

FIN2010 Financial Management

Lecture 23:

Personal Investment -Rational Decisions



Review—Financial Products

- Mutual Funds
 - Advantage over direct investment: 1) diversification. 2) expertise
 - Fee: management (0.5~1.5%), front/rear load
- ETFs
 - Mostly index funds, low fee, instant transactions
- Private Equity
 - Private funds with less regulation, cannot market to the public
 - Better performance
 - Higher fees: management fee (1.5%-2%) + incentive fee (15%-20%)
- Real Estate
 - Embedded leverage: $\text{return on equity} = \text{leverage} * \text{return on asset}$
 - Alternative way: REITS



Agenda

- Foreword
- Characterizing Your Preferences
 - Risk Attitude
 - Expected Utility
- Portfolio Choice Problems
 - One-period Portfolio Choice
 - Life-cycle Portfolio Choice
- Insurance



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Foreword—What is Rational?

- A rational agent makes decisions based on
 - Logic or reasons
- The decisions are not based on
 - Heuristics (past experience)
 - Emotion
- This lecture: provide a framework for you to compare investment outcomes, so that ex ante, you can make rational choices
 - It does not mean you have to think in this way. We only provide an example of a rational framework



Roles of Economic Models

- Descriptive
 - Designed to fit empirical observations or experimental results
- Normative
 - Able to provide guidance on making optimal choices
- This lecture
 - Discuss the rational objective when making investment decisions
 - Introduce a normative model as to how should you allocate your money between consumption and saving over time.



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Understanding Your Preference

What is your goal of investing? Is it to have the highest expected value of wealth?

Consider the following options:

1. You get ¥10 million for sure
2. There is 10% chance you get ¥150 million, and 90% chance you get ¥0 ($E[\text{wealth}] = ¥15\text{M}$)

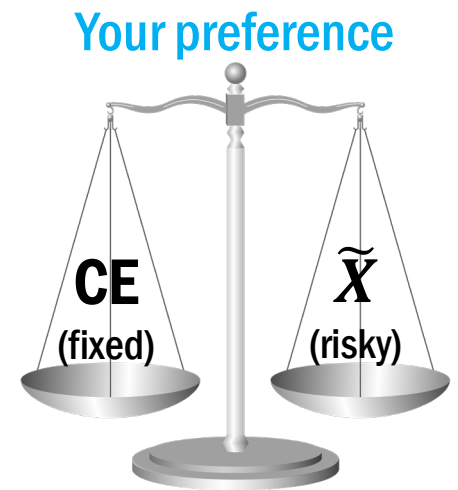
You can think of (1) as you work for a company and earn a steady income of 10M over the course of your lifetime, and (2) as you start your own business.

Our goal is to maximize the utility: your pleasure or satisfaction level with a certain choice!



Risk Attitude

- Certainty Equivalent (CE) is the amount that makes you *indifferent* between:
 - An fixed cash income with the amount of CE
 - An uncertain cash income \tilde{X}
- One's utility function is determined by his/her risk attitude.
 - Risk seeking: $CE(\tilde{X}) > E[\tilde{X}]$
 - Risk neutral: $CE(\tilde{X}) = E[\tilde{X}]$
 - Risk averse: $CE(\tilde{X}) < E[\tilde{X}]$
- Most individuals are Risk Averse. Therefore, we demand a premium for risk.



Risk Attitude Example

- You have the opportunity to play coin-flip game: 50% chance to win \$100,000 or 50% chance to win \$0. Someone offers to give you a guaranteed reward of \$X in exchange for the coin-flip game. What is the lowest X that will make you accept the offer?
 - Expected payoff of the game: \$50,000
 - X: certainty equivalent of the game.
- Mr. A requires \$30,000: risk averse ($CE < E(\text{payoff})$)
- Mr. B requires \$50,000: risk neutral ($CE = E(\text{payoff})$)
- Mr. C requires \$52,000: risk seeking ($CE > E(\text{payoff})$)
- Mr. A is willing to give up \$20,000 in expected payoff. Why?
 - He does not like risks. A risky game will have to offer \$20,000 more than a guaranteed offer to make her indifferent between the two.



