Glossary of Terms in Combinatorics

collected and maintained by [Douglas B. West](#)

This glossary accompanies a web site of [Open Problems in Graph Theory and Combinatorics](#). The glossary collects terms and gives quick reminders of definitions, too abbreviated in most cases to give full introduction. An attempt has been made to list standard notations. Typically, *G* denotes a graph, *D* a directed graph, *H* a graph or hypergraph, *P* a poset, *L* a lattice, *M* a matroid. Also *u*, . . .,*z* are vertices or elements, *uv* is an edge, *n* is the number of vertices or elements, and *e* is an edge or (as a function) is the number of edges. The set of the first *n* natural numbers is denoted *[n]*.

Certainly many terms are missing, and some terms are lacking definitions due to lack of time. Also, the conversion of tex commands leaves much to be desired. Please send notice of items that should be added (or corrected or clarified) to Douglas B. West at *west@math.uiuc.edu*. This glossary is based on the glossary of *Combinatorial Mathematics*, a graduate textbook expected to be published in 2004.

For ease in accessing desired definitions, click below on the first letter or first two letters of the term sought.
[A](#) [B](#) [Ca](#) [Ch](#) [Co](#) [D](#) [E](#) [F](#) [G](#) [H](#) [I](#) [J](#) [K](#) [L](#) [M](#) [N](#) [O](#) [P](#) [Q](#) [R](#) [S](#) [T](#) [U](#) [V](#) [W](#) [X](#) [Y](#) [Z](#)
To search for a specific string on this page, use the "Edit" menu on your browser and select "Find".

A

Abstract dual - a graph whose cycles correspond to the bonds of the original graph
Acyclic - without cycles
Acyclic orientation - an orientation of a graph that is an acyclic digraph
Admissible transposition - applied to a permutation, a transposition that does not change the *P*-symbol
Adjacency matrix *A* - entry *i,j* is number of edges from vertex *i* to vertex *j* (symmetric if graph is undirected)
Adjacency set *N(v)* - the set of vertices adjacent to *v*
Adjacent - vertices joined by an edge ("<-> " means "is adjacent to", "\nadj " means "is not adjacent to")
Advance - for a permutation σ, a value *x* such that $\sigma^{-1}(x) < \sigma^{-1}(x+1)$
Affine - 1) for a vector space, ...; 2) for a geometry, ...; 3) for a lattice
Ahlswede-Daykin inequality - if $\alpha,\beta,\gamma,\delta$ are non-negative functions satisfying $\alpha(x)\beta(y) \leq \gamma(x \wedge y)\delta(x \vee y)$ for all $x,y \in L$, then $\alpha(X)\beta(Y) \leq \gamma(X \wedge Y)\delta(X \vee Y)$ for all $X,Y \subseteq L$, where $f(X) = \sum_{x \in X} f(x)$
Algebraic dependence - satisfies a polynomial with coefficients over a field
Almost always - having probability asymptotic to 1
α-critical - deletion of any edge increases the independence number
Alteration principle - the two-step randomization in probabilistic methods
Alternating cycle - in a poset, a collection of *k* incomparable pairs $(x_1,y_1),...,(x_k,y_k)$ such that $y_i \leq x_{i+1}$ for all *i* (modulo *k*); they cannot occur together in an extension
Alternating group - the group of permutations with even sign
Alternating path - a path alternating between edges in some set and not in the set (used mostly when the set is a matching)
Angle order - containment order on a collection of angles in the plane
Antiblocker - the hypergraph on *V(H)* whose edges are the sets intersecting each edge of *H* at most once
Antichain - set of pairwise incomparable elements
Anticlique - stable set
Antihole - induced subgraph whose complement is C_k for some $k \geq 4$
Antisymmetric relation - *(x,y),(y,x)∈R* imply *x=y*
Arborescence - a directed forest in which every vertex has out-degree at most one
Arboricity *Y(G)* - minimum number of forests covering the edges of *G*
Arc - directed edge (ordered pair)
Arithmetic progression - sequence of integers differing by a constant
Articulation point - a vertex whose deletion increases the number of components
Ascent - for a permutation σ, a value *i* such that $\sigma(i) < \sigma(i+1)$

Assignment problem - minimize (or maximize) the sum of the edge weights in a perfect matching of a complete bipartite graph with equal part-sizes

Asteroidal triple - three vertices such that each pair is connected by a path containing no neighbor of the third vertex

Asymmetric - having no automorphisms other than the identity

Atom - element of rank 1

Augmentation property - ability to augment one independent set from a larger one

Augmenting path - 1) for a matching, an alternating path connecting unsaturated vertices; 2) for a flow, a path to increase its value

Automorphism - an edge- or arc-preserving permutation of the vertices

Automorphism group Γ - the group of automorphisms, under composition

B

Balanced k-partite - having part-sizes differing by at most one (see equipartite)

Balanced graph - among subgraphs, average vertex degree is maximized by the full graph

δ-Balanced comparison - a pair of incomparable elements such that $x < y$ in a fraction between δ and 1-δ of the linear extensions

Ballot list - list of n 0s and n 1s in which each prefix has at least as many 0s as 1s

Bandwidth - minimum, over injective numberings of vertices, of the largest absolute difference between adjacent labels

Barycenter - in a graph, the subgraph induced by vertices with minimum sum of distances to all other vertices

Base - 1) maximal independent set of a matroid; 2) maximal element of an ideal

2-Basis - basis for the cycle space of a graph in which every element appears in at most two elements

BCH (Bose-Chaudhuri-Hocquenghem) code - type of multiple-error-correcting polynomial code

Bell number - number of partitions of $[n]$

Berge graph - a graph having no odd hole or odd antihole

Bertrand's Ballot Problem - probability that candidates in an election never change order during the counting

β-perfection - for each induced subgraph H, $\omega(H)\ \alpha(H) \geq n(H)$

Biadjacency matrix - matrix recording the edges of a bipartite graph

Bicentral tree - a tree with two vertices in its center

X,Y-bigraph - bipartite graph with partite sets X,Y

Binary (matrix, vector, list) - having all entries 0 or 1

Binary matroid - representable over the two-element field

Binary tree - a rooted tree with vertex degrees 1 or 3, except for a root of degree 2

Binomial coefficient - number of ways to select k of n distinct objects

Binomial sequence - sequence of polynomials, p_n of degree n, with $p_n(x+y)=\sum_{k=0}^{n}\{n\backslash choose\ k\}p_k(x)p_{n-k}(y)$

k-binomial expansion - expression of an integer as $\sum_{i=j}^{k}\{m_i\backslash choose\ i\}$ with $m_k > \ldots > m_j$

Bidimension - (also biorder dimension) Ferrers dimension: minimum number of biorders whose intersection is D

Bijection - function that yields each element of its target exactly once

Binomial Theorem - $(x+y)^n \geq \sum_{k\geq 0}\{n\backslash choose\ k\}x^k y^{n-k}$

Biorder - name for Ferrers digraph from viewpoint of relations

Bipartite graph - having a vertex partition into (at most) two independent sets

Bipartite Ramsey number - for a bipartite G, the minimum n such that 2-coloring the edges of $K_{n,n}$ forces a monochromatic G

Bipartite poset - poset whose comparability graph is bipartite

Bipartition - a partition of the vertex set into two independent sets

Birkhoff diamond - a reducible configuration for 4-coloring of planar graphs

Block - 1) maximal (sub)graph with no cutvertex. 2) member of a partition of a set

Block code - a code in which all codewords have the same length

Block design - a family of k-sets from a v-set such that every pair of elements appears together λ times

Block graph - intersection graph of the blocks

Block-cutpoint graph - bipartite graph on the blocks and cut-vertices of G where adjacency represents membership

Board (forbidden positions) - set of positions in a grid

Bond - a minimal edge cut

Bond space - cocycle space

Book embedding - a decomposition of G into outerplanar graphs with a consistent ordering of the vertices (as on the spine of a book)

Boolean algebra - a common name for the subset lattice

Boolean function - function from subset lattice to $\{0,1\}$ (False/True)

Boundary - the cycle bounding a region in an embedding of a graph on a surface

Bounded poset - having one minimal element 0 and one maximal element 1

Box - a rectilinear parallelopiped with sides parallel to the coordinate axes

Boxicity - minimum dimension in which G is the intersection graph of boxes

Bracketing structure - encoding of subsets describing an explicit symmetric chain decomposition

Branch of tree - a subtree obtained by deleting the central edge or by taking a maximal subtree with the central vertex having degree 1

Branch vertex, Branchpoint - vertex of degree at least 3

Branching - a directed graph in which each vertex has indegree one except for a single vertex (root) with indegree 0

Branching forest - arborescence

Breadth-first search - exploration of a graph in order of distance from a fixed vertex

Bridge - 1) an edge whose deletion increases the number of components of a graph. 2) for a subgraph H (usually a cycle), a component of G-H together with the edges to its vertices of attachment

Brooks' Theorem - $\chi(G) \leq \Delta(G)$, except for cliques and odd cycles

Broom - union of a path and a star at a common leaf

deBruijn graph - describes the possible transitions between words in a stream of letters from an alphabet

