STA2001 Probability and statistical Inference I
Tutorial 4

1. 2.3-17. Let X equal the number of flips of a fair coin that are required to observe heads-tails on consecutive fips. (Similar to 2.3-16)

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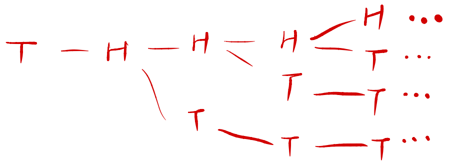
   (a) Find the pmf of X. Hint: Draw a tree diagram.

   (b) Show that the mgf of X is $M(t) = e^{2t}/(e^t - 2)^2$. $\Rightarrow$ 高中方法

   (c) Use the mgf to find the values of (i) the mean and (ii) the variance of X.

   (d) Find the values of (i) $P(X \leq 3)$, (ii) $P(X \geq 5)$, and (iii) $P(X = 3)$.

a) X^{th} $(X-1)^{th}$ ---- $(X-4)^{th}$



terminate at X flips, will have $(X-1)$ possible outcomes

For each outcome: prob. $(\frac{1}{2})^X$

$\Rightarrow f(x) = (\frac{1}{2})^x \cdot (x-1)$, $x \in S_x = \{2, 3, 4 \cdots\}$

c) $M'(t) = \frac{8e^{2t} - 4e^{3t}}{(e^t - 2)^4}$ $M''(t) = \cdots$ 略

2. 2.4-15. A hospital obtains 40% of its flu vaccine from Company A, 50% from Company B, and 10% from Company C. From past experience, it is known that 3% of the vials from A are ineffective, 2% from B are ineffective, and 5% from C are ineffective. The hospital tests five vials from each shipment (Each shipment comes from a single company). If at least one of the five is ineffective, find the conditional probability of that shipment's having come from C. (Please recall the fair/unfair coin problem" in the tutorial of Bayes' Theorem)

A	B	C
40%	50%	10%
3%	2%	5%

$0.4 \times 0.97 + 0.5 \times 0.98 + 0.1 \times 0.95 = 0.973$

$1 - 0.973 = 0.027$

$\frac{\cancel{0.1} \times 0.05}{\cancel{0.0}0.27} = \frac{5}{27}$

I: event that ≥ 1 out 5 is ineffective

$P(I|C) = P(X_C \geq 1) = 1 - P(X_{C=0}) = 1 - \binom{5}{0} 0.05^0 \cdot 0.95^5 = 1 - 0.95^5$

$P(I|A) = 1 - 0.97^5$

$P(I|B) = 1 - 0.98^5$

$P(C|I) = \frac{P(C)P(I|C)}{P(A)P(I|A) + \cdots} = 0.178$

3. 2.5-9. (Coupon collector's problem) One of four different prizes was randomly put into each box of a cereal. If a family decided to buy this cereal until it obtained at least one of each of the four different prizes, what is the expected number of boxes of cereal that must be purchased? (Similar to 2.5-10)

A B C D

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

Let:

X_1 = # boxes to obtain the 1st price

X_2 = — — 2nd, we got 1st

X_3 = — — 3rd, — 1st & 2nd

X_4 = — — 4th, — 1st & 2nd & 3rd

$X_1 = 1$

$X_2, X_3, X_4 \sim$ Geometric dis

$\frac{3}{4}$ $\frac{2}{4}$ $\frac{1}{4}$

$E(X) = \frac{1}{p}$

$$E(X) = E(X_1 + X_2 + X_3 + X_4) = 1 + \frac{1}{\frac{3}{4}} + \frac{1}{\frac{2}{4}} + \frac{1}{\frac{1}{4}} = \frac{25}{3} \approx 8$$

4. 2.6-9. A store selling newspapers orders only $n = 4$ of a certain newspape because the manager does not get many calls for that publication. If the number of requests per day follows a Poisson distribution with mean 3,

(a.) What is the expected value of the number sold?

(b.) What is the minimum number that the manager should order so that the chance of having more requests than available newspapers is less than 0.05?

Let X: # of requests/Day, $X \sim$ Poisson (3), $f(x) = \frac{3^k e^{-3}}{k!}$, $x \in S_x = \{0, 1, 2, 3 \dots\}$

Let $u(x)$: # of newspaper sold: $\begin{cases} x & , x = 0, 1, 2, 3 \\ 4 & , x \geq 4 \end{cases}$

$$E[u(x)] = \sum_{x \in S_x} u(x) f(x) = \sum_{k=0}^{3} k \cdot \frac{3^k e^{-3}}{k!} + \sum_{k=4}^{\infty} 4 \cdot \frac{3^k e^{-3}}{k!} \approx 2.681$$

Let y: minimum number

$P(X > y) < 0.05 \iff 1 - P(X \leq y) < 0.05$

$\iff P(X \leq y) > 0.95$

$\iff F(y) > 0.95 \implies y = 6$