How do We Quantify Preference?

- Recall from microeconomics:

Utility function: $u(C)$

- Intuition: Assign a “happiness” level to a consumption level
- Note: Utility function does not measure the happiness level. It only compares the happiness between two choices



So the latter should be assigned a higher utility level

Properties of Utility Functions

What properties should the utility functions satisfy?

- Monotonically increases with C: $u'(C) > 0$
 - More consumption is always preferable

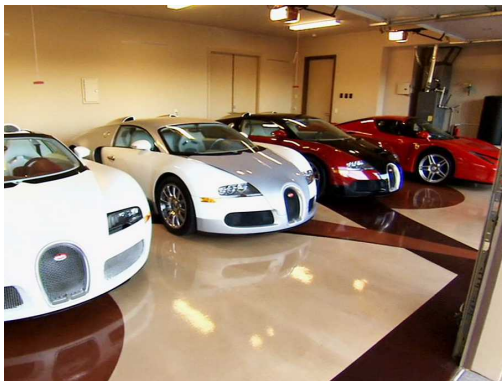


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Properties of Utility Functions—cont.

- Diminishing marginal return: $u''(C) < 0$
 - The more money you spend, additional \$1 will create less happiness
 - $u(.)$ is concave



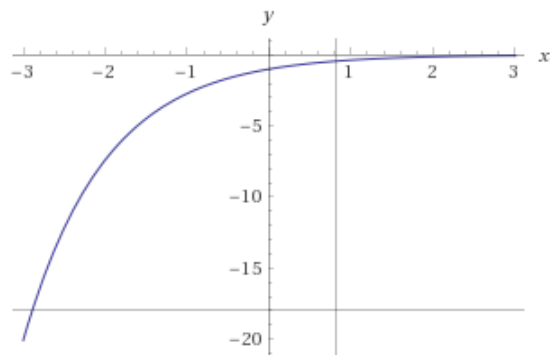
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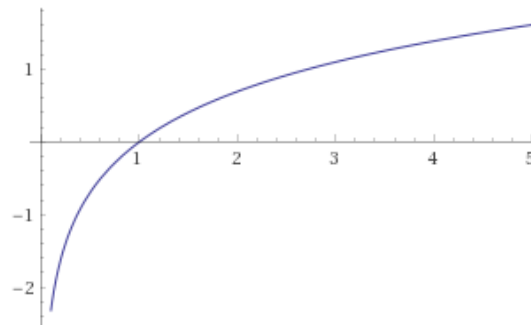
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Commonly Used Utility Functions

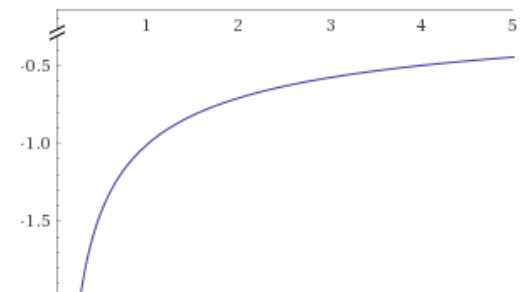


$$u(C) = -e^{-C}$$



$$u(C) = \ln(C)$$

A negative value doesn't mean you are unhappy



$$u(C) = -\frac{1}{\sqrt{C}}$$



Utility with Uncertainty

- Measuring utility of a given level of consumption is straightforward.
- How do we quantify the utility when the consumption level is uncertain?
 1. You get ¥10 million for sure
 2. There is 10% chance you get ¥150 million, and 90% chance you get ¥0

- Expected utility: Given an uncertain consumption level \tilde{C} ,

$$U(\tilde{C}) = E[u(\tilde{C})]$$

- The utility of an uncertain consumption is equal to the average utility across all possible states

- Example: $u(C) = \ln\left(\frac{C}{10M} + 1\right)$

1. $U(C_1) = \ln(1 + 1) = 0.6931$

2. $U(\tilde{C}_2) = \frac{1}{10}\ln(15 + 1) + \frac{9}{10}\ln(0 + 1) = 0.2773$



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Application—Portfolio Choice

