Advanced Data Analysis

Spring 2012

STAT W4201 ADVANCED DATA ANALYSIS

Instructor: E-mail: alem@stat.columbia.edu

Office Hours: Friday: 5:00 PM - 6 PM,

and by appointment

TAs:

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TA Office Hours: Wed and Thursday 5 PM – 7 PM

Prerequisites:

At least two of the following courses are prerequisites: W4315/4200, W4220/W4325, W4437, W4413, W4543, W4290, W4240 and W4330

Course Objectives: Emphasis will be on hands-on experience with data analysis, involving case studies and using common statistical packages.

Method of Evaluation:

- Homework 30%.
 - Assigned weekly, on Fridays after class and is due the following Friday. Homework should be turned in at designated boxes in Room 904, before 5 PM each Friday.
- Midterm/Test 30%.
 - Date TBD
- Project 40%.
 - Due on Monday April 30, 2012; 5 PM.

Homework must be placed in designated boxes in Room 904 each Friday, before class.

Suggested Reference Books:

Because of the nature of the course, no single text book is required.

However, the following text is suggested:

- The Statistical Sleuth: A Course in Methods of Data Analysis. Ramsey & Schafer 1st Duxbury 534386709
- RECOMMENDED:
 - Introduction Time Series and Forecasting Brockwell & Davis 2nd Springer-Verlag 0387953515
 - Generalized Linear Models Peter Mccullagh, John A. Nelder CRC Press 412317605
 - An Introduction to S and S-Plus Spector 1st Duxbury 053419866X

The following are also useful references

- 1. Miller, R. Survival Analysis. 1981, Wiley
- 2. McCullagh and Nedler. Generalized Linear Models. Chapman/Hall.
- 3. Selvin. *Practical Biostatistical Methods*, Duxbury
- 4. Hoaglin, et al. Fundamentals of Explor. Analysis of Variance, Wiley
- 5. Hosmer and Lemshow. Applied Logistic Regression, Wiley
- 6. Neter, Wasserman and Kutner. Applied Linear Statistical Models. Wiley.
- 7. Chambers and Hastie. Statistical Models in S. Wadsworth.

GSAS Statement on Academic Honesty

Students should be aware that academic dishonesty (for example, plagiarism, cheating on an examination, or dishonesty in dealing with a faculty member or other University official) or the threat of violence or harassment are particularly serious offenses and will be dealt with severely under Dean's Discipline. Graduate students are expected to exhibit the high level of personal and academic integrity and honesty required of all members of an academic community as they engage in scholarly discourse and research.

Scholars draw inspiration from the work done by other scholars; they argue their claims with reference to others' work; they extract evidence from the world or from earlier scholarly works. When a student engages in these activities, it is vital to credit properly the source of his or her claims or evidence. To fail to do so would violate one's scholarly responsibility.

In practical terms, students must not cheat on examinations, and deliberate plagiarism is of course prohibited. Plagiarism includes buying, stealing, borrowing, or otherwise obtaining all or part of a paper (including obtaining or posting a paper online); hiring someone to write a paper; copying from or paraphrasing another source without proper citation or falsification of citations; and building on the ideas of another without citation. Students also should not submit the same paper to more than one class.

Graduate students are responsible for proper citation and paraphrasing, and must also take special care to avoid even accidental plagiarism. The best strategy is to use great caution in the handling of ideas and prose passages: take notes carefully and clearly mark words and ideas not one's own. Failure to observe these rules of conduct will result in serious academic consequences, which can include dismissal from the university.

