香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

Introduction to Computer Science: Programming Methodology

Lecture 10
Linked List

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Why we need another list data type

- Python's list class is highly optimized, and often a great choice for storage

- However, many programming languages do not support this kind of optimized list data type

指涉结构

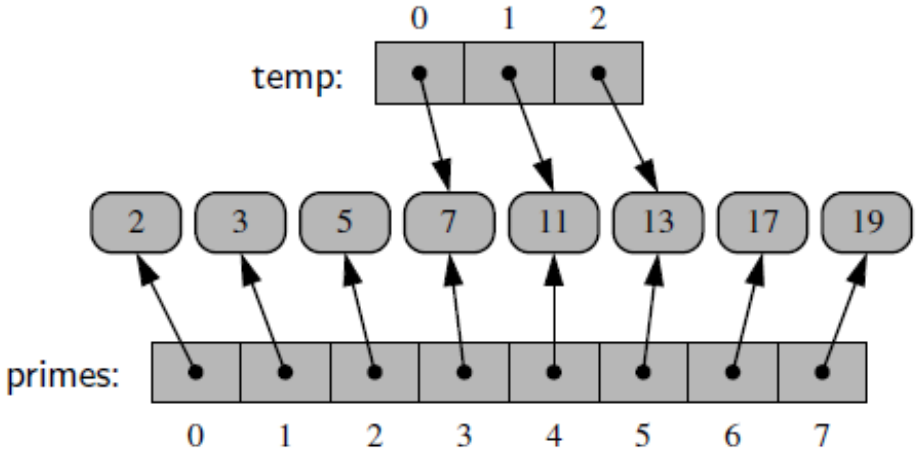
List in Python is a referential structure

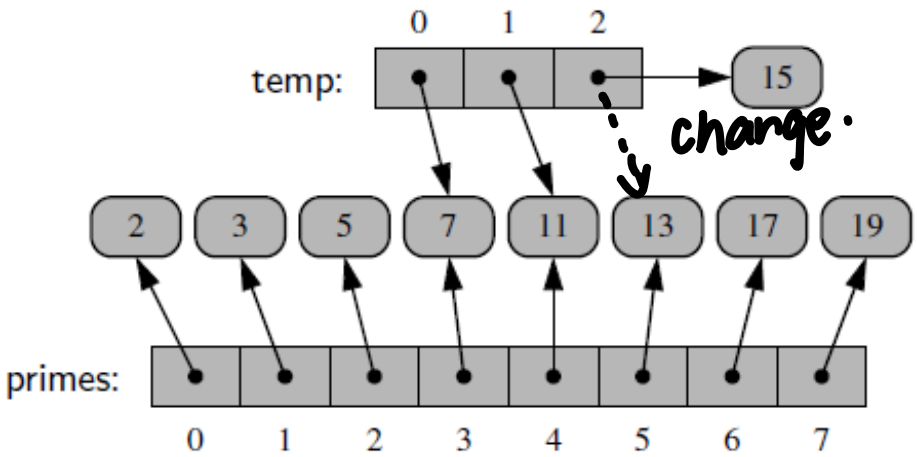
```
>>> a=[1,2,3,4,5]
>>> for i in range(0,5):
        print(id(a[i]))

1546964720
1546964752
1546964784
1546964816
1546964848
```

```
>>> a.insert(2,10)
>>> a
[1, 2, 10, 3, 4, 5]
>>> for i in range(0,6):
        print(id(a[i]))

1546964720
1546964752
1546965008
1546964784
1546964816
1546964848
```

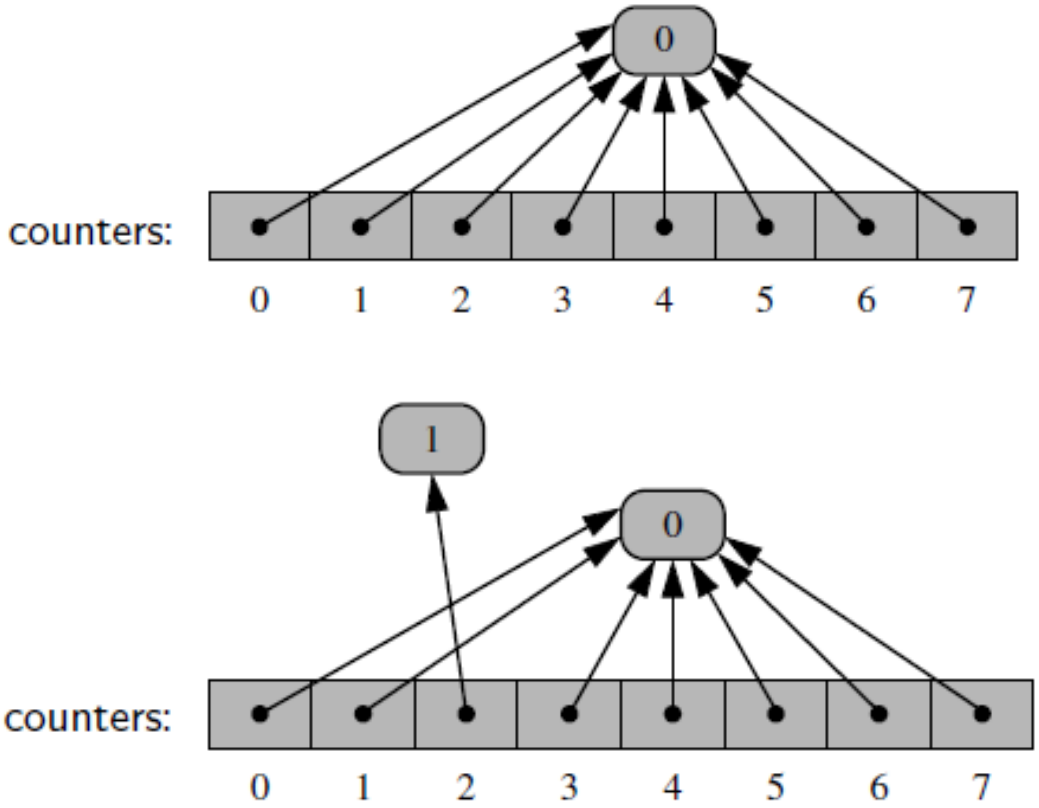
others' id won't change.