- Suppose your utility function is $\ln(C)$. You now have \$100. You consume all your wealth after a year. You can pick between investing in a stock or a bond:

State	Probability	Stock Return	Bond Return
Recession	1/3	-20%	5%
Normal	1/3	10%	4%
Boom	1/3	31%	3%

- If you invest X in stock, and $100 - X$ in bond, your expected utility is

$$\begin{aligned}
 E[U] &= \frac{1}{3}u[C_{Recession}] + \frac{1}{3}u[C_{Normal}] + \frac{1}{3}u[C_{Boom}] \\
 &= \frac{1}{3}\ln[0.8X + 1.05(100 - X)] + \frac{1}{3}\ln[1.1X + 1.04(100 - X)] + \frac{1}{3}\ln[1.31X \\
 &\quad + 1.03(100 - X)]
 \end{aligned}$$

Achieves maximum at $X=68.63$



Why don't we put all money in stock?

- $E[R_S]=7\%$, $E[R_B]=4\%$

State	Probability	Stock Return	Bond Return
Recession	1/3	-20%	5%
Normal	1/3	10%	4%
Boom	1/3	31%	3%

State	Wealth (All stock)	U (All Stock)	Wealth (Optimal: 68.63 in stock, 31.39 in bonds)	U (Optimal: 68.63 in stock, 31.39 in bonds)
Recession	80	4.382027	87.84	4.475553
Normal	110	4.70048	108.12	4.68322
Boom	131	4.875197	122.22	4.805788

$$E[U]=4.652568$$

$$E[U]=4.654853$$



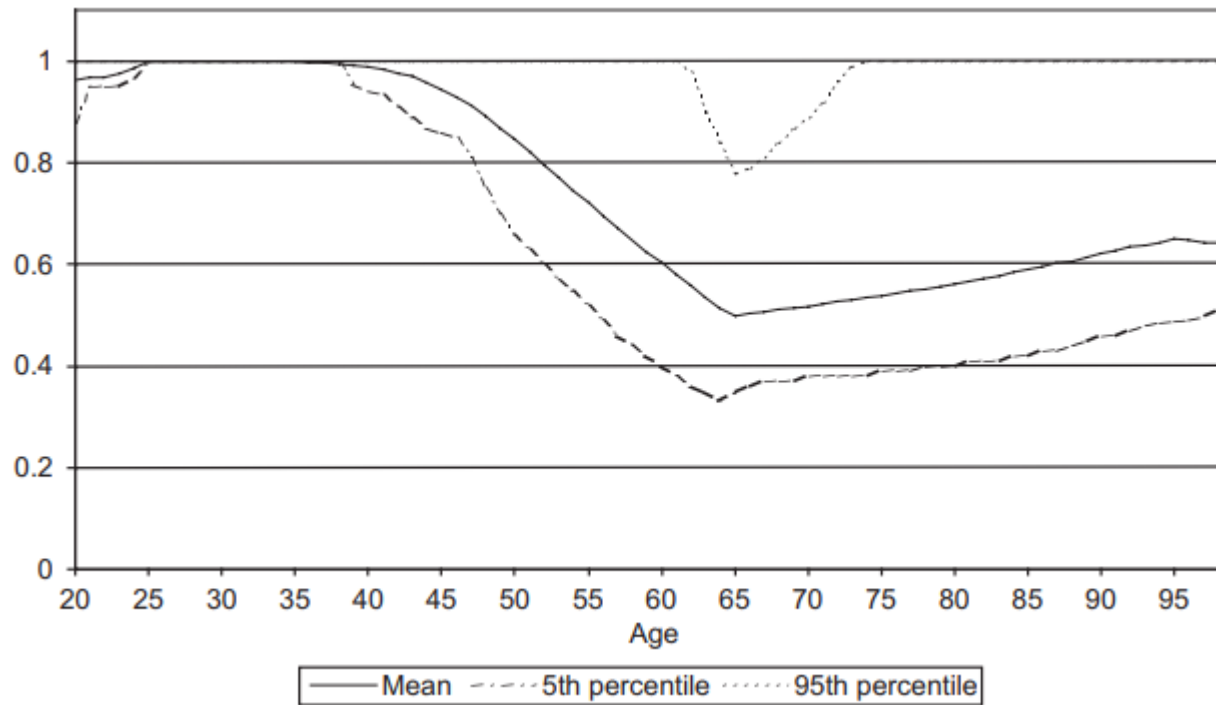
Life-cycle Portfolio Choice

- [Coco et al. \(2005\)](#)

