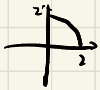
Assignment 10 杜奕璇 122090095

15.4

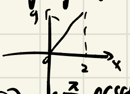
(3) $\frac{\pi}{4} \le \theta \le \frac{3}{4}\pi \quad y=1 \Rightarrow r = \csc\theta \quad \therefore 0 \le r \le \csc\theta$

(6) $-\frac{\pi}{3} \le \theta \le \frac{\pi}{3} \quad \sec\theta \le r \le 2$

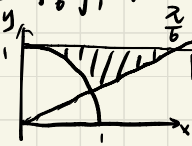
(11)  $\int_0^2\int_0^{\sqrt{4-y^2}}(x^2+y^2)\,dx\,dy = \int_0^{\frac{\pi}{2}}\int_0^2 r^2\cdot r\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\left[\frac{r^4}{4}\right]_0^2 d\theta = \int_0^{\frac{\pi}{2}} 4\,d\theta = (4\theta)_0^{\frac{\pi}{2}} = 2\pi$

$x = r\cos\theta,\ y = r\sin\theta$

(14) $\int_0^2\int_0^x y\,dy\,dx = \int_0^{\frac{\pi}{4}}\int_0^{2\sec\theta} r^2\sin\theta\,dr\,d\theta = \int_0^{\frac{\pi}{4}}\left[\frac{r^3\sin\theta}{3}\right]_0^{2\sec\theta} d\theta = \int_0^{\frac{\pi}{4}} \frac{8}{3}\sec^2\theta\tan\theta\,d\theta = \frac{8}{3}\int_1^{\sqrt{2}}\frac{u^2}{2}du = \frac{4}{3}$

 $y = r\sin\theta$

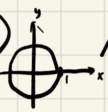
(24) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_1^{\csc\theta} r^2\cos\theta\,dr\,d\theta$



$\frac{\pi}{6} \le \theta \le \frac{\pi}{2} \quad 0 \le \cos\theta \le \frac{\sqrt{3}}{2} \quad \frac{1}{2} \le \sin\theta \le 1$

$1 \le r \le \csc\theta$

$\int_{\frac{1}{2}}^1\int_{\sqrt{1-y^2}}^{\sqrt{3}y} x\,dx\,dy$, $\int_0^{\frac{\sqrt{3}}{2}}\int_{\sqrt{1-x^2}}^1 x\,dy\,dx + \int_{\frac{\sqrt{3}}{2}}^{\sqrt{3}}\int_{\frac{x}{\sqrt{3}}}^1 x\,dy\,dx$

(36)  $\text{Average} = \frac{1}{\pi}\iint_R [(1-x)^2+y^2]\,dy\,dx = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 [(1-r\cos\theta)^2 + r^2\sin^2\theta]\,r\,dr\,d\theta =$

$= \frac{1}{\pi}\int_0^{2\pi}\int_0^1 (r^3 - 2r^2\cos\theta + r)\,dr\,d\theta = \frac{1}{\pi}\left[\frac{3}{4}\theta - \frac{2\sin\theta}{3}\right]_0^{2\pi} = \frac{3}{2}$

(41) (a) $I^2 = \int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy = \int_0^{\frac{\pi}{2}}\int_0^\infty (e^{-r^2})\,r\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\left[\lim_{b\to\infty}\int_0^b r e^{-r^2}dr\right]d\theta = \int_0^{\frac{\pi}{2}}\left[\frac{e^{-r^2}}{-2}\right]_0^b d\theta$

$x = r\cos\theta,\ y = r\sin\theta \qquad = \int_0^{\frac{\pi}{2}}\frac{1}{2}\,d\theta = \frac{\pi}{4} \qquad \therefore I = \frac{\sqrt{\pi}}{2}$

(b) $\lim_{x\to\infty}\int_0^x \frac{2e^{-t^2}}{\sqrt{\pi}}\,dt = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2}dt = \left(\frac{2}{\sqrt{\pi}}\right)\left(\frac{\sqrt{\pi}}{2}\right) = 1$

15.5

(11) $\int_0^{\frac{\pi}{6}}\int_0^1\int_{-2}^3 y\sin z\,dx\,dy\,dz = \int_0^{\frac{\pi}{6}}\int_0^1 (y\sin z\, x)_{-2}^3\,dy\,dz = \int_0^{\frac{\pi}{6}}\int_0^1 (5y\sin z)\,dy\,dz = \int_0^{\frac{\pi}{6}}\left[\frac{5\sin z\, y^2}{2}\right]_0^1 dz$