List in Python is a referential structure





List in Python is a referential structure



紧凑型数组

Compact array

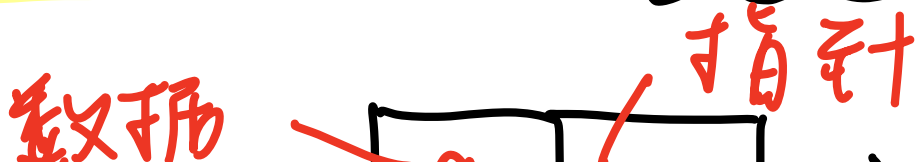
将数据紧密存到，减少存储空间，提高访问速度

- A collection of numbers are usually stored as a compact array in languages such as C/C++ and Jaya

- A compact array is storing the bits that represent the primary data (not reference)

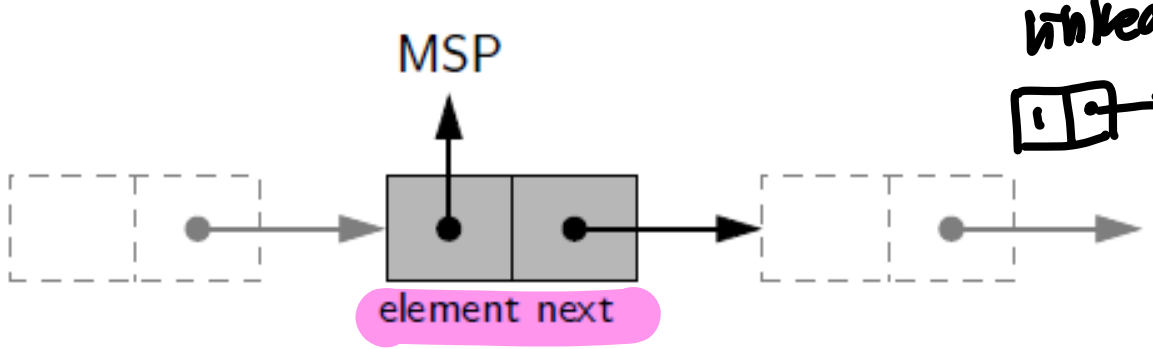
总体内存使用量低

- The overall memory usage will be much lower for a compact structure because there is no overhead devoted to the explicit storage of the sequence of memory references (in addition to the primary data)



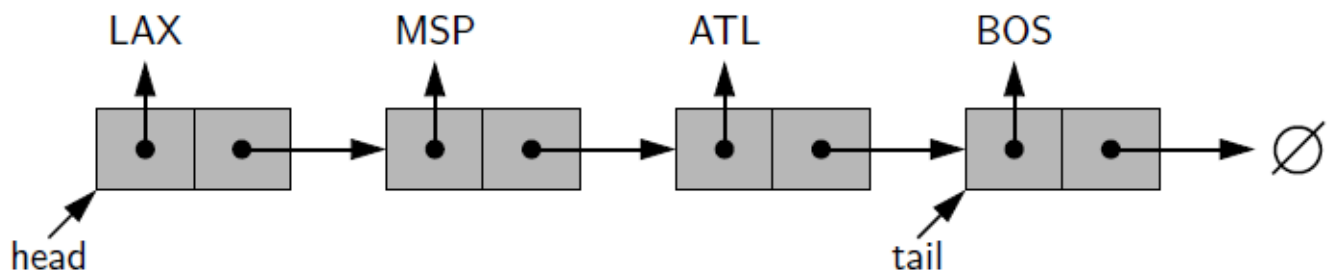
Linked List

- A singly linked list, in its simplest form, is a collection of nodes that collectively form a linear sequence
- Each node stores a reference to an object that is an element of the sequence, as well as a reference to the next node of the list

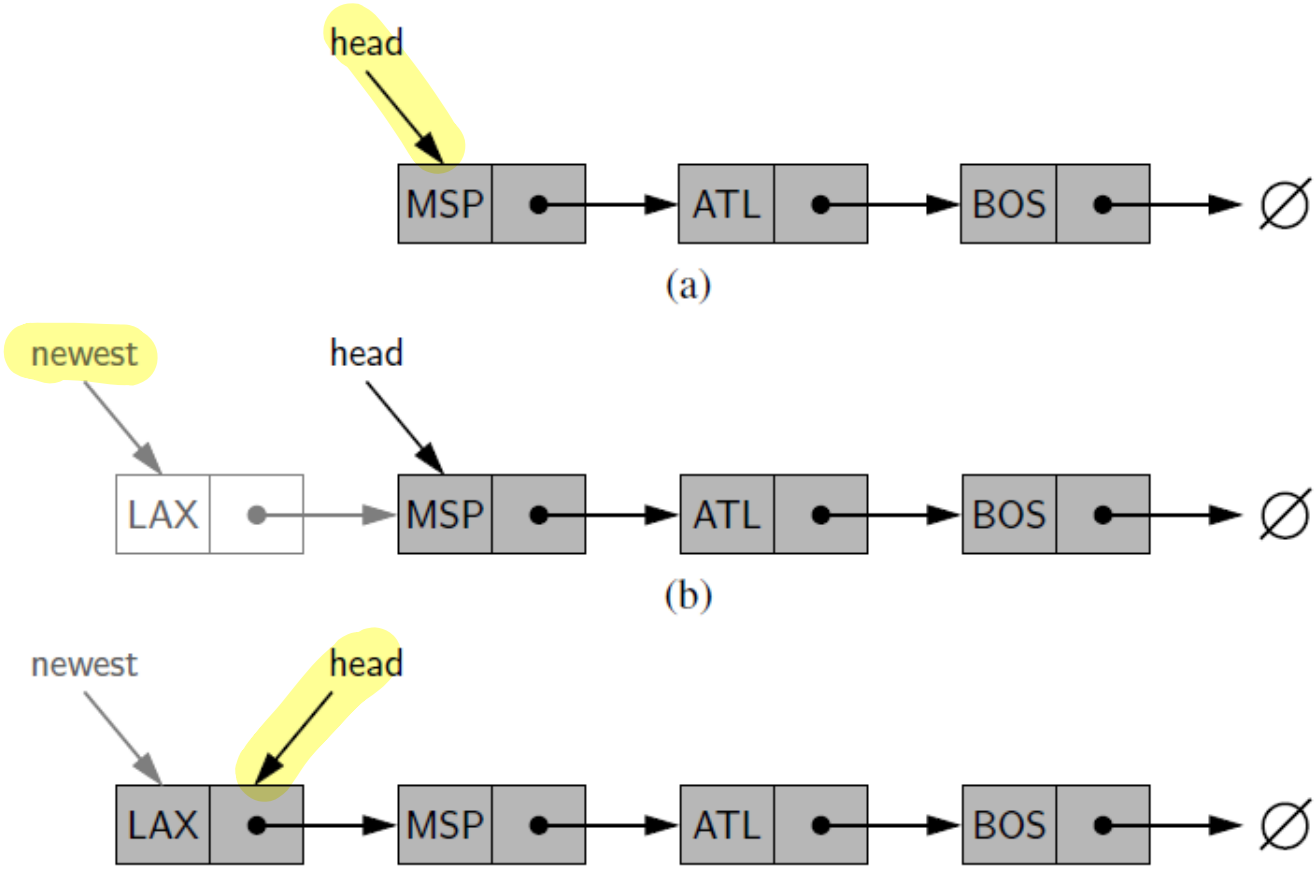


Linked List

- The first and last nodes of a linked list are known as the head and tail of the list, respectively
- By starting at the head, and moving from one node to another by following each node's next reference, we can reach the tail of the list
- We can identify the tail as the node having None as its next reference. This process is commonly known as traversing the linked list.
- Because the next reference of a node can be viewed as a link or pointer to another node, the process of traversing a list is also known as link hopping or pointer hopping



