IEOR 4150 Homework 1

Mengqi Zong $< mz2326@columbia.edu >$

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1.

Problem 3

a)

$$\begin{aligned} EF &= \{1,3,5,7\} \cap \{7,4,6\} = \{7\} \end{aligned}$$

b)

$$\begin{aligned} E \cup FG &= \{1,3,5,7\} \cup (\{7,4,6\} \cap \{1,4\}) \\ &= \{1,3,5,7\} \cup \{4\} \\ &= \{1,3,4,5,7\} \end{aligned}$$

c)

$$\begin{aligned} EG^C &= \{1,3,5,7\} \cap \{1,4\}^C \\ &= \{1,3,5,7\} \cap \{2,3,5,6,7\} \\ &= \{3,5,7\} \end{aligned}$$

d)

$$\begin{aligned} EF^C \cup G &= (\{1,3,5,7\} \cap \{7,4,6\}^C) \cup \{1,4\} \\ &= (\{1,3,5,7\} \cap \{1,2,3,5\}) \cup \{1,4\} \\ &= \{1,3,5\} \cup \{1,4\} \\ &= \{1,3,4,5\} \end{aligned}$$

e)

$$\begin{aligned} E^C(F \cup G) &= \{1,3,5,7\}^C \cap (\{7,4,6\} \cup \{1,4\}) \\ &= \{2,4,6\} \cap \{1,4,6,7\} \\ &= \{4,6\} \end{aligned}$$

f)

$$\begin{aligned} EG \cup FG &= (\{1,3,5,7\} \cap \{1,4\}) \cup (\{7,4,6\} \cap \{1,4\}) \\ &= \{1\} \cup \{4\} \\ &= \{1,4\} \end{aligned}$$

Problem 6

a) $EF^C G^C$
b) $EF^C G$
c) $E \cup F \cup G$
d) $EF \cup EG \cup FG$
e) EFG
f) $E^C F^C G^C$
g) $E^C F^C G^C \cup EF^C G^C \cup E^C F G^C \cup E^C F^C G$
h) $(EFG)^C$
i) $E^C FG \cup EF^C G \cup EFG^C$
j) S

Problem 7
a) S
b) $\varnothing$
c) E
d) S
e) $EG \cup F$

Problem 14

$$\begin{aligned} P(\text{exactly one of the events E or F occurs}) &= P(E \cup F) - P(EF) \\ &= P(E) + P(F) - P(EF) - P(EF) \\ &= P(E) + P(F) - 2P(EF) \end{aligned}$$

Problem 23

$$\begin{aligned} S &= \{rr, rb, bb\} \\ P(RR|R) &= \frac{|\{rr\}|}{|\{rr, rb\}|} = 1/2 \end{aligned}$$

Problem 24

$$\begin{aligned} S &= \{gg, gb, bg, bb\} \\ P(\text{two girls}|\text{eldest is girl}) &= \frac{|\{gg\}|}{|\{gg, gb\}|} = 1/2 \end{aligned}$$

Problem 29
Let A denote the event the sickly plant will die without water.
Let B denote the event the sickly plant will die with water.
Let C denote the neighbor will remember to water the plan.

$$\begin{aligned} P(\text{alive}) &= P(A^C C^C) + P(B^C C) \\ &= P(A^C)P(C^C) + P(B^C)P(C) \\ &= 0.2 \times 0.1 + 0.85 \times 0.9 \\ &= 0.785 \end{aligned}$$

Problem 33

$$\begin{aligned} S &= \{As_1, As_2, Bs, Bg\} \\ P(A|s) &= \frac{P(As)}{P(s)} \\ &= \frac{|\{As_1, As_2\}|}{|\{As_1, As_2, Bs\}|} \\ &= 2/3 \end{aligned}$$

Problem 48

$$
\begin{aligned}
P(C) &= 0.02 \\
P(P|C) &= 0.9 \\
P(P|C^C) &= 0.1 \\
P(C|P) &= \frac{P(CP)}{P(P)} \\
&= \frac{P(C)P(P|C)}{P(C)P(P|C) + P(C^C)P(P|C^C)} \\
&= \frac{0.02 \times 0.9}{0.02 \times 0.9 + 0.98 \times 0.1} \\
&= \frac{0.018}{0.018 + 0.098} \\
&= 0.155
\end{aligned}
$$

2

a)

$$
\begin{aligned}
P(AB) &= P(1) \\
&= 0.1 \\
P(A)P(B) &= P(1,2)P(1,3) \\
&= (1/10 + 2/10)(1/10 + 3/10) \\
&= 0.12
\end{aligned}
$$

Because $P(AB) \neq P(A)P(B)$, A and B are not independent.

b)

$$
\begin{aligned}
P(AB) &= P(1) \\
&= 0.25 \\
P(A)P(B) &= P(1,2)P(1,3) \\
&= (1/4 + 1/4)(1/4 + 1/4) \\
&= 1/4
\end{aligned}
$$

Because $P(AB) = P(A)P(B)$, A and B are independent.

c)

$$\begin{aligned}
P(AB) &= P(1) \\
&= 0.25 \\
P(A)P(B) &= P(1,2)P(1,3) \\
&= (1/4+1/4)(1/4+1/4) \\
&= 1/4 \\
P(AC) &= P(1) \\
&= 0.25 \\
P(A)P(C) &= P(1,2)P(1,4) \\
&= (1/4+1/4)(1/4+1/4) \\
&= 1/4 \\
P(BC) &= P(1) \\
&= 0.25 \\
P(B)P(C) &= P(1,3)P(1,4) \\
&= (1/4+1/4)(1/4+1/4) \\
&= 1/4
\end{aligned}$$

Since $P(AB) = P(A)P(B)$, $P(AC) = P(A)P(C)$, $P(BC) = P(B)P(C)$, A, B, and C are pair-wise independent.

d)

$$\begin{aligned}
P(A \cap B \cap C) &= P(1) \\
&= 0.25 \\
P(A)P(B)P(C) &= P(1,2)P(1,3)P(1,4) \\
&= (1/4+1/4)(1/4+1/4)(1/4+1/4) \\
&= 0.125
\end{aligned}$$

To sum up, $P(A \cap B \cap C) \neq P(A)P(B)P(C)$. We can conclude that events that are pair-wise independent do not indicate all the events are independent.