Exercises for the course
Machine Learning 2
Summer semester 2017

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Exercise Sheet 9

Exercise 1: Dual of One-Class SVM (60 P)

The spherical version of the one-class SVM (also called "support vector data description") is given by the minimization problem:

$$\min_{R,c,(\xi_i)_{i=1}^n} \quad R^2 + \frac{1}{n\nu}\sum_{i=1}^n \xi_i$$

subject to

$$\forall_{i=1}^n: \quad \|\phi(x_i) - c\|^2 \leq R^2 + \xi_i \quad \text{and} \quad \xi_i \geq 0$$

where $x_1, \ldots, x_n$ are the training data and $\phi(x_i) \in \mathbb{R}^d$ is a feature space representation.

(a) *Derive* the dual program for the one-class SVM.

(b) *Show* that the kernelized dual has the form

$$\begin{aligned} \max_{\alpha} \quad & \sum_{i=1}^n \alpha_i k(x_i, x_i) - \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j k(x_i, x_j) \\ \text{subject to} \quad & \sum_{i=1}^n \alpha_i = 1 \quad \text{and} \quad \forall_{i=1}^n: \ 0 \leq \alpha_i \leq \frac{1}{n\nu} \\ \text{and with center} \quad & c = \sum_{i=1}^n \alpha_i \phi(x_i) \end{aligned}$$

where k is the kernel associated to the feature map ϕ.

Exercise 2: Quadratic Programming (40 P)

Show that the dual program derived in Exercise 1 is a linearly constrained quadratic program, by writing it in the matrix form

$$\begin{aligned} \min_{\alpha} \quad & \alpha^\top P \alpha + q^\top \alpha \\ \text{subject to} \quad & G\alpha \leq h \quad \text{and} \quad A\alpha = b \end{aligned}$$

with matrices P, G, A and vectors q, h, b. That is, *express* the matrices P, G, A and vectors q, h, b in terms of the solution of Exercise 1, and *specify* their dimensions.