Inserting an Element at the Head of a Singly Linked List



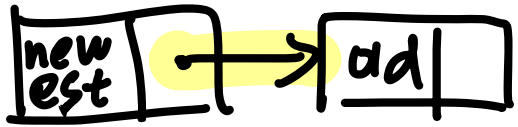
(Pseudo code)for inserting a node at the head

在头部插入节点

(node: 节点)

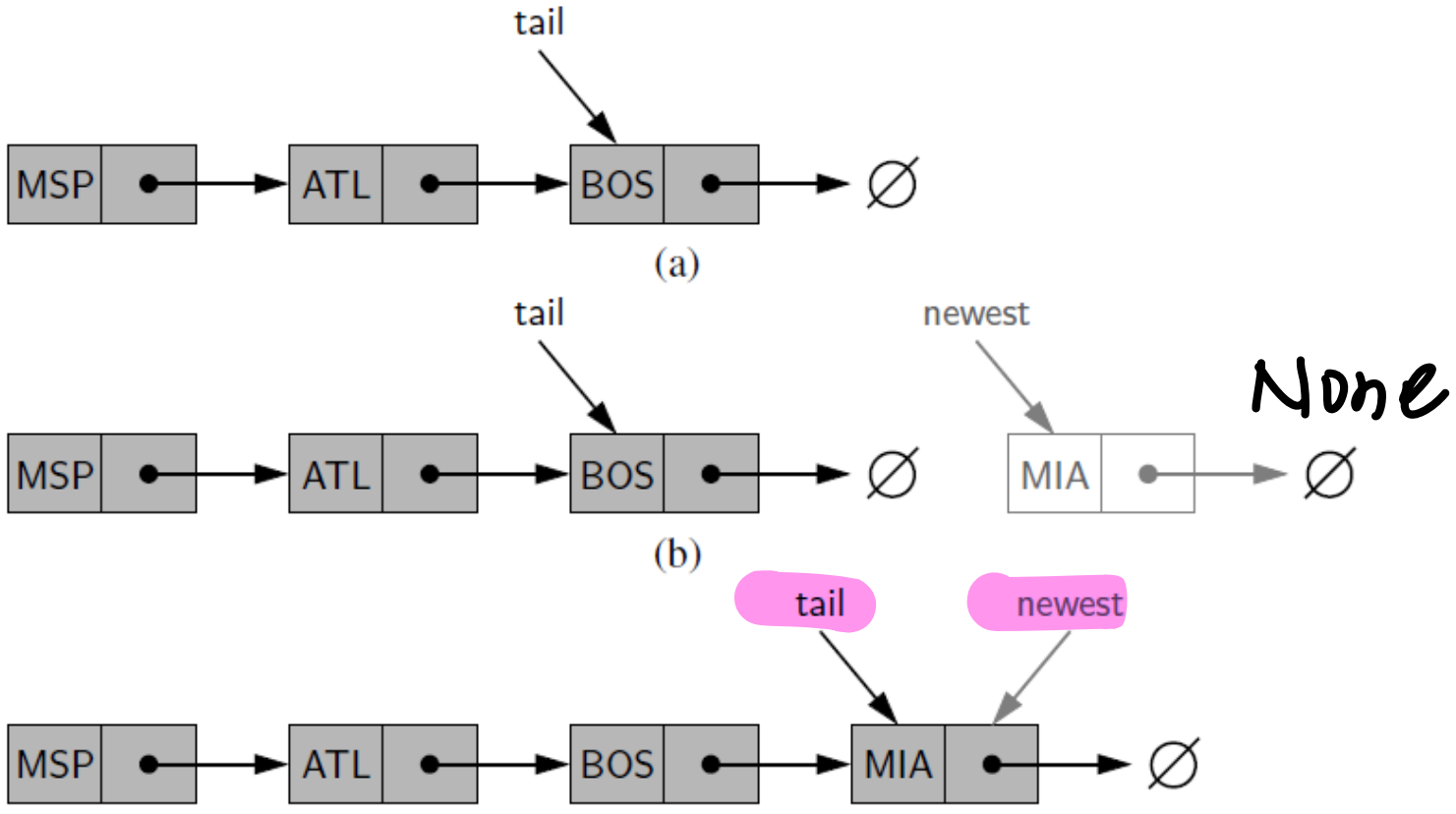
Algorithm add_first(L, e):

```
newest = Node(e)      {create new node instance storing reference to element e}
newest.next = L.head  {set new node's next to reference the old head node}
L.head = newest       {set variable head to reference the new node}
L.size = L.size + 1   {increment the node count}
```



思路: ① creat Node(e)
② 判断 newest 方向 (前 + 后)
③ L.head/L.tail = newest
④ L.size + 1

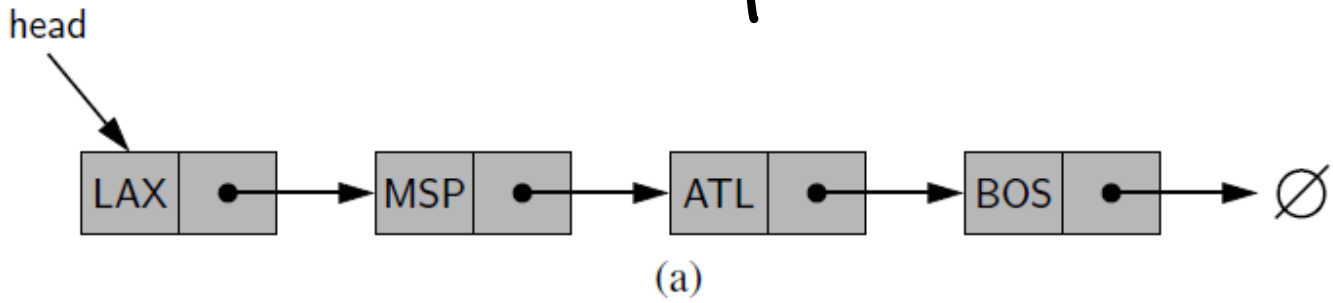
Inserting an Element at the Tail of a Singly Linked List