Students engaging in research must be aware of and follow university policies regarding intellectual and financial conflicts of interest, integrity and security in data collection and management, intellectual property rights and data ownership, and necessary institutional approval for research with human subjects and animals.

http://www.columbia.edu/cu/gsas/rules/chapter-9/pages/honesty/index.html

Ground Rules

- Attendance mandatory
 - surprise quizzes may be given to encourage attendance
 - No cellphones, web surfing or chatting during lecture
- No late homework or make-up test or project
- All homework, tests and project reports should reflect individual effort
- Active participation in class expected
 - Questions and answers
 - Project presentations

Topics:

- Exploratory data analysis
- Model formulation, goodness of fit testing
- Standard and non-standard statistical procedures, including:
 - Linear regression,
 - Analysis of variance
 - Nonlinear regression
 - Generalized linear models
 - Survival analysis
 - Time series analysis
 - Bayesian methods
- Each student will be asked to propose a data set, with the approval of the instructor, define a research problem, develop an analysis plan, and submit a report at the end of the semester. Only original (and NO group) projects will be accepted

Generating and Summarizing Data

Experiments: Performed to generate data to help make decisions.

- 1. Clinical Research: Is a new therapy superior to the standard?
- 2. Genetics: Is there any association between genetic make-up and occurrence of a certain type of disease?
- 3. Agriculture: What is the effect of soil types on crop yield?
- 4. Finance: What factors affect the performance of a company's stock?
- 5. Weather. What is the forecast in the next quarter?

Generating and Summarizing Data (cont'd)

- 6. Political Science: How do the polls predict election results?
- 7. Game Theory: Why does the casino make a profit at a roulette?
- 8. Manufacturing: What is the reliability of a certain manufacturing process?
- 9. Demography: What is the growth rate of a population in a given region?

Types of Studies

- Controlled
- Observational studies

Types of Studies

- Randomized, controlled, double-blind
 - Randomization guards against selection bias
 - Ensures that groups are comparable.
 - Double-blind: Minimizes bias, either in the response or in the evaluation of the experimental outcomes.
- Observational studies:
 - Assignment of experimental subjects to study groups not done by the investigator.
 - May lack advantages of controlled trials
 - May help establish association when RCTs not feasible

Questions to Ask in Data Analysis

- What is the objective of the analysis and/or the original experiment?
- What was the design of the study?
 - Randomized controlled or observational?
 - If a controlled trial, how were subjects assigned to the different groups?
 - Was the assignment process controlled by the investigator?
 - If an observational study: Are the groups comparable? What factors are confounded with treatment?
- What procedure would be appropriate for the data?
 - Exploratory data analysis techniques?
 - Inferential statistical techniques?
 - Model building?
- Implementation of analysis plan?
- Interpretation of Results?
 - Are the results relevant?

Analysis Plan

- Was the planned analysis followed?
 - If a large number of analyses are performed, some of them will be sure to show structure.

``If you torture the data long enough, they'll admit to anything"

- Were assumptions validated?
- Were there any confounding factors. If so, were appropriate measures taken?
- Were multiple procedures/subgroup analyses performed?
 - If so, what adjustments were made for multiplicity?

Exploratory Data Analysis (EDA)

Preliminary look at data:

- Evaluating data quality
 - Missing values
 - Outliers/Influential points
- Checking assumptions: Distributions, relationships, etc.
- Measures of location & dispersion

EDA cont.

Approaches:

- Descriptive Statistics
 - Measures of location and dispersion
- Graphical
 - Histograms, box-plots, Q-Q plots, etc.

EDA cont.

Measures of Location

- Properties of the sample mean
 - It is easy to compute
 - It is easy to interpret/understand
 - Its variance has a simple expression
 - It is susceptible to outliers
- Properties of the sample median:
 - Relatively more complex to compute and understand,
 - Strongly resistant to outliers.
 - Variability does not have a simple expression.

EDA cont.

Measures of Dispersion

Properties of the sample variance

$$S^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (X_{i} - \bar{X})^{2}$$

- Properties of the IQR, Range
 - Sampling distribution?

Robustness

Robustness relates to lack of susceptibility to departures from underlying model assumptions.

Robustness of validity.

This often relates to tolerance of non-normal tails.

Little or no effect on validity of inferential results (e.g., level of confidence intervals or p-values).

Robustness of efficiency:

High effectiveness in the face of non-normality. E.g., length of confidence intervals or power of tests not affected.