$= \int_0^{\frac{\pi}{6}}\frac{5}{2}\sin z\,dz = \left(-\frac{5}{2}\cos z\right)_0^{\frac{\pi}{6}} = -\frac{5}{2}\left(\frac{\sqrt{3}}{2} - 1\right) = \frac{5}{2} - \frac{5\sqrt{3}}{4}$

(16) $\int_0^1\int_0^{1-x^2}\int_3^{4-x^2-y^2} x\,dz\,dy\,dx = \int_0^1\int_0^{1-x^2} x(1-x^2-y^2)\,dy\,dx = \int_0^1\left[x(1-x^2)(1-x^2) - \frac{x(1-x^2)^3}{3}\right]dx = \int_0^1 \frac{1}{2}x(1-x^2)^2dx$

$= -\left[\frac{1}{12}(1-x^2)^3\right]_0^1 = \frac{1}{12}$

$\int_{-1}^1\int_{x^2}^1\int_0^{1-y} dz\,dy\,dx$

(21) a. $\int_{-1}^1\int_0^{1-x^2}\int_{x^2}^{1-z} dy\,dz\,dx$ b. $\int_0^1\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\,dx\,dz$ (c) $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy\,dz$

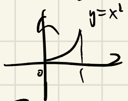
d. $\int_0^1\int_0^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$ e. $\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}}\int_0^{1-y} dz\,dx\,dy$

(27) $\int_0^1\int_0^{2-2x}\int_0^{\frac{3-3x-3y}{2}} dz\,dy\,dx = \int_0^1\int_0^{2-2x}(3-3x-\frac{3}{2}y)\,dy\,dx = \int_0^1[6(1-x)^2-\frac{3}{4}\times4(1-x)^2]\,dx = \int_0^1 3(1-x)^2\,dx = [-(1-x)^3]_0^1 = 1$

(29) $V = 8\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} dz\,dy\,dx = 8\int_0^1\int_0^{\sqrt{1-x^2}}\sqrt{1-x^2}\,dy\,dx = 8\int_0^1(1-x^2)\,dx = \frac{16}{3}$

(42) $\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dy\,dx\,dz = \int_0^1\int_0^1\int_0^{\sqrt{y}} 12xye^{zy^2}\,dx\,dy\,dz = \int_0^1\int_0^1 6yze^{zy^2}\,dy\,dz = \int_0^1[3e^{zy^2}]_0^1\,dz$

$= 3\int_0^1(e^z-1)\,dz = 3[e^z-z]_0^1 = 3e-6$



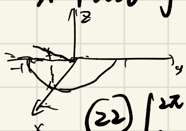
15.7

(4) $\int_0^\pi\int_0^{\frac{\theta}{\pi}}\int_{-\sqrt{4-r^2}}^{3\sqrt{4-r^2}} z\,dz\,r\,dr\,d\theta = \int_0^\pi\int_0^{\frac{\theta}{\pi}} 4(4-r^2)r\,dr\,d\theta = \int_0^\pi \frac{8}{\pi^2}\theta^2-\frac{\theta^4}{\pi^4}\,d\theta = (\frac{8}{3\pi^2}\theta^3-\frac{\theta^5}{5\pi^4}]_0^\pi = \frac{8}{3}\pi-\frac{\pi}{5} = \frac{37}{15}\pi$

(10) $\int_0^2\int_{r-2}^{\sqrt{4-r^2}}\int_0^{2\pi}(r\sin\theta+1)\,r\,d\theta\,dz\,dr = \int_0^2\int_{r-2}^{\sqrt{4-r^2}} 2\pi r\,dz\,dr = \int_0^2 2\pi r[\sqrt{4-r^2}-(r-2)]\,dr = 8\pi$

(14) $\int_{-1}^1\int_0^{\sqrt{1-y^2}}\int_0^x(x^2+y^2)\,dz\,dx\,dy = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1\int_0^{r\cos\theta} r^3\,dz\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1 r^4\cos\theta\,dr\,d\theta = \frac{2}{5}$

$x=r\cos\theta \quad y=r\sin\theta$



(18) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\cos\theta}^{2\cos\theta}\int_0^{3-r\sin\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$

(22) $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^2(\rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}[\frac{\rho^4}{4}\cos\phi\sin\phi]_0^2\,d\phi\,d\theta$