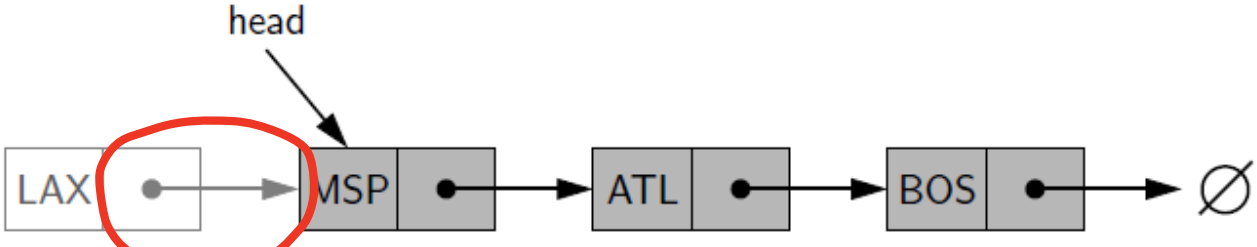
Pseudo code for inserting at the tail

```
Algorithm add_last(L, e):
    newest = Node(e)      {create new node instance storing reference to element e}
    newest.next = None    {set new node's next to reference the None object}
    L.tail.next = newest  {make old tail node point to new node}
    L.tail = newest       {set variable tail to reference the new node}
    L.size = L.size + 1   {increment the node count}
```

Removing an Element from the head of a Singly Linked List 单链表中移除元素

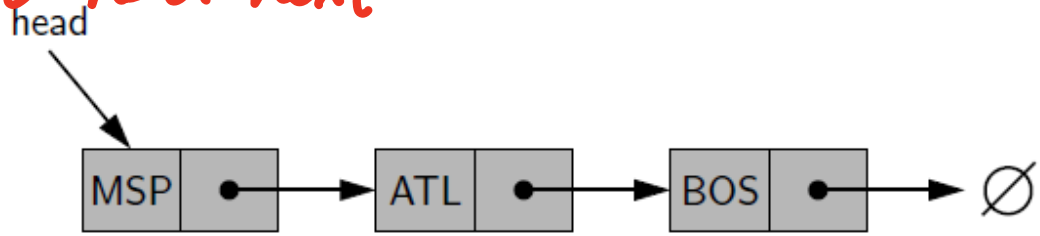


(a)



(b)

v = head.next



Pseudo code for removing a node from the head

移除，先判断是否为空.

```
Algorithm remove_first(L):
    if L.head is None then
        (Indicate an error:) the list is empty.
    L.head = L.head.next
    L.size = L.size − 1
```

check if empty.

下一位成为 L.head

{make head point to next node (or None)}

{decrement the node count}

reference to 1st node

把指向赋给 L.head.

first node 仍然在，但无指向，可下一步.

与上节课对比：

(push, top, pop)

Practice: Implement stack with a singly linked list

list 与 linked list

2 data fields

```
class Node:
    def __init__(self, element, pointer):
        self.element = element
        self.pointer = pointer
```

pointer

element

(last in, first out)

```
class LinkedStack:

    def __init__(self):
        self.head = None
        self.size = 0

    def __len__(self):
        return self.size

    def is_empty(self):
        return self.size == 0

☆ always has a head pointer!!
    def push(self, e):
        self.head = Node(e, self.head)
        self.size += 1
```

element, pointer

node1 = Node(1, None)

1 → None

node2 = Node(2, node1)

旧的 self.head

2 → 1 → None

head

head = node2

最初：head → ∅ size = 0

(⇒ head, node2 id 相同)

```
def top(self):
    if self.is_empty():
        print('Stack is empty.')
    else:
        return self.head.element
```

a reference to the node in the list

☆ return the last in element

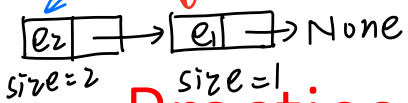
```
def pop(self):
    if self.is_empty():
        print('Stack is empty.')
    else:
        answer = self.head.element
        self.head = self.head.pointer
        self.size -= 1
        return answer
```

remove → update the pointer

print(head.element) = 2

head = head.pointer

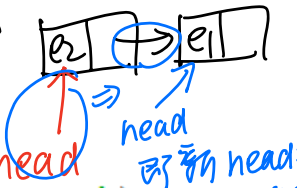
print(head.element) = 1



{ head pointer, node2 pointer

head 指向改变了

Practice: Implement queue with a singly linked list



即新 head 为 e1，即 e2 无access，相当于remove e2.

```
class LinkedQueue:

    def __init__(self):
        self.head = None
        self.tail = None
        self.size = 0

    def __len__(self):
        return self.size

    def is_empty(self):
        return self.size == 0

    def first(self):
        if self.is_empty():
            print('Queue is empty.')
        else:
            return self.head.element
```

) empty queue

比 stack 多出的操作：关注 tail.

优点：不用考虑 maximum.

返回并移除

```
    def dequeue(self):
        if self.is_empty():
            print('Queue is empty.')
        else:
            answer = self.head.element
            self.head = self.head.pointer
            self.size -= 1
            if self.is_empty():
                self.tail = None
            return answer
```

remove

update self.head 的 value，改在 self.head 下面代表下一个元素.

移除后，若列表为空，self.tail = None.

尾部添加.

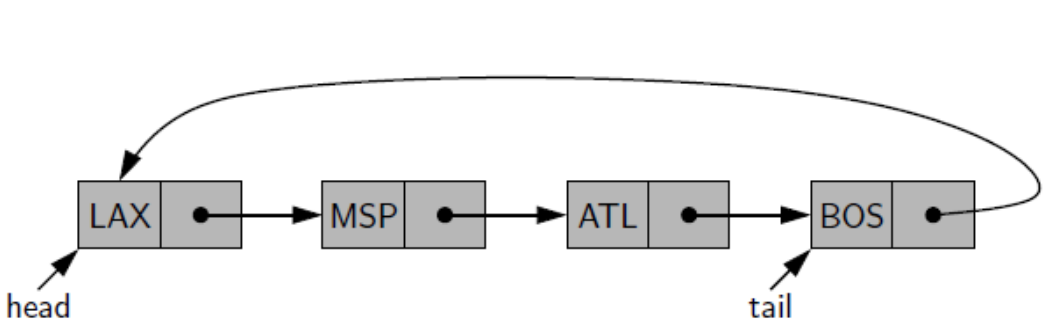
```
    def enqueue(self, e):
        newest = Node(e, None)
        if self.is_empty():
            self.head = newest
        else:
            self.tail.pointer = newest
        self.tail = newest
        self.size += 1
```

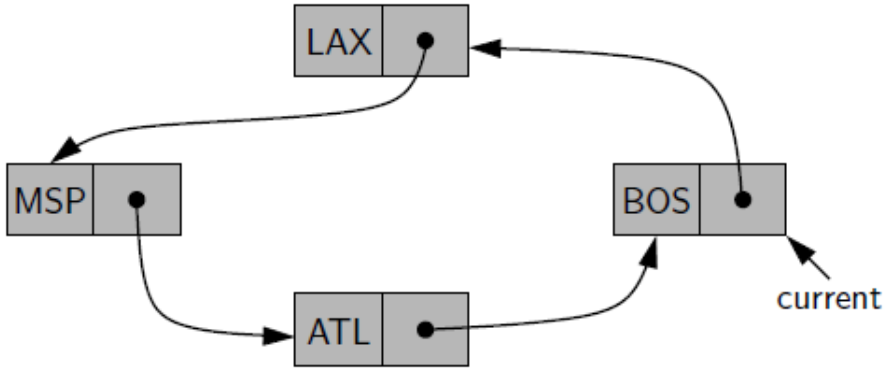
若新加入一个节点，要创建 newest = Node(e, None).