Robustness (cont'd)

Breakdown Point

 Fraction of data that could be made arbitrarily large, without making the estimator useless

Breakdown point for sample mean vs. median

Some examples of robust procedures

- Sample trimmed mean

$$\bar{X}_t = \frac{X_{(g+1)} + \dots + X_{(n-g)}}{n - 2q}$$

where $g = [\gamma n]$, $0 \le \gamma \le 0.5$. Usually, $\gamma = 0.2$ is used.

$$Var(\bar{X}_t) \approx \frac{1}{n^2(1-2\gamma)^2} \sum (W_I - \bar{W})^2$$

where

$$W_{i} = \begin{cases} X_{(g+1)}, & X_{i} \leq X_{(g+1)} \\ X_{i}, & X_{(g+1)} < X_{i} < X_{(n-g)} \\ X_{(n-g)}, & otherwise \end{cases}$$

 $\bar{W} = \frac{\sum W_i}{n}$ is called the winsorized sample mean.

- M Estimates

M estimates are obtained as minimizers of the quantity

$$\sum_{j=1}^{n} \rho\left(\frac{X_i - \mu}{\sigma}\right)$$

A common choice for the weight function ρ is the Huber weight functions,

$$\rho(u) = \begin{cases} \frac{u^2}{2} & |u| \le k \\ k |u| & |u| > k \end{cases}$$

which are quadratic near zero, and linear beyond a prespecified cutoff point k. When $k=\infty$ we get the sample mean, while a value of k=0 gives the median.

- Median absolute deviation

 $median\{\mid X_j - \tilde{X}\mid, j=1,\cdots,n\}$

The Jackknife Method

Let θ be an unknown population parameter of interest.

Let $\hat{\theta}$ be a statistic or estimator of θ .

Then the bias of $\hat{\theta}$ is given by $E[\hat{\theta}] = \theta$.

Generally, the bias and variance of an estimator may not be readily computable.

A method, due to Quenouille (1956), that may be used to compute bias and variance is the *jackknife* procedure.

Let $\hat{\theta}_{(j)}$ be an estimator computed based on all but X_j , i.e., leaving out the j'th observation.

Then the jackknife estimator of bias is given by

$$B_{JACK} = (n-1) \left[\frac{\sum_{j=0}^{n} \hat{\theta}_{(j)}}{n} - \hat{\theta} \right]$$

The bias reduced jackknife estimator is given by

$$\hat{\theta}_{JACK} = \hat{\theta} - B_{JACK}$$

and the variance

$$V_{JACK} = \frac{n-1}{n} \sum_{i} (\hat{\theta}_{(j)} - \frac{\sum_{j} \hat{\theta}_{(j)}}{n})^{2}$$

Caution in the use of the jackknife:

- Jackknife may not be appropriate in the presence of outliers or for markedly skewed distributions
- The jackknife may not be appropriate when θ has restricted values, e.g., $\theta \in [0, 1]$.

Computer-Intensive Statistical Methods

- Monte Carlo Methods
- Bootstrap Methods
- Randomization Tests

Motivations

- Classical statistics mostly based on idealized assumptions
- Advance in computation helps replace complex theoretical analysis by computationally intensive methods

Monte Carlo Methods

- Requirement:
 - Knowledge of the distribution to easily generate new samples

Example:

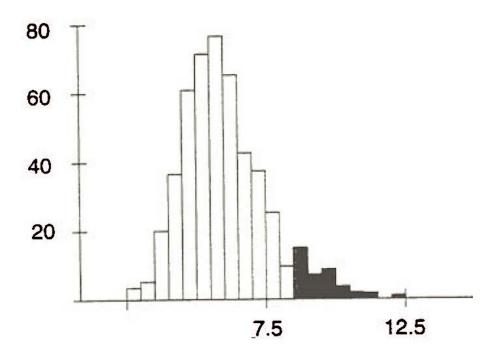
- Simulate the sampling distribution of the interquartile range (IQR) of scores
 - Sampling distribution unknown

Procedure Monte Carlo Sampling: IQR

1. Let F be Normal($\mu = 50$, $\sigma = 5$).

Let \mathfrak{S} be the original sample of size N = 20 students, and let $IQR = IQR(\mathfrak{S}) = 8.1$ be our sample statistic.

