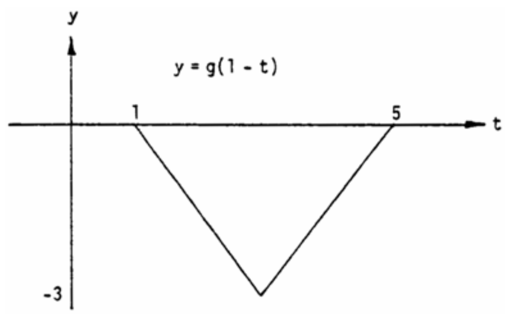
(e) domain: [2, 4]; range: [–3, 0]



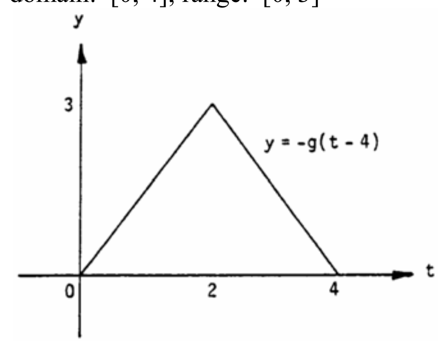
(f) domain: [–2, 2]; range: [–3, 0]



(g) domain: [1, 5]; range: [–3, 0]



(h) domain: [0, 4]; range: [0, 3]



57. $y = 3x^2 - 3$

58. $y = (2x)^2 - 1 = 4x^2 - 1$

59. $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

60. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

61. $y = \sqrt{4x+1}$

62. $y = 3\sqrt{x+1}$

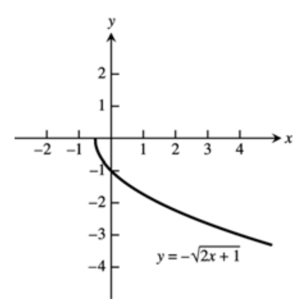
63. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

64. $y = \frac{1}{3}\sqrt{4 - x^2}$

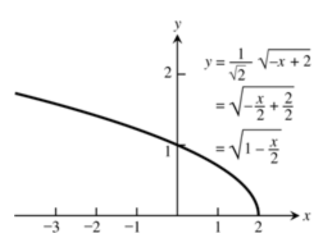
65. $y = 1 - (3x)^3 = 1 - 27x^3$

66. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

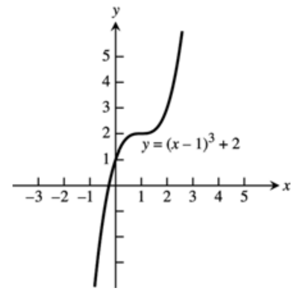
67. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$, $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$ is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x-axis.



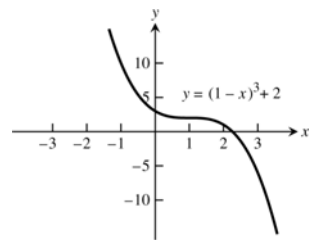
68. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$, $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} = \sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x-axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.



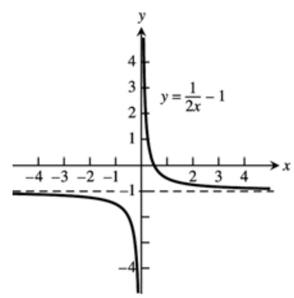
69. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.



70. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x-axis.

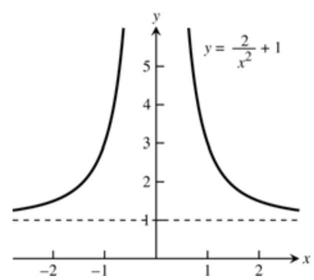


71. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.

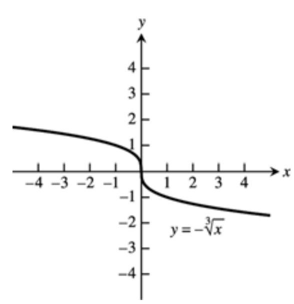


72. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x^2}{2}\right)} + 1$

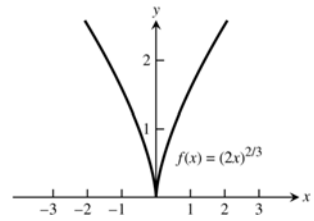
$= \frac{1}{\left(x/\sqrt{2}\right)^2} + 1 = \frac{1}{\left[\left(1/\sqrt{2}\right)x\right]^2} + 1$. Since $\sqrt{2} \approx 1.4$, we see that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.



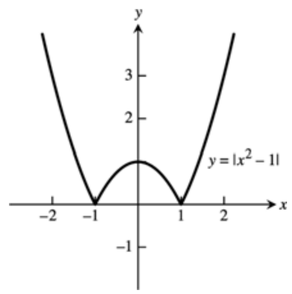
73. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x-axis to get $g(x) = -\sqrt[3]{x}$.



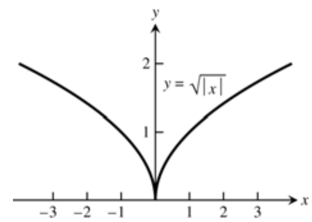
74. $y = f(x) = (-2x)^{2/3} = [(-1)(2)x]^{2/3} = (-1)^{2/3}(2x)^{2/3}$
$= (2x)^{2/3}$. So the graph of $f(x)$ is the graph of
$g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



75.



76.



77. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd

(b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd

(c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd

(d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even

(e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even

(f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even

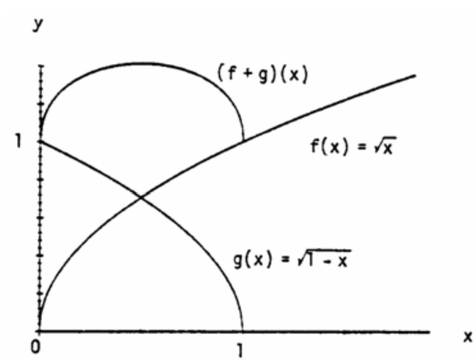
(g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even

(h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even

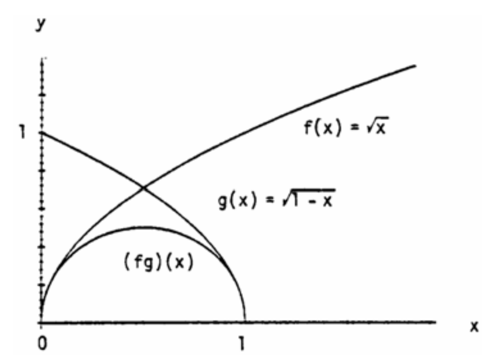
(i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

78. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

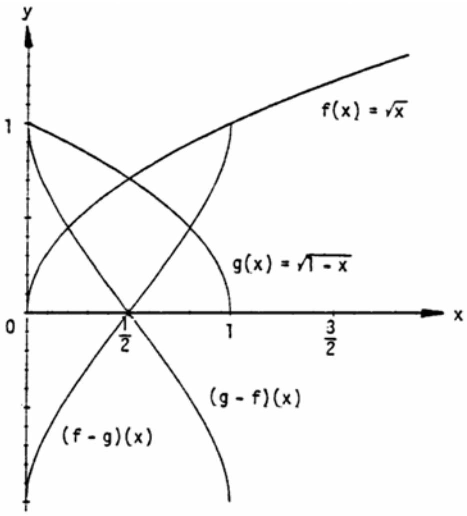
79. (a)



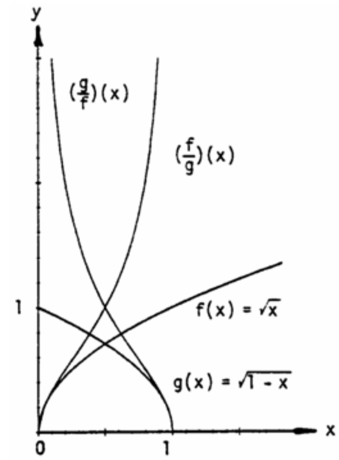
(b)



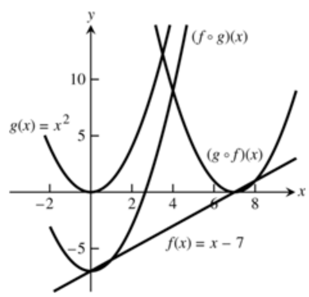
(c)



(d)



80.



1.3 TRIGONOMETRIC FUNCTIONS

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}\ m$

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ cm. (since the diameter $= 12$ cm. $\Rightarrow$ radius $= 6$ cm.)

4. $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\ \left(\frac{180°}{\pi}\right) \approx 34°$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

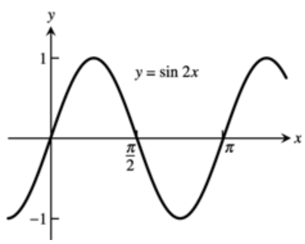
9. $\sin x = -\frac{\sqrt{8}}{3}$, $\tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}$, $\tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$

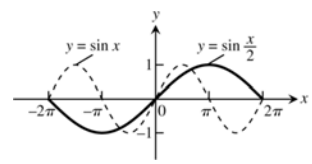
12. $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

13.