若原 queue 为空，指向 add.

Circularly Linked List

- The tail of a linked list can use its next reference to point back to the head of the list
- Such a structure is usually called a circularly linked list





循环调度.

Example: Round-robin scheduler

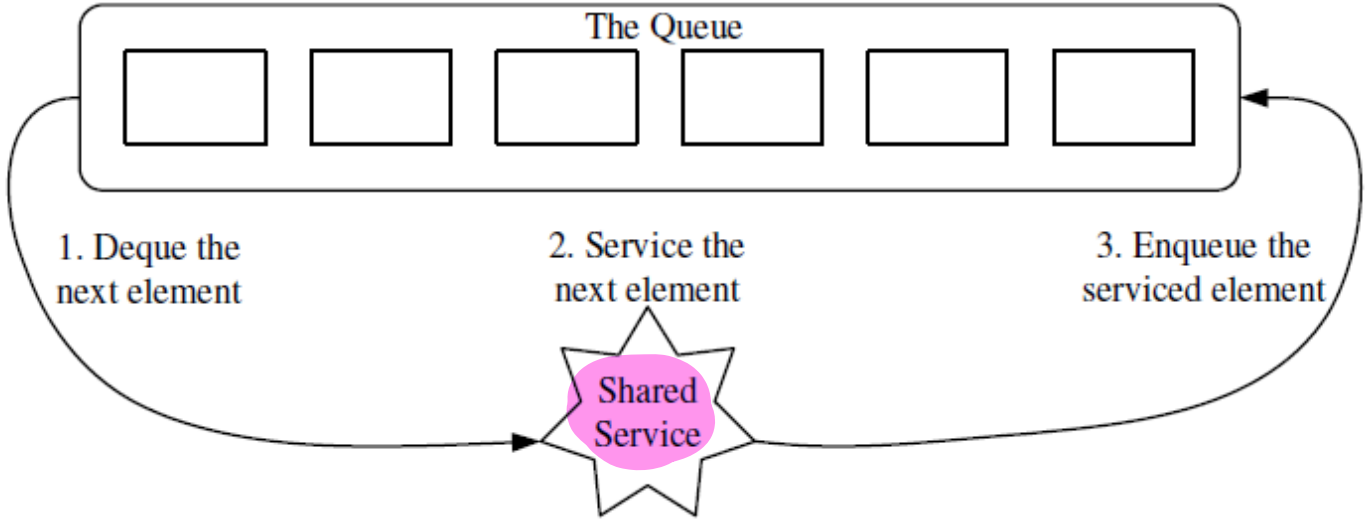
- A round-robin scheduler iterates through a collection of elements in a circular fashion and "serves" each element by performing a given action on it

- Such a scheduler is used, for example, to fairly allocate a resource that must be shared by a collection of clients

- For instance, round-robin scheduling is often used to allocate slices of CPU time to various applications running concurrently on a computer

Implementing round-robin scheduler using standard queue

- A round-robin scheduler could be implemented with the standard queue, by repeatedly performing the following steps on queue Q:
  1) e = Q.dequeue()
  2) Service element e
  3) Q.enqueue(e)



Implement a Queue with a Circularly Linked List

执行复杂

删减3

self.__head, 均用 tail 的 pointer 来指代

```
class Node:
    def __init__(self, element, pointer):
        self.element = element
        self.pointer = pointer
```

```
class CQueue:

    def __init__(self):
        self.__tail = None
        self.__size = 0

    def __len__(self):
        return self.__size

    def is_empty(self):
        return self.__size == 0

    def first(self):

        if self.is_empty():
            print('Queue is empty.')
        else:
            head = self.__tail.pointer
            return head.element
```

⇒ 无用定义 self.__head, 因为 self.__tail.pointer

⇒ 指回自己.

head

```
def dequeue(self):
    if self.is_empty():
        print('Queue is empty.')
    else:
        oldhead = self.__tail.pointer
        if self.__size == 1:
            self.__tail = None
        else:
            self.__tail.pointer = oldhead.pointer
        self.__size -= 1
        return oldhead.element
```

after removing

(only 1 node, pointer points to itself)

(*)

↓

tail pointer将指向原来第二个元素.

```
def enqueue(self,e):
    newest = Node(e,None)
    if self.is_empty():
        newest.pointer = newest
    else:
        newest.pointer = self.__tail.pointer
        self.__tail.pointer = newest
    self.__tail = newest
    self.__size += 1
```

对比单链表：

self.head = newest ←

(only 1 element, points back to itself!!)

理解 ☆

↓

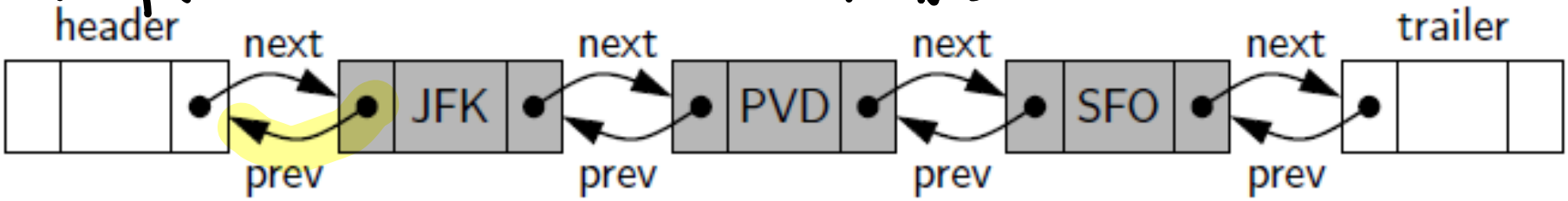
指回第一个元素的pointer.