Bumping procedure - produces a pair of Young tableaux from a permutation

Burnside's Lemma - tool for counting equivalence classes under group action

C

Cactus - a graph in which every edge appears in at most one cycle

Cage - a regular graph of given degree and girth having minimal number of vertices

Canonical cycle representation - listing of cycles of a permutation, each cycle with least element first, cycles in decreasing order of least element

Canonical simplex - simplex in the order polytope associated with a particular linear extension

Canonical tableaux - tableau encoding linear programs using inequality constraints, to emphasize duality

Capacity - a limit on flow through an arc in a network

Cartesian product $G_1 \backslash cart\ G_2$ - graph with $V(G)_1 \times V(G)_2)$ as vertices, having $(u_1,u_2)<\rightarrow (v_1,v_2)$ if 1) $u_1=v_1$ and $u_2<\rightarrow v_2$ in G_2 or 2) $u_2=v_2$ and $u_1<\rightarrow v_1$ in G_1

Cartesian product $P \times Q$ - poset with $\{(p,q): p \in P, q \in Q\}$ as elements, having $(p,q) \leq (p',q')$ if and only if $p \leq p'$ in P and $q \leq q'$ in Q

Catalan numbers $C_n = \{2n \backslash choose\ n\}/(n+1)$ - sequence with $C_0=1$ and $C_n=\Sigma_{i=1}^n C_{i-1}C_{n-i}$, solution for many counting problems such as ballot lists

Caterpillar - tree with one path (the spine) containing or incident to every edge

Cayley graph - given a group Γ and a set S of generators, this is graph with vertex set Γ defined by $u<\rightarrow v$ if $u=gv$ or $u=g^{-1}v$ for some $g \in S$

Cayley's formula - there are n^{n-2} labeled trees on n vertices

2-cell - on a surface, a region homeomorphic to a disc, i.e. having no handle and having a simple closed curve as boundary

2-cell embedding - an embedding in which every boundary encloses a 2-cell

Center - subgraph induced by the vertices of minimum eccentricity

Central tree - a tree with one vertex in its center

δ-central element - an element belong to between δ and 1-δ of the ideals

Centroid - for a region in $\mathbf{R}^n$, the vector whose ith coordinate is $\int x_i dV / \int dV$

Chain - 1) set of pairwise comparable elements (totally ordered poset); 2) in graphs, used by some authors to mean a walk

Chain hypergraph - hypergraph on the elements of a poset in which the edges are the sets forming chains

Characteristic equation - polynomial equation related to a linear constant-coefficent recurrence

Characteristic function - for a set, has value 1 on the set, 0 off it

Characteristic polynomial ϕ - 1) for a graph, the characteristic polynomial of the adjacency matrix (roots are the eigenvalues); 2) for a poset, the generating function of the Möbius function by co-rank

Characteristic roots - roots of the characteristic equation

Characteristic vector - indexed by the elements of a universe, records the characteristic function for a set

χ-bounded - family with chromatic number bounded by a function of clique number

Chinese postman problem - problem of finding the cheapest closed walk covering all the edges in an edge-weighted graph

Choice function - function mapping each subset of *[n]* to one of its members

Choosability - list-chromatic number

Chord - edge not in a path or cycle but joining its vertices

Chordal - having no chordless cycle

Chordal poset - having a chordal comparability graph

Chordless cycle - an induced subgraph isomorphic to a cycle of length at least 4

Chromatic index - edge-chromatic number

Chromatic number χ - 1) minimum number of colors in a proper coloring. 2) for a surface, the minimum k such that all graphs embeddable on it are k-colorable).

Chromatic polynomial χ *(G;λ)* - a polynomial whose value at an integer *λ=k* is the number of proper colorings of *G* using any or all of colors *1,...,k*.

k-chromatic - having chromatic number *k*

Chvátal conjecture - 1) in an ideal of sets, the largest intersecting family consists of those containing a single element; 2) bounded toughness yields spanning cycle

Circle graph - an intersection graph of chords of a circle

Circuit - 1) for graphs, a closed trail without distinguishing start, used to mean "cycle" by some authors; 2) for matroids, a minimal dependent set

Circle order - containment order on a collection of circles

Circular arc graph - an intersection graph of arcs of a circle

Circulation - an assignment of flows to the arcs of a directed graph such that the net flow at each vertex is 0

Circumference - the length of the longest cycle

Class 1 - simple graph with chromatic index *Δ(G)*

Class 2 - simple graph with chromatic index *Δ(G)+1*

Claw - the graph $K_{1,3}$

Claw-free - having no induced claw

Clique - 1) a pairwise adjacent set of vertices; 2) a complete (sub)graph

Clique cover number \theta - 1) minimum number of cliques to cover *V(G)*. 2) sometimes used for minimum number of cliques to cover *E(G)* (intersection number)

Clique number ω - maximum size of a clique in *G*

Clique partition number - minimum number of cliques to partition *E(G)*

Clique tree - minimal representation of a chordal graph as an intersection graph of subtrees of a tree

Clique-vertex incidence matrix - entry *(i,j)* is 1 if vertex *j* belongs to maximal clique *i*, otherwise 0

Closed ear - a cycle being added to a 2-edge-connected graph

Closed neigborhood - a vertex and all its neighbors

Closed walk - a walk with last vertex the same as the first

Closed set - set of matroid elements whose span is itself; more generally, a set invariant under a closure operator

Closure operator - an expansive, order-preserving, idempotent function from subset lattice to itself

Co-atom - element of co-rank 1

Cobase - base of the dual (complement of a base)

Cocircuit - circuit in the dual (matroid)

Cocycle matroid - matroid whose circuts are the bonds of *G*

Code - a set of binary vectors

Coding polynomial - used to encode messages in a polynomial code

k-cofamily - a family containing no *k+1*-element antichain

Cograph - a graph with no induced P_4

Cointerval graph - complement of an interval graph

Colexicographic order - $x < y$ if $x_i i$ in the last place where they differ

k-colorable - chromatic number $\leq k$

Color class - in a coloring, a set of vertices receiving the same color

Color critical - deletion of any edge or vertex reduces chromatic number

Coloring - a mapping to the integers

Column matroid - matroid whose independent sets are the independent sets of columns of a matrix

Column-canonical permutation - permutation obtained from its P-symbol by reading columns in order

Column-strict tableau - placement of integers in the positions of a Ferrers diagram so that rows are strictly increasing and columns are nondecreasing

Columns condition - Rado's condition for monochromatic integer solutions to a matrix equation

k-coloring - a partition of the vertices into k independent sets

Combinatorial geometry - matroid

Common system of distinct representatives - a set of elements that is a system of distinct representatives for each of two families of sets

Comparable - $x < y$ or $y < x$

Comparability digraph - the order relation of a poset, viewed as a digraph

Comparability graph - graph obtained from a partial order by letting elements be adjacent if they are comparable in the partial order

Competition graph - graph obtained from a directed graph D by $u<->v$ if u,v have a common out-neighbor in D

Complement \ov{G} (or G^c) - 1) for graphs, the graph with the same vertices as G and $u<->v$ in G^c iff $u\nadj$ v in G; 2) for digraphs, $u->v$ in D^c iff $u\nadj$ v in D

Complement reducible graph - reducible to isolated vertices by complementation within components

Complementary slackness - condition that in pairs of variables in dual linear programs, at most one is nonzero in optimal solutions

Complete bipartite graph - m by n bipartite graph with m_n edges

Complete digraph - for every pair v,w of vertices, at least one of vw and wv appears as an arc

Complete graph K_n - every pair of the n vertices is adjacent

Complete k-partite graph $K_{n_1,...,n_k}$ - k-partite graph in which every pair of vertices not belonging to the same part is adjacent (part-sizes are $n_1,...,n_k$)

Complete matching - edge set of a 1-factor

Complete r-uniform hypergraph - hypergraph having vertex partition into r sets so that edges are the sets consisting of one vertex from each partite set

Complete symmetric digraph - every ordered pair of vertices is an edge (once)

Component - maximal connected subgraph

Composition $G_1[G_2]$ - graph whose vertex set is the cartesian product of the vertex sets of the factors, with $(u_1,u_2)<->(v_1,v_2)$ if and only if $u_1<->v_1$ in G_1, or $u_1=v_1$ and $u_2<->v_2$ in G_2

Conditional probability $Prob(A|B)$ - probability of the intersection divided by the probability of B

Cone - 1) in a poset, the elements comparable to a given element; 2) in linear programming, the positive linear combinations of a set of vectors

Congruence relation mod n - related pairs of integers are those differing by a multiple of n

Conjugate partition - partition whose Ferrers diagram is transpose of Ferrers diagram of original partition

Connected - 1) for graphs and digraphs, having a (u,v)-path for every pair of vertices u,v; 2) for posets, having a connected comparability graph; 3) for matroids, each pair of elements lies in a circuit

k-connected - having connectivity $\geq k$

k-connected component - a maximal k-connected subgraph

Connection coefficients - matrix to convert between expansions in terms of various binomial sequences

Connectivity $\varkappa(G)$ - the minimum number of vertices whose deletion disconnects the graph (sometimes called "vertex connectivity" for clarity)

Consecutive chain - a maximal chain between its endpoints (i.e., skipping no ranks)

Consecutive ones property for columns - existence of a row permutation such that ones appear consecutively in each column