- An individual with age between 20 to 100; he plans to retire at 65; he may die at any time (calibrated using actual data)
- His wants to achieve the maximum expected utility from consumption and bequest during his life
- His wealth comes from two components:
 - Labor income: random variable, rising with age, correlated with stock
 - Financial assets: stock ($\mu = 6\%, \sigma = 15.7\%$), bond ($\mu = 2\%, \sigma = 0$)
- His utility function:
 - In each period, $U_t(C) = -C^{-9}$
 - Overall utility $U = U_0(C_0) + \delta U_1(C_1) + \delta^2 U_2(C_2) + \dots$, where $\delta = 0.96$ is the discount factor for utility
- Decisions to make each period
 - How much to consume
 - How much to invest in stocks vs. bonds



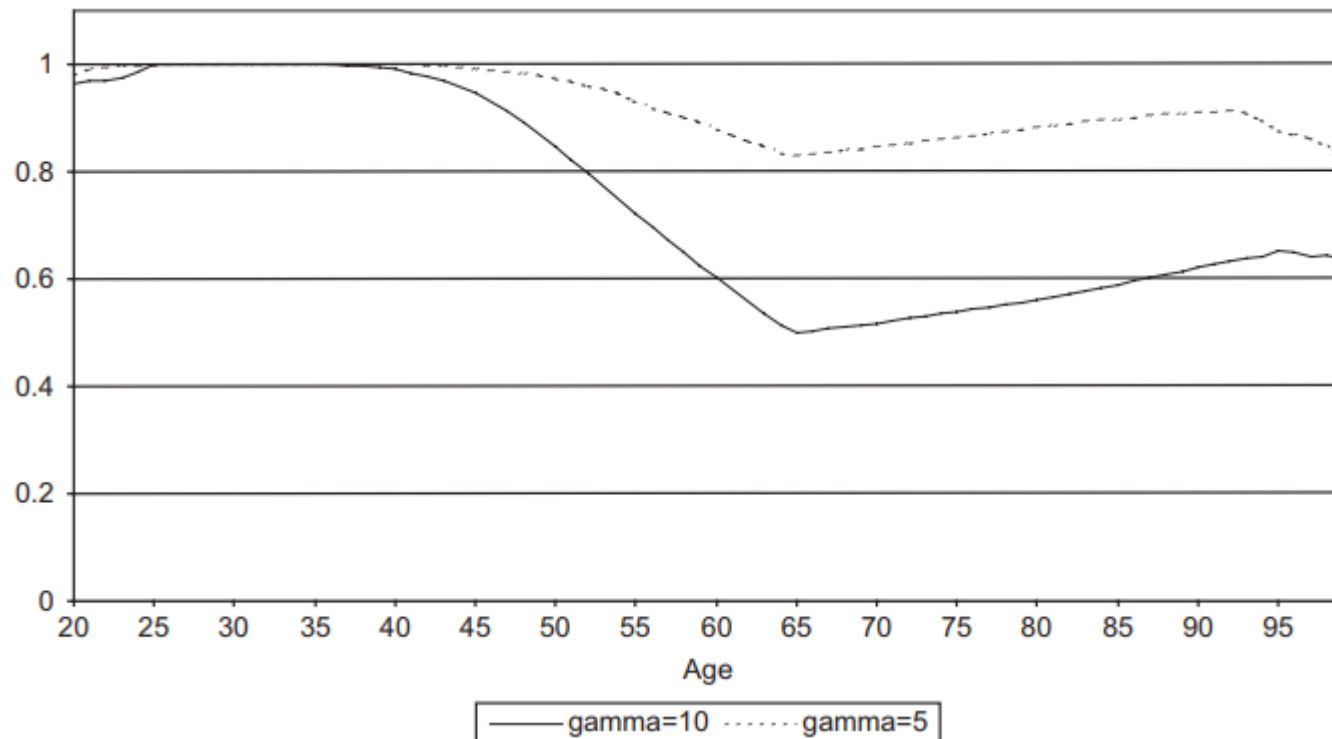
Optimal Life Cycle Portfolio Choice



Optimal decisions depend on labor income and predicted stock return. The graphs plots the optimal portfolio choice for lucky simulation rounds (95%), average rounds (mean) and unlucky rounds (5%).