Doubly linked list

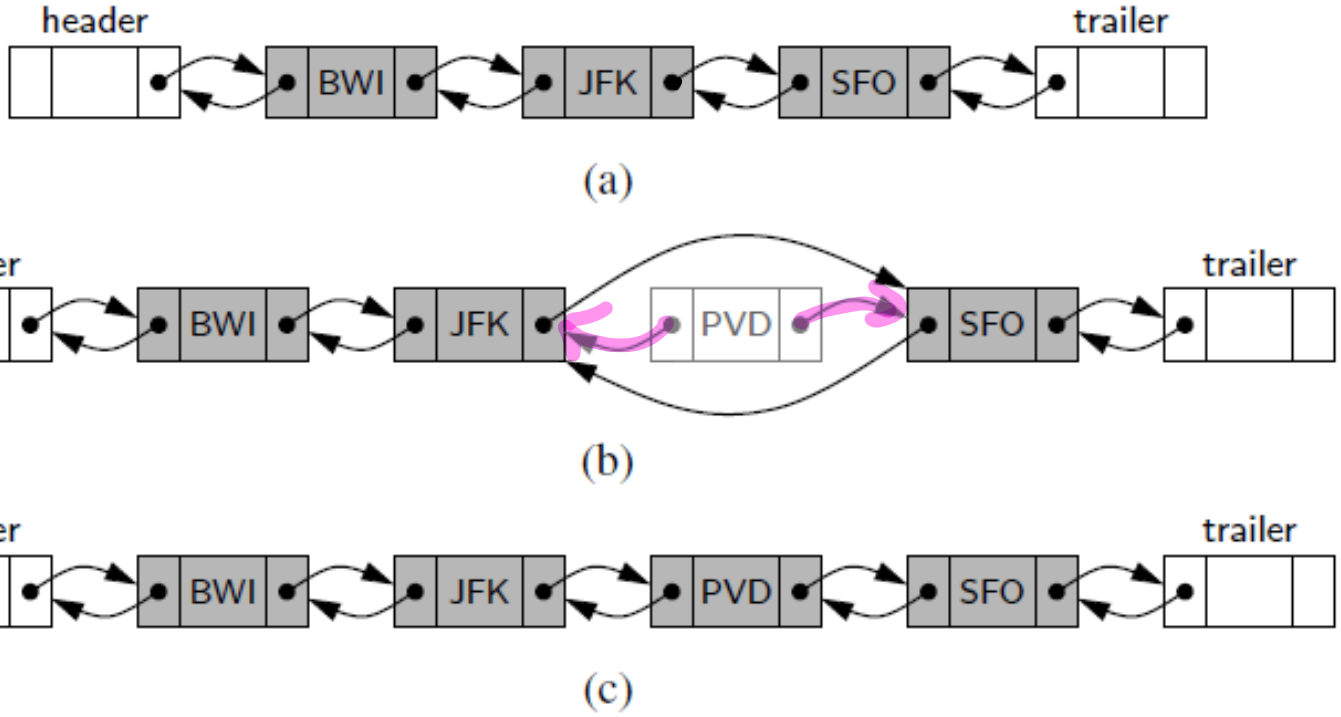
- For a singly linked list, we can efficiently insert a node at either end of a singly linked list, and can delete a node at the head of a list
- But we cannot efficiently delete a node at the tail of the list
- We can define a linked list in which each node keeps an explicit reference to the node before it and a reference to the node after it
- This kind of data structure is called doubly linked list

Head and tail sentinels

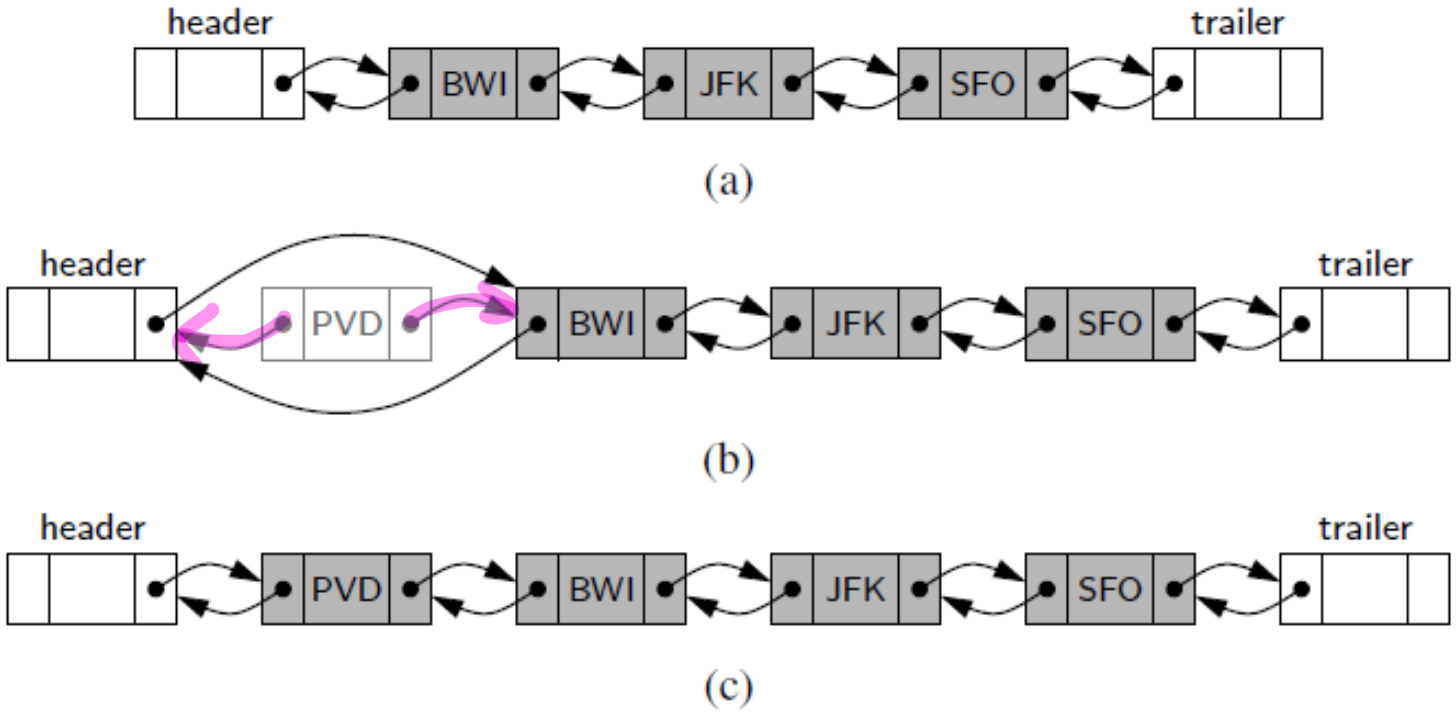
- In order to avoid some special cases when operating near the boundaries of a doubly linked list, it helps to add special nodes at both ends of the list: a header node at the beginning of the list, and a trailer node at the end of the list
- These "dummy" nodes are known as sentinels (or guards), and they do not store elements of the primary sequence