Conservation - for a flow, the condition of net flow 0 at a vertex

Containment graph - subgraph of an intersection graph consisting of edges generated by containment

Containment order - poset generated by sets $f(x)$ assigned to elements x by $x < y$ if $f(x) \subseteq f(y)$

Contraction - 1) a graph obtained by a sequence of elementary contractions; 2) contraction of matroid M to F is the matroid on F whose spanning sets are the subsets of F that together with \Fb are spanning in M

Converse D^{-1} - u -> v in D^{-1} iff v -> u in D

Convex family - intersection of an ideal and a dual ideal

Convex polyhedron - the set of points satisfying a set of linear inequalities in $\mathbf{R}^d$

Convex polytope - the set of convex combinations of a finite set of points in $\mathbf{R}^d$

Convex property - holds for all objects between two objects where it holds

Convolution - 1) for sequences $<a>,<b>$, new sequence given by $c_k=\sum_i a_i b_{k-i}$; 2) for incidence functions f,g, new function given by $h(x,y)=\sum_{x\leq z\leq y}f(x,z)g(z,y)$

Cocycle space - orthogonal complement to cycle space, generated by cocycles

Co-rank - distance from the top in a lattice

Correlational inequality - a statement of positive correlation, as in $E(XY) \geq E(X)E(Y)$ or $Prob(A\cap B) \geq Prob(A)Prob(B)$

Cospectral - having the same spectrum

Cothreshold graph - complement of a threshold graph

Cotriangulated graph - complement of a triangulated graph

Cotree - with respect to a graph, the edges not belonging to a given spanning tree

Countable - set with same cardinality as N

Counting two ways - proving identity by finding a set counted by both sides

Cover diagram - cover graph drawn with vertical displacement encoding cover relation

Cover graph - the underlying graph of the cover relation

Cover relation - the order relation consisting of the pairs (x,y) such that y covers x

Covering problem - in hypergraphs, find the minimum number of edges in H that cover $V(H)$

Covers - y covers x if $x<y$ in a poset and no element is between them

Critical classes - sets of positions in the Hales - Jewes Theorem

Critical graph - used with respect to many graph properties, indicating that the deletion of any vertex (or edge, depending on context) destroys the property

Critical pair - an incomparable pair not forced by any other pair

Crossing number cr(G) - minimum number of edge crossings in a drawing of G in the plane

k-cube - graph with vertex set $\{0,1\}^k$, defined by $u<->v$ if and only if u and v differ in exactly one place

Crown - the poset with elements i,i' for $1\leq i\leq n+k$ related by $i\|j'$ for $i\leq j\leq i+k$ (modulo $(n+k)$) and otherwise $i<j'$

Cryptomorphism - map from one aspect of a matroid to another

Cubic graph - a regular graph of degree 3

Cubicity - minimum dimension in which G is the intersection graph of cubical boxes

Cut $[S,\ov\{S\}]$ - the edges from S to $\backslash Sb$

Cut-edge - an edge whose deletion disconnects a connected graph

Cutset - a set of vertices whose deletion disconnects a graph

Cut-vertex - vertex whose deletion increases number of components

Cutwidth - minimum over vertex orderings $v_1,...,v_n$ of maximum over j of the number of edges joining vertices earlier than v_j to vertices later than v_j in the ordering

Cycle - 1) an orbit in a permutation; 2) a simple graph whose vertices can be cyclically arranged so that the edges are the pairs of consecutive vertices

k-cycle - a cycle of length k, i.e., consisting of k vertices and k edges

Cycle double cover conjecture - a bridgeless graph has a collection of cycles together covering each edge exactly twice

Cycle matroid - for graph G, the matroid on $E(G)$ whose circuits are the cycles in G

Cycle rank - dimension of cycle space; $|E(G)|-|V(G)|+k$ for graph G with k components

Cycle space - the nullspace of the boundary operator, also the set of linear combinations of cycles (modulo 2)

Cyclic codes - codes in which all blocks are translates of a single block

Cyclic edge-connectivity - number of edges that must be deleted to disconnect a component so that every remaining component contains a cycle

Cyclically k-edge-connected - cyclic edge-connectivity at least k

D

Decomposition - an expression of G as an edge-disjoint union of subgraphs

Decomposition number - minimum size of a decomposition of G using subgraphs in a specified class

Defect - for a partial transversal, the number of sets not represented

Deficiency - for a set in a bipartite graph, *min{0,|Y|-|N(Y)|}*

Degree *d(v)* - 1) for a vertex, the number of edges containing it (loops count once or twice, depending on context); 2) for a regular graph, the degree of each vertex; 3) for a face of an embedding, the length of the boundary; 4) for a poset element, its degree in the comparability graph

Degree-majorization - one degree sequence degree-majorizes another if it dominates it; i.e., the *i*th largest entry in the first is at least as large as the corresponding entry in the second, for all *i*

Degree sequence $d_1 \geq \ldots \geq d_n$ - the list of vertex degrees, usually in nonincreasing order

Degree set - the set of vertex degrees (listed once each)

Dependent set - contains a circuit

Deletion method - modification technique after generating a random object

Demand - a constraint in the Transportation Problem

Density - 1) in graphs, ratio of number of edges to number of vertices;

Dependency graph - graph on a set of events in which independent sets of vertices correspond to mutually independent sets of events

Derangements - permutations with no fixed point

t-design - a family of *k*-sets from a *v*-set such that every *t*-set of elements appears together λ times

Diameter - the maximum distance between pairs of vertices in a graph

Digraph - directed graph

Dijkstra's algorithm - finds shortest paths from a vertex, in increasing order of distance

Dilworth decomposition - a partition into *w(P)* chains

Dilworth's Theorem - a poset of width *k* has a covering by *k* chains

Dimension - for posets, order dimension

k-dimension - minimum number of chains of size *k* in whose product *P* embeds

Direct product - Cartesian product, especially for posets

Direct sum - the matroid union of matroids on pairwise disjoint sets

Directed edge - ordered pair of vertices, also called arc or edge

Directed graph - model in which edges are ordered pairs of vertices

Directed walk, trail, path, cycle, etc. - one that respects edge orientations in a digraph, dropping "directed" does not change the meaning

Disc - in a surface of genus 0, the region bounded by a simple closed curve

Discrepancy - the minimum, over all plus/minus colorings of the vertices, of the maximum sum on an edge

Disconnected - a graph with more than one component

Disjoint union G_1+G_2 - the union of two graphs with disjoint vertex sets (or the union of posets with disjoint sets of elements)

Distance *d(u,v)* - the minimum number of edges (arcs) in a *(u,v)*-path

Distribution models - classical enumeration problems for objects in boxes

Distributive lattice - a lattice in which *x∧ (y∨ z) = (x∧ y)∨ (x∧ z)* for all *x,y,z*

Division algorithm - finds quotient and remainder

Domain - set on which a function is defined

Dominance ordering - ordering on partitions in which $\mu \leq \lambda$ if every partial sum in μ is at most the corresponding partial sum in λ

Domination number - the minimum size of a dominating set of vertices

Dominating set - a set $S \subseteq V$ such that every vertex outside *S* has a neighbor in *S*

Double shift graph - graph on triples of numbers from *[n]* such that *i,j,k* is adjacent to *l,m,n* if *i < j < k* and *l < m < n* and *(j,k)=(l,m)*

Double star - a tree with at most two nonleaf vertices

Doubly stochastic matrix - matrix in which every row and every column sums to 1

Double torus - the (orientable) surface with two handles

Double width - maximum size of a double antichain

Down-set - ideal

Dual code - the code whose code words form the orthogonal complement of the space of code words of the original code

Dual graph *G** - for a plane graph, the graph with a vertex for each region of *G*, in which vertices are adjacent if the boundaries of the corresponding regions of *G* share an edge (also makes sense for 2-cell embeddings on any surface)

Dual poset *P** - obtained by reversing all the relations

Dual program - a special linear (or integer) program bounding the original optimization problem

Dual matroid M^* - the bases are the complements of the bases of M

Dual ideal - an ideal in the dual of P

Duplication of vertices - adding a vertex whose neighborhood is that of the vertex duplicated

E

Ear - maximal path such that internal vertices have degree 2

Ear decomposition - successive removal of ears in a 2-connected graph

Eccentricity - for a vertex, the maximum distance to other vertices

Edge - 1) in a graph, a pair of vertices ($E(G)$ denotes the edge set). 2) in a hypergraph, a subset of the vertices

Edge chromatic number χ' - the minimum number of colors in a proper edge-coloring

Edge color class - the edges assigned a given color in a proper edge coloring

k-edge colorable - edge chromatic number $\leq k$

Edge coloring - an assignment of labels to the edges

k-edge connected - edge connectivity $\geq k$

Edge connectivity $\varkappa'(G)$ - minimum number of edges whose deletion disconnects G

Edge cover - a set of edges incident to all the vertices

Edge cut $[S,\ov\{S\}]$ - the set of edges joining a vertex in S to a vertex not in S

Edge-deleted subgraph - a subgraph obtained by deleting an edge (but not deleting its endpoints)

Edge independence number - the maximum size of a matching

Edge-induced subgraph - the subgraph consisting of a set of edges and all vertices contained in them