$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}}(4\cos\phi\sin\phi)\,d\phi\,d\theta = \int_0^{2\pi} 1\,d\theta = 2\pi$

(26) $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi}(\rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{1}{4}\int_0^{2\pi}[\frac{1}{2}\tan^2\phi]_0^{\frac{\pi}{4}}\,d\theta = \frac{1}{8}\int_0^{2\pi}d\theta = \frac{\pi}{4}$

(37) (a) $x=\rho\sin\phi\cos\theta \quad y=\rho\sin\phi\sin\theta \quad z=\rho\cos\phi$

$z = \rho\sin\phi = \rho\cos\phi \quad \therefore \phi = \frac{\pi}{4}$

$V = \int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{2\cos\phi}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{8}{3}\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^3\phi\sin\phi\,d\phi\,d\theta = \frac{8}{3}\int_0^{2\pi}[-\frac{\cos^4\phi}{4}]_{\frac{\pi}{4}}^{\frac{\pi}{2}}\,d\theta$

$= (\frac{8}{3})(\frac{1}{16})\int_0^{2\pi}d\theta = \frac{1}{6}(2\pi) = \frac{\pi}{3}$

(48) $V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\cos\theta}\int_0^{3\sqrt{1-r^2}} dz\,r\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\int_0^{\cos\theta} 3r\sqrt{1-r^2}\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[-(1-r^2)^{\frac{3}{2}}]_0^{\cos\theta}\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\sin^3\theta)\,d\theta$

$x=r\cos\theta \quad y=r\sin\theta, \quad z=3\sqrt{1-r^2}$

$= [\theta+\frac{\sin^2\theta\cos\theta}{3}]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{2}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin\theta\,d\theta$

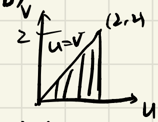
$= \frac{\pi}{2}-(-\frac{\pi}{2})-\frac{2}{3}[\cos\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \pi$

15.8

(2) (a) $y=\frac{1}{3}(u-v)$ $\quad x=\frac{1}{3}(u+2v)$

$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix}\frac13 & \frac23\\ \frac13 & -\frac13\end{vmatrix} = -\frac19-\frac29=-\frac13$

(b)



$x-y=0 \Rightarrow v=0$

$y=0$ from $(0,0)$ to $(2,0)$ $\Rightarrow u=v$

$x+2y=2$ from $(\frac23,\frac23)$ to $(2,0)$ $\Rightarrow u=2$

(6) $\iint_R (2x^2-xy-y^2)\,dxdy = \iint_R (x-y)(2x+y)\,dxdy = \iint_G uv\left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv = \frac13\iint_G uv\,dudv$

$u=x-y$, $v=2x+y$ $\quad x=\frac13(u+v)$ $\quad y=\frac13 v-\frac23 u$ $\quad \therefore \Rightarrow \frac13\iint_G uv\,dudv$

$y=-2x+4$ $\quad \frac13(-2u+v)=-\frac23(u+v)+4$ $\quad v=4$

$y=-2x+7$ $\quad \frac13(-2u+v)=-\frac23(u+v)+7$ $\quad v=7$ $\quad =\frac13\int_{-1}^{2}\int_4^7 uv\,dvdu=\frac13\int_{-1}^{2}u\left[\frac{v^2}{2}\right]_4^7 du$

$y=x-2$ $\quad \frac13(-2u+v)=\frac13(u+v)-2$ $\quad u=2$ $\quad =\frac{11}{2}\left[\frac{u^2}{2}\right]_{-1}^{2}=\frac{33}{4}$

$y=x+1$ $\quad \frac13(-2u+v)=\frac13(u+v)+1$ $\quad u=-1$

(14) $x=u+\frac v2$ $\quad y=v$ $\quad \Rightarrow 2x-y=2u$

$\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}1&\frac12\\0&1\end{vmatrix}=1$

$x=\frac y2$ $\quad u+\frac v2=\frac v2$ $\quad u=0$

$x=\frac y2+2$ $\quad u+\frac v2=\frac v2+2$ $\quad u=2$

$y=0$ $\quad v=0$ $\quad v=0$

$y=2$ $\quad v=2$ $\quad v=2$

$\therefore \int_0^2\int_{\frac y2}^{\frac y2+2} y^3(2x-y)e^{(2x-y)^2}dxdy=\int_0^2\int_0^2 v^3(2u)e^{4u^2}=\frac14\int_0^2 v^3(e^{16}-1)dv$