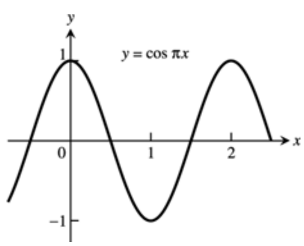
period $= \pi$

14.



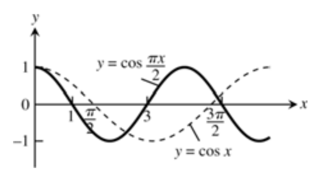
period $= 4\pi$

15.



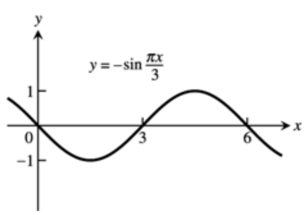
period $= 2$

16.



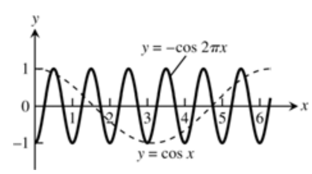
period $= 4$

17.



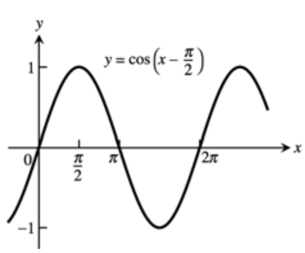
period $= 6$

18.



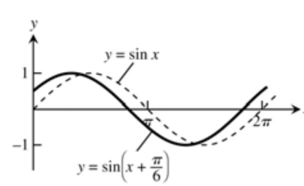
period $= 1$

19.



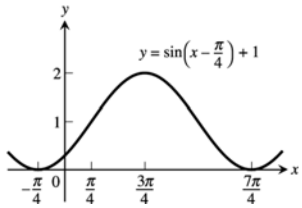
period $= 2\pi$

20.



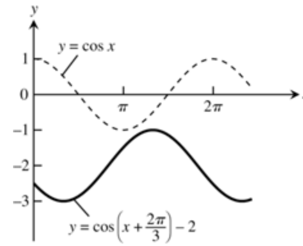
period $= 2\pi$

21.



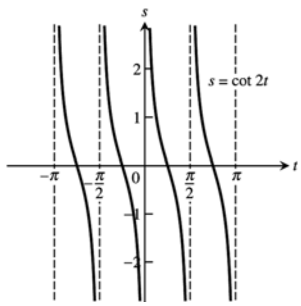
period $= 2\pi$

22.

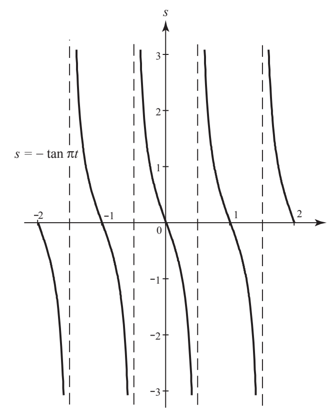


period $= 2\pi$

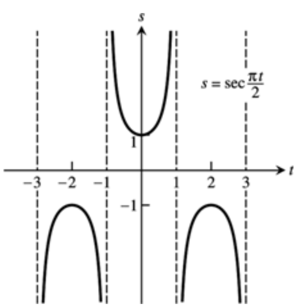
23. period $= \frac{\pi}{2}$, symmetric about the origin



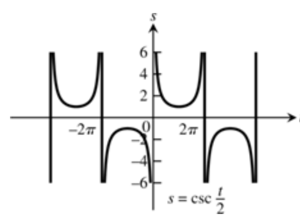
24. period $= 1$, symmetric about the origin



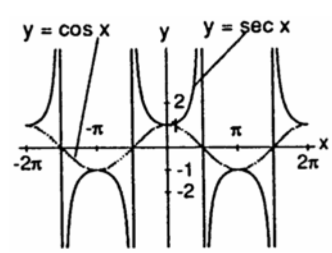
25. period $= 4$, symmetric about the s-axis



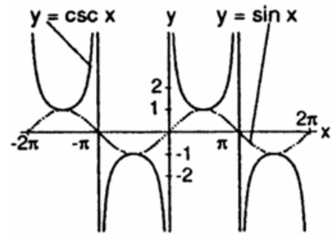
26. period $= 4\pi$, symmetric about the origin



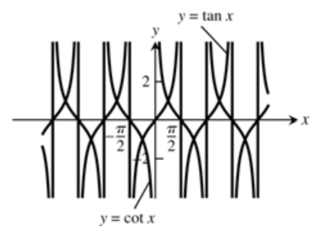
27. (a) Cos x and sec x are positive for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; and cos x and sec x are negative for x in the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.



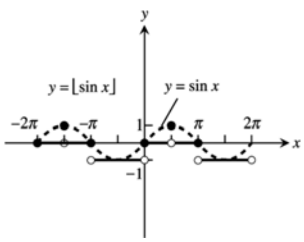
(b) Sin x and csc x are positive for x in the intervals $\left(-\frac{3\pi}{2}, -\pi\right)$ and $(0, \pi)$; and sin x and csc x are negative for x in the intervals $(-\pi, 0)$ and $\left(\pi, \frac{3\pi}{2}\right)$. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



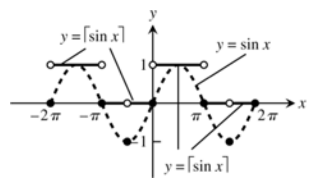
28. Since $\cot x = \frac{1}{\tan x}$, cot x is undefined when $\tan x = 0$ and is zero when tan x is undefined. As tan x approaches zero through positive values, cot x approaches infinity. Also, cot x approaches negative infinity as tan x approaches zero through negative values.



29. $D: -\infty < x < \infty;\ R: y = -1, 0, 1$



30. $D: -\infty < x < \infty;\ R: y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

39. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

40. $\sin(2\pi - x) = \sin 2\pi\cos(-x) + \cos(2\pi)\sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

41. $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$

42. $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

43. $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

44. $\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$

45. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{3}}{2\sqrt{2}}$

46. $\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{3}}{2\sqrt{2}}$

47. $\cos^2\frac{\pi}{8} = \frac{1 + \cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}$

48. $\cos^2\frac{5\pi}{12} = \frac{1 + \cos\left(\frac{10\pi}{12}\right)}{2} = \frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{2 - \sqrt{3}}{4}$

49. $\sin^2\frac{\pi}{12} = \frac{1 - \cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$

50. $\sin^2\frac{3\pi}{8} = \frac{1 - \cos\left(\frac{6\pi}{8}\right)}{2} = \frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2} = \frac{2 + \sqrt{2}}{4}$

51. $\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

52. $\sin^2\theta = \cos^2\theta \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} \Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

53. $\sin 2\theta - \cos\theta = 0 \Rightarrow 2\sin\theta\cos\theta - \cos\theta = 0 \Rightarrow \cos\theta(2\sin\theta - 1) = 0 \Rightarrow \cos\theta = 0$ or $2\sin\theta - 1 = 0$
$\Rightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$, or $\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

54. $\cos 2\theta + \cos\theta = 0 \Rightarrow 2\cos^2\theta - 1 + \cos\theta = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0 \Rightarrow (\cos\theta + 1)(2\cos\theta - 1) = 0$
$\Rightarrow \cos\theta + 1 = 0$ or $2\cos\theta - 1 = 0 \Rightarrow \cos\theta = -1$ or $\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

55. $\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A\cos B + \cos A\cos B}{\cos A\cos B - \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

56. $\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{\sin A\cos B - \cos A\cos B}{\cos A\cos B + \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

57. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\theta$ $= 1^2 + 1^2 - 2\cos(A - B) = 2 - 2\cos(A - B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ $= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B = 2 - 2(\cos A\cos B + \sin A\sin B)$. Thus $c^2 = 2 - 2\cos(A - B) = 2 - 2(\cos A\cos B + \sin A\sin B) \Rightarrow \cos(A - B) = \cos A\cos B + \sin A\sin B$.

58. (a) $\cos(A-B)=\cos A\cos B+\sin A\sin B$
$\sin\theta=\cos\left(\frac{\pi}{2}-\theta\right)$ and $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$
Let $\theta=A+B$
$\sin(A+B)=\cos\left[\frac{\pi}{2}-(A+B)\right]=\cos\left[\left(\frac{\pi}{2}-A\right)-B\right]=\cos\left(\frac{\pi}{2}-A\right)\cos B+\sin\left(\frac{\pi}{2}-A\right)\sin B$
$=\sin A\cos B+\cos A\sin B$

(b) $\cos(A-B)=\cos A\cos B+\sin A\sin B$
$\cos(A-(-B))=\cos A\cos(-B)+\sin A\sin(-B)$
$\Rightarrow\cos(A+B)=\cos A\cos(-B)+\sin A\sin(-B)=\cos A\cos B+\sin A(-\sin B)=\cos A\cos B-\sin A\sin B$
Because the cosine function is even and the sine functions is odd.

59. $c^2=a^2+b^2-2ab\cos C=2^2+3^2-2(2)(3)\cos(60°)=4+9-12\cos(60°)=13-12\left(\frac{1}{2}\right)=7.$
Thus, $c=\sqrt{7}\approx 2.65$.

60. $c^2=a^2+b^2-2ab\cos C=2^2+3^2-2(2)(3)\cos(40°)=13-12\cos(40°)$. Thus, $c=\sqrt{13-12\cos 40°}\approx 1.951$.