Edge-reconstructible - a graph that can be determined (up to isomorphism) by knowing the multiset of subgraphs obtained by deleting single edges

Edge-reconstruction conjecture - the conjecture that every graph with at least four edges is edge-reconstructible

Edge-transitive - a graph where the automorphism group act transitively on the edges (as unordered pairs)

Eigenvalue - for a graph, an eigenvalue of the adjacency matrix

$EKR(k,t)$-family - antichain of sets in which all have size at most k and pairs have at least t common elements

Elementary contraction - "shrinking" an edge by replacing the endpoints with a single vertex incident to all other edges incident to the original endpoints

Elementary cycle - 1) boundary of a region in a plane graph. 2) used to mean (simple) cycle by some authors who use cycle to mean circuit

Elementary subdivision - replacement of an edge by a path of two edges connecting the endpoints of the original edge (also called "edge subdivision")

Embedding - a mapping of a graph into a surface, such that (the images of) its edges do not intersect except at endpoints

Empty graph - having no edges

End-block - a block that intersects only one other

Endpoint - for an edge, either of its members

End-vertex - a vertex of degree 1

Equipartite - having part-sizes differing by at most one

Equivalence relation - reflexive, symmetric, and transitive relation

Equivalence class - a block of the partition induced by an equivalence relation

Equivalent codes - having the same set of codewords

k-error detecting - a code with minimum distance $k+1$

k-error correcting - a code with minimum distance $2k+1$

Erdös number - distance from Erdös in the collaboration graph of mathematicians

Erdös-Ko-Rado theorem - the largest collection of sets of size at most k in an intersecting family is the collection of k-sets containing a single element, if $k \leq n/2$

Euclidean algorithm - procedure for extracting greatest common denominator of a and b and expressing it as an integer combination of a and b

Euler characteristic χ - alternating sum of number of faces of each dimension in a simplicial complex

Euler totient - number of elements in $[n]$ relatively prime to $[n]$

Euler tour - Eulerian circuit

Eulerian - having an Eulerian circuit

Eulerian circuit - a closed trail containing every edge

Eulerian (di)graph - a graph or digraph having an Eulerian circuit

Eulerian trail - a trail containing every edge

Euler characteristic - for a genus-γ surface, 2-2γ

Euler's formula - the formula n-m+f=2-2γ for any 2-cell embedding of a connected n-vertex graph with m edges and f faces on a surface of genus γ

Even cycle - cycle with an even number of edges (or vertices)

Even graph - graph with all vertex degrees even

Even vertex - vertex of even degree

Expansive property - for set functions, $X \subseteq \sigma(X)$

Expectation - for random variable X taking integer values, $\sum kProb(X=k)$

Exponential generating function - formal power series expressing a sequence $\{a_n\}$ as $\sum a_n x^n/n!$

Exponential formula - a relation between generating functions for connected objects and arbitrary objects of the same type

Extension - for a poset, a poset on the same elements obtained by adding relations

Exterior region - the unbounded region in a plane graph

F

Face - a region of an embedding

Factor - a spanning subgraph

k-factor - a regular spanning subgraph of degree k

k-factorable - having a k-factorization

Factor-critical - graph where each single-vertex-deleted subgraph has a 1-factor

Factorial $n!$ - $\prod_{i=1}^{n} i$

Factorization - an expression of G as the edge-disjoint union of spanning subgraphs

Falling factorial $n_{(k)}$ - $\prod_{i=0}^{k-1} (n$-$i)$

Family - a collection of elements in a poset

k-family - a family containing no k+1-chain

Fano plane / Fano matroid - design with blocks 124, 235, 346, 457, 561, 672, 713

Fáry's Theorem - a planar graph has a straight-line embedding in the plane

k-factorization - a decomposition of a graph into k-factors

Feasible flow - a network flow satisfying edge-constraints and having net flow 0 at each vertex

Fermat's Little Theorem - if a is an integer not divisible by a prime p, then a^{p-1}-1 is divisible by p

Ferrer's diagram - an arrangement of unit squares with λ_i left-justified positions in the ith row, where λ is a partition of n

Ferrer's digraph - a digraph with no x,y,z,w (not necessarily distinct) such that x -> y and z -> w but failure of both z -> y and x -> w; equivalently, the successor sets or predecessor sets are ordered by inclusion; equivalently, the adjacency matrix has no 2 by 2 permutation submatrix.

Fibonacci numbers - sequence with F_0=0, F_1=1, F_n=F_{n-1}+F_{n-2} for $n \geq 2$

Field - set endowed with addition and multiplication to form an additive group with identity 0 and a multiplicative group on the nonzero elements

Filter - a dual ideal

Fixed point - an element mapped to itself

FKG inequality - positive correlation for monotone functions on a distributive lattice with a log-supermodular weight function

Flat - a closed set in a matroid

Flow - an assignment of values to variables for each arc of a network

Flower snark -

Forcibly Hamiltonian - a degree sequence such that every simple graph with that degree sequence is Hamiltonian

Forcing relation - $\{((a,b),(c,d))\}$ such that $c < d$ in every extension having $a < b$

Forest - a disjoint union of trees

Four color theorem - the theorem that planar graphs are 4-colorable

Four function inequality - Ahlswede-Daykin inequality

Fractional - for various packing and covering parameters, the solution to the linear programming relaxation of

the integer program for the unmodified parameter
H-Free - a graph not containing *H* as an induced subgraph
Free matroid - uniform matroid of rank |*E*|
Fundamental cycle - for a spanning tree, a cycle formed by adding an edge to it

G

Gammoid - a matroid induced on a subset of the vertices of a digraph
Generalized partition matroid - a direct sum of uniform matroids
Genus γ - 1) for a surface, the number of handles in its topological description; 2) for a graph, the minimum genus surface on which it embeds
Geodesic - a shortest path between its endpoints
Geodetic - a graph in which each pair of vertices *u,v* are the endpoints of a unique path of length *d(u,v)*
Geometric lattice - a semimodular lattice without infinite chains in which every element is a join of atoms
Girth *g* - the length of the shortest cycle in *G*
k-gon - in an embedding, a *k*-cycle bounding a region
k-gon order - a containment order of a collection of *k*-gons in the plane
Good coloring - often means proper coloring
Graceful labeling - injective vertex labeling from *{0,...,|E(G)|}* such that the differences between the labels on endpoints of edges form *1,...,|E(G)|*
Graceful graph - a graph with a graceful labeling
Graceful tree - a tree with a graceful labeling
Graded poset - all maximal chains have the same length
Graeco-Latin square - an orthogonal pair of Latin squares
Graph - a collection of pairs of elements from some set
Graphic matroid - a matroid that is the cycle matroid of some graph
Graphic sequence - a list of integers realizable as the vertex degrees in a simple graph
Graph Ramsey number $r(G_1,...,G_k)$ - the minimum *n* such that coloring the edges of K_n forces an *i*-monochromatic copy of G_i for some *i*
Greatest lower bound $x \wedge y$ - a lower bound of *x* and *y* that is greater than any other lower bound of *x* and *y*
Greedy algorithm - a fast non-backtracking algorithm to find a good feasible solution by iteratively making a heuristically good choice
Greedy coloring - given a vertex ordering, color each vertex with the least-indexed color not appearing on earlier neighbors
Greedy dimension - minimum number of greedy extensions realizing *P*
Greedy extension - linear extension such that x_{k+1} covers x_k if some minimal element of the subposet induced by $x_{k+1},...,x_n$ covers x_k
Greene-Kleitman Theorem - for every poset and every *k*, there is a chain partition that is both *k* and *k+1*-saturated
Grötzsch graph - the smallest triangle-free 4-chromatic graph
Grundy number - the maximum number of colors in an application of the greedy coloring algorithm

H

Hadamard matrix - an *n*-by-*n* matrix with entries in {+1,-1} 1 whose product with its transpose is *n* times the identity
Hadwiger conjecture - a *k*-chromatic graph has a subgraph contractible to K_k (true for "almost all" graphs)
Hajós conjecture - a *k*-chromatic graph has a subgraph homeomorphic to K_k (false for *k≥7*)
Halin graph - obtained from a planar embedding of a tree by adding a cycle through the leaves in order
Hall's Condition - 1) |*N(S)*| ≥ |*S*| for all *S⊆X*; 2) |$\cup_{i \in I} A_i$| ≥ |*I*| for all *I⊆[m]*
Hall's Theorem - Hall's Condition is necessary and sufficient condition for the existence of a 1) matching, 2) system of distinct representatives
Hamilton tour - Hamiltonian cycle
Hamiltonian - having a Hamiltonian cycle
Hamiltonian-connected - having a spanning path from each vertex to every other

Hamiltonian cycle - a cycle containing each vertex

Hamiltonian path - a path containing each vertex

Hamming distance - number of positions in which coordinates differ

Harary graphs - graphs of minimum size for given order and connectivity

Hasse diagram - cover diagram

Head - the second vertex of an edge in a digraph

Head partition matroid - the partition matroid induced on the edges of a digraph by the partition according to heads

Heawood Theorem - the chromatic number of a graph embedded on the oriented surface with γ handles is at most $\lfloor \frac{1}{2}(7+\sqrt{1+48\gamma}) \rfloor$.