$=\frac14(e^{16}-1)\left[\frac{v^4}{4}\right]_0^2=e^{16}-1$

(17) $\int_0^3\int_0^4\int_{\frac y2}^{1+\frac y2}\left(\frac{2x-y}{2}+\frac z3\right)dxdydz=\int_0^3\int_0^4\left[\frac{x^2}{2}-\frac{xy}{2}+\frac{xz}{3}\right]_{\frac y2}^{1+\frac y2}dydz=\int_0^3\int_0^4\left[\frac12(y+1)-\frac y2+\frac z3\right]dydz$

$=\int_0^3\left[\frac{(y+1)^2}{4}-\frac{y^2}{4}+\frac{yz}{3}\right]_0^4 dz=\int_0^3\left(2+\frac{4z}{3}\right)dz=\left[2z+\frac{2z^2}{3}\right]_0^3=12$

(19) $J(u,v,w)=\begin{vmatrix}a&0&0\\0&b&0\\0&0&c\end{vmatrix}=abc$ $\quad \iiint_R |xyz|\,dxdydz\ \iiint_G a^2b^2c^2uvw\,dwdvdu$

$=8a^2b^2c^2\int_0^{\frac\pi2}\int_0^{\frac\pi2}\int_0^1 \rho(\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)(\rho\cos\phi)(\rho^2\sin\phi)\,d\rho\, d\phi\, d\theta$

$=\frac{4a^2b^2c^2}{3}\int_0^{\frac\pi2}\int_0^{\frac\pi2}\sin\theta\cos\theta\sin^3\phi\cos\phi\,d\phi\, d\theta=\frac{a^2b^2c^2}{3}\int_0^{\frac\pi2}\sin\theta\cos\theta\,d\theta=\frac{a^2b^2c^2}{6}$

(22) $u=x,\ v=xy,\ w=3z \Rightarrow x=u,\ y=\frac vu,\ z=\frac13 w \Rightarrow J(u,v,w)=\begin{vmatrix}1&0&0\\-\frac{v}{u^2}&\frac1u&0\\0&0&\frac13\end{vmatrix}=\frac{1}{3u}$

$\iiint_D (x^2y+3xyz)\,dxdydz=\frac13\int_0^3\int_0^2\int_1^2\left(v+\frac{vw}{u}\right)dudvdw=\frac23\left[w+\frac{w^2}{2}\ln2\right]_0^3=2+3\ln2$

(26) $x=r\cos\theta,\ y=y,\ z=r\sin\theta$

$V=\iiint_G r\,dyd\theta dr=\int_a^b\int_0^{2\pi}\int_0^{f(r)} r\,dyd\theta dr=\int_a^b\int_0^{2\pi}[ry]_0^{f(r)}d\theta dr=\int_a^b\int_0^{2\pi} rf(r)\,d\theta dr$

$=\int_a^b[r\theta f(r)]_0^{2\pi}dr=\int_a^b 2\pi r f(r)\,dr$

$\therefore V=\int_a^b 2\pi x f(x)\,dx$

16.1

(12) $\vec{r}(t)=(4\cos t)\vec{i}+(4\sin t)\vec{j}+3t\vec{k}$, $-2\pi\le t\le 2\pi$ $\therefore \frac{d\vec{r}}{dt}=(-4\sin t)\vec{i}+(4\cos t)\vec{j}+3\vec{k}$

$\Rightarrow \left|\frac{d\vec{r}}{dt}\right|=\sqrt{16\sin^2 t+16\cos^2 t+9}=5$, $\sqrt{x^2+y^2}=\sqrt{16\cos^2 t+16\sin^2 t}=4$

$\Rightarrow \int_C f(x,y,z)\,ds=\int_{-2\pi}^{2\pi}(4)(5)\,dt=[20t]_{-2\pi}^{2\pi}=80\pi$

(15) C_1: $\vec{r}(t)=t\vec{i}+t^2\vec{j}$, $0\le t\le 1$ $\therefore \frac{d\vec{r}}{dt}=\vec{i}+2t\vec{j}$ $\left|\frac{d\vec{r}}{dt}\right|=\sqrt{1+4t^2}$ $x+\sqrt{y}-z^2=t+\sqrt{t^2}=2t$

since $t\ge 0 \Rightarrow \int_{C_1} f(x,y,z)\,ds=\int_0^1 2t\sqrt{1+4t^2}\,dt=\left[\frac{1}{6}(1+4t^2)^{\frac{3}{2}}\right]_0^1=\frac{1}{6}(5)^{\frac{3}{2}}-\frac{1}{6}=\frac{1}{6}(5\sqrt{5}-1)$