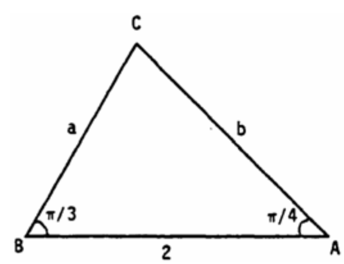
61. From the figures in the text, we see that $\sin B=\frac{h}{c}$. If C is an acute angle, then $\sin C=\frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right in the text), then $\sin C=\sin(\pi-C)=\frac{h}{b}$. Thus, in either case, $h=b\sin C=c\sin B\Rightarrow ah=ab\sin C=ac\sin B$.

By the law of cosines, $\cos C=\frac{a^2+b^2-c^2}{2ab}$ and $\cos B=\frac{a^2+c^2-b^2}{2ac}$. Moreover, since the sum of the interior angles of triangle is π, we have $\sin A=\sin(\pi-(B+C))=\sin(B+C)=\sin B\cos C+\cos B\sin C$
$=\left(\frac{h}{c}\right)\left[\frac{a^2+b^2-c^2}{2ab}\right]+\left[\frac{a^2+c^2-b^2}{2ac}\right]\left(\frac{h}{b}\right)=\left(\frac{h}{2abc}\right)(2a^2+b^2-c^2+c^2-b^2)=\frac{ah}{bc}\Rightarrow ah=bc\sin A.$

Combining our results we have $ah=ab\sin C$, $ah=ac\sin B$, and $ah=bc\sin A$. Dividing by abc gives
$\frac{h}{bc}=\underbrace{\frac{\sin A}{a}=\frac{\sin C}{c}=\frac{\sin B}{b}}_{\text{law of sines}}.$

62. By the law of sines, $\frac{\sin A}{2}=\frac{\sin B}{3}=\frac{\sqrt{3}/2}{c}$. By Exercise 59 we know that $c=\sqrt{7}$. Thus $\sin B=\frac{3\sqrt{3}}{2\sqrt{7}}\simeq 0.982$.

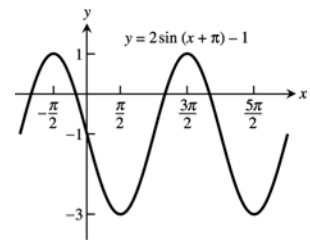
63. From the figure at the right and the law of cosines,
$b^2=a^2+2^2-2(2a)\cos B$
$=a^2+4-4a\left(\frac{1}{2}\right)=a^2-2a+4.$



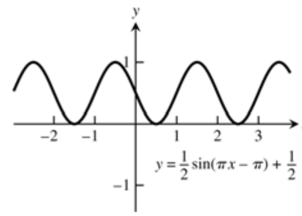
Applying the law of sines to the figure, $\frac{\sin A}{a}=\frac{\sin B}{b}$
$\Rightarrow\frac{\sqrt{2}/2}{a}=\frac{\sqrt{3}/2}{b}\Rightarrow b=\sqrt{\frac{3}{2}}\,a$. Thus, combining results,
$a^2-2a+4=b^2=\frac{3}{2}a^2\Rightarrow 0=\frac{1}{2}a^2+2a-4\Rightarrow 0=a^2+4a-8$. From the quadratic formula and the fact that $a>0$, we have $a=\frac{-4+\sqrt{4^2-4(1)(-8)}}{2}=\frac{4\sqrt{3}-4}{2}\simeq 1.464.$

64. (a) The graphs of $y=\sin x$ and $y=x$ nearly coincide when x is near the origin (when the calculator is in radians mode).
(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

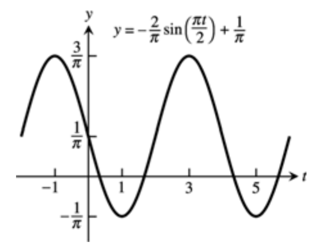
65. $A = 2, B = 2\pi, C = -\pi, D = -1$



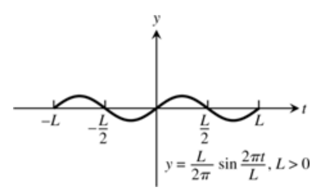
66. $A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$



67. $A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$



68. $A = \frac{L}{2\pi}, B = L, C = 0, D = 0$

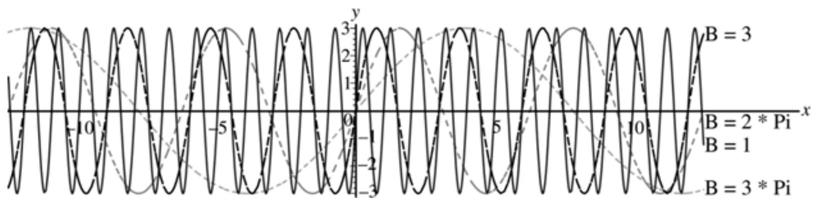


69–72. Example CAS commands:

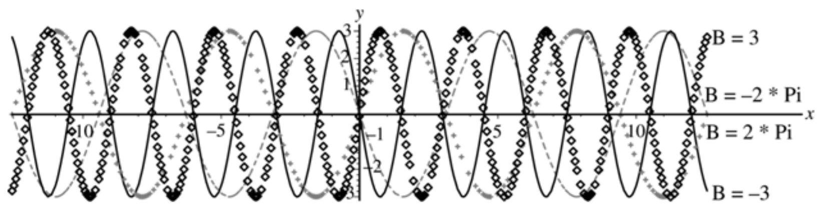
Maple:
```
f :=x -> A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list := [seq(f(x), B=[1,3,2*Pi,5*Pi])];
plot(f_list, x=-4*Pi..4*Pi, scaling=constrained,
     color=[red,blue,green,cyan], linestyle=[1,3,4,7],
     legend=["B=1", "B=3","B=2*Pi","B=3*Pi"],
     title="#69 (Section 1.3)");
```

Mathematica:
```
Clear[a, b, c, d, f, x]
f[x_]:=a Sin[2π/b (x − c)] + d
Plot[f[x]/.{a → 3, b → 1, c → 0, d → 0}, {x, −4π, 4π}]
```

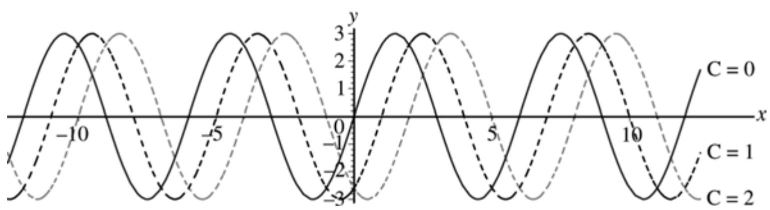
69. (a) The graph stretches horizontally.



(b) The period remains the same: period $= |B|$. The graph has a horizontal shift of $\frac{1}{2}$ period.

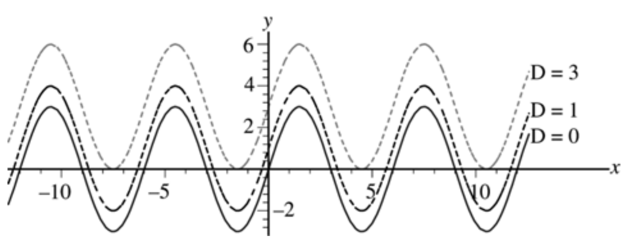


70. (a) The graph is shifted right C units.



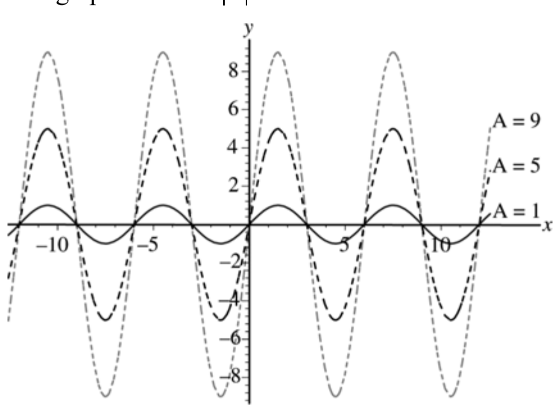
(b) The graph is shifted left C units.
(c) A shift of $\pm$ one period will produce no apparent shift. $|C| = 6$

71. (a) The graph shifts upwards $|D|$ units for $D > 0$
(b) The graph shifts down $|D|$ units for $D < 0$.



72. (a) The graph stretches $|A|$ units.

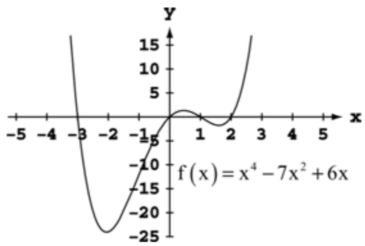
(b) For $A < 0$, the graph is inverted.



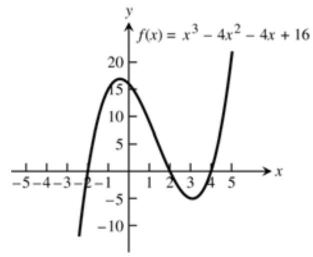
1.4 GRAPHING WITH SOFTWARE

1–4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

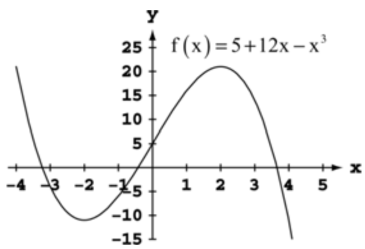
1. d.