Height - size (or one less than size) of largest chain in poset (or having x at the top

Helly property - the property of the real line (or trees) that any collection of pairwise intersecting subsets has a common intersection point

Helly number - for some universe, the number k such that the sets in F have a common intersection if every k sets of F have a common intersection

Hereditary class - a class F for which every subgraph of a graph in F is also in F

Hereditary hypergraph - every subset of an edge is an edge (an ideal of sets)

Hereditary property - a graph property preserved by taking induced subgraphs

Hereditary system - on a ground set of elements E, may be specified by an order ideal I, the antichain of maximal elements of I, the antichain of minimal elements not in I, a rank function r, a span function σ, or other "aspects"

Homeomorphic - obtainable from the same graph by subdivision of edges

Homogeneous set - in Ramsey theory, a set whose subsets receive the same color

Homomorphism - a map $f: V(G) \rightarrow V(H)$ that preserves adjacency

Hook - L-shaped subset of positions in a tableau

Hook length - size of hook

Hook length formula - formula for number of tableaux of a given shape

Hungarian method - an algorithm for solving the assignment problem

Husimi tree - a graph in which every block is a clique

Hypergraph - a generalization of graph in which edges may be any subset of the vertices, equivalently, a set system

Hyperplane - a maximal closed nonspanning set of matroid elements

Hypohamiltonian - a non-Hamiltonian graph whose vertex-deleted subgraphs are all Hamiltonian

Hypotraceable - a non-traceable graph whose vertex-deleted subgraphs are all traceable

I

Ideal I - a collection of elements such that $x \leq y \in I$ implies $x \in I$

Idempotence - $f^2 = f$, where f is a function

Identification - an operation replacing two vertices by a single vertex with the combined incidences (similar to contraction if the vertices were adjacent

Implication class - a collection of edges such that the orientation of any one determines the orientation of the others in any transitive orientation

Incidence algebra - functions defined on intervals in a poset, with fg defined by $fg(x,y)=\sum_{x\leq z\leq y} f(x,z)g(z,y)$

Incidence matrix - the matrix in which entry (i,j) is 1 if vertex j belongs to edge i, and 0 if not (for a digraph, 1 if vertex j is the head of edge i, -1 if it is the tail, 0 otherwise)

Incident - 1) for a vertex v and edge e, v belongs to e. 2) edges are incident if they intersect

Incomparable $x \| y$ - neither $x < y$ nor $y < x$

Incomparable pair - ordered pair of incomparable elements

Incomparability graph - obtained from a partial order by letting elements be adjacent if they are incomparable in the partial order

Indegree - for a vertex in a digraph, the number of edges of which it is the head

Independence number α - maximum size of an independent set of vertices

Independent domination number - minimum size of an independent dominating set

Independent events - $Prob(A \cap B)=Prob(A)Prob(B)$

Independent set - 1) in graphs, a set of pairwise nonadjacent vertices; 2) in hereditary systems and matroids,

an element of the ideal

Indifference graph - representable by assigning weights to vertices such that $u<-> v$ if and only if $|f(u)-f(v)| \leq 1$

Induced sub(di)graph (G_A or $G[A]$) - the sub(di)graph on vertex set $A \subset V$ obtained by taking all edges of the original graph that join two vertices in A

Integer program - a combinatorial optimization problem in integer variables

Internal vertices - 1) for a path, the non-endvertices. 2) for a plane graph, the vertices that do not belong to the boundary of the exterior face

Internally disjoint paths - paths intersecting only at end-vertices

Intersecting family - a pairwise intersecting collection of sets

S-intersecting family - a family whose pairwise intersections have sizes only in S

t-intersecting family - a family whose pairwise intersections have size at least t

Intersection - 1) for posets P,Q on the same set, the poset on that set having $x \leq y$ if and only if $x \leq y$ in both P and Q; 2) for matroids, the hereditary system whose independent sets are the sets independent in both matroids

Intersection graph - for a collection of sets, the graph with a vertex for each set, in which vertices are adjacent if the sets intersect

Intersection number - minimum size of a set such that G is an intersection graph of subsets of it (equals minimum number of cliques covering edges)

Interval count - minimum number of distinct interval lengths in a representation of an interval graph

Interval dimension - minimum size of a realizer by interval extensions

Interval extension - interval order that is an extension

Interval graph - a graph having an interval representation

Interval number - minimum t such that G (or P) has a t-interval representation

Interval order - a poset representable by real intervals such that $x < y$ if and only if I_x is completely to the left of I_y.

Interval representation - a collection of intervals whose intersection graph is G (or that represent P)

t-Interval representation f - a mapping $f:V(G) \to \mathbf{R}$ such that each image consists of at most t intervals and $v<->w$ iff $f(v) \cap f(w) \neq \varnothing$

Inversion - 1) a pair of elements k,l in a permutation σ of $[n]$ such that $\sigma(i)=k$ and $\sigma(j)=l$ with k,l in the opposite order to i,j

Involution - a permutation of order two (fixed points and disjoint transpositions)

Involution principle - a procedure for generating bijective proofs from involutions

Irreducible poset - deletion of any element reduces the dimension

Isolated - vertex (or edge) incident to no (other) edge

Isometric embedding - a distance-preserving mapping of $V(G)$ into $V(H)$

Isomorphism - a bijection between sets of vertices (or elements) that preserves structure; graphs (preserves adjacency), digraphs (preserves succession), posets (preserves order), lattices (preserves meets and joins), etc.

J

Jeu de Taquin - procedure of reducing a skew tableau to a column-strict tableau

Join $G \vee H$ - a graph combination obtained by taking disjoint copies of G,H and putting $u<->v$ for every $u \in V(G), v \in V(H)$

Join $x \vee y$ - the least upper bound

Joined to - adjacent to

K

Kempe chain - a path alternate between two colors

Kernel - independent dominating set

Kirchhoff's current law - net flow around a closed walk is 0

Kirkman triple system - resolvable Steiner triple system

Kleitman's inequality - membership in ideals is positively correlated (holds as generally as distributive lattices

Knotting graph - a modification of a graph that is bipartite if and only if the graph is a comparability graph

König's Theorem - independence number equals edge cover number for bigraphs

König-Hall-Egervary Theorem - maximum matching equals vertex cover number for bipartite graphs

Kronecker product - weak product

Kruskal's algorithm - greedy algorithm for minimum weighted spanning tree

Kruskal-Katona theorem - the collection of m k-sets having the minimum $k-1$-shadow is the first m k-sets in the reverse lexicographic order

Kuratowski's theorem - a graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$

L

Labeling - a mapping to the integers

Latin square - an n by n arrangement of n copies each of n symbols, such that each symbol appears exactly once in each row and column

Lattice - poset where each two elements have a greatest lower bound and a least upper bound

Lattice path - a path from one integer point to another, moving by a specified set of integer steps, usually $+1$ in one coordinate

Leaf - an endvertex of a tree

Least upper bound $x \wedge y$ - an upper bound of x and y that is less than any other upper bound of x and y

Length - 1) in graphs, the number of edges (counted with multiplicity, if necessary); 2) in posets, height; 3) in codes, the number of code digits (n)

Lexicographic product - composition

Line - another name for edge

Line graph $L(G)$ - the intersection graph of the edges of G, i.e. vertices correspond to edges of G and are adjacent if the corresponding edges intersect

Linear code - a code f in which message words and codewords belong to vector spaces, $f(x+y)=f(x)+f(y)$, and $f(ax)=af(x)$

Linear extension - an extension of a poset that is a chain

Linear order - totally ordered set (chain)

Linear program - maximization (or minimization) of a linear objective function of n variables, subject to linear constraints

q-Linear matroid - matroid whose independent sets are representable as the sets of independent columns of a matrix over a field of q elements

Link - edge

k-linked - a stronger condition than k-connected, in which for every choice of two k-tuples of vertices $(u_1,...,u_k)$ and $(v_1,...,v_k)$, there exists a set of k internally disjoint paths connecting corresponding vertices u_i,v_i.

Locally finite - every interval has finitely many elements

Log-concave sequence - $a_k^2 \geq a_{k+1}a_{k-1}$

Log-supermodular function - $\mu(x \wedge y)\mu(x \vee y) \geq \mu(x)\mu(y)$

Loop - an edge joining a vertex to itself; a circuit of size 1 in a matroid

Loop-graph - pseudograph

Lower bound - 1) for elements in a poset, an element that is less than or equal to all of them; 2) for a poset, an element less than all others

Lower extension of $Q \subset P$ - an extension of Q such that $x>y$ whenever $x \in Q$, $y \in P-Q$, and $x \| y$

Lower semimodular lattice - $y \backslash prec \; x \vee y$ implies $x \wedge y \backslash prec \; x$ for all x,y

Lucas numbers - satisfy the Fibonacci recurrence and have $L_1=1$ and $L_2=3$.