C_2: $\vec{r}(t)=\vec{i}+\vec{j}+t\vec{k}$, $0\le t\le 1 \Rightarrow \frac{d\vec{r}}{dt}=\vec{k}$ $\left|\frac{d\vec{r}}{dt}\right|=1$, $x+\sqrt{y}-z^2=1+1-t^2=2-t^2$

$\Rightarrow \int_{C_2} f(x,y,z)\,ds=\int_0^1(2-t^2)(1)\,dt=\left[2t-\frac{1}{3}t^3\right]_0^1=\frac{5}{3}$

therefore $\int_C f(x,y,z)\,ds=\int_{C_1} f(x,y,z)\,ds+\int_{C_2} f(x,y,z)\,ds=\frac{5}{6}\sqrt{5}+\frac{3}{2}$

(20) (a) $\vec{r}(t)=t\vec{i}+4t\vec{j}$, $0\le t\le 1$ $\frac{d\vec{r}}{dt}=\vec{i}+4\vec{j}$ $\left|\frac{d\vec{r}}{dt}\right|=\sqrt{17}$

$\int_C\sqrt{x+2y}\,ds=\int_0^1\sqrt{t+2(4t)}\,\sqrt{17}\,dt=\sqrt{17}\int_0^1\sqrt{9t}\,dt=3\sqrt{17}\int_0^1\sqrt{t}\,dt=\left[2\sqrt{17}\,t^{\frac{3}{2}}\right]_0^1=2\sqrt{17}$

(b) C_1: $\vec{r}(t)=t\vec{i}$, $0\le t\le 1 \Rightarrow \frac{d\vec{r}}{dt}=\vec{i}$ $\therefore \left|\frac{d\vec{r}}{dt}\right|=1$

C_2: $\vec{r}(t)=\vec{i}+t\vec{j}$, $0\le t\le 1$ $\frac{d\vec{r}}{dt}=\vec{j}$ $\left|\frac{d\vec{r}}{dt}\right|=1$

$\int_C\sqrt{x+2y}\,ds=\int_{C_1}\sqrt{x+2y}\,ds+\int_{C_2}\sqrt{x+2y}\,ds=\int_0^1\sqrt{t+2(0)}\,dt+\int_0^2\sqrt{1+2t}\,dt$

$=\int_0^1\sqrt{t}\,dt+\int_0^2\sqrt{1+2t}\,dt=\left[\frac{2}{3}t^{\frac{3}{2}}\right]_0^1+\left[\frac{1}{3}(1+2t)^{\frac{3}{2}}\right]_0^2=\frac{5\sqrt{5}+1}{3}$

(32) $2x+3y=6$, $0\le x\le 6$ $\vec{r}(t)=t\vec{i}+\left(2-\frac{2}{3}t\right)\vec{j}$ $0\le t\le 6$ $\therefore \left|\frac{d\vec{r}}{dt}\right|=\frac{\sqrt{13}}{3}$

$\therefore A=\int_C f(x,y)\,ds=\int_C(4+3x+2y)\,ds=\int_0^6\left(4+3t+2\left(2-\frac{2}{3}t\right)\right)\frac{\sqrt{13}}{3}\,dt=\frac{\sqrt{13}}{3}\int_0^6\left(8+\frac{5}{3}t\right)dt=\frac{\sqrt{13}}{3}\left[8t+\frac{5}{6}t^2\right]_0^6$
$=26\sqrt{13}$

(35) $\vec{r}(t)=\sqrt{2}t\vec{i}+\sqrt{2}t\vec{j}+(4-t^2)\vec{k}$, $0\le t\le 1$

$\frac{d\vec{r}}{dt}=\sqrt{2}\vec{i}+\sqrt{2}\vec{j}-2t\vec{k}$ $\therefore \left|\frac{d\vec{r}}{dt}\right|=\sqrt{2+2+4t^2}=2\sqrt{1+t^2}$

(a) $M=\int_C\delta\,ds=\int_0^1(3t)\,2\sqrt{1+t^2}\,dt=\left[2(1+t^2)^{\frac{3}{2}}\right]_0^1=4\sqrt{2}-2$