Less Risk Averse



A less risk averse investor allocates a larger share to stocks.



Average Consumption, Income and Wealth



Income: increases until retirement, stays steady afterwards

Wealth: increases until retirement, quickly disappears afterwards

Consumption: stays smooth across the entire life span



Main Take Away

If you hope to smooth your consumption and maximize the life-time utility from consumption:

- Since stock outperform bond in the long run, it is almost always better to invest heavily ($>80\%$ of assets) in the stock market
 - Especially true when one is young. Should invest 100% in stock below age 40
- When one is approaching retirement and will soon lose labor income, he/she should start allocating more to the bond
 - A common advise in the advisory industry: $(100 - \text{Age})\%$ in stock
 - Will start withdrawing from the retirement savings, which means he/she is particularly worried that he/she withdraw at the worst possible time



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Insurance – What and Why

- While you are saving up for the future, do not forget to hedge extreme risks in life!
- Life is full of risks: sickness, car accidents, fire... Insurance is a great tool to reduce risks and make your life smoother!
- What is insurance: Insurance is a contract, represented by a policy, in which an individual or entity receives financial protection or reimbursement against losses from an insurance company.
- How does it work: individuals pay small premiums and get compensated when disasters happen.



Insurance – An example

- A golfer earns \$1 million if healthy. However, there is a 10% probability that he gets injured and earns \$0. His utility function is $U=W^{0.5}$. An insurance firm offers to pay him \$1 million if he gets injured. But he needs to pay a premium of \$150,000.
- Will he accept the offer?

• **Solution:**

	Golfer's payoff		
	Healthy (90%)	Injured (10%)	Expected payoff
W/O insurance	\$1,000,000	0	900,000
W/ insurance	1,000,000- 150,000 =850,000	1,000,000- 150,000 =850,000	850,000

- His expected utility w/o insurance = $90\% * \sqrt{1,000,000} + 10\% * 0 = 900$
- His utility w/ insurance = $\sqrt{850,000} = 922.0 > 900$
- Although the insurance generate negative expected payoff, he still wants to buy it. **People are risk averse!**



Insurance - Choices

- **Health insurances:** reimburse the insured for expenses incurred from illness or injury
- **Critical illness insurance:** pays a lump sum amount if an insured person is diagnosed with a wide range of major illness
- **Life insurances:** pay a designated beneficiary a sum of money upon the death of an insured person
- **Personal Accident Insurance:** reimburse the insured for expenses from accidents
- **Liability insurance:** reimburse the insured for losses from liabilities such as lawsuits and similar claims
- **Property insurance:** reimburse the insured in the event of accidental physical loss, destruction or damage to property insured
- ...



What Insurance should I Buy?

- **What risks are you most concerned about?**
- **How much is the premium?**
- **Medical insurance:** everybody needs it!
- **Life insurance:** gives your family income in case you die
 - One may NOT want/need life insurance if:
 - One is single and do not have any dependents
 - One is married, a double income couple, with no children
 - Consider insurance if:
 - One has children: one should have coverage for raising and educating one's children until they are financially self-sufficient.
 - One is married, a single income couple, with no children: one should have insurance to allow his/her surviving spouse to maintain her/his lifestyle until she/he is self-sufficient
 - One owns a business: a life insurance can allow ones family to pay off any debt if one dies.
 - ...



