

Exercise Sheet 7

Exercise 1: Weighted Degree Kernels (30 P)

We would like to implement a classifier for genes sequences (a sequence of symbols $\{A, C, T, G\}$). The weighted degree kernel is proposed for such task and is defined as:

$$k(x, x') = \sum_{m=1}^M \beta_m \sum_{n=1}^{N-m+1} I(u_{m,n}(x) = u_{m,n}(x')).$$

where $u_{m,n}(x)$ is a string of length m which starts at position n in sequence x , and $\beta_m \geq 0$. The symbol $I(\cdot)$ denotes the indicator function which returns 1 if the input argument is true and 0 otherwise.

x AAACAAATAAGTAACTAATCTTTTAGGAAGAACGTTTC AACCATTTTGAG
#1-mers .|. |.| |.| |.| |.| |.| |.| |.| |.| |.| |.
#2-mers|.....|.....|.....|.....
#3-mers|.....|.....|.....
 x' TACCTTAATTATGAAATTA AATTTCA GTGTGCTGATGGAAACGGAGAAGTC

- (a) Show that k is a positive semi-definite kernel. That is, show that

$$\sum_{i=1}^K \sum_{j=1}^K \alpha_i \alpha_j k(x_i, x_j) \geq 0$$

for all inputs x_1, \dots, x_K and choice of real numbers $\alpha_1, \dots, \alpha_K$.

- (b) Give a feature map associated to this kernel for the special case $M = 1$.
- (c) Give a feature map associated to this kernel for the special case $M = 2$ with $\beta_1 = 0$ and $\beta_2 = 1$.

Exercise 2: Programming (70 P)

Download the programming files on ISIS and follow the instructions.