(b) $M=\int_C\delta\,ds=\int_0^1(1)(2\sqrt{1+t^2})\,dt=\left[t\sqrt{1+t^2}+\ln\left(t+\sqrt{1+t^2}\right)\right]_0^1=\left[\sqrt{2}+\ln(1+\sqrt{2})\right]-(0+\ln 1)=\sqrt{2}+\ln(1+\sqrt{2})$

16.2

(11) (a) $\vec{F}=(3t^2-3t)\vec{i}+3t\vec{j}+\vec{k}$ $\quad \frac{d\vec{r}}{dt}=\vec{i}+\vec{j}+\vec{k}$

$\Rightarrow \vec{F}\cdot\frac{d\vec{r}}{dt}=3t^2+1 \Rightarrow \int_0^1(3t^2+1)dt=[t^3+t]_0^1=2$

(b) $\vec{F}=(3t^2-3t)\vec{i}+3t^4\vec{j}+\vec{k}$ $\quad \frac{d\vec{r}}{dt}=\vec{i}+2t\vec{j}+4t^3\vec{k}$

$\Rightarrow \vec{F}\cdot\frac{d\vec{r}}{dt}=6t^5+4t^3+3t^2-3t \quad \therefore \int_0^1(6t^5+4t^3+3t^2-3t)dt=[t^6+t^4+t^3-\frac{3}{2}t^2]_0^1=\frac{3}{2}$

(c) $\vec{r_1}=t\vec{i}+t\vec{j}$ $\quad \vec{r_2}=\vec{i}+\vec{j}+t\vec{k}$ $\quad \vec{F_1}=(3t^2-3t)\vec{i}+\vec{k}$ $\quad \frac{d\vec{r_1}}{dt}=\vec{i}+\vec{j}$ $\quad \vec{F_1}\cdot\frac{d\vec{r_1}}{dt}=3t^2-3t$

$\int_0^1(3t^2-3t)dt=[t^3-\frac{3}{2}t^2]_0^1=-\frac{1}{2}$ $\quad \vec{F_2}=3t\vec{j}+\vec{k}$ $\quad \frac{d\vec{r_2}}{dt}=\vec{k}$ $\quad \vec{F_2}\cdot\frac{d\vec{r_2}}{dt}=1$

$\therefore \int_0 dt=1 \quad -\frac{1}{2}+1=\frac{1}{2}$

(14) $x=t,\ y=t^2 \quad 1\le t\le 2 \quad dy=2t\,dt$

$\int_C \frac{x}{y}dy=\int_1^2\frac{t}{t^2}(2t)dt=\int_1^2 2\,dt=[2t]_1^2=2$

(28) $\vec{r}=(2\cos t)\vec{i}+(2\sin t)\vec{j} \quad 0\le t\le 2\pi \quad \vec{F}=\nabla f=2(x+y)\vec{i}+2(x+y)\vec{j}$

$\Rightarrow \vec{F}=4(\cos t+\sin t)\vec{i}+4(\cos t+\sin t)\vec{j} \quad \frac{d\vec{r}}{dt}=(-2\sin t)\vec{i}+(2\cos t)\vec{j}$

$\vec{F}\cdot\frac{d\vec{r}}{dt}=-8(\sin t\cos t+\sin^2 t)+8(\cos^2 t+\cos t\sin t)=8(\cos^2 t-\sin^2 t)=8\cos 2t$

$\text{work}=\int_C\nabla f\,d\vec{r}=\int_C\vec{F}\cdot\frac{d\vec{r}}{dt}dt=\int_0^{2\pi}8\cos 2t\,dt=[4\sin 2t]_0^{2\pi}=0$

(29) (a) $\vec{r}=(\cos t)\vec{i}+(\sin t)\vec{j}$, $0\le t\le 2\pi$, $\vec{F_1}=x\vec{i}+y\vec{j}$ $\quad \vec{F_2}=-y\vec{i}+x\vec{j}$ $\quad \frac{d\vec{r}}{dt}=(-\sin t)\vec{i}+(\cos t)\vec{j}$

$\vec{F_1}=(\cos t)\vec{i}+(\sin t)\vec{j}$ $\quad \vec{F_2}=(-\sin t)\vec{i}+(\cos t)\vec{j}$ $\quad \vec{F_1}\cdot\frac{d\vec{r}}{dt}=0$ $\quad \vec{F_2}\cdot\frac{d\vec{r}}{dt}=\sin^2 t+\cos^2 t=1$