LYM inequality - the sum of reciprocals of rank-sizes is at most 1

LYM property - every antichain satisfies the LYM inequality

M

Majorization - for random variables, $Prob(G>r) \geq Prob(H>r)$ for all r

Martingale - sequence of random variables in which the expectation of X_i given the values of $X_1,...,X_{i-1}$ is the value of X_{i-1}

Matching - a set of pairwise disjoint edges in a graph or hypergraph

b-matching - given a constraint vector b, a subgraph H with $d_H(v) \leq b(v)$ for all v

Matric matroid - representable matroid

Matrix game - 2-person 0-sum game, with payoff from one player to other given by a matrix indexed by the player's pure options

Matrix-tree theorem - counting of spanning trees of G as cofactor of the matrix D-$A(G)$

Matroid - a hereditary system satisfying any one of a plethora of equivalent useful additional conditions

Matroid intersection theorem - the maximum size of a common independent set in matroids M_1 and M_2 on E is the minimum of $r_1(X)+r_2(E$-$X)$ over sets $X{\subseteq}E$

Max-flow min-cut theorem - maximum flow value equals minimum cut value

Maximal - used with sets satisfying any property, meaning that addition of anything destroys the property (examples follow)

Maximal chain - no additional element is comparable to all elements of the chain

Maximal clique - a maximal vertex set inducing a clique

Maximal element - an element of a poset that is not covered by any other

Maximal planar graph - a simple planar graph where adding any edge destroys planarity

Maximum - a "maximum set" of some type is a maximum-sized set of that type (implies maximal) (examples - maximum antichain, maximum clique)

Maximum degree Δ - maximum of the vertex degrees

Maximum flow - a feasible flow of maximum value, or the value of such a flow

Maximum genus $\gamma_M(G)$ - the maximum genus of a surface on which G is 2-cell-embeddable

Meet $x {\wedge} y$ - the greatest lower bound

Metric - a real-valued symmetric non-negative binary function that is 0 only when the arguments are equal and satisfies the triangle inequality

Menger's theorems - minimax characterizations of connectivity by number of internally-disjoint or edge-disjoint paths between pairs of vertices

Minimal - used with any property, meaning that deletion of anything destroys the property

Minimum - a "minimum set" of some type is a minimum-sized set of that type (implies minimal) (examples - minimum chain cover)

Minimum cost flow problem - miinimize the cost of a feasible flow

Minimum cut - a network cut having minimum value, or the value of such a cut

Minimum cutset - a minimum-sized set of elements meeting all maximal chains

Minimum degree $\delta(G)$ - minimum of the vertex degrees

Minimum spanning tree - spanning tree with minimum sum of edge weights

Minimum vertex cover - a minimum-sized set of vertices covering the edges

Min-max relation - a universal equality between the solution values to a pair of natural dual combinatorial optimization problems, typically dual linear integer programming problems

Minor - 1) a contraction of a subgraph; 2) a restriction of a contraction

Mixed graph - a graph concept allowing directed and undirected edges

Möbius function - the inverse of ζ in the incidence algebra, defined on intervals of P by $\mu(x,x)=1$ and $\mu(x,y)=$-$\sum_{x \leq z < y}\mu(x,z)$

Möbius inversion formula - generalization of inclusion-exclusion to posets, in which f is obtained from g satisfying $g(x)=\sum_{y \leq x}f(y)$ by $f(x)=\sum_{y \leq x}g(y)\mu(y)$, where $\mu(y)$ is the value $\mu(0,y)$ for the Möbius function of the poset

Möbius ladder - obtained by adding to an even cycle the chords between diametrically opposite vertices (a ladder with a twist)

Möbius strip - the non-orientable surface obtained by identifying two opposite sides of a rectangle using opposite orientation

Modular lattice - $x {\wedge} (y {\vee} z)=(x {\wedge} y) {\vee} z$ whenever $z \leq x$

Monochromatic - in a coloring, a set having all elements the same color

Monotone property - a graph property preserved by taking arbitrary subgraphs

Multigraph - allows multiple edges (arcs), but generally no loops

Multiple edges - repeated pairs of vertices in the edge set

N

Nash equilibrium - optimal strategies for players in a matrix game

Neighbors - (noun) the vertices adjacent to a given vertex; (verb) "is adjacent to"

Neighborhood $N(v)$ - if "open," same as adjacency set; if "closed," then includes in addition the vertex itself (and notation becomes $N[v]$)

Net flow - at a vertex, the sum of flows on exiting edges minus the sum of flows on entering edges

Network - a directed graph with a distinguished initial vertex (set) and a distinguished terminal vertex (set), in which each edge is assigned a flow capacity and sometimes also a flow demand (lower bound)

Node - vertex, especially in network flow problems

Node potential - dual variable in min cost flow problem

Normal product - strong product

Normalized matching property - for any set of elements A at rank k with shadow $A*$ at rank $k+1$, $|A*|/N_{k+1} \geq |A|/N_k$

Null graph - graph having no vertices

O

Odd anti-hole - complement of an odd hole

Odd component - component with an odd number of vertices

Odd cycle - cycle with an odd number of edges (vertices)

Odd hole - chordless odd cycle

Odd vertex - vertex of odd degree

On-line algorithm - algorithm that only learns the input one item at a time

Open walk - walk in which the end-vertices differ

Optimal tour - solution to Traveling Salesman problem or Chinese Postman problem

Order - the number of vertices, sometimes called "size" when no confusion is (hopefully) possible

Order dimension - minimum number of linear extensions whose intersection is P

Ordered set - partially ordered set

Order-reversing - a function from the elements of one ordered set to another such that $x \leq y$ implies $f(x) \geq f(y)$

Order polynomial - value at k counts the order-preserving functions from P to $[k]$

Order polytope - the order-preserving functions from a poset to the interval $[0,1]$, viewed as a subset of $\mathbf{R}^{|P|}$

Order-preserving - a function from the elements of one ordered set to another such that $x \leq y$ implies $f(x) \leq f(y)$

Ordinal sum $P \oplus Q$ - a poset obtain from the disjoin union $P+Q$ by placing each element of Q above each element of P

Ordinary generating function - formal power series expressing $<a>$ as $\sum a_n x^n$

Orientable surface - a surface with two distinct sides

Orientation - an assignment of order to each of the edge pairs in an undirected graph, making it a directed graph

Orthogonal - 1) for two chain decompositions, having no pair on the same chain in both decompositions; 2) for a chain partition and an antichain partition, each chain intersects each antichain; 3) for Latin squares, each pair of elements appears in corresponding positions exactly once

Outdegree - for a vertex, the number of arcs of which it is the tail

Outerplanar graph - a planar graph embeddable in the plane so that all the vertices belong to the boundary of the exterior region

Out-of-Kilter algorithm - algorithm to solve min-cost flow problems

Overlap graph - subgraph of the intersection graph of a set of intervals, obtained by discarding edges generated by one interval contained in another

P

Packing number - the maximum number of something findable in a graph without violating some condition such as pairwise disjointness, nonadjacency, etc.

Packing problem - in hypergraphs, find the largest edge in the antiblocker

Page - one of the outerplanar subgraphs in a book embedding

Pagenumber - minimum number of pages in a book embedding

Pan-connected - the condition of having, for every pair of vertices u,v, (u,v)-paths of all lengths at least $d(u,v)$

Pancylic - having cycles of all lengths at least 3

Parallel edges - multiple edges

Parallel elements - circuits of size two in a matroid

k-partite - having a vertex partition into k parts such that no edges join vertices in the same part; same as k-colorable

Partial extension - an extension of a poset, usually not a linear extension

Partially ordered set $P=(X,R)$ - formally, a set of elements X and binary order relation R that is reflexive, antisymmetric, and transitive

Partite set - what are usually called the "parts" in a k-partite graph

Partition matroid - matroid with independent sets being those with at most one element in each block of a given partition of E

Partitionable graph - G such that for any $x \in V(G)$, G-x is partitionable into $\alpha(G)$ cliques of size $\omega(G)$ and into $\omega(G)$ stable sets of size $\alpha(G)$

Part-sizes - sizes of the partite sets

2-part Sperner property - largest semiantichain in product poset is a single rank

Path - an open walk with no repeated vertices (in a digraph, must follow arrows)

(u,v)-path - a path with u and v as endpoints

Pattern inventory - a generating function in many variables in which the coefficient of $x_j^{e_j}$ is the number of patterns in which type j appears with weight e_j

Pendant edge - incident with a pendant vertex

Pendant vertex - 1-valent vertex

Perfect code - single-error correcting code in which every possible received word is within distance one of a codeword

Perfect graph - chromatic number equals clique number for all induced subgraphs

α-perfect - same as perfect

γ-perfect - a graph whose complement is perfect

Perfect graph theorem - a graph is perfect if and only if its complement is perfect

Perfect matching - edge set of a 1-factor

Permanent - for an n-by-n matrix, the sum over all collections of n independent positions of the product of the entries in those positions

Permutation graph - representable by a permutation σ such that $v_i <\text{-}> v_j$ if and only if σ reverses the order of i and j

Petersen graph - a graph disproving many reasonable conjectures, it is the complement of the intersection graph of the 2-sets of a 5-element set

Pigeonhole principle - in a set of numbers, there is one that is at least the average

Planar - embeddable in the plane 1) for graphs, drawable in plane without crossings; 2) for posets, having a planar cover diagram

Plane graph - a particular planar embedding of a planar graph

Plane partition - placement of integers in the position of a Ferrers diagram so that rows and columns are nondecreasing