- 2. Repeat i = 1 ... K times:
 - a. Draw a pseudosample \mathfrak{S}_i^* of size n from F by random sampling.
 - b. Calculate and record $IQR_i^* = IQR(\mathcal{S}_i^*)$.
- 3. The distribution of IQR* is an empirical sampling distribution.



- MC sampling distribution of IQRs for K = 500.
- Dark: 40 scores greater than 8.1

Bootstrap Methods

Idea:

- Now suppose F is arbitrary, unknown distribution.
- Resample from the sample, treating the sample as the population.
 - The sample should be representative of the population.
- Approximate sampling distribution of statistics based on corresponding pseudo-sample quantities.

Bootstrap Methods (cont'd)

Let X_1, \dots, X_n be a random sample from F_{θ} .

Suppose an estimator of θ is $\hat{\theta}_n$.

When θ is the median, the sample median is approximately $N(\theta, \frac{1}{4nf^2(\theta)})$

The bootstrap may be used to perform valid statistical inference about θ .

A simple bootstrap procedure involves drawing B samples, with replacement, from the empirical distribution \hat{F}_n of the data

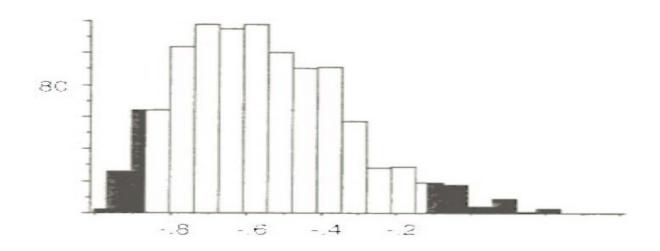
Bootstrap Methods (cont'd)

For each sample, compute a statistic of interest, say $\hat{\theta}_n^*$.

- Assess the variability of $\hat{\theta}_n$ about θ by that of $\hat{\theta}_n^*$ about $\hat{\theta}_n$
- Estimate the bias $\hat{\theta}_n \theta$ by the mean of $\hat{\theta}^* \hat{\theta}$
- Estimate the distribution of $\hat{\theta}$ by the e.d.f. of $\hat{\theta}^*$.

Bootstrapping the sample correlation coefficient

X	5	1.75	0.8	5	1.75	5	1.75	1	5	1.75
V	27.8	20.82	44.12	29.41	31.19	28.68	29.53	34.62	20	41.54



Bootstrap sampling distribution of r (K = 1000).

Remarks

- ❖In practice, the bootstrap works well in many situations, but not in all
- ❖ Assumes: The original sample is representative of the population.

Randomization Tests

- The bootstrap treats samples as "proxies" for populations.
- Sometimes may wish to determine whether two samples are related without any reference to population parameters.

Randomization Tests (cont'd)

Example: Is the performance of this year's students significantly more variable (IQR) than the performance of last year's students?

Sample 1: last year						
48.35	53.93	55.48	45.67	52.82		
49.47	57.00	53.61	57.69	51.34		
44.98	54.70	59.32	51.70	50.73		
46.84	63.13	52.50	49.67	54.07		
44.84	48.68	53.94	59.00	50.92		

Sample 2: this year						
64.82	51.69	57.00	58.17	40.63		
50.90	48.77	40.33	50.76	49.64		
56.25	65.68	57.50	47.45	46.78		
61.34	53.66	49.10	54.49	54.15		

- Let d_{IQR} denote the difference between the IQRs of the samples, $d_{IQR} = 6 8.5 = -2.5$.
 - ❖What is the probability that this difference occurs by chance?
- Claim: students' performance is no more variable this year than last.
- **\bigstar** If claim is really true, then randomly swapping scores between the samples will not influence d_{IQR} .

