STA2001 Tutorial 12

1. 5.4-10. Let X equal the outcome when a fair four-sided die that has its faces numbered 0, 1, 2, and 3 is rolled. Let Y equal the outcome when a fair four-sided die that has its faces numbered 0, 4, 8, and 12 is rolled.

   (a) Define the mgf of X.

   (b) Define the mgf of Y.

   (c) Let $W = X + Y$, the sum when the pair of dice is rolled. Find the mgf of W.

   (d) Give the pmf of W; that is, determine $P(W = w), w = 0, 1, \cdots, 15$, from the mgf of W. $\frac{1}{16}$?

(a) $P(X=0) = P(X=1) = P(X=2) = P(X=3) = \frac{1}{4}$

$E[e^{tX}] = \frac{1}{4}(1 + e^{t} + e^{2t} + e^{3t})$

(b) $\frac{1}{4}(1 + e^{4t} + e^{8t} + e^{12t})$

0	0	0	4	8	12	
1	4	1	5	9	13	$\frac{1}{16}$
2	8	2	6	10	14	
3	12	3	7	11	15	

$\frac{1}{16}(1 + e^{t} + \cdots + e^{15t})$

$f(x) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\theta}}$ $\quad \frac{1}{(1-\theta t)^{\alpha}} \quad \alpha\theta \quad \alpha\theta^2$

$x \geq 0, \alpha > 0, \theta > 0$

2. 5.4-20. The time X in minutes of a visit to a cardiovascular disease specialist by a patient is modeled by a gamma pdf with $\alpha = 1.5$ and $\theta = 10$. Suppose that you are such a patient and have four 4 patients ahead of you. Assuming independence, what integral gives the probability that you will wait more than 90 minutes?

$$\frac{1}{\Gamma(1.5) \times 10^{1.5}} x^{0.5} e^{-\frac{x}{10}}$$

$\mu = 800$ $\sigma = 100$

3. 5.5-10. A consumer buys n light bulbs, each of which has a lifetime that has a mean of 800 hours, a standard deviation of 100 hours, and a normal distribution. A light bulb is replaced by another as soon as it burns out. Assuming independence of the lifetimes, find the smallest n so that the succession of light bulbs produces light for at least 10,000 hours with a probability of 0.90.

$$\frac{\bar{X} - 800}{\frac{100}{\sqrt{n}}} \sim N(0,1)$$

$f(x) = \frac{3}{2}x^2$

4. 5.6-2. Let $Y = X_1 + X_2 + \cdots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose pdf is $f(x) = (3/2)x^2$, $-1 < x < 1$. Using the pdf of Y, we find that $P(-0.3 \le Y \le 1.5) = 0.22788$. Use the central limit theorem to approximate this probability.

$P(Y \le 1.5) - P(Y \le -0.3) = 0.22788$

$P(\sum X_i \le 1.5) - \underbrace{P(\sum X_i \le -0.3)}_{=0}$

$P(\sum X_i \le 1.5) = 0.22788$

0.2313