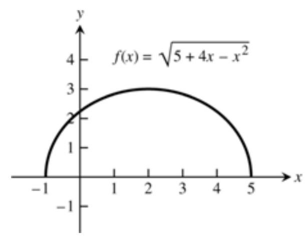
2. c.



3. d.

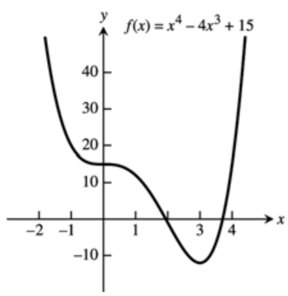


4. b.

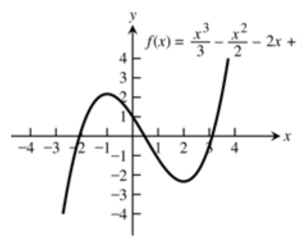


5–30. For any display there are many appropriate display widows. The graphs given as answers in Exercises 5–30 are not unique in appearance.

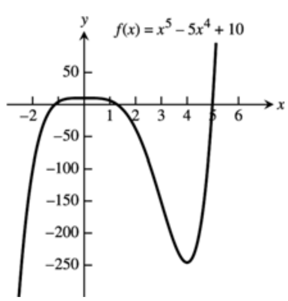
5. $[-2, 5]$ by $[-15, 40]$



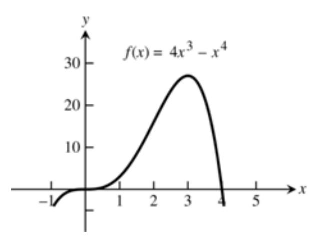
6. $[-4, 4]$ by $[-4, 4]$



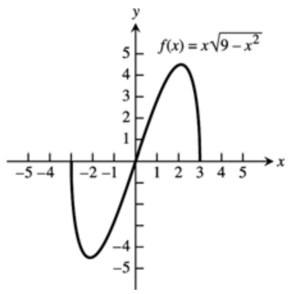
7. $[-2, 6]$ by $[-250, 50]$



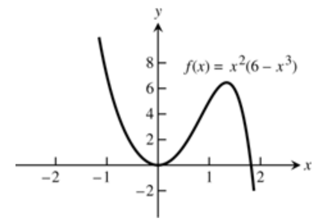
8. $[-1, 5]$ by $[-5, 30]$



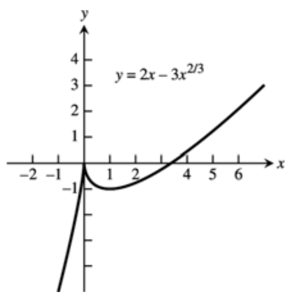
9. $[-4, 4]$ by $[-5, 5]$



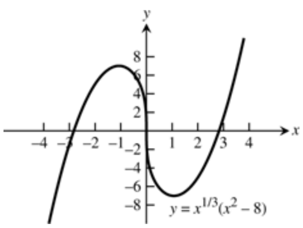
10. $[-2, 2]$ by $[-2, 8]$



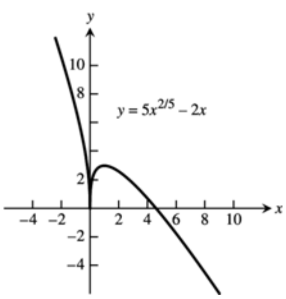
11. $[-2, 6]$ by $[-5, 4]$



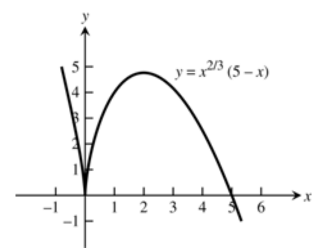
12. $[-4, 4]$ by $[-8, 8]$



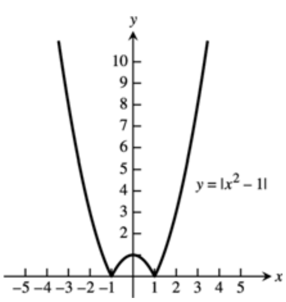
13. $[-1, 6]$ by $[-1, 4]$



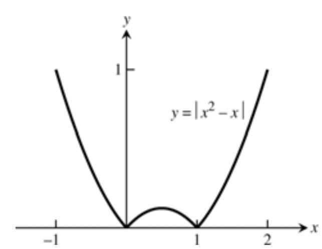
14. $[-1, 6]$ by $[-1, 5]$



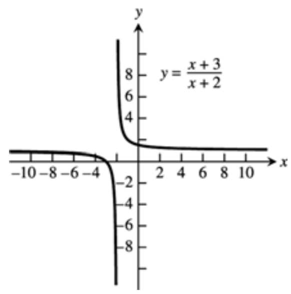
15. $[-3, 3]$ by $[0, 10]$



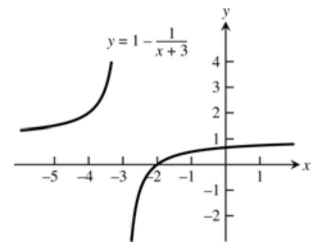
16. $[-1, 2]$ by $[0, 1]$



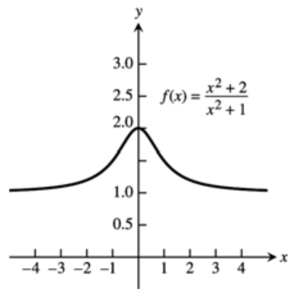
17. $[-5, 1]$ by $[-5, 5]$



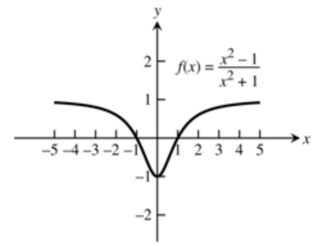
18. $[-5, 1]$ by $[-2, 4]$



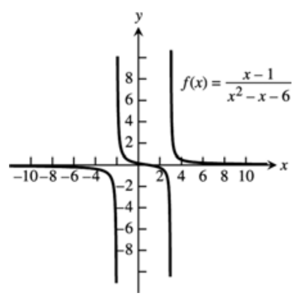
19. $[-4, 4]$ by $[0, 3]$



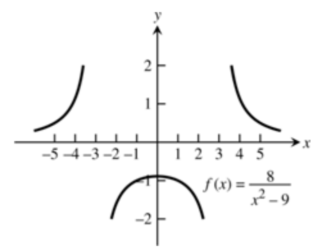
20. $[-5, 5]$ by $[-2, 2]$



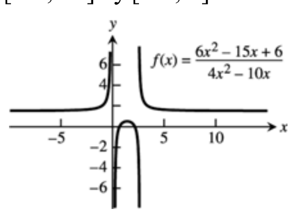
21. $[-10, 10]$ by $[-6, 6]$



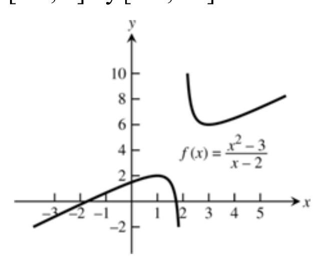
22. $[-5, 5]$ by $[-2, 2]$



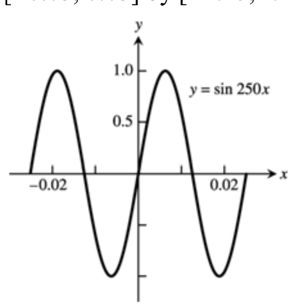
23. $[-6, 10]$ by $[-6, 6]$



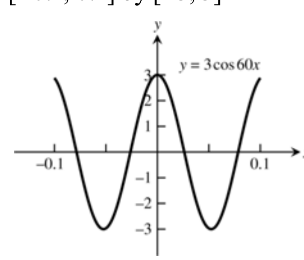
24. $[-3, 5]$ by $[-2, 10]$



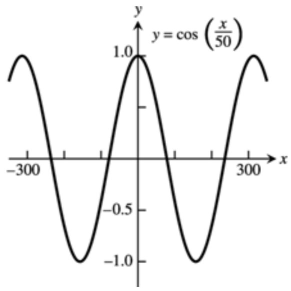
25. $[-0.03, 0.03]$ by $[-1.25, 1.25]$



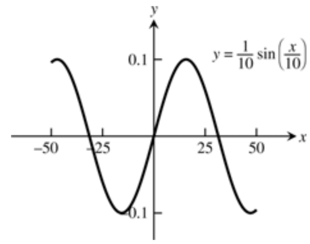
26. $[-0.1, 0.1]$ by $[-3, 3]$



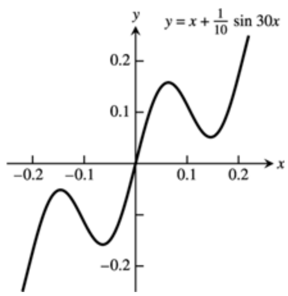
27. $[-300, 300]$ by $[-1.25, 1.25]$



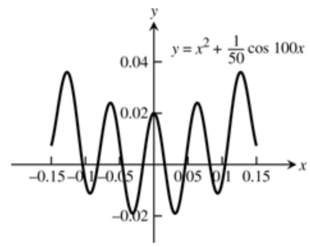
28. $[-50, 50]$ by $[-0.1, 0.1]$



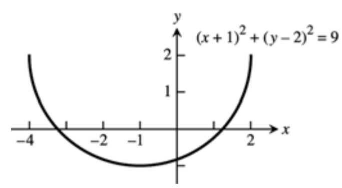
29. $[-0.25, 0.25]$ by $[-0.3, 0.3]$



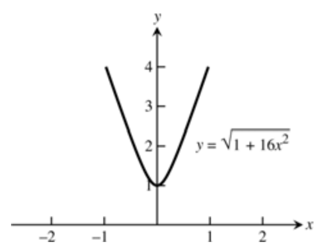
30. $[-0.15, 0.15]$ by $[-0.02, 0.05]$



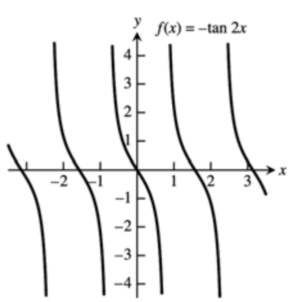
31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$.
The lower half is produced by graphing
$y = 2 - \sqrt{-x^2 - 2x + 8}$.