Platonic solid - bounded regular polyhedron

Point - vertex

Poisson paradigm - technique for obtaining sharp threshold functions

Polya's Theorem - technique for counting equivalence classes by patterns

Polygon matroid - cycle matroid

Polyhedron - an intersection of half-spaces

Polynomial code - linear codes encoded by polynomial multiplication

Polytope - the convex hull of a set of vertices

Poset - partially ordered set

Potential - a vertex labeling used in the dual to the min-cost flow problem

kth-power (G^k) - the graph on the same vertices as G in which vertices are joined by an edge if G has a walk of length k thbetween them

Predecessor - for v in a digraph, a vertex u with $u \text{-}> v$

Predecessor set - for v in a digraph, the set of precedessors

Principle ideal - an ideal generated by one element (having one maximal element)

Product - obtained by combining factors in several possible ways - see Cartesian product, direct product,

normal product, strong product, weak product

Product dimension - minimum number of cliques whose weak product contains G as an induced subgraph

Projective plane - n^2+n+1 points and n^2+n+1 lines with each two points on one common line and each two lines having one common point

Proper coloring - 1) for vertices, a coloring in which no edge is monochromatic. 2) for edges, a coloring in which no intersecting edges get the same color

Proper interval graph - having an interval representation in which no interval contains another

Proper subgraph - a subgraph not equal to the graph itself

Prüfer code - for a labeled tree, a sequence of length $n-2$ obtained by successively deleting the leaf with smallest label and recording its neighbor's label

Pseudograph - graph model with both loops and multiple edges permitted

Q

R

Radius - the minimum eccentricity of the vertices

Ramsey number - the minimum number of vertices such that assigning colors to all pairs of those vertices produces a monochromatic clique of specified size (or a specified graph) in one of the colors

Random graph - a graph from a probability space, most often the space in which each labeled pair of vertices independently has probability p of adjacency; typically, $p=1/2$ or p is a function of n

Rank - 1) in posets, length of longest chain, a set of elements with the same height, or the maximum size of a minimal relaizer by linear extensions

Rank function - 1) in a poset, increases by 1 with cover relations; 2) in a matroid, gives largest size of an independent set in X; 3) for a hypergraph, gives largest size of an edge contained in S

Ranked - having a rank function

Ranking - a ranked poset having all relations between ranks

Reachability matrix - for a directed graph, the matrix in which entry i,j is 1 if there is a path from vertex i to vertex j, otherwise 0

Realizer - a collection of extensions whose intersection is P

Reciprocity theorem - in general, a relationship between two functions f,f' stating $f'(n,k)=(-1)^n f(n,-k)$

Reconstructible - a graph determined (up to isomorphism) by its multiset of subgraphs obtainable by deleting a single vertex

Reconstruction conjecture - the conjecture that all graphs with at least 3 vertices are reconstructible

Rectilinear crossing number - the minimum number of crossings in a drawing of the graph in the plane in which all edges appear as straight line segments

Reflexive - 1) for a digraph, having a loop at every vertex; 2) for a binary relation R, having xRx for all x

Region - for an embedding of a graph on a surface, a maximal connected subset of the surface that does not contain any part of the graph

Regular - 1) for a graph, having all vertex degrees equal; 2) for a poset, having each element at rank k cover the same number of elements and be covered by the same number of elements, for all k; 3) for a matroid, being representable as a linear matroid over any field.

Regular covering - nonempty list of maximal chains such that every element appears as often as as every other element in its rank

k-regular - having all vertex degrees equal k

Removable pair - pair of elements whose deletion reduces dimension by at most 1

Representable matroid - a matroid whose independent sets are the independent sets of columns of some matrix (over some field)

Restriction - restriction of a matroid M to elements F is the matroid on F whose independent sets are the subsets of F independent in M

Reverse lexicographic order - $x < y$ if $x_i<y_i$ in last component where they differ

Rigid circuit graph - chordal graph

Root - 1) a distinguished vertex; 2) for a directed tree, a vertex from which every other is reachable

Rooted - having a root specified

Rotation scheme - a description of a 2-cell embedding; a circular permutation of the edges at each vertex, giving their counter-clockwise order around the vertex

RSK (Robinson-Shensted-Knuth) correspondence - a bijection from permutations of *[n]* to pairs of Young tableaux on *n* elements having the same shape; extended by Knuth beyond permutations

S

Saturated arc - for a network flow, an arc in which the flow equals the capacity

k-Saturated partition - a partition whose *k*-norm equals the largest size of a *k*-family

Saturated vertex - 1) for a matching, a matched vertex. 2) for a *b*-matching, a vertex with *b(v)* incident edges.

Score sequence - the sequence of outdegrees in a tournament

k-scrambling - a family of sets where any *k* yield a Venn diagram with all cells nonempty

Second moment method - method for obtaining threshold functions

Self-complementary - isomorphic to the complement

Self-converse - isomorphic to the converse

Self-dual - isomorphic to the dual

Semiantichain - in $P \times Q$, a collection in which comparable pairs must differ in both coordinates

Semi-strong perfect graph theorem - intermediate between the perfect graph theorem and strong perfect graph theorem, concerns the P_4-structure of a graph

Semilattice - poset where each pair of elements has a greatest lower bound (meet semilattice), or where each pair has a least upper bound (join semilattice)

Semimodular lattice - (also "upper semimodular") $x \wedge y \backslash prec\ x$ implies $y \backslash prec\ x \vee y$ for all *x,y*

Semiorder - a poset representable by a function *f* and threshold δ such that $x < y$ if and only if $f(x) < f(y)-\delta$

Semipath - for a digraph, a path in the underlying graph

Semiwalk - for a digraph, a walk in the underlying graph

Separating set - a vertex set whose deletion increases the number of components

Separator theorem - for a hereditary class of graphs, specifies a small function of *n* such that deleting at most that many vertices from a graph in the class splits the remaining vertices in a balanced way

k-Shadow - for a family *F*, the set of elements with rank *k* that are related to elements of *F*

Shannon capacity - limit of *k*th root of independence number of *k*th power of graph

Signed (di)graph - special case of weighted (di)graph, assinging + or - to each edge

Simple - 1) graph having no loops or multiple edges; 2) hypergraph having no loops or edges sharing two vertices; 3) matroid having no loops or parallel elements; 4) poset having width equal to the number of maximal chains; 5) polytope having every vertex of degree equal to the number of dimensions

Simple game - an *n*-person game in which every subset of players is winning or losing, equivalent to an order-preserving function on B_n

Simplex - 1) method for solving linear programs; 2) the convex hull of *d+1* nondegenerate points in a *d*-dimensional metric space

Simplicial - 1) vertex whose neighbors induce a clique; 2) hypergraph where every subset of an edge is also an edge; 3) polytope where every face is a simplex

Sink - the distinguished terminal vertex (set) in a network

Size - the number of edges, sometimes used for the number of vertices

Source - the distinguished initial vertex (set) in a network

Span function $\sigma(X)$ - in hereditary systems and matroids, *X* together with the elements that complete circuits with elements of *X*

Spanning - 1) subgraph containing each vertex; 2) subset of matroid elements that span the entire ground set

Spectrum - the set of eigenvalues

Sperner property - for a ranked poset, having a maximum antichain consisting of a single rank

k-Sperner property - having a maximum *k*-family consisting of the union of *k* ranks

Sperner's lemma - a properly labeled simplicial decomposition of a simplex has a completely labeled fundamental simplex

Sperner's theorem - B_n has the Sperner property

Split graph - having a vertex partition into a clique and an independent set

Splitting element - comparable to every other

Square of a graph - the second power

Star - 1) in graphs, a tree with at most one non-leaf ($K_{1,n-1}$); 2) in hypergraphs (or families of sets), a collection all containing a single vertex (element)

Steiner triple system - block design using blocks of size 3

Steinitz(-Maclane) exchange property - for a set function σ, $e \notin \sigma(X)$ and $e \in \sigma(X \cup f)$ imply $f \in \sigma(X \cup e)$

Stirling's formula - $n! = n^n e^{-n} \sqrt{2\pi n}(1+o(1))$

Stirling number (of the second kind) $S(n,k)$ - counts partitions of $[n]$ into k blocks

Stirling numbers of the first kind $s(n,k)$ - the number of permutations of $[n]$ with k cycles, multiplied by $(-1)^{n-k}$

Strict chain - $x_1 < x_2 < \ldots < x_n$

Strict correlation - $Prob(A|B) > Prob(A)$

Strict Sperner property - all maximum antichains are single ranks

Strictly balanced - average vertex degree is strictly greater than average vertex degree of any subgraph

Strong component - maximal strongly connected subdigraph

Strong digraph - strongly connected

Strong order on S_n - $\sigma \prec \tau$ if τ is obtained from σ by transposing any pair of elements to increase the number of inversions

Strong orientation - strongly connected orientation

Strong perfect graph theorem - a graph is perfect if and only if it has no odd hole or odd antihole

Strong product $G_1 \cdot G_2$ - a graph product with vertex set $V(G_1) \times V(G_2)$ and edge set $(u_1,v_1) <-> (u_2,v_2)$ if $u_1 = u_2$ or $u_1 <-> u_2$ and $v_1 = v_2$ or $v_1 <-> v_2$