$\therefore \text{Circ}_1=\int_0^{2\pi}0\,dt=0$, $\text{circ}_2=\int_0^{2\pi}dt=2\pi$ $\quad \vec{n}=(\cos t)\vec{i}+(\sin t)\vec{j}$

$\vec{F_1}\cdot\vec{n}=\cos^2 t+\sin^2 t=1$ and $\vec{F_2}\cdot\vec{n}=0$

$\text{Flux}_1=\int_0^{2\pi}dt=2\pi$ and $\text{Flux}_2=\int_0^{2\pi}0\,dt=0$

(b) $\vec{r}=(\cos t)\vec{i}+(4\sin t)\vec{j} \quad 0\le t\le 2\pi \quad \frac{d\vec{r}}{dt}=(-\sin t)\vec{i}+(4\cos t)\vec{j}$

$\vec{F_1}=(\cos t)\vec{i}+(4\sin t)\vec{j}$ $\quad \vec{F_2}=(-4\sin t)\vec{i}+(\cos t)\vec{j}$ $\quad \vec{F_1}\cdot\frac{d\vec{r}}{dt}=15\sin t\cos t$ $\quad \vec{F_2}\cdot\frac{d\vec{r}}{dt}=4$

$\Rightarrow \text{Circ}_1=\int_0^{2\pi}15\sin t\cos t\,dt=[\frac{15}{2}\sin^2 t]_0^{2\pi}=0$

$\text{Circ}_2=\int_0^{2\pi}4\,dt=8\pi$ $\quad \vec{n}=(\frac{4}{\sqrt{17}}\cos t)\vec{i}+(\frac{1}{\sqrt{17}}\sin t)\vec{j}$ $\quad \vec{F_1}\cdot\vec{n}=\frac{4}{\sqrt{17}}\cos^2 t+\frac{4}{\sqrt{17}}\sin^2 t$

$\vec{F_2}\cdot\vec{n}=-\frac{15}{\sqrt{17}}\sin t\cos t$

$\text{Flux}_1=\int_0^{2\pi}(\vec{F_1}\cdot\vec{n})|\vec{v}|dt=\int_0^{2\pi}(\frac{4}{\sqrt{17}})\sqrt{17}\,dt=8\pi$

$\text{Flux}_2=\int_0^{2\pi}(\vec{F_2}\cdot\vec{n})|\vec{v}|dt=\int_0^{2\pi}(-\frac{15}{\sqrt{17}}\sin t\cos t)\sqrt{17}\,dt=[-\frac{15}{2}\sin^2 t]_0^{2\pi}=0$

(30) $\vec{r}=(a\cos t)\vec{i}+(a\sin t)\vec{j},\ 0\le t\le 2\pi,\ \vec{F_1}=2x\vec{i}-3y\vec{j}\quad \vec{F_2}=2x\vec{i}+(x-y)\vec{j}$

$\frac{d\vec{r}}{dt}=(-a\sin t)\vec{i}+(a\cos t)\vec{j}$

$\vec{F_1}=(2a\cos t)\vec{i}-(3a\sin t)\vec{j}\qquad \vec{F_2}=(2a\cos t)\vec{i}+(a\cos t-a\sin t)\vec{j}$

$\vec{n}|\vec{v}|=(a\cos t)\vec{i}+(a\sin t)\vec{j}$

$\vec{F_1}\cdot\vec{n}|\vec{v}|=2a^2\cos^2 t-3a^2\sin^2 t\qquad \vec{F_2}\cdot\vec{n}|\vec{v}|=2a^2\cos^2 t+a^2\sin t\cos t-a^2\sin^2 t$

$\Rightarrow Flux_1=\int_0^{2\pi}(2a^2\cos^2 t-3a^2\sin^2 t)\,dt=2a^2\left[\frac{t}{2}+\frac{\sin 2t}{4}\right]_0^{2\pi}-3a^2\left[\frac{t}{2}-\frac{\sin 2t}{4}\right]_0^{2\pi}=-\pi a^2$

$Flux_2=\int_0^{2\pi}(2a^2\cos^2 t-a^2\sin t\cos t-a^2\sin^2 t)\,dt$

$=2a^2\left[\frac{t}{2}+\frac{\sin 2t}{4}\right]_0^{2\pi}+\frac{a^2}{2}\left[\sin^2 t\right]_0^{2\pi}-a^2\left[\frac{t}{2}-\frac{\sin 2t}{4}\right]_0^{2\pi}=\pi a^2$