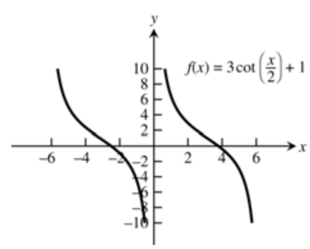
32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm\sqrt{1+16x^2}$. The upper branch is produced by graphing $y = \sqrt{1+16x^2}$.



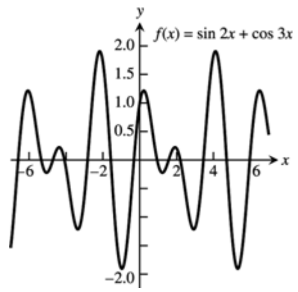
33.



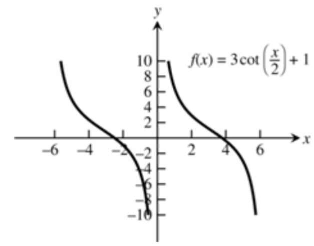
34.



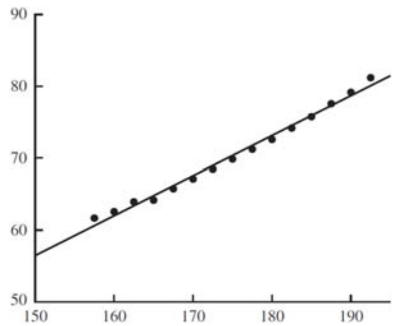
35.



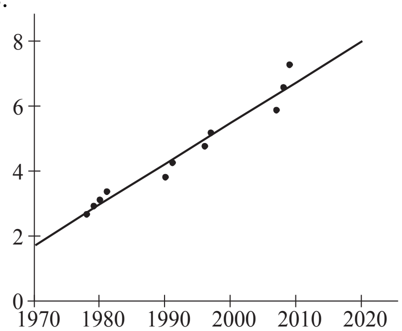
36.



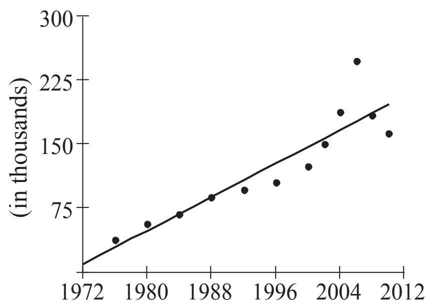
37.



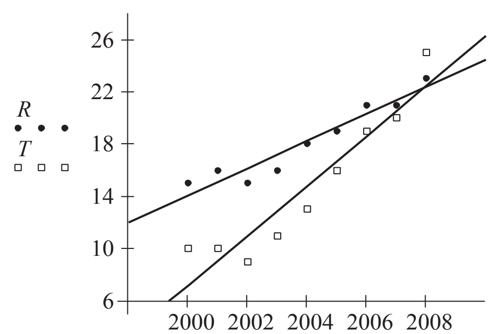
38.



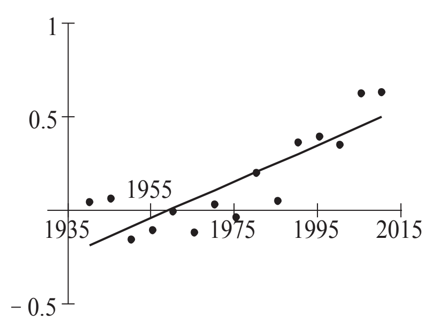
39.



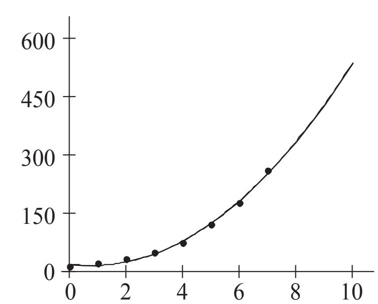
40.



41.



42.



CHAPTER 1 PRACTICE EXERCISES

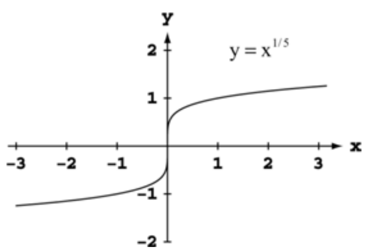
1. The area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. Thus, $r = \frac{C}{2\pi} \Rightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$.

2. The surface area is $S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}$. The volume is $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. Substitution into the formula for surface area gives $S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$.

3. The coordinates of a point on the parabola are (x, x^2). The angle of inclination θ joining this point to the origin satisfies the equation $\tan\theta = \frac{x^2}{x} = x$. Thus the point has coordinates $(x, x^2) = (\tan\theta, \tan^2\theta)$.

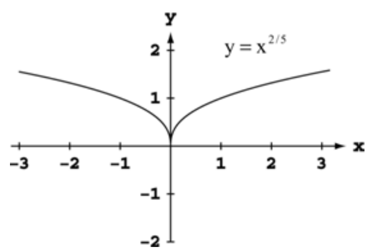
4. $\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500\tan\theta$ m .

5.



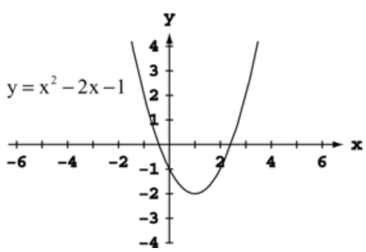
Symmetric about the origin.

6.



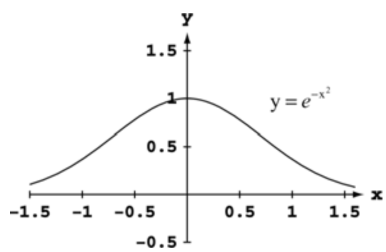
Symmetric about the y-axis.

7.



Neither

8.



Symmetric about the y-axis.

9. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$. Even.

10. $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$. Odd.

11. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$. Even.

12. $y(-x) = \sec(-x)\tan(-x) = \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x} = -\sec x \tan x = -y(x)$. Odd.

13. $y(-x) = \frac{(-x)^4 + 1}{(-x)^3 - 2(-x)} = \frac{x^4 + 1}{-x^3 + 2x} = -\frac{x^4 + 1}{x^3 - 2x} = -y(x)$. Odd.

14. $y(-x) = (-x) - \sin(-x) = (-x) + \sin x = -(x - \sin x) = -y(x)$. Odd.

15. $y(-x) = -x + \cos(-x) = -x + \cos x$. Neither even nor odd.

16. $y(-x) = (-x)\cos(-x) = -x\cos x = -y(x)$. Odd.

17. Since f and g are odd $\Rightarrow f(-x) = -f(x)$ and $g(-x) = -g(x)$.
    (a) $(f \cdot g)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (f \cdot g)(x) \Rightarrow f \cdot g$ is even.
    (b) $f^3(-x) = f(-x)f(-x)f(-x) = [-f(x)][-f(x)][-f(x)] = -f(x) \cdot f(x) \cdot f(x) = -f^3(x) \Rightarrow f^3$ is odd.
    (c) $f(\sin(-x)) = f(-\sin(x)) = -f(\sin(x)) \Rightarrow f(\sin(x))$ is odd.
    (d) $g(\sec(-x)) = g(\sec(x)) \Rightarrow g(\sec(x))$ is even.
    (e) $|g(-x)| = |-g(x)| = |g(x)| \Rightarrow |g|$ is even.

18. Let $f(a-x) = f(a+x)$ and define $g(x) = f(x+a)$. Then $g(-x) = f((-x)+a) = f(a-x) = f(a+x) = f(x+a) = g(x) \Rightarrow g(x) = f(x+a)$ is even.

19. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

20. (a) Since the square root requires $1 - x \geq 0$, the domain is $(-\infty, 1]$.
    (b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.