Approximate Randomization to Test Whether Two Samples Are Drawn from the Same Population

1. Let S_A and S_B be two samples of sizes n_A and n_B , respectively. Let $\theta = f(S_A, S_B)$ a statistic calculated from the two samples. Let S_{A+B} be the merge of S_A and S_B .

2. Do i = 1 ... K times:

- a. Shuffle the elements of S_{A+B} thoroughly.
- b. Assign the first n_A elements of S_{A+B} to a randomized pseudosample A_i^* and the remaining n_B elements to B_i^* .
- c. Calculate $\theta_i^* = f(A_i^*, B_i^*)$ and record the result.
- 3. The distribution of θ_i^* can now be used to find the probability of the sample result θ under the hypothesis that the samples are drawn from the same population.

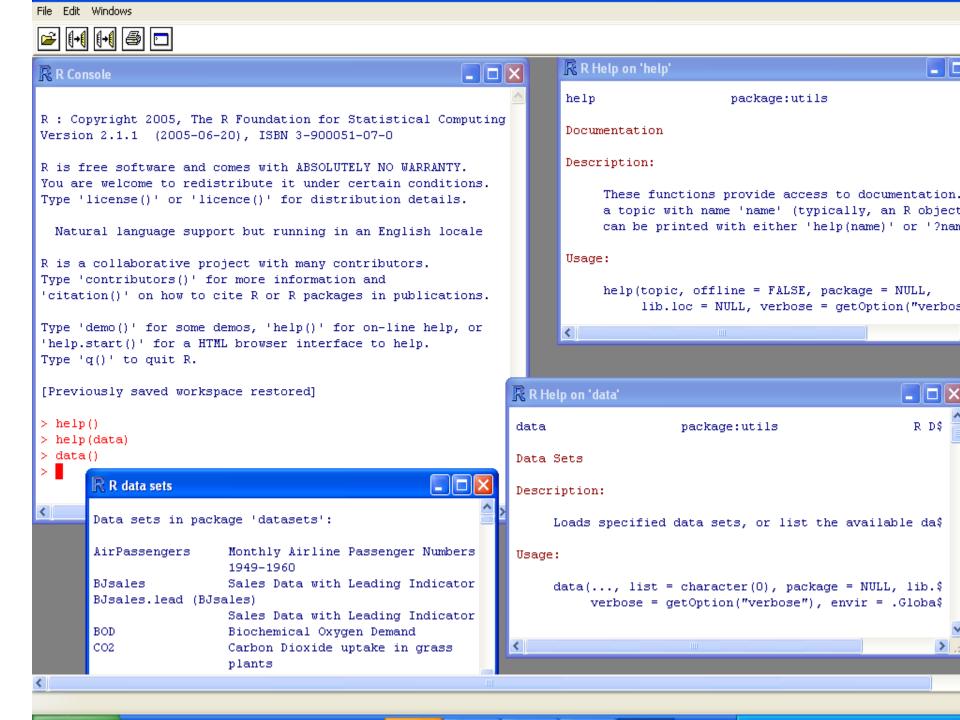
- Approximate randomization: K iterations do not exhaust the space of all possible assignments of elements of S_{A+B} to A_i^* and B_i^* .
- *Exact randomization: If can find exact probability by generating all possible outcomes.

Comparing Bootstrap and Randomization Procedures

- Both generate the distribution of a statistic by resampling from the original sample.
 - Bootstrap resamples with replacement.
 - Randomization resamples without replacement.
- Bootstrap simulates the process of drawing samples from a population, while randomization does not.
- They produce different distributions!
- Randomization <u>cannot</u> be used to draw inferences about population parameters (e.g., confidence intervals).