(36) $\vec{r_1}=(1-t)\vec{i}+t\vec{j},\ 0\le t\le 1,\quad \vec{F}=(x+y)\vec{i}-(x^2+y^2)\vec{j}\qquad \frac{d\vec{r_1}}{dt}=-\vec{i}+\vec{j}$

$\vec{F}=\vec{i}-(1-2t+2t^2)\vec{j}\quad \vec{n_1}|\vec{v_1}|=\vec{i}+\vec{j}\Rightarrow \vec{F}\cdot\vec{n_1}|\vec{v_1}|=2t-2t^2$

$Flux_1=\int_0^1(2t-2t^2)\,dt=\left[t^2-\frac{2}{3}t^3\right]_0^1=\frac{1}{3}$

from $(0,1)$ to $(-1,0)$ $\quad\vec{r_2}=-t\vec{i}+(1-t)\vec{j},\ 0\le t\le 1\quad \vec{F}=(x+y)\vec{i}-(x^2+y^2)\vec{j}\Rightarrow \frac{d\vec{r_2}}{dt}=-\vec{i}-\vec{j}$

$\vec{F}=(1-2t)\vec{i}-(1-2t+2t^2)\vec{j}\qquad \vec{n_2}|\vec{v_2}|=-\vec{i}+\vec{j}\qquad \vec{F}\cdot\vec{n_2}|\vec{v_2}|=(2t-1)+(-1+2t-2t^2)=-2+4t-2t^2$

$Flux_2=\int_0^1(-2+4t-2t^2)\,dt=\left[-2t+2t^2-\frac{2}{3}t^3\right]_0^1=-\frac{2}{3}$

From $(-1,0)$ to $(1,0)$ $\quad\vec{r_3}=(-1+2t)\vec{i},\ 0\le t\le 1,\ \vec{F}=(x+y)\vec{i}-(x^2+y^2)\vec{j}\qquad \frac{d\vec{r_3}}{dt}=2\vec{i}$

$\vec{F}=(-1+2t)\vec{i}-(1-4t+4t^2)\vec{j}\qquad \vec{n_3}|\vec{v_3}|=-2\vec{j}\qquad \vec{F}\cdot\vec{n_3}|\vec{v_3}|=2(1-4t+4t^2)$

$Flux_3=2\int_0^1(1-4t+4t^2)\,dt=2\left[t-2t^2+\frac{4}{3}t^3\right]_0^1=\frac{2}{3}$

$Flux=Flux_1+Flux_2+Flux_3=\frac{1}{3}-\frac{2}{3}+\frac{2}{3}=\frac{1}{3}$

(22) $\vec{r}=(\cos t)\vec{i}+(\sin t)\vec{j}+\frac{t}{6}\vec{k},\quad \vec{F}=2y\vec{i}+3x\vec{j}+(x+y)\vec{k}$

$\vec{F}=(2\sin t)\vec{i}+(3\cos t)\vec{j}+(\cos t+\sin t)\vec{k}\qquad \frac{d\vec{r}}{dt}=(-\sin t)\vec{i}+(\cos t)\vec{j}+\frac{1}{6}\vec{k}$

$\vec{F}\cdot\frac{d\vec{r}}{dt}=3\cos^2 t-2\sin^2 t+\frac{1}{6}\cos t+\frac{1}{6}\sin t\qquad \therefore W=\int_0^{2\pi}(3\cos^2 t-2\sin^2 t+\frac{1}{6}\cos t+\frac{1}{6}\sin t)\,dt$

$=\left[\frac{3}{2}t+\frac{3}{4}\sin 2t-t+\frac{\sin 2t}{2}+\frac{1}{6}\sin t-\frac{1}{6}\cos t\right]_0^{2\pi}=\pi$

(26) $\vec{r}=(\cos t)\vec{i}+(\sin t)\vec{j}\quad 0\le t\le\frac{\pi}{2}\qquad \vec{F}=y\vec{i}-x\vec{j}$

$\vec{F}=(\sin t)\vec{i}-(\cos t)\vec{j}\qquad \frac{d\vec{r}}{dt}=(-\sin t)\vec{i}+(\cos t)\vec{j}$

$\vec{F}\cdot\frac{d\vec{r}}{dt}=-\sin^2 t-\cos^2 t=-1\qquad \int_C\vec{F}\cdot d\vec{r}=\int_0^{\frac{\pi}{2}}(-1)\,dt=-\frac{\pi}{2}$