21. (a) Since the square root requires $16 - x^2 \geq 0$, the domain is $[-4, 4]$.
    (b) For values of x in the domain, $0 \leq 16 - x^2 \leq 16$, so $0 \leq \sqrt{16 - x^2} \leq 4$. The range is $[0, 4]$.

22. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.

23. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

24. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.
    (b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

25. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The sine function attains values from -1 to 1, so $-2 \leq 2\sin(3x + \pi) \leq 2$ and hence $-3 \leq 2\sin(3x + \pi) - 1 \leq 1$. The range is $[-3, 1]$.

26. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

27. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.
    (b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

28. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

29. (a) Increasing because volume increases as radius increases.
 (b) Neither, since the greatest integer function is composed of horizontal (constant) line segments.
 (c) Decreasing because as the height increases, the atmospheric pressure decreases.
 (d) Increasing because the kinetic (motion) energy increases as the particles velocity increases.

30. (a) Increasing on $[2, \infty)$
 (b) Increasing on $[-1, \infty)$
 (c) Increasing on $(-\infty, \infty)$
 (d) Increasing on $\left[\frac{1}{2}, \infty\right)$

31. (a) The function is defined for $-4 \le x \le 4$, so the domain is $[-4, 4]$.
 (b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \le x \le 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

32. (a) The function is defined for $-2 \le x \le 2$, so the domain is $[-2, 2]$.
 (b) The range is $[-1, 1]$.

33. First piece: Line through (0, 1) and (1, 0). $m = \frac{0-1}{1-0} = \frac{-1}{1} = -1 \Rightarrow y = -x + 1 = 1 - x$
 Second piece: Line through (1, 1) and (2, 0). $m = \frac{0-1}{2-1} = \frac{-1}{1} = -1 \Rightarrow y = -(x-1) + 1 = -x + 2 = 2 - x$
 $$f(x) = \begin{cases} 1-x, & 0 \le x < 1 \\ 2-x, & 1 \le x \le 2 \end{cases}$$

34. First piece: Line through (0, 0) and (2, 5). $m = \frac{5-0}{2-0} = \frac{5}{2} \Rightarrow y = \frac{5}{2}x$
 Second piece: Line through (2, 5) and (4, 0). $m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2} \Rightarrow y = -\frac{5}{2}(x-2) + 5 = -\frac{5}{2}x + 10 = 10 - \frac{5x}{2}$
 $$f(x) = \begin{cases} \frac{5}{2}x, & 0 \le x < 2 \\ 10 - \frac{5x}{2}, & 2 \le x \le 4 \end{cases} \quad \text{(Note: } x = 2 \text{ can be included on either piece.)}$$

35. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$
 (b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2}+2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$
 (c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x, x \ne 0$
 (d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x+2}}+2}} = \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$

36. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\sqrt[3]{-1+1}\right) = f(0) = 2 - 0 = 2$
 (b) $(g \circ f)(2) = f(g(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$
 (c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2 - (2-x) = x$
 (d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{x+1}\right) = \sqrt[3]{\sqrt[3]{x+1}+1}$

37. (a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x+2}\right) = 2 - \left(\sqrt{x+2}\right)^2 = -x, \ x \ge -2.$
 $(g \circ f)(x) = g(f(x)) = g(2-x^2) = \sqrt{\left(2-x^2\right)+2} = \sqrt{4-x^2}$
 (b) Domain of $f \circ g$: $[-2, \infty)$.
 Domain of $g \circ f$: $[-2, 2]$.
 (c) Range of $f \circ g$: $(-\infty, 2]$.
 Range of $g \circ f$: $[0, 2]$.

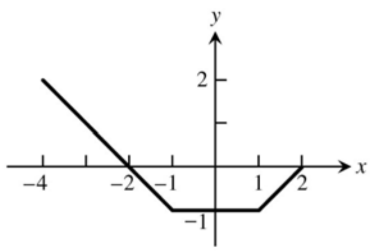
38. (a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{1-x}\right) = \sqrt{\sqrt{1-x}} = \sqrt[4]{1-x}.$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{1-\sqrt{x}}$

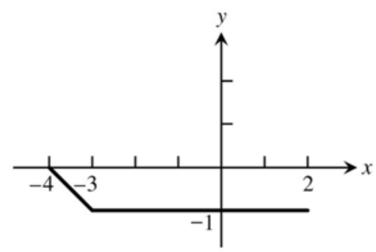
(b) Domain of $f \circ g$: $(-\infty, 1]$.
Domain of $g \circ f$: $[0, 1]$.

(c) Range of $f \circ g$: $[0, \infty)$.
Range of $g \circ f$: $[0, 1]$.

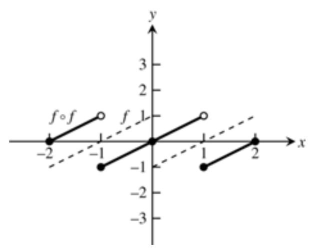
39. $y = f(x)$



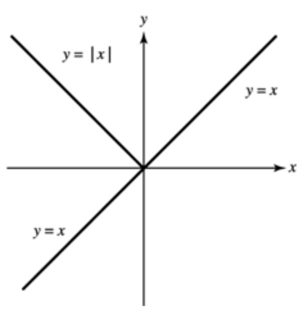
$y = (f \circ f)(x)$



40.

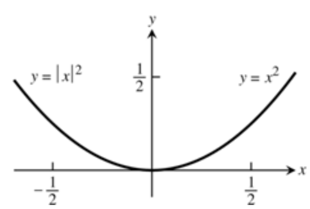


41.



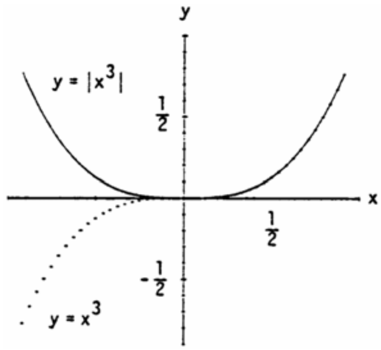
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x), x \ge 0$ across the y-axis.

42.



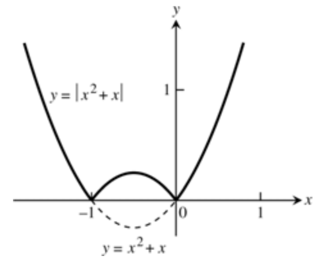
It does not change the graph.

43.



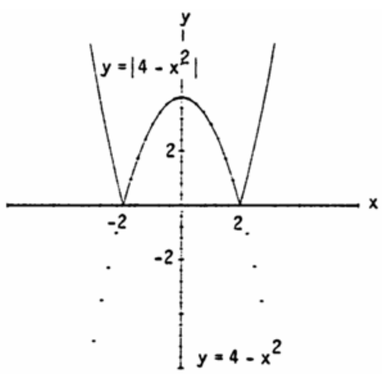
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

44.



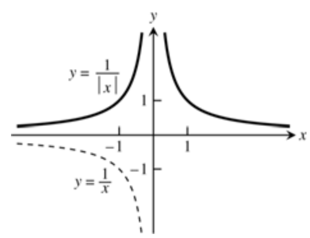
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

45.



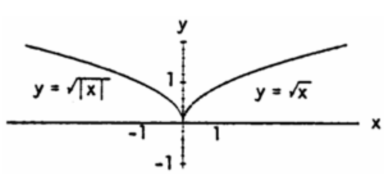
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

46.



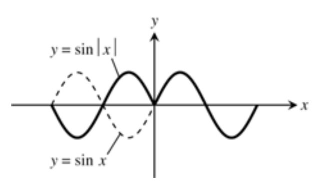
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

47.



The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

48.

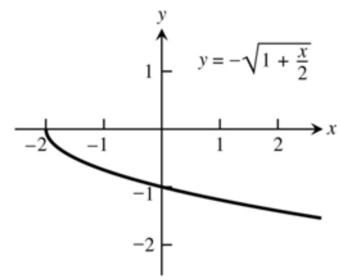


The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

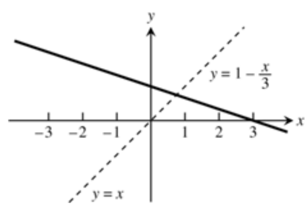
49. (a) $y = g(x-3) + \frac{1}{2}$
(b) $y = g\left(x + \frac{2}{3}\right) - 2$
(c) $y = g(-x)$
(d) $y = -g(x)$
(e) $y = 5 \cdot g(x)$
(f) $y = g(5x)$

50. (a) Shift the graph of f right 5 units (b) Horizontally compress the graph of f by a factor of 4
(c) Horizontally compress the graph of f by a factor of 3 and then reflect the graph about the y-axis
(d) Horizontally compress the graph of f by a factor of 2 and then shift the graph left $\frac{1}{2}$ unit.
(e) Horizontally stretch the graph of f by a factor of 3 and then shift the graph down 4 units.
(f) Vertically stretch the graph of f by a factor of 3, then reflect the graph about the x-axis, and finally shift the graph up $\frac{1}{4}$ unit.

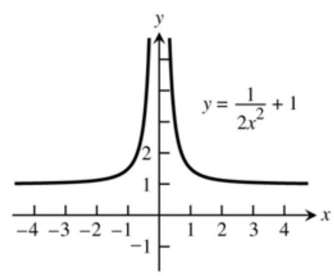
51. Reflection of the graph of $y = \sqrt{x}$ about the x-axis followed by a horizontal compression by a factor of $\frac{1}{2}$ then a shift left 2 units.