Computer-Intensive vs. Parametric Procedures

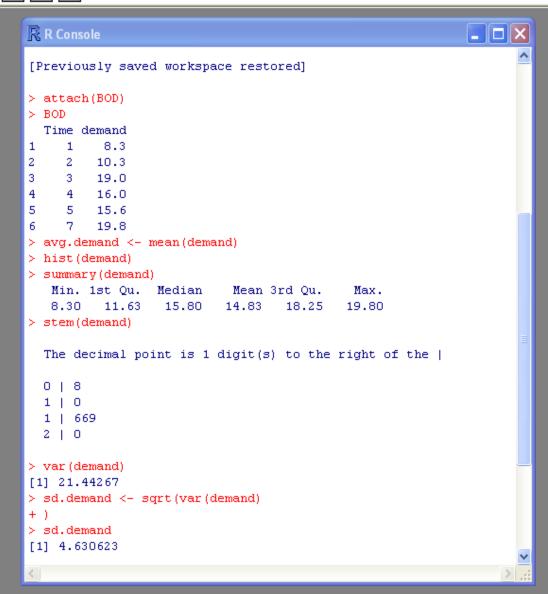
- Computer-intensive methods most desirable when:
 - No parametric sampling distribution exists for a statistic.
 - Assumptions of underlying a parametric test are violated and procedure not robust.