Strong Sperner property - k-Sperner for all k

Strongly connected - digraph having a (u,v)-path for each ordered pair (u,v) of vertices

Strongly chordal - a chordal graph having a perfect elimination scheme in which the neighbors of the vertex to be deleted have neighborhoods forming a chain under inclusion; equivalent to forbidding trampolines as induced subgraphs

Strongly perfect - a graph in which some stable set meets every maximal clique

Subdigraph - a subgraph of a directed graph

Subdivision - replacement of edges by paths

Subgraph - a graph whose vertices and edges all belong to G

Sublattice - a subposet of a lattice that is a lattice and inherits meets and joins from the full lattice

Submodular function - $r(x \wedge y) + r(x \vee y) \leq r(x) + r(y)$

Subposet - a poset on a subset of the elements that inherits all relations among the elements (analogous to induced sugraph)

Subspace - a closed set in a matroid

Successor - for u in a digraph, a vertex v with $u \to v$

Successor set - for u in a digraph, the set of successors

Sum - 1) for cycles and cocycles, taken modulo 2. 2) for a graph, the disjoin union. 3) for matroids on disjoint sets, the matroid on their union whose independent sets are all unions of an independent set from each

Supergraph - a graph of which G is a subgraph

Supermodular function - $r(x \wedge y) + r(x \vee y) \geq r(x) + r(y)$

2-switch - replacing two independent edges with two edges forming a 4-cycle with them

Symmetric - 1) for a graph, having a non-trivial automorphism; 2) for a digraph, having $u \to v$ iff $v \to u$ (equivalent to an undirected graph); 3) for a binary relation R, having xRy iff yRx

Symmetric chain - in a poset of rank n, meeting ranks j and $n-j$ for some set of indices j

Symmetric chain decomposition - partition of P into symmetric consecutive chains

Symmetric chain order - having a symmetric chain decomposition

System of distinct representatives - from a collection of sets, a choice of one member from each set so that all the representatives are distinct

T

Tail - the first vertex of an edge in a digraph

Tail partition matroid - the partition matroid induced on the edges of a digraph by the partition according to tails

Tait coloring - for a planar cubic graph, a proper 3-edge-coloring

Tensor product - weak product

Terminal edge - a cut edge incident with an endvertex

Thickness θG - the minimum number of planar graphs whose union (G) G

Threshold graph - having a threshold and a vertex weighting such that $u \nadj v$ iff $w(u) + w(v) \leq t$

Threshold dimension - minimum number of threshold graphs whose union is G

Threshold function - a parametrized expression for probability in a sequence of random variables that almost ensures or almost forbids a property, depending on the value of the parameter

Topological minor - a graph for which a subdivision occurs as a subgraph of G

Toroidal - 1) graph having a 2-cell embedding on the torus; 2) topological parameter on the torus in place of the plane (toroidal thickness, crossing number, etc.)

Torus - the (orientable) surface with one handle

Total coloring - a labeling of both the vertices and edges

Total domination number - minimum number of vertices such that

Total interval number - minimum of the total number of intervals used to represent G as the intersection graph of unions of intervals on the real line

Total graph - the intersection graph of the sets in $V(G) \cup E(G)$, for some G

Total order - chain

Totally unimodular - all square submatrices have determinant in $\{0,-1,+1\}$

Toughness - minimum t such that $|S| \geq tc(G\text{-}S)$ for all $S \subseteq V(G)$, where $c(G\text{-}S)$ is the number of components of $G\text{-}S$

Tournament - an orientation of the complete graph

Traceable - having a Hamiltonian path

Trail - a walk in which no edge appears more than once

Trampoline - a split graph consisting of a clique on $x_1,...,x_k$, a stable set on $y_1,...,y_k$, and the edges y_ix_i and y_ix_{i+1} (cyclically) for all i

Transitive - 1) for a digraph, uw must be an arc whenever uv and vw are arcs. 2) for a group action, such as the automorphism group acting on the vertice set, the existence of a group operator mapping each element to any other

Transitive closure - 1) for a digraph D, the digraph with $u \to w$ whenever there is a path from u to w in D; 2) for a relation R, the relation S with xSy whenever there is a sequence $x_0,...,x_k$ with $x=x_0Rx_1R \ldots Rx_k=y$

Transitive orientation - an orientation of an undirected graph that makes it a transitive digraph

Transposition - interchange of two elements in a permutation

Transportation problem - generalization of the assignment problem with supplies at each source and demands at each destination

Transversal - set of vertices meeting each edge of a hypergraph, sometimes

Transversal matroid - matroid on E whose independent sets are the partial systems of distinct representatives of a fixed set system on E

Tree - a connected graph with no cycles

Triangle - a cycle of length 3; i.e., K_3

Triangle inequality - $d(x,y)+d(y,z) \geq d(x,z)$

Triangle-free - having no induced triangle

Triangulated - having no chordless cycle

Triangulation - a graph embedding on a surface such that every region is a 3-gon

Trivalent - 3-regular

Trivial - having no edges

Turán number - 1) for a specified hypergraph, -. 2) for parameters n,k,b,

Turán's theorem - charcterization of the complete equipartite r-partite graphs as the largest graphs of a given order with no $r+1$-clique

Tutte's Theorem - 1) characterization of graphs with 1-factors (also f-factors); 2) 3-connected planar graphs have embeddings with all bounded faces convex; 3) characterization of 3-connected graphs by contractions to wheels.

U

Underlying graph - the undirected graph obtained by removing the orientation from the edges of a directed graph

Unforced pair - critical pair, minimal element of forcing relation

Unichain - in a product, a chain that is constant in one coordinate

Unicyclic - having exactly one cycle

k-Uniform hypergraph - having each edge of size k

Uniform matroid $U_k(n)$ - has as bases all k-sets of the n-element ground set

Unimodal - a sequence such that $a_1 \leq \ldots \leq a_k \geq a_{k+1} \geq \ldots \geq n$ for some k

Unimodular - for matrices, having determinant 0, +1, or -1

Union 1) for graphs, $G_1 \cup G_2$ is a graph whose vertex set is the union of the vertices in G_1 and G_2 and whose edge set is the union of the edges in G_1 and G_2 (written $G_1 + G_2$ if the vertex sets are disjoint); 2) for matroids, $M_1 \cup M_2$ is the matroid whose independent sets are the unions of independent sets in M_1 and M_2

Unit interval graph - having a representation using intervals of the same length

Universally correlated - events positively correlated in every poset

Up-set - a dual ideal

Upper bound - 1) for elements in a poset, an element greater than or equal to all of them; 2) for a poset, an element dominating all others

Upper bound graph - undirected graph on the elements of poset such that vertices are adjacent if and only if the corresponding elements of the poset have a common upper bound

Upper extension of $Q \subset P$ - an extension of Q such that $x < y$ for all $x \in Q$, $y \in P-Q$, $x \| y$

V

Valence - degree

Value - 1) for a flow, the net flow out of the source or into the sink; 2) for a matrix game, the best result that each player can guarantee

Van der Waerden number - minimum *[n]* such that k-colorings of the integers *[n]* must have a monochromatic arithmetic sequence with l terms

Vectorial matroid - representable matroid

Vertex - element of $V(G)$, the vertex set

Vertex cut - a separating set of vertices

Vertex set $V(G)$ - the set of elements on which the graph is defined

Vertex chromatic number - chromatic number

Vertex connectivity - connectivity

Vertex cover - a set of vertices containing at least one endpoint of every edge

Vizing's Theorem - $\chi'(G) \leq \Delta(G)+1$ for graphs; $\chi'(G) \leq \Delta(G)+\mu(G)$ for multigraphs, where $\mu(G)$ is the maximum multiplicity of an edge

Voltage graph - a directed graph with edges labeled by elements of a group; used to study embeddings of a larger graph derived from the voltage graph.

W

Walk - a sequence of vertices and edges in a graph such that each vertex belongs to the edge before and after it (in a digraph, must follow arrows)

Weak chain - $x_1 \leq x_2 \leq \ldots \leq x_n$

Weak order - a ranking

Weak order on S_n - $\sigma \backslash prec\ \tau$ if τ is obtained from σ by transposing adjacent elements to increase the number of inversions

Weak product $G_1 \otimes G_2$ - a graph product with vertices $V(G_1) \times V(G_2)$, and edges $(u_1,v_1)<->(u_2,v_2)$ iff $u_1<->u_2$ and $v_1<->v_2$

Weakly connected - a directed graph whose underlying graph is connected

Weight - 1) a real number; 2) for a binary vector, the number of ones

Weighted - having an assignment of weights (to edges and/or vertices)

Wheel - a graph obtained by taking the join of a cycle and a single vertex

Whitney numbers (of the second kind) - rank sizes of poset

Whitney numbers of the first kind - coefficients of the characteristic polynomial

Width $w(P)$ - size of largest antichain

X

X-join - lexicographic product
XYZ inequality - the events $x < y$ and $x < z$ are positively correlated (in any poset)

Y

Young lattice - lattice of partitions of all integers, ordered componentwise
Young tableau - placement of the integers *[n]* in the positions of a Ferrers diagram so that entries are increasing in every row and column

Z

Zeta function - incidence function defined by $\zeta(x,y)=1$ for all $x{\leq}y$

[Index Page](...)