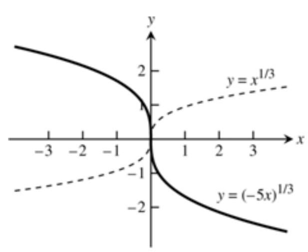
52. Reflect the graph of $y = x$ about the x-axis, followed by a vertical compression of the graph by a factor of 3, then shift the graph up 1 unit.



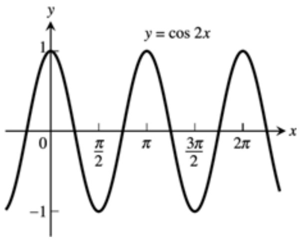
53. Vertical compression of the graph of $y = \frac{1}{x^2}$ by a factor of 2, then shift the graph up 1 unit.



54. Reflect the graph of $y = x^{1/3}$ about the y-axis, then compress the graph horizontally by a factor of 5.

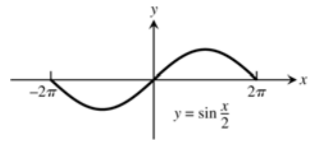


55.



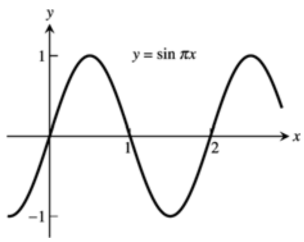
period $= \pi$

56.



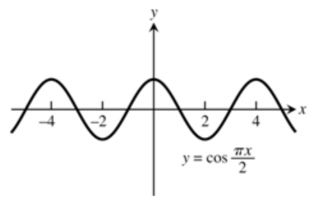
period $= 4\pi$

57.



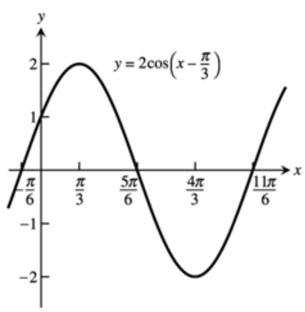
period $= 2$

58.



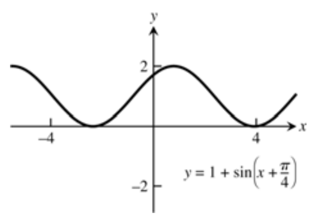
period $= 4$

59.



period $= 2\pi$

60.



period $= 2\pi$

61. (a) $\sin B = \sin\frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2\sin\frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$. By the theorem of Pythagoras, $a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4-3} = 1$.

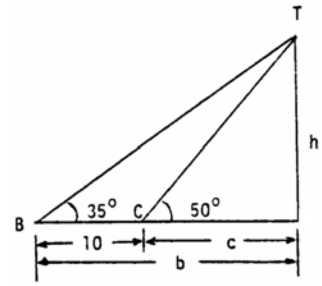
(b) $\sin B = \sin\frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin\frac{\pi}{3}} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

62. (a) $\sin A = \frac{a}{c} \Rightarrow a = c\sin A$ (b) $\tan A = \frac{a}{b} \Rightarrow a = b\tan A$

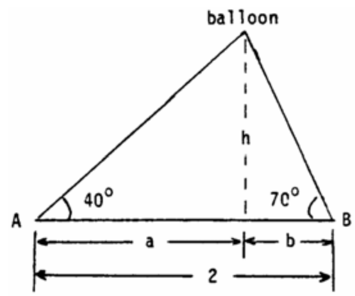
63. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$ (b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

64. (a) $\sin A = \frac{a}{c}$ (b) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

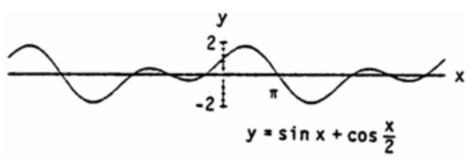
65. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flat ground, respectively. Then, $\tan 50° = \frac{h}{c}$, $\tan 35° = \frac{h}{b}$, and $b - c = 10$.
Thus, $h = c\tan 50°$ and $h = b\tan 35° = (c+10)\tan 35°$
$\Rightarrow c\tan 50° = (c+10)\tan 35°$
$\Rightarrow c(\tan 50° - \tan 35°) = 10\tan 35°$
$\Rightarrow c = \frac{10\tan 35°}{\tan 50° - \tan 35°} \Rightarrow h = c\tan 50°$
$= \frac{10\tan 35°\tan 50°}{\tan 50° - \tan 35°} \approx 16.98$ m.



66. Let h = height of balloon above ground. From the figure at the right, $\tan 40° = \frac{h}{a}$, $\tan 70° = \frac{h}{b}$, and $a + b = 2$. Thus, $h = b\tan 70° \Rightarrow h = (2-a)\tan 70°$ and $h = a\tan 40° \Rightarrow (2-a)\tan 70° = a\tan 40°$
$\Rightarrow a(\tan 40° + \tan 70°) = 2\tan 70°$
$\Rightarrow a = \frac{2\tan 70°}{\tan 40° + \tan 70°} \Rightarrow h = a\tan 40°$
$= \frac{2\tan 70°\tan 40°}{\tan 40° + \tan 70°} \approx 1.3$ km.

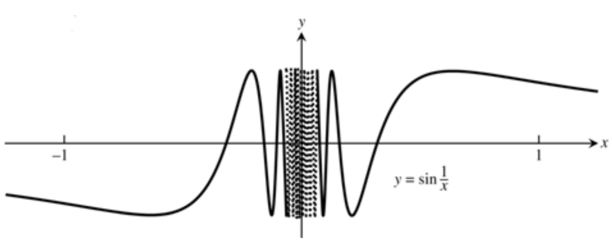


67. (a)



(b) The period appears to be 4π.
(c) $f(x+4\pi) = \sin(x+4\pi) + \cos\left(\frac{x+4\pi}{2}\right) = \sin(x+2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos\frac{x}{2}$
since the period of sine and cosine is 2π. Thus, $f(x)$ has period 4π.

68. (a)



(b) $D = (-\infty, 0) \cup (0, \infty)$; $R = [-1, 1]$
(c) f is not periodic. For suppose f has period p. Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k.
Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/(2\pi)) + kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/(2\pi)) + kp}\right) > 0$
which is a contradiction. Thus f has no period, as claimed.

CHAPTER 1 ADDITIONAL AND ADVANCED EXERCISES

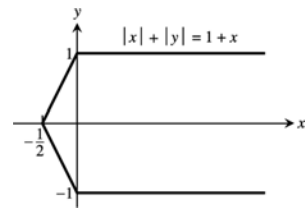
1. There are (infinitely) many such function pairs. For example, $f(x)=3x$ and $g(x)=4x$ satisfy $f(g(x))=f(4x)=3(4x)=12x=4(3x)=g(3x)=g(f(x))$.

2. Yes, there are many such function pairs. For example, if $g(x)=(2x+3)^3$ and $f(x)=x^{1/3}$, then $(f\circ g)(x)=f(g(x))=f((2x+3)^3)=((2x+3)^3)^{1/3}=2x+3$.

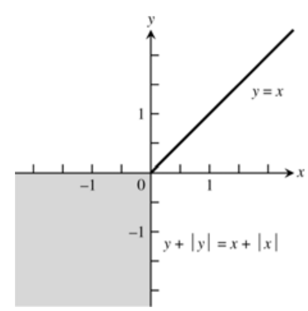
3. If f is odd and defined at x, then $f(-x)=-f(x)$. Thus $g(-x)=f(-x)-2=-f(x)-2$ whereas $-g(x)=-(f(x)-2)=-f(x)+2$. Then g cannot be odd because $g(-x)=-g(x)\Rightarrow -f(x)-2=-f(x)+2$ $\Rightarrow 4=0$, which is a contradiction. Also, $g(x)$ is not even unless $f(x)=0$ for all x. On the other hand, if f is even, then $g(x)=f(x)-2$ is also even: $g(-x)=f(-x)-2=f(x)-2=g(x)$.

4. If g is odd and $g(0)$ is defined, then $g(0)=g(-0)=-g(0)$. Therefore, $2g(0)=0\Rightarrow g(0)=0$.

5. For (x, y) in the 1st quadrant, $|x|+|y|=1+x$ $\Leftrightarrow x+y=1+x\Leftrightarrow y=1$. For (x, y) in the 2nd quadrant, $|x|+|y|=x+1\Leftrightarrow -x+y=x+1$ $\Leftrightarrow y=2x+1$. In the 3rd quadrant, $|x|+|y|=x+1$ $\Leftrightarrow -x-y=x+1\Leftrightarrow y=-2x-1$. In the 4th quadrant, $|x|+|y|=x+1\Leftrightarrow x+(-y)=x+1$ $\Leftrightarrow y=-1$. The graph is given at the right.