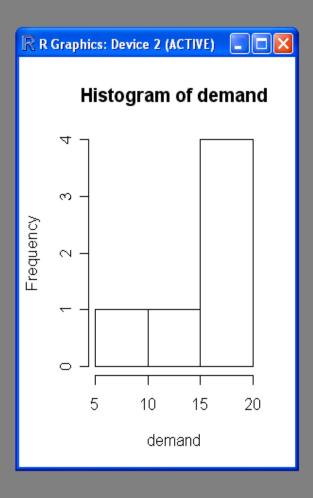


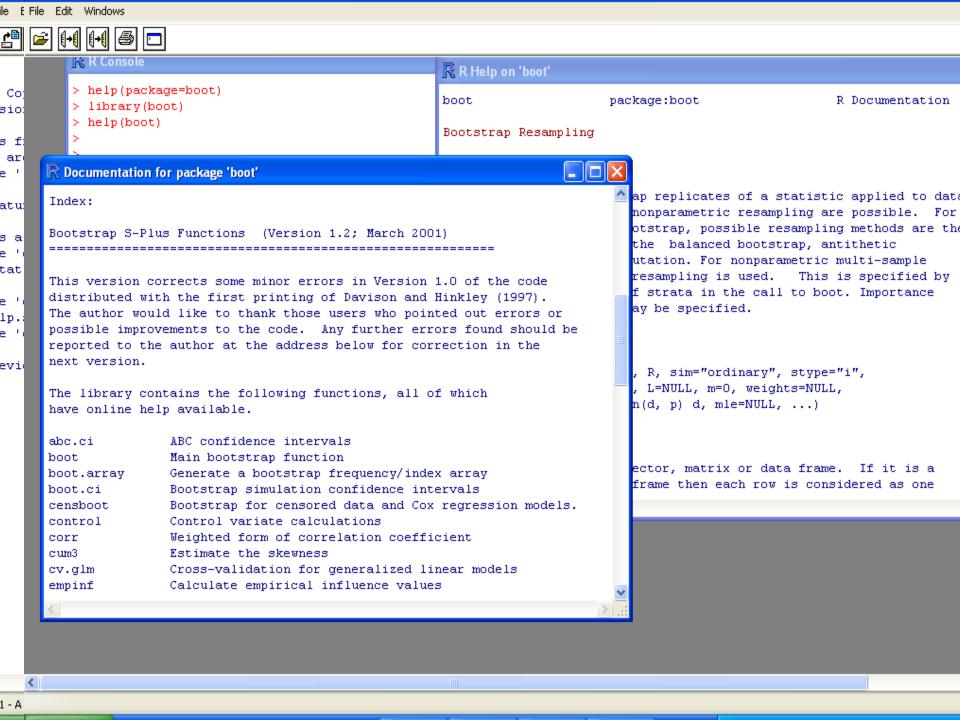


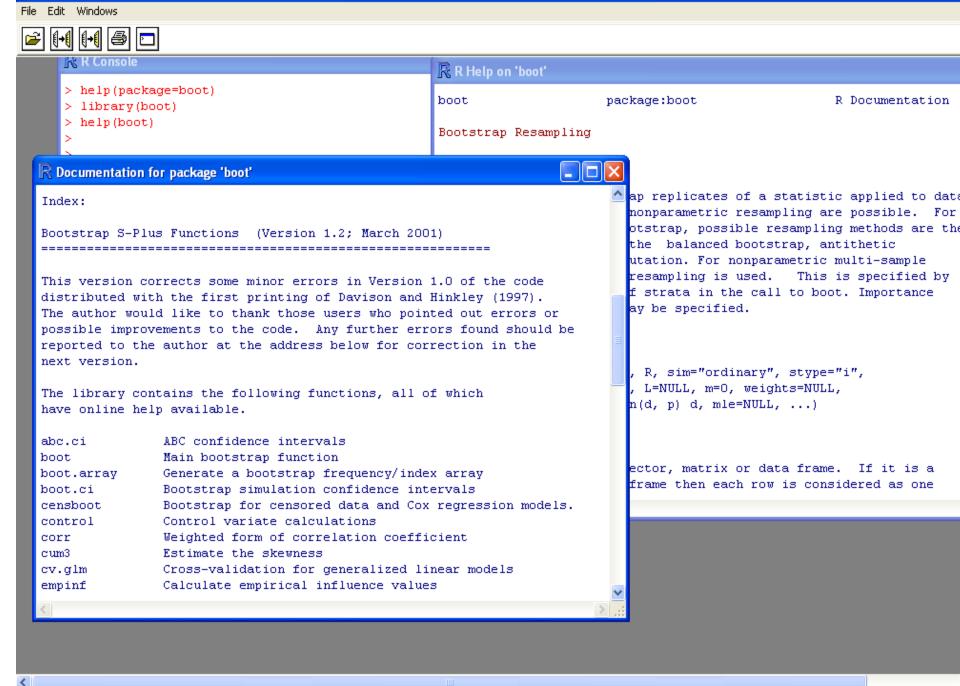




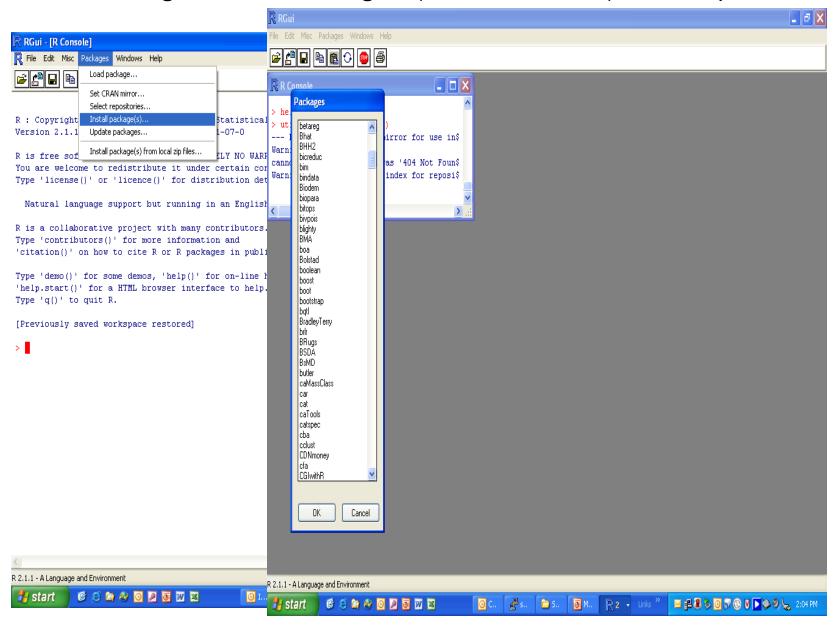


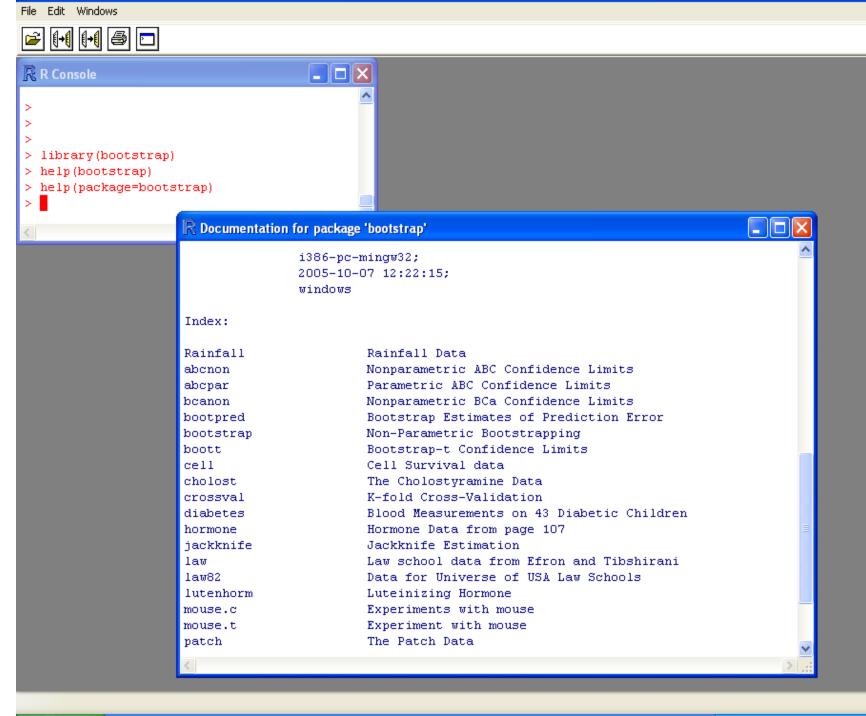






#Go to Packages, Install Packages, (select USA1, etc.) Bootstrap





Problem Set 1

Reading Assignment 1

Chapter 1. The Statistical Sleuth: A Course in Methods of Data Analysis. Ramsey & Schafer

Reading Assignment 2

An Introduction to R: http://cran.r-project.org/doc/manuals/R-intro.pdf

Consider the Salary Data (Display 1.3) in Ramsey &Schafer, Chapter 1, and partially appended below.

- Determine whether there are outliers in the combined data, using boxplots.
- ii) Perform separate EDA, and compute appropriate measures of dispersion for the data in each group (i.e., Males and Females).