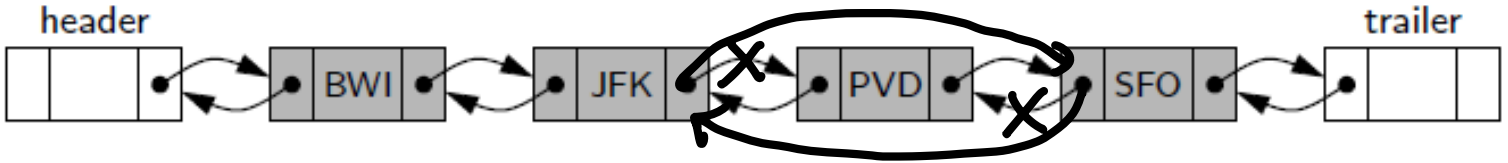
Inserting in the middle of a doubly linked list



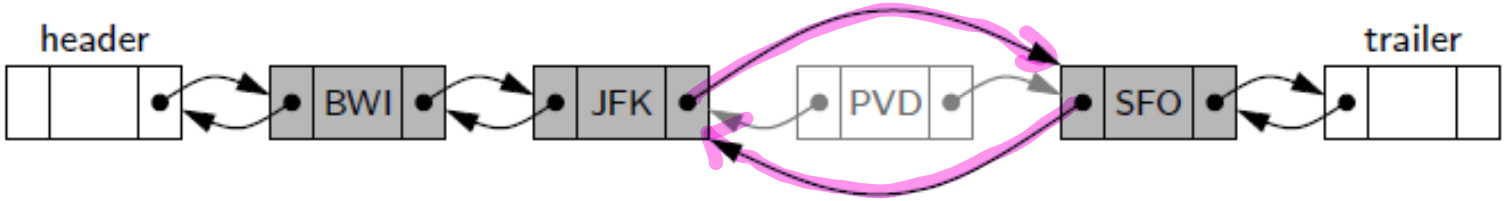
Inserting at the head of the doubly linked list



Deleting from the doubly linked list



(a)



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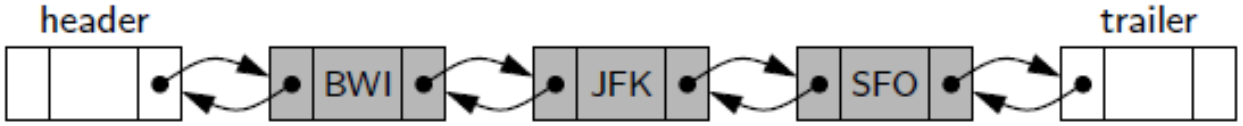
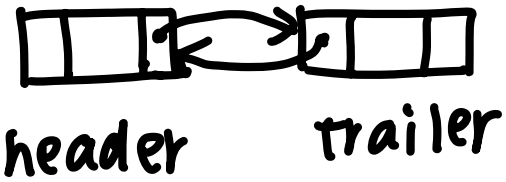


(c)

Code for the doubly linked list

```
class Node:
    def __init__(self, element, prev, nxt):
        self.element = element
        self.prev = prev
        self.nxt = nxt

class DLList:

    def __init__(self):
        self.header = Node(None, None, None)
        self.trailer = Node(None, None, None)
        self.header.nxt = self.trailer
        self.trailer.prev = self.header
        self.size = 0

    def __len__(self):
        return self.size

    def is_empty(self):
        return self.size == 0
```




```
def insert_between(self, e, predecessor, successor):
    newest = Node(e, predecessor, successor)
    predecessor.nxt = newest
    successor.prev = newest
    self.size+=1
    return newest
```

⇒理解：前一项的下一项为newest，后项的前一项为newest

```
def delete_node(self, node):
    predecessor = node.prev
    successor = node.nxt
    predecessor.nxt = successor
    successor.prev = predecessor
    self.size -=1
    element = node.element
    node.prev = node.nxt = node.element = None
    return element
```

Node(None, None, None)

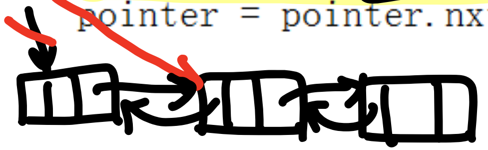
* header, trailer为 None

```
def iterate(self):
    pointer = self.header.nxt
    print('The elements in the list:')
    while pointer != self.trailer:
        print(pointer.element)
        pointer = pointer.nxt
```

打印list所有elements (包含内容的) (iterate, 迭代)

→末指向最后元素

head



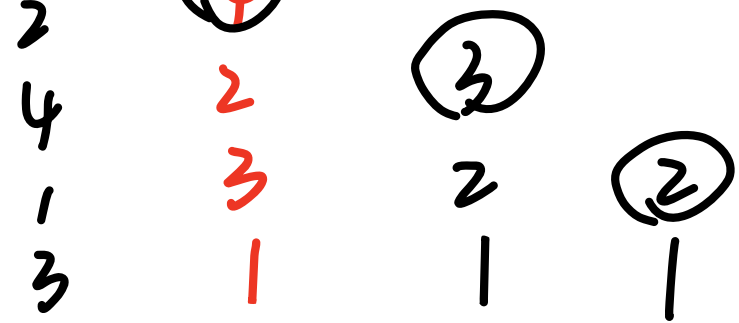
```
def main():
    d=DLList()
    d.__len__()

    newNode = d.insert_between(10, d.header, d.trailer)
    newNode = d.insert_between(20, newNode, d.trailer)
    newNode = d.insert_between(30, newNode, d.trailer)
    d.iterate()
    d.delete_node(d.header.nxt.nxt)
    d.iterate()
```

△理解 删除第一个元素

updating the value for newNode.

[2,4,1,3]

e.g.



Bubble sort

冒泡排序：重复遍历列表，

- Bubble sort is a simple sorting algorithm

比较相邻元素并交换位置来实现排序，每一轮把循环都能确定将来

- Its general procedure is:

排序部分中的最大元素移动到数组的末尾.

1) Iterate over a list of numbers, compare every element i with the following element i+1, and swap them if i is larger
2) Iterate over the list again and repeat the procedure in step 1, but ignore the last element in the list
3) Continuously iterate over the list, but each time ignore one more element at the tail of the list, until there is only one element left

(最后的元素会是最大的数)

Practice: Bubble sort over a standard list

```
def bubble(bubbleList):
    listLength = len(bubbleList)
    while listLength > 0:
        for i in range(listLength - 1):
            if bubbleList[i] > bubbleList[i+1]:
                buf = bubbleList[i]
                bubbleList[i] = bubbleList[i+1]
                bubbleList[i+1] = buf
        listLength -= 1
    return bubbleList

def main():
    bubbleList = [3, 4, 1, 2, 5, 8, 0, 100, 17]
    print(bubble(bubbleList))
```

Practice: Bubble sort over a singly linked list

对单链表进行冒泡排序

- time complexity of bubble sort: $O(n^2)$

n为排序的元素数量.

在最坏情况下，要进行(n-1)轮(比较与交换)，

每一轮要遍历整个数列，TC为O(n).

所以总的TC为$O(n^2)$.