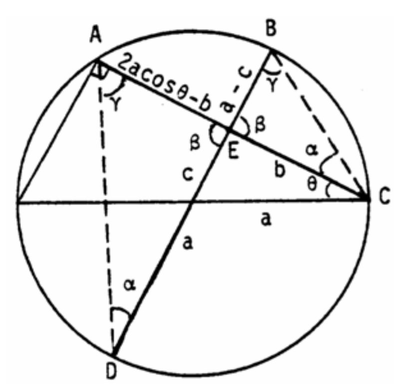


6. We use reasoning similar to Exercise 5.
   (1) 1st quadrant: $y+|y|=x+|x|$
   $\Leftrightarrow 2y=2x\Leftrightarrow y=x$.
   (2) 2nd quadrant: $y+|y|=x+|x|$
   $\Leftrightarrow 2y=x+(-x)=0\Leftrightarrow y=0$.
   (3) 3rd quadrant: $y+|y|=x+|x|$
   $\Leftrightarrow y+(-y)=x+(-x)\Leftrightarrow 0=0$
   $\Rightarrow$ all points in the 3rd quadrant satisfy the equation.
   (4) 4th quadrant: $y+|y|=x+|x|$
   $\Leftrightarrow y+(-y)=2x\Leftrightarrow 0=x$. Combining these results we have the graph given at the right:



7. (a) $\sin^2 x+\cos^2 x=1\Rightarrow \sin^2 x=1-\cos^2 x=(1-\cos x)(1+\cos x)\Rightarrow (1-\cos x)=\frac{\sin^2 x}{1+\cos x}\Rightarrow \frac{1-\cos x}{\sin x}=\frac{\sin x}{1+\cos x}$
   (b) Using the definition of the tangent function and the double angle formulas, we have
   $$\tan^2\left(\frac{x}{2}\right)=\frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}=\frac{\frac{1-\cos\left(2\left(\frac{x}{2}\right)\right)}{2}}{\frac{1+\cos\left(2\left(\frac{x}{2}\right)\right)}{2}}=\frac{1-\cos x}{1+\cos x}.$$

8. The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD. Similarly, the two angles labeled α are equal since they both subtend arc AB. Thus, triangles AED and BEC are similar which implies $\frac{a-c}{b}=\frac{2a\cos\theta-b}{a+c}$
   $\Rightarrow (a-c)(a+c)=b(2a\cos\theta-b)$
   $\Rightarrow a^2-c^2=2ab\cos\theta-b^2$
   $\Rightarrow c^2-a^2+b^2-2ab\cos\theta$.



9. As in the proof of the law of sines of Section 1.3, Exercise 61, $ah = bc\sin A = ab\sin C = ac\sin B$ $\Rightarrow$ the area of $ABC = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}ah = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$.

10. As in Section 1.3, Exercise 61, $(\text{Area of } ABC)^2 = \frac{1}{4}(\text{base})^2(\text{height})^2 = \frac{1}{4}a^2h^2 = \frac{1}{4}a^2b^2\sin^2 C$ $= \frac{1}{4}a^2b^2(1-\cos^2 C)$. By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow \cos C = \frac{a^2+b^2-c^2}{2ab}$. Thus,
$(\text{area of } ABC)^2 = \frac{1}{4}a^2b^2(1-\cos^2 C) = \frac{1}{4}a^2b^2\left(1-\left(\frac{a^2+b^2-c^2}{2ab}\right)^2\right) = \frac{a^2b^2}{4}\left(1-\frac{(a^2+b^2-c^2)^2}{4a^2b^2}\right)$
$= \frac{1}{16}\left(4a^2b^2-(a^2+b^2-c^2)^2\right) = \frac{1}{16}[(2ab+(a^2+b^2-c^2))\,(2ab-(a^2+b^2-c^2))]$
$= \frac{1}{16}[((a+b)^2-c^2)(c^2-(a-b)^2)] = \frac{1}{16}[((a+b)+c)((a+b)-c)(c+(a-b))(c-(a-b))]$
$= \left[\left(\frac{a+b+c}{2}\right)\left(\frac{-a+b+c}{2}\right)\left(\frac{a-b+c}{2}\right)\left(\frac{a+b-c}{2}\right)\right] = s(s-a)(s-b)(s-c)$, where $s = \frac{a+b+c}{2}$.
Therefore, the area of ABC equals $\sqrt{s(s-a)(s-b)(s-c)}$.

11. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x) \Rightarrow f(x) = -f(x)$ for all x in the domain of f. Thus $2f(x) = 0 \Rightarrow f(x) = 0$.

12. (a) As suggested, let $E(x) = \frac{f(x)+f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x)+f(-(-x))}{2} = \frac{f(x)+f(-x)}{2} = E(x) \Rightarrow E$ is an even function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x)+f(-x)}{2} = \frac{f(x)-f(-x)}{2}$. Then $O(-x) = \frac{f(-x)-f(-(-x))}{2}$ $= \frac{f(-x)-f(x)}{2} = -\left(\frac{f(x)-f(-x)}{2}\right) = -O(x) \Rightarrow O$ is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an even and an odd function.

(b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0$ $= (E_1(x) + O_1(x)) - (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$ (since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x)$ $= -O_1(x) - (-O(x))$ (since O and O_1 are odd) $= -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd. Therefore, $E - E_1$ and $O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by Exercise 11. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.

13. $y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

(a) If $a > 0$ the graph is a parabola that opens upward. Increasing a causes a vertical stretching and a shift of the vertex toward the y-axis and upward. If $a < 0$ the graph is a parabola that opens downward. Decreasing a causes a vertical stretching and a shift of the vertex toward the y-axis and downward.

(b) If $a > 0$ the graph is a parabola that opens upward. If also $b > 0$, then increasing b causes a shift of the graph downward to the left; if $b < 0$, then decreasing b causes a shift of the graph downward and to the right. If $a < 0$ the graph is a parabola that opens downward. If $b > 0$, increasing b shifts the graph upward to the right. If $b < 0$, decreasing b shifts the graph upward to the left.

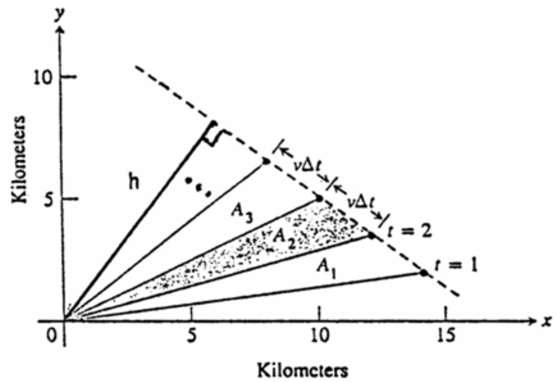
(c) Changing c (for fixed a and b) by Δc shifts the graph upward Δc units if $\Delta c > 0$, and downward $-\Delta c$ units if $\Delta c < 0$.

14. (a) If $a > 0$, the graph rises to the right of the vertical line $x = -b$ and falls to the left. If $a < 0$, the graph falls to the right of the line $x = -b$ and rises to the left. If $a = 0$, the graph reduces to the horizontal line $y = c$. As $|a|$ increases, the slope at any given point $x = x_0$ increases in magnitude and the graph becomes steeper. As $|a|$ decreases, the slope at x_0 decreases in magnitude and the graph rises or falls more gradually.

(b) Increasing b shifts the graph to the left; decreasing b shifts it to the right.

(c) Increasing c shifts the graph upward; decreasing c shifts it downward.

15. Each of the triangles pictured has the same base $b = v\Delta t = v(1\text{ s})$. Moreover, the height of each triangle is the same value h. Thus $\frac{1}{2}$ (base)(height) $= \frac{1}{2}bh = A_1 = A_2 = A_3 = \ldots$. In conclusion, the object sweeps out equal areas in each one second interval.



16. (a) Using the midpoint formula, the coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of $\overline{OP} = \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.

(b) The slope of $\overline{AB} = \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments $\overline{AB}$ and $\overline{OP}$ are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, $\overline{AB}$ is perpendicular to $\overline{OP}$ when $a = b$.

17. From the figure we see that $0 \le \theta \le \frac{\pi}{2}$ and $AB = AD = 1$. From trigonometry we have the following: $\sin\theta = \frac{EB}{AB} = EB$, $\cos\theta = \frac{AE}{AB} = AE$, $\tan\theta = \frac{CD}{AD} = CD$, and $\tan\theta = \frac{EB}{AE} = \frac{\sin\theta}{\cos\theta}$. We can see that:
area $\Delta AEB <$ area sector $\widehat{DB} <$ area $\Delta ADC \Rightarrow \frac{1}{2}(AE)(EB) < \frac{1}{2}(AD)^2\theta < \frac{1}{2}(AD)(CD)$
$\Rightarrow \frac{1}{2}\sin\theta\cos\theta < \frac{1}{2}(1)^2\theta < \frac{1}{2}(1)(\tan\theta) \Rightarrow \frac{1}{2}\sin\theta\cos\theta < \frac{1}{2}\theta < \frac{1}{2}\frac{\sin\theta}{\cos\theta}$

18. $(f\circ g)(x) = f(g(x)) = a(cx+d)+b = acx+ad+b$ and $(g\circ f)(x) = g(f(x)) = c(ax+b)+d = acx+cb+d$
Thus $(f\circ g)(x) = (g\circ f)(x) \Rightarrow acx+ad+b = acx+bc+d \Rightarrow ad+b = bc+d$. Note that $f(d) = ad+b$ and $g(b) = cb+d$, thus $(f\circ g)(x) = (g\circ f)(x)$ if $f(d) = g(b)$.