- iii) For each of the estimates computed in (ii) above, determine the bias and variance using each of the following methods:
 - Jackknife
 - Bootstrap

SALARY SEX		4620 MALE
3900 FEMALE		5040 MALE
4020 FEMALE	5100 FEMALE	5100 MALE
4290 FEMALE	5100 FEMALE	5100 MALE
	5160 FEMALE	5220 MALE
4380 FEMALE	5220 FEMALE	5400 MALE
4380 FEMALE	5220 FEMALE	5400 MALE
4380 FEMALE	5280 FEMALE	5400 MALE
4380 FEMALE	5280 FEMALE	5400 MALE
4380 FEMALE	5280 FEMALE	5400 MALE
4440 FEMALE	5400 FEMALE	5700 MALE
4500 FEMALE	5400 FEMALE	6000 MALE
4500 FEMALE	5400 FEMALE	6000 MALE
4620 FEMALE	5400 FEMALE	6000 MALE
4800 FEMALE	5400 FEMALE	6000 MALE
4800 FEMALE	5400 FEMALE	6000 MALE
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4800 FEMALE	5520 FEMALE	6000 MALE
4980 FEMALE	5520 FEMALE	6300 MALE
5100 FEMALE		6600 MALE
5100 FEMALE		6600 MALE
5100 FEMALE		6600 MALE
5100 FEMALE		6840 MALE
		6900 MALE
		6900 MALE
		8100 MALE

4620 MALE