Be Cautious about Insurances with Savings Functions

- Some insurance products serve as both investments and insurance.
- You should be very cautious of these insurance products as they usually provide very low returns.
- **Let insurance be insurance, and let investment be investment.**



Example #1

- E.g. an education insurance by Ping'an:
全能英才保险产品计划(平安)

40095-95511
平安将为您提供专业投保咨询服务

保费试算

您每年需支付保费: ¥12345.00元

您的孩子将可获得以下保障:

保障权益	保障详情	可获保障
成长关爱金	第五个保单周年日开始	900元/年
教育关爱金	15周岁至24周岁	0.9万元/年
学业有成金	18、21、24周岁	0.9万元/年
成家立业金	30周岁	3万元

*以上金额仅供参考, 具体请以实际保险条款为准

重测保费

Return is only 2%!

Year	Cash outflows	Cash inflows 1	Cash inflows 2	Cash inflow 3	Net cash flows
1	-12345				-12345
2	-12345				-12345
3	-12345				-12345
4	-12345				-12345
5	-12345				-12345
6	-12345	900			-11445
7	-12345	900			-11445
8	-12345	900			-11445
9	-12345	900			-11445
10	-12345	900			-11445
11		900			900
12		900			900
13		900			900
14		900			900
15		900	9000		9900
16		900	9000		9900
17		900	9000		9900
18		900	9000	9000	18900
19		900	9000		9900
20		900	9000		9900
21		900	9000	9000	18900
22		900	9000		9900
23		900	9000		9900
24		900	9000	9000	18900
25		900			900
26		900			900
27		900			900
28		900			900
29		900			900
30		900		30000	30900



Example #2

- E.g. an annuity plan from CPIC (太平洋保险公司鑫满意保险产品计划) [\[link\]](#)

投保示例



王先生, 30周岁,
大型跨国企业部门经理

王先生, 30周岁, 大型跨国企业部门经理, 为自己投保了20份“鑫满意年金保险(分红型)”, 选择保险期间至70周岁, 年交保费200,000元, 5年交清, 基本保险金额3,500元; 同时投保了“传世赢家终身寿险(万能型)”, 保险期间为终身, 趸交保费100元, 该笔趸交保费在扣除初始费用后进入“传世赢家”保单账户(简称“万能账户”)参与投资结算。

“鑫满意”的祝贺金、祝福金、祝寿金和红利也均进入万能账户参与投资结算。

王先生的保单利益如下:

祝贺金: 王先生可分别于第5个及第6个合同生效日对应日领取200,000元的祝贺金。

祝福金: 自第7个合同生效日对应日起及以后的每个合同生效日对应日, 王先生每年可领取3,500元, 直至69周岁的合同生效日对应日。

祝寿金: 鑫满意保险期间届满时, 王先生可领取一笔金额等于鑫满意已支付的保险费总额的祝寿金1,000,000元。

红利: 鑫满意保单有效期内, 可参与公司红利分配。红利分配是不确定的。

上述祝贺金、祝福金、祝寿金、红利以及趸交保费在扣除相应初始费用后进入万能账户参与投资结算, 持续增值。账户价值预测数据举例如下:

Age	Cash outflows	Cash inflows 1	Cash inflows 2	Cash inflow 3	Net cash flows
30	-200000				-200000
31	-200000				-200000
32	-200000				-200000
33	-200000				-200000
34	-200000	200,000			0
35		200,000			200000
36-69			3500		3500
70			1000000		1000000

Return is only 2%!



Summary

- Utility function: compare different consumptions
 - Expected utility: $U(\tilde{C}) = E[u(\tilde{C})]$
 - Interpretation: the utility of an uncertain consumption is equal to the average utility across all possible states
 - Risk aversion: naturally arises due to diminishing margin return
- Portfolio choice
 - Life cycle choice: hold mostly stock when young, add bonds when old
- Other applications
 - Insurance: can still increase utility even with negative expected payoff
 - Life insurance: need to consider your dependents
 - Beware of insurances of saving features!



Next Time—Behavioral Biases

- Efficient Market Hypotheses (EMH)
- Behavioral Biases
 - Non-Participation in Stock Market
 - Home-bias
 - Disposition Effect
 - Prospect Theory
 - Excessive trading
 - Overconfidence
 - Ignorance of transaction costs
 - Attention Buying