(平均)，最好O(n).

or: ① (n-1)次

② (n-2)次

$\frac{(n-1+1)(n-1)}{2} = \frac{n^2-n}{2} \rightarrow O(n^2)$

③ (n-3)次

Solution:

1次，共(n-1)轮

```
from LinkedQueue import LinkedQueue

def LinkedBubble(q):
    listLength = q.size

    while listLength > 0:
        index = 0
        pointer = q.head
        while index < listLength-1:
            if pointer.element > pointer.pointer.element:
                buf = pointer.element
                pointer.element = pointer.pointer.element
                pointer.pointer.element = buf
            index += 1
            pointer = pointer.pointer
        listLength -= 1
    return q
```

相当于 b[i] > b[i+1]

swap.

让pointer指向下一个node.

```
def outputQ(q):
    pointer = q.head

    while pointer:
        print(pointer.element)
        pointer = pointer.pointer

def main():
    oldList = [9, 8, 6, 10, 45, 67, 21, 1]
    q = LinkedQueue()

    for i in oldList:
        q.enqueue(i)

    print('Before the sorting...')
    outputQ(q)

    q = LinkedBubble(q)
    print()
    print('After the sorting...')
    outputQ(q)
```

update

若为while False. 跳出loop

若指向None.

加入queue

为什么要创造index:

Quick sort

- Quick sort is a widely used algorithm, which is more efficient than bubble sort

- The main procedure of quick sort algorithm is:

1) Pick an element, called a pivot, from the array
2) Partitioning: reorder the array so that all elements with values less than the pivot come before the pivot, while all elements with values greater than the pivot come after it (equal values can go either way). After this partitioning, the pivot is in its final position. This is called the partition operation
3) Recursively apply the above steps to the sub-array of elements with smaller values and separately to the sub-array of elements with greater values

in a recurise way.

time for comparison reflects time complexity

每次，size减半：
有 log n 个 recursive call，(layers)
⇒ n·log n
(平均)
⇒ worst case，已排完，
1+..+(n-1)
O(n²)

pick as a pint

Practice: Quick sort over a standard list

list　two index：

```
def quickSort(L, low, high):
    i = low
    j = high
    if i >= j:
        return L
    key = L[i]
    while i < j:
        while i < j and L[j] >= key:
            j = j-1
        L[i] = L[j]
        while i < j and L[i] <= key:
            i = i+1
        L[j] = L[i]
    L[i] = key
    quickSort(L, low, i-1)
    quickSort(L, j+1, high)
    return L
```

完整的一个循环 组合相对互补 重组了一次

pick the starting index as a pivot

→ 即只有一个元素，完成 sub array 的排序。

look at the righthand side numbers

update the j

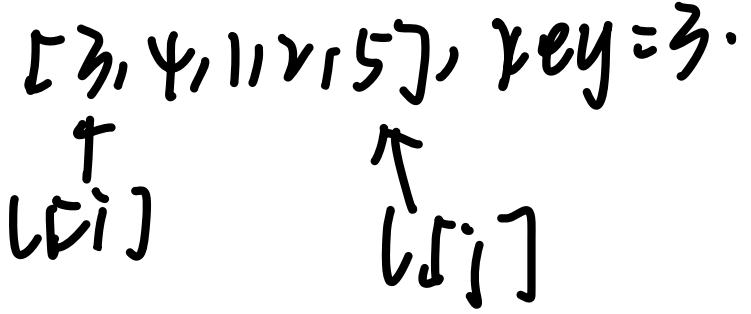
lefthand side

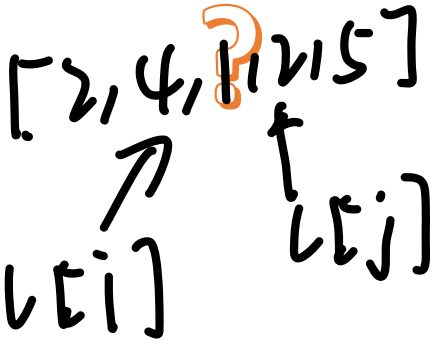
直到 i=j，L[i]=key

mylist = [3,4,1,2,5]

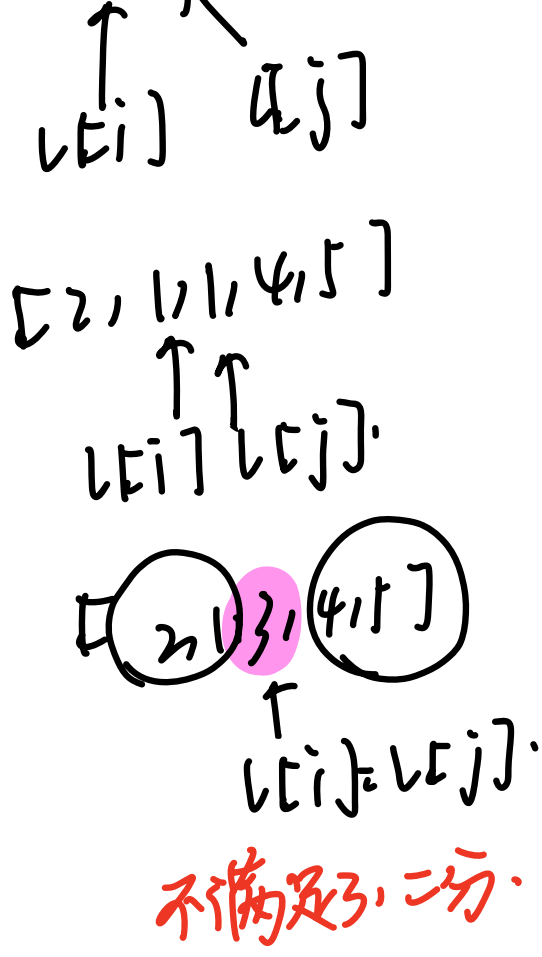
```
rankedlist = quickSort (mylist, 0, le(mylist)-1)
print(rankedlist)
```

Practice: Quick sort over a singly linked list





★每进行完一个 while，进入下一个 while，一定是 j 满足的，可进入的（刚刚换完）.



不满足3,二分.

△单链表快速排序.

i j

3 5 4 1 2 6 8 7 9

(i) ①找pivot ⇒左边第一个 (j) ⇒这种不适用于单向链表.

② i j 位置 ⇒ pivot 后一个

两个指针

③ 移动两个指针: { 1)让i左边数<pivot

2)让[i,j]> pivot

不把pivot

//

的数

(如果 num[j] > pivot,
j++

如果 num[j] < pivot,
① 交换 i, j 的值
② i++

提到之外

此时 i 前面的
均小于 pivot,
i 后面的均
大于 pivot.

j 从头到尾遍历一次为止.
但 j 触碰最后一个节点, 终止.

④ 交换 pivot 和 num[i-1]

